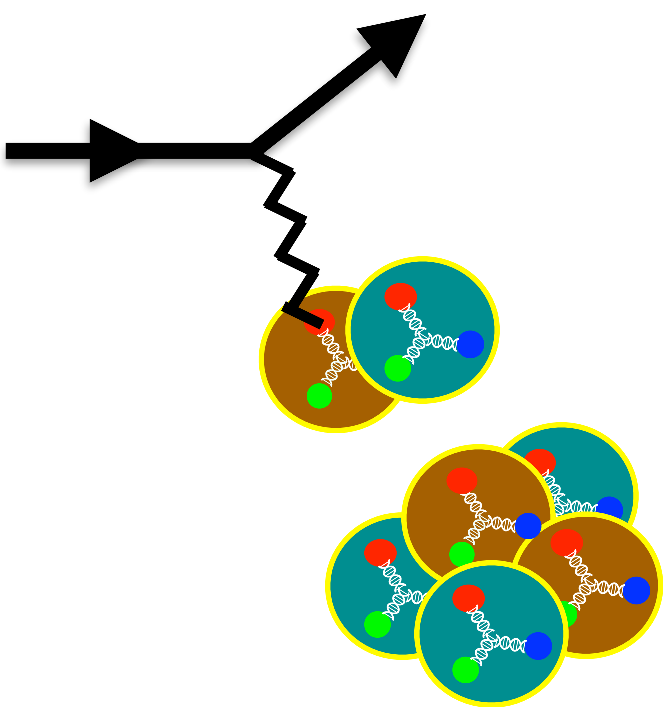
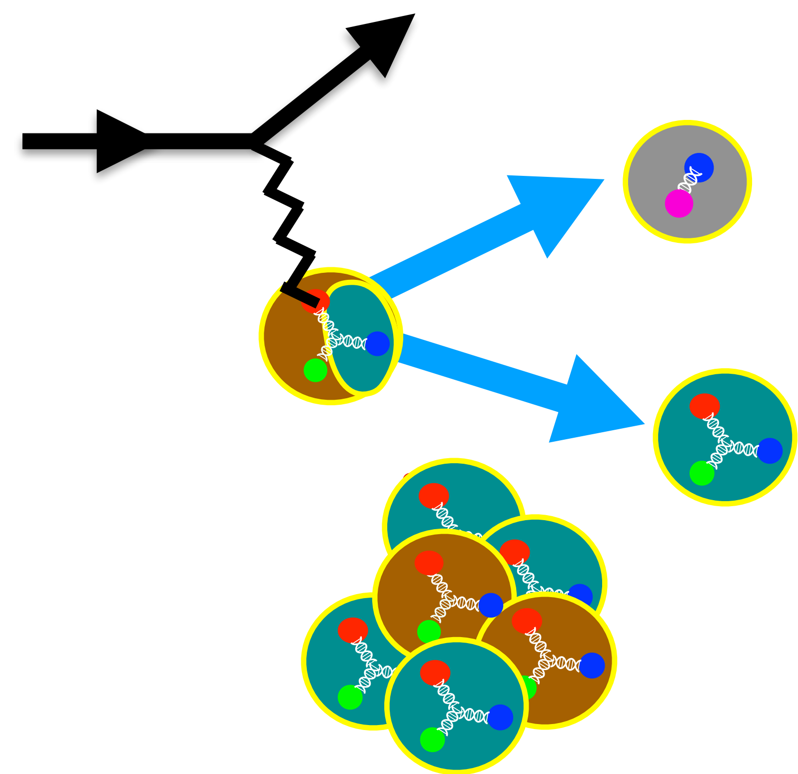
Toward lattice QCD studies of two-body currents and resonance production



Michael Wagman

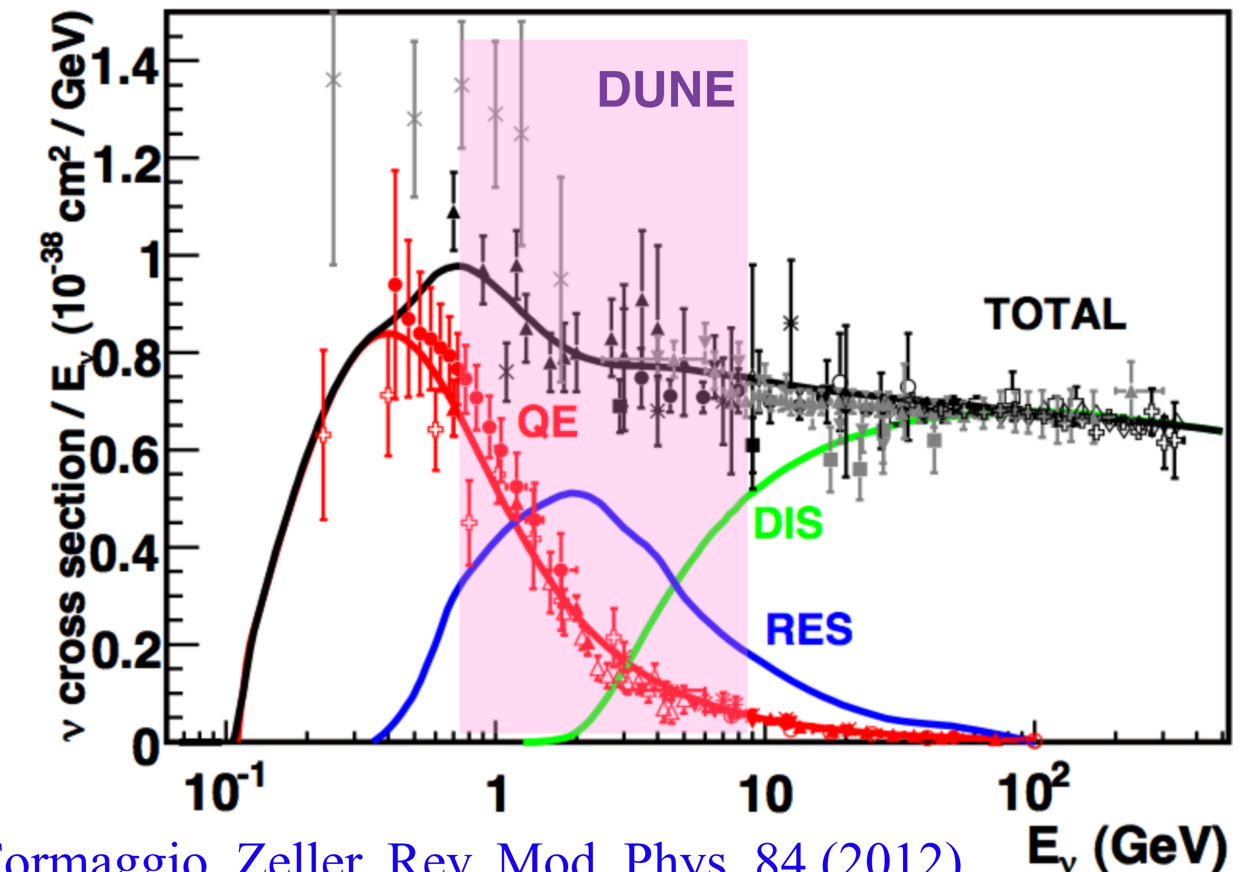


Neutrino Joint Meeting

February 10, 2022

Fermilab

Neutrino-nucleus scattering

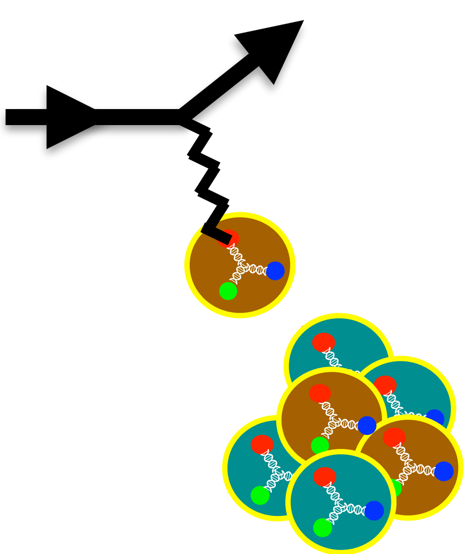


Formaggio, Zeller, Rev. Mod. Phys. 84 (2012)

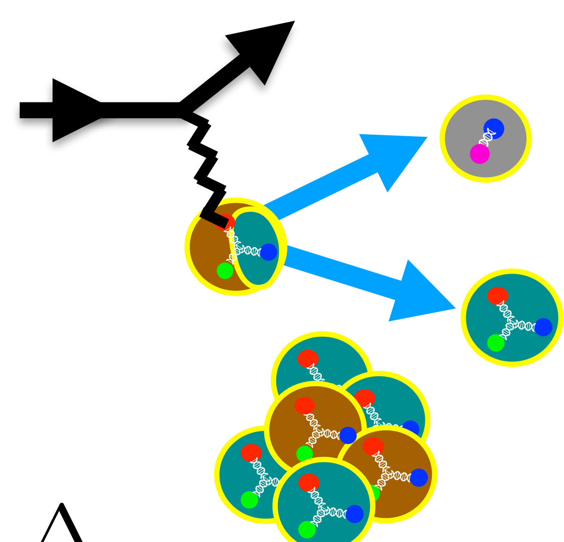
Accelerator neutrino flux covers a wide range of energies with different dominant physics processes:

- Quasi-elastic
- Resonance production
- Transition region
- Deep inelastic scattering

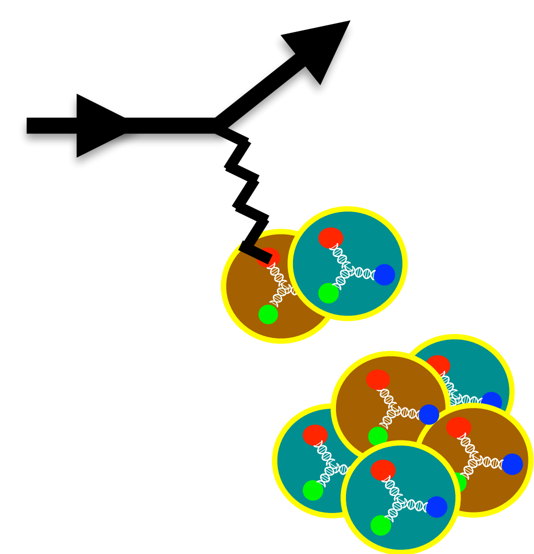
Nuclear models exist that can describe most of these regions, but precision x-sec predictions require precise knowledge of few-nucleon input parameters



Nucleon form factors



$N, \ \pi, \ \Delta$ **transition form factors**



Two-body currents

Lattice QCD, EFT, and νA

LQCD can provide results for few-nucleon quantities that inform nuclear effective theories and are complementary to experiment

LQCD → Nuclear EFT and many-body → Event generators

See USQCD νA white paper: Kronfeld et al Eur. Phys. J. A 55 (2019)

Easy for LQCD:

- Axial vs vector currents
- Isovector vs isoscalar
- Pions

Hard for LQCD:

- Large baryon number
- Real-time dynamics
- Multi-hadron states
- (Light quark masses)

Lattice QCD and νA

νA scattering amplitudes factorize into leptonic and hadronic parts

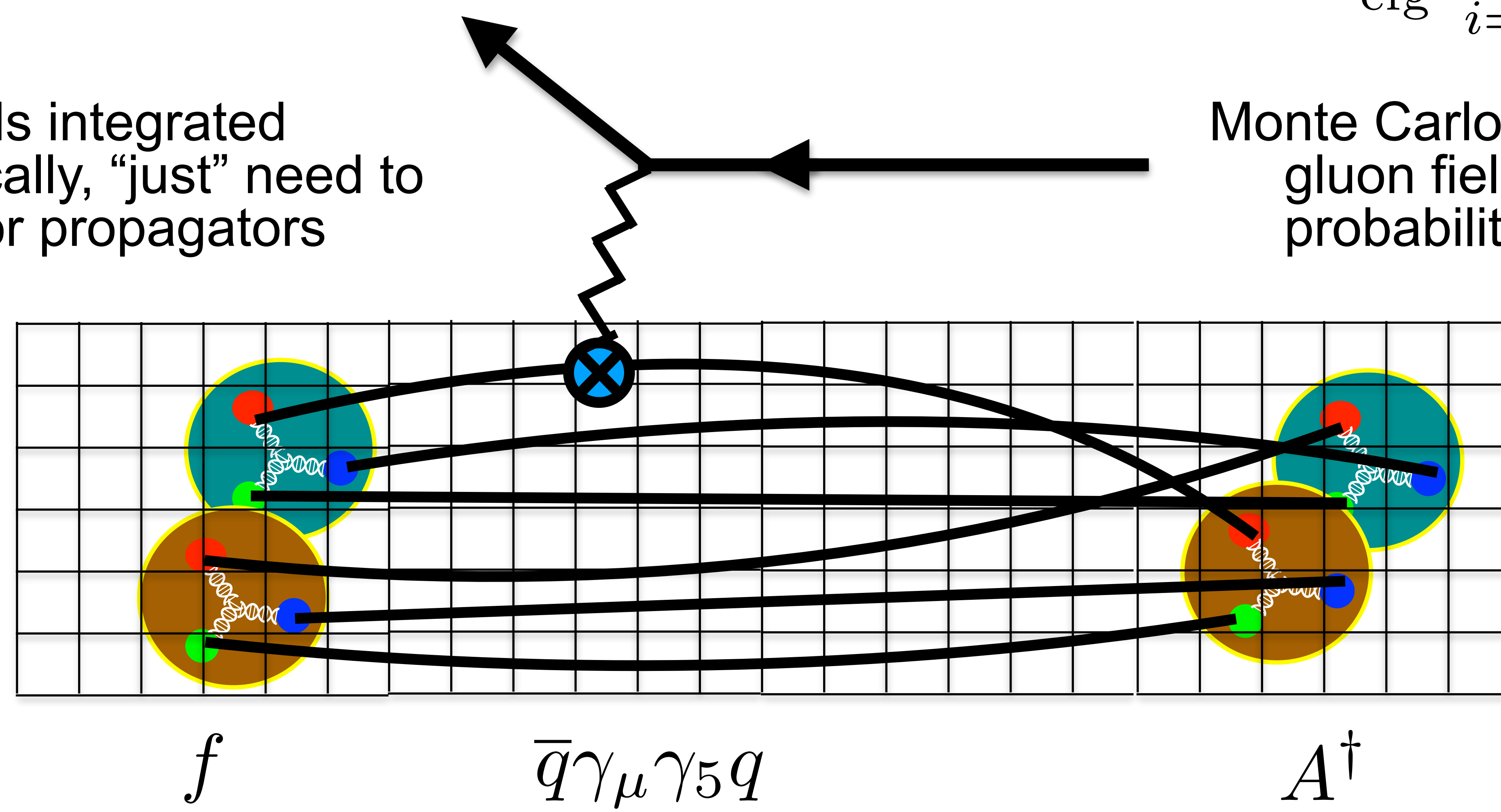
$$\mathcal{M}_{\nu A \to \ell f} \propto \left(\overline{u}_\ell \gamma_\mu \gamma_5 u_\nu\right) \langle f | \, \overline{q} \gamma_\mu \gamma_5 q \, | A \rangle$$

Generic Euclidean hadronic matrix elements calculable (in principle) using lattice QCD

$$\langle \mathcal{O} \rangle = \int \mathcal{D}U \mathcal{D}\overline{q} \mathcal{D}q \; e^{-S_{QCD}(U,q,\overline{q})} \; \mathcal{O}(U,q,\overline{q}) \approx \frac{1}{N_{\mathrm{cfg}}} \sum_{i=1}^{N_{\mathrm{cfg}}} \mathcal{O}(U_i)$$

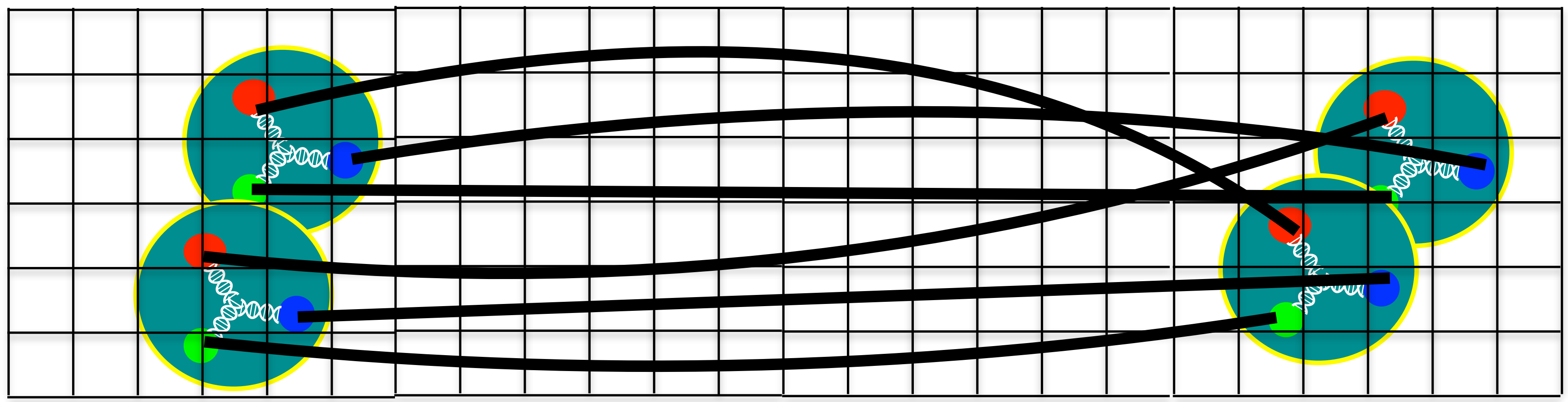
Quark fields integrated analytically, “just” need to solve for propagators

Monte Carlo sample gluon fields with probability $\propto e^{-S_{eff}}$



LQCD and nuclear matrix elements

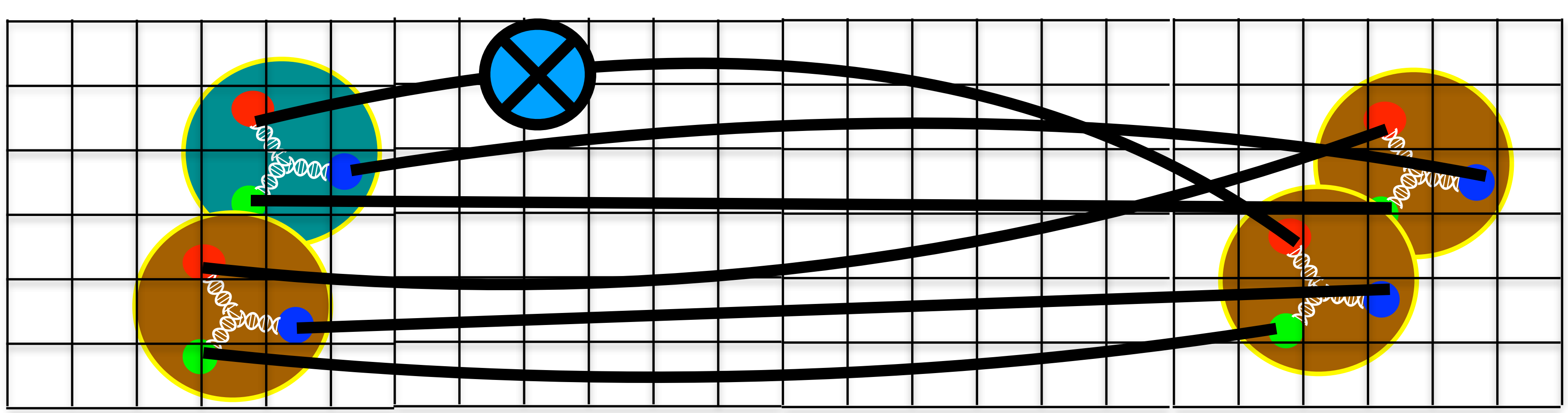
LQCD spectrum determined from 2-point correlation functions



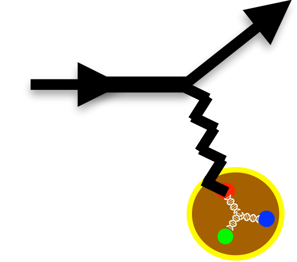
$$C_A(t) = \langle A(t) A^\dagger(0) \rangle = \sum_n \langle 0|A(0)e^{-Ht}|n\rangle \langle n|A^\dagger(0)|0\rangle + \ldots$$

$$= \sum_n |Z_n|^2 e^{-E_n t}$$

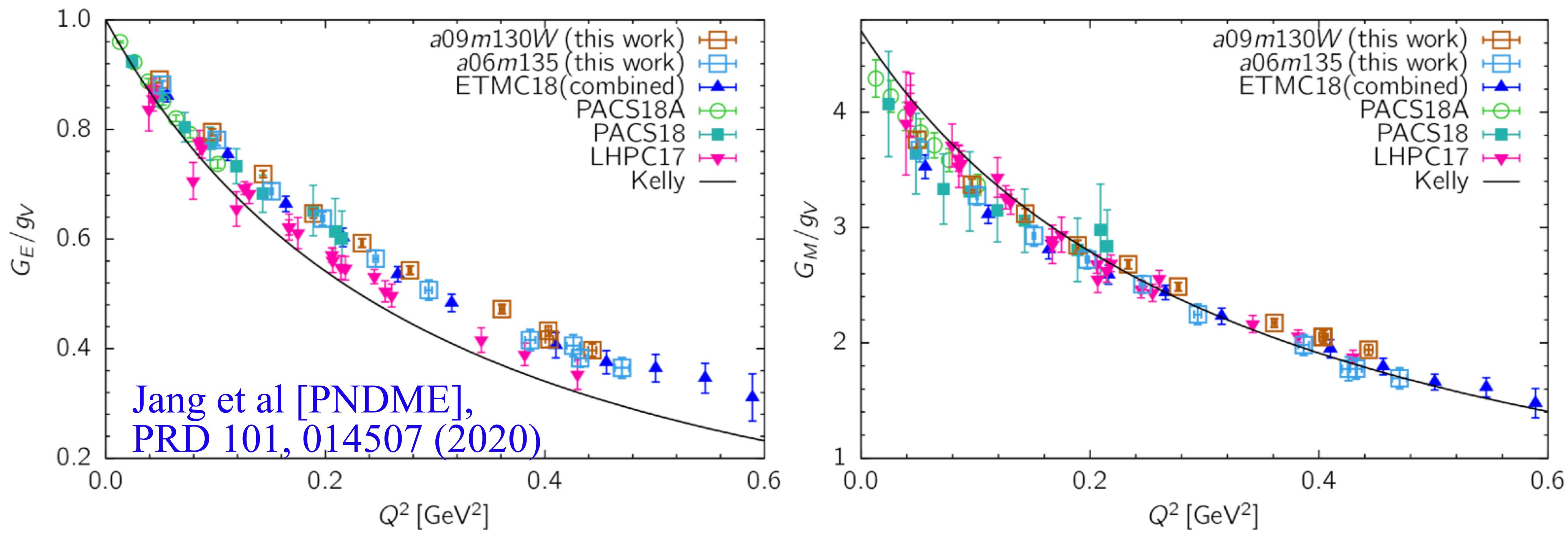
Nuclear matrix elements determined from 3-point correlation functions including a local operator insertion



Nucleon form factors



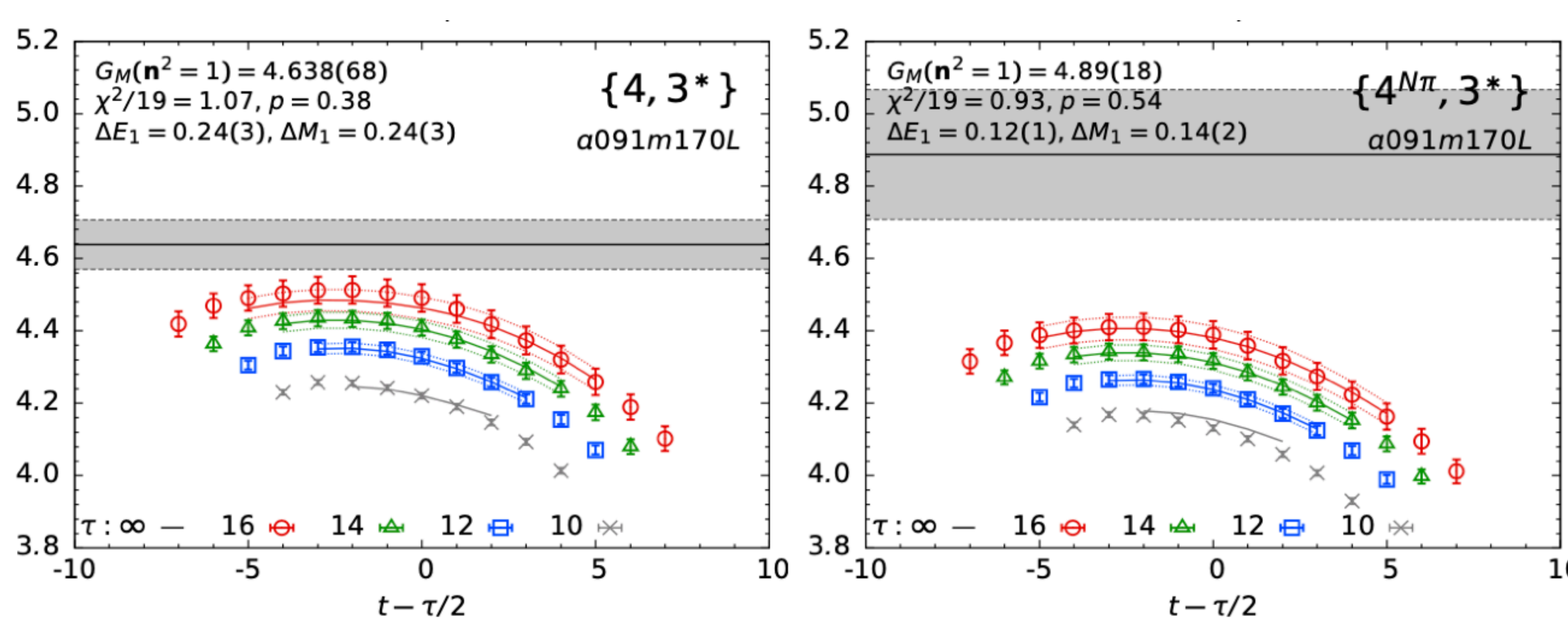
Vector and axial form factors recently calculated using nearly physical quark masses:



Jang et al [PNDME], PRD 101, 014507 (2020)

Remaining LQCD systematic uncertainties arise from lattice spacing effects, finite-volume effects, **excited-state effects**

Recent vector / axial form factor studies show the importance of $N\pi$ excited-state effects in nucleon form factor calculations with light quark masses

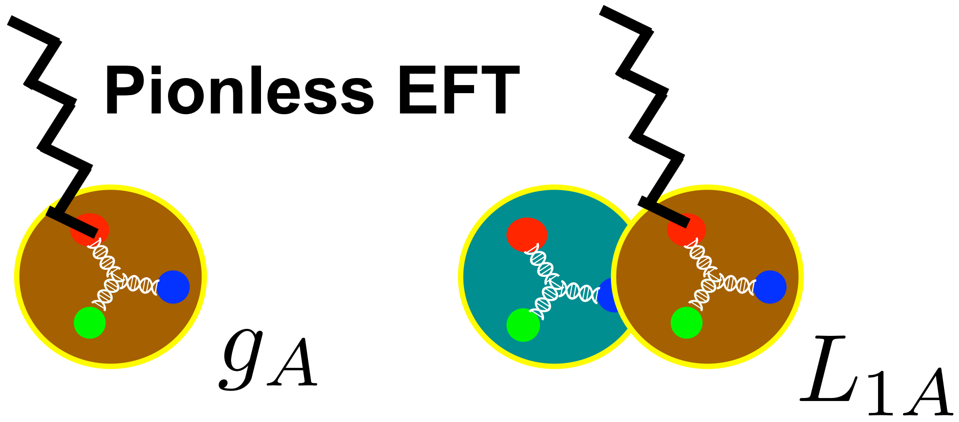


Park et al [NME], arXiv:2103.05599

Axial currents in nuclei

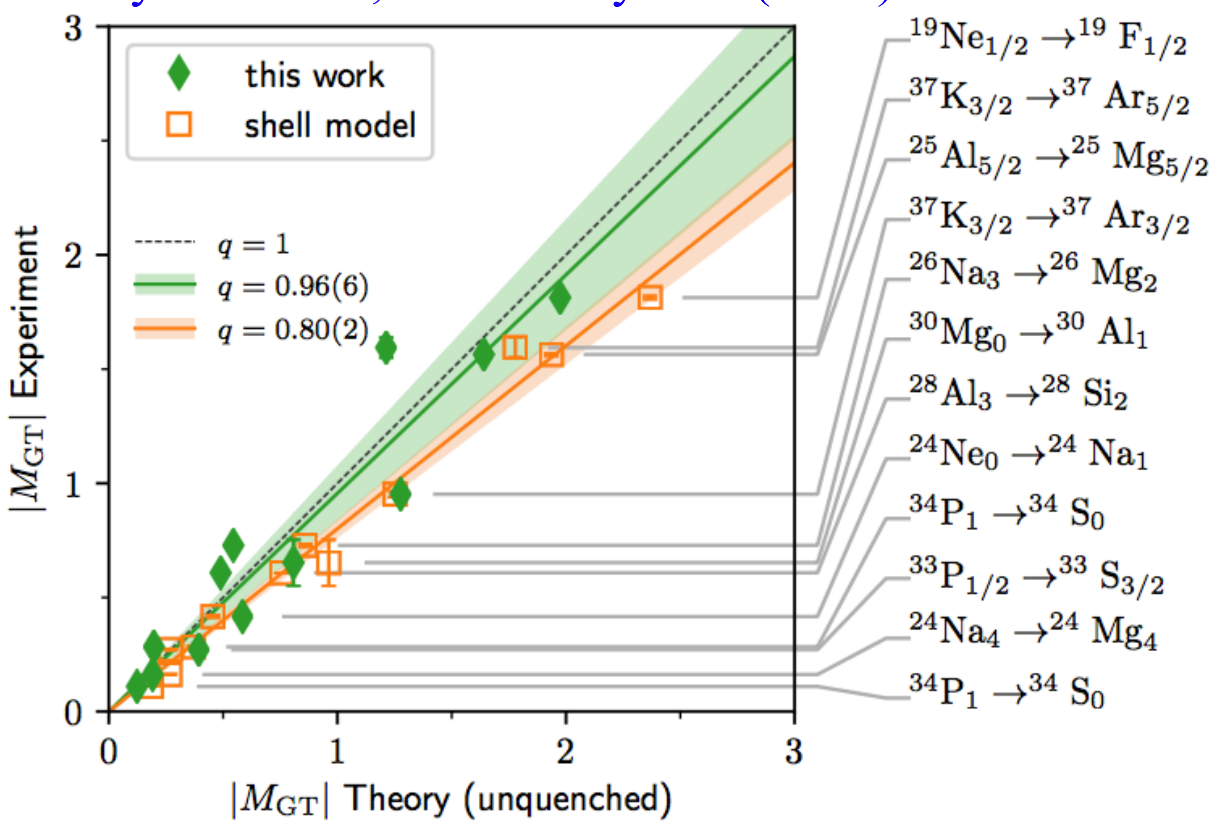
Axial-current responses of nucleons are modified in nuclei

Modern calculations including multi-nucleon correlations and currents with e.g. chiral EFT can reproduce experiment without “quenching” g_A

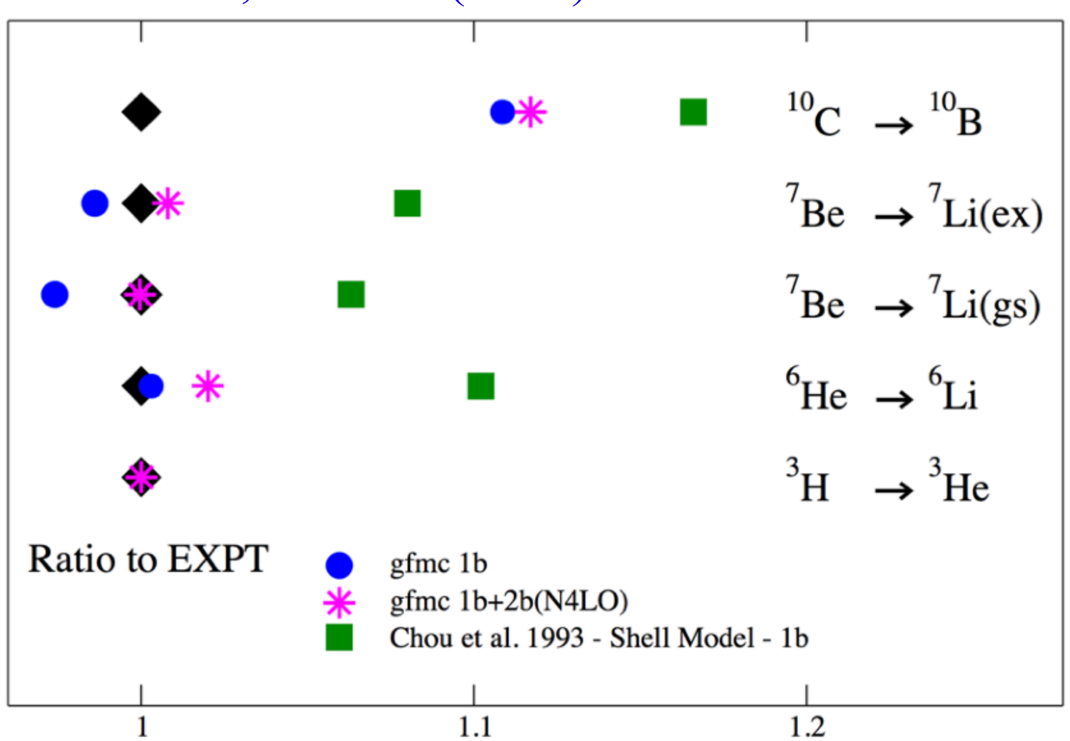


Few-nucleon LQCD results can constrain two-body currents in nuclear models and EFTs

Gysbers et al, Nature Phys. 15 (2019)



Pastore et al, PRC 97 (2018)

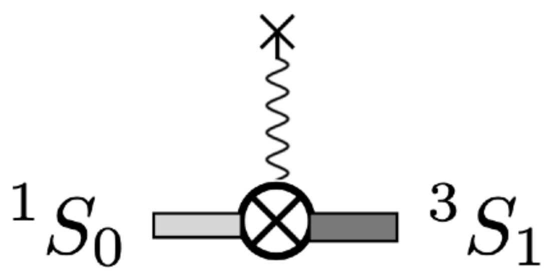


Proton-proton fusion

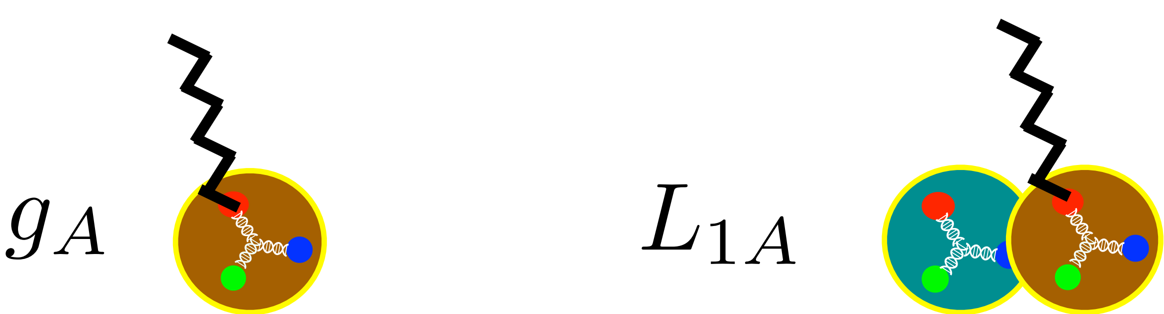
Axial current transition matrix element between spin-singlet and spin-triplet np systems computed using fixed-order background fields

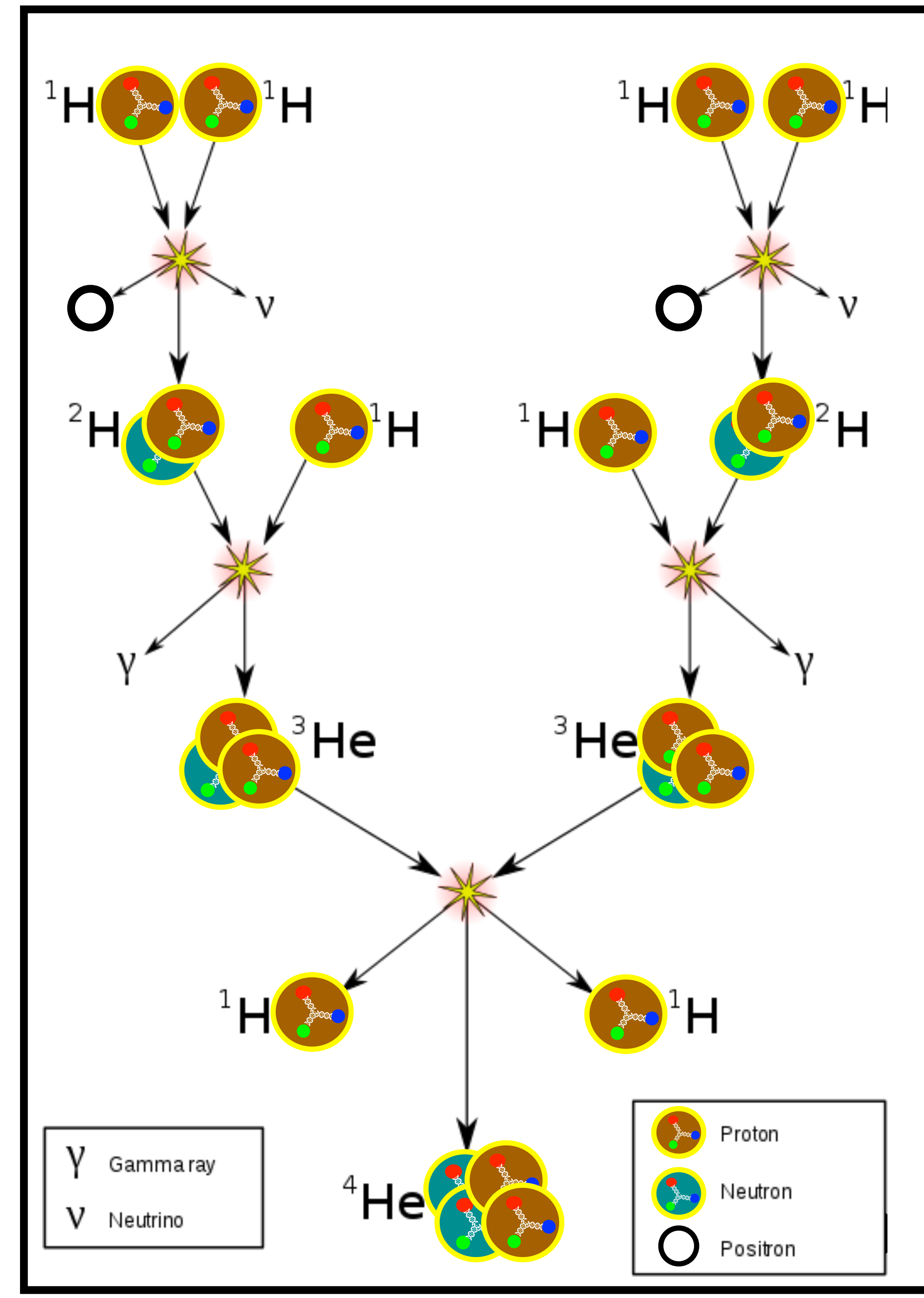
Savage, MW et al [NPLQCD], PRL 119 (2017)

LQCD results matched to pionless EFT by computing same background-field correlation function



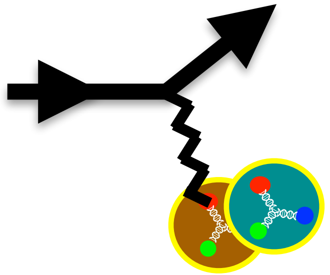
Results used to constrain LEC for two-body axial current operator in pionless EFT





Same operator relevant for proton-proton fusion and other reactions, future LQCD calculations could improve phenomenological predictions

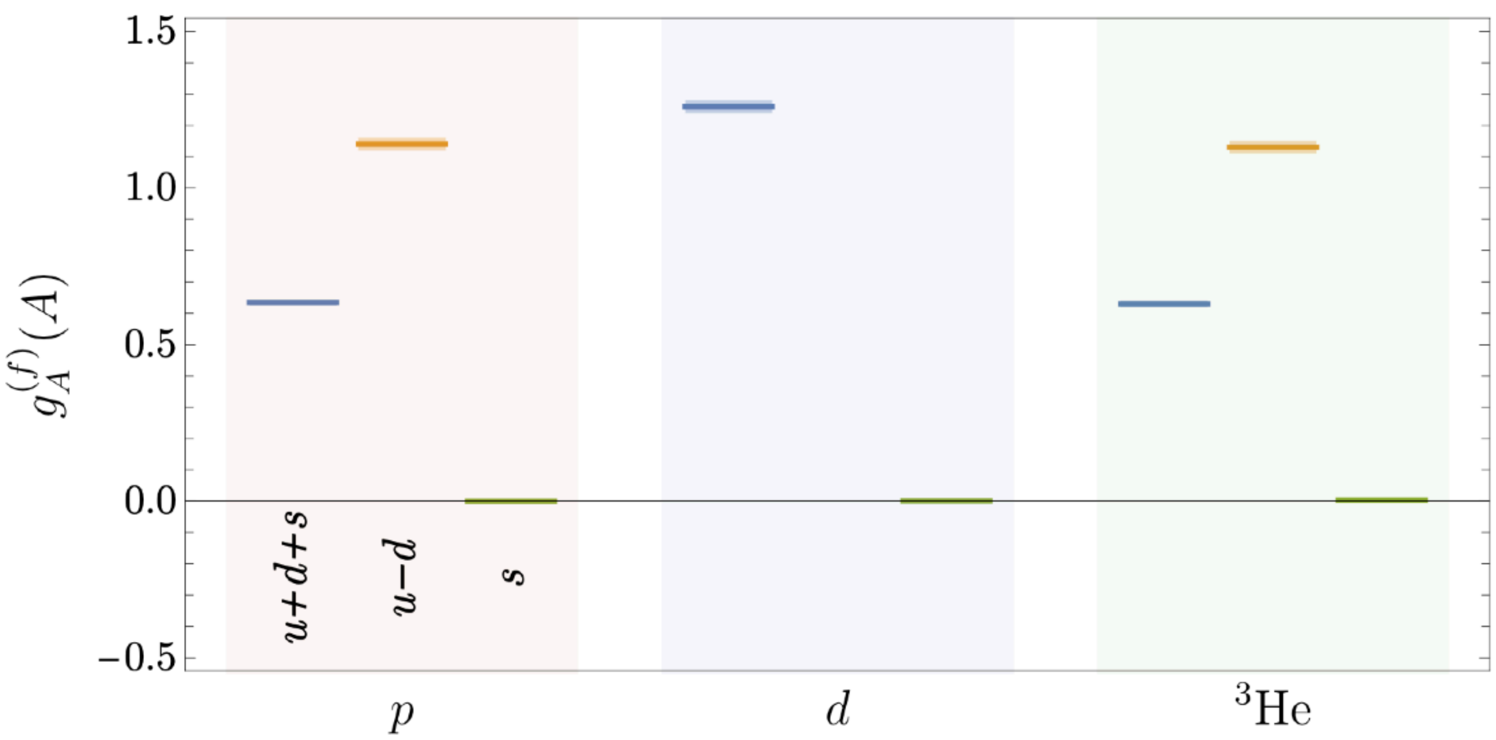
Axial matrix elements

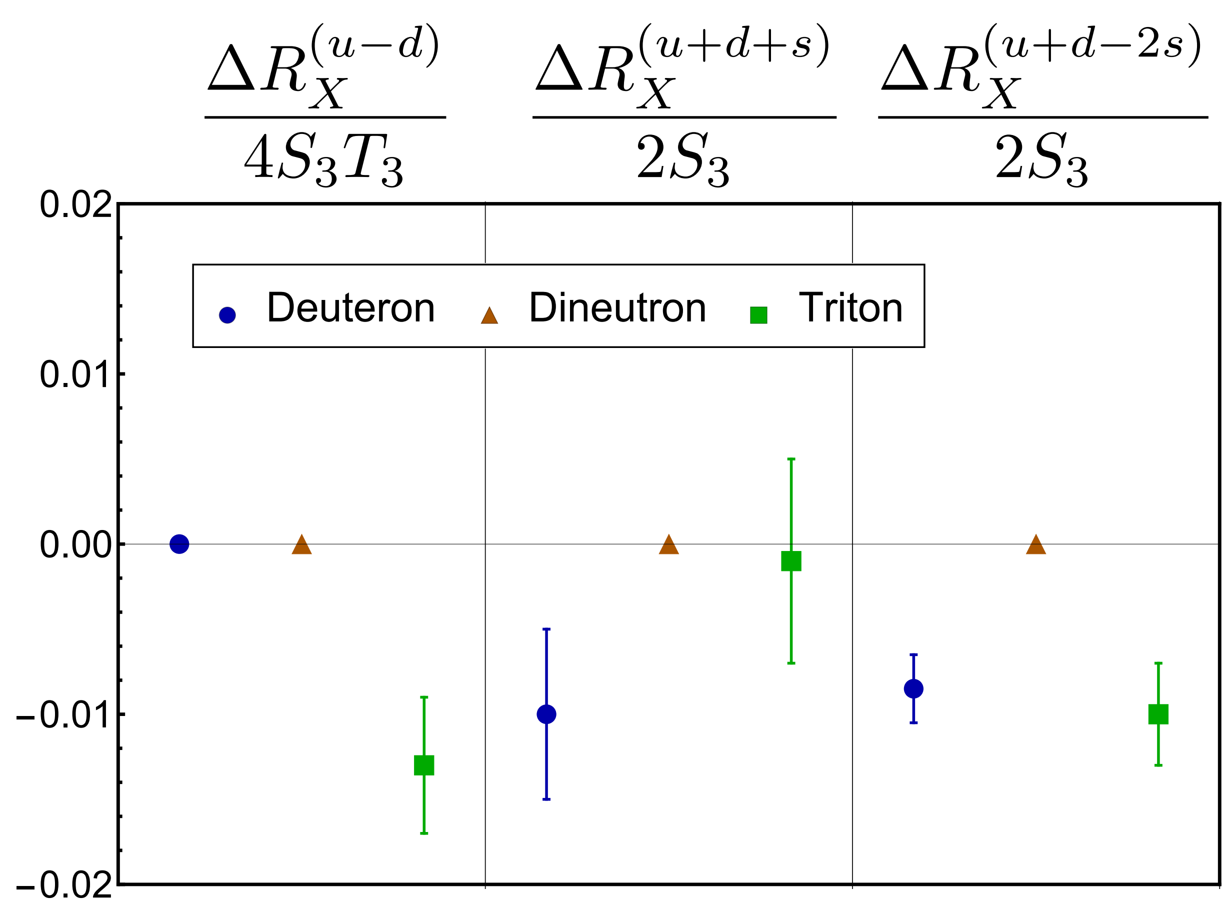


Flavor decomposition of axial matrix elements of up to three nucleon systems computed with

$m_\pi = 806$ MeV

Chang, MW et al [NPLQCD], PRL 120 (2018)

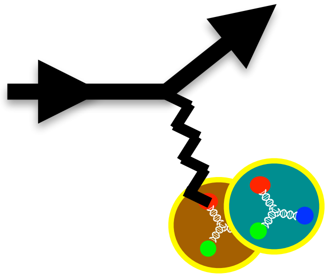


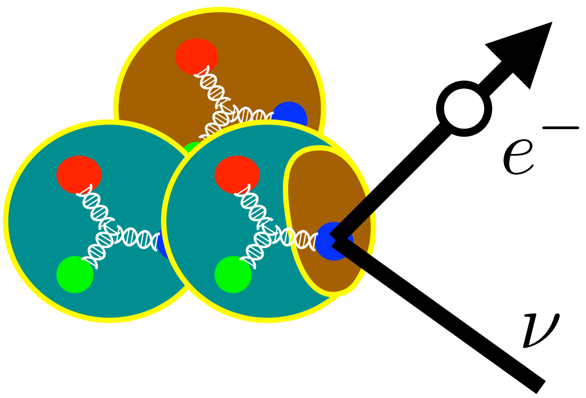


$N_f = 3,\ m_\pi = 806(9)$ MeV$,\ a = 0.145(2)$ fm

Fractional differences from naive shell model expectations show that multi-nucleon correlations lead to percent-level effects on axial charges of light nuclei for these quark masses

Triton β decay





Triton β - decay rate governed by Gamow-Teller matrix element

$$g_A(^3\mathrm{H}) = |\langle ^3\mathrm{He}|A_3^+|^3\mathrm{H}\rangle| = |\langle ^3\mathrm{H}|A_3^+|^3\mathrm{H}\rangle|$$

Computed in ChEFT   Baroni et al, PRC 98 (2018)

After fitting LECs to experimental triton β-decay rate predicts

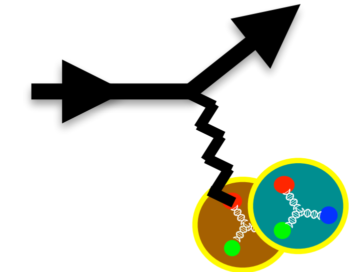
$$\frac{|\langle ^3\mathrm{He}|A_3^+|^3\mathrm{H}\rangle|}{g_A} = 0.951(13)$$

Deviations from 1 arise from two-body currents and multi-nucleon interactions

NLO calculations in pionless EFT relate nuclear effects to the two-body axial current coupling L_{1A} appearing in proton-proton fusion

De-Leon, Platter, Gazit (2016)

Triton β decay from LQCD

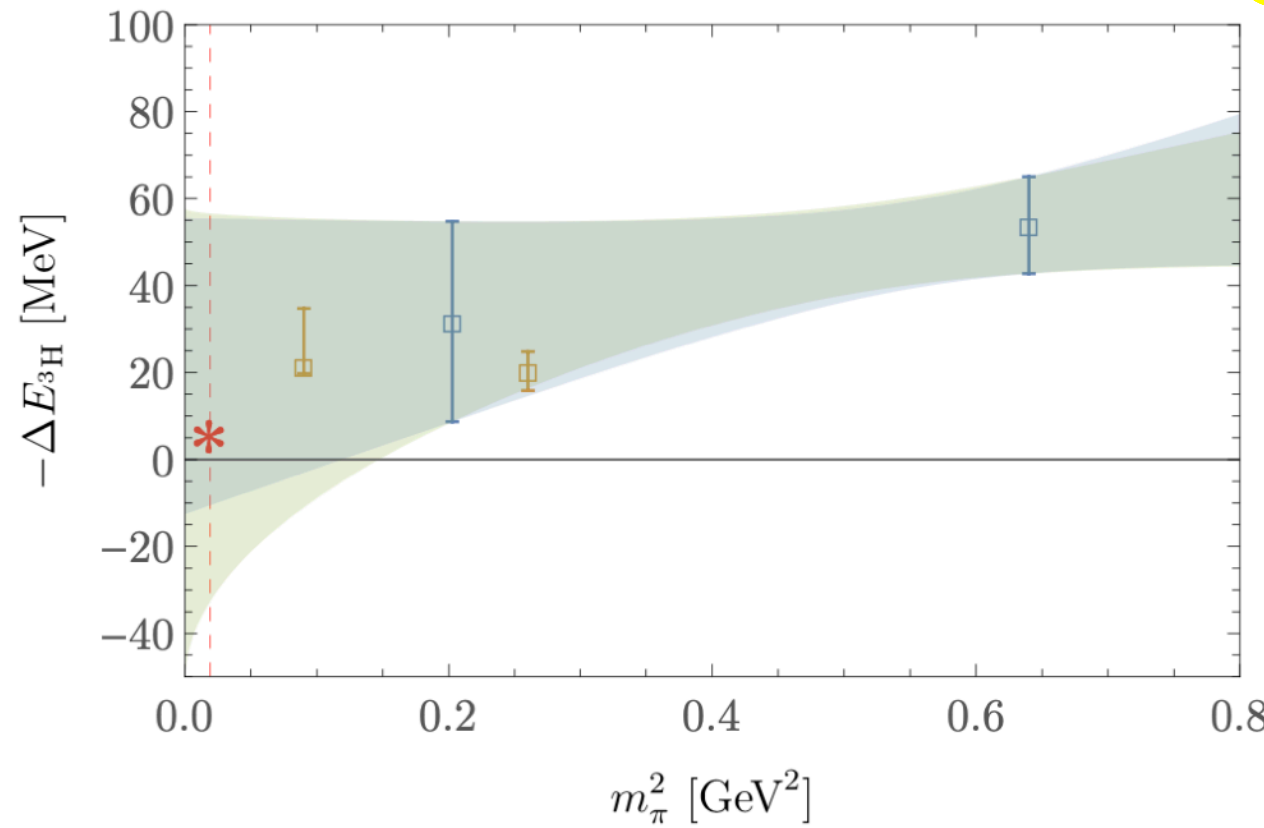


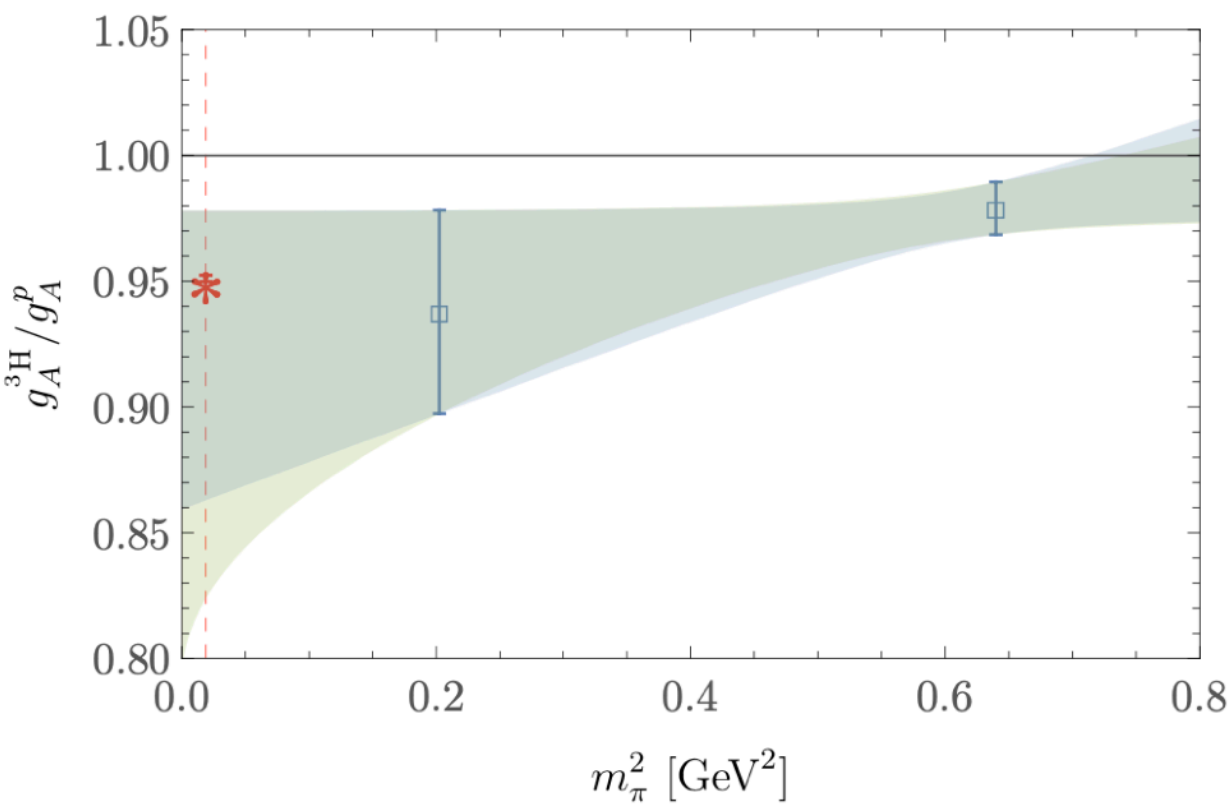
- LQCD calculations of triton recently performed using $m_\pi = 450$ MeV

Parreño, MW et al [NPLQCD] PRD 103 (2021)

- Signal-to-noise problem makes calculations exponentially noisier at lighter quark masses

- Results consistent with bound triton obtained on 3 volumes





Parreño, MW et al [NPLQCD] PRD 103 (2021)

- Axial current matrix element calculations with $m_\pi = 450$ MeV permit preliminary extrapolation to physical point

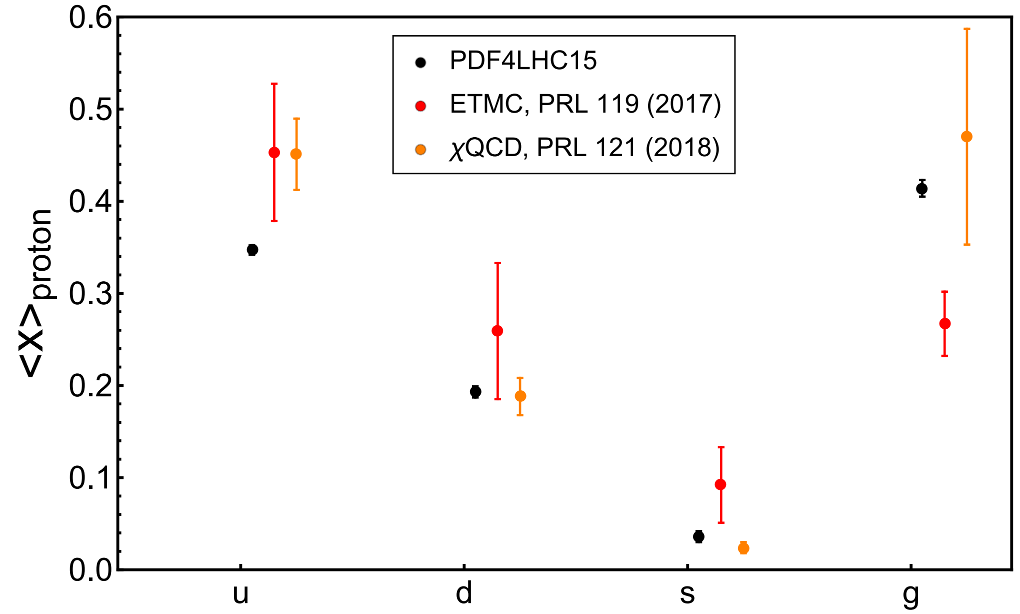
- Several systematic uncertainties remain, but encouraging agreement with experiment seen

- Matching to finite-volume pionless EFT used to constrain L_{1A}

Detmold and Shanahan, PRD 103 (2021)

DIS and LQCD

LQCD can also compute quantities relevant to neutrino DIS

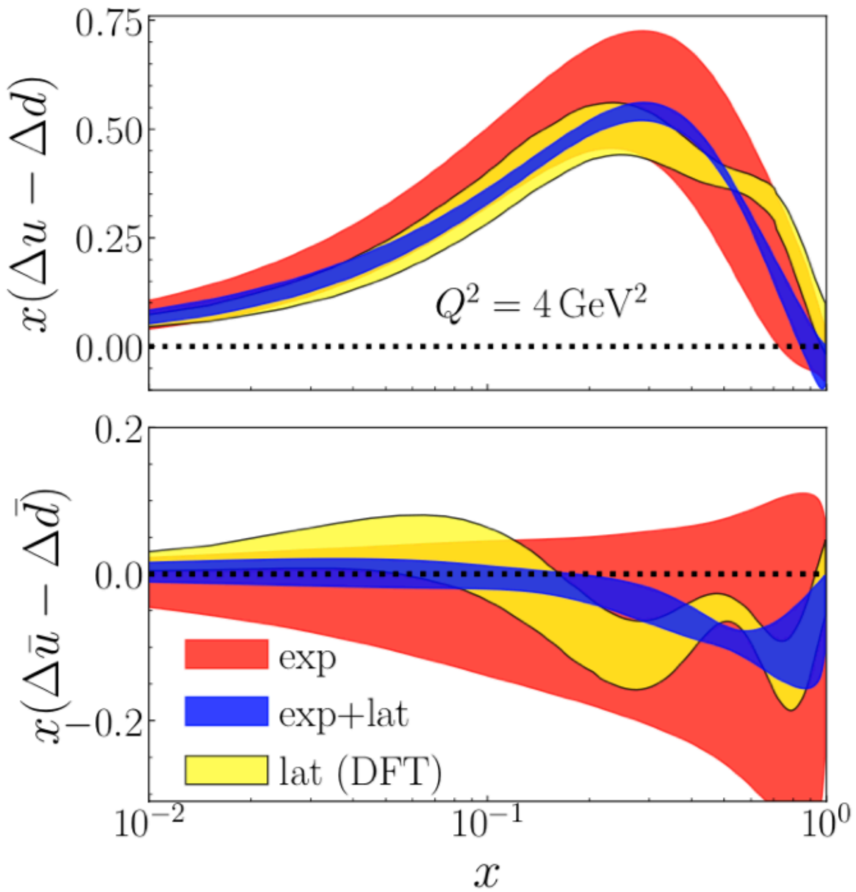


$\langle x\rangle^q_{\rm proton}$ and $\langle x\rangle^g_{\rm proton}$ calculated by several groups

Review: Lin et al, Prog. Part. Nucl. Phys. 100 (2018)

Large momentum effective theory connects Euclidean matrix elements to light-cone PDFs

Review: Ji et al, Rev. Mod. Phys. 93 (2021)



Current LQCD results can improve global analyses of isovector polarized PDFs

Bringewatt et al [JAM], arXiv:2010.00548

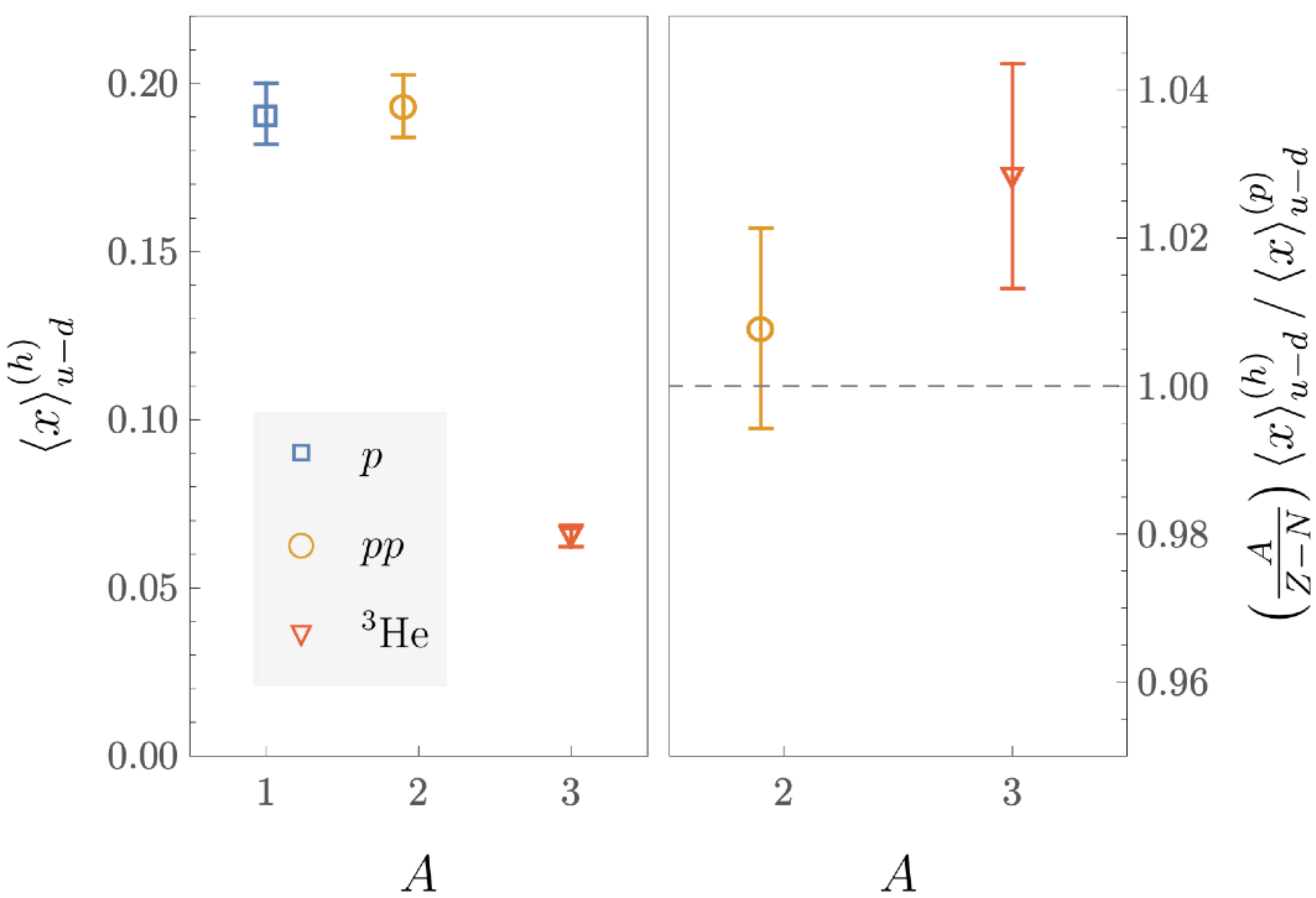
Nuclear momentum fractions

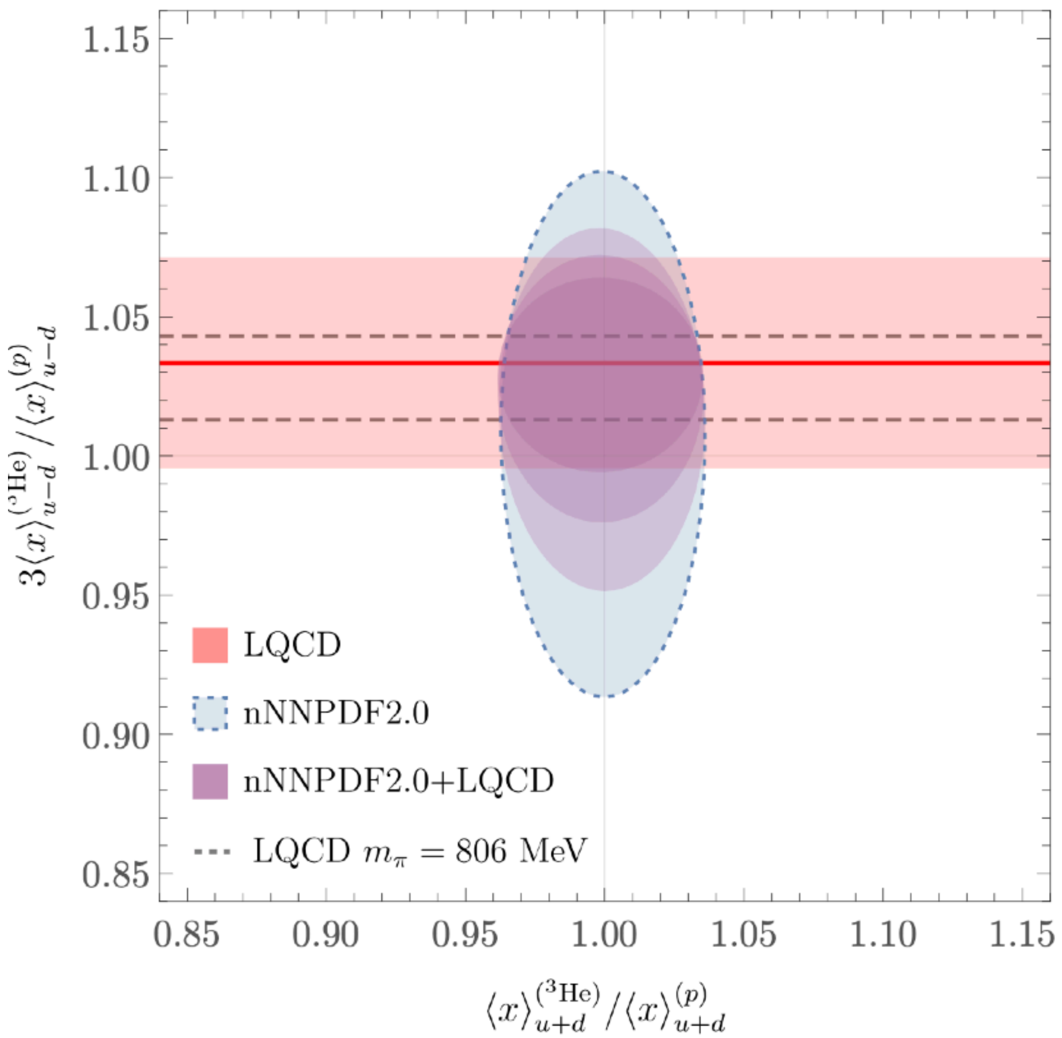
First calculations of gluon and isovector quark momentum fractions of light nuclei

Winter, MW et al [NPLQCD], PRD 96 (2017)   Detmold, MW et al [NPLQCD] PRL 126 (2021)

Results matched to poinless EFT to determine two-body current operator relevant to isovector EMC effects

Although systematic uncertainties are not fully controlled (one lattice spacing, volume, quark mass, …) demonstrates potential for LQCD to usefully constrain nuclear PDFs





Systematic uncertainties

Several systematic uncertainties remain to be quantified in detail

- Heavier than physical quark masses only
- One lattice spacing
- Excited-state effects

Systematic uncertainties

Several systematic uncertainties remain to be quantified in detail

- Heavier than physical quark masses only
- One lattice spacing
- Excited-state effects

Gap between ground and two-nucleon finite-volume “scattering” states becomes small for large volumes, ground-state dominance relies on overlap factors

$$Z_0 e^{-E_0 t}\left(1+\frac{Z_1}{Z_0}e^{-\delta t}+\ldots\right) \qquad \delta \sim \frac{4\pi^2}{ML^2}$$

For non-positive-definite correlation functions, cancellations between the ground and excited-state could in principle conspire to form a “false plateau”

See e.g. Iritani et al, JHEP 10 (2016)

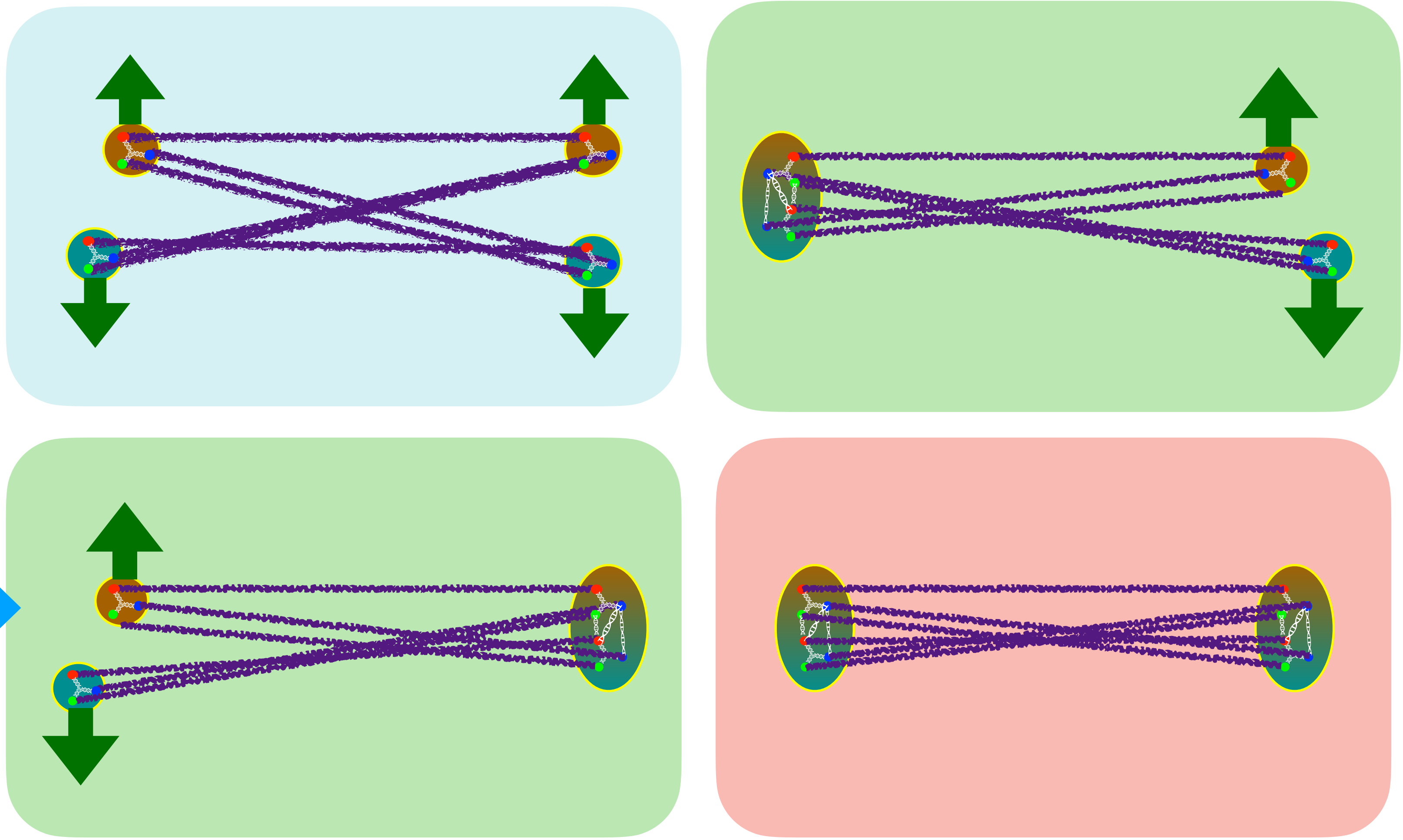
First studies using positive-definite correlation functions (enabled by distillation / stochastic LapH) give results that suggest tensions with previous studies

Francis et al, PRD 99 (2019) Hörz et al, PRC 103 (2021) Green et al, PRL 127 (2021)

The variational method

Correlation-function matrices for an interpolator set including both local “hexaquark” and bilocal “dibaryon” operators can generalize calculations performed to date

Variational bounds on energy spectrum obtained by diagonalizing these matrices



All LQCD nuclear matrix element calculations

Although application of variational methods to multi-nucleon systems has long been advocated, it has only recently become computationally feasible through methods such as distillation and propagator sparsening

Peardon et al PRD 80 (2009)

Morningstar et al PRD 83 (2011)

Detmold, MW, PRD 104 (2021)

Li et al, PRD 103 (2021)

Interpolating operators

Known from $\pi\pi$ scattering studies near the ρ resonance that local and nonlocal operators can be nearly orthogonal

Dudek et al, PRD 87 (2013)

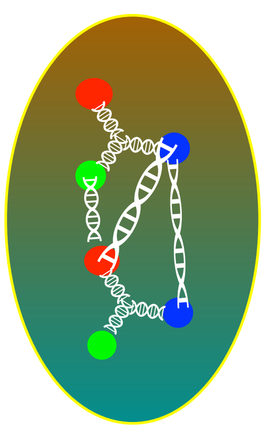
Wilson et al, PRD 92 (2015)

$$\overline{q}(x)\Gamma q(x) \qquad \pi(\vec{p}_1)\pi(\vec{p}_2)$$

Calculations with $t \sim$ few fm neglecting one type of operator show plateau-like behavior but energy spectra with "missing levels" (compared to more complete calculations)

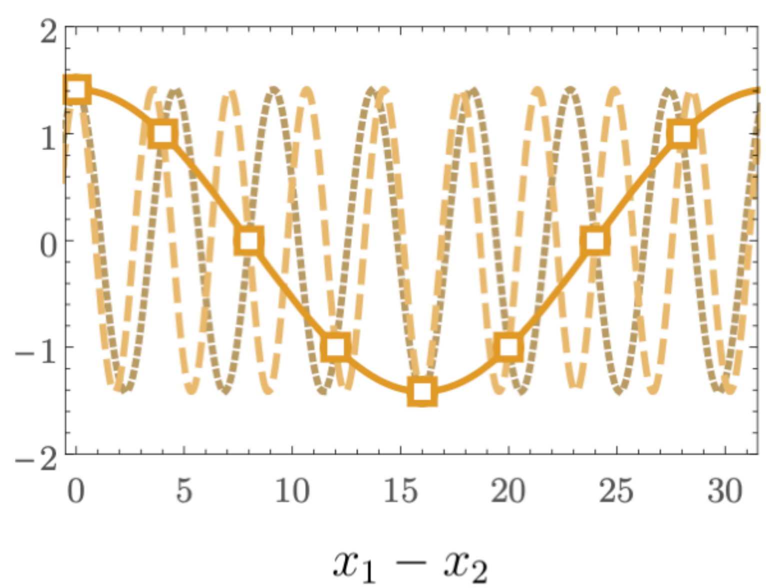
Wide range of two-nucleon operators explored Amarasinghe, MW et al, arXiv:2108.10835

Hexaquark:



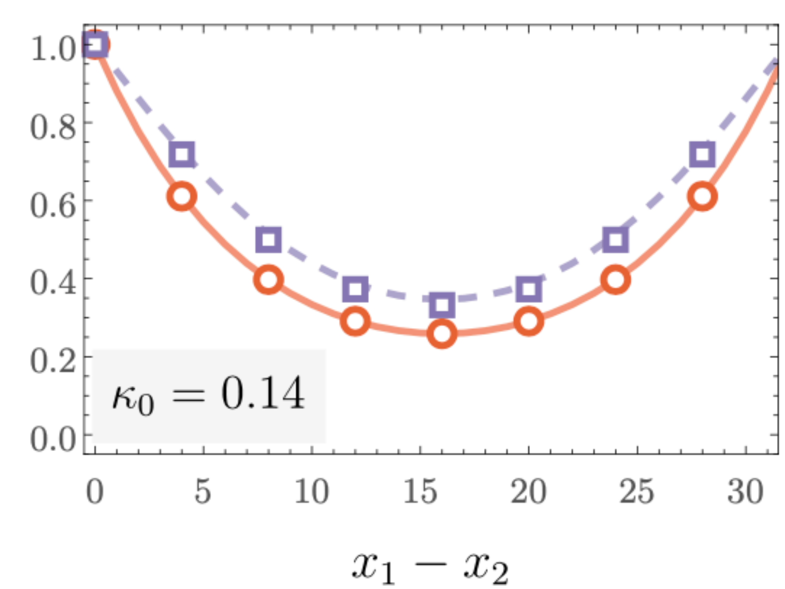
Six Gaussian smeared quarks

Dibaryon:



Two plane-wave baryons with relative momenta

Quasi-local:

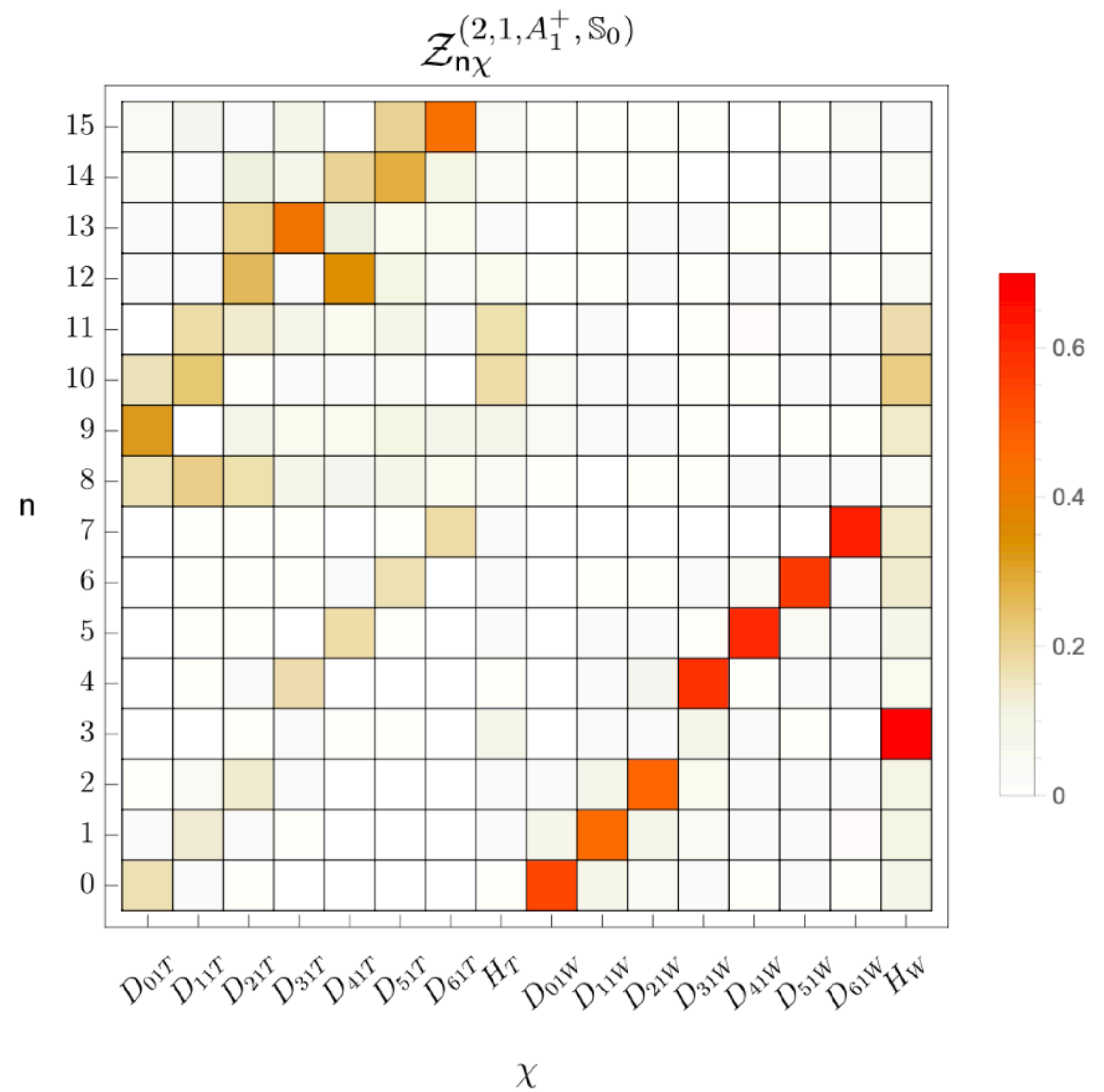


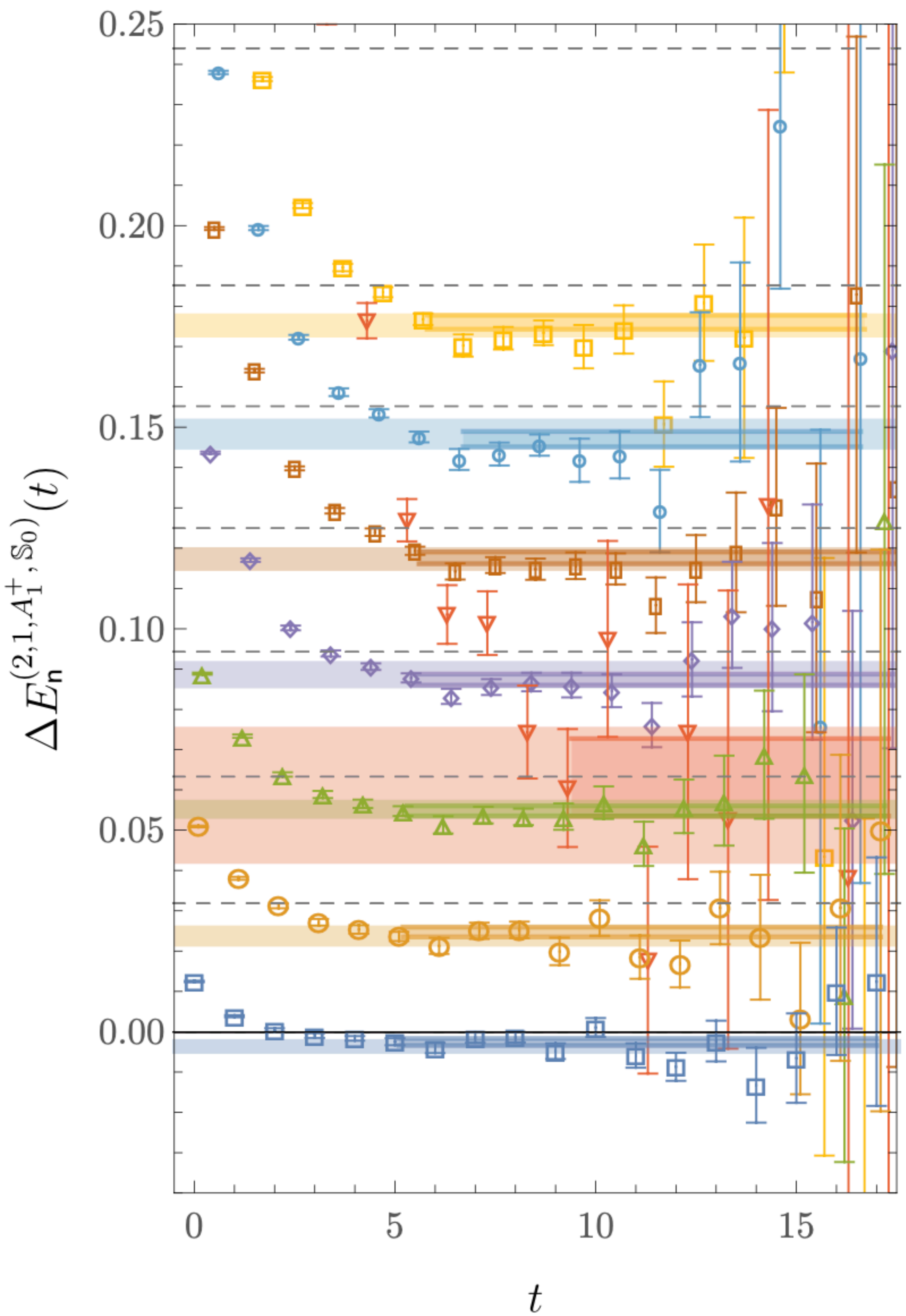
Two exponentially localized baryons

Two nucleons in a box

Diagonalization of correlation-function matrices can be used to remove excited-state contamination from states strongly overlapping with other operators

Each energy level dominantly overlaps with one operator structure, sub-dominant operators collectively 30%

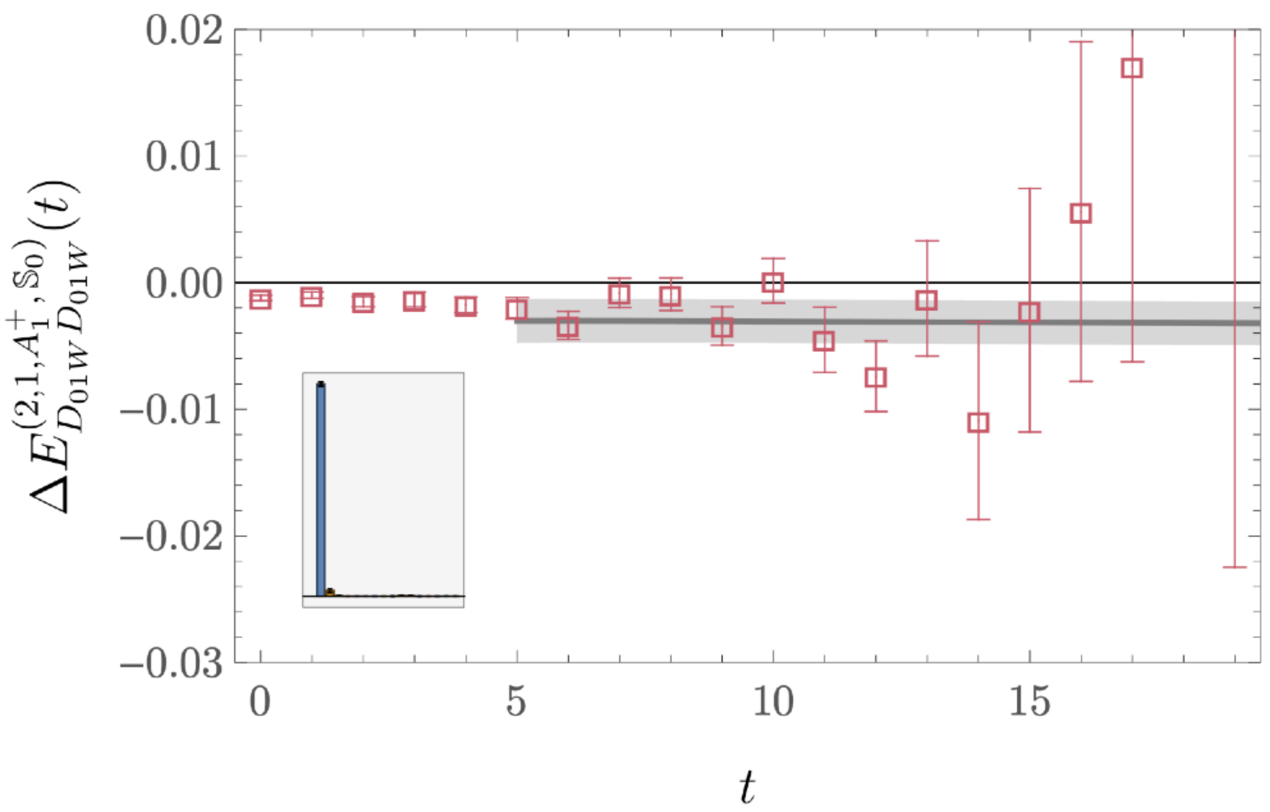




Amarasinghe, MW et al, arXiv:2108.10835

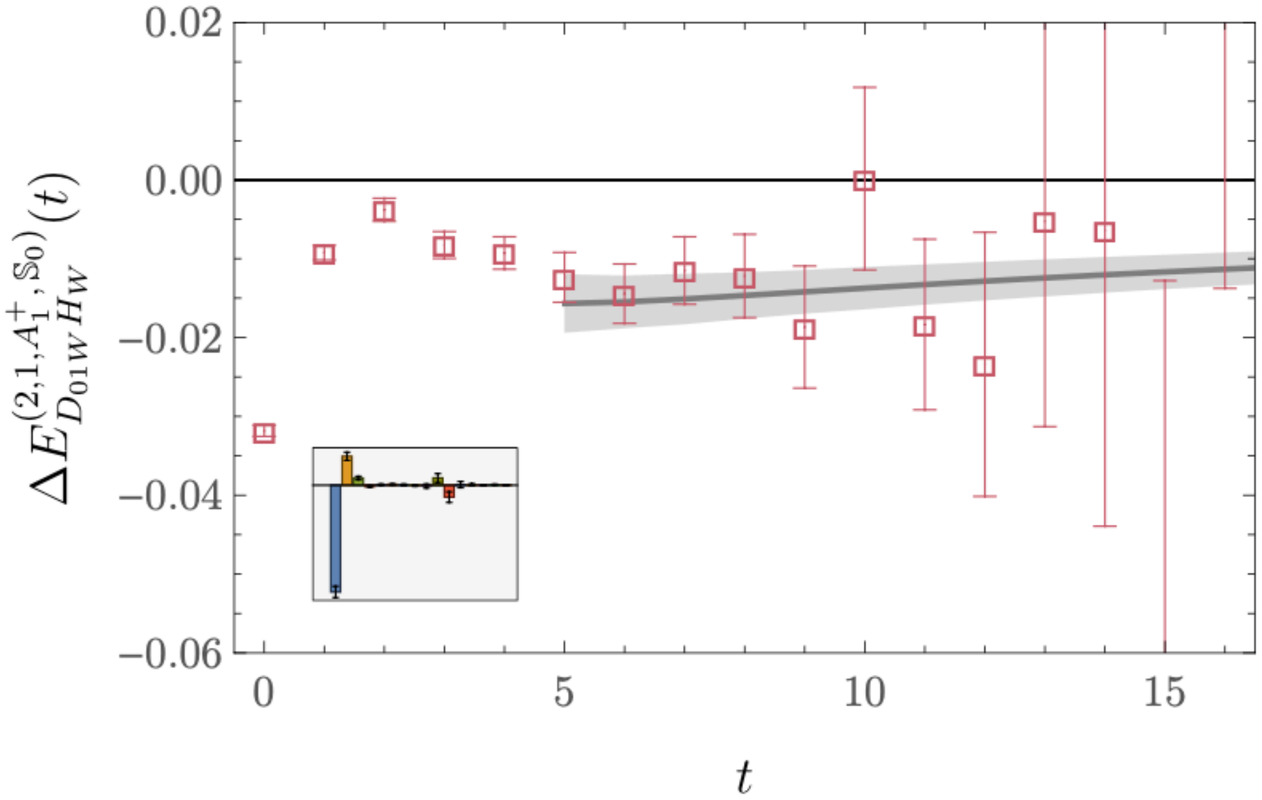
Can heavier quarks build nuclei?

Dibaryon-dibaryon, [*D*, *D*], correlation functions ~ 95% ground-state contribution in reconstruction using same operator set



Reconstructions of [*D*, *H*] correlation functions using spectrum from variational methods can reproduce LQCD results

Variational method results provide model of spectrum in which [*D*, *H*] correlation functions approach ground-state from below



Amarasinghe, MW et al, arXiv:2108.10835

Interpolating-operator dependence

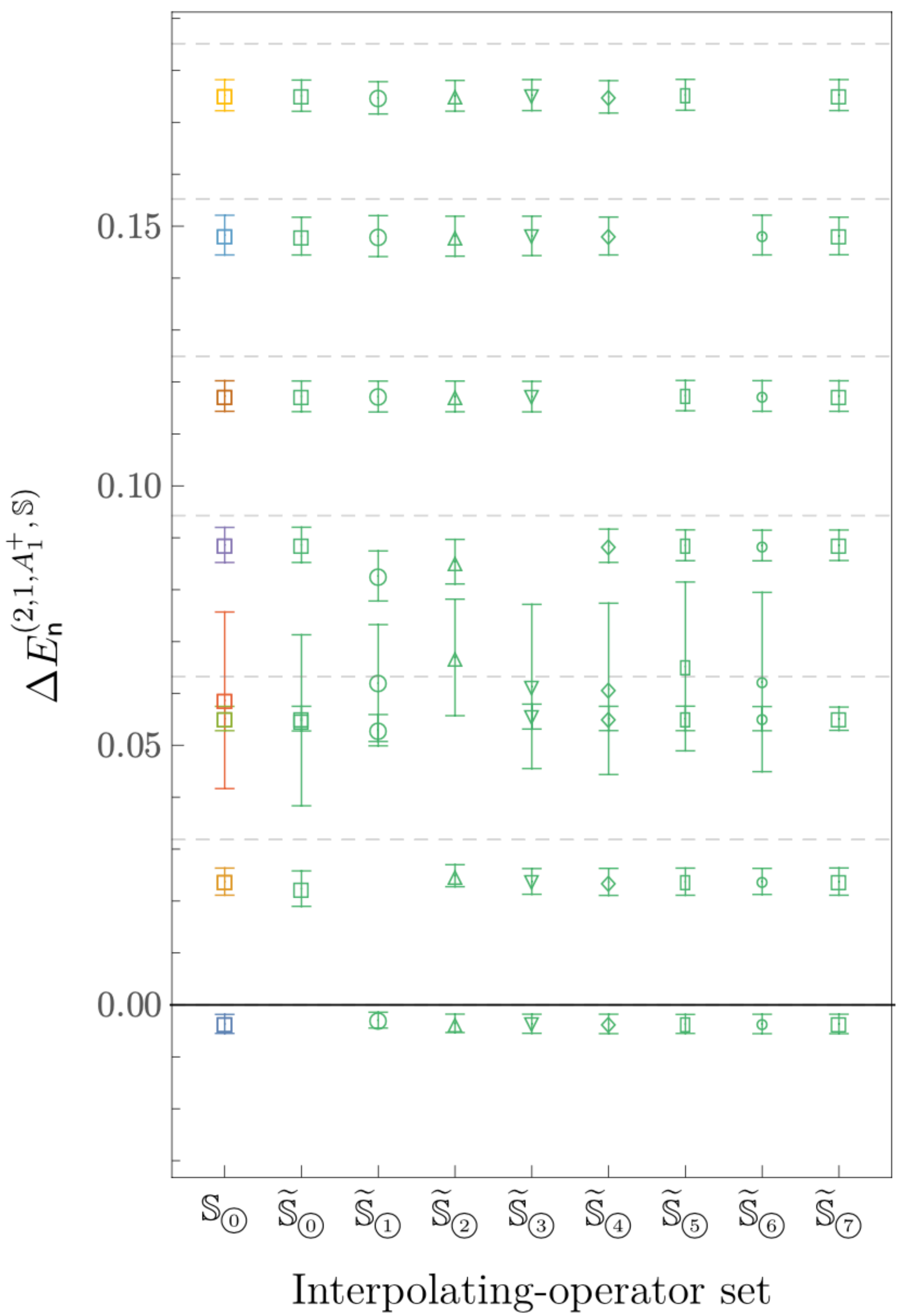
Removing the operator structure with maximum overlap on to a given energy level leads to “missing energy levels”

Even with 10s of interpolating operators, possible to “miss” ground-state

— valid lower bound on ground-state energy, but best-fit results can differ by 5+ σ

Consistent with various dibaryon and hexaquark operators being approximately orthogonal

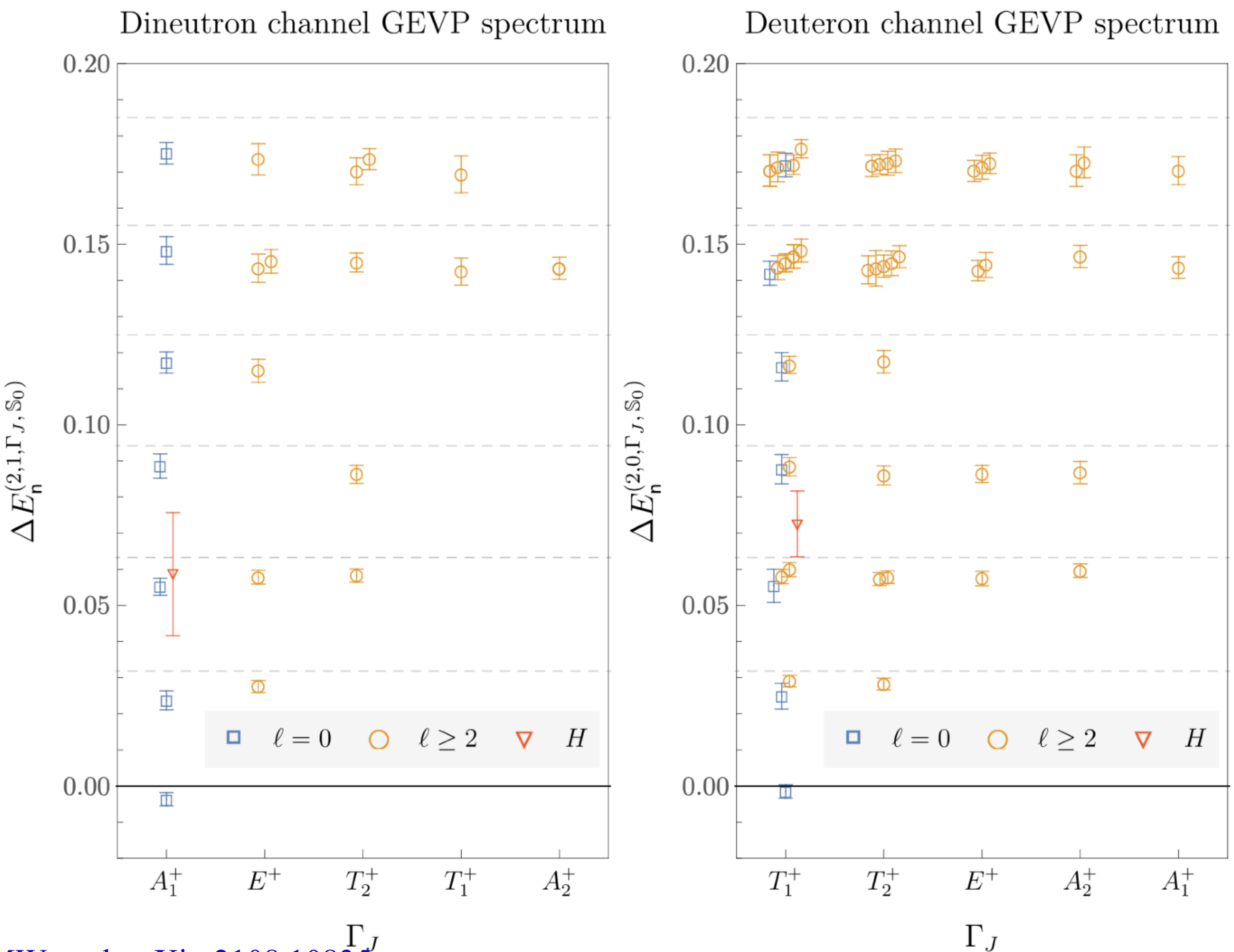
Much larger ($t \gtrsim 1/\delta \sim 5 \text{ fm}$) source/sink separations would be needed to resolve spectrum using interpolating-operator set missing dominant operators



Amarasinghe, MW et al, arXiv:2108.10835

Variational energy spectrum

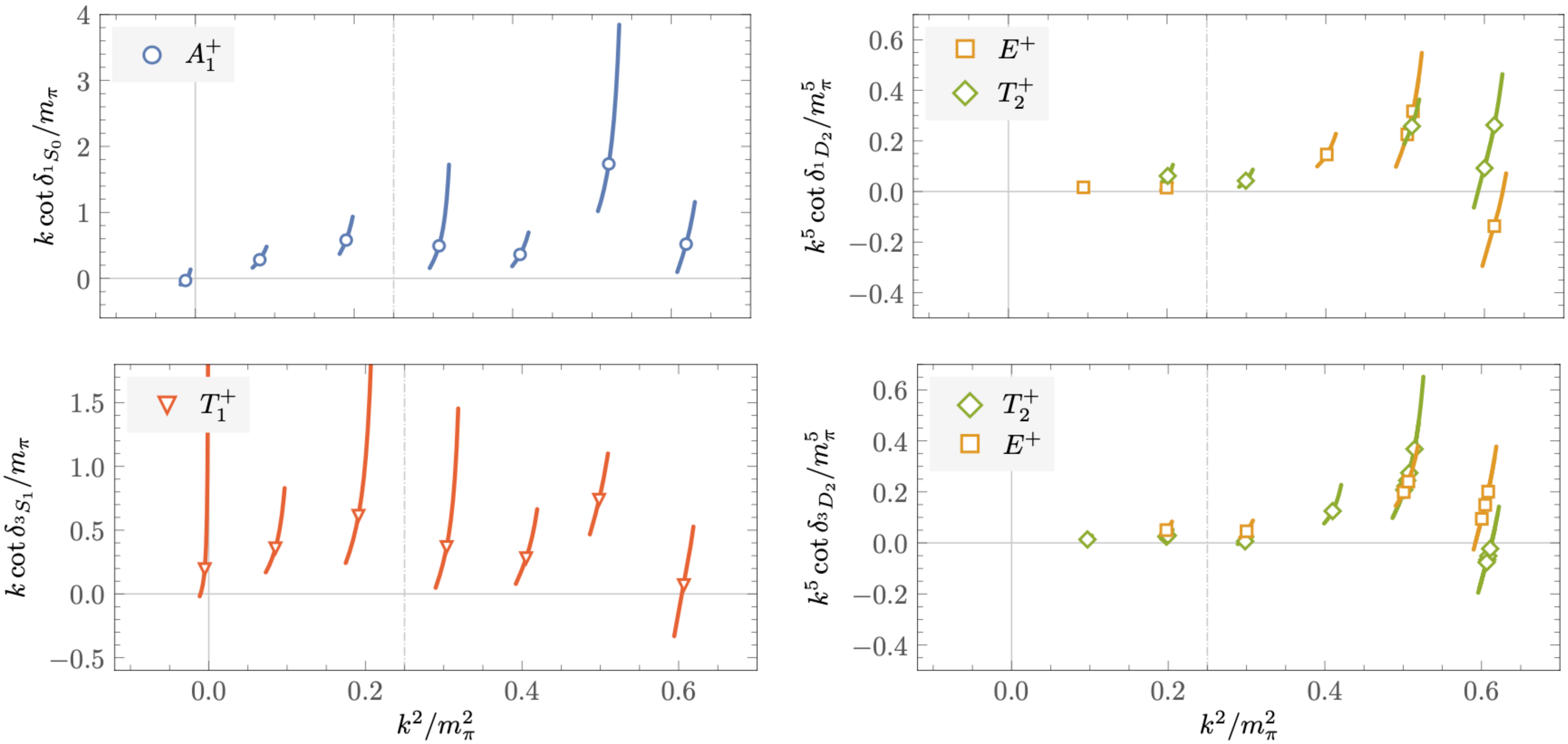
For a given interpolating-operator set, two-nucleon finite-volume energy spectrum can be extracted in various cubic irreps associated with *S*-wave, *D*-wave, and higher-partial-wave interactions



Amarasinghe, MW et al, arXiv:2108.10835

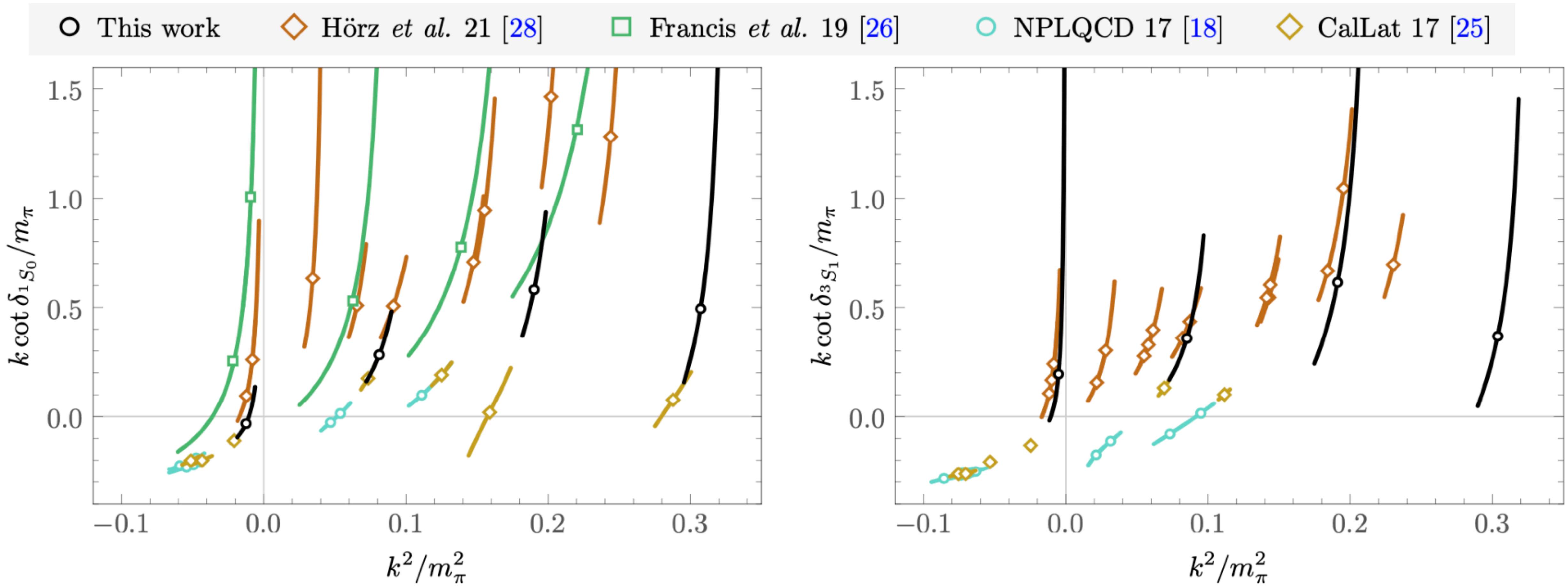
Variational phase shift results

Finite-volume spectrum can be mapped to *S*-wave, *P*-wave and higher-partial-wave scattering phase shifts using generalizations of Lüscher's quantization condition



Amarasinghe, MW et al, arXiv:2108.10835

NN phase shift comparisons



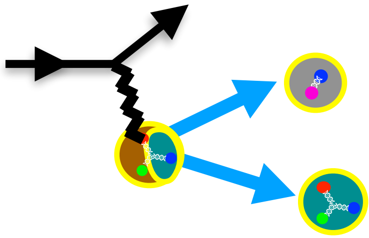
S-wave phase shift results using variational methods and symmetric dibaryon correlation functions consistent among several groups

Discrepancies with previous results using dibaryon-hexaquark correlation functions on the same gauge-field ensemble from multiple groups

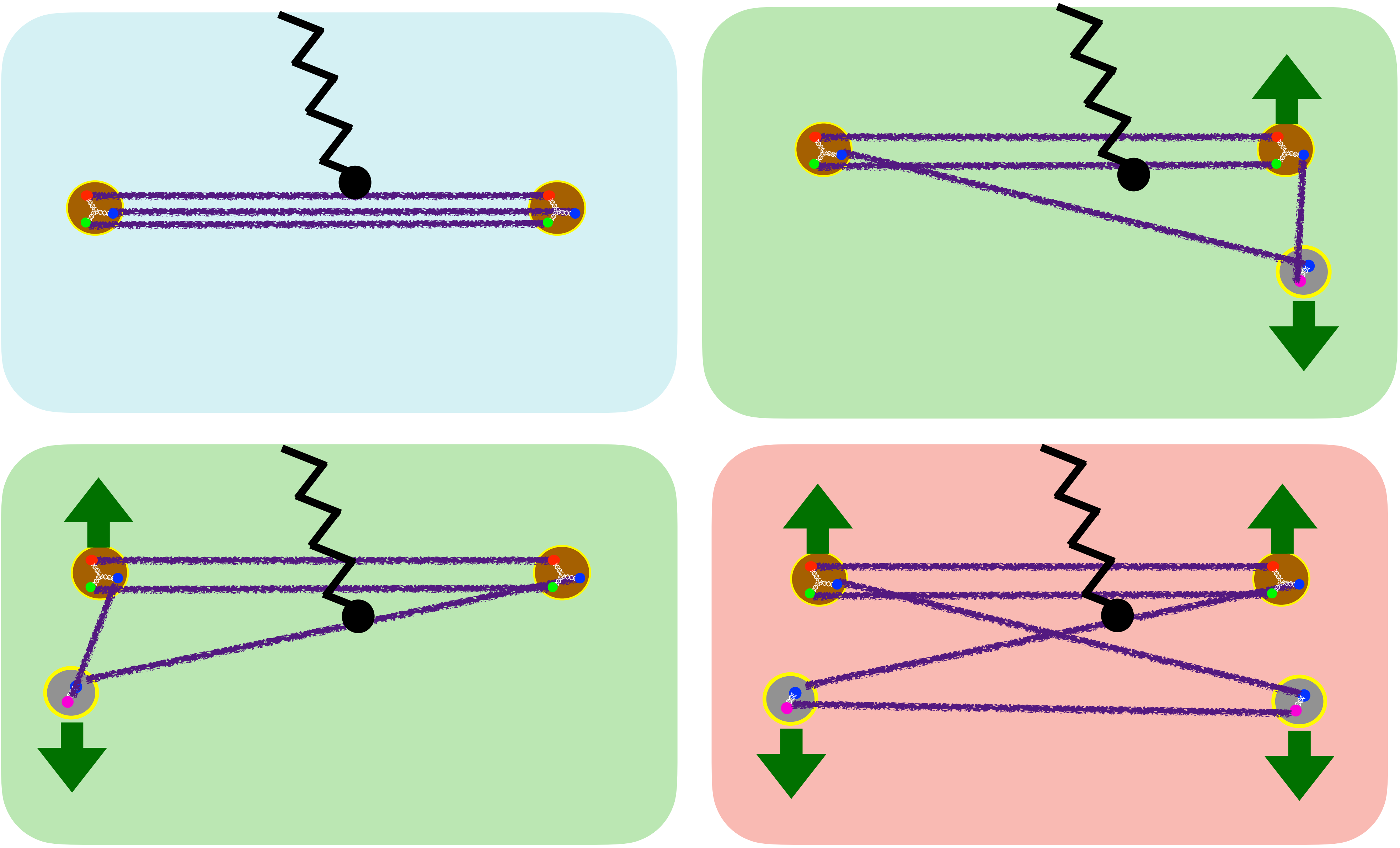
Further variational studies are needed to conclusively determine whether two-nucleon systems bind with heavier-than-physical quark masses

Amarasinghe, MW et al, arXiv:2108.10835

Variational methods for $N\pi$

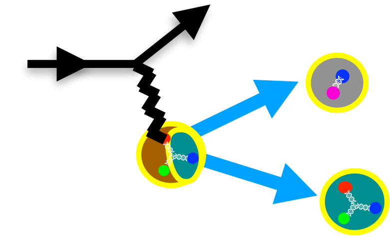


Analogous variational methods can be applied to study $N\pi$ systems



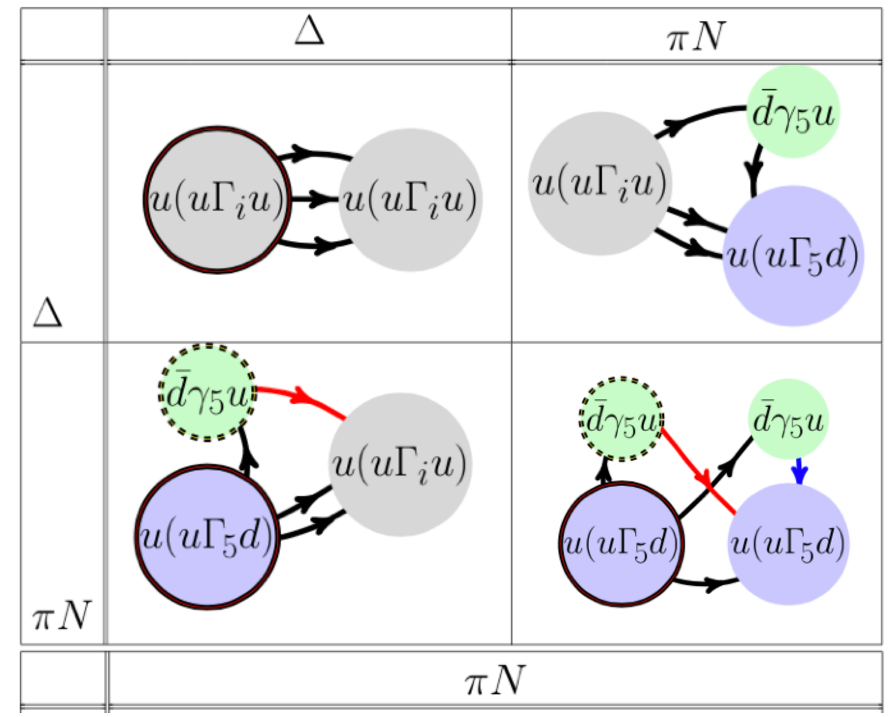
Same calculation can be used to explicitly remove excited-state contamination from elastic nucleon form factors and access pion-production amplitudes

$N\pi$ scattering

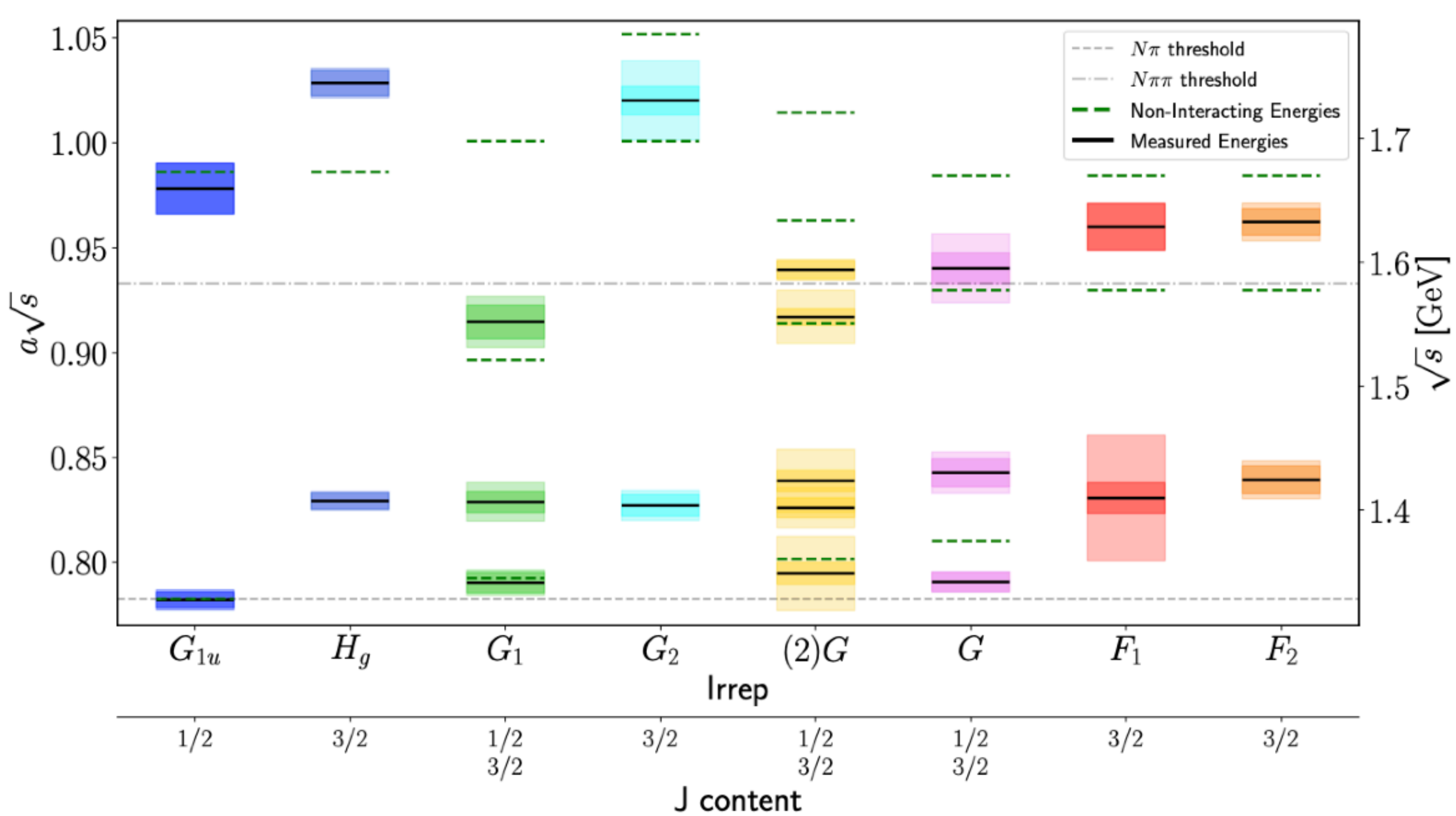


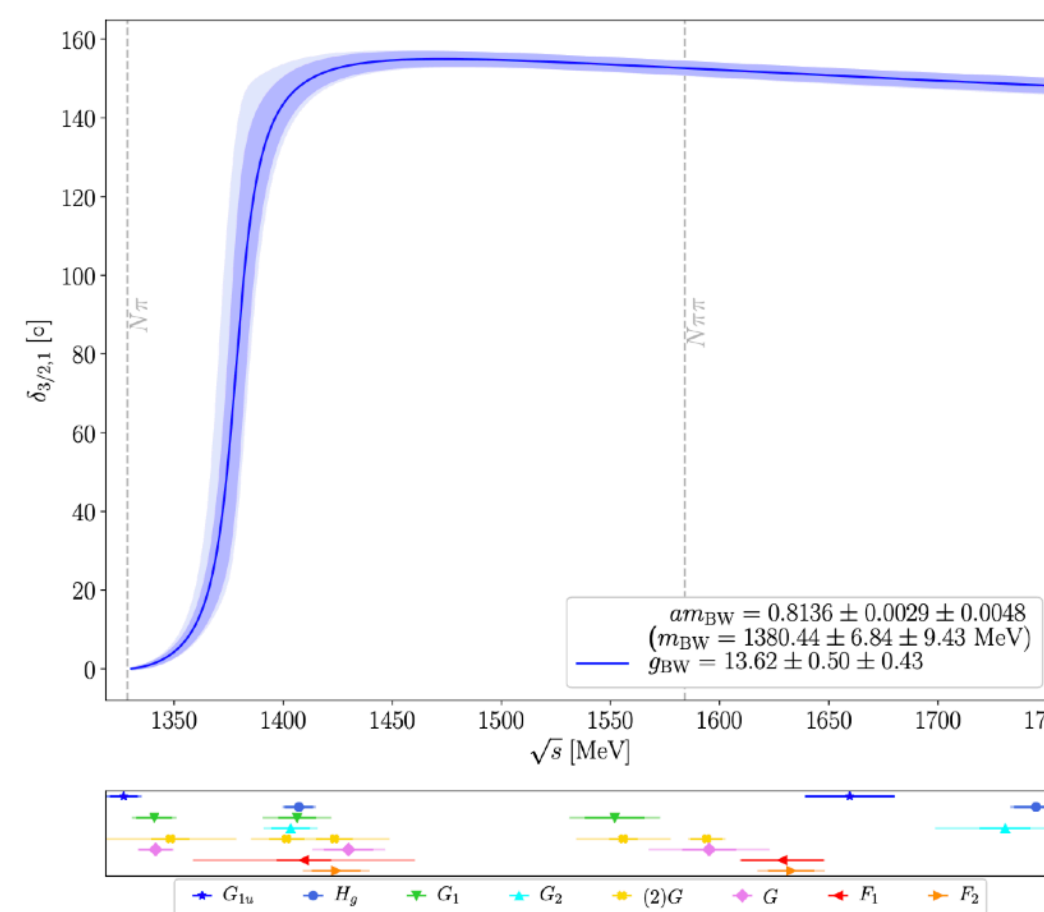
Recent calculations in *I* = 3/2 channel including Δ and $N\pi$ interpolating operators use variational methods to extract ground- and excited-states in many angular momentum channels (cubic irreps)

Silvi et al, PRD 23 (2021)

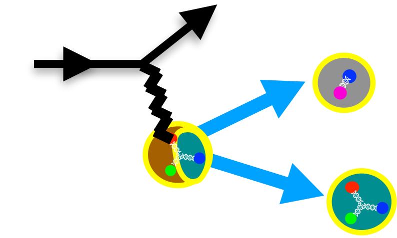


Finite-volume quantization conditions relate energy levels to p-wave $N\pi$ scattering phase shifts

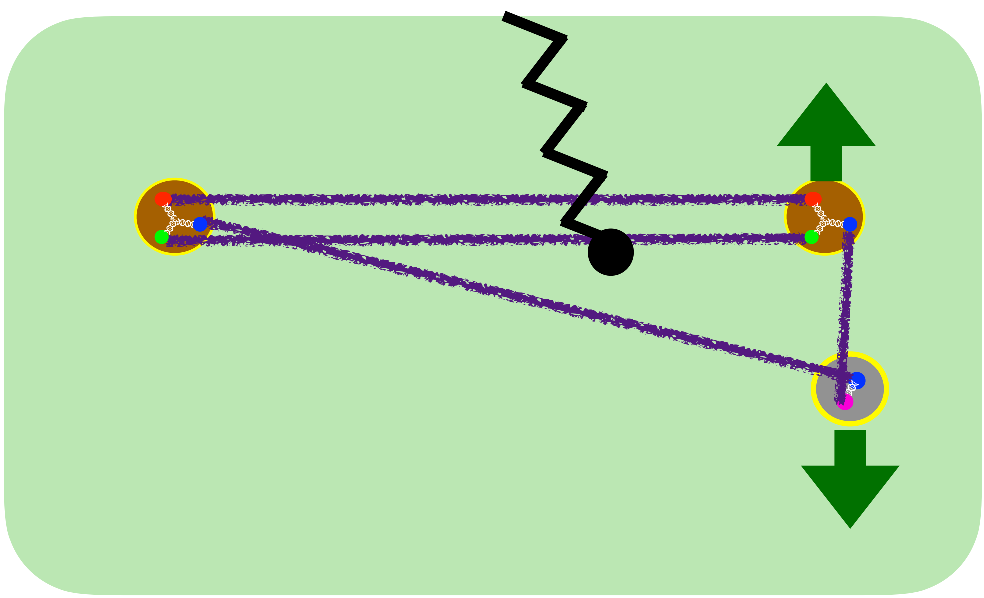


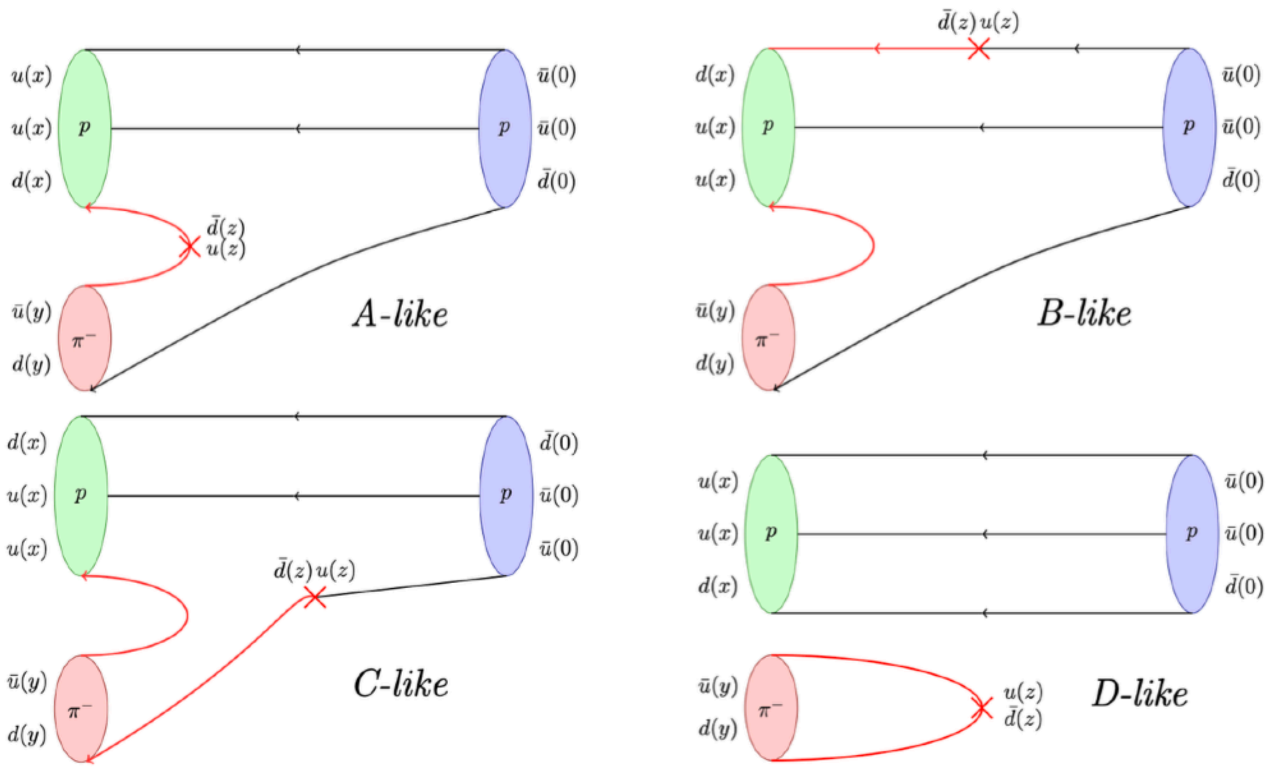


Towards pion production



Exploratory work for pion-production amplitudes in progress
by multiple groups





Challenges:

Barca, Bali, and Collins, arXiv:2110.11908 [hep-lat]

- Approximating all-to-all quark propagators
- Computing all Wick contractions
- Wavefunction construction and finite-volume spin-orbit group theory
- Analyzing rich spectrum of resonant and non-resonant states
- Relating finite-volume LQCD observables to nuclear many-body inputs
- …

Questions