Neutrino and antineutrino interactions with nuclei

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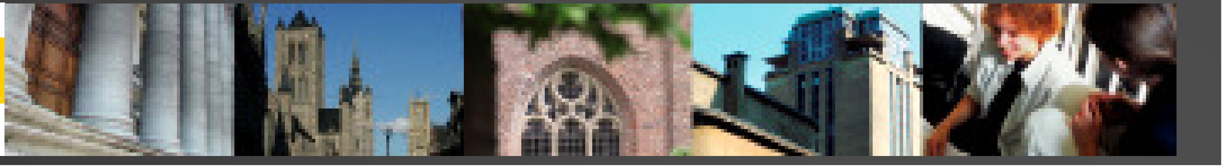
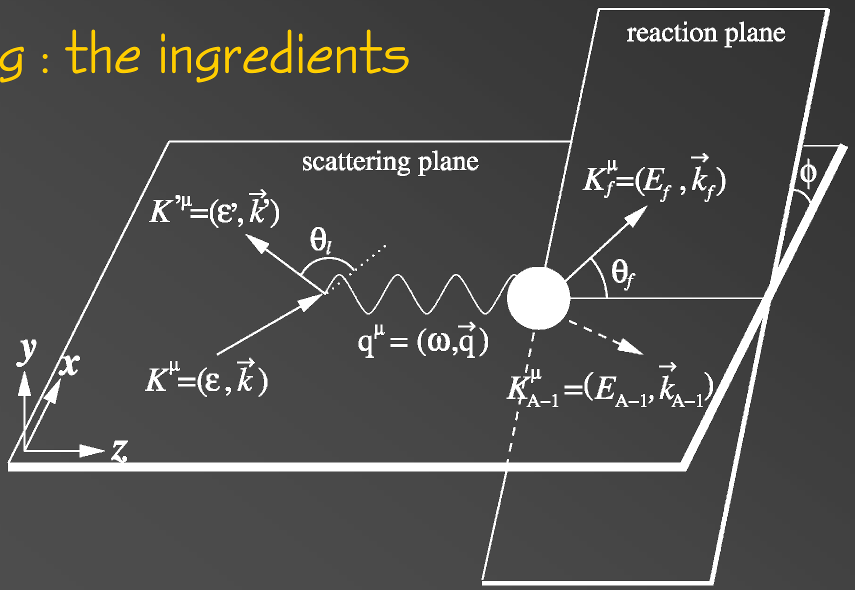


Neutrino-nucleus scattering : the ingredients



Cross section :

$$d\sigma = \frac{1}{\beta}\overline{\sum_{if}}|M_{fi}|^2 \frac{M_l}{\varepsilon'}\frac{M_{A-1}}{E_{A-1}}\frac{M_N}{E_f} d^3\vec{k}_{A-1} d^3\vec{k}' d^3\vec{k}_f$$

$$(2\pi)^{-5}\delta^4(K^\mu + K^\mu_A - K'^\mu - K^\mu_{A-1} - K^\mu_f)$$

$$\frac{d^5\sigma}{d\varepsilon' d^2\Omega_l d^2\Omega_f} = \frac{M_l M_N M_{A-1}}{(2\pi)^5 M_A \varepsilon'} k'^2 k_f f_{rec}^{-1} \overline{\sum_{if}}|M_{fi}|^2$$

with $$\overline{\sum_{if}}|M_{fi}|^2 = \frac{G_F^2}{2}\left[\frac{M_B^2}{Q^2+M_B^2}\right]^2 l_{\alpha\beta}W^{\alpha\beta}$$

lepton tensor

hadron tensor

lepton tensor

$$l_{\alpha\beta} \equiv \overline{\sum_{s,s'}}[\overline{u}_l \gamma_\alpha (1-\gamma_5) u_l]^\dagger [\overline{u}_\nu \gamma_\beta (1-\gamma_5) u_\nu]$$

hadron tensor

$$W^{\alpha\beta} = \overline{\sum_{if}} \langle \Delta^{\alpha\mu} J_\mu \rangle^\dagger \langle \Delta^{\beta\nu} J_\nu \rangle = \overline{\sum_{if}} \langle \mathcal{J}^\alpha \rangle^\dagger \langle \mathcal{J}^\beta \rangle$$

$$\downarrow$$

$$\langle \mathcal{J}^\alpha \rangle \equiv \left\langle (A-1)(J_R M_R), K_f(E_f, \vec{k}_f) m_s \middle| \Delta_{\alpha\mu} \hat{J}^\mu \middle| A(0^+, g.s.) \right\rangle$$

$$\downarrow$$

$$J^\mu = F_1(Q^2)\gamma^\mu + i\frac{\kappa}{2M_N} F_2(Q^2)\sigma^{\mu\nu} q_\nu + G_A(Q^2)\gamma^\mu\gamma_5 + \frac{1}{2M_N} G_P(Q^2) q^\mu \gamma_5$$

Cross section :

$$\frac{d^5\sigma}{d\varepsilon' d^2\Omega_l d^2\Omega_f} = \frac{M_N M_{A-1}}{(2\pi)^3 M_A} k_f f_{rec}^{-1} \sigma_M^{Z,W^\pm}$$

$$[v_L R_L + v_T R_T + v_{TT} R_{TT} \cos 2\phi + v_{TL} R_{TL} \cos\phi + h(v'_T R'_T + v'_{TL} R'_{TL} \cos\phi)]$$

Kinematic factors	Response functions

$$\begin{aligned} v_L &= 1, \\ v_T &= \tan^2\frac{\theta_l}{2} + \frac{Q^2}{2|\vec{q}|^2}, \\ v_{TT} &= -\frac{Q^2}{2|\vec{q}|^2}, \\ v_{TL} &= -\frac{1}{\sqrt{2}}\sqrt{\tan^2\frac{\theta_l}{2} + \frac{Q^2}{|\vec{q}|^2}}, \\ v'_T &= \tan\frac{\theta_l}{2}\sqrt{\tan^2\frac{\theta_l}{2} + \frac{Q^2}{|\vec{q}|^2}}, \\ v'_{TL} &= \frac{1}{\sqrt{2}}\tan\frac{\theta_l}{2}. \end{aligned}$$

$$\begin{aligned} R_L &= \left|\langle\mathcal{J}^0(\vec{q})\rangle - \frac{\omega}{|\vec{q}|}\langle\mathcal{J}^z(\vec{q})\rangle\right|^2, \\ R_T &= \left|\langle\mathcal{J}^+(\vec{q})\rangle\right|^2 + \left|\langle\mathcal{J}^-(\vec{q})\rangle\right|^2, \\ R_{TT}\cos 2\phi &= 2\Re\left\{\langle\mathcal{J}^+(\vec{q})\rangle^*\langle\mathcal{J}^-(\vec{q})\rangle\right\}, \\ R_{TL}\cos\phi &= -2\Re\left\{\left[\langle\mathcal{J}^0(\vec{q})\rangle - \frac{\omega}{|\vec{q}|}\langle\mathcal{J}^0(\vec{q})\rangle\right]\left[\langle\mathcal{J}^+(\vec{q})\rangle - \langle\mathcal{J}^-(\vec{q})\rangle\right]^*\right\} \\ R'_T &= \left|\langle\mathcal{J}^+(\vec{q})\rangle\right|^2 - \left|\langle\mathcal{J}^-(\vec{q})\rangle\right|^2, \\ R'_{TL}\cos\phi &= -2\Re\left\{\left[\langle\mathcal{J}^0(\vec{q})\rangle - \frac{\omega}{|\vec{q}|}\langle\mathcal{J}^z(\vec{q})\rangle\right]\left[\langle\mathcal{J}^+(\vec{q})\rangle + \langle\mathcal{J}^-(\vec{q})\rangle\right]^*\right\} \end{aligned}$$

⬆ transition matrix elements

$$\langle J^{\mu} \rangle = \int d\vec{r}\, \overline{\phi}_F(\vec{r}) \hat{J}^{\mu}(\vec{r}) e^{i\vec{q}.\vec{r}} \phi_B(\vec{r})$$

RPWIA : relativistic wave functions obtained within the Hartree-approximation to the Walecka-Serot σ-ω model

\+

final state interactions :

RMSGA

relativistic multiple scattering Glauber approximation

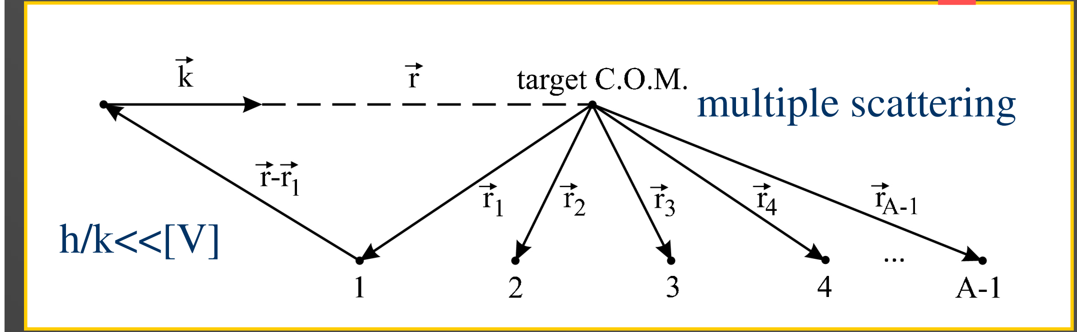
- semi-classical approach
- ‘high’ energies
- nucleon-nucleon data

M.C. Martínez et al PRC73, 024607 (2006)

Final State Interactions

$$\langle J^\mu \rangle = \int d\vec{r}\, \overline{\phi}_F(\vec{r}) \hat{J}^\mu(\vec{r}) e^{i\vec{q}.\vec{r}} \phi_B(\vec{r})$$

scattering wave function



$$\phi_F(\vec{r}) \equiv \mathcal{G}(\vec{b}, z) \phi_{k_f,\, s_f}(\vec{r})$$

Glauber phase

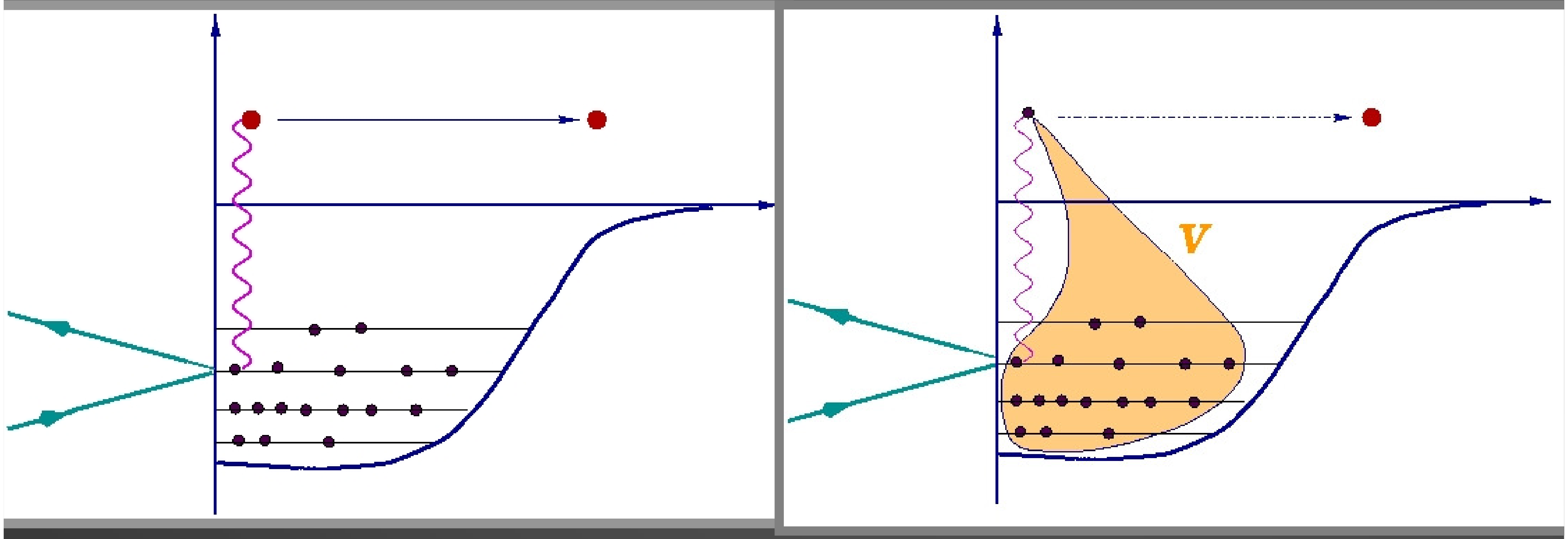
- eikonal approach
- linear trajectories
- ‘frozen spectators’ approximation

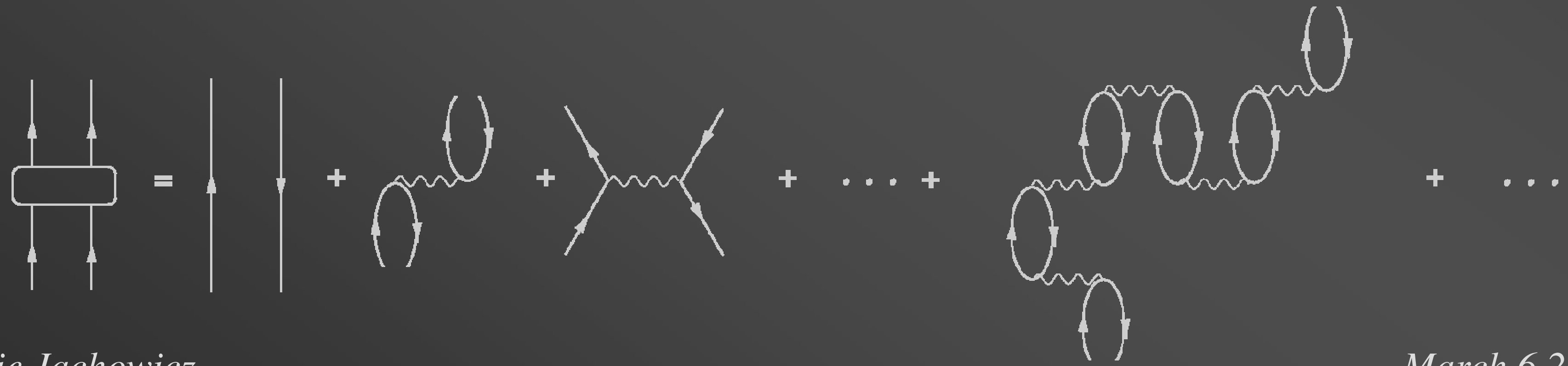
$$\mathcal{G}(\vec{b}, z) = \prod_{\alpha \neq B} \left[1 - \int d\vec{r}\,' |\phi_\alpha(\vec{r}\,')|^2 \theta(z' - z) \Gamma(\vec{b}' - \vec{b}) \right]$$

$$\mathcal{G}(\vec{b}, z) \approx \left\{ 1 - \frac{\sigma_{NN}^{tot}(1 - i\epsilon_{NN})}{4\pi\beta_{NN}^2} \int_0^\infty b' db' T_B(b', z) \exp\left[-\frac{(b - b')^2}{2\beta_{NN}^2} \right] \int_0^{2\pi} d\phi_{b'} \exp\left[\frac{-2bb'}{\beta_{NN}^2} \sin^2\left(\frac{\phi_b - \phi_{b'}}{2} \right) \right] \right\}^{A-1}$$

Modeling neutrino-nucleus cross sections :

RPA





Summarizing : included in the description of the nucleus and the reaction :

-nuclear binding : missing momentum distributions for the bound nucleons

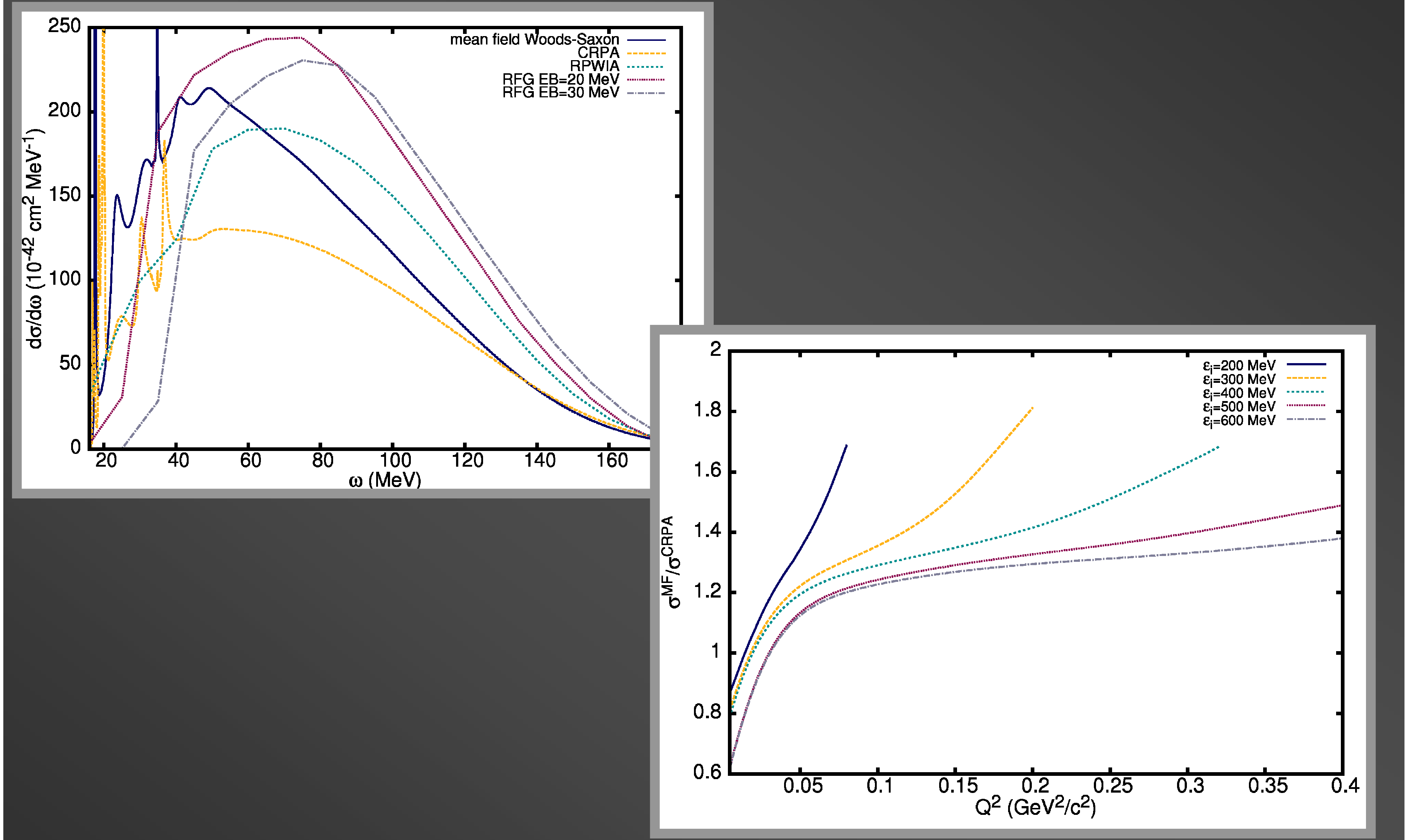
-Pauli-blocking

-fully relativistic description of the nuclear dynamics

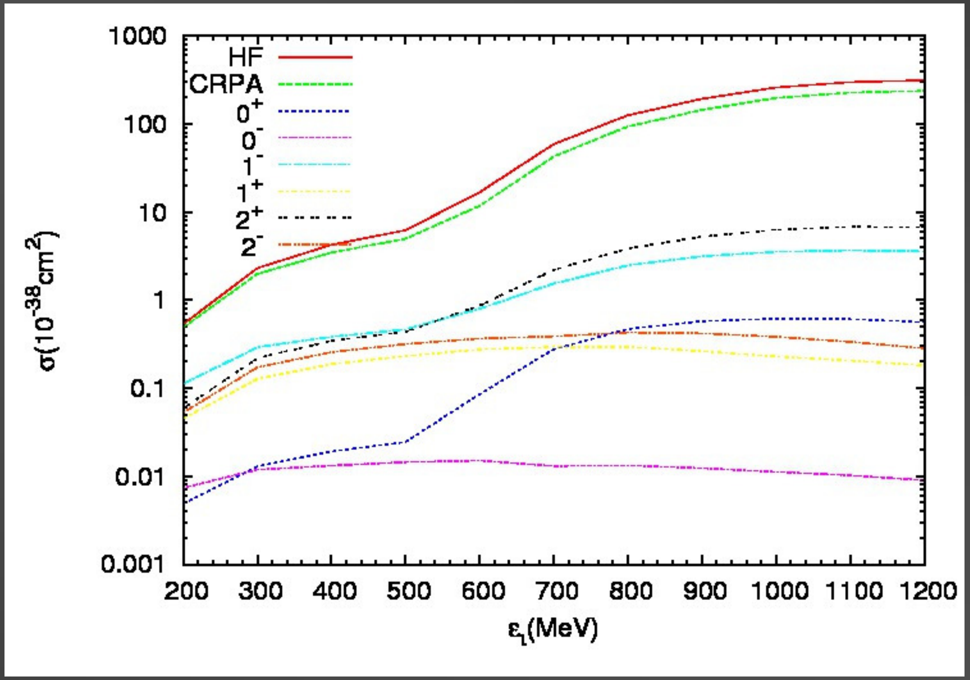
-final state interactions

-full implementation of hadronic current with axial, and weak vector contributions, momentum distribution for the form factors

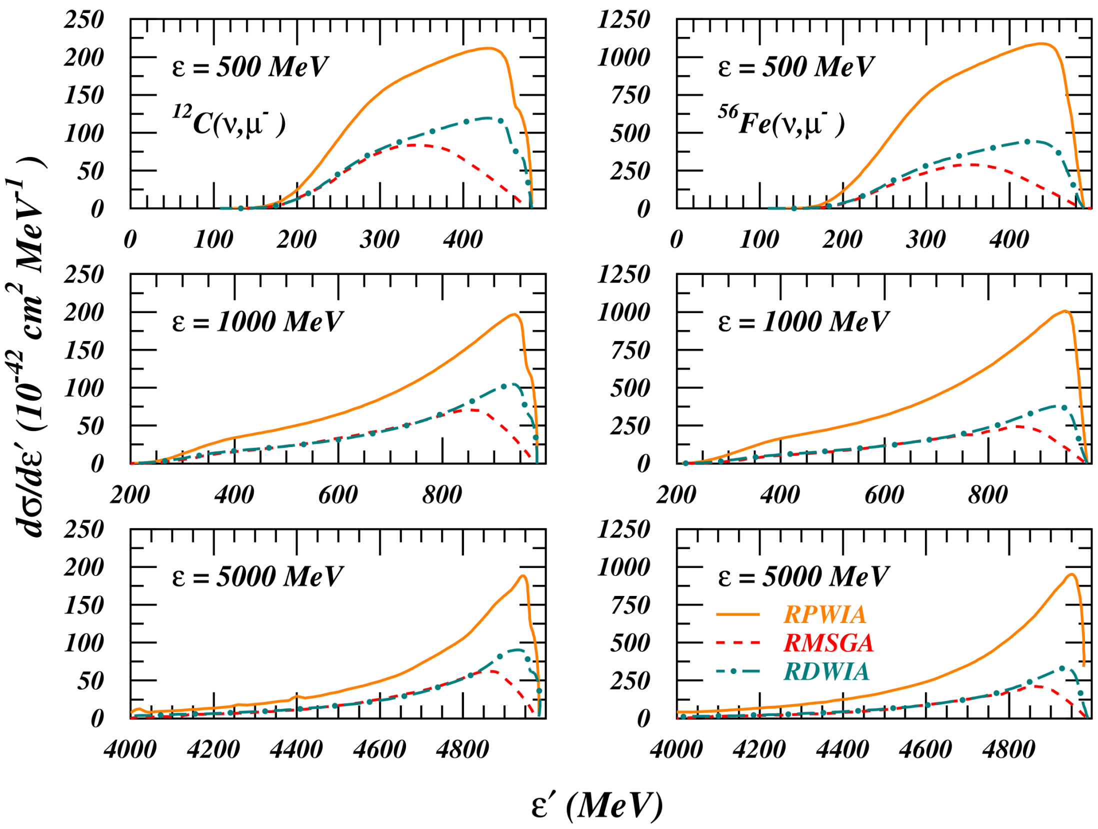
-long range RPA correlations, important at lower energies


dσ/dω (10^{-42} cm^2 MeV^{-1})
ω (MeV)
0
50
100
150
200
250
20
40
60
80
100
120
140
160
mean field Woods-Saxon
CRPA
RPWIA
RFG EB=20 MeV
RFG EB=30 MeV
$\sigma^{MF}/\sigma^{CRPA}$
Q^2 (GeV^2/c^2)
0.6
0.8
1
1.2
1.4
1.6
1.8
2
0.05
0.1
0.15
0.2
0.25
0.3
0.35
0.4
ε_i=200 MeV
ε_i=300 MeV
ε_i=400 MeV
ε_i=500 MeV
ε_i=600 MeV

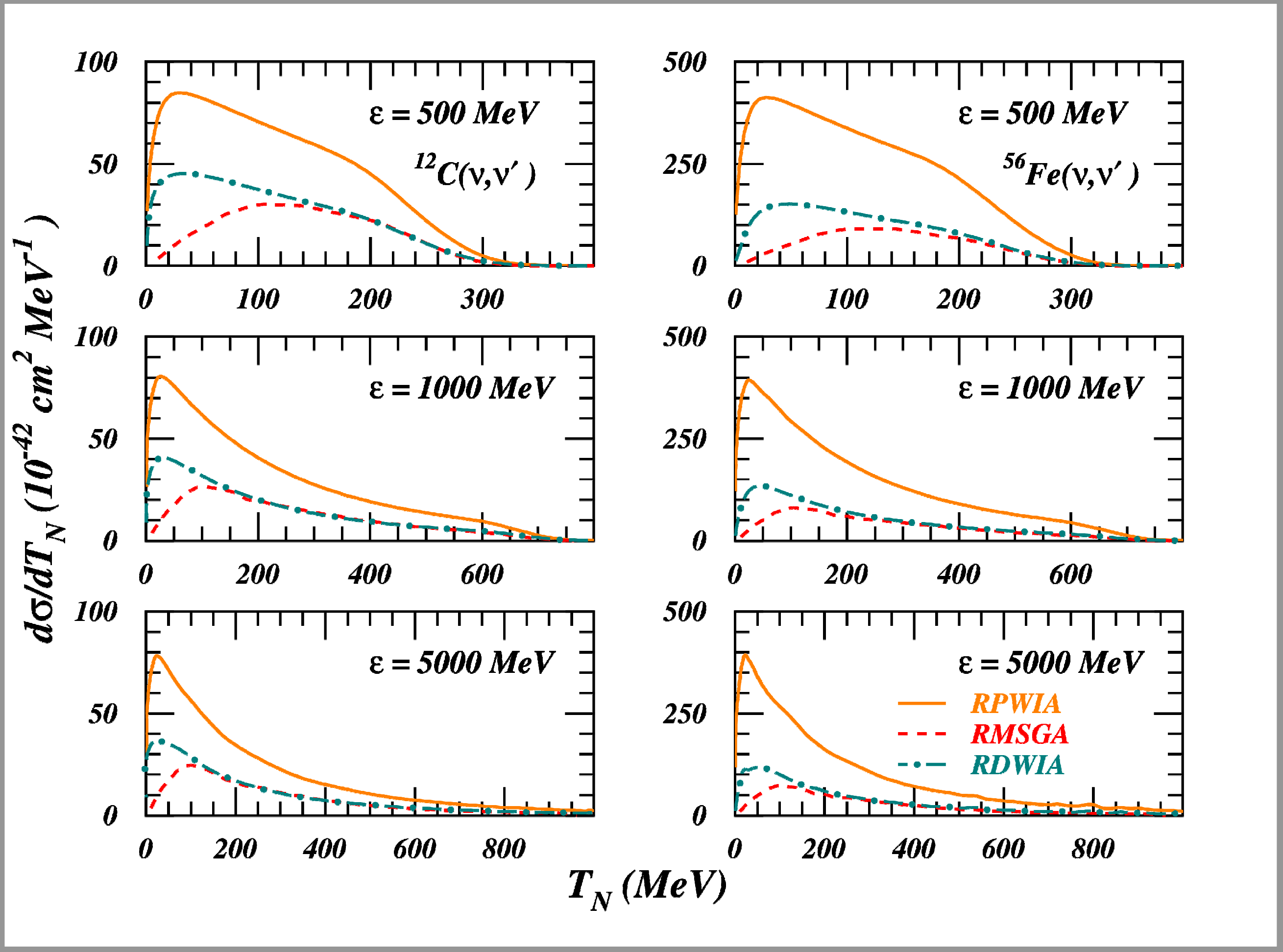
Influence of RPA calculations on cross section folded with MiniBoone spectrum :



Charged current results



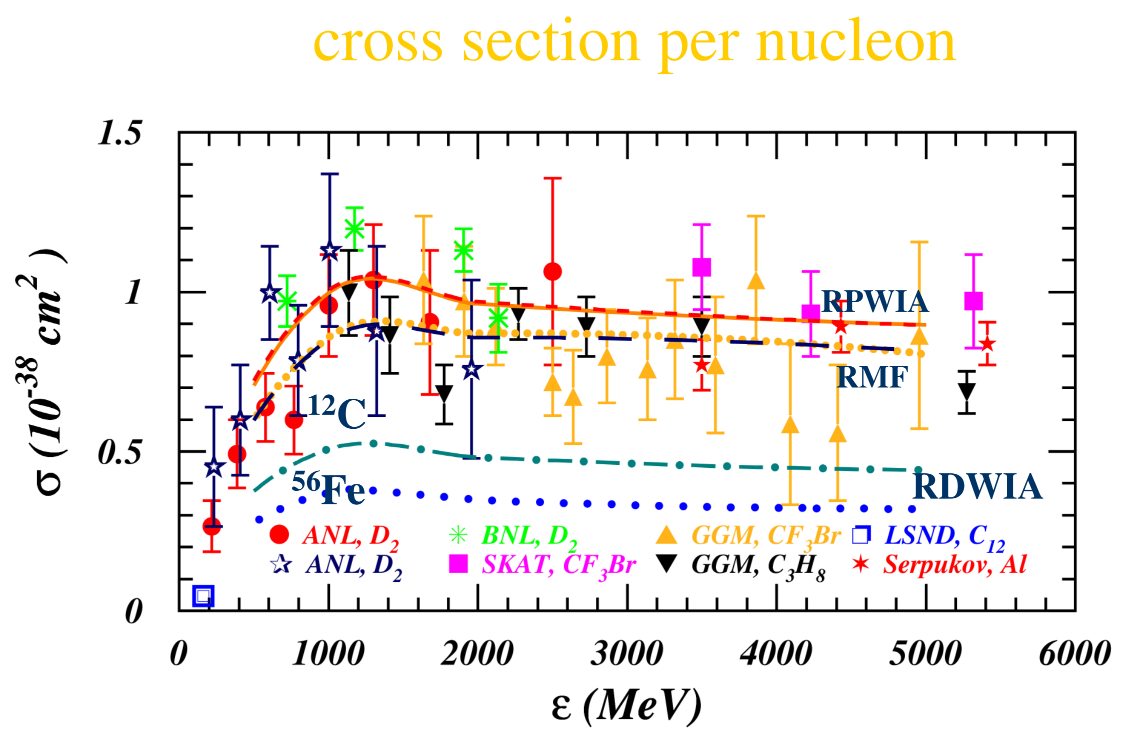
Neutral current results



Comparison with data

- The results are in good agreement with those obtained with other models e.g. M.C. Martínez et al., PRC73, 024607 (2006) ; A. Meucci et al., NPA773, 250 (2006) ; J.E. Amaro et al., PRL98, 242501 (2007).

- cross sections scale with target mass
- cross sections saturate at high incoming neutrino energies



cross section per nucleon

→ approximately half of the measured strength can be attributed to single-step nucleon knockout

Transparencies

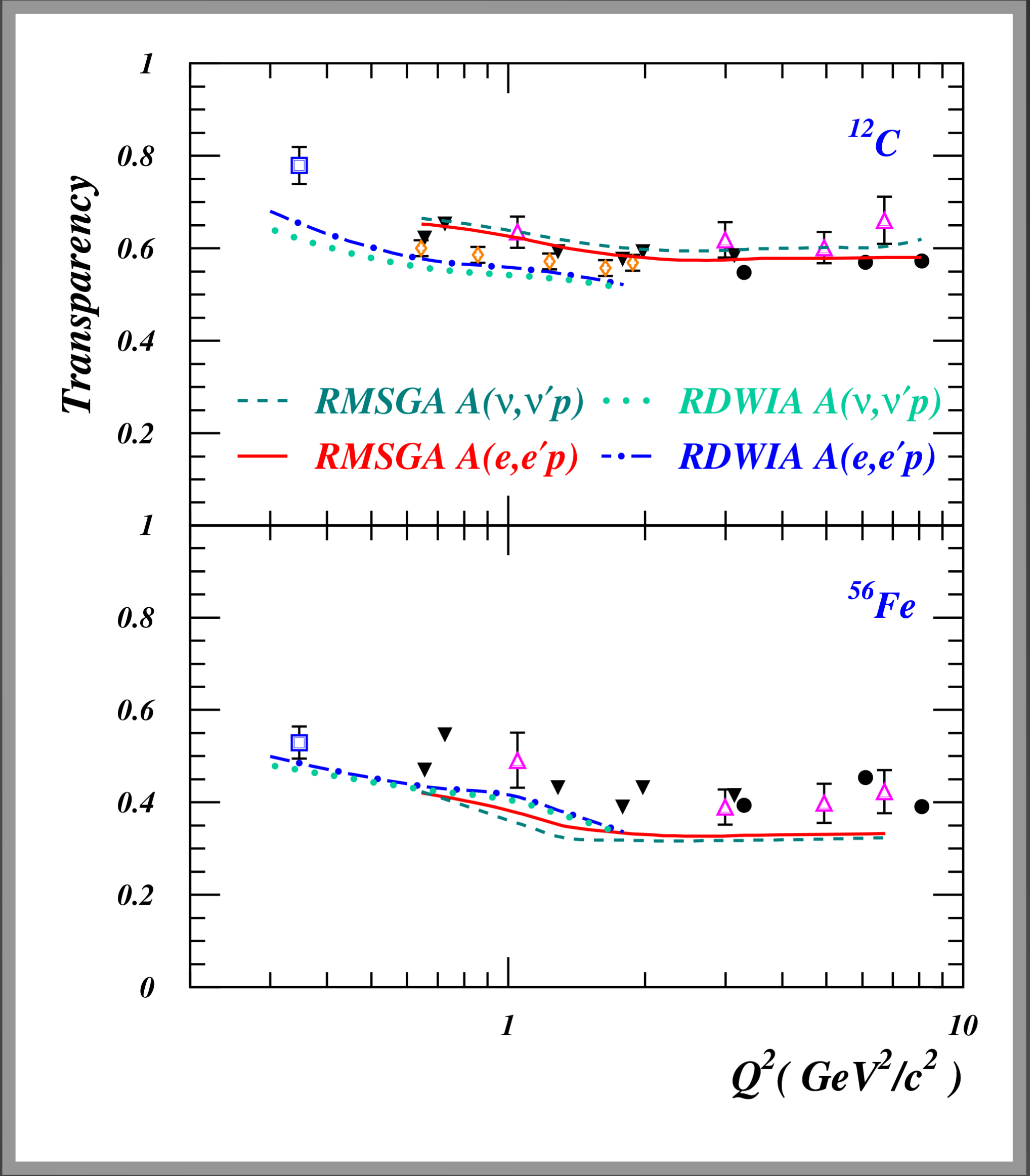
→ measure for the probability that a nucleon can escape from the nucleus without further interactions

$$T_{exp}(Q^2) = \frac{\int_V d^3p_m dE_m Y_{exp}(E_m, \vec{p}_m, \vec{k}_f)}{c(A)\int_V d^3p_m dE_m Y_{PWIA}(E_m, \vec{p}_m)}$$

- data from electron scattering
- neutrino transparencies ?

Transparencies

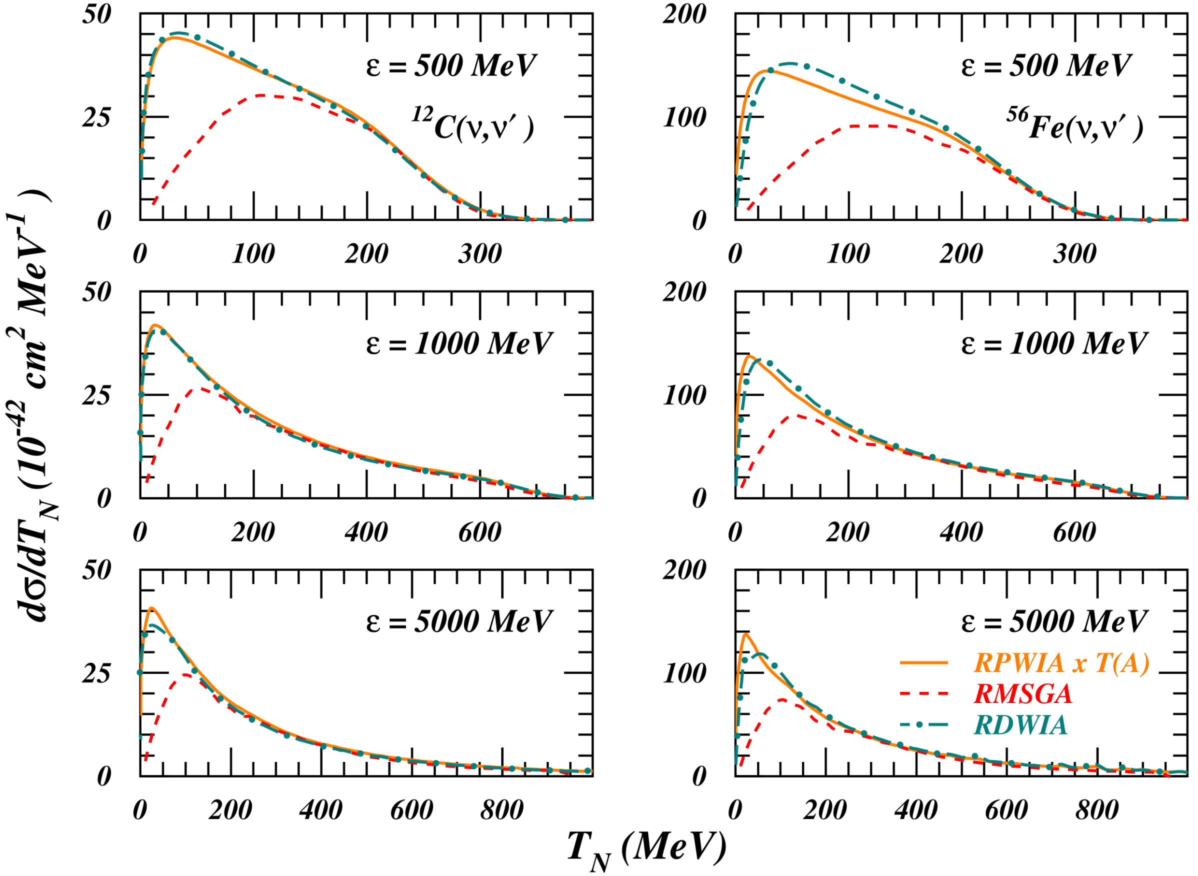
- (e, e’ p) ⟷ (ν, ν’ p)
- quasi-elastic kinematics
- comparison with data



Transparencies

neutral current

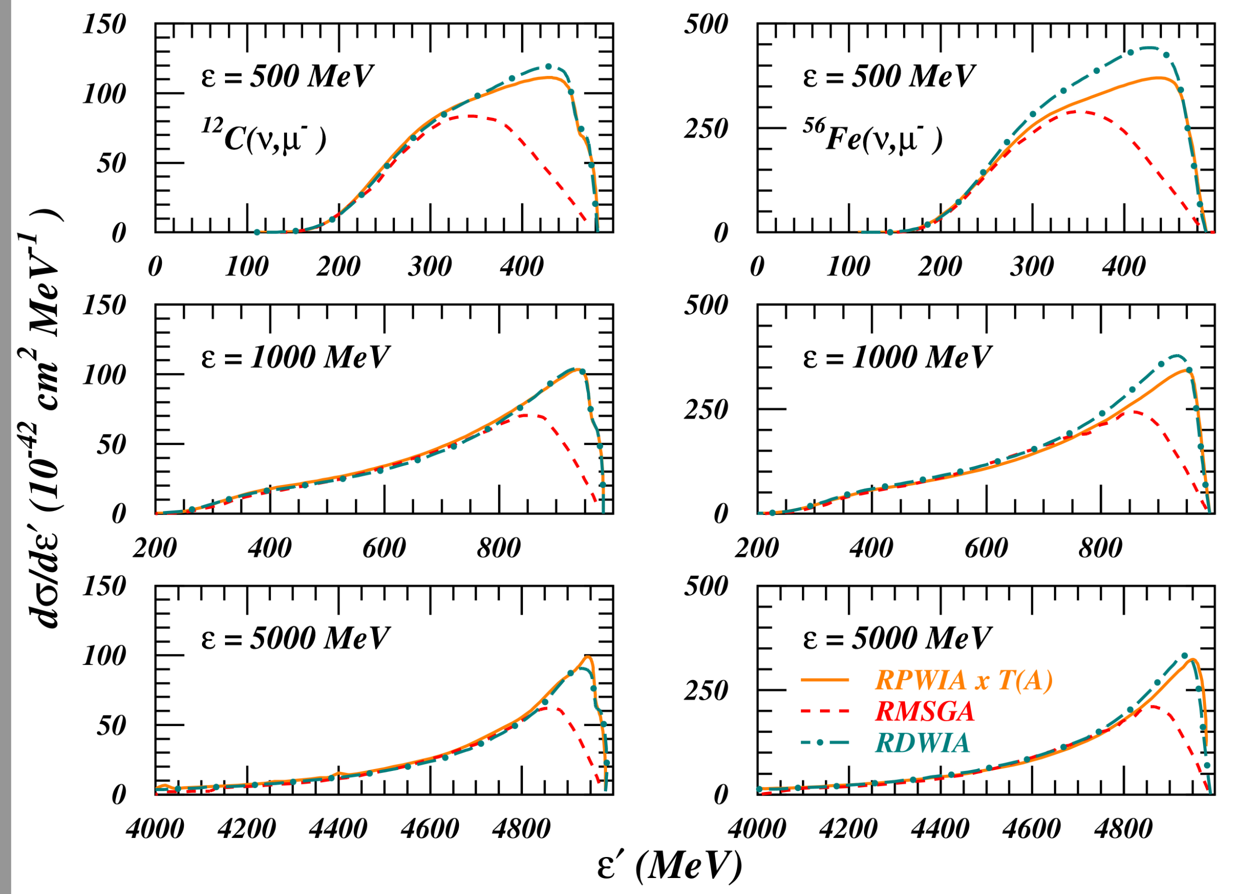
- (ν, ν’ p)
- inclusive cross sections
- RPWIA x T(A) vs RMSGA
- RPWIA x T(A) vs RMSGA



Transparencies

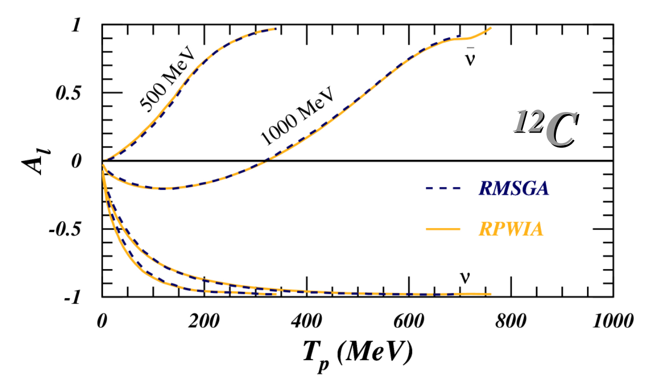
charged current

- (ν, ν' p)
- inclusive cross sections
- RPWIA x T(A) vs RMSGA
- RPWIA x T(A) vs RMSGA

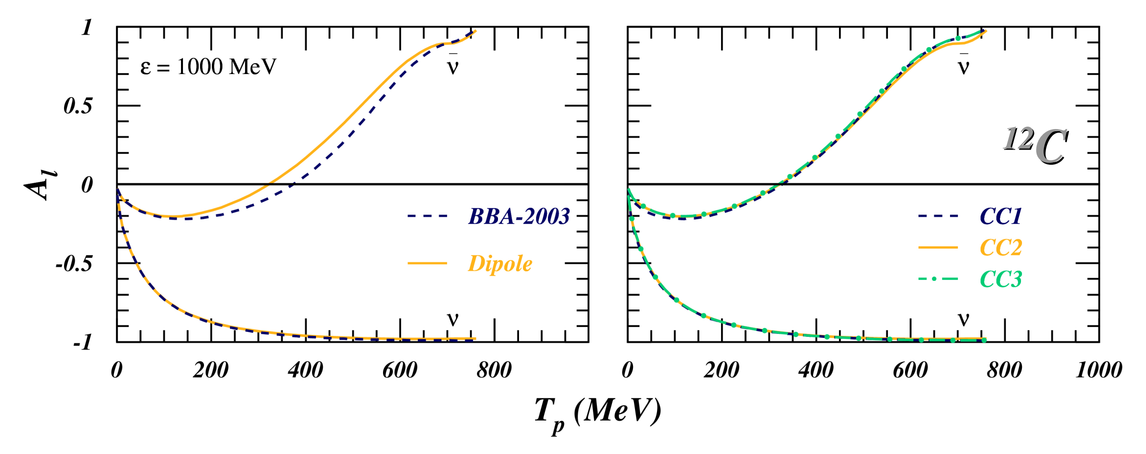


Cross section ratios

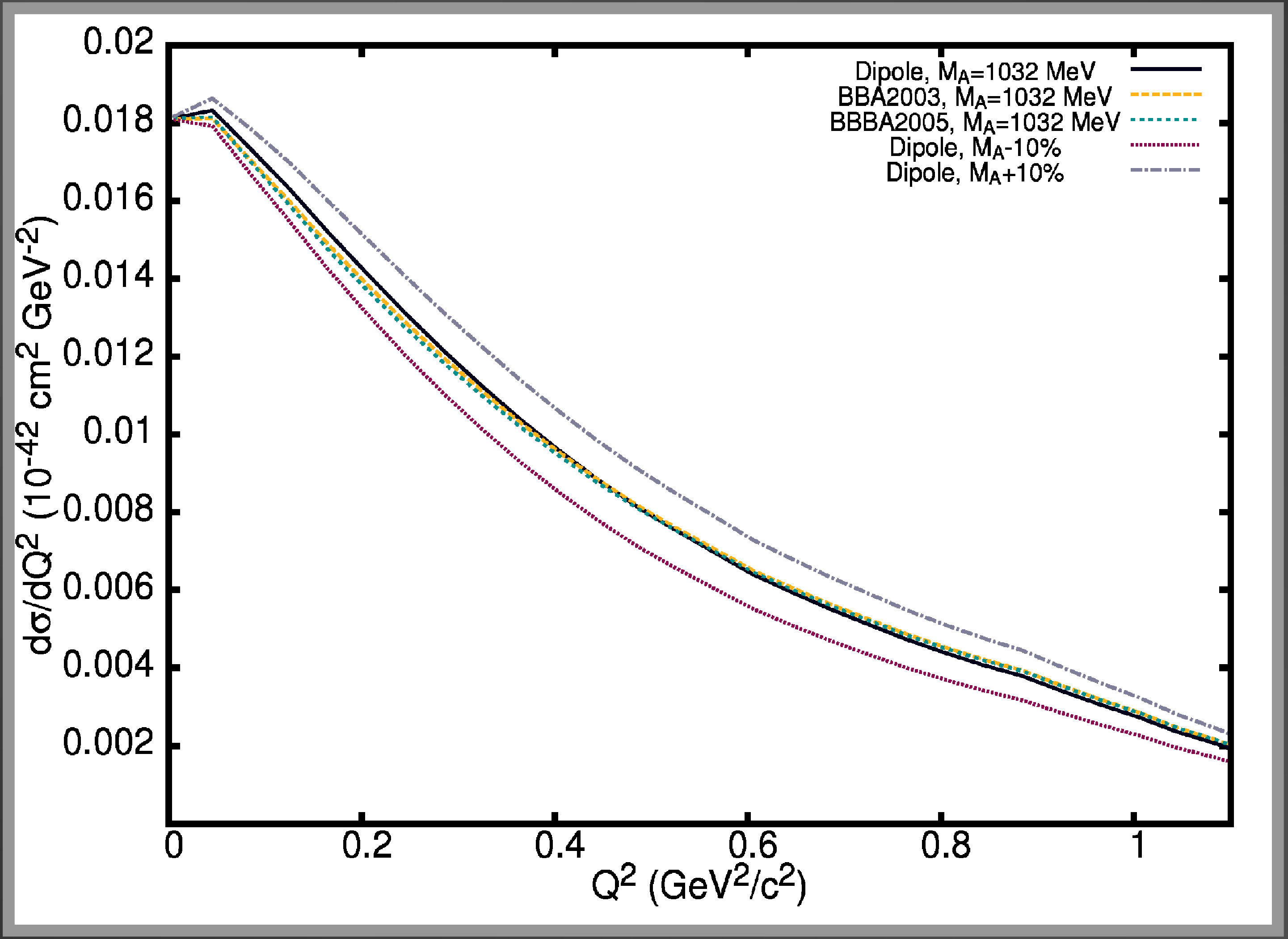
- ratios stable against the influence of final state interactions, uncertainties in the description of the Q^2 dependence of the vector form factors, off-shell ambiguities.



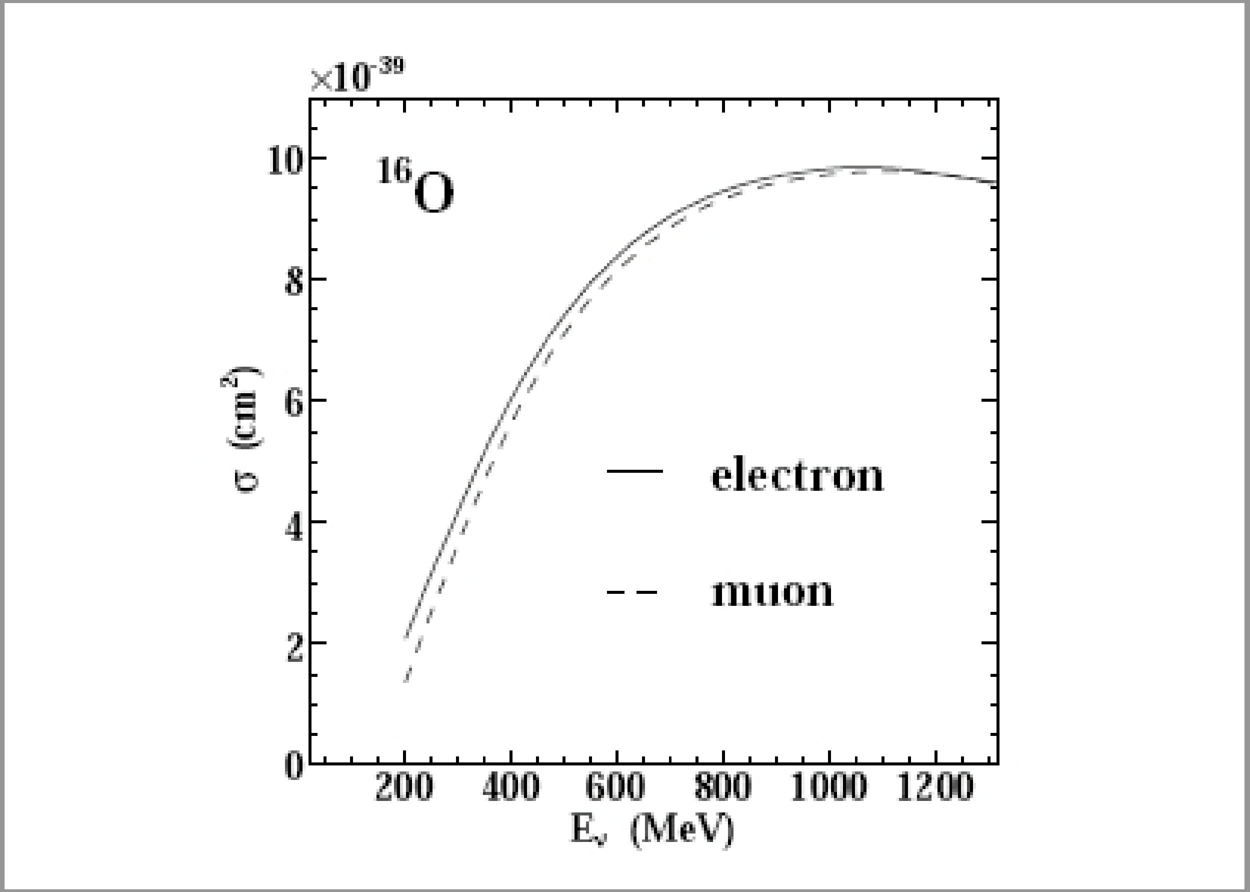
Ratios are quite robust observables !



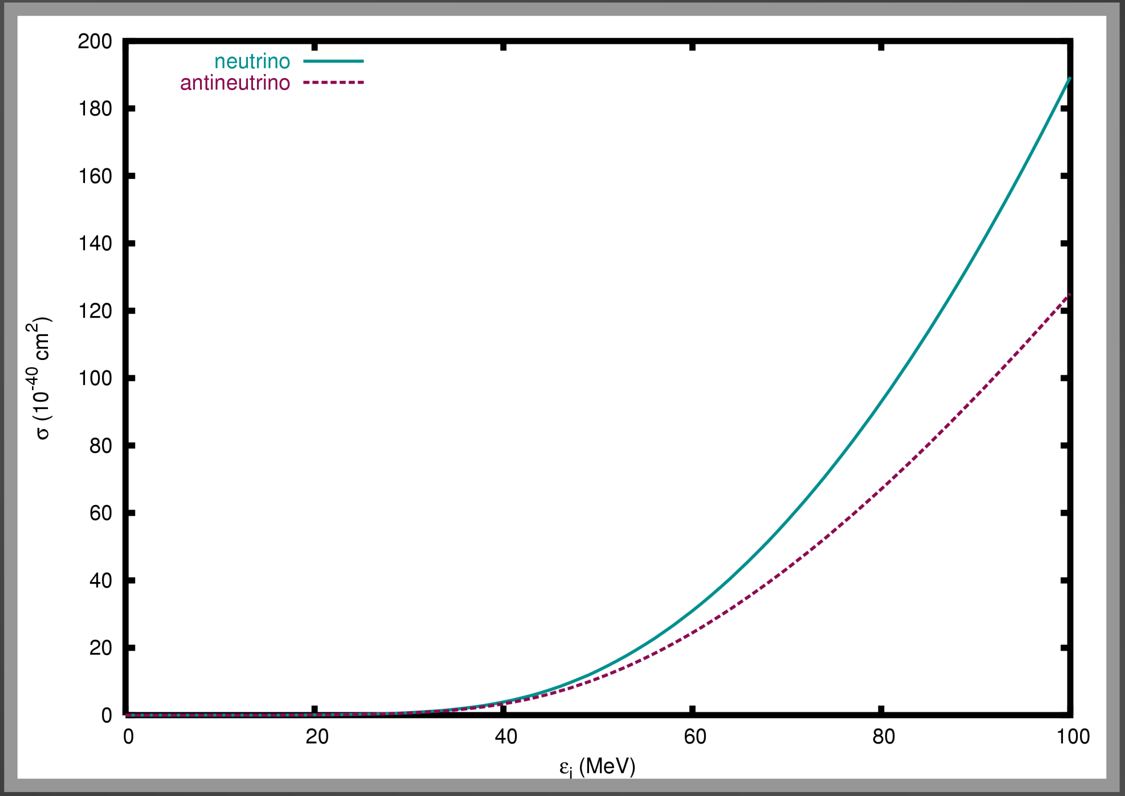
M_Δ ……


0.02
0.018
0.016
0.014
0.012
0.01
0.008
0.006
0.004
0.002
$d\sigma/dQ^2$ (10^{-42} cm^2 GeV^{-2})
0
0.2
0.4
0.6
0.8
1
Q^2 (GeV^2/c^2)
Dipole, M_A=1032 MeV
BBA2003, M_A=1032 MeV
BBBA2005, M_A=1032 MeV
Dipole, M_A-10%
Dipole, M_A+10%

Electron vs muon neutrino cross sections



Neutrino vs antineutrino cross sections



With the transition matrix element :

$$|\langle f|\widehat{H}_W|i\rangle|^2 = v_l R_l + v_{TL} R_{TL} + v_T R_T + V_{TT} R_{TT} + h(v_{TL'} R_{TL'} + v_{T'} R_{T'})$$

h= -1 for neutrinos, ***h***= +1 for antineutrinos,

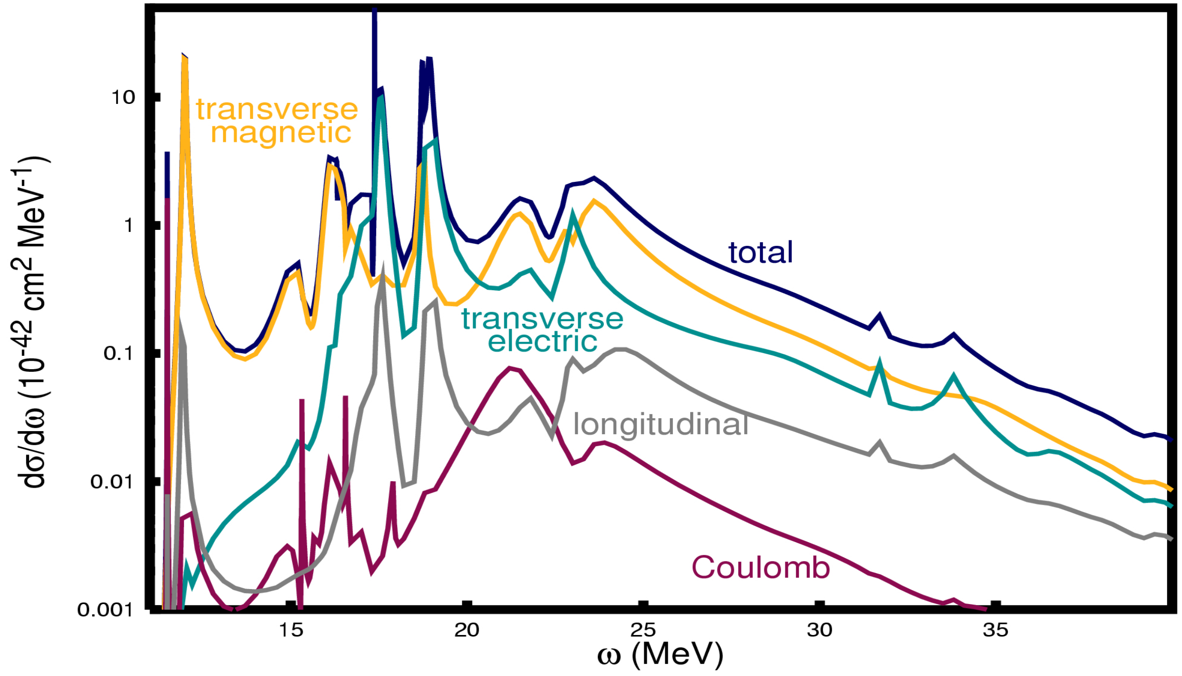
and the hadronic response :

$$\begin{aligned} R_L &= |h_0 - \frac{\omega}{\kappa} h_z|^2 \\ R_{TL} &= 2\Re[h_o(h_+{}^* - h_-{}^*)] - \frac{\omega}{\kappa} 2\Re[h_z(h_+{}^* - h_-{}^*)] \\ R_T &= h_+ h_+{}^* + h_- h_-{}^* \\ R_{TT} &= 2\Re(h_+ h_-{}^*) \\ R_{TL'} &= 2\Re[h_o(h_+{}^* + h_-{}^*)] - \frac{\omega}{\kappa} 2\Re[h_z(h_+{}^* + h_-{}^*)] \\ R_{T'} &= h_+ h_+{}^* - h_- h_-{}^* \end{aligned}$$

With the transition matrix element :

$$|\langle f|\widehat{H}_W|i\rangle|^2 = v_l R_l + v_{TL} R_{TL} + v_T R_T + V_{TT} R_{TT} + h(v_{TL'} R_{TL'} + v_{T'} R_{T'})$$

and the hadro



Continuum Random Phase Approximation (CRPA)

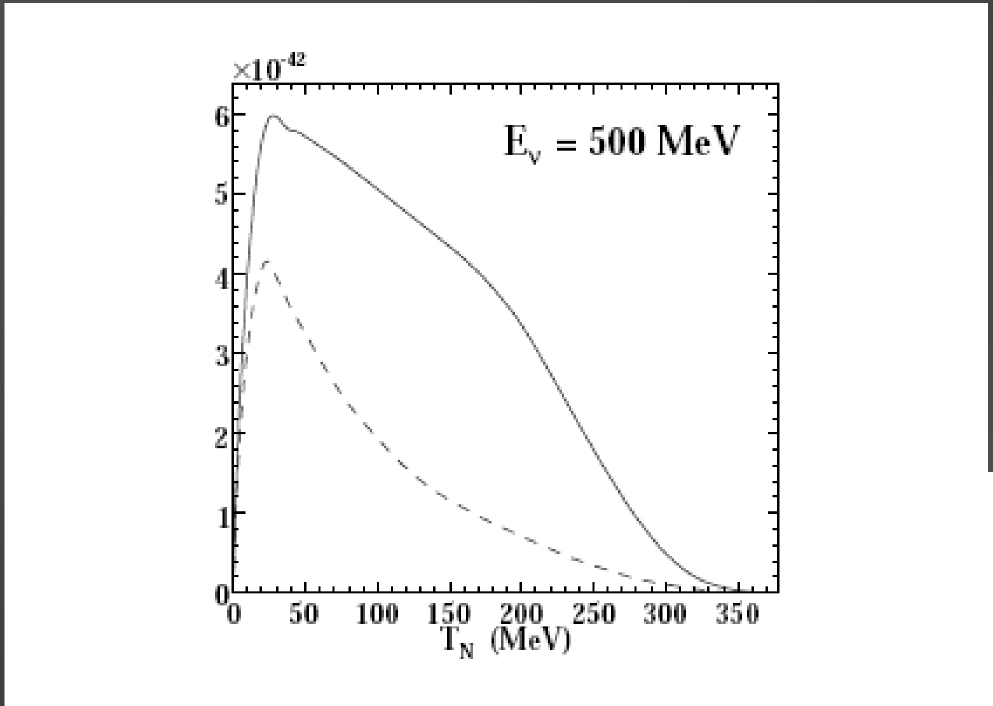
$^{16}O(\nu_{50\,MeV}, \nu')\,^{16}O^{*}$

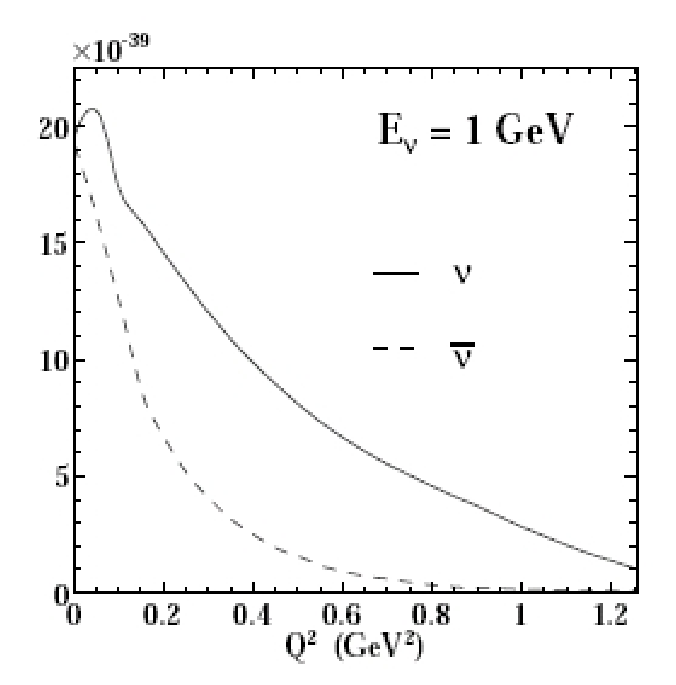
Neutrino and antineutrino interactions with nuclei

$$\begin{aligned} & l_- l_-{}^* h_+ h_+{}^* + l_+ l_+{}^* h_- h_-{}^* \\ & \quad = v_T (h_+ h_+{}^* + h_- h_-{}^*) \\ & \qquad\qquad + h\, v_{T'} (h_+ h_+{}^* - h_- h_-{}^*) \\ & \quad = (v_T + h v_{T'}) h_+ h_+{}^* + (v_T - h v_{T'}) h_- h_-{}^* \end{aligned}$$

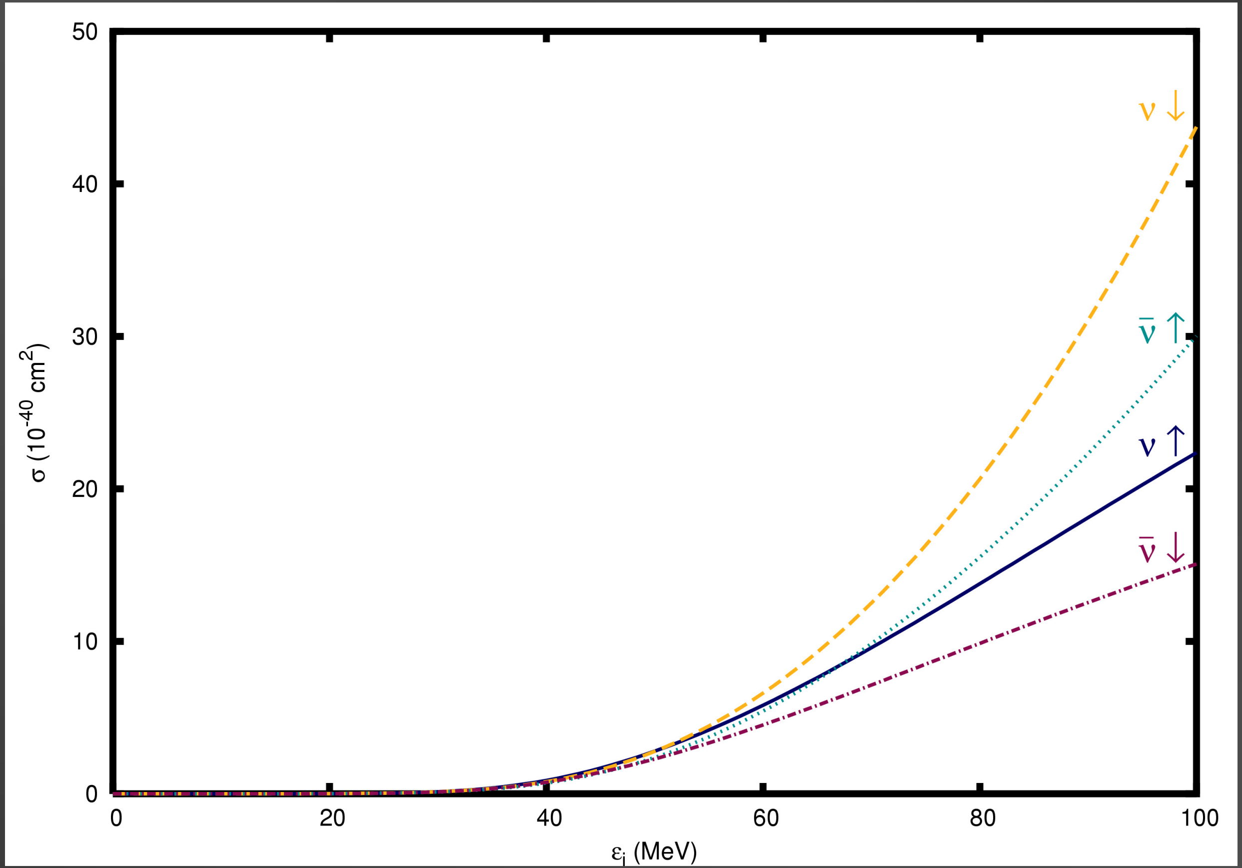
$$\begin{aligned} v_T &= \frac{2\sin^2\frac{\theta}{2}(\epsilon_i^2 + \epsilon_f^2 + 2\sin^2\frac{\theta}{2}\epsilon_i\epsilon_f)}{(2\pi)^6(\epsilon_f^2 + \epsilon_i^2 - 2\epsilon_i\epsilon_f\cos\theta)}, \\ v_{T'} &= \frac{2\sin^2\frac{\theta}{2}(\epsilon_i + \epsilon_f)}{(2\pi)^6\sqrt{\epsilon_f^2 + \epsilon_i^2 - 2\epsilon_i\epsilon_f\cos\theta}}. \end{aligned}$$

	$h_+ h_+{}^*$	$h_- h_-{}^*$
G_E^2	$\frac{-k_+ k_-}{m_N^2}\delta_{r,m_s}\delta_{r,m_s'}$	$\frac{-k_+ k_-}{m_N^2}\delta_{r,m_s}\delta_{r,m_s'}$
$G_E G_A$	$\frac{-k_+\sqrt{2}}{m_N}\delta_{r,\frac{1}{2}}\delta_{m_s,\frac{1}{2}}\delta_{m_s',-\frac{1}{2}}$	$\frac{-k_+\sqrt{2}}{m_N}\delta_{r,-\frac{1}{2}}\delta_{m_s,\frac{1}{2}}\delta_{m_s',-\frac{1}{2}}$
	$\frac{+k_-\sqrt{2}}{m_N}\delta_{r,\frac{1}{2}}\delta_{m_s',\frac{1}{2}}\delta_{m_s,-\frac{1}{2}}$	$\frac{+k_-\sqrt{2}}{m_N}\delta_{r,-\frac{1}{2}}\delta_{m_s',\frac{1}{2}}\delta_{m_s,-\frac{1}{2}}$
$G_E G_M \frac{\kappa}{2m_N}$	$\frac{-k_+\sqrt{2}}{m_N}\delta_{r,\frac{1}{2}}\delta_{m_s,\frac{1}{2}}\delta_{m_s',-\frac{1}{2}}$	$\frac{+k_+\sqrt{2}}{m_N}\delta_{r,-\frac{1}{2}}\delta_{m_s,\frac{1}{2}}\delta_{m_s',-\frac{1}{2}}$
	$\frac{+k_-\sqrt{2}}{m_N}\delta_{r,\frac{1}{2}}\delta_{m_s',\frac{1}{2}}\delta_{m_s,-\frac{1}{2}}$	$\frac{-k_-\sqrt{2}}{m_N}\delta_{r,-\frac{1}{2}}\delta_{m_s',\frac{1}{2}}\delta_{m_s,-\frac{1}{2}}$
G_A^2	$2\delta_{r,\frac{1}{2}}\delta_{m_s,-\frac{1}{2}}\delta_{m_s',-\frac{1}{2}}$	$2\delta_{r,-\frac{1}{2}}\delta_{m_s,\frac{1}{2}}\delta_{m_s',\frac{1}{2}}$
$G_M^2 \frac{\kappa^2}{4m_N^2}$	$2\delta_{r,\frac{1}{2}}\delta_{m_s,-\frac{1}{2}}\delta_{m_s',-\frac{1}{2}}$	$2\delta_{r,-\frac{1}{2}}\delta_{m_s,\frac{1}{2}}\delta_{m_s',\frac{1}{2}}$
$G_A G_M \frac{\kappa}{2m_N}$	$2\delta_{r,\frac{1}{2}}\delta_{m_s,-\frac{1}{2}}\delta_{m_s',-\frac{1}{2}}$	$-2\delta_{r,-\frac{1}{2}}\delta_{m_s,\frac{1}{2}}\delta_{m_s',\frac{1}{2}}$

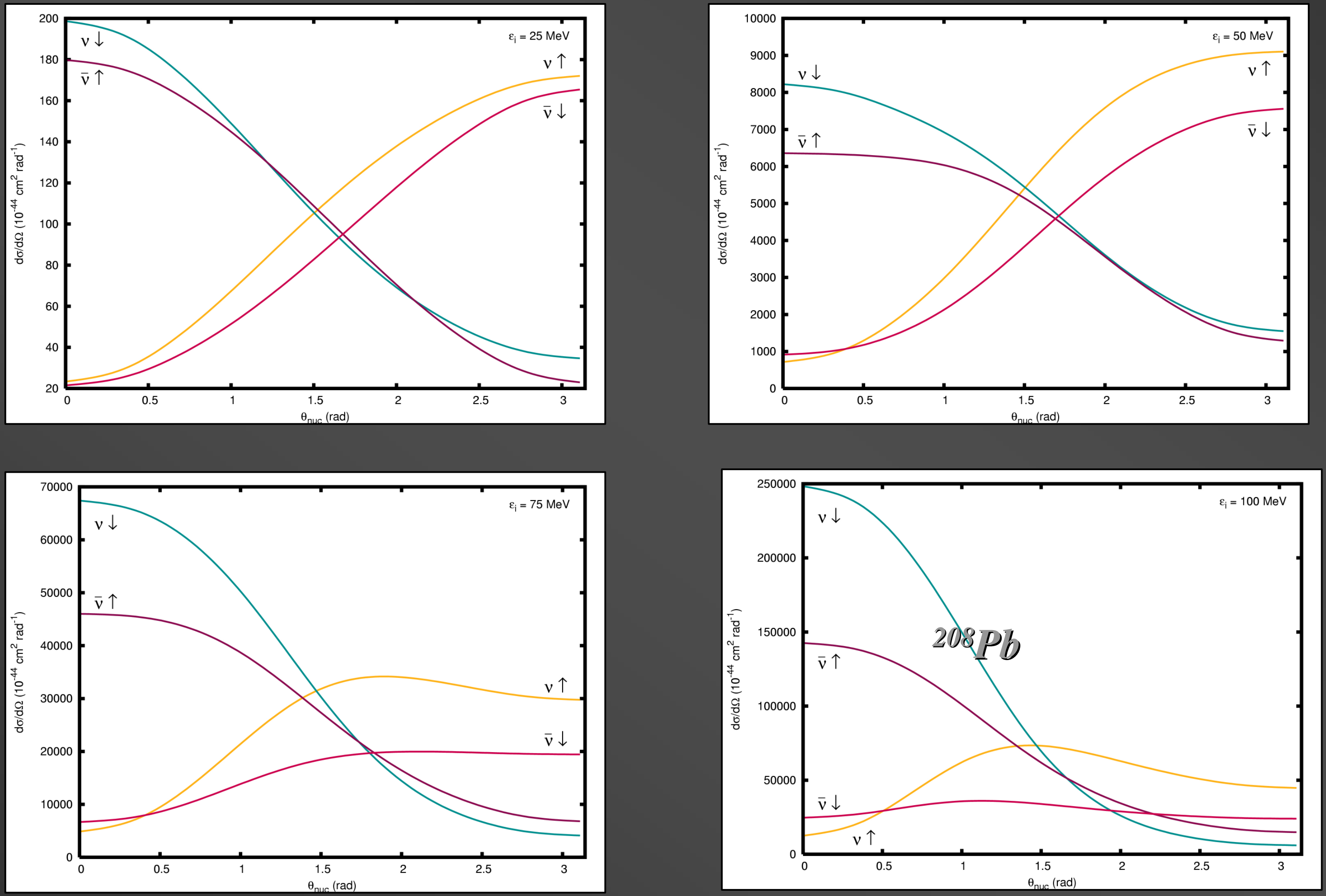

×10^-42
6
5
4
3
2
1
0
0 50 100 150 200 250 300 350
T_N (MeV)
E_ν = 500 MeV


×10^-39
20
15
10
5
0
0 0.2 0.4 0.6 0.8 1 1.2
Q² (GeV²)
E_ν = 1 GeV
ν
ν̄

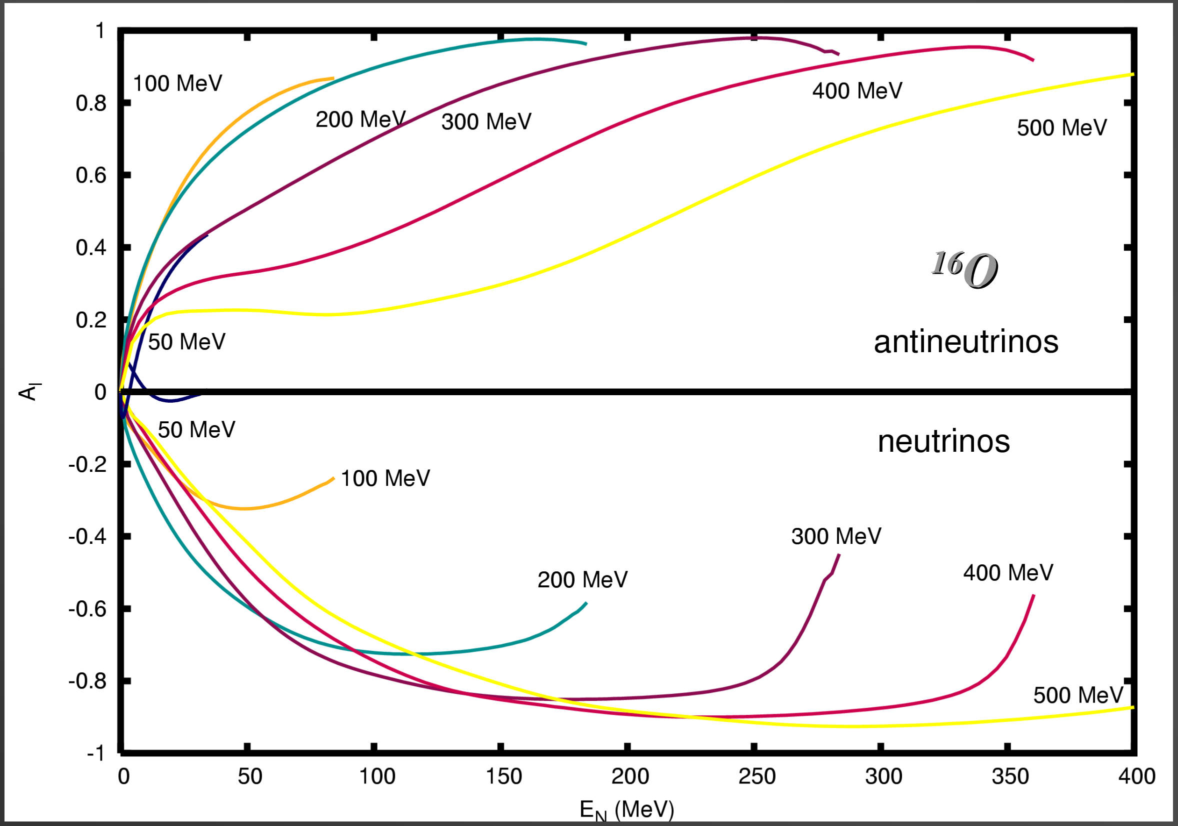
Neutrino vs antineutrino cross sections



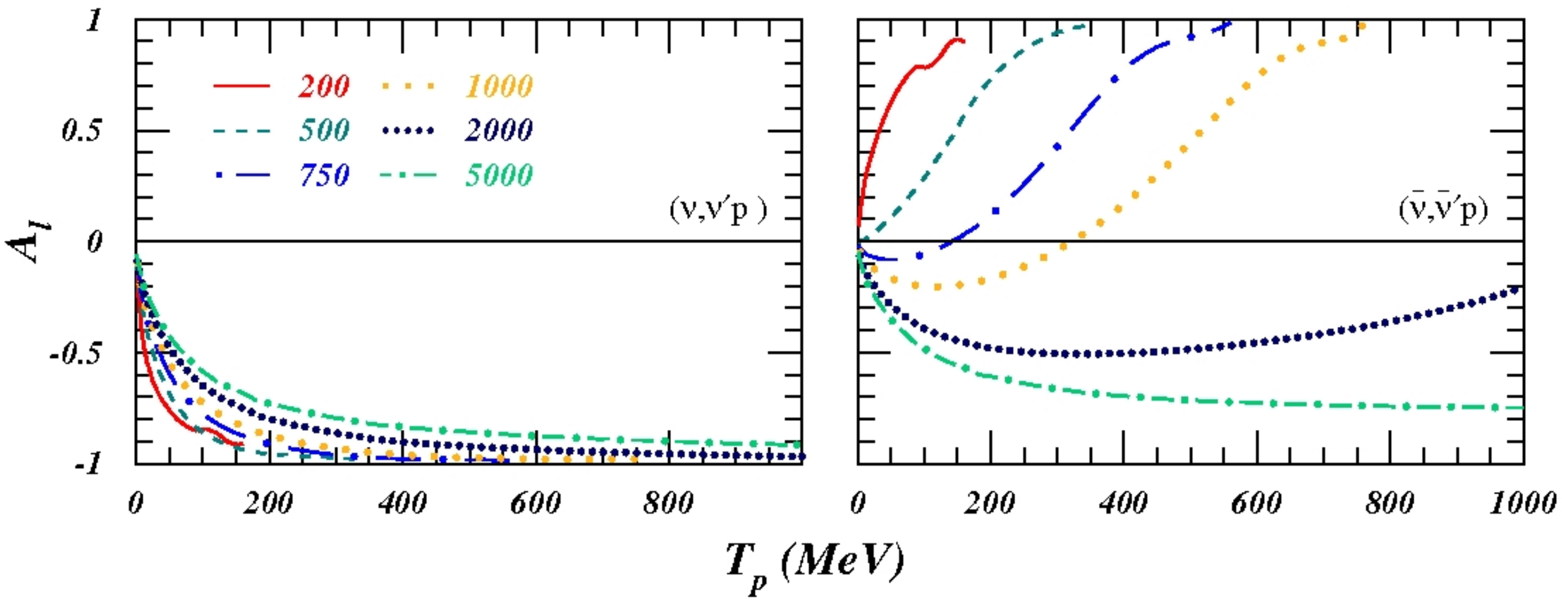
Clear differences in differential cross sections



Longitudinal polarization asymmetry : $$A_l = \frac{\sigma(s_N^l = \uparrow) - \sigma(s_N^l = \downarrow)}{\sigma(s_N^l = \uparrow) + \sigma(s_N^l = \downarrow)}$$

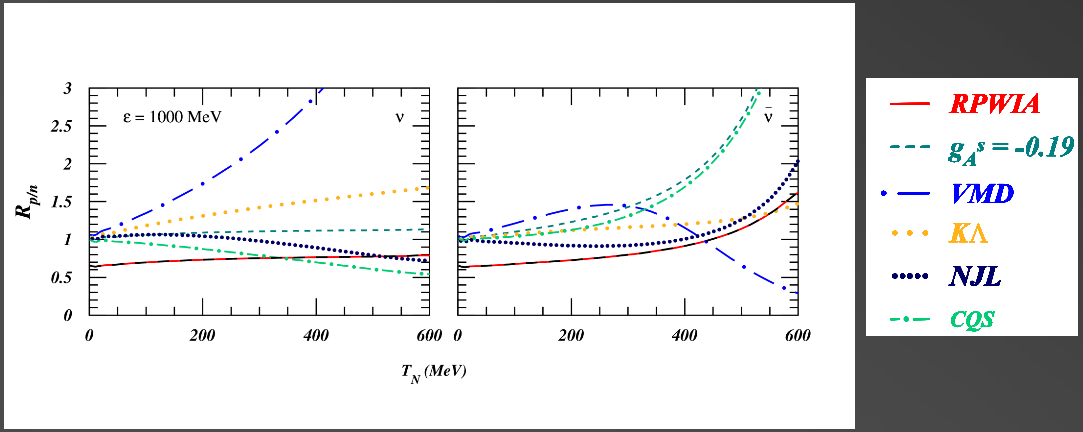


- For antineutrinos, A_l is large and positive
- For neutrinos, A_l is large and negative

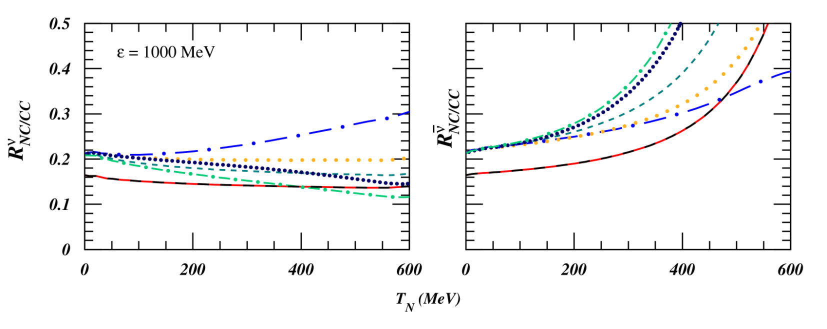

1
0.5
0
-0.5
-1
A_l
200
500
750
1000
2000
5000
(ν,ν′p)
($\bar{\nu}$,$\bar{\nu}$′p)
0
200
400
600
800
1000
T_p (MeV)

Influence strangeness content of the nucleon on cross section ratios for neutral current scattering :

- proton-to-neutron knockout ratio



- neutral-to-charged current cross section ratio

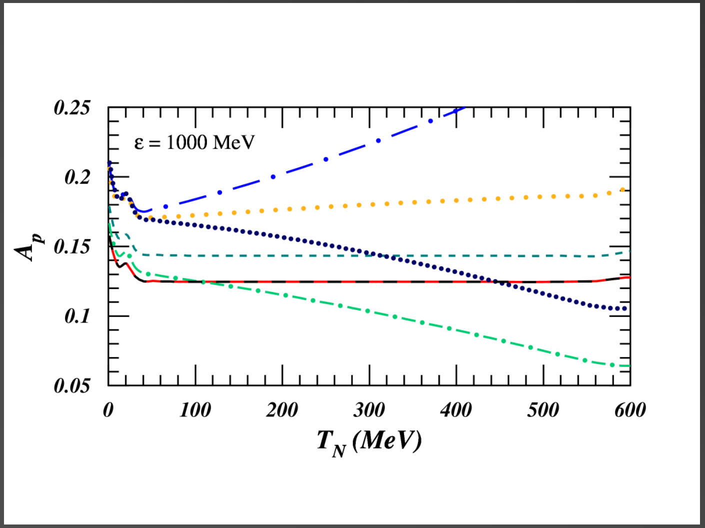


Neutrino and antineutrino interactions with nuclei

- **Paschos-Wolfenstein relation**

$$R^p_{PW} = \frac{\frac{d\sigma}{dT_N}(\nu p \rightarrow \nu p) - \frac{d\sigma}{dT_N}(\overline{\nu} p \rightarrow \overline{\nu} p)}{\frac{d\sigma}{dT_N}(\nu n \rightarrow \mu^- p) - \frac{d\sigma}{dT_N}(\overline{\nu} p \rightarrow \mu^+ n)}$$

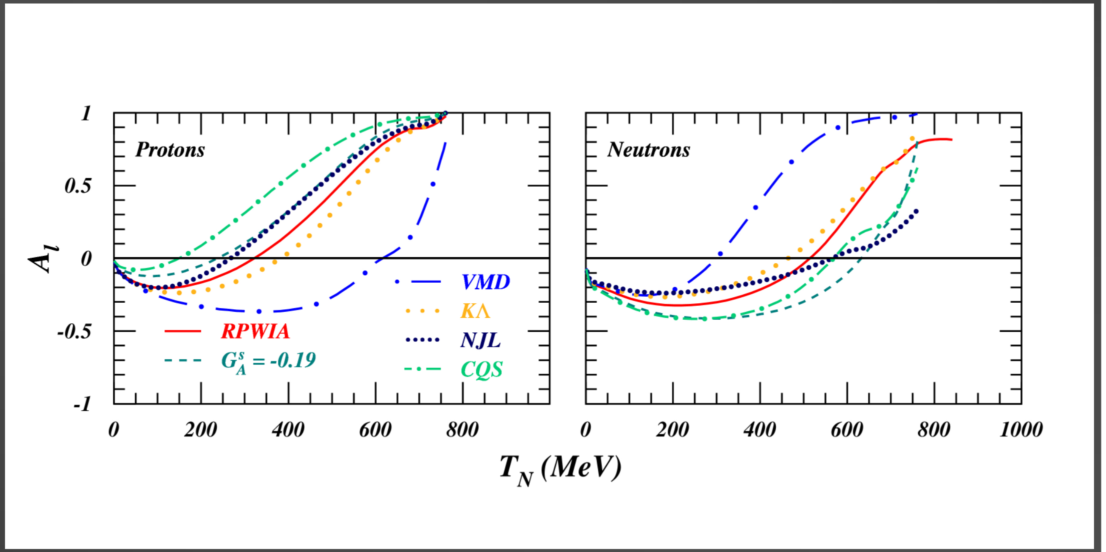
$$R^n_{PW} = \frac{\frac{d\sigma}{dT_N}(\nu n \rightarrow \nu n) - \frac{d\sigma}{dT_N}(\overline{\nu} n \rightarrow \overline{\nu} n)}{\frac{d\sigma}{dT_N}(\nu n \rightarrow \mu^- p) - \frac{d\sigma}{dT_N}(\overline{\nu} p \rightarrow \mu^+ n)}$$



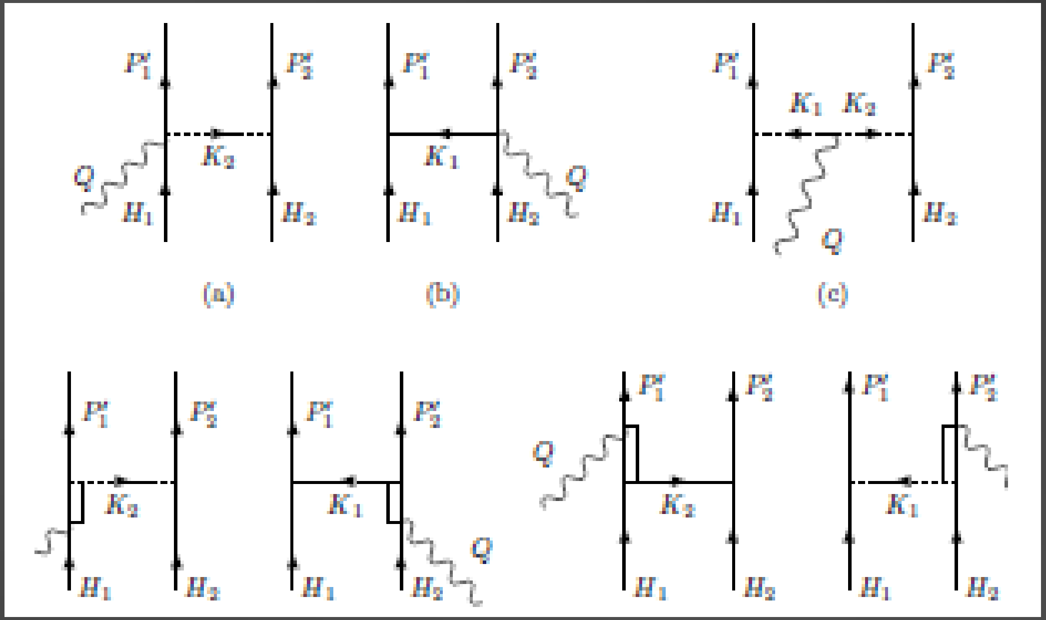
- **longitudinal helicity asymmetry**

$$A^\nu_h = \frac{\frac{d\sigma}{dT_N}(\nu p \rightarrow \nu p,\, h_p = +1) - \frac{d\sigma}{dT_N}(\nu p \rightarrow \nu p,\, h_p = -1)}{\frac{d\sigma}{dT_N}(\nu p \rightarrow \nu p,\, h_p = +1) + \frac{d\sigma}{dT_N}(\nu p \rightarrow \nu p,\, h_p = -1)}$$

$$A^{\overline{\nu}}_h = \frac{\frac{d\sigma}{dT_N}(\overline{\nu} p \rightarrow \overline{\nu} p,\, h_p = +1) - \frac{d\sigma}{dT_N}(\overline{\nu} p \rightarrow \overline{\nu} p,\, h_p = -1)}{\frac{d\sigma}{dT_N}(\overline{\nu} p \rightarrow \overline{\nu} p,\, h_p = +1) + \frac{d\sigma}{dT_N}(\overline{\nu} p \rightarrow \overline{\nu} p,\, h_p = -1)}$$

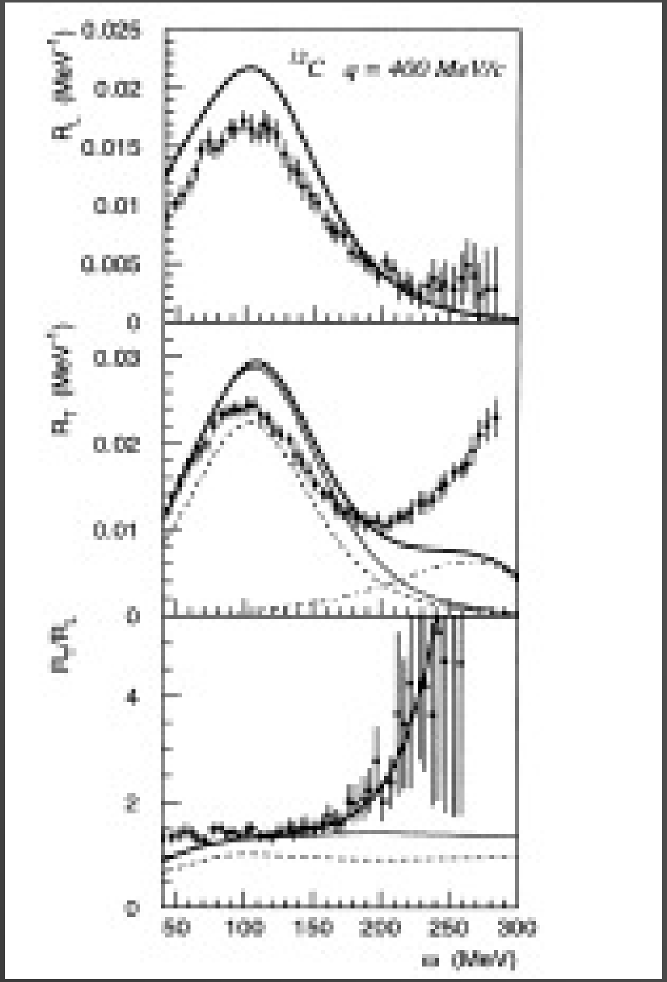


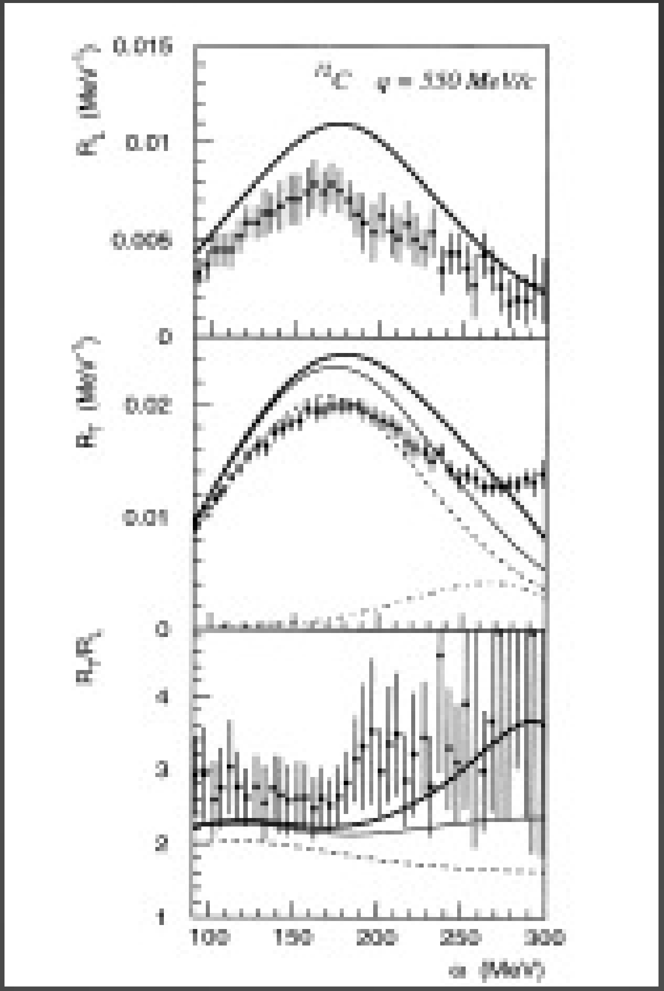
Meson-exchange contributions and 2-body currents



Meson-exchange contributions and 2-body currents

(e,e')
differences
of 20-30%
in
transverse
channel in
QE region





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Concluding … ?!

References : P.Lava, N.Jachowicz, M.C. Martínez, J. Ryckebusch, 'Nucleon helicity asymmetries in quasielastic neutrino-nucleus scattering', PRC73, 064605 (2006) ; M.C. Martínez, P.Lava, N. Jachowicz, J.Ryckebusch, J.M. Udías, 'Relativistic models for quasielastic neutrino scattering', PRC73, 024607 (2006) ; C.Praet, N.Jachowicz, J.Ryckebusch 'Paschos-Wolfenstein relation in a hadronic picture', PRC74, 065501 (2006) ; N. Jachowicz, P. Vancraeyveld, P. Lava, C. Praet, J. Ryckebusch, 'Strangeness content of the nucleon in quasielastic neutrino-nucleus reactions', PRC76, 055501 (2008),…