Transverse Beam Echoes In IOTA

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Abstract

Beam echo provides a sensitive probe for measuring the rate of diffusion in transverse phase space. We investigate beam echoes in the IOTA (Integrable Optics Test Accelerator) at Fermi National Accelerator Laboratory. Simulations are performed using the codes `MADX` and `pyORBIT` using typical IOTA machine parameters. In the absence of space charge, theory and experiment are found to be in good agreement. As expected, echo amplitude is found to be dependent on dipole and quadrupole kick strengths; however, in 4-dimensional transverse phase space the presence of coupling between the horizontal and vertical planes has a significant effect on the observed echo amplitude. Finally, we show that multiple echoes may be observed provided the dipole kick is sufficiently large.

1 Introduction

The Integrable Optics Test Accelerator (IOTA) is a 40 meter storage ring located at Fermilab and dedicated to advanced experimental beam physics. Its small size and layout make it versatile and scientists can quickly modify or reconfigure it as they see fit. While either electrons or protons maybe stored, beam echoes is a phenomena specific to protons; therefore we shall be referring exclusively to proton beams in what follows.

Due to unavoidable magnetic guiding field imperfections and individual particles finite momentum spread, in addition to bending magnets accelerators must resort to focusing elements in order to confine particles within the vacuum chamber. In a conventional machine, the focusing elements are quadrupoles and the focusing is linear i.e. the focusing force is proportional to the distance from the reference trajectory. The optical design is based on idealized linear single particle motion i.e. the particle beam is considered as an ensemble of independent non-interacting particles. In reality, interactions between particles and nonlinearities in the external focusing elements are always present and are treated as perturbations. The effect of such perturbations can be significant.

Space charge, beam-beam interactions, nonlinearities etc . . . are perturbations that may be complex to analyze in details. Collectively, their overall effect manifests itself as diffusion in phase space. Diffusion causes the beam phase space volume (emittance) to increase and may eventually lead to particle loss which in turn cause activation of the elements. For safety reasons, such activation must be minimized. In this context, it is useful and important to understand and measure phase space diffusion.

2 Beam Echoes

Phase space diffusion is typically a relatively slow phenomenon. As a result, the methods available for a direct measurement tend to be cumbersome and time-consuming. Because echo amplitude is known to be strongly dependent on phase space diffusion rates, beam echoes may provide the foundation for a quick and accurate technique to measure diffusion rates.

To produce an echo, one starts by applying a dipole kick. The kick triggers an oscillation of the centroid which subsequently decays as the beam decoheres. The decoherence results from individual particles oscillating at slightly different betatron frequencies (due for example to finite momentum spread) causing phases to gradually become evenly distributed. Shortly after the beam has decohered, a quadrupole kick is applied. While the quadrupole kick does not affect the beam centroid, interestingly, the particles oscillate back in phase and experience temporary re-coherence. The latter manifests itself as a blip in the centroid signal, as shown in figure 1. Beam Position Monitors (BPMs) in the accelerator can measure the centroid position with a typical resolution of one-hundred microns; this sets the scale for the minimum amplitude of usable echo signal.

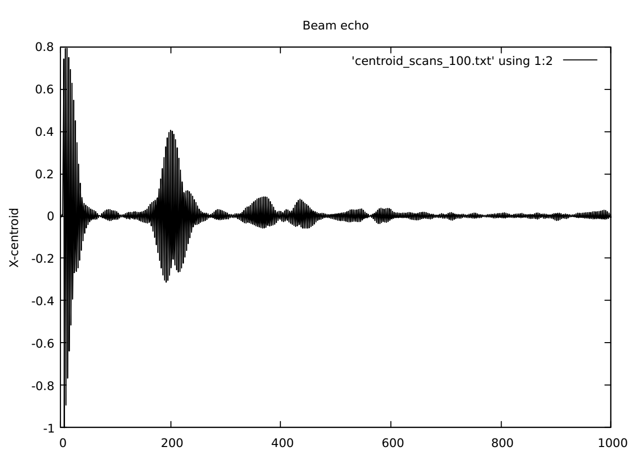


Figure 1: Horizontal centroid signal for 1000 turns. The dipole kick is applied at turn 0. Beam echo occurs near turn 200.

3 Space Charge

A beam is made of individual particles. Each of these particles produce electromagnetic fields felt by all others. For a given particle inside the beam, it can be shown that the net force F due to all others is $F = \frac{e^2 N}{2\pi\varepsilon_0 a^2 \gamma^2} r$ while for a particle outside the beam the force is $F = \frac{e^2 N}{2\pi\varepsilon_0 a^2 \gamma^2}$ [3]. We refer to the effects arising from self electromagnetic forces as space charge effects. Though small at relativistic velocities, the perturbations due to space charge lead to emittance growth and particle loss.

4 Theory

To predict and analyze beam echo amplitudes in the absence of space charge, we resort to the nonlinear theory of echoes presented in [1]. See also [2] for a previous study of echoes in IOTA. The theory predicts that the relative echo amplitude, i.e. the absolute echo amplitude scaled by the dipole kick, is given by:

$$A = \frac{\langle x \rangle}{\beta_k \theta} \approx \frac{Q}{(1+Q^2)^{3/2}} \tag{1}$$

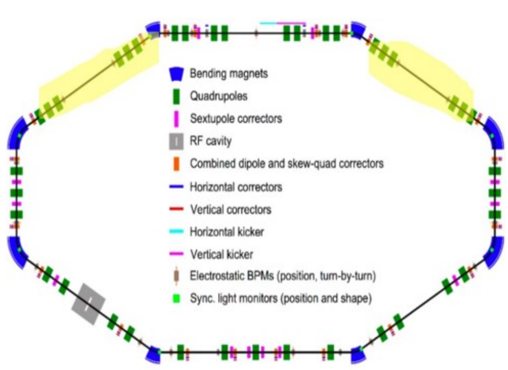
where A is the echo amplitude, $Q = q\omega' \epsilon_f \tau$, $q = \frac{\beta_{\text{quad}}}{f}$[1], β_{quad} is the quadrupole strength and f is the focal length of the quadrupole kicker lens. In the thin lens approximation, $f = \frac{1}{kl}$ where l is the length of the quadrupole and $k = \frac{1}{B\rho}\frac{dB}{dx}$ is the quadrupole strength; ω' is the derivative of the angular betatron frequency with respect to action, τ is the time between the dipole and quadrupole kick, and ϵ_f is the final emittance. Setting the derivative of this expression to 0, we find that by adjusting Q, A reaches a maximum value A^{max}

$$A^{max} = \frac{2}{3^{3/2}} \approx 0.38 \tag{2}$$

at the optimal value $Q_{opt} = 1/\sqrt{2}$ This result assumes the dipole kicks are small enough to be treated as a linear perturbation. When extended to larger dipole kicks, the theory shows that the maximum echo amplitude can be larger than 0.38 and also predicts the occurrence of later echos at multiples of 2τ.

5 Method

Two simulation codes were used in this research. `MADX` (Methodical Acceleration Design) a program developed at CERN to simulate beam dynamics and optimize beam optics was used to perform simulations without space charge while `Pyorbit` developed at ORNL was used used for simulations involving the presence of space charge. Both codes require as input a detailed lattice description of an accelerator as well as initial parameters (see figure 2), e.g. particle mass, energy, initial beam emittance, dipole and quadrupole strengths etc.



(a)

IOTA proton parameters	
Circumference	39.97 [m]
Kinetic Energy	2.5 [MeV]
Maximum bunch intensity /current	9×10^{10} / 8 [mA]
Transverse normalized rms emittance	(0.3, 0.3) [mm-mrad]
Betatron tunes	(5.3, 5.3)
Natural chromaticities	(-8.2, -8.1)
Max /min β values	6.9 / 0.28 [m]
Average transverse beam sizes (rms)	(2.22, 2.22) [mm]
Kinematic γ / Transition γ_T	1.003 / 3.75
Rf voltage	400 [V]
Rf frequency / harmonic number	2.2 [MHz] / 4
Bucket wavelength	~ 10 [m]
Bucket half height in $\Delta p/p$	3.72×10^{-3}
rms bunch length	1.7 [m]
rms energy /momentum spread	1.05×10^{-5} / 1.99×10^{-3}
Beam pipe radius	25 [mm]
Bunch density	6.9×10^{14} [m^{-3}]
Plasma period τ_p	0.18 [μ-sec] / 0.1 [turns]
Average Debye length λ_D	559 [μm]

Table 1: Machine and beam parameters of the IOTA proton ring

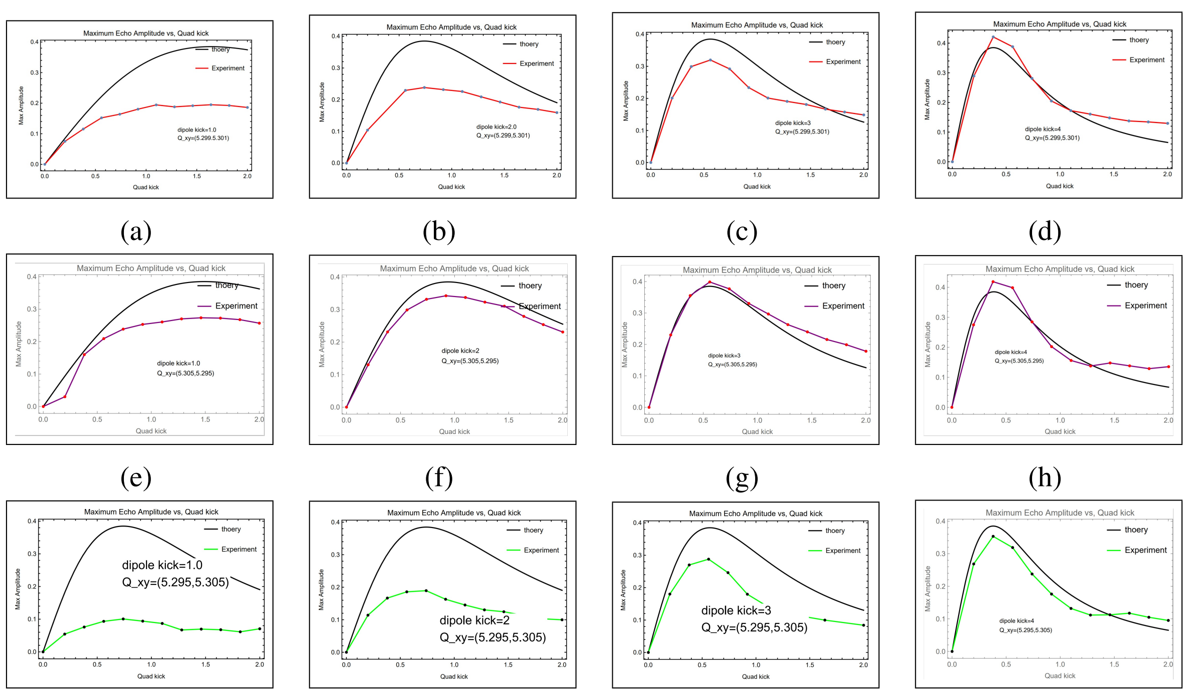
(b)

Figure 2: (a) is the IOTA lattice and (b) is the IOTA proton parameters

6 Results

6.1 Theory vs. Simulations

Running simulations for a range of dipole and quadrupole kick strengths, beam sizes (from 1 to 4 rms) and betatron tune splits ($q_x - q_y$), we found simulation and theory in good agreement. A seen in Figure 3) the maximum echo amplitude consistently occur near A≈ 0.38. We observe a weak dependence of A, and therefore Q, on the dipole kick strength. We remind the reader that the theory does not account for space charge effects.



(a) (b) (c) (d)

(e) (f) (g) (h)

(i) (j) (k) (l)

Figure 3: Theory vs. simulation at different tune splits

6.2 Maximum Echo Amplitude

In the absence of space charge, simulations predict the maximum echo amplitude increases as the dipole kick strength increases, as seen in figure 4. This further affirms the result of section 6.2: the variable Q in the theory depends on the dipole kick strength. At low dipole kick strengths, the quadrupole kick strength does not affect the maximum echo amplitude; however at high dipole quick strength, the quadrupole kick strength has a noticeable effect on the maximum echo amplitude.

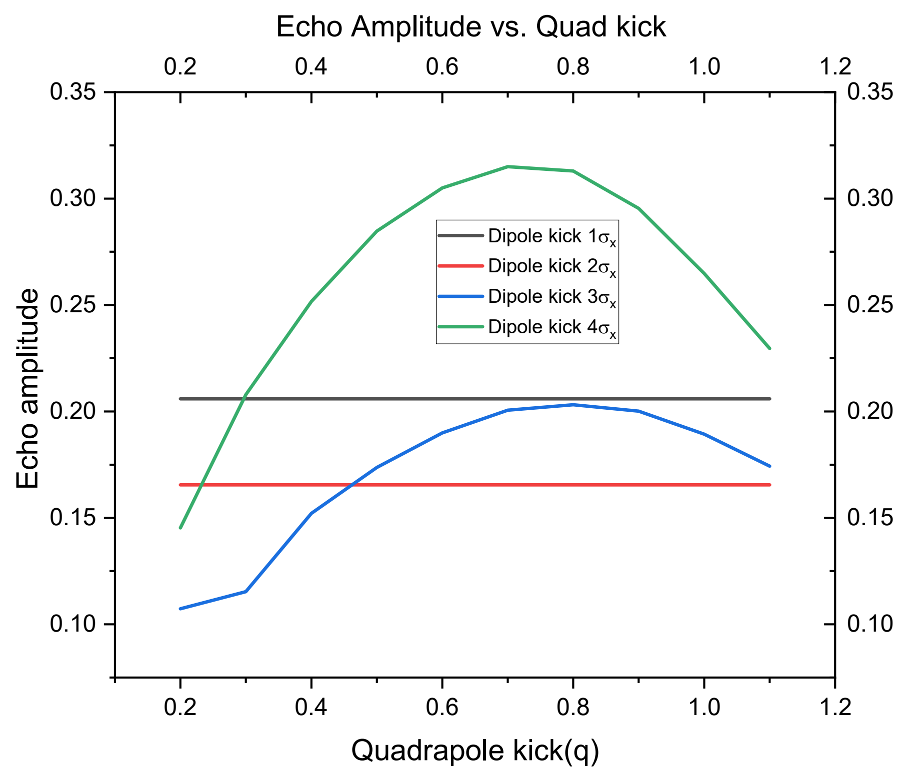


Figure 4: Maximum echo amplitude at increasing quadrupole kick strengths and dipole kick strengths of 1 to $4\sigma_x$.

6.3 Coupling

Evidence of the impact of coupling was found in the simulations as can been seen in Figure 5. Coupling occurs when the horizontal tune and vertical tunes are nearly equal. As energy is transferred between the horizontal and vertical planes the (horizontal) echo amplitude is reduced. One can clearly see this effect in figure 5 : the maximum amplitude decreases as the transverse tunes get closer.

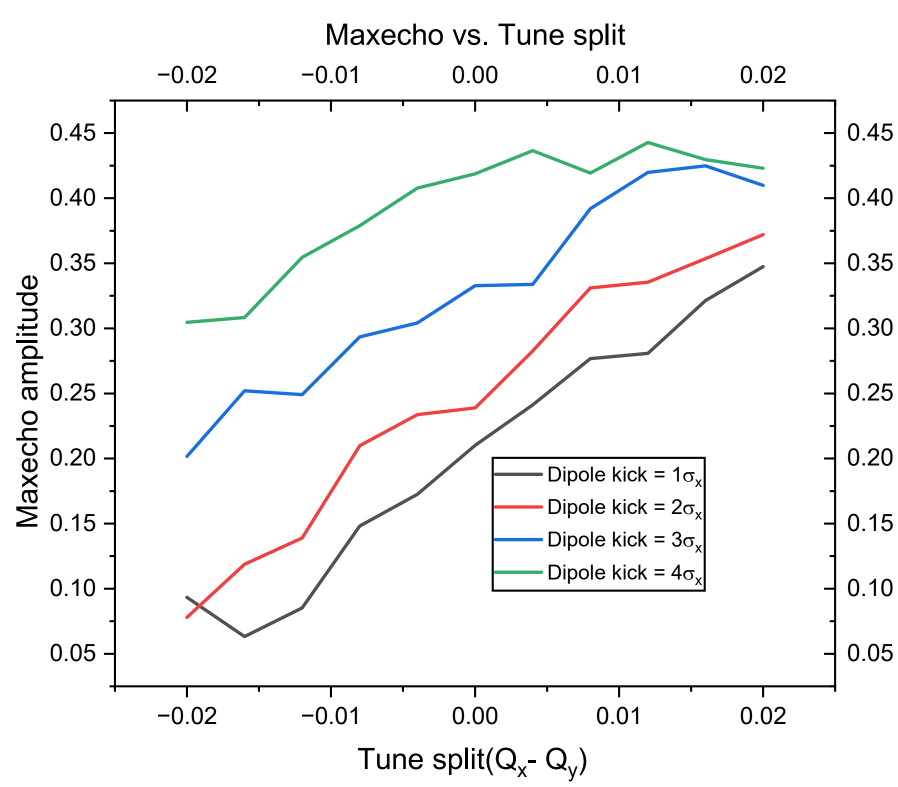


Figure 5: Coupling effect in simulations, observed at dipole kick strengths from 1 to $4\sigma_x$ and increasing tune split.

6.4 Decoherence Time

The decoherence time is defined as the time at which the centroid signal decays to $1/e$ times its initial value; typically the quadrupole kick is applied after near complete decoherence has been achieved. Figure 6 shows the decoherence time as a function of the dipole kick (in units of σ_x) for tune split values. We observe that the decoherence time decreases with increasing dipole kick as expected from one dimensional theory [1]. However, we also observe that the decoherence time depends on the tune split in a complex way, e.g. the time is not symmetric about zero tune split. This dependence on the transverse coupling needs to be understood.

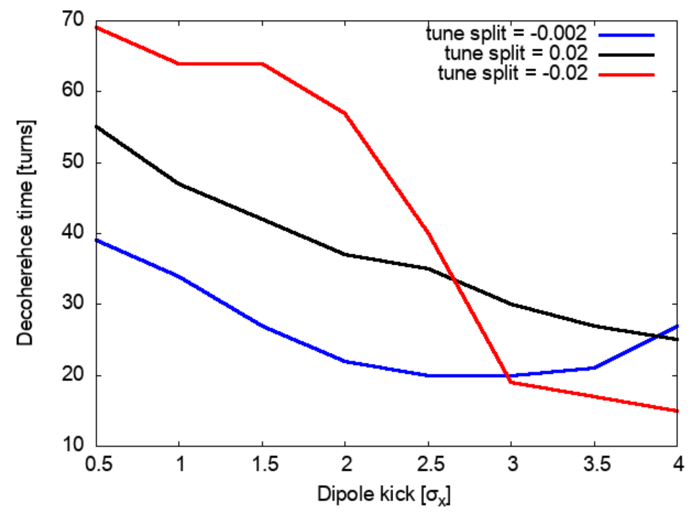
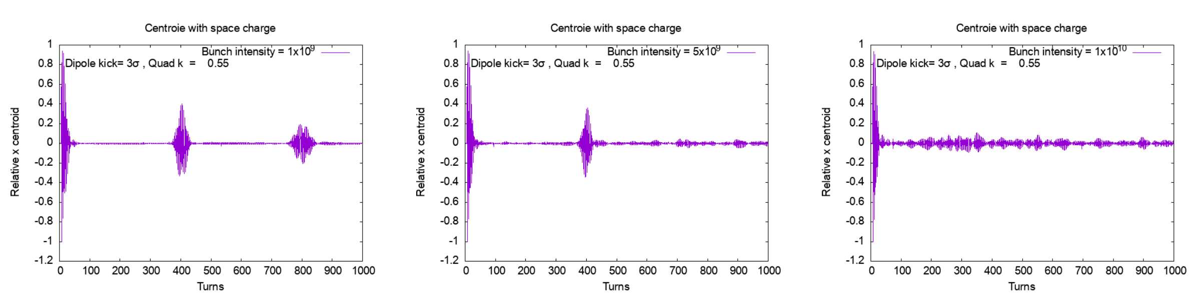


Figure 6: Decoherence time vs. dipole kick strengths from 0.5 to $4\sigma_x$ for different values of the tune split. 1 turn = 2μ seconds

7 Space Charge Effects

Figure 7 shows the evolution of the x-centroid in the presence of space charge at three different bunch intensities. The external parameters such as dipole and quadrupole kick strengths and the tune split were held constant in these cases. We observe that as the intensity increases, the amplitude of the first echo drops while the second echo disappears at intensities at and above 5×10^9. This could be due to a combination of factors: the optimum parameter values change substantially with the intensity and beam diffusion increases with intensity. Figure 8 shows the



(a) (b) (c)

Figure 7: X-Centroid with space charge at intensities (a) 1×10^9 (b) 5×10^9 (c) 1×10^{10}

echo amplitude as a function of the quadrupole strength at two intensities at constant tune split and time delay. We observe that at the higher intensity, the maximum echo amplitude drops by more than a factor of two; the optimum quadrupole strength is also lower.

7.1 Coupling with space charge

With space charge we witness a different trend in the coupling from the results without space charge. As seen in figure 9, for a delay of 100 turns we see the maximum echo increases as the tune split $Q_x - Q_y$ increases. We see from figure 9 that for a 200 turns delay the echo amplitude is more symmetric about the zero tune split value; this suggests that the echo dynamics with space charge depends in a complicated way on the coupling and the delay.

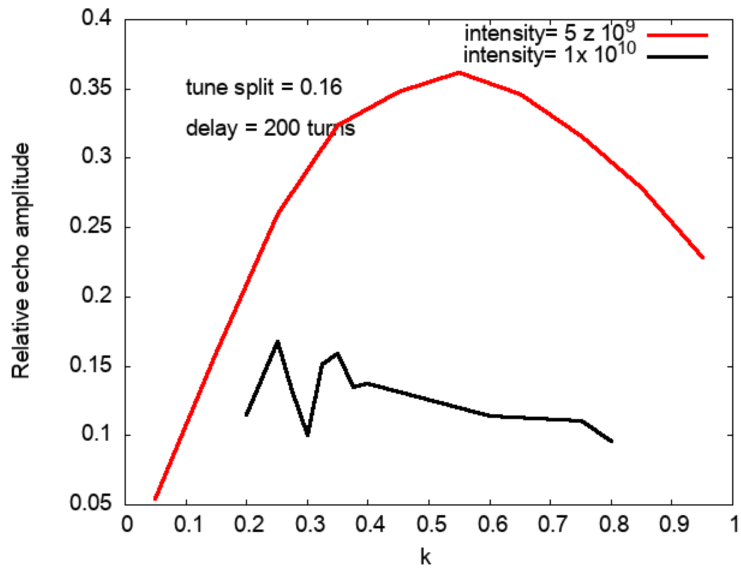
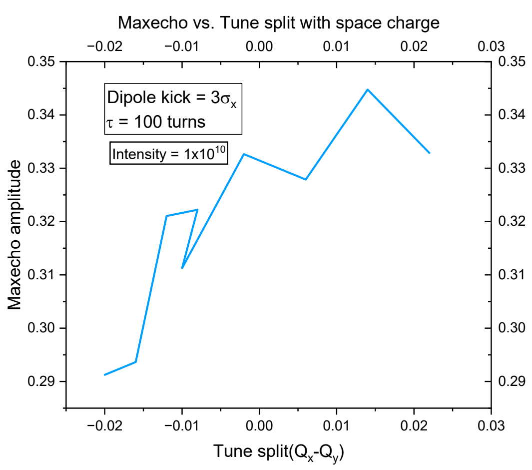
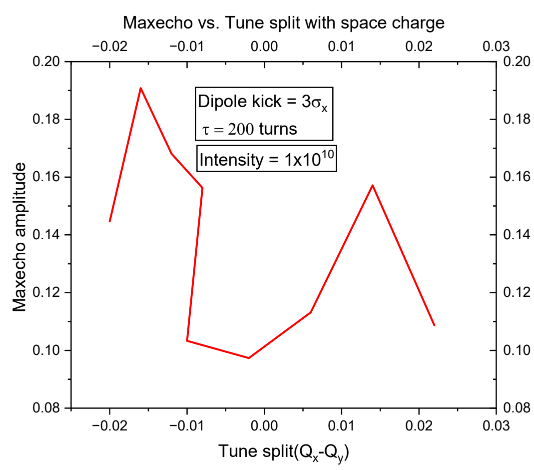


Figure 8: The echo amplitude as a function of the quadrupole strength at two intensities.



(a)



(b)

Figure 9: Maximum echo amplitude as a function of tune split at intensity 1×10^{10}, (a) $\tau = 100$ (b) $\tau = 200$

7.2 Influence of the delay

To illustrate how the delay alone impacts the echo in the presence of space charge, Fig. 10 shows the echo as a function of the quadrupole strength at delays of 100 and 200 turns keeping the tune split constant. We observe that the echo amplitude is significantly larger at the smaller delay. This is expected if diffusion plays a role since that causes irreversible mixing between the dipole and quadrupole kicks and limits the degree of possible recoherence. It is expected that experimental studies of the echo amplitude as a function of the delay will allow a measurement of the diffusion rates [4].

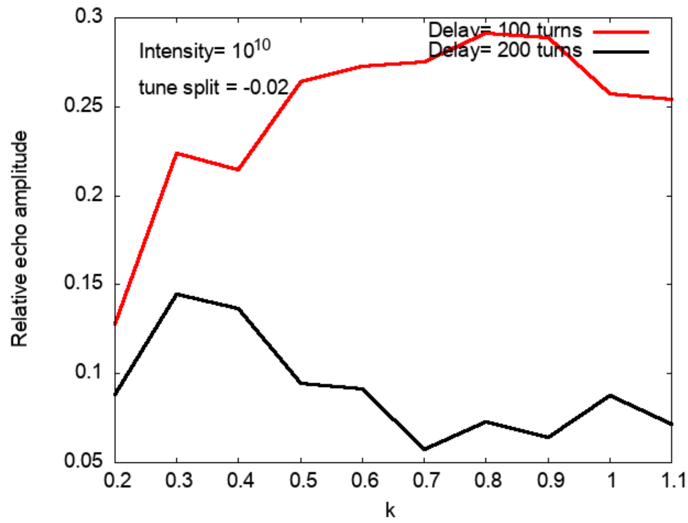


Figure 10: Echo amplitude as a function of quadrupole kick at intensity 10^{10} and constant tune split at two delays.

7.3 Summary of results with space charge

We have simulated beam echoes with space charge at intensities up to 10^{10} particles/bunch. We find that

- The echo amplitude decreases with increasing intensity.
- The optimum value of the quadrupole kick, tune split and the delay that maximize the echo depend strongly on the intensity. In general, the optimum quadrupole strength decreases with intensity.
- At the bunch intensity of 10^{10}, the echo amplitude is larger at a positive tune split $Q_x - Q_y$ than at negative values.
- The echo increases with a smaller delay which shows that diffusion is important even at intensities $\sim 10^{10}$.

8 Conclusions

In the absence of space charge, agreement between theory and simulations is generally very good. Using simulations we observed that the echo amplitude depends on both dipole and quadrupole kick strengths as well as on transverse coupling. Finally we observed that as predicted by theory, large enough dipole kicks produce multiple measurable echoes. In the presence of space charge, we find that choosing the optimal quadrupole kick strengths, tune splits and delays between the dipole and quadrupole kicks yield echoes of sufficient amplitude to be measurable. Our results affirm the ability of simulations to determine experimental parameters that maximize the echo signal; doing should provide a way to effectively and efficiently measure diffusion rates. That said, work is still needed to fully characterize and understand the intricate dynamic of echoes in the presence of space charge effects.

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