# Quantitative Modeling of SCRF Accelerator Cost and Performance

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## Problem in 1999, Jefferson lab:

2 linacs of 160 cavities each; each cavity has nonlinear performance limits.

Want to optimize 160 degrees of freedom to give best total performance, without searching 160-dimensional vector space. (In essence, the operator crew was searching this space by hand)

## Answer:

"Operational Optimization of Large-Scale SRF Accelerators," Delayen et al., PAC 1999

Search 3-dimensional nonlinear space, from which (by variational means) we can constructively find the 160 cavity voltages.

# Problem in 2011, NGLS:

Predict total cost and total performance of a hypothetical machine, where each of about  $\sim 200$  cavities will have restrictions similar to above.

### Answer:

Assume each performance parameter (e.g., max gradient, max RF power) has a statistical fluctuation. Use Monte-Carlo techniques to build a virtual accelerator, then use the analysis from 1999 to predict the performance of the ensemble.

### Input data in 1999:

Each cavity's  $Q_0$  and  $Q_L$  measured at commissioning. Q slope ignored.

Each klystron's  $P_{\text{max}}$  estimated from high voltage power supply.

Microphonics  $\Delta f$  surveyed in-place

Window arc trip model developed by data-mining months of accelerator fault logs

# Input "data" in 2011:

Niobium surface resistance a combination of BCS (temperature-dependent) and extrinsic (temperature-independent) terms. Q slope from Weingarten model

$$R_{\rm S} = \frac{2}{3}\pi\mu_0 \alpha f B^2 \cdot \sum_{i=0}^{\infty} \frac{2(\beta B^2)^i}{i+2}$$

Parameters estimated from literature search (but here to collaborate on quantitative extrapolations from Fermilab facilities).

Electromagnetic parameters (r/Q, G) from ILC/XFEL LL design.

Wild guess on microphonic distribution.

Window design assumed arc-free.

Parameterized initial cost estimates for cryomodule, refrigerator, RF, tunnel.

Cavity shape and processing technology

$(r/Q)_{ m mid}$	118	Ω		
$(r/Q)_{ m end}$	118	Ω		
$E_{\rm peak}/E_{\rm acc}$	2			
$B_{\rm peak}/E_{\rm acc}$	4.26	$\mathrm{mT}/(\mathrm{MV}/\mathrm{m})$		
G	284	Ω		
$r_{ m resid}$	6	$n\Omega/\Box$	$\pm 20\%$	depends on magnetic shielding
lpha	$2.2 \times 10^{-10}$	${ m m}{\cdot}{ m T}^{-2}$	$\pm 30\%$	Weingarten[3]
eta	68	$T^{-2}$	$\pm 30\%$	Weingarten[3]
$E_{\max}$	50	MV/m	$\pm 30\%$	surface field

Cryomodule design and technology

$n_{\mathrm{cell}}$	7			per cavity	
$n_{ m cav}$	8			per module	
$w_{\parallel}$	459	V/pC	wake/module, $60 \mathrm{mm}$ iris		
$P_{ m mod}$	23	W		static losses	
$l_1$	0.3	m		between cavities	
$l_2$	1.5	m		between modules	
$\eta_0$	$5 \times 10^{-5}$			fundamental coupler	
$\eta_{ m HOM}$	0.1				
$\Delta f$	20	Hz	$\pm 30\%$	microphonic	

# Global design and operations

f	1300	MHz		
T	1.8	Κ		
$n_{ m mod}$	18			
$V_B$	2400	$\mathrm{MeV}$		beam energy goal
$I_B$	1	${ m mA}$		
q	250	pC		
RF design				
$P_{\max}$	21.58	$\rm kW$	$\pm 5\%$	
$Q_{ m L}$	$1.689{\times}10^7$		$\pm 10\%$	

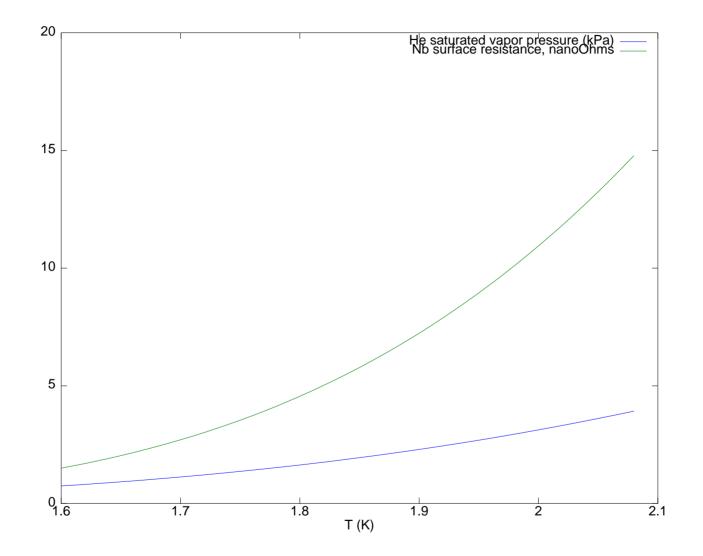
# Example run

Ensemble linac setup with component fluctuations Linac Energy setup goal: 2400 MeV, 1.0 mA with 144 cavities 3108 kW nominal RF power available Setup parameter dH/dV = 1.311e-05 W/V Cavities limited by RF power: 69, cryo load: 40, max field: 35 Gradient statistics: mean 20.65 std 2.79 MV/m Total dynamic cryo heat load 3957 Watts at 1.80 K (1639 Pa) Wakefield 459 V/pC/module, bunch charge 250 pC, 10% dissipated in cold mass Additional 1.80 K losses: static 414 W, wake field 103 W, coupler 155 W

Cryoplant xx.xx M\$ (4629.6 W at 1.80 K, 2 refrigerators)

- Modules xx.xx M\$ (18 x 8-cavity 7-cell 1300 MHz)
- Tunnel xx.xx M\$ (181.0 m, 64.2% packing efficiency)
- RF power xx.xx M\$ (144 klystrons operated at 21.6 kW max)
- Total xxx.xx M\$ (without contingency)

**Pressure and**  $R_S$  vs. temperature



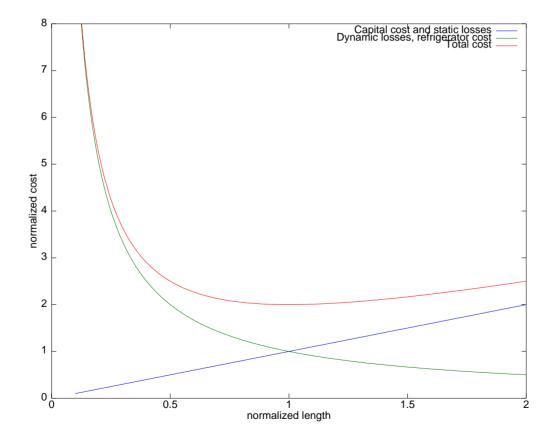
#### Simplest possible optimization

For a desired total linac voltage V, wish to optimize length l.

Capital cost and static thermal losses scale as l.

Gradient scales as V/l, so for constant  $Q_0$  the dynamic losses scale as  $l \cdot (V/l)^2/Q_0$ , or  $V^2/l$ .

Balance between l and 1/l terms gives an optimal length.



Unless otherwise stated,  $V = V_{\text{acc}}$  and  $E = E_{\text{acc}}$ , where V = lE and l is the length of a single cavity with  $n_{\text{cell}}$  cells. Also,  $\omega_0 = 2\pi f_0$  is the operating frequency of the SRF accelerator.

To understand the behavior of a hypothetical accelerator, we start by preparing an ensemble of cavities with a distribution of parameters. This ensemble can be set up with gradient setpoints according to the process described in Delayen *et al.*[1] Two limits are simple, based on the maximum voltage rating of the cavity, and the maximum power available from the klystron. The third is a variational equilibrium to give the best total voltage for a finite refrigeration capacity, where dH/dV is a given constant. The total accelerator voltage and needed refrigeration capacity can then be computed. Scans and adjustments for target values can take place by adjusting dH/dV and other parameters, such as beam current.

Shunt impedance r/Q relates the stored energy in the cavity to the accelerating voltage, according to  $U = V^2/(r/Q)\omega_0$ , where  $V = E \cdot l$ . The basic useful relationship for us is the power emitted from the fundamental power port

$$P = E^2 l/Q_L(r/Q) = V^2/Q_L(r/Q)$$

Some literature uses a definition of shunt impedance which is twice this one.

Cavity  $Q_0$  is related to the surface resistance according to  $Q_0 = G/r_s$ . The four parameters R/Q,  $E_{\text{peak}}/E_{\text{acc}}$ ,  $B_{\text{peak}}/E_{\text{acc}}$ , and G are readily computed from cell geometry using electromagnetic analysis codes such as Superfish.

Surface resistance has units of  $\Omega/\Box$ , often shortened to just  $\Omega$ .

$$r_{\rm S} = r_{\rm resid} + r_{\rm BCS} + r_{\rm W}$$

One source of residual resistance  $r_{\text{resid}}$  is the magnetic field present in the niobium at the time it cools

past  $T_{\rm C}$ , leading to trapped flux vortices that act as loss centers. Magnetic shields are built into the cryomodule to minimize this term.

I willfully ignore the possible losses due to field emission. Usually concerns that the radiation will damage surrounding equipment lead to accelerator cavity operation with field emission losses smaller than the other losses considered. I simply set a (statistical) upper bound  $E_{\text{max}}$  on surface electric field, forcing  $E < E_{\text{max}}/(E_{\text{peak}}/E_{\text{acc}})$ .

The basic equation for BCS loss is not in doubt, but the exact numbers seem to vary slightly from sample to sample. I follow the version in the NLS design report[2]

$$r_{\rm BCS} = 0.89 \times 10^{-22} (\Omega {\rm K/GHz}^2) \frac{f^2}{T} e^{-(17.67 {\rm K})/T}$$

Weingarten[3] gives a model explaining SRF "Q slope" as a surface resistance term

$$r_{\rm W} = \frac{2}{3}\pi\mu_0 \alpha f B^2 \cdot \sum_{i=0}^{\infty} \frac{2(\beta B^2)^i}{i+2}$$

where  $\alpha$  and  $\beta$  are fit parameters that can be interpreted in terms of materials and defect properties. Weingarten and I take  $B = (B_{\text{peak}}/E_{\text{acc}}) \cdot E_{\text{acc}}$ , an approximation at best because surface B field varies across the surface of the cavity. Weingarten also gives a surface resistance term proportional to  $B^{-2}$ , which I do not use.

If surface resistance were constant, dH/dE would be linear in E and it would be trivial to solve for E. After including the Weingarten losses, that equation becomes cubic, still tractable for constructive determination of E.

The full vector (complex) expression for cavity fields, combining shunt impedance, detuning (primarily microphonics), and beam loading, but ignoring cavity dissipation, is

$$\left(1 - j\frac{\omega_d}{\omega_f}\right)\vec{V} + \frac{1}{\omega_f}\frac{d\vec{V}}{dt} = 2\vec{K}\sqrt{R_c} - R_c\vec{I}$$

where  $\vec{K}$  is the incident wave amplitude in  $\sqrt{\text{Watts}}$ ,  $R_c = Q_L(r/Q)$  is the coupling impedance,  $\vec{I}$  is the DC beam current,  $\omega_d = 2\pi\Delta f$  is the (time varying) detune angle, and  $\omega_f = \omega_0/2Q_L$  is the cavity bandwidth. The derivative term is assumed to be zero, at which point it is possible to solve for the maximum field achievable given the other conditions. That expression can be rearranged to give the power required

$$|\vec{K}|^2 = \frac{|\vec{V}|^2}{4(r/Q)} \left| \frac{1 + BQ_L - jDQ_L}{\sqrt{Q_L}} \right|^2$$

where  $B \equiv (r/Q)\vec{I}/\vec{V}$  and  $D \equiv 2\omega_d/\omega_0$ . Power is minimized when  $Q_L = (B^2 + D^2)^{-1/2}$ . Note that if an average value of  $|\vec{V}|$  is used in these equations, the ensemble will underperform. Since some cavities will perform below average because of other effects, the remaining cavities (limited by RF power) need to run at above average voltage.

Losses due to wakefields are difficult to understand quantitatively, and few experiments or analyses cover the relevant frequency band for this accelerator, that reaches up to about 1 THz. I use results from Zagorodnov and Solyak[4] for the ILC cavities with 60 mm diameter iris. The monopole losses from the beam in each cryomodule are given by

Power 
$$= \frac{1}{2} w_{\parallel} q I_{\text{beam}}$$

which I then multiply by  $\eta_{HOM}$  to estimate the power dissipated in the low temperature (nominally 1.8 K) bath.

Cost model for the refrigerator is due to Niinikoski,[5]

$$Cost = 0.3 + 6.80 \cdot P^{0.6}$$

where cost is in million 1998 CHF, and P is power in kW at 1.8 K, up to a maximum 2.5 kW. This model needs attention to add a dependence on temperature (actually intake pressure). I have multiplied 1998 CHF by 2 to get approximate current US\$.

The cost model for Klystrons involves simply a cost per unit, and a cost per watt of high voltage supply, up to the maximum for that particular design. A choice between different models or styles of tube (e.g., IOT *vs.* klystron) would be made using this model. In place of real quotes from a manufacturer, I have used an early estimate from Ken Baptiste[6] for 1.3 GHz IOTs.

To compute the length of accelerator tunnel, we not only need to know the operating frequency  $f_0$ , cells/cavity  $n_{cell}$ , cavities/module  $n_{cav}$ , and modules in the accelerator  $n_{mod}$ , but also the separation between cavities  $l_1$  within the cold mass, and the separation between cold masses  $l_2$ . These gaps have to be big enough to hold couplers, flanges, instrumentation, cryogenic transitions, and transverse optical elements. The cost model for tunnel construction is simply linear in overall length.

Yet to be accounted for: electric power costs, transfer line costs, and cavity production yield.

#### References

- [1] J. Delayen et al., "Operational Optimization of Large-scale SRF Accelerators," 1999 PAC.
- [2] Jon Marangos et al., "New Light Source (NLS) Project: Conceptual Design Report," May 2010
- [3] W. Weingarten, "On the Dependence of the Q-Value on the Accelerating Gradient for Superconducting Cavities," 2007 SRF Workshop.
- [4] I. Zagorodnov and N. Solyak, "Wakefield Effects of New ILC Cavity Shapes," 2006 EPAC.
- [5] Tapio Niinikoski, private communication.
- [6] Ken Baptiste, private communication.