SuperScaling in electron-nucleus scattering and its link to CC and NC QE neutrino-nucleus scattering

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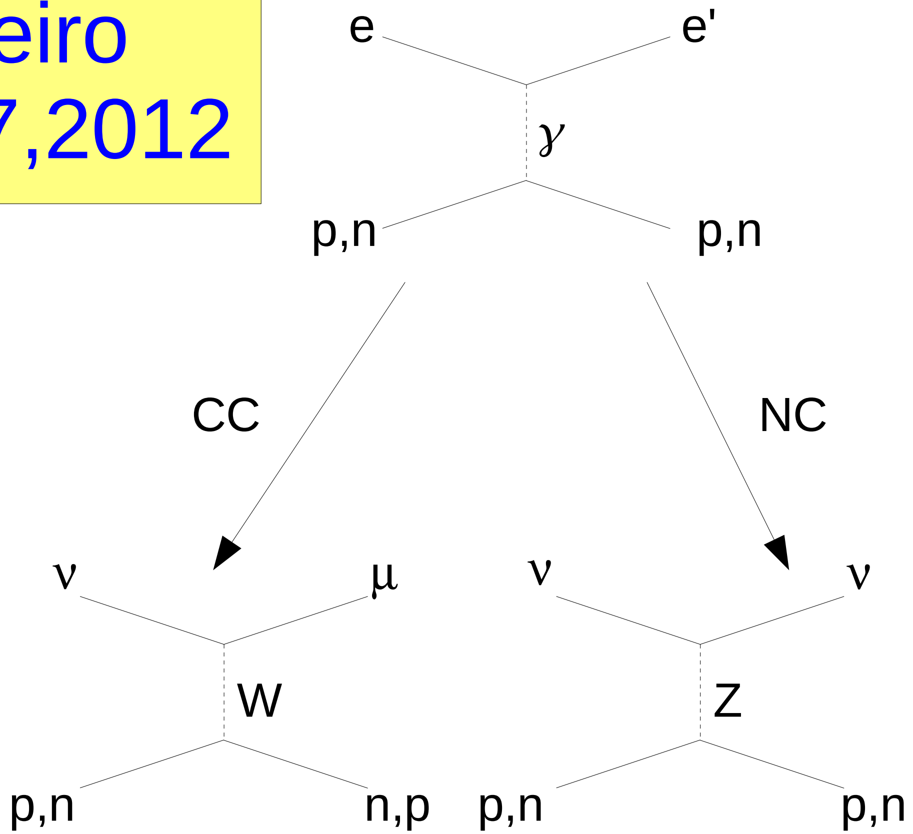

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C. Williamson (MIT, USA)

Outline

- Review of SuperScaling in quasi-elastic inclusive electron scattering
- Connecting electron and neutrino scattering via SuperScaling: the “SuSA” approach
- Extended SuSA model: Meson Exchange Currents
- Application to CC quasielastic neutrino scattering and comparison with MiniBooNE cross sections
- Application to NC quasielastic neutrino scattering and comparison with MiniBooNE cross sections and ratio p/N
- Predictions for antineutrino scattering (see Joe Grange's talk for comparison with data)

SuperScaling Approximation

- Many high quality data are available for quasi-elastic electron scattering

  [O. Benhar, D. Day and I. Sick, Rev. Mod. Phys. 80 (2008) , http://faculty.virginia.edu/qes-archive/]

- Any reliable nuclear model must reproduce these data
- Is there a way to use (e,e') data to predict CC and NC ν-scattering cross sections in the QE region?
- Answer: yes, exploiting Super-Scaling properties of QE electron scattering

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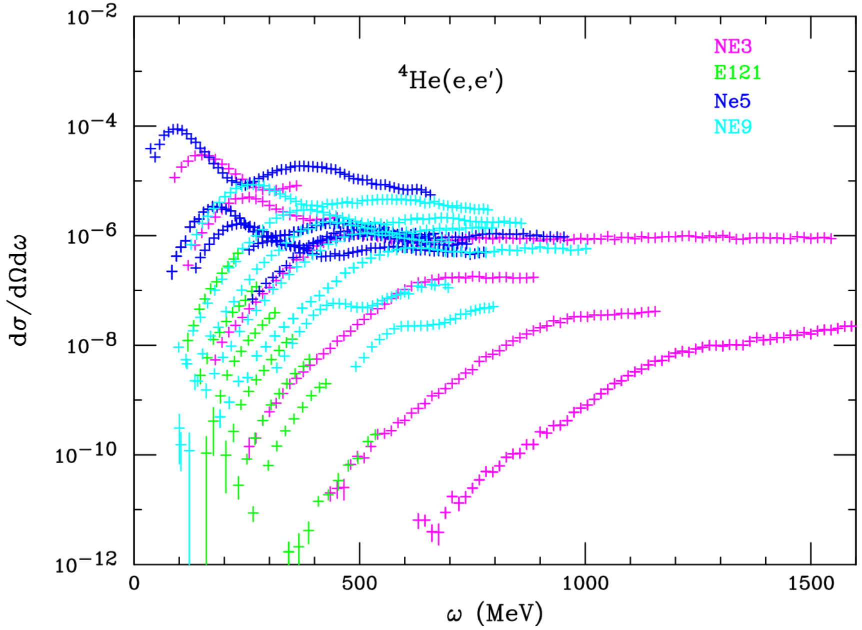

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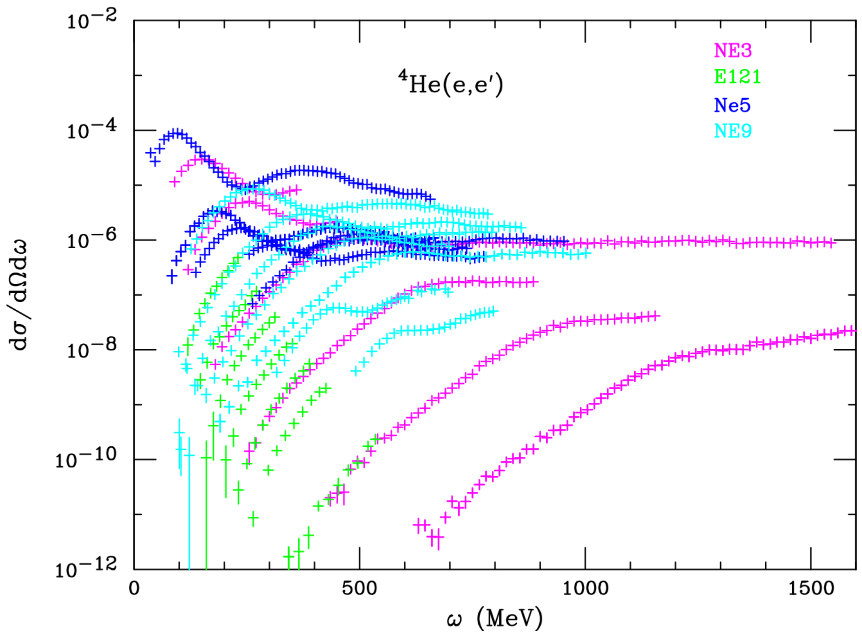

Assume that QE scattering is dominated by (e,e'N)

$$F(q,y) = \frac{d\sigma/d\Omega d\omega}{\sigma_{Mott}(v_L G_L + v_T G_T)}$$ scaling function

$$y(q,\omega) = -p_{min}$$ scaling variable (y=0 QEP)

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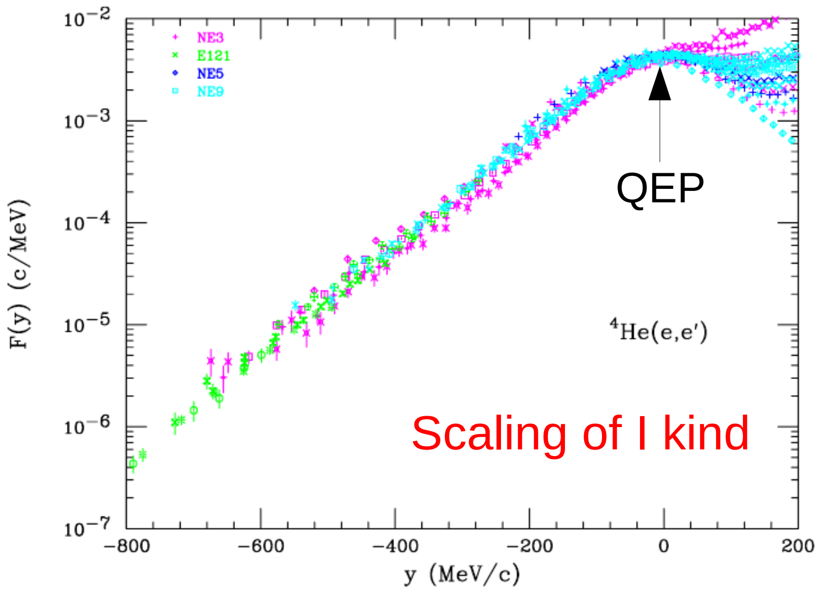

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$$F(q,y) \rightarrow F(y)$$ *q* sufficiently large (*q>400 MeV/c roughly*)

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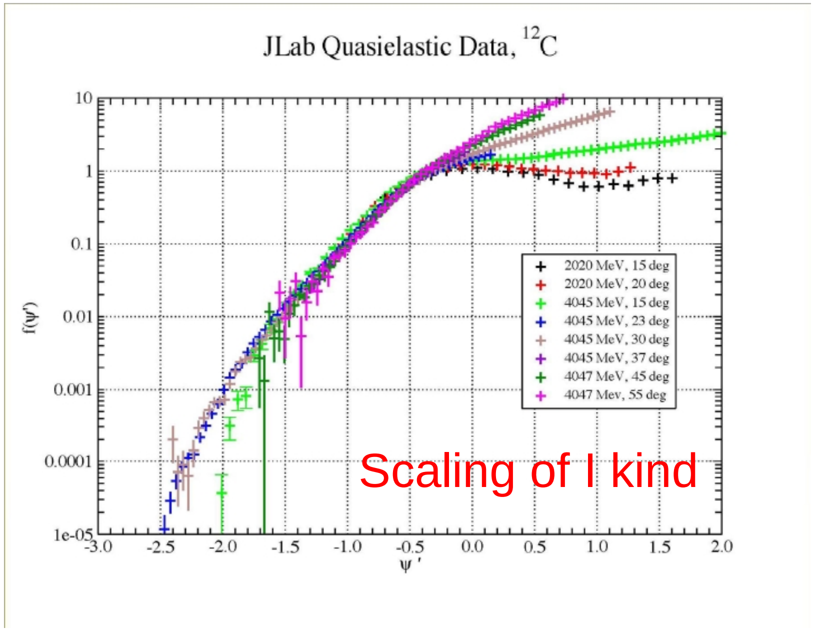

$$\psi(q,\omega)=\pm\sqrt{T_{min}/T_F}\simeq y/k_F$$ scaling variable (relativistic)

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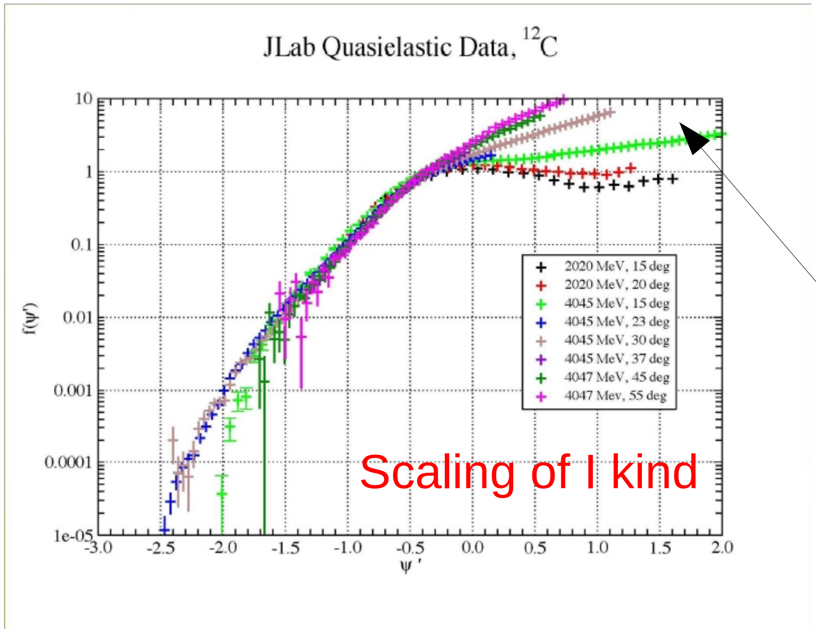

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At ψ>0 scaling is broken by resonances, meson production, etc., mainly in the transverse channel (from analysis of L/T separated data)

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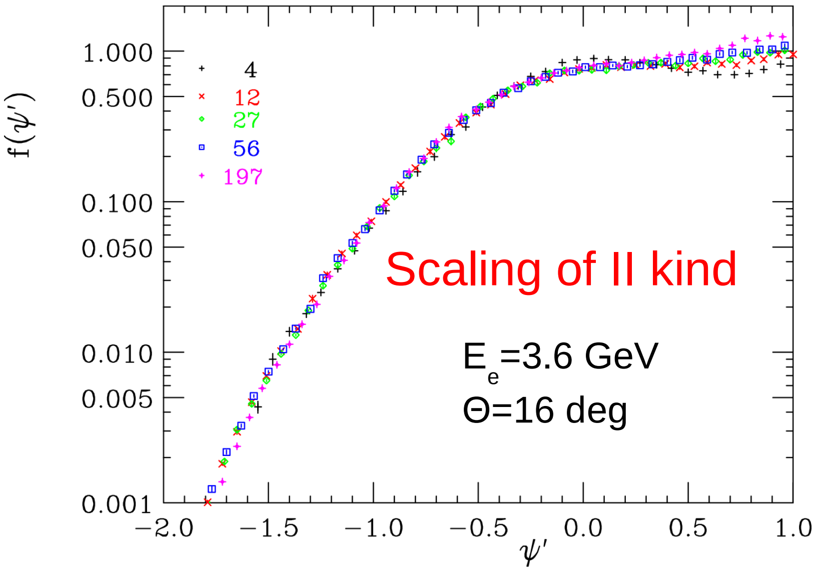
$$f = k_F * F$$

super-scaling function

k_F Fermi momentum

Plot f as a function of ψ
for different nuclei at fixed kinematics:



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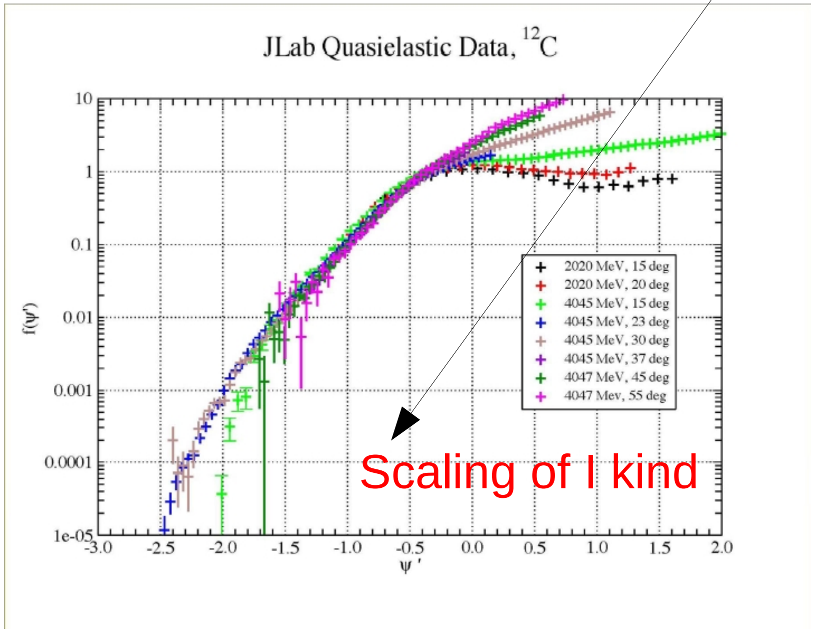

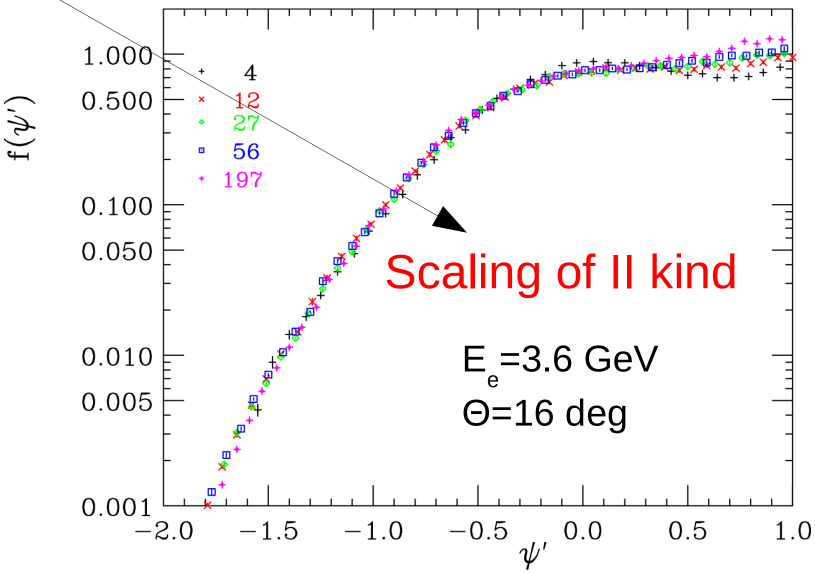

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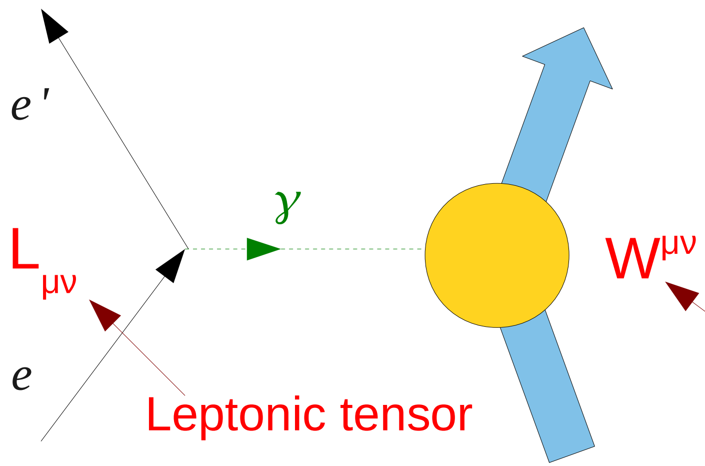
“SuSA” approach

1) extract the super-scaling function from QE electron scattering data
2) plug it into neutrino cross sections

$$f(\psi) \sim R^{em}/G^{em}_{s.n.} \longrightarrow R^{weak} \sim G^{weak}_{s.n.} * f(\psi)$$

Formalism: (l,l') inclusive scattering

(e,e')



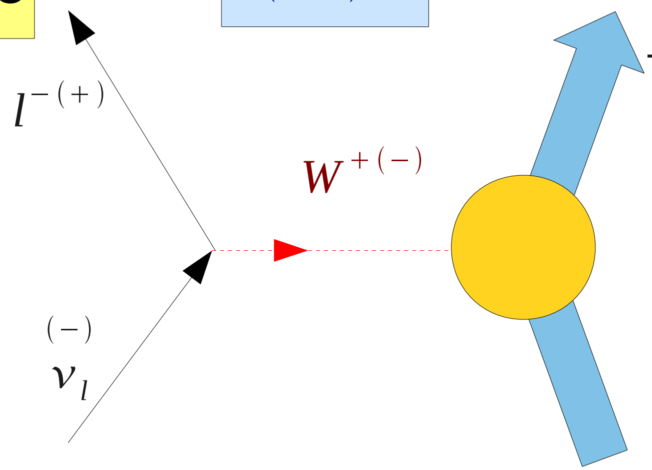
$$\frac{d^2\sigma}{d\epsilon' d\Omega'} = \sigma_{Mott}(v_L R_L + v_T R_T)$$

2 electromagnetic response functions

Hadronic tensor

CC

(ν, l)



$$\frac{d^2\sigma}{d\epsilon' d\Omega'} = \sigma_0(\boxed{V_{CC}R_{CC} + 2V_{CL}R_{CL} + V_{LL}R_{LL}} + V_T R_T \pm 2V_{T'}R_{T'})$$

$$V_L R_L$$

$l = \mu, e, \tau$

5 (3) weak response functions

$+\ \nu$

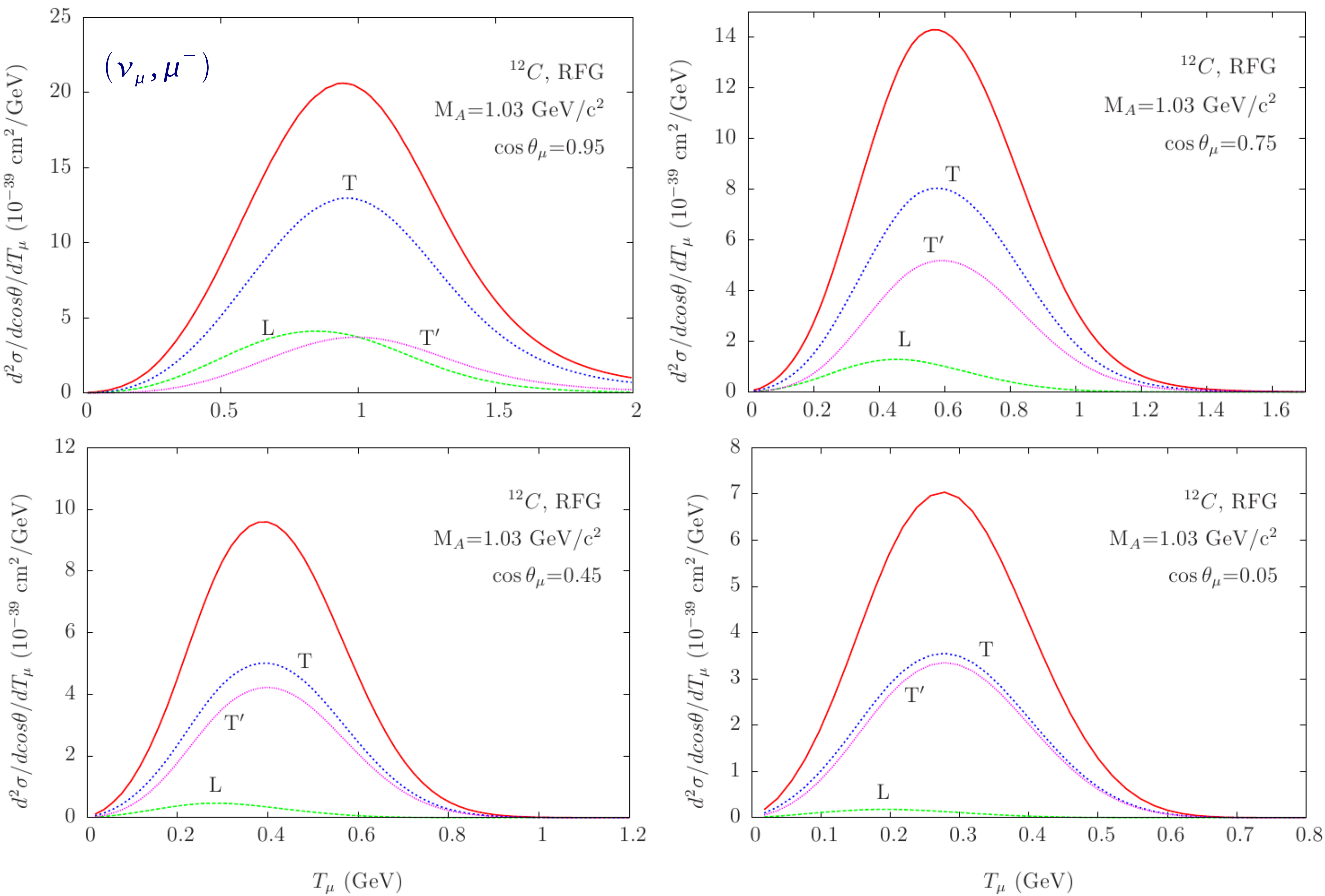
$-\ \nu$

Purely isovector
Typically transverse (CC,CL,LL small)
Have VV, AA and VA components generated by

$J_\mu = J_\mu^V + J_\mu^A$

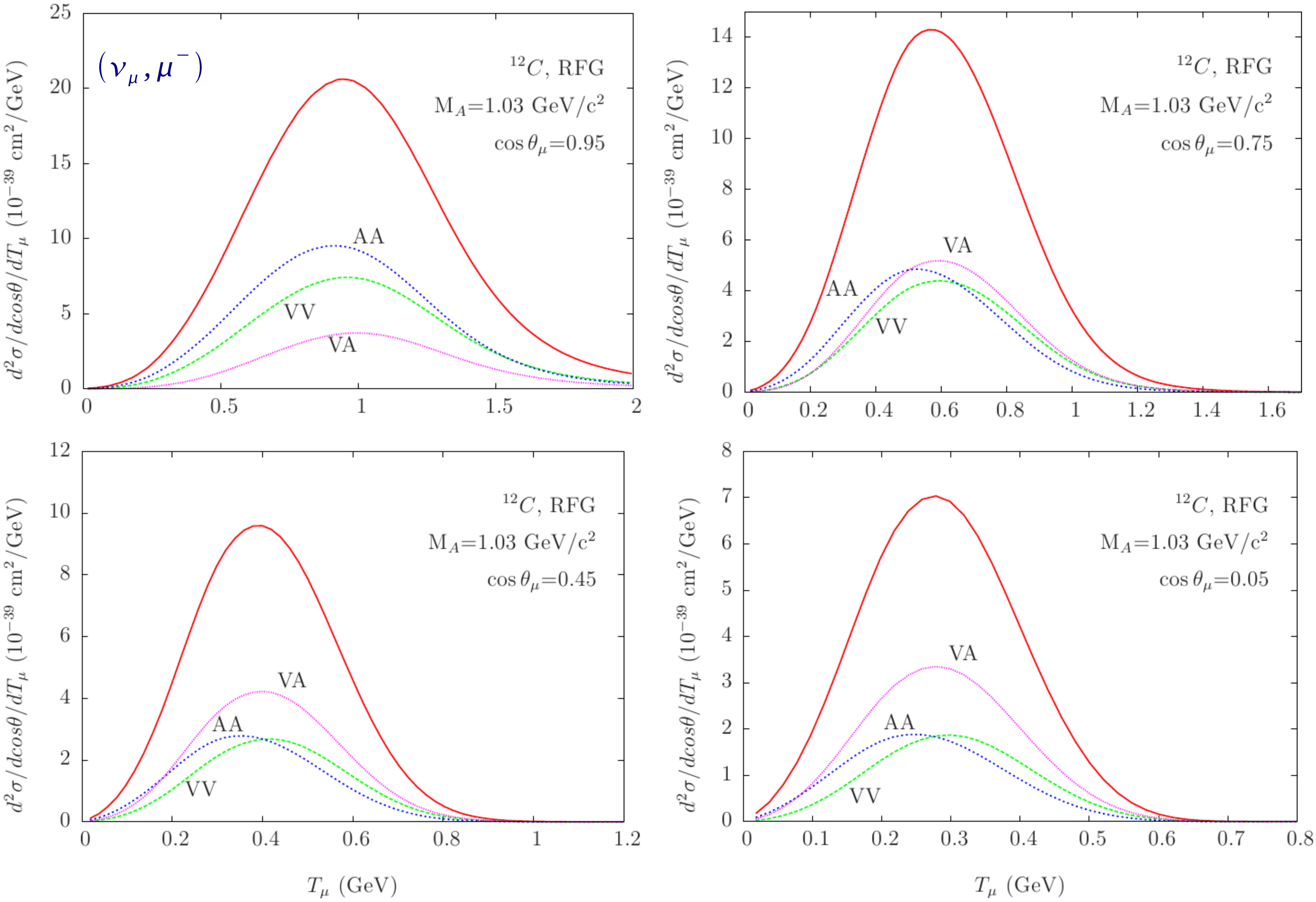
L-T-T' separation

MiniBooNE kinematics



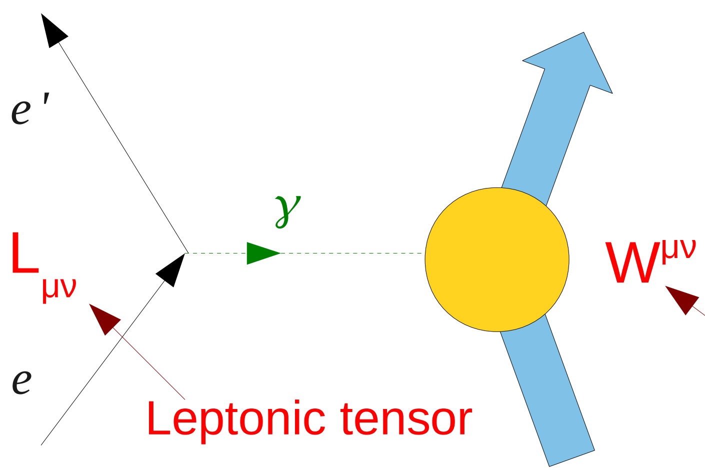
VV-AA-VA separation

MiniBooNE kinematics



Formalism: (l,l') inclusive scattering

(e,e')



$$\frac{d^2\sigma}{d\epsilon' d\Omega'} = \sigma_{Mott}(v_L R_L + v_T R_T)$$

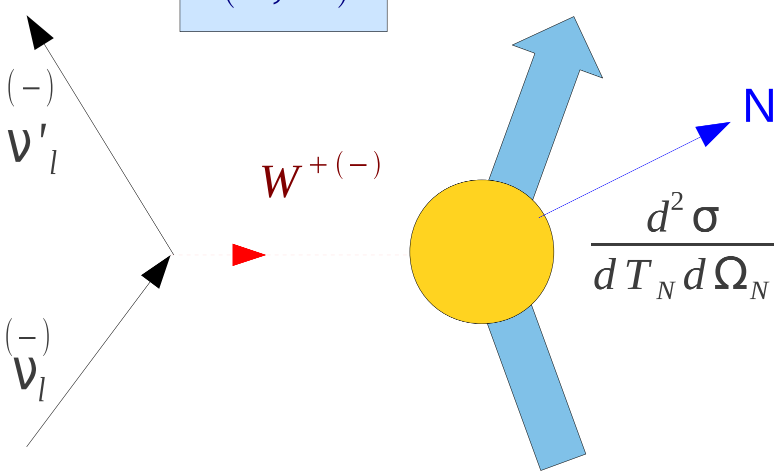
2 electromagnetic response functions

Hadronic tensor

t-inclusive
$t = (k_e - k'_e)^2$

NC

(ν,ν')



$$\frac{d^2\sigma}{dT_N d\Omega_N} = \sigma_0 [V_L R_L + V_T R_T + V_{TT} R_{TT} + V_{TL} R_{TL} \pm (2 V_{T'} R_{T'} + 2 V_{TL'} R_{TL'})]$$

$l = \mu, e, \tau$

6 weak response functions

u-inclusive
$u = (k_\nu - p_N)^2$

Phenomenological super-scaling function

- The analysis of (e,e') world data shows that:

  1. Scaling of I kind is reasonably good below the QE peak (ψ<0)

  2. Scaling of II kind is excellent in the same region

  3. Scaling violations, particularly of I kind, occur above the QEP and reside mainly in the transverse response

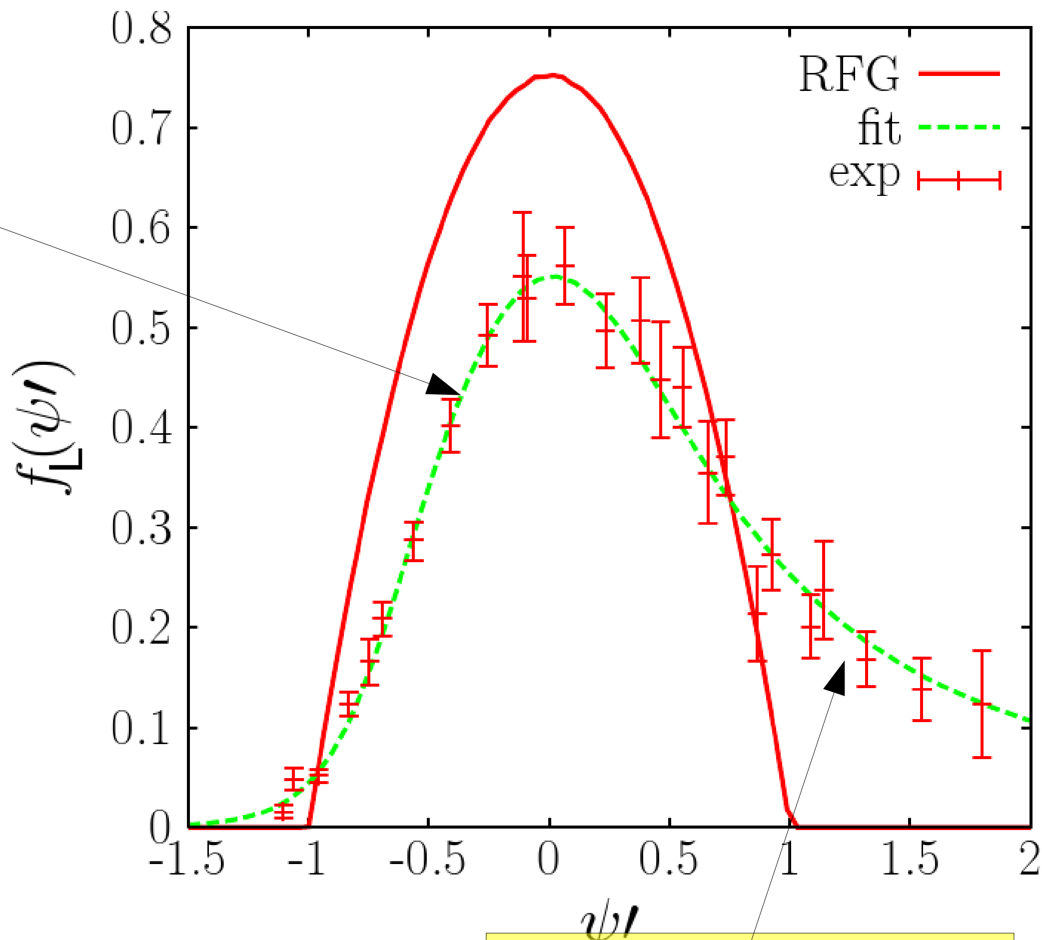
  4. The longitudinal response super-scales

- A phenomenological super-scaling function has been extracted from the (e,e') world data [Jourdan, NPA603, 117 ('96)]

  - Asymmetric shape: long tail at high energy transfer

  - Only 4 parameters for all kinematics and all nuclei

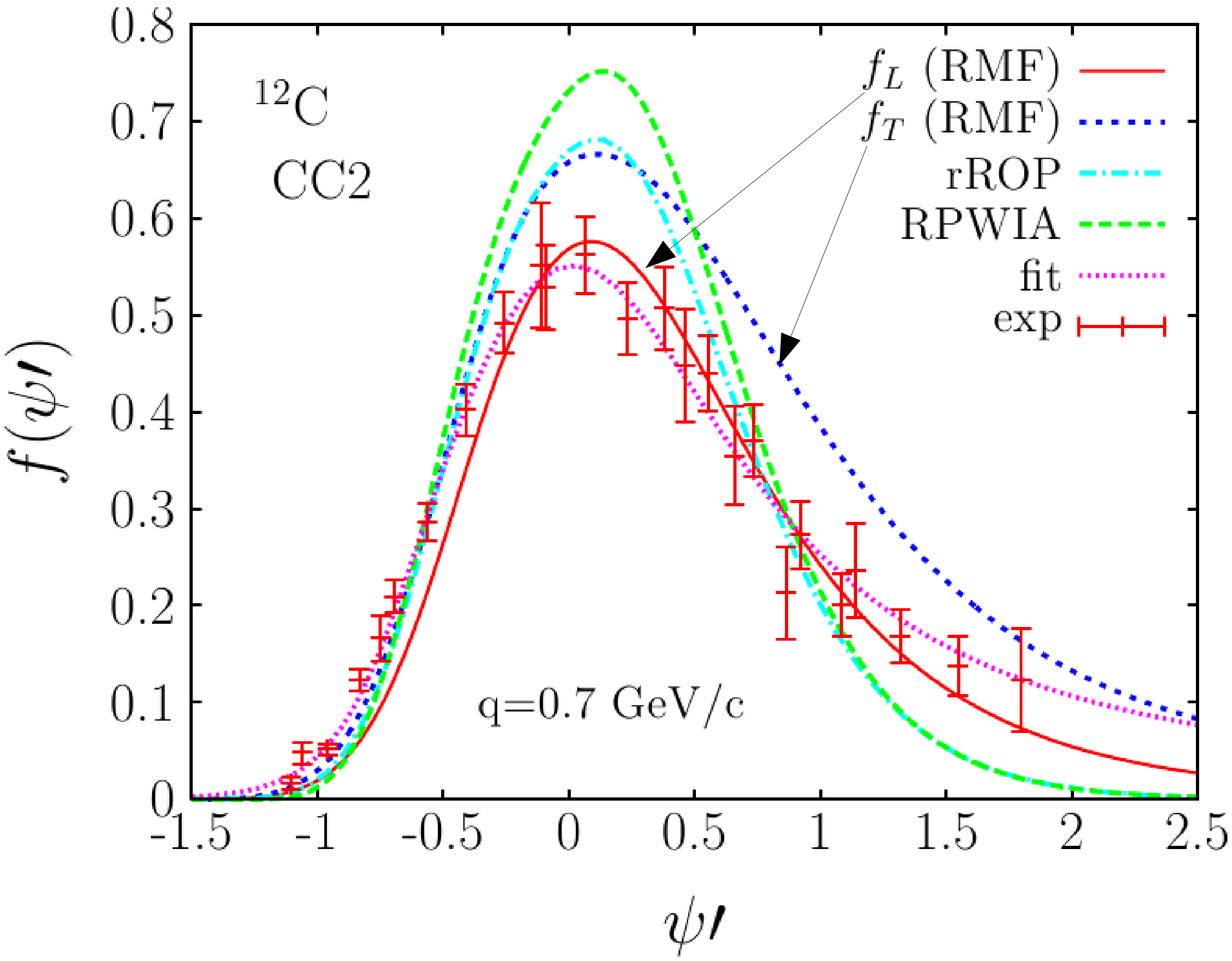
  - Represents a strong constraint on nuclear models



- The RFG is very poor: it does super-scale, but to the wrong function!

$$f_{RFG}(\psi)=\frac{3}{4}(1-\psi^2)\theta(1-\psi^2)$$

Scaling function(s) and microscopic models

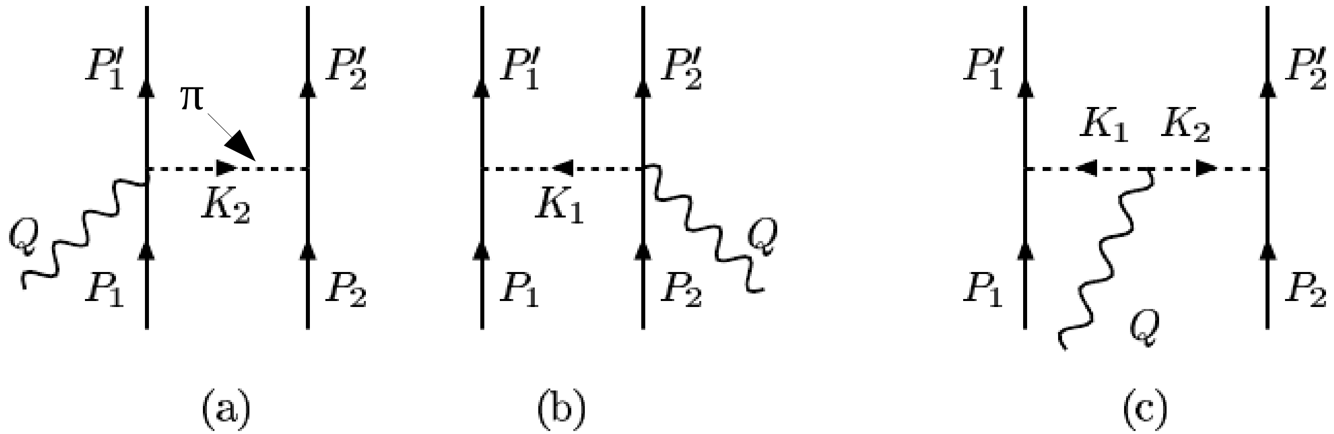


- The Relativistic Mean Field model successfully reproduces the experimental f_L
- $f_T^{RMF} > f_L^{RMF}$ by ~ 20%, in agreement with exp'tal evidence
- RPWIA and rROP give a symmetric scaling function and $f_L = f_T$

Modified SuSA model: Meson Exchange Currents

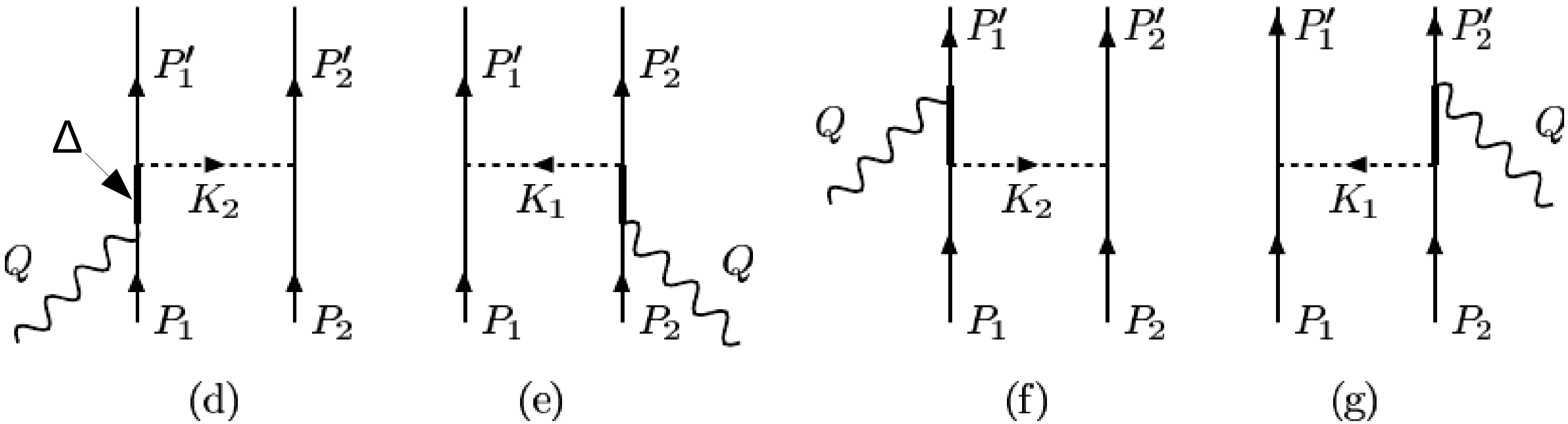
MEC are two-body currents involving 2 nucleons exchanging a meson.
Currents induced by the pion occur (up to higher order relativistic corrections) in the transverse channel and violate superscaling.
We follow a perturbative scheme, considering all the diagrams involving one pion in the two-body current. The calculation is fully relativistic and performed on the basis of RFG.

"contact" or "seagull"



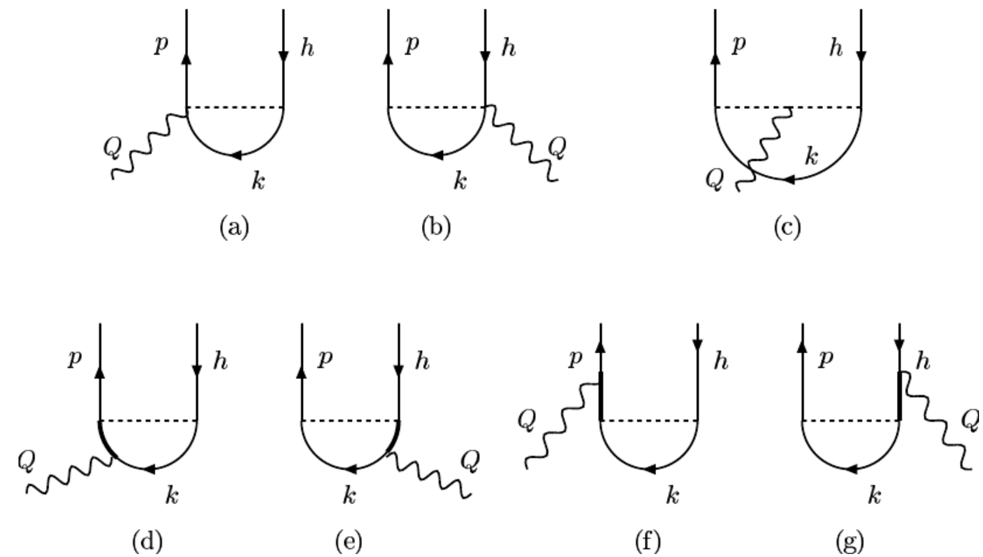
"pion-in-flight"

"Δ-MEC"



Meson Exchange Currents: 1p1h and 2p2h many-body diagrams

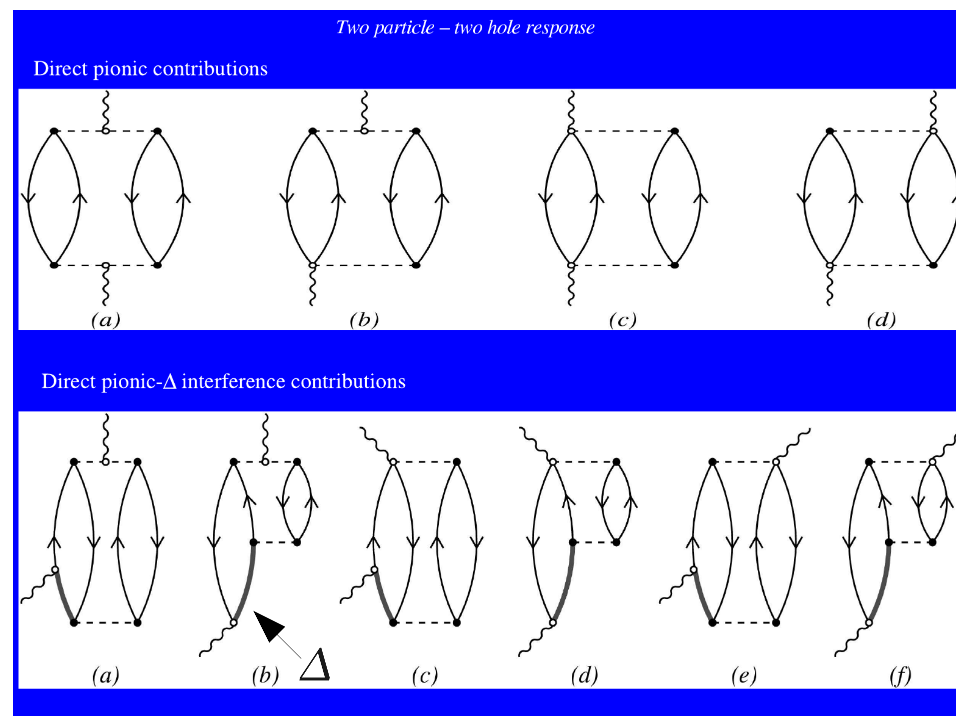
1p-1h sector:



Only contribute inside the RFG response region $-1<\psi<1$.
The net contribution to (e,e') QEP is small due to cancellations between MEC and correlations
[Amaro et al., Phys.Rept.368(2002),NPA723 (2003)]

2p-2h sector

(just a subset of all the 28 many-body diagrams involving two pionic lines) ⇒



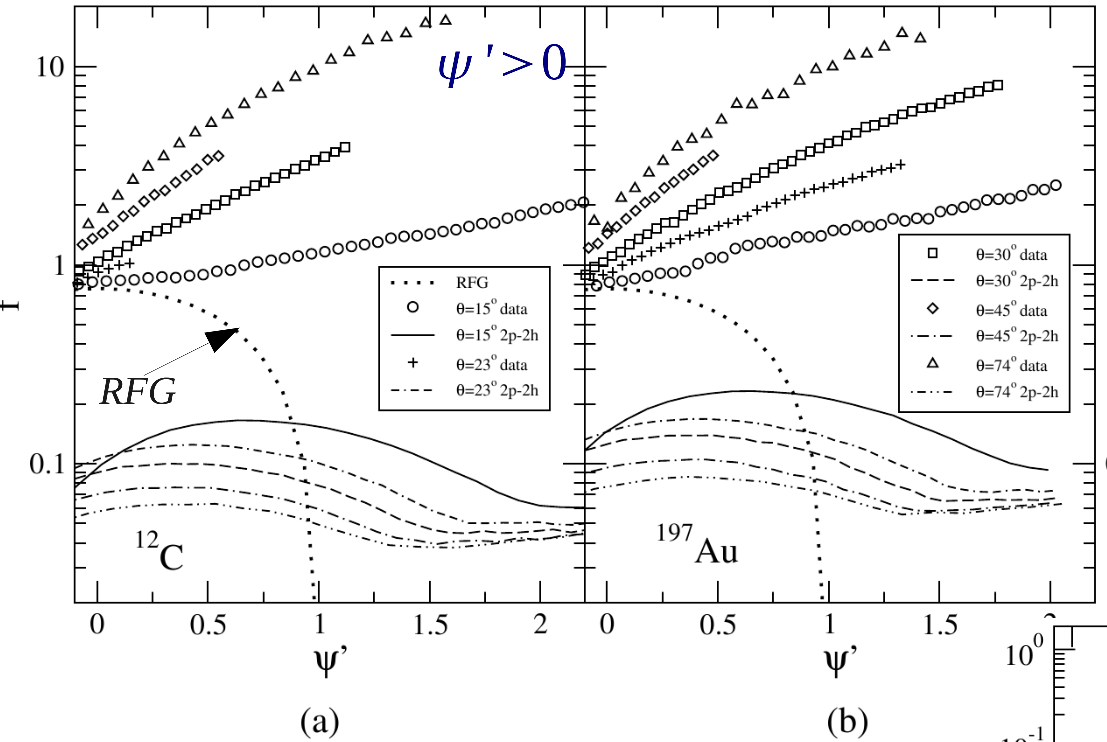
Contribute also outside the RFG response region: $\psi<-1$ and $\psi>1$

J.W.Van Orden, T.W.Donnelly, Ann. Phys. 131 (1981)
Non relativistic calculation with relativistic corrections

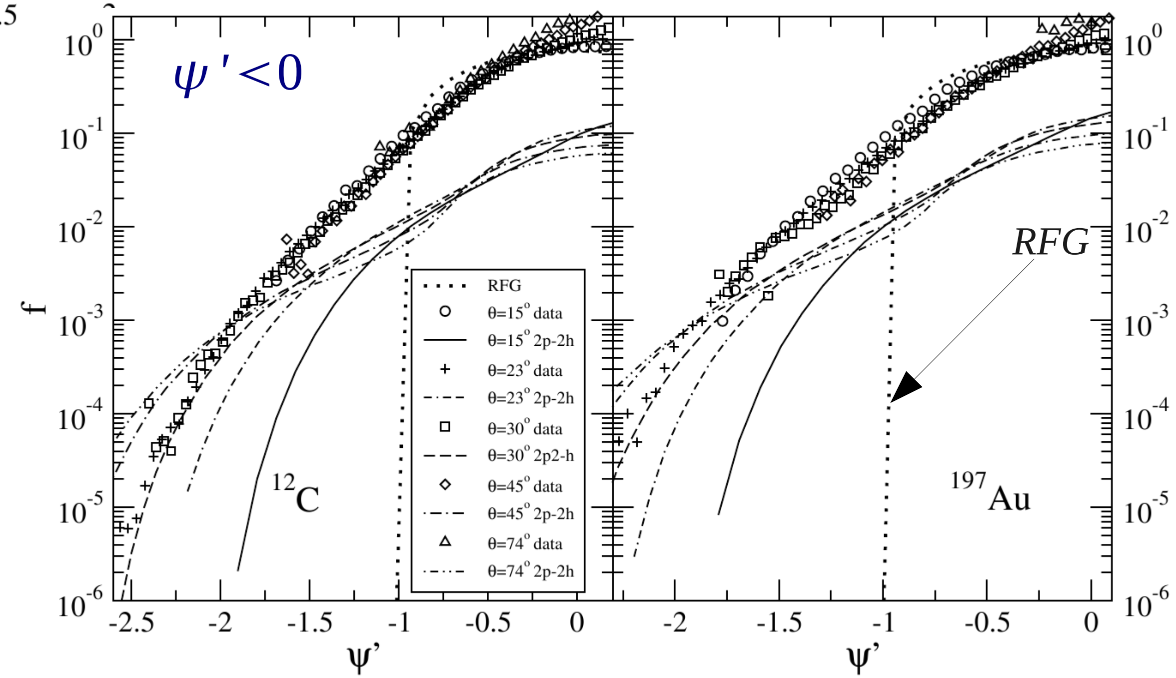
A.De Pace et al., NPA741 (2004)
Fully relativistic calculation

2p-2h MEC in electron scattering

De Pace et al., NPA741, 249 (2004), RFG-based calculation



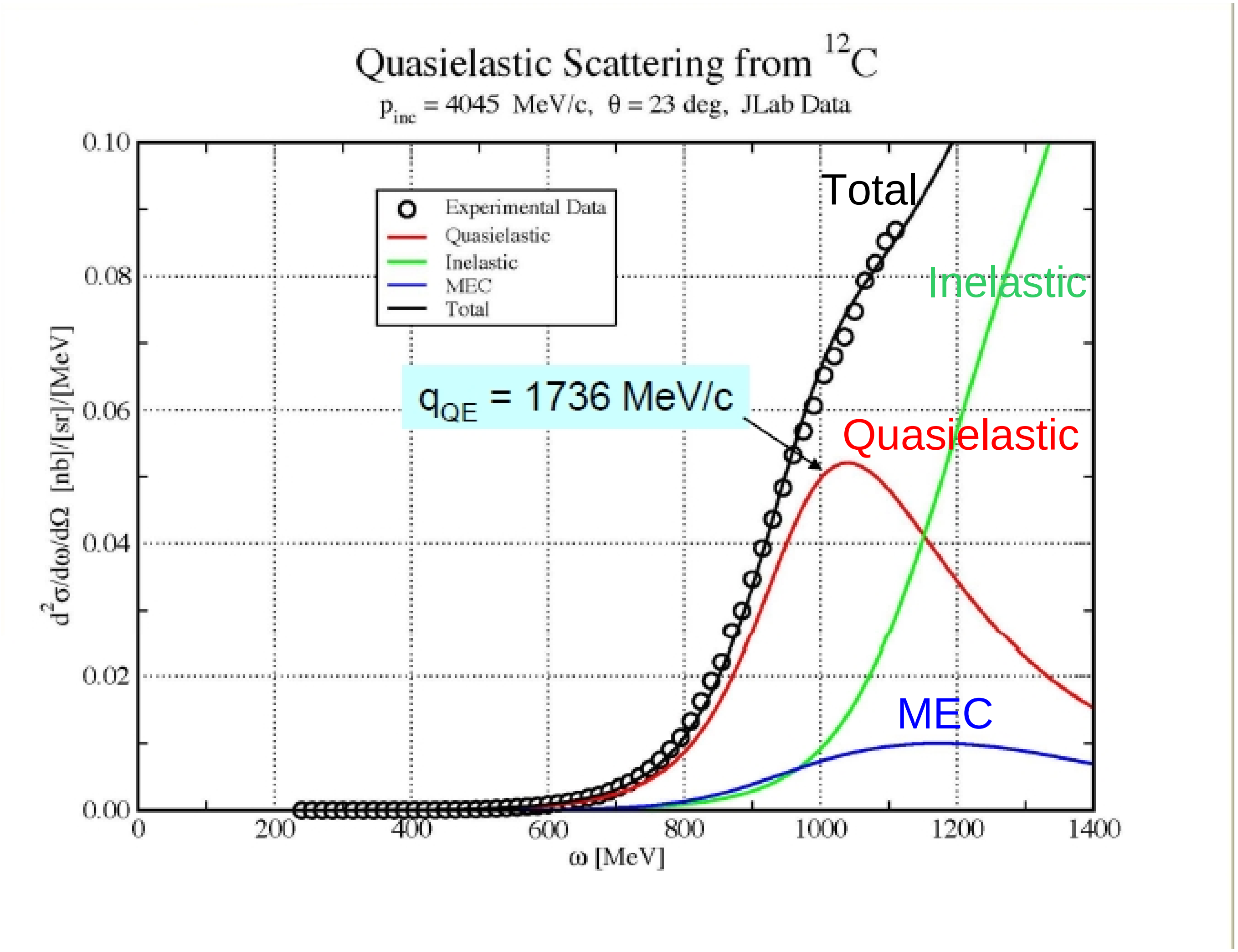
Scaling is broken both above and below the QEP



2p-2h MEC give a positive contribution of ~10-20% outside the QEP, filling the “dip” between the QE and Δ peak.

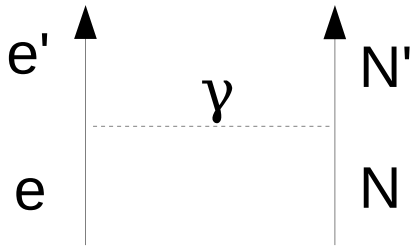
Test of the modified SuSA model: (e,e') cross section

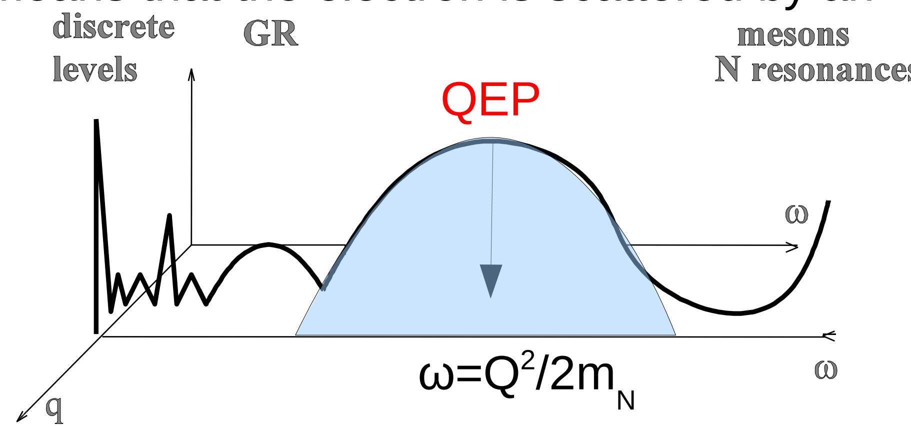
An example:



MEC in “quasielastic” neutrino scattering

In (e,e') experiments E_e is well-known and “QE” means that the electron is scattered by an individual nucleon moving inside the nucleus

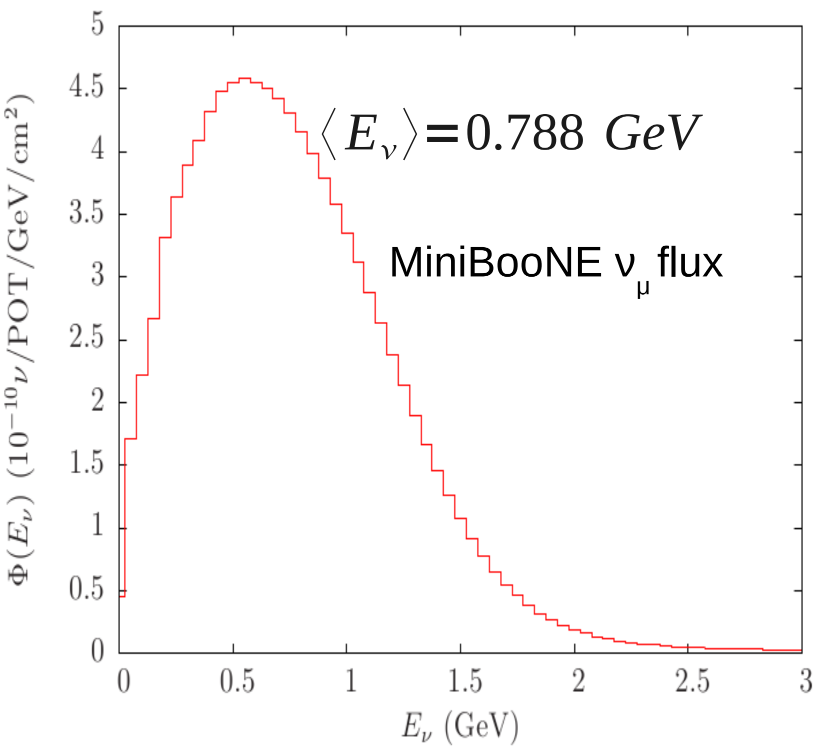




In (ν_μ,μ) the neutrino beam is not monochromatic, but it spans a wide range of energies

“Flux-averaged” cross section:

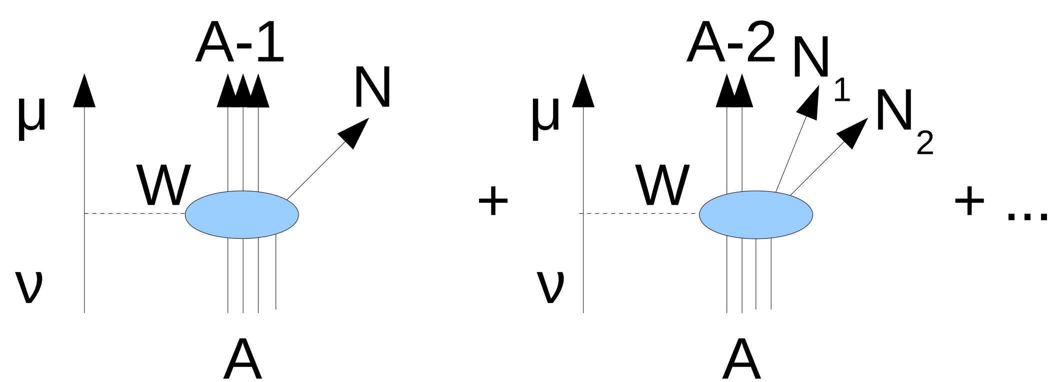
$$\left\langle \frac{d^2\sigma}{d\cos\theta\, dT_\mu} \right\rangle = \frac{1}{\Phi_{tot}} \int \frac{d^2\sigma(E_\nu)}{d\cos\theta\, dT_\mu} \phi(E_\nu)\, dE_\nu$$



Each experimental point (θ_μ,T_μ) takes contribution from different regions in the (q,ω) plane, corresponding to different reaction mechanisms.

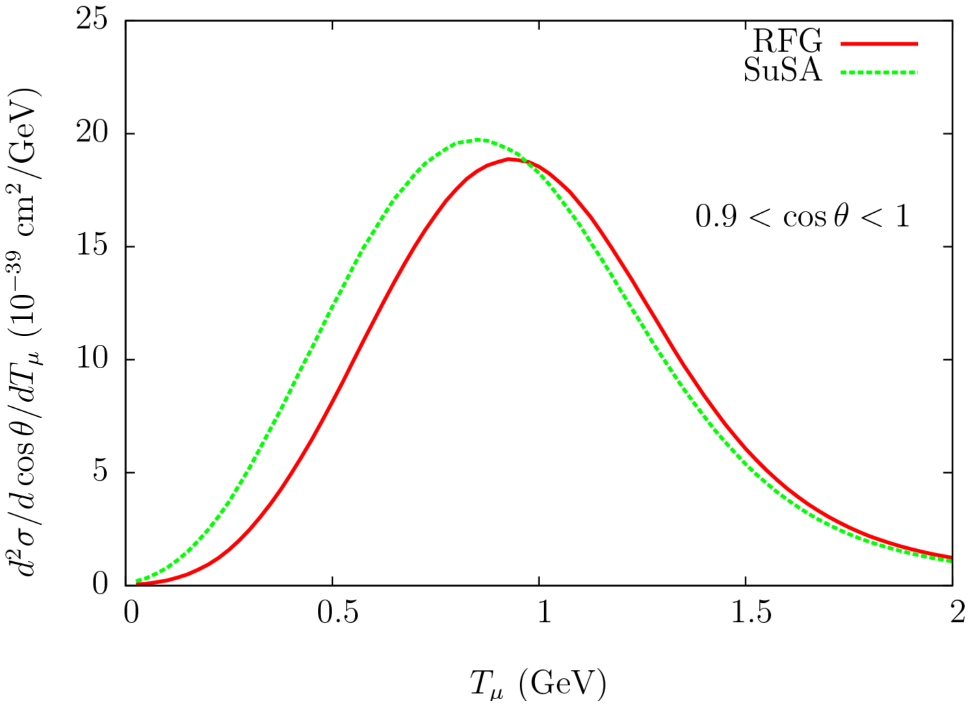
“QE”=no pions in the final state

Processes involving scattering off two or more nucleons must also be considered [Martini et al, Nieves et al]



Results for CCQE neutrino scattering

MiniBooNE double differential CCQE cross sections at forward angles



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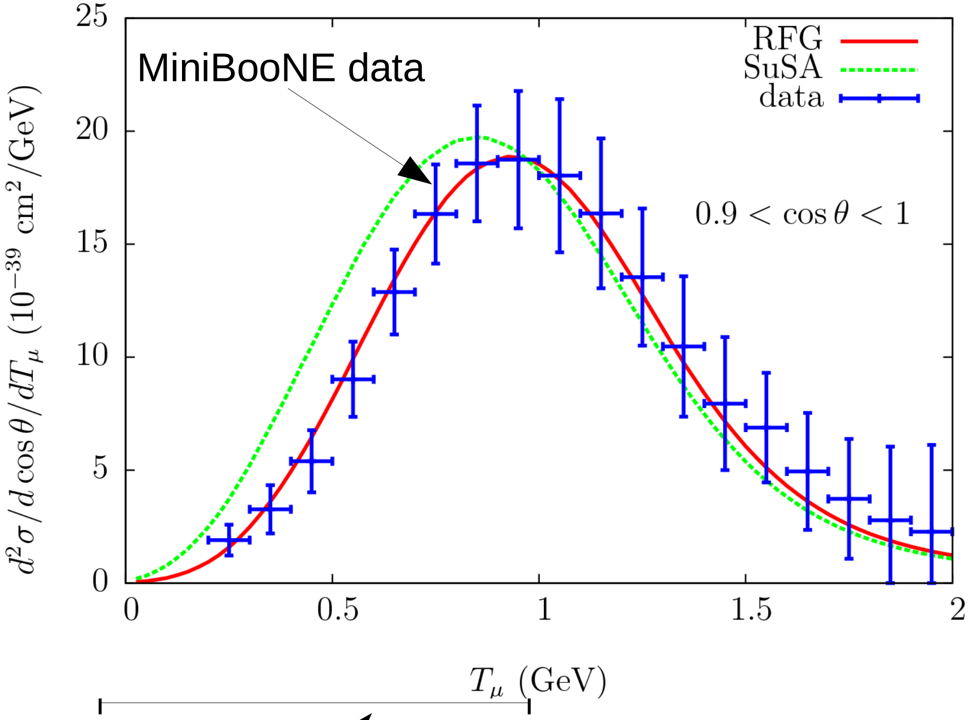

Pauli blocking is active in this region (low momentum transfers, q≲0.4 GeV/c): this explains the big difference between the RFG (where PB is included by definition) and the SuSA (which has no PB) results.

At very low angles both RFG and SuSA are compatible with the data, except for the Pauli-blocked region, where super-scaling ideas are not applicable.

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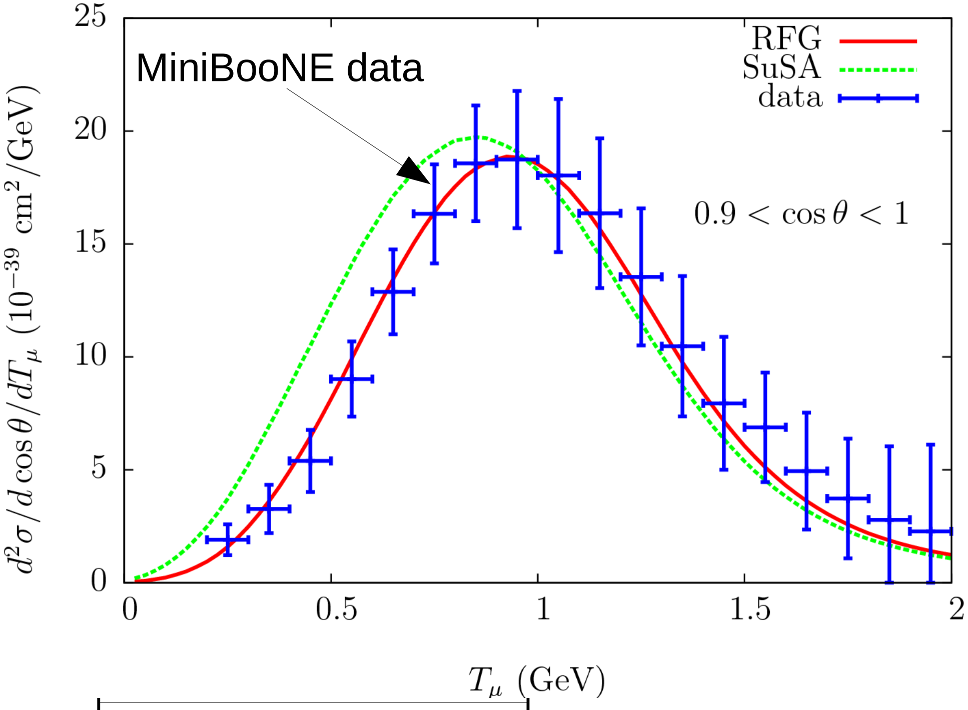
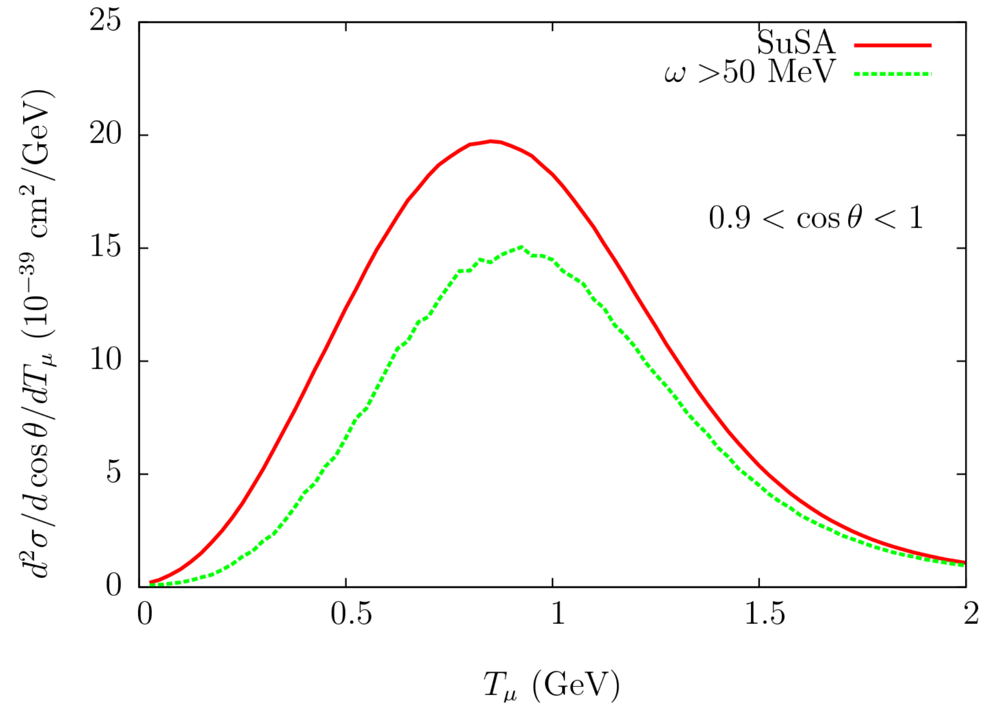


However: about ½ of the cross section for such kinematics arises from the first 50 MeV of excitation, where none of the two approaches should be trusted.

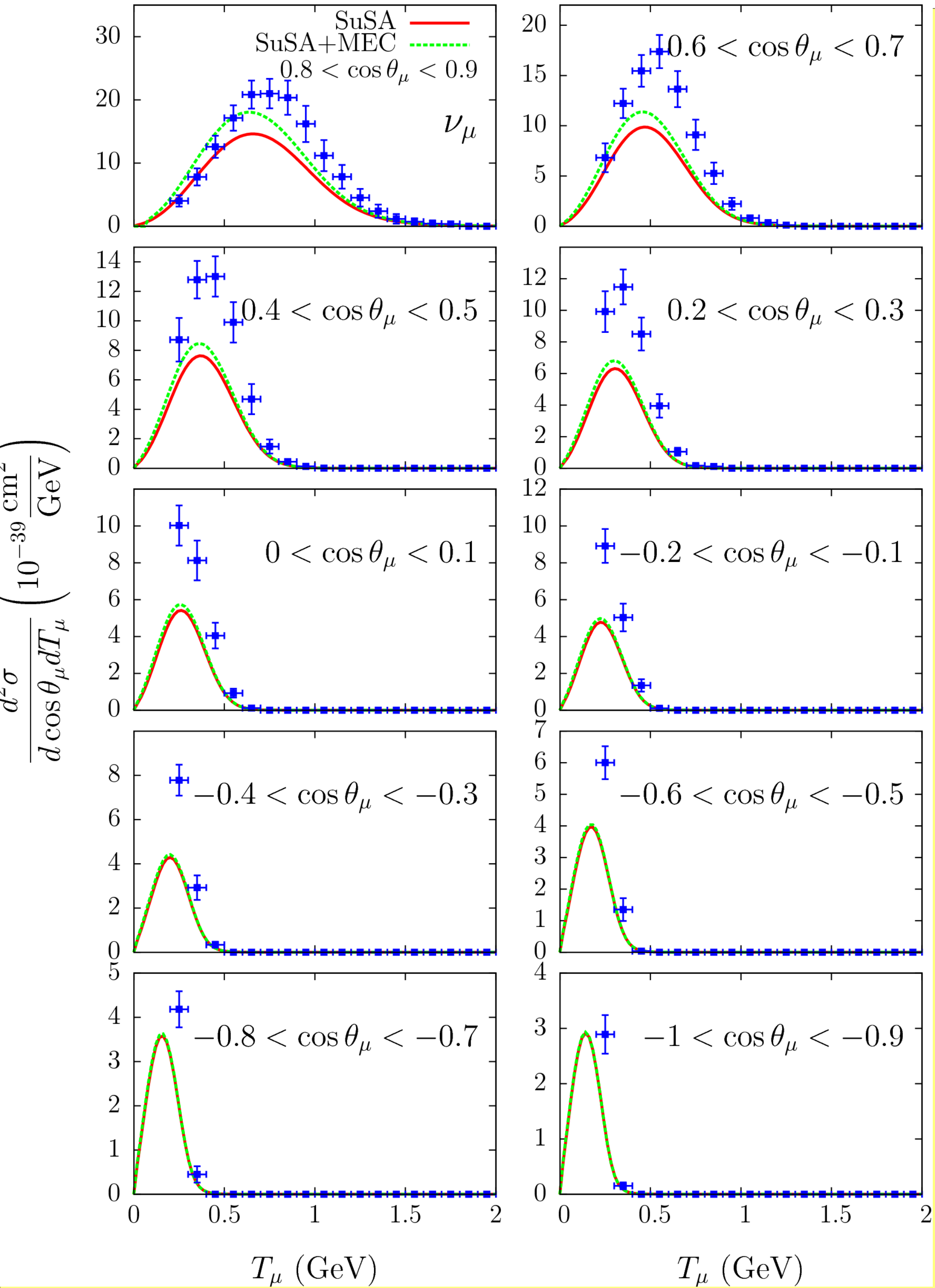


Pauli blocking is active in this region (low momentum transfers, q≤0.4 GeV/c): this explains the big difference between the RFG (where PB is included by definition) and the SuSA (which has no PB) results.

At very low angles both RFG and SuSA are compatible with the data, except for the Pauli-blocked region, where super-scaling ideas are not applicable.

Here a proper treatment of collective excitations, like RPA with realistic nuclear wave functions, is required.
[Amaro et al., PLB696 (2011)]

Comparison with MiniBooNE differential CC cross sections



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- 2p2h MEC improve the agreement but are not enough to explain the data

- The effect of 2p2h diagrams decreases with increasing scattering angle

Amaro et al., PLB696 (2011)

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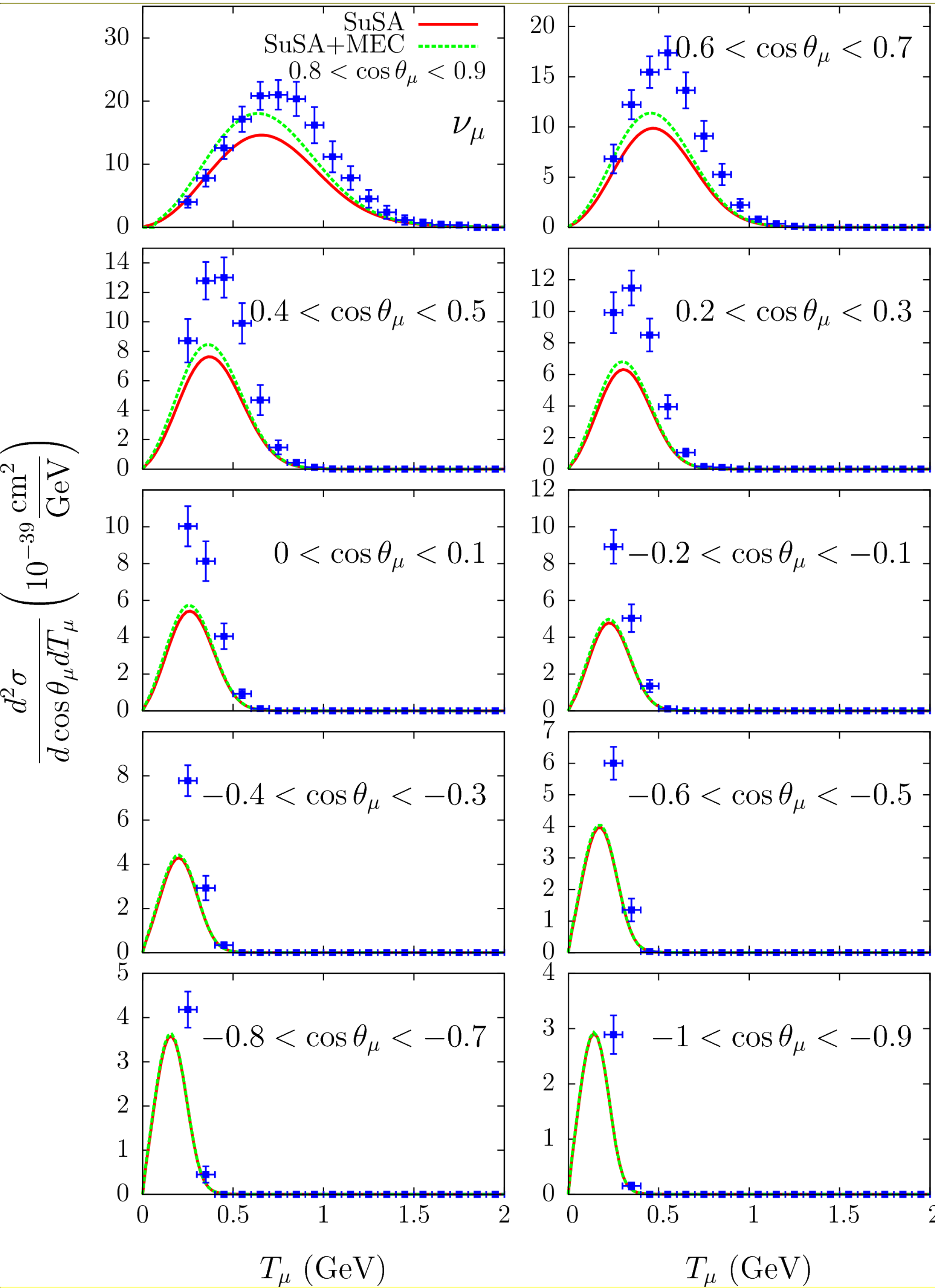

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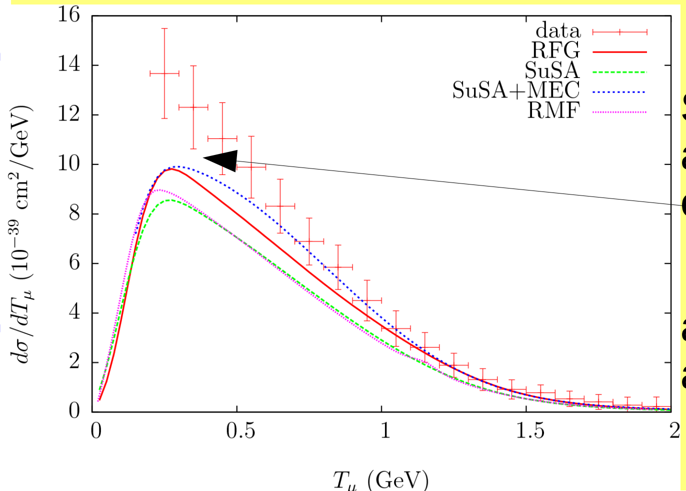
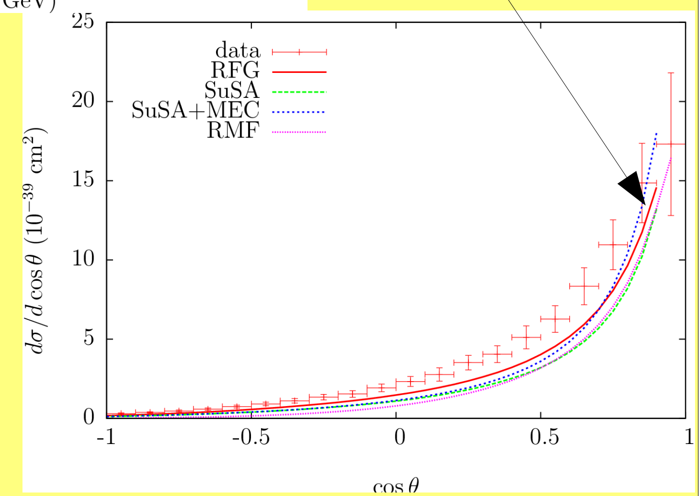


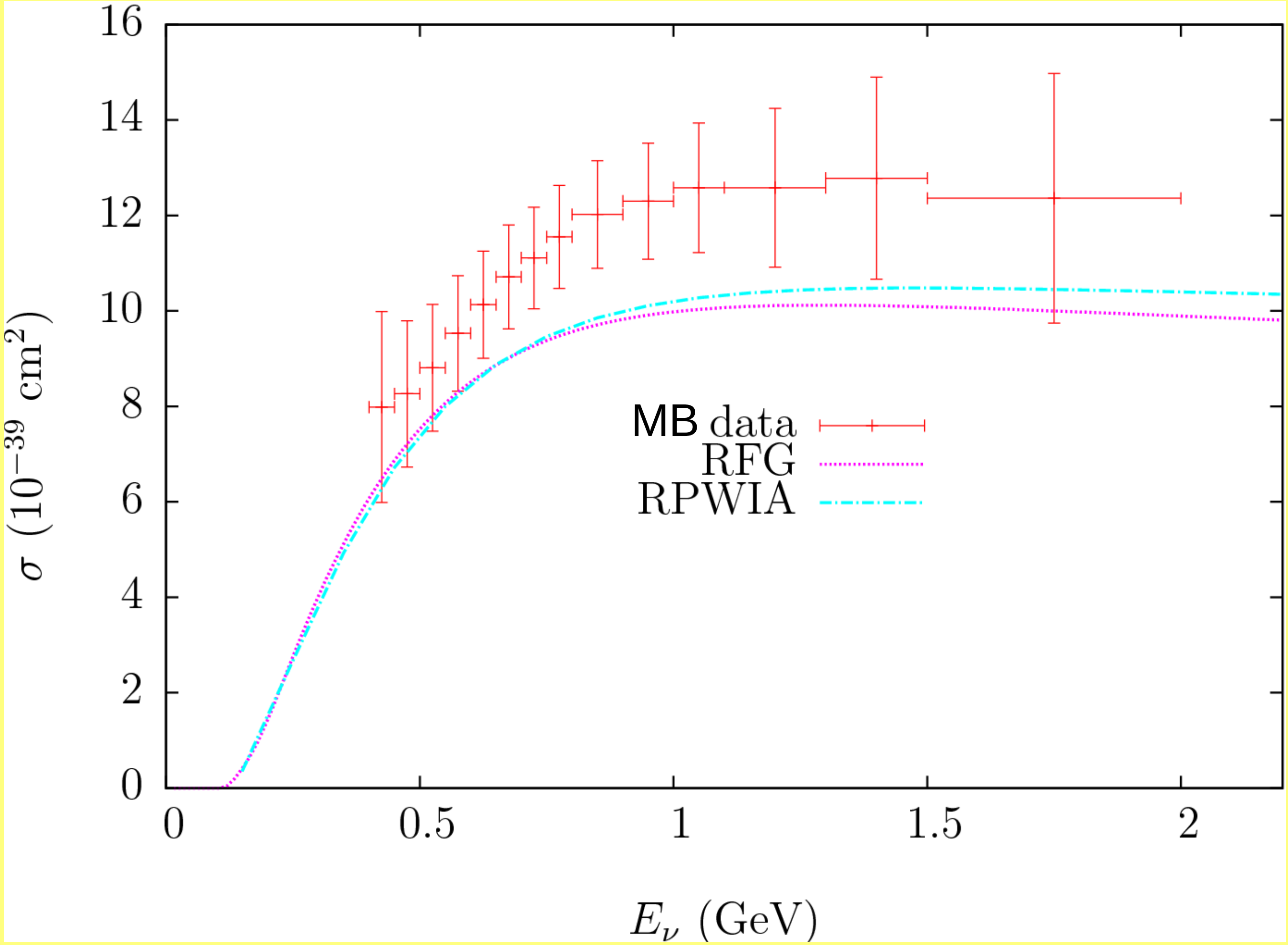
Strength is missing at lower muon energies

and larger scattering angles



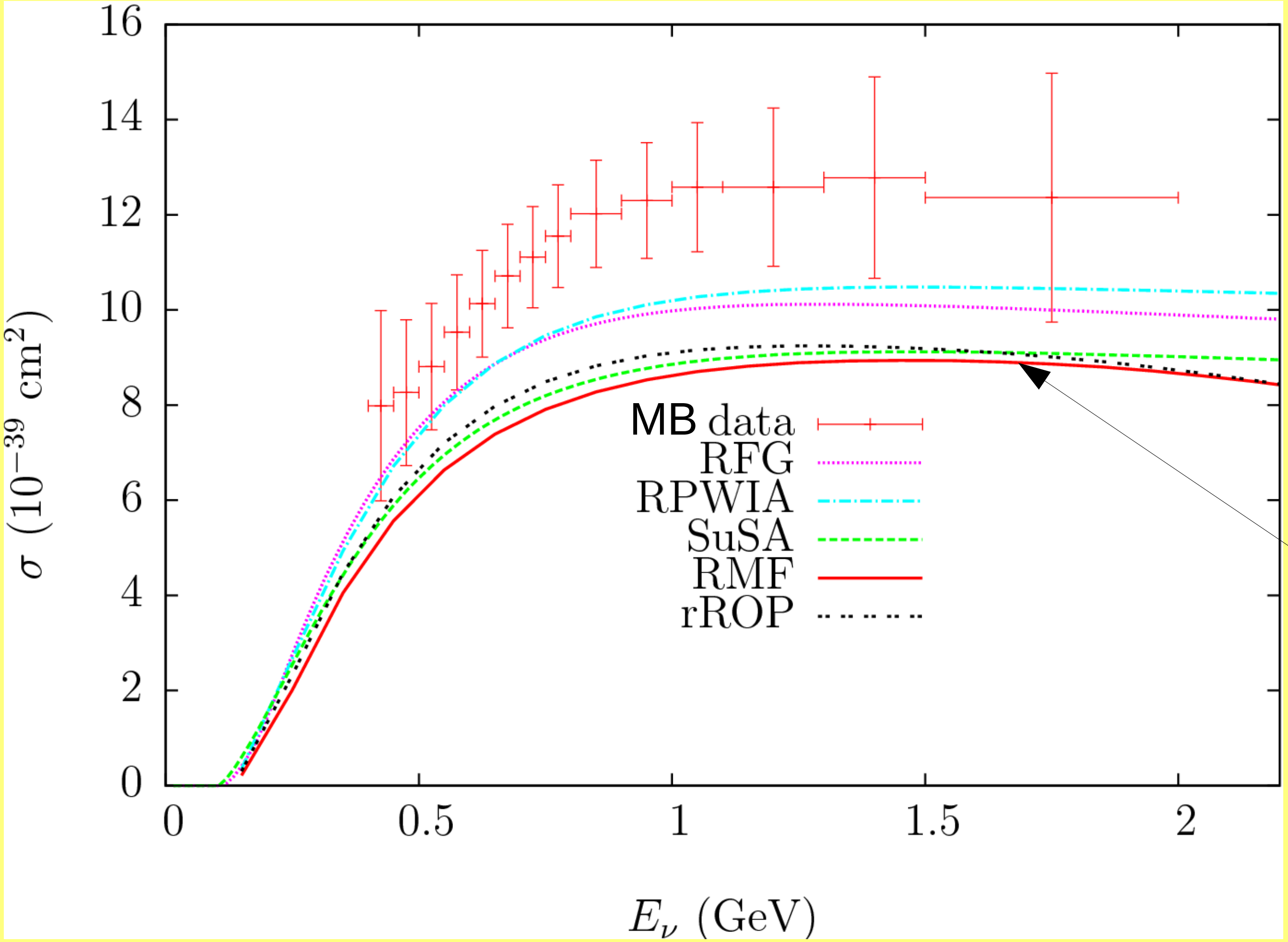
Amaro et al., PLB696 (2011)

Total CC cross section



Amaro et al., PLB696 (2011)

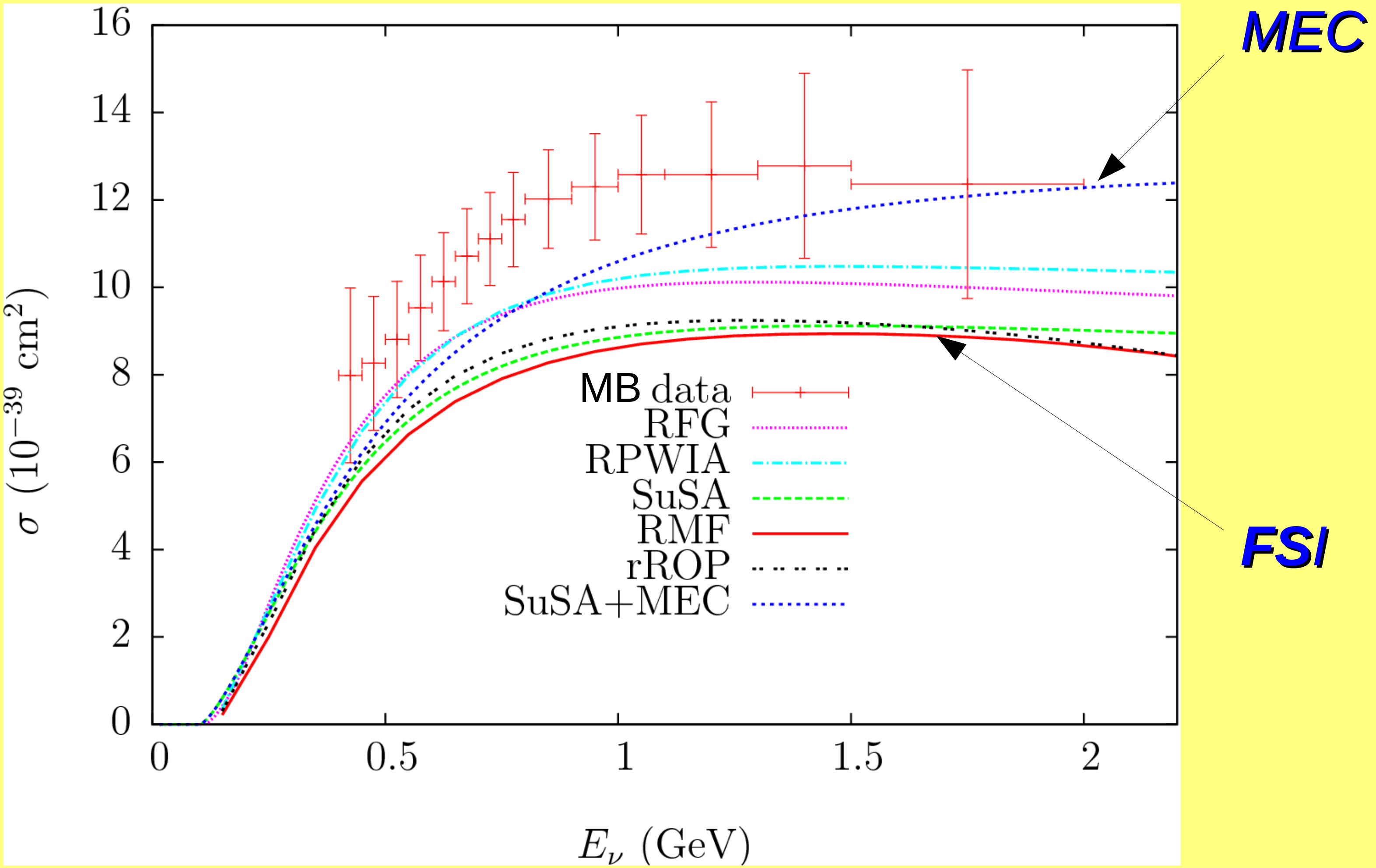
Total CC cross section



FSI

Amaro et al., PLB696 (2011)

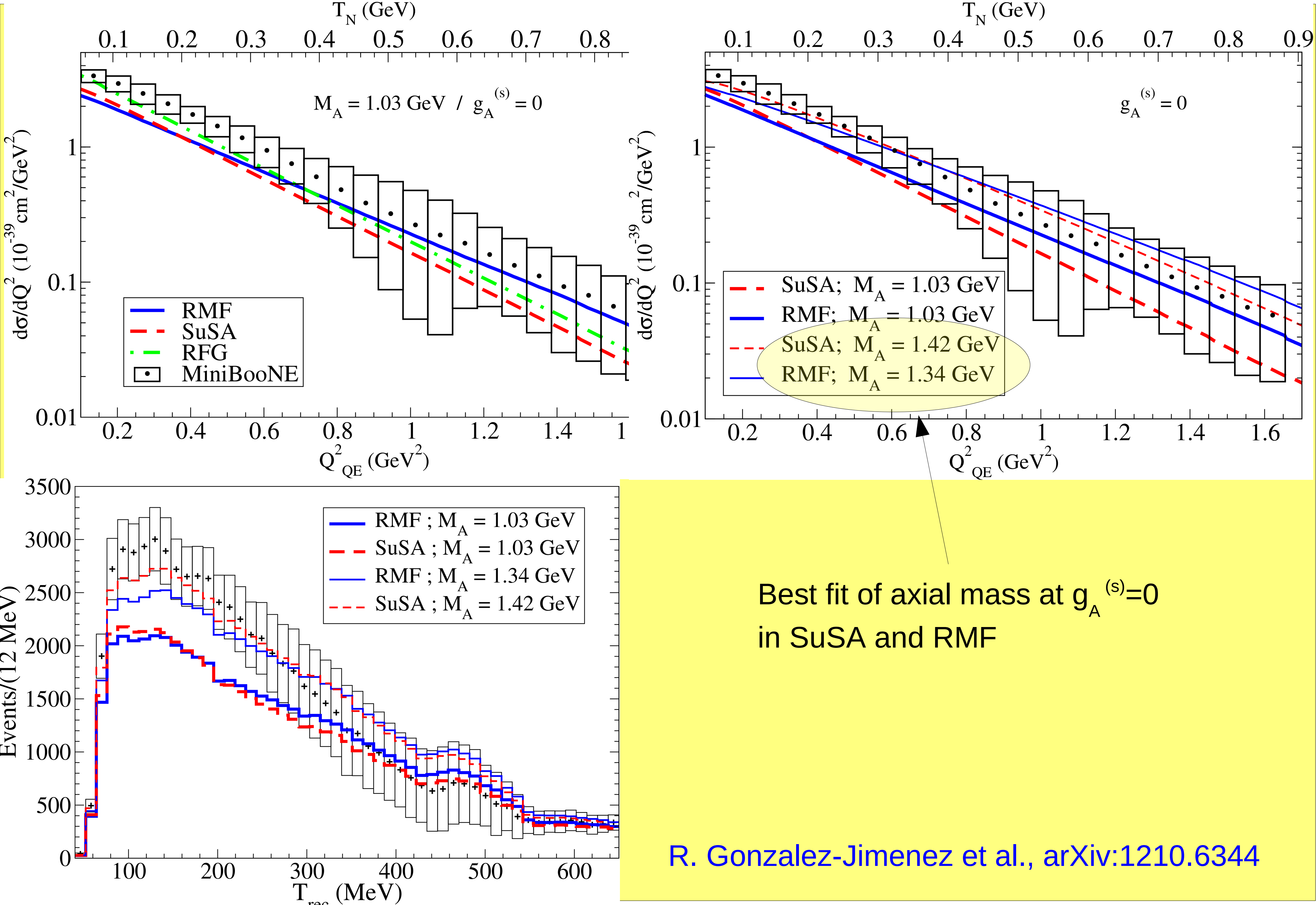
Total CC cross section



Amaro et al., PLB696 (2011)

Results for NCQE neutrino scattering

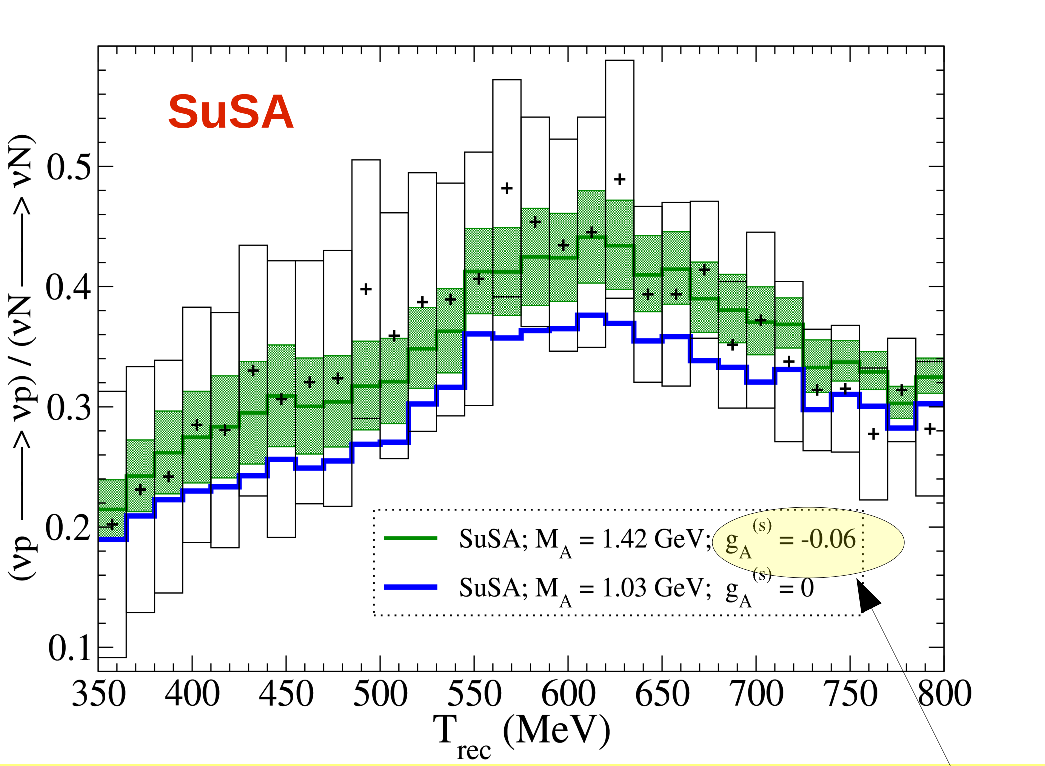
NC neutrino cross section

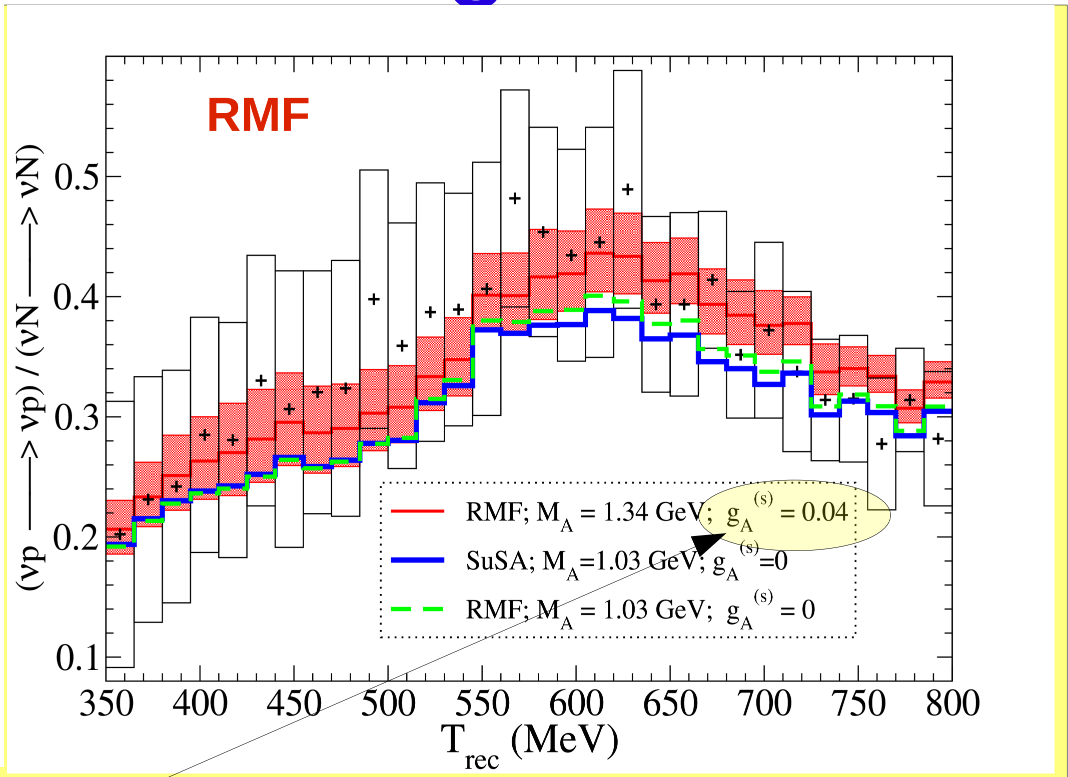


Best fit of axial mass at $g_A^{(s)}$=0 in SuSA and RMF

R. Gonzalez-Jimenez et al., arXiv:1210.6344

NC p/N ratio: axial strangeness





Best fits of $g_A^{(s)}$ at fixed M_A

$g_A^{(s)}$ = -0.06±0.31 SuSA (χ^2/DOF=31.3/29)

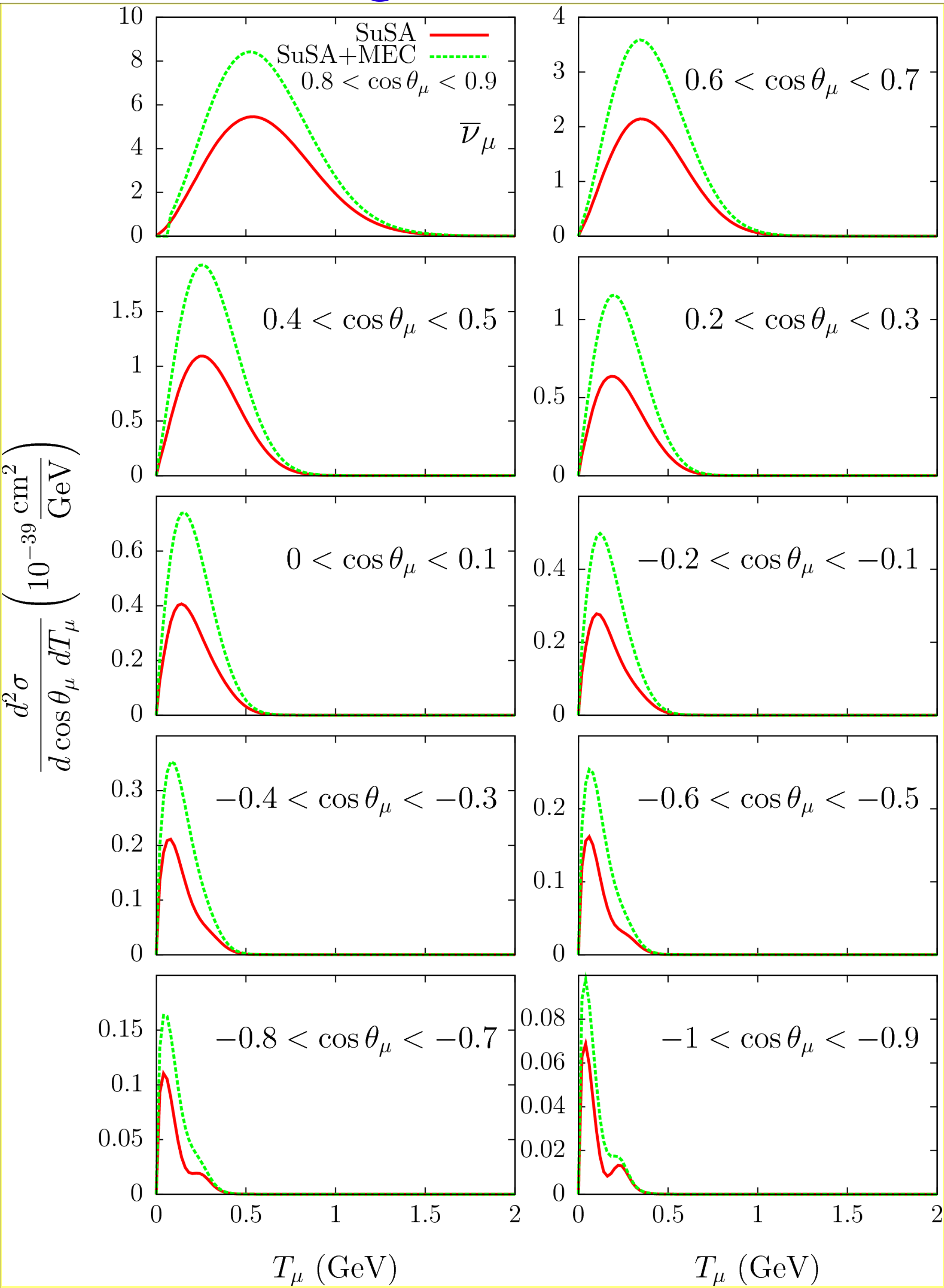
$g_A^{(s)}$ = +0.04±0.28 RMF (χ^2/DOF=33.6/29)

The dependence upon the nuclear model is essentially canceled in the ratio

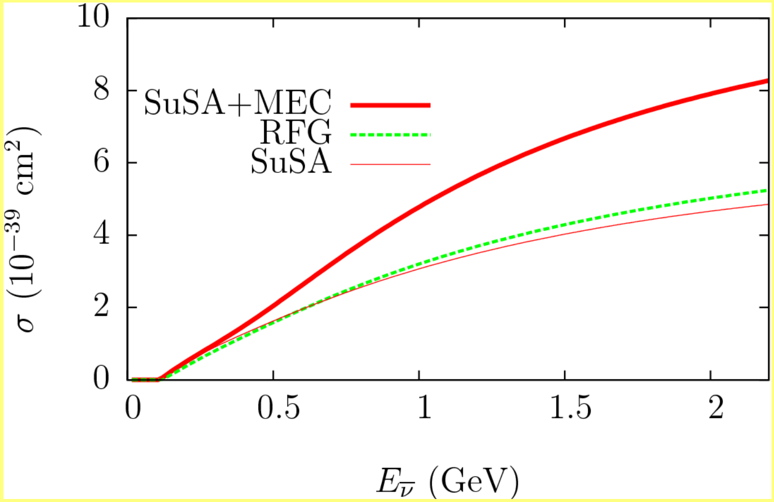
R. Gonzalez-Jimenez et al., arXiv:1210.6344

Predictions for antineutrino scattering

CCQE antineutrino cross section

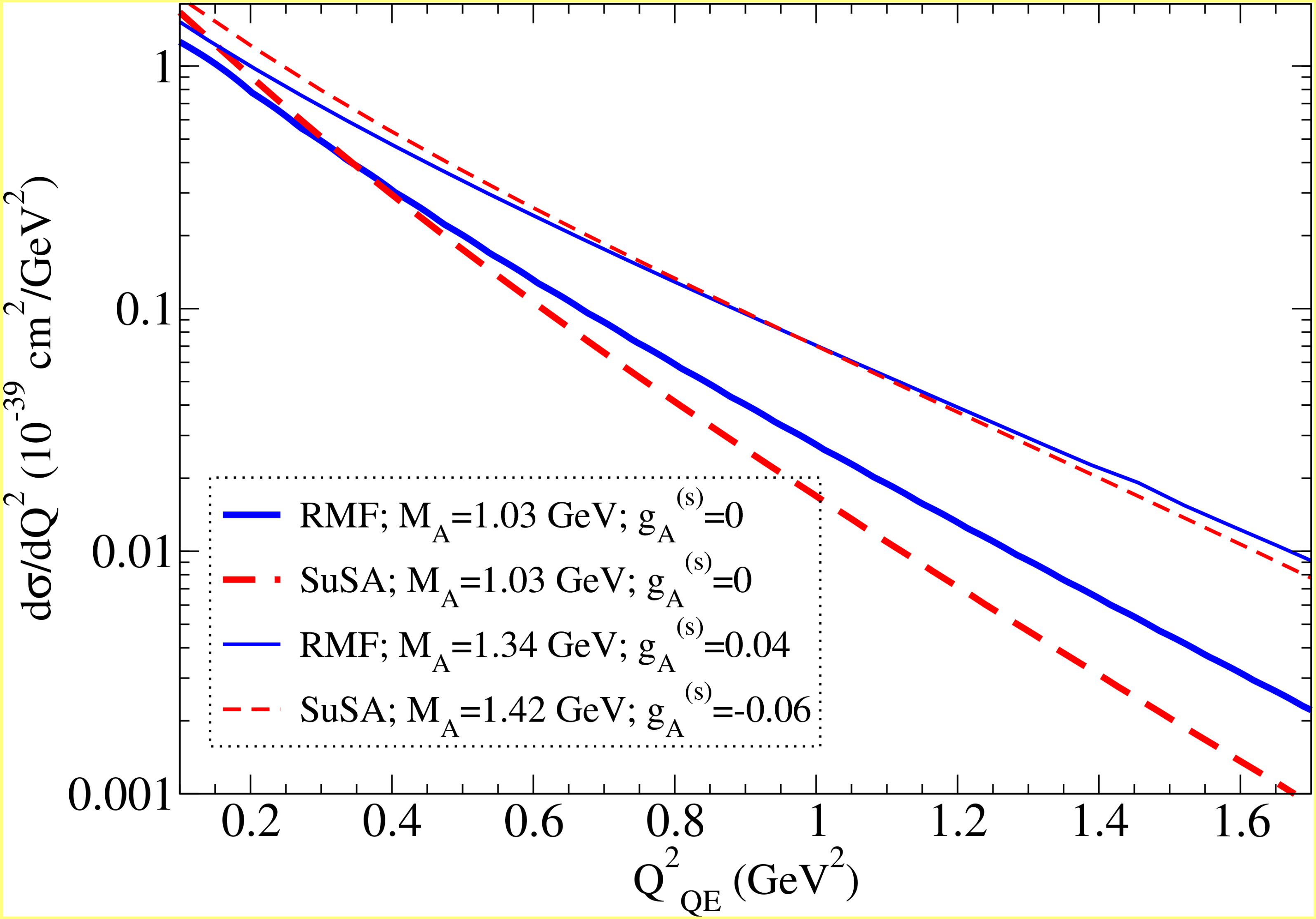


The effects of MEC in the present model are found to be very important and significantly larger than for neutrino scattering



Amaro et al., PRL 208 (2012)

NCQE antineutrino cross section



R. Gonzalez-Jimenez et al., arXiv:1210.6344

Summary

The SuperScaling approach to neutrino scattering:

- agrees by construction with electron scattering data in a wide range of kinematics;
- it can be applied to all nuclei (II kind scaling);
- the superscaling function is phenomenological, but it is well reproduced by the relativistic mean field model.

- It is based on some assumptions:

1. The superscaling function is extracted from longitudinal data and the approach assumes $f_L=f_T=f_{T'}$ (true in some, but not all, microscopic models)
2. Superscaling violations are not accounted for and must be added (MEC)

- Application to the CCQE process leads to cross sections lower than the MiniBooNE data
- Addition of 2p2h MEC diagrams improves the agreement but still misses the data at higher scattering angles and lower muon energies (however, the axial MEC are missing)
- Application to the NCQE process gives results lower than the MiniBooNE data at low Q^2 (but no MEC in the present model)

- Improvements of the model (in progress):
  1. Inclusion of axial MEC in the 2p2h sector
  2. Inclusion of correlations associated to MEC, necessary to preserve gauge invariance

Backup Slides

Relativistic effects

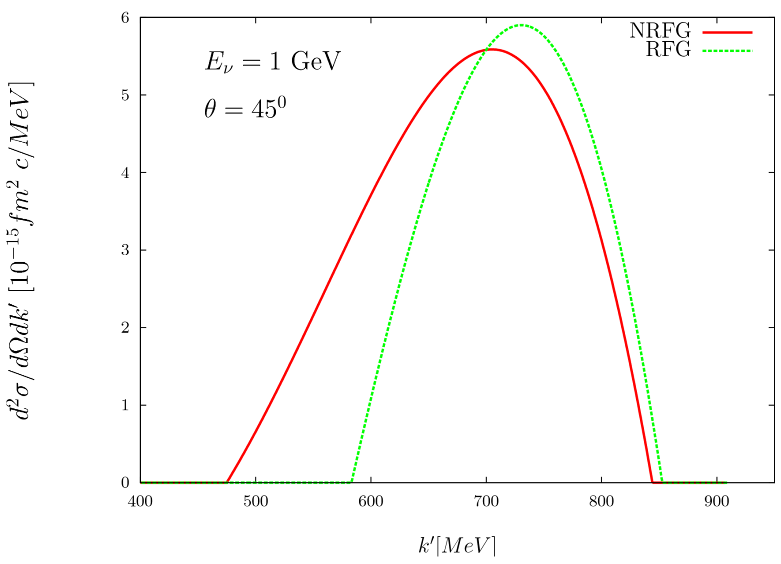
Reliable nuclear modeling in the 1 GeV region cannot neglect relativistic effects

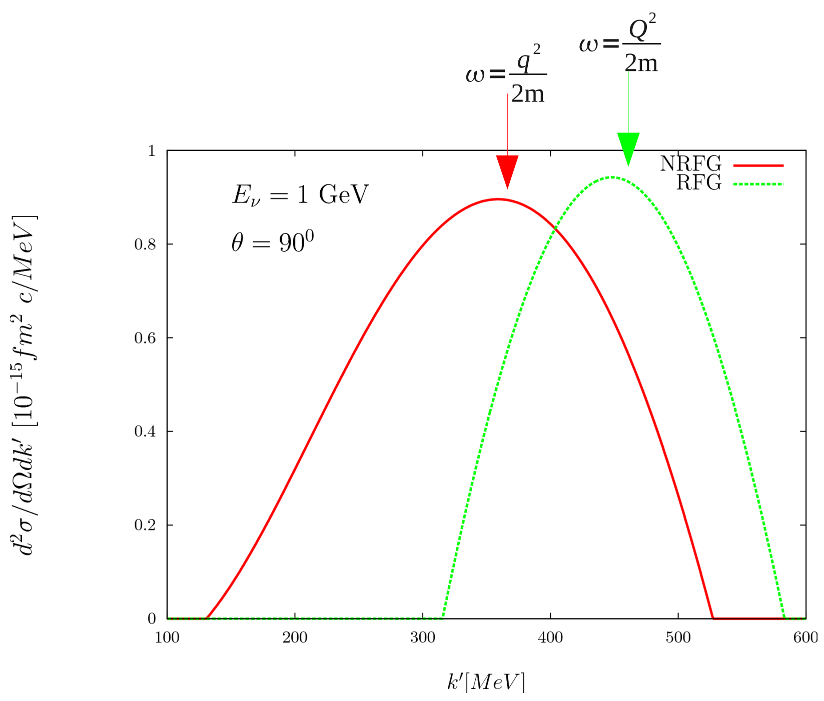
1. Relativistic Kinematics: $E_p = m + p^2/(2m) \longrightarrow E_p = \sqrt{p^2 + m^2}$

2. Relativistic Current Operators: Dirac spinors and γ-matrices

3. Relativistic treatment of the nuclear dynamics

All these effects are large at the typical neutrino energies of current experiments ($E_\nu \sim$ 1-few GeV)





Expressions for the π-exchange currents

$$J^{\mu}_{(2)} = J^{\mu}_{(s)} + J^{\mu}_{(\pi)} + J^{\mu}_{(\Delta)}$$

“Seagull”:

$$J^{\mu}_{(s)}(p'_1, p'_2; p_1, p_2) = \frac{f^2}{m_\pi^2} i\epsilon_{3ab} \bar{u}(p'_1)\tau_a \gamma_5 \gamma^\nu K_{1\nu} u(p_1) \frac{F_1^V}{K_1^2 - m_\pi^2} \bar{u}(p'_2)\tau_b \gamma_5 \gamma^\mu u(p_2) + (1 \Leftrightarrow 2)$$

“Pion-in-flight”:

$$J^{\mu}_{(\pi)}(p'_1, p'_2; p_1, p_2) = \frac{f^2}{m_\pi^2} i\epsilon_{3ab} \bar{u}(p'_1)\tau_a \gamma_5 \gamma^\nu K_{1\nu} u(p_1) \frac{F_\pi (K_1 - K_2)^\mu}{(K_1^2 - m_\pi^2)(K_2^2 - m_\pi^2)} \bar{u}(p'_2)\tau_b \gamma_5 \gamma^\rho K_{2\rho} u(p_2)$$

“Δ-MEC”:

$$J^{\mu}_{(\Delta)}(p'_1, p'_2; p_1, p_2) = \frac{f_{\pi N \Delta} f}{m_\pi^2} \bar{u}(p'_1) T_a^\mu(1) \gamma_5 u(p_1) \frac{1}{K_2^2 - m_\pi^2} \bar{u}(p'_2)\tau_a \gamma_5 \gamma^\nu K_{2\nu} u(p_2) + (1 \Leftrightarrow 2)$$

$$T_a^\mu(1) = K_{2\alpha} \Theta^{\alpha\beta} G^{\Delta}_{\beta\rho}(H_1 + Q) S_f^{\rho\mu}(H_1) T_a T_3 + T_3 T_a S_b^{\rho\mu}(P'_1) G^{\Delta}_{\rho\beta}(P'_1 - Q) \Theta^{\beta\alpha} K_{2\alpha}$$

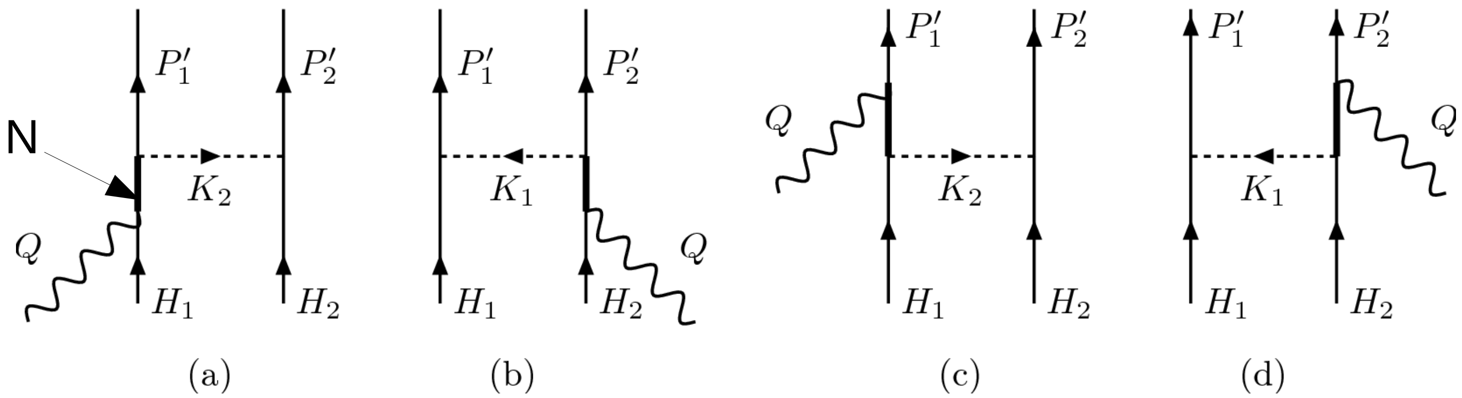
$$\Theta^{\alpha\beta} = g^{\alpha\beta} - \frac{1}{4}\gamma^\alpha \gamma^\beta$$

Rarita-Schwinger propagator

Forward and backward Δ-electroexcitation tensors

Correlation Currents

In order to preserve gauge invariance correlation diagrams, where the virtual boson attaches to one of two interacting nucleons, must be also considered:



The total two-body current is conserved:

$$\boxed{\partial^{\mu} J^{(2)}_{\mu} = 0}$$

Correlation currents contribute to both longitudinal and transverse channels.

Relativistic Impulse Approximation Models

RIA: Scattering off a nucleus ⇒Incoherent sum of single-nucleon scattering processes

Nuclear current ⇒ One-body operator

$$J_N^\mu(\omega,q)=\int d\vec{p}\,\overline{\Psi}_F(\vec{p}+\vec{q})\,J_N^\mu\,\Psi_B(\vec{p})$$

1) Relativistic Mean Field Model - RMF

2) Semi-relativistic Shell Model - RSM

4) Relativistic Green's Function - RGF

In the RMF approach

Ψ_B: bound nucleon w.f. ⇒ Relativistic Mean Field (strong S and V potentials)

Ψ_F: ejected nucleon w.f. ⇒ Final State Interaction, treated in different approaches:

RPWIA: relativistic plane wave (no FSI)

rROP: real relativistic optical potential

RMF: uses the same RMF employed for the initial state

Relativistic Fermi Gas and super-scaling

- The nucleus is a collection of free nucleons described by Dirac spinors u(p,s)
- The only correlations between nucleons are the Pauli correlations
- Lorentz covariance and Gauge invariance exactly fulfilled

dimensionless variables

$$\lambda = \frac{\omega}{2m_N}, \kappa = \frac{q}{2m_N}, \tau = \kappa^2 - \lambda^2$$

Response functions: $$R_K(\kappa,\lambda) = G_K(\kappa,\lambda) f_{RFG}(\psi)$$

single-nucleon functions

"super-scaling" function (universal)

$$f_{RFG}(\psi) = \frac{3}{4}(1-\psi^2)\theta(1-\psi^2)$$

$$\psi = 0 \rightarrow QEP$$

parabola

$$\psi(\lambda,\tau) = \frac{1}{\sqrt{\xi_F}} \frac{\lambda - \tau}{\sqrt{\tau(1+\lambda) + \kappa(1+\tau)}}$$

RFG scaling variable

ξ_F Fermi kinetic energy

Two parameters: k_F $\quad$ $Fermi\ momentum$

E_s $\quad$ $energy\ shift\ (typically \sim 20\ MeV)$: $\quad \omega \rightarrow \omega' = \omega - E_s$

$\psi \rightarrow \psi' = \psi(\lambda', \tau')$

fixed by fitting the position and the width of the QEP.

The RFG predicts that if the nuclear cross section is divided by the single nucleon one and plotted versus ψ for different values of the momentum transfer q and k_F, the result is

1) independent of q: scaling of first kind

2) independent of k_F: scaling of second kind → The RFG exactly super-scales
All scaling functions (L,T,T') are equal

Scaling in the Delta region

1) subtract the QE contribution obtained from
Superscaling hypothesis

$$\left[\frac{d^2\sigma}{d\omega\, d\Omega}\right]_{'\Delta'} = \left[\frac{d^2\sigma}{d\omega\, d\Omega}\right]_{\exp} - \left[\frac{d^2\sigma}{d\omega\, d\Omega}\right]_{QE}$$

2) divide by the elementary N $\rightarrow\Delta$
cross section

$$F_{'\Delta'} = \frac{\left[\frac{d^2\sigma}{d\omega\, d\Omega}\right]_{'\Delta'}}{\sigma_M \left(v_L G_L^{\Delta} + v_T G_T^{\Delta}\right)}$$

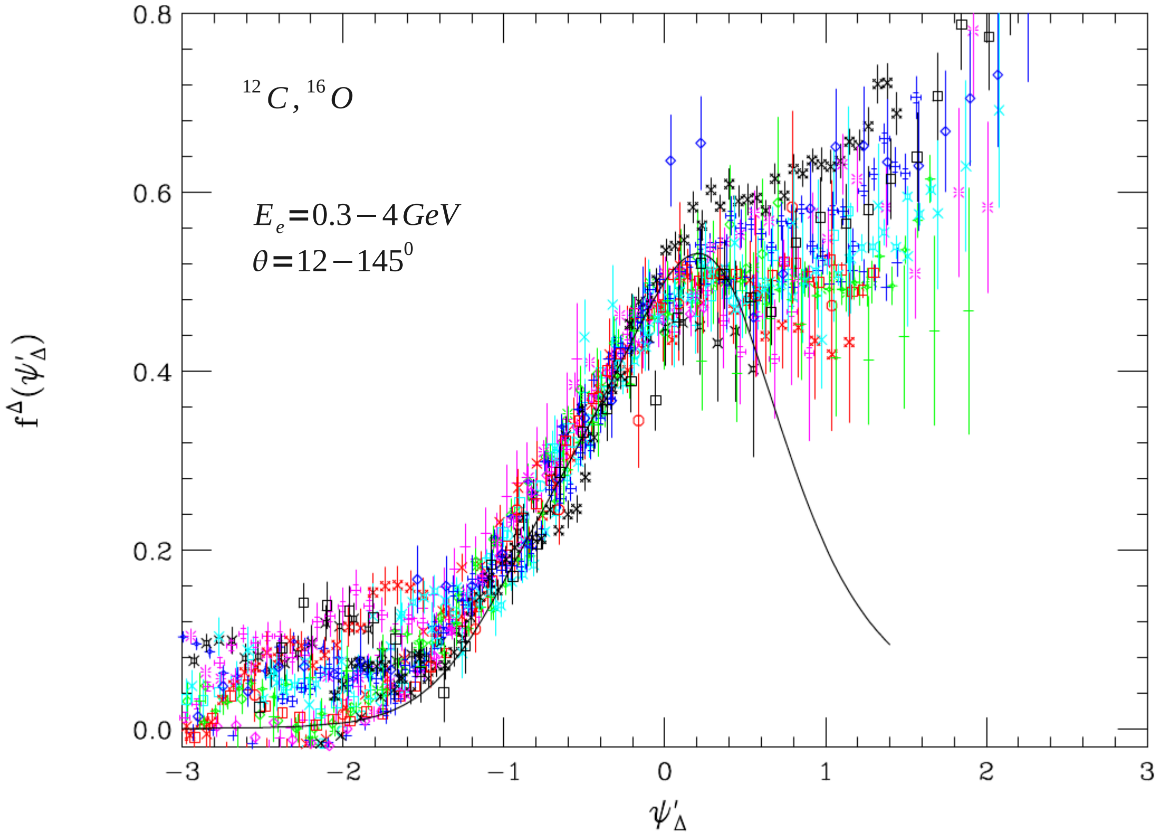
3) multiply by the Fermi momentum

$$f_{'\Delta'} = k_F F_{'\Delta'}$$

4) plot versus the appropriate scaling variable

$$\psi_{\Delta} = \psi(q\rho, \omega\rho)$$

$$\rho = 1 + \frac{1}{4\tau}\left(m_{\Delta}^2 / m_N^2 - 1\right)$$ inelasticity



Amaro, Barbaro, Caballero, Donnelly, Molinari, Sick, PRC71 (2005)

This approach can work only at Ψ$_\Delta$<0, since at Ψ$_\Delta$>0 other resonances and the tail of DIS contribute