Homework Problems IV
Accelerator Physics

1. Show the following properties of the unit symplectic matrix **U**:

$$\mathbf{U}^T = -\mathbf{U}$$
$$\mathbf{U}\mathbf{U} = -\mathbf{I}$$
$$\mathbf{U}\mathbf{U}^T = \mathbf{I}$$

$$\mathbf{U} = \begin{bmatrix} 0 & 1 & 0 & 0 \\ -1 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & -1 & 0 \end{bmatrix}$$

2. A commonly applied focusing system in accelerators is the so-called FODO system. For a thin lens approximation to this system, the one period transfer matrix starting from the middle of the focusing lens is

$$M = \begin{pmatrix} 1 & 0 \\ -1/(2f) & 1 \end{pmatrix}\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 1/f & 1 \end{pmatrix}\begin{pmatrix} 1 & L \\ 0 & 1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ -1/(2f) & 1 \end{pmatrix}.$$

where f is the lens focal length and L is the distance between lenses. Evaluate the total transfer matrix.

What is the result of a similar calculation to obtain the one period transfer map starting at the middle of the defocusing lens? (Hint: You don't have to perform the whole matrix multiplication again. Change a relevant parameter in the solution you've already obtained!).

Compare the matrix traces of the two results you have obtained. How must one choose the ratio L / f to obtain a phase advance of 60 degrees? In this case, what are the beta-functions and alpha-functions for the periodic solutions in the middle of the focusing lens and in the middle of the defocusing lens.

3. To match the dispersion from a regular FODO arc cell (with phase advance/cell μ and bend angle θ) to a dispersion free straight section, one needs a dispersion suppressor composed of two reduced bending FODO cells with the same phase advance/cell μ) and with reduced bending angles θ_1 and θ_2 for each cell. Show that the conditions for zero dispersion and its derivative after the dispersion suppressor are:

$$\theta_1 = \left(1 - \frac{1}{4\sin^2\frac{\mu}{2}}\right)\theta \qquad \theta_2 = \left(\frac{1}{4\sin^2\frac{\mu}{2}}\right)\theta \qquad \text{and} \qquad \theta = \theta_1 + \theta_2$$