Quasielastic scattering with the Relativistic Green Function approach

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Introduction

- ♡ : we are interested in electron and neutrino-nucleus scattering
- ♠ : relativistic models for quasielastic scattering
- ♢ : the Relativistic Green's Function
- ♣ : some results for electron scattering
- ♡ : results for CCQE and NCE neutrino-nucleus scattering $\Longrightarrow$ MiniBooNE data
- ♠ : Collaborators C. Giusti (Pavia), F.D. Pacati (Pavia)
  M.B. Barbaro (Torino)
  J.A. Caballero (Sevilla)
  J.M. Udías (Madrid)

ν-nucleus scattering

$$\left.\begin{array}{l} \nu_\mu(\bar{\nu}_\mu) + A \longrightarrow \mu^-(\mu^+) + N + (A-1) \\ \nu_\mu(\bar{\nu}_\mu) + A \longrightarrow \nu_\mu(\bar{\nu}_\mu) + N + (A-1) \end{array}\right\} \quad \text{CC and NC scattering!}$$

$$\mathrm{d}\sigma = \frac{G_\mathrm{F}^2}{2}\, 2\pi\, L^{\mu\nu}\, W_{\mu\nu}\, \frac{\mathrm{d}^3 k}{(2\pi)^3}\, \frac{\mathrm{d}^3 p_\mathrm{N}}{(2\pi)^3}$$

♡ : lepton tensor

$$L^{\mu\nu} \quad = \quad \frac{2}{\varepsilon_i \varepsilon}\left[l_S^{\mu\nu} \mp l_A^{\mu\nu}\right] \quad \left(\begin{array}{l} \nu \text{ scattering} \\ \bar{\nu} \text{ scattering} \end{array} \right)$$

♠ : $G_\mathrm{F} \simeq 1.16639 \times 10^{-11}$ MeV^{-2} Fermi constant ($\times \cos^2\vartheta_\mathrm{C} \simeq 0.9749$ for CC scattering)

♢ : hadron tensor

$$W^{\mu\nu}(q,\omega) = \overline{\sum_i} \, \sum\!\!\!\!\!\!\!\int_f \langle \Psi_f \mid \hat{J}^\mu(\boldsymbol{q}) \mid \Psi_0 \rangle \langle \Psi_0 \mid \hat{J}^{\nu\dagger}(\boldsymbol{q}) \mid \Psi_f \rangle \, \delta(E_0 + \omega - E_f),$$

the transition amplitude in RDWIA

♡ : The components of the hadron tensor can be expressed as

$$W^{\mu\nu}(q,\omega) = \overline{\sum_i} \sum\!\!\!\!\!\!\int_f \langle \Psi_f \mid \hat{J}^{\mu}(\boldsymbol{q}) \mid \Psi_0 \rangle \langle \Psi_0 \mid \hat{J}^{\nu\dagger}(\boldsymbol{q}) \mid \Psi_f \rangle \, \delta(E_0 + \omega - E_f),$$

♠ : The matrix elements of the nuclear current operator are expressed in the IA as the sum, over all the single-particle states, of the squared absolute value of the transition matrix elements of the single-nucleon current

$$\langle \chi^{(-)}(E) \mid j^{\mu} \mid \varphi_n(E) \rangle \ ,$$

and are calculated with relativistic wave functions $\Longrightarrow$ same treatment as in $(e, e'p)$ calculations

♢ : $\varphi_n \Longrightarrow$ Dirac-Hartree solutions of a relativistic mean field theory Lagrangian

♣ : Scattering wave function $\chi^{(-)}(E) \Longrightarrow$ we write it in terms of its positive energy components

$$\chi^{(-)}(E) = \begin{pmatrix} \Psi_{\mathrm{f}+} \\ \dfrac{1}{M + E + S^{\dagger}(E) - V^{\dagger}(E)} \boldsymbol{\sigma} \cdot \boldsymbol{p} \Psi_{\mathrm{f}+} \end{pmatrix} \ ,$$

♡ : Ψ_{f+} is related to a Schrödinger-like wave function, Φ_f, by the Darwin factor, i.e.,

$$\Psi_{f+} = \sqrt{D^{\dagger}(E)} \Phi_f \ , \quad D(E) = 1 + \frac{S(E) - V(E)}{M + E}$$

♠ : RDWIA $\Longrightarrow$ FSI effects are accounted for by solving the Dirac equation with strong relativistic scalar and vector optical potentials $\Longrightarrow$ suitable for exclusive processes but not for inclusive ones

♢ : RPWIA $\Longrightarrow$ FSI effects are neglected
rROP $\Longrightarrow$ only the real part of the optical potential is retained

the Relativistic Green's Function approach

♡ : suitable approximations which are basically related to the impulse approximation

♠ : For an inclusive process the components of the hadron tensor can be expressed as

$$W^{\mu\nu} = \langle \Psi_0 \mid J^{\nu\dagger} \delta\,(E_{\mathrm{f}} - H)\, J^{\mu} \mid \Psi_0 \rangle$$

♢ : The components of $W^{\mu\nu}$ can be re-expressed in terms of the Green's operators related to the nuclear Hamiltonian H. For example (for $\mu = 0, x, y, z$)

$$\omega^{\mu\mu} = W^{\mu\mu} = -\frac{1}{\pi}\mathrm{Im}\langle \Psi_0 \mid J^{\mu\dagger} G(E_{\mathrm{f}}) J^{\mu} \mid \Psi_0 \rangle$$

♣ : The components of the nuclear response can be written in terms of the single-particle Green's function $\mathcal{G}(E)$, whose self-energy is the Feshbach's optical potential.

♡ : An explicit calculation of the Green's function can be avoided by its spectral representation, which is based on a biorthogonal expansion in terms of the eigenfunctions of the non-Hermitian optical potential $\mathcal{V}$, and of its Hermitian conjugate $\mathcal{V}^{\dagger}$, i.e.,

$$\left[\mathcal{E} - T - \mathcal{V}^{\dagger}(E)\right] \mid \chi_{\mathcal{E}}^{(-)}(E)\rangle = 0\,, \quad [\mathcal{E} - T - \mathcal{V}(E)] \mid \tilde{\chi}_{\mathcal{E}}^{(-)}(E)\rangle = 0\,.$$

spectral representation of the hadron tensor

♡ : spectral representation of the Green's function:

$$\mathcal{G}(E) = \int_M^\infty \mathrm{d}\mathcal{E} \mid \tilde{\chi}_{\mathcal{E}}^{(-)}(E)\rangle \frac{1}{E - \mathcal{E} + i\eta} \langle \chi_{\mathcal{E}}^{(-)}(E) \mid$$

♠ : expanded form for the single particle expressions of the hadron tensor components

$$\omega^{\mu\nu}(\omega, q) = -\frac{1}{\pi} \sum_n \mathrm{Im} \left[\int_M^\infty \mathrm{d}\mathcal{E} \frac{1}{E_\mathrm{f} - \epsilon_n - \mathcal{E} + i\eta} T_n^{\mu\nu}(\mathcal{E}, E_\mathrm{f} - \epsilon_n) \right]$$

$$T_n^{\mu\mu}(\mathcal{E}, E) = \lambda_n \langle \varphi_n \mid j^{\mu\dagger}(\boldsymbol{q}) \sqrt{1 - \mathcal{V}'(E)} \mid \tilde{\chi}_{\mathcal{E}}^{(-)}(E)\rangle \langle \chi_{\mathcal{E}}^{(-)}(E) \mid \sqrt{1 - \mathcal{V}'(E)} j^{\mu}(\boldsymbol{q}) \mid \varphi_n \rangle$$

♢ : similar expressions for the terms with $\mu \neq \nu$

♣ : expanded form for the hadron tensor components

$$\omega^{\mu\nu}(\omega, q) = \sum_n \left[\mathrm{Re}\, T_n^{\mu\nu}(E_\mathrm{f} - \epsilon_n, E_\mathrm{f} - \epsilon_n) - \frac{1}{\pi} \mathcal{P} \int_M^\infty \mathrm{d}\mathcal{E} \frac{1}{E_\mathrm{f} - \epsilon_n - \mathcal{E}} \mathrm{Im}\, T_n^{\mu\nu}(\mathcal{E}, E_\mathrm{f} - \epsilon_n) \right]$$

- $\lim_{\eta \to 0} \frac{1}{E - \mathcal{E} + i\eta} = \mathcal{P}\left(\frac{1}{E - \mathcal{E}} \right) - i\pi\delta\left(E - \mathcal{E}\right)$
- $\sqrt{1 - \mathcal{V}'(E)}$ accounts for interference effects between different channels and allows the replacement of the mean field $\mathcal{V}$ with the phenomenological optical potential $\mathcal{V}_\mathrm{L}$

comments ...

♡ : $\sqrt{\lambda_n}\langle\chi_{\mathcal{E}}^{(-)}(E) \mid j^{\mu}(\boldsymbol{q}) \mid \varphi_n\rangle \Longrightarrow$ transition amplitude for the single-nucleon knockout from a nucleus in the state $\mid \Psi_0\rangle$ leaving the residual nucleus in the state $\mid n\rangle$
$\Longrightarrow$ attenuation of the strength due to imaginary part of $\mathcal{V}^{\dagger}$

♠ : in the case of inclusive reactions this attenuation must be compensated

♢ : $\sqrt{\lambda_n}\langle\varphi_n \mid j^{\mu\dagger}(\boldsymbol{q})\sqrt{1-\mathcal{V}'(E)} \mid \tilde{\chi}_{\mathcal{E}}^{(-)}(E)\rangle \Longrightarrow$ gain due to the flux lost towards the channel n by the other final states asymptotically originated by the channels different from n

♣ : usual shell-model calculation: sum, over all the single-particle shell-model states, of the squared absolute value of the transition matrix elements
$\Longrightarrow$ loss of flux that is inconsistent with the inclusive process, where all the inelastic channels must be considered and the total flux must be conserved

♡ : Green's function approach $\Longrightarrow$ the flux is conserved: the loss of flux, produced by the negative imaginary part of the optical potential in χ, is compensated by the gain of flux, produced in the first matrix element by the positive imaginary part of the Hermitian conjugate optical potential in $\tilde{\chi}$

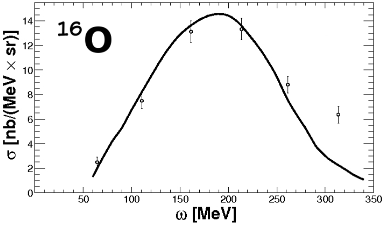
♠ : relativistic Green's function $\Longrightarrow$ requires matrix elements of the same type as usual RDWIA models, but involves eigenfunctions of both $\mathcal{V}$ and $\mathcal{V}^{\dagger}$ $\Longrightarrow$ FSI are described by the same complex optical potential as in RDWIA but the imaginary part is used in a different way

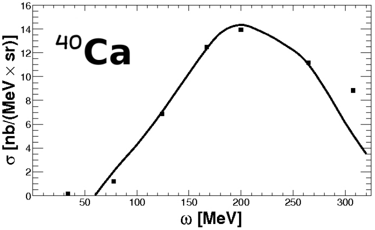
♢ : relativistic Green's function $\Longrightarrow$ consistent treatment of FSI in the exclusive and in the inclusive scattering

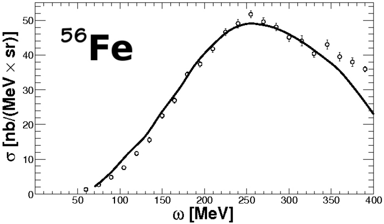
♣ : The imaginary part of the optical potential plays a decisive role and makes the RGF very different from models like rROP, where only the real part of the optical potential is retained

♡ : see A. Meucci, F. Capuzzi, C.Giusti, F.D. Pacati, Phys.Rev. C **67**, 054601 (2003); A. Meucci, C.Giusti, F.D. Pacati, Nucl. Phys. A 739, 277 (2004)

electron scattering







♡ : RGF and (e, e') cross section (examples)

♣ : ^{16}O $\Longrightarrow$ $E_e = 1080$ MeV and $\vartheta_e = 32^o$

♢ : data M. Anghinolfi, *et al.*, Nucl. Phys. **A602**, 405 (1996)

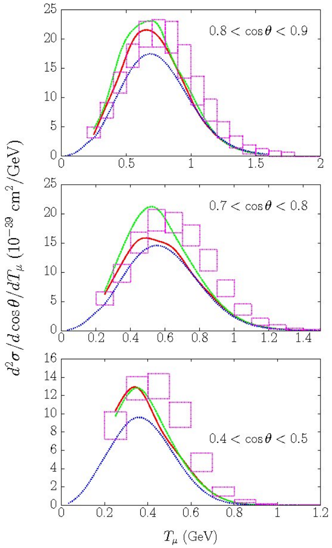
♠ : ^{40}Ca $\Longrightarrow$ $E_e = 841$ MeV and $\vartheta_e = 45^o$

♡ : data C. Williamson, *et al.*, Phys. Rev. C **56**, 3152 (1997)

♣ : ^{56}Fe $\Longrightarrow$ $E_e = 2020$ MeV and $\vartheta_e = 20^o$

♢ : data D.B. Day, *et al.*, Phys. Rev. C **48**, 1849 (1993)

MiniBooNE CCQE double differential cross section



♡ : CCQE double differential cross section averaged over the neutrino flux as a function of the muon kinetic energy

♣ : 3 bins of $\cos\theta$ and $M_A = 1.03$ GeV

♢ : RMF: reasonable agreement with data for small θ and low T_μ

♠ : RMF: the scattering states are described by using the same real scalar and vector relativistic mean field potentials considered in the description of the initial bound state

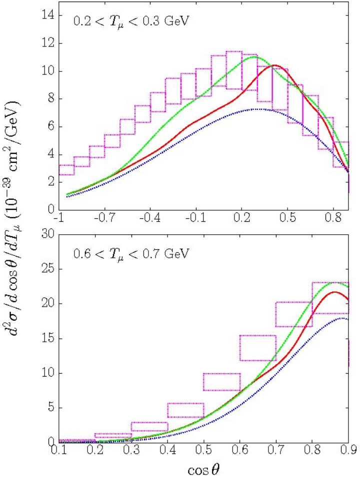
♡ : RGF-EDAD1 and RGF-EDAI: results larger than the RMF and in reasonable agreement with data

♣ : RGF can include also contributions of channels that are not included in the model but can be recovered by the imaginary part of the optical potential (e.g., re-scattering, non-nucleonic Δ excitations, multinucleon processes)

♢ : see A. Meucci, M.B. Barbaro, J.A. Caballero, C.Giusti, J.M. Udías, Phys. Rev. Lett. **107**, 172501 (2011)

♠ : data from A.A. Aguilar-Arevalo, *et al.*, Phys. Rev. D **81**, 092005 (2010)

MiniBooNE CCQE double differential cross section



♡ : CCQE double differential cross section averaged over the neutrino flux as a function of $\cos\theta$

♣ : 2 bins of T_μ

♢ : $M_A = 1.03$ GeV

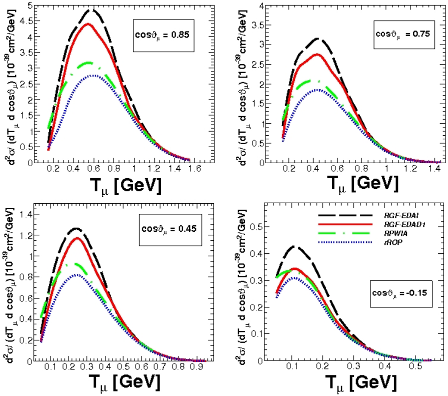
♠ : RMF: underestimates the data for low T_μ and better agreement as T_μ increases

♡ : RGF-EDAD1 and RGF-EDAI: results larger than the RMF and in reasonable agreement with data for small θ

♣ : see A. Meucci, M.B. Barbaro, J.A. Caballero, C.Giusti, J.M. Udías, Phys. Rev. Lett. **107**, 172501 (2011)

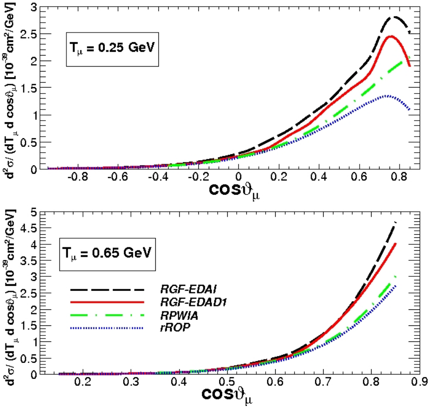
♢ : data from A.A. Aguilar-Arevalo, *et al.*, Phys. Rev. D **81**, 092005 (2010)

MiniBooNE CCQE $\bar{\nu}$ double differential cross section



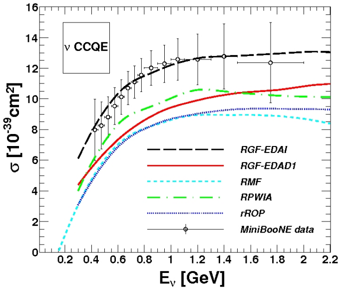
- ♡ : CCQE double differential cross section averaged over the antineutrino flux as a function of the muon kinetic energy
- ♣ : 4 bins of $\cos\theta$
- ♢ : $M_A = 1.03$ GeV
- ♠ : RPWIA and ROP
- ♡ : RGF $\longrightarrow$ results larger than the RPWIA
- ♣ : see A. Meucci, C.Giusti, Phys. Rev. D **85**, 093002 (2012)

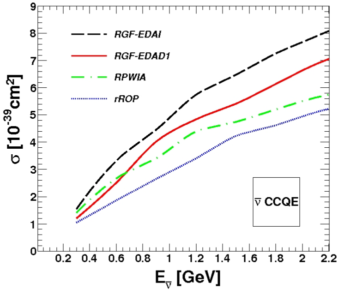
MiniBooNE CCQE $\bar{\nu}$ double differential cross section



- ♡ : CCQE double differential cross section averaged over the antineutrino flux as a function of $\cos\theta$
- ♣ : 2 bins of T_μ
- ♢ : $M_A = 1.03$ GeV
- ♠ : RPWIA and ROP
- ♡ : RGF $\longrightarrow$ results larger than the RPWIA
- ♣ : see A. Meucci, C.Giusti, Phys. Rev. D **85**, 093002 (2012)

Flux-unfolded MiniBooNE ν and $\bar{\nu}$ CCQE cross section





♡ : total CCQE cross section per neutron as a function of the neutrino or antineutrino energy

♣ : RMF and RGF

♢ : RPWIA and rROP

♠ : $M_A = 1.03$ GeV

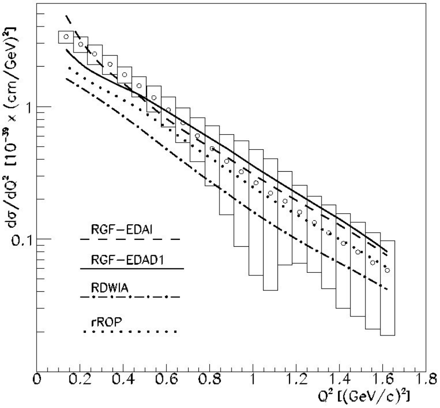
♡ : differences between RGF-EDAI and RGF-EDAD1 $\longrightarrow$ different values of the imaginary parts of both potentials, particularly for the energies considered in kinematics with the lowest θ and the largest T_μ

♣ : ν CCQE see A. Meucci, M.B. Barbaro, J.A. Caballero, C.Giusti, J.M. Udías, Phys. Rev. Lett. **107**, 172501 (2011)

♢ : $\bar{\nu}$ CCQE see A. Meucci, C.Giusti, Phys. Rev. D **85**, 093002 (2012)

♠ : ν data from A.A. Aguilar-Arevalo, *et al.*, Phys. Rev. D **81**, 092005 (2010)

MiniBooNE NCE cross section: relativistic Green's function



♡ : flux-averaged differential cross section as a function of Q^2 for NCE scattering on CH_2

♣ : see A.A. Aguilar-Arevalo, et al., Phys. Rev. D **82**, 092005 (2010)

♣ : RGF, RDWIA, and rROP

♢ : $M_A = 1.03$ GeV and $\Delta s = 0$

♠ : The RGF is suitable for an inclusive process like (e, e') or (ν_μ, μ^-)

♡ : NCE scattering $\Longrightarrow$ RGF can include also contributions of channels that are not included in the NCE MiniBooNE cross section

♣ : possible contribution of reaction channels like re-scattering processes of the nucleon in its way out of the nucleus or non-nucleonic Δ excitations, which may arise during nucleon propagation, with or without real pion production, or also multinucleon processes $\Longrightarrow$ they are not included explicitly in the Green's function but can be recovered, to some extent, by the imaginary part of the phenomenological optical potential

♢ : see A. Meucci, C.Giusti, F.D. Pacati, Phys. Rev. D **84**, 113003 (2011)

Conclusions

♡ : relativistic models for neutrino-nucleus scattering

♠ : Relativistic Green's Function

♢ : RGF: matrix elements of the same type as usual RDWIA models, but eigenfunctions of both $\mathcal{V}$ and $\mathcal{V}^{\dagger}$ are involved $\Longrightarrow$ the imaginary part of the optical potential is very important $\Longrightarrow$ RGF very different from models like rROP

♣ : results for the (e, e') cross section

♡ : results for the CCQE and NCE cross sections from MiniBooNE

♠ : Thank you very much!