S(1080) Gluon Representation in Lattice QCD Computer Simulations

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Quantum computers continue to develop and promise to model quantum physics better than classical computers. This makes quantum computers ideal for lattice QCD computer simulations. Unfortunately, current lattice QCD computer simulations require more computer storage than quantum computers support. Our results from Monte-Carlo lattice QCD simulations show that a gluon field made up of S(1080)-element gauge links, rather than SU(3)-element gauge links, work in lattice QCD simulations with an improved gluon action, the Lüscher-Weisz action. The S(1080) gluon representation reduces the cost of the gluon field in lattice QCD simulations by a factor of more than 100, a size that will fit on soon-to-come quantum computers.

I. INTRODUCTION

The Standard Model, today's most unifying particle physics theory, describes three of four known fundamental forces and all known elementary particles–including quarks and gluons, which behave according to quantum chromodynamics (QCD). Lattice QCD computer simulations numerically solve problems in QCD, namely those that involve strong coupling and confinement. But, lattice QCD simulations have their limitations. They can only calculate static, not dynamical, quantities [1].

Current lattice QCD simulations run on classical computers, but a new technology, the quantum computer, continues to develop and promises to model quantum physics better than a classical computer [2]. Quantum computers will expand the capabilities of lattice QCD simulations and could be used to calculate still-unknown dynamical QCD quantities [1].

Several hurdles prevent lattice QCD simulations from running on quantum computers. For one, lattice QCD simulations require more storage than quantum computers support [3].

In lattice QCD, gauge links (the arrows in Fig. 1) connect spacetime coordinates and make up the gluon field. Each gauge link is a special, unitary, three-by-three matrix or SU(3) group element. These matrices cost 1,152 bits–128 bits for each of 9 double-precision complex numbers–on a classical computer and 1,152 qubits on a quantum computer. Current quantum computers only support around 100 qubits, and soon-to-come quantum computers will only support on the order of 1,000 [4]. 1,152 qubits far exceed the former and rival the latter. For lattice QCD simulations to run on quantum computers in a remotely near future, their storage requirements must be reduced.

The S(1080) group is a subgroup of SU(3) with 1,080 elements. Each element of S(1080) can be stored with 11 bits or qubits, so using S(1080) group elements in place of SU(3) matrices reduces the cost of the gluon field in a lattice QCD simulation by a factor of about 100. Of course, such a wild reduction comes with losses like gluon field freeze-out. In Fig. 2, the gluon field freezes out in the navy region, and in Fig. 3, it freezes out where the blue data points drop to zero.

II. GLUON FIELD FREEZE-OUT

In lattice QCD, a sum over *every possible* field configuration gives a QCD observable or measurable quantity (like a mass ratio or average plaquette energy). This means that if a field can take on a mere 2 values at every spacetime coordinate on a 2^4 lattice, a sum over $2^{16} = 65,536$ field configurations must be calculated. Of course, the gluon field can take on far more than 2 values at each spacetime coordinate, and many lattice QCD simulations use lattices larger than 2^4. Instead of calculating such an near-impossible sum over all field configurations, lattice QCD computer simulations use Monte-Carlo methods to sum and average over a *sample* of them. Field configurations get sampled according to their Boltzmann factor $e^{-S[U_N]}$, where S denotes an action and U_N denotes a possible field configuration. In the simulations we ran, we used the Wilson action or the gluon action of lattice spacetime:

$$S_W = -\sum_x \beta_1 \mathrm{Re}[\mathrm{Tr}(P_x)] \tag{1}$$

β_1 denotes a constant coefficient and P denotes a product of gauge links around a closed loop of perimeter $4a$. (See Fig. 1). The Wilson action sums over all spacetime coordinates (indexed by x).

In both the SU(3) and S(1080) groups, the identity element or (identity matrix) has the largest trace, so Monte-Carlo sampling favors identity or near-identity gauge-links. While the SU(3) group contains infinite elements arbitrarily close to the identity, the S(1080) subgroup of SU(3) does not. S(1080)'s lack of near-identity elements creates a problem at large enough β_1. As β_1 increases, the favoring of near-identity gauge-links becomes more pronounced and forces each S(1080) gauge link to the identity, called the gluon field freeze-out [3]. See Fig. 2 and 3.

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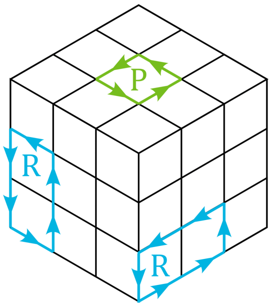

FIG. 1. A spacetime lattice with lattice constant a. Gauge links–either SU(3) matrices or S(1080) group elements–connect each spacetime coordinate and represent the gluon field. The product of gauge links around a closed loop with perimeter $4a$ (in green) or $6a$ (in blue) give P and R, respectively.

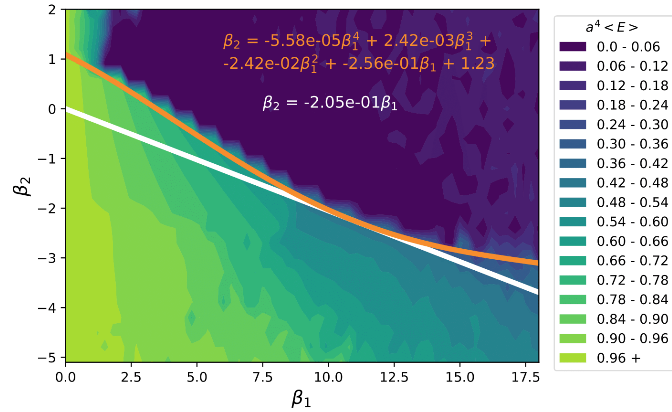

FIG. 2. $a^4\langle E\rangle$ results from a Monte-Carlo simulation on a 2^4 lattice with the Lüscher-Weisz action and S(1080) gluon representation. The orange and white curve and line trace the outskirts of a first-order phase transition in $a^4\langle E\rangle$.

III. THE LÜSCHER-WEISZ ACTION

The gluon field freeze-out can be avoided with an improved gluon action. Unless calculated in the limit that the lattice constant a goes to 0, the aforementioned Wilson action only approximates the true gluon action of continuous spacetime and has errors that scale with a. An improved action, the Lüscher-Weisz (LW) action, adds a term to the Wilson action that cancels its errors proportional to a^2 and looks like:

$$S_{LW} = -(\sum_x \beta_1 \mathrm{Re}[\mathrm{Tr}(P_x)] + \beta_2 \mathrm{Re}[\mathrm{Tr}(R_x)]) \tag{2}$$

β_2 denotes a constant coefficient and R denotes a product of gauge links around a closed loop of perimeter $6a$. (See Fig. 1). Like the Wilson action, the Lüscher-Weisz action sums over all spacetime coordinates (indexed by x).

IV. LATTICE QCD SIMULATION RESULTS

To test S(1080) gluon representation with the Lüscher-Weisz action, we ran a Monte-Carlo lattice simulation on a 2^4 lattice and measured average plaquette energy $\langle E\rangle$ times a^4. The average plaquette energy is given by:

$$\langle E\rangle = \frac{1}{N}\sum_x \mathrm{Re}[\mathrm{Tr}(P_x)] \tag{3}$$

N denotes the number of gluon field configurations.

Plaquette energy data shows that the gluon field freezes out even with the Lüscher-Weisz action. (See Fig. 2). However, the freeze-out can be avoided by fixing β_2 as a function of β_1. The gluon field freezes out according to a first-order phase transition, so to find a function to fix β_2, we took the derivative of $a^4\langle E\rangle$ with respect to β_1, found peaks in the derivative, then fit a function to the locations of those peaks. We then shifted the function down by 0.147 β_2. The orange curve in Fig. 2 traces the shifted function. In addition to the aforementioned curve, we found a β_2-as-a-function-of-β_1 line that avoids the gluon field freeze-out. The line and curve we found both successfully avoid gluon field freeze-out as indicated by Fig. 2 and 3.

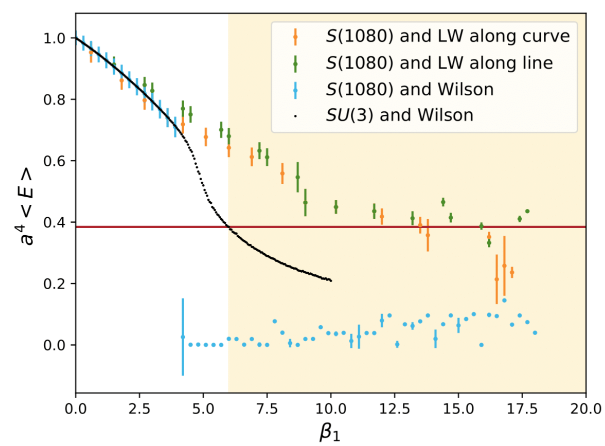

FIG. 3. $a^4\langle E\rangle$ results plotted against β_1 for the Lüscher-Weisz and the Wilson actions. The data along the white line and orange curve in Fig. 2 give the Lüscher-Weisz data.

V. THE SCALING REGIME

In order for lattice QCD simulations to be used for meaningful calculations, the simulations must enter the scaling regime. In the scaling regime, errors in observables behave polynomially with a so can be minimized by running simulations with smaller and smaller lattice constants. The scaling regime is a region where observables can be tuned to desired accuracies.

For the Wilson action, the scaling regime begins where

$\beta_1 \gtrapprox 6$–the region shaded yellow in Fig. 3. At β_1 equals 6, $a^4\langle E\rangle$ equals 0.384 for known-to-work SU(3) and Wilson data (plotted in black). We argue that because the S(1080) data reaches a similar $a^4\langle E\rangle$ value where $\beta_1 \gtrapprox 6$ (see where the green and orange points intersect the red line in Fig. 3), simulations with S(1080) gluon representation and the Lüscher-Weisz action enter a scaling regime.

VI. CONCLUSIONS

Current lattice QCD simulations do not fit on today's near 100-qubit quantum computers in large-part due to their expensive gluon representation. SU(3) gauge links in the gluon field each cost 1,152 qubits whereas S(1080) gauge links only cost 11. Using S(1080) gauge links to represent the gluon field reduces the size of lattice QCD simulations to a size that soon-to-come quantum computers support. As shown by our results, lattice QCD simulations with an S(1080) gluon representation and the Lüscher-Weisz action avoid problematic gluon field freeze-out and enter a scaling regime. Further, these simulations will fit on the 1,000-qubit quantum computers to be made within the decade so can be used for new QCD calculations in the near future.

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