Gluon Representation for Lattice QCD Computer Simulations

Anna Janni, University of Minnesota | Hank Lamm, Fermilab | Ruth Van de Water, Fermilab

FERMI-POSTER-22-131-STUDENT-T

Lattice QCD Computer Simulations

Quarks and gluons, both elementary particles, make up protons, neutrons, and other matter. Lattice quantum chromodynamics (or LQCD) computer simulations explore and uncover details about these fundamental building blocks. They enable theory predictions about Large Hadron Collider, LIGO gravitational wave, and other experiments. Currently, classical computers run LQCD simulations, but quantum computers continue to develop and promise to emulate certain quantum physics better. LQCD simulations cannot run on quantum computers until they require less computer storage.

Gluon Representation

In LQCD, $SU(3)$ matrices (or group elements) at each spacetime lattice link (see **Fig. 1**) represent gluons, the force carrier that quarks exchange. Current LQCD simulations represent $SU(3)$ matrices to high precision with $1{,}152$ bits—128 bits for each of 9 double-precision complex numbers. Though a qubit—the storage unit of quantum computers—can store a superposition of computational basis elements and reduce storage requirements, the computational basis elements themselves (in this case $SU(3)$ matrices) require the same number of qubits as bits—$1{,}152$ qubits. Current quantum computers only support 100.

$S(1080)$ group elements (used to form a computational basis) require up to 11 rather than $1{,}152$ bits or qubits, but when they represent the gluon field, LQCD simulations with the Wilson action become inaccurate [1]. Their gluon field values freeze out or wrongfully drop to zero at a certain β_1 (**Equ. 1**). See how $S(1080)$ Wilson data freeze out and diverge from $SU(3)$ Wilson data at $\beta_1 \approx 4$ in **Fig. 3**.

The Lüscher-Weisz Action

$$S_{LW} = \sum_x \beta_1 \mathrm{Re}[\mathrm{Tr}\,(P_x)] + \beta_2 \mathrm{Re}[\mathrm{Tr}(R_x)]$$

Equ. 1 – The Lüscher-Weisz action. β_1 and β_2 denote constant coefficients, and P and R denote gauge link products around closed loops with perimeter $4a$ and $6a$, respectively. The LW action sums over loops at all spacetime coordinates, indexed by x. The Wilson action (S_W) equals S_{LW} with $\beta_2 = 0$.

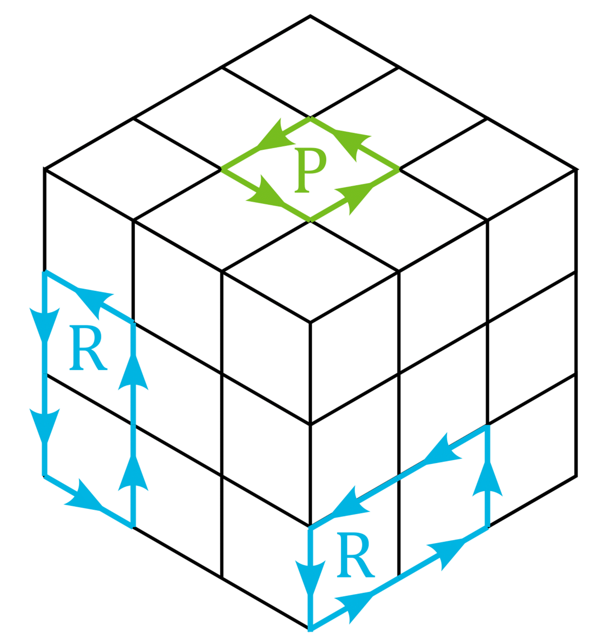

Fig. 1 – spacetime lattice with lattice constant a. Gauge links—either $SU(3)$ matrices or $S(1080)$ group elements—connect each spacetime coordinate and represent the gluon field. The product of gauge links around a closed loop with perimeter $4a$ or $6a$ give P and R, respectively.

The discrete Wilson gluon action of LQCD approximates its continuous counterpart of QCD and introduces errors that scale with the lattice constant a. The LW action cancels the $O(a^2)$ errors in the Wilson action [2].

To reduce LQCD simulation storage requirements and maintain simulation accuracy, we propose an $S(1080)$ representation of the gluon and the Lüscher-Weisz (LW) action be implemented into simulations.

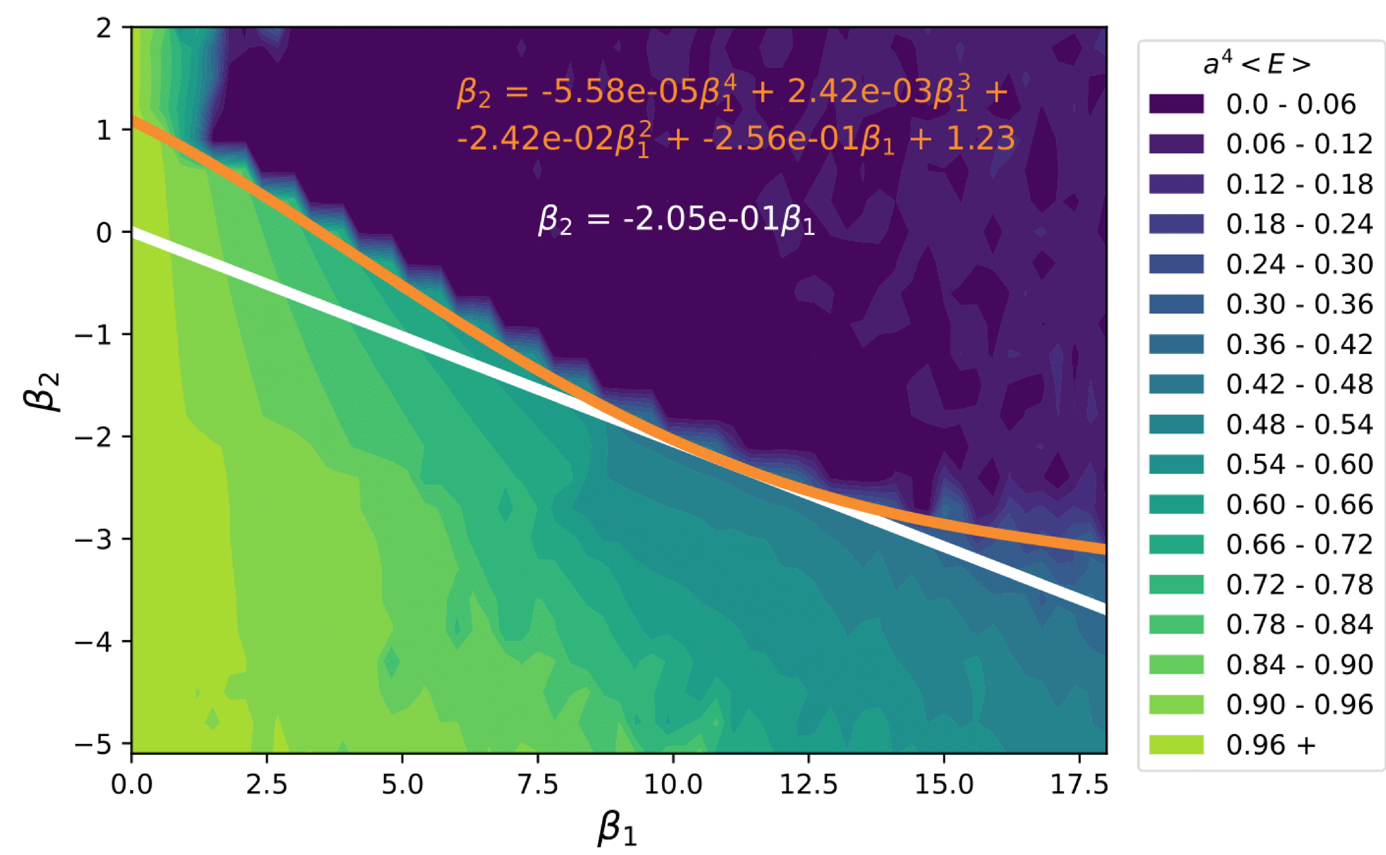

Fig. 2 – $a^4\langle E\rangle$ ($\langle E\rangle$ denotes average energy of P loops, a metric of gluon field strength) results from a Monte-Carlo simulation on a 2^4 lattice with the LW action and $S(1080)$ gluon representation. The orange and white lines trace the outskirts of a first-order phase transition in $a^4\langle E\rangle$.

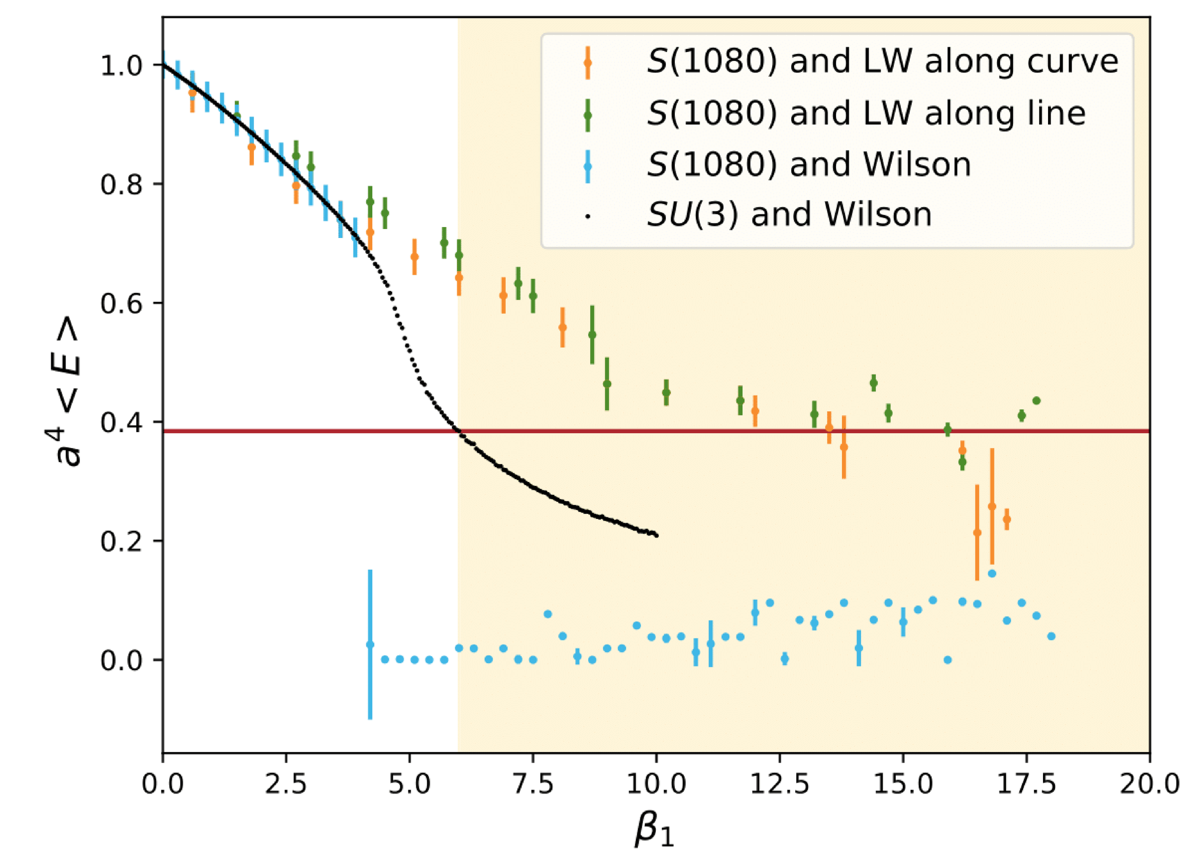

Fig. 3 – $a^4\langle E\rangle$ results plotted against β_1 for the LW and the Wilson actions. The data along the white line and orange curve in Fig. 4 give the LW data.

References

[1] A. Alexandru et al. "Gluon Field Digitization for Quantum Computers". Phys. Rev. D 100, 114501 (2019).
[2] Gattringer C., Lang C.B., *Quantum Chromodynamics on the Lattice: An Introductory Presentation*, Lect. Notes Phys. 788 (Springer, Berlin Heidelberg 2010), DOI 10.1007/978-3-642-01850-3.

Analysis of Results

LQCD calculations discretize space and time into chunks of a and introduce errors dependent on a. Knowing how these errors behave enables LQCD results to be extrapolated from lattice spacetime to continuous spacetime (to the limit $a \to 0$). For $SU(3)$ and Wilson action calculations, we know errors behave polynomially with a in the region $\beta_1 \gtrsim 6$, called the scaling regime (shaded yellow in **Fig. 3**). At the border of the scaling regime, $a^4\langle E\rangle \approx 0.384$ (the line drawn in **Fig. 3**). While $S(1080)$ and Wilson simulations freeze out and do not produce $a^4\langle E\rangle \approx 0.384$, $S(1080)$ and LW simulations do and thereby enter a scaling regime. $S(1080)$ and LW simulation results can be extrapolated to the limit $a \to 0$. The $S(1080)$ gluon representation and LW gluon action reduce storage requirements for LQCD simulations by a factor of 100 and output accurate simulation results making them a viable combination for LQCD simulations for quantum computers.

Future Work

Continued runs of LQCD simulations with $S(1080)$ gluon fields and the LW action will be used to find the temperature (T_c) where quarks transition from a confined to deconfined phase at finite lattice spacing and to measure observables like static potential and Wilson flow. Observables will be compared to results from the known-to-work $SU(3)$ simulations to determine the lattice constant a for accurate simulations.

This manuscript has been authored by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics