Many-body Electroweak Currents and e/ν Inclusive Scattering

- Nuclear interactions and EM currents: an update
- Inclusive (e, e') scattering in light nuclei
- Summary
- Outlook: ν inclusive scattering by CC and NC

In collaboration with:

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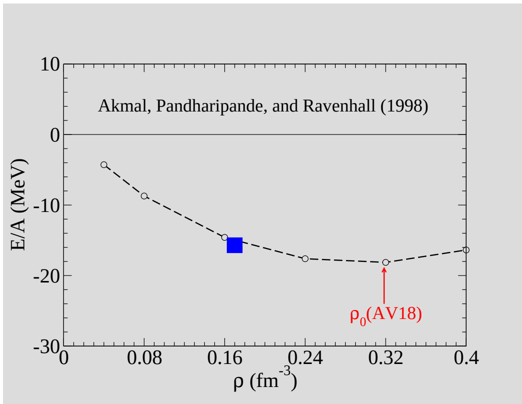
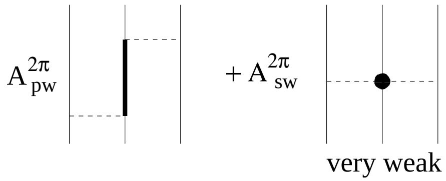
Nuclear Interactions

- $v = v_0(\text{static}) + v_p(\text{momentum dependent}) \to v(\text{OPE})$ fits large NN database with $\chi^2 \simeq 1$

- NN interactions alone fail to predict:
  1. spectra of light nuclei
  2. Nd scattering
  3. nuclear matter $E_0(\rho)$



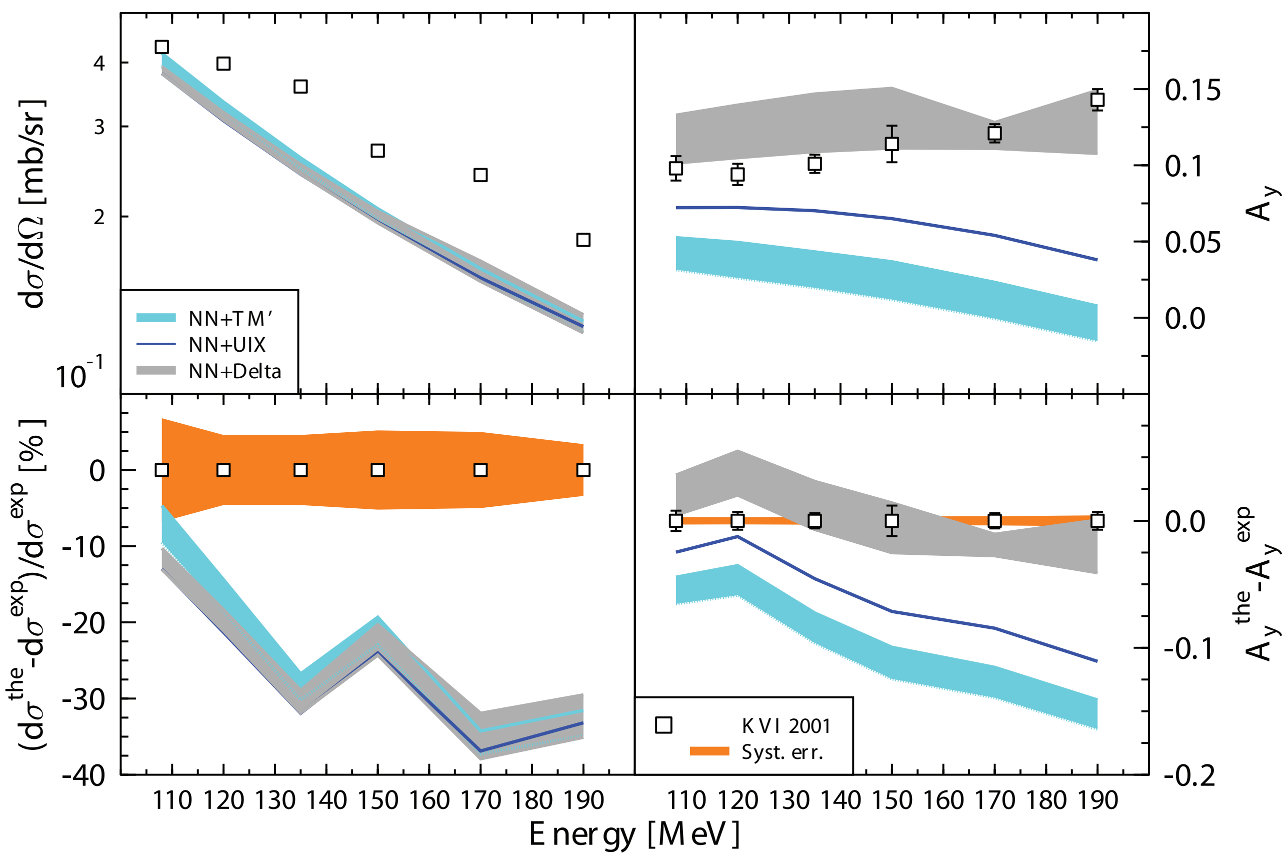
- 2π-NNN interactions:



Proton-Deuteron Elastic Scattering

Ermisch *et al.* (KVI collaboration) (2005) and Kalantar-Nayestanaki, private communication

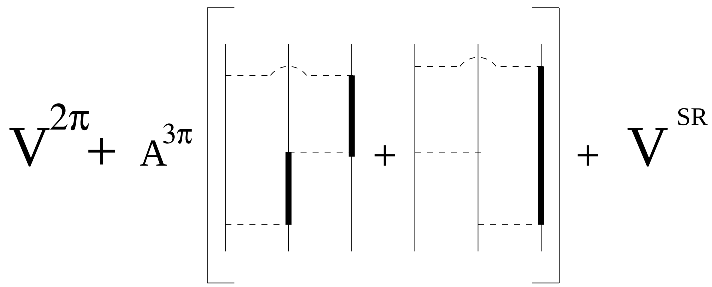
$V^{2\pi}$ only resolves some of the problems above . . .



Beyond 2π-exchange

Pieper and Wiringa, private communication

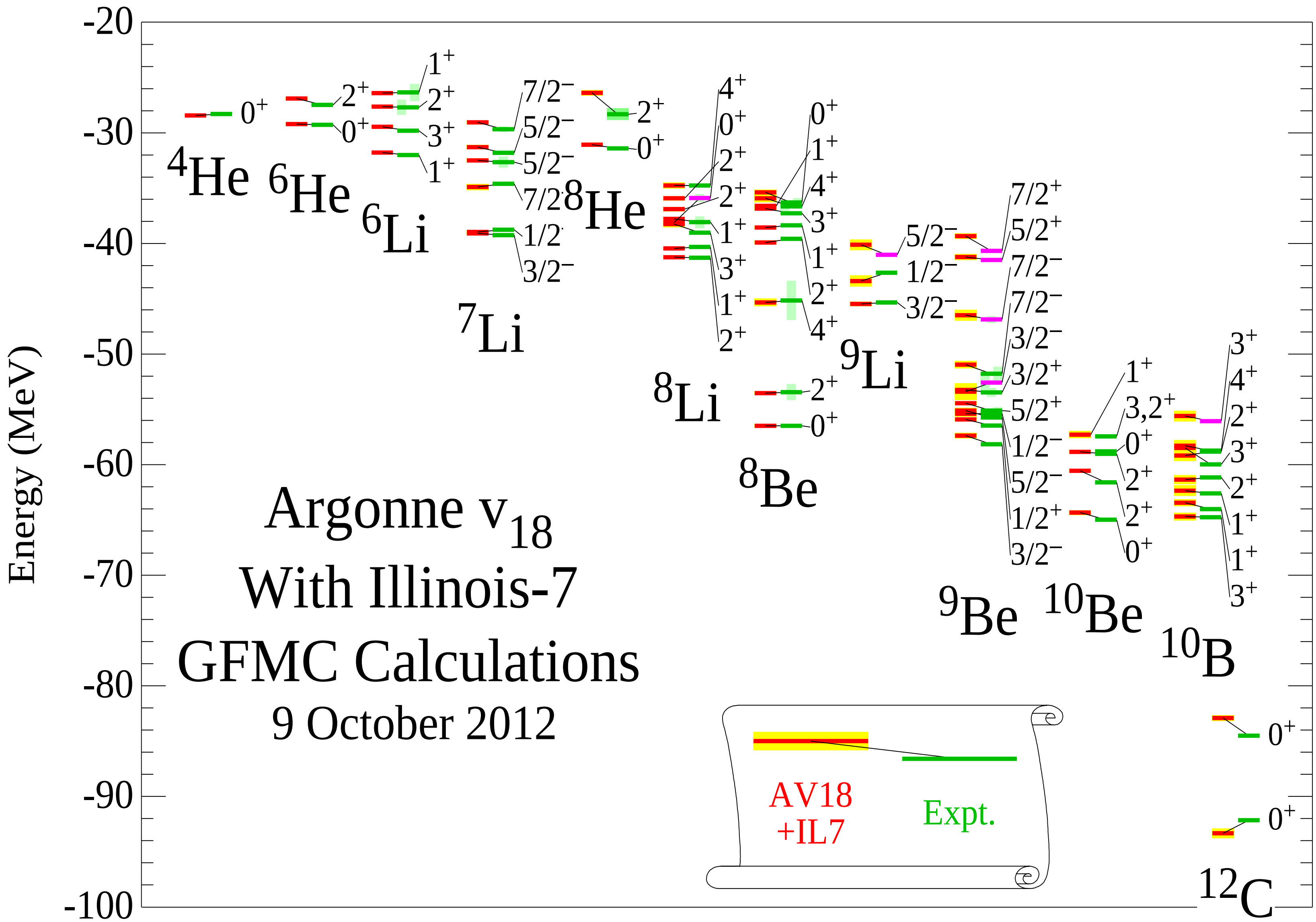
IL7 model has important $T = 3/2$ terms



parameters (~ 4) fixed by a best fit to the energies of low-lying states (~ 17) of nuclei with $A \leq 10$

AV18/IL7 Hamiltonian reproduces well:

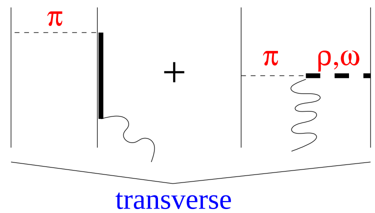
- spectra of A=9–12 nuclei (attraction provided by IL7 in $T = 3/2$ triplets crucial for p-shell nuclei)
- low-lying p-wave resonances with J^π=$3/2^-$ and $1/2^-$ respectively, as well as low-energy s-wave $(1/2^+)$ scattering


Argonne v_{18}
With Illinois-7
GFMC Calculations
9 October 2012
Energy (MeV)
-20
-30
-40
-50
-60
-70
-80
-90
-100
^{4}He
^{6}He
^{6}Li
^{7}Li
^{8}He
^{8}Li
^{8}Be
^{9}Li
^{9}Be
^{10}Be
^{10}B
^{12}C
AV18
+IL7
Expt.

EM Current Operators I

Marcucci *et al.* (2005)

$$\mathbf{j} = \mathbf{j}^{(1)} + \mathbf{j}^{(2)}(\mathrm{v}) + \ldots + \mathbf{j}^{(3)}(\mathrm{V}^{2\pi})$$

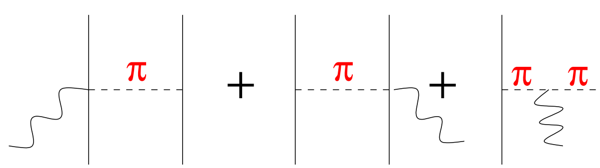


- Static part v_0 of v from π-like (PS) and ρ-like (V) exchanges
- Currents from corresponding PS and V exchanges, for example

$$\begin{aligned}\mathbf{j}_{ij}(v_0; PS) &= \mathrm{i}\, G_E^V(Q^2)\, (\boldsymbol{\tau}_i \times \boldsymbol{\tau}_j)_z\, v_{PS}(k_j) \Big[\boldsymbol{\sigma}_i \\ &- \frac{\mathbf{k}_i - \mathbf{k}_j}{k_i^2 - k_j^2} (\boldsymbol{\sigma}_i \cdot \mathbf{k}_i) \Big] (\boldsymbol{\sigma}_j \cdot \mathbf{k}_j) + i \leftrightharpoons j\end{aligned}$$

with $v_{PS}(k) = v^{\sigma\tau}(k) - 2\, v^{t\tau}(k)$ projected out from v_0 terms

$$\mathbf{j}^{(2)}(\mathrm{v}) \xrightarrow[\text{long range}]{}$$



EM Current Operators II

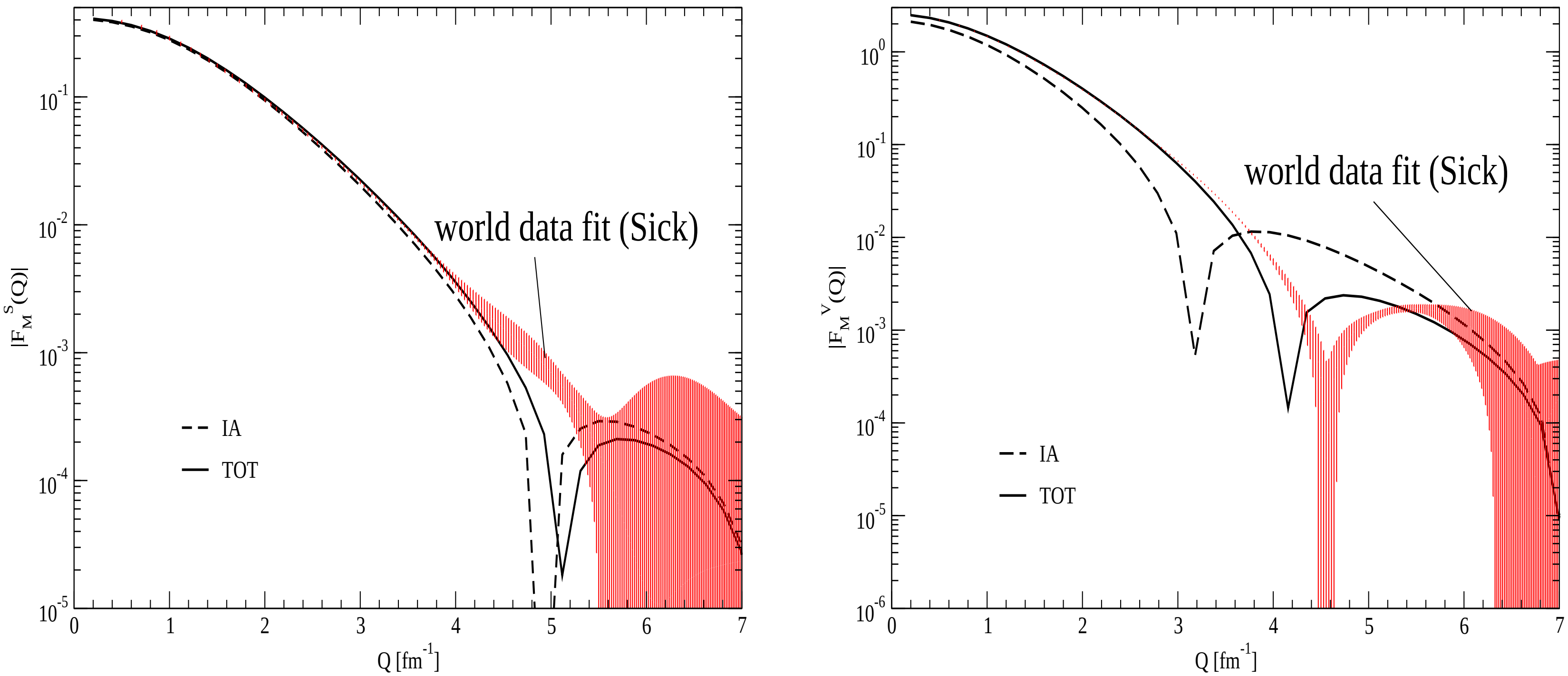
- Currents from v_p via minimal substitution in i) explicit and ii) implicit p-dependence, the latter from

$$\boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j = -1 + (1 + \boldsymbol{\sigma}_i \cdot \boldsymbol{\sigma}_j)\, \mathrm{e}^{\mathrm{i}(\mathbf{r}_{ji} \cdot \mathbf{p}_i + \mathbf{r}_{ij} \cdot \mathbf{p}_j)}$$

- Currents are conserved, contain no free parameters, and are consistent with short-range behavior of v and $V^{2\pi}$, but are not unique

$$\mathbf{q} \cdot \left[\mathbf{j}^{(1)} + \mathbf{j}^{(2)}(v) + \mathbf{j}^{(3)}(V^{2\pi})\right] = \left[T + v + V^{2\pi}\,,\, \rho\right]$$

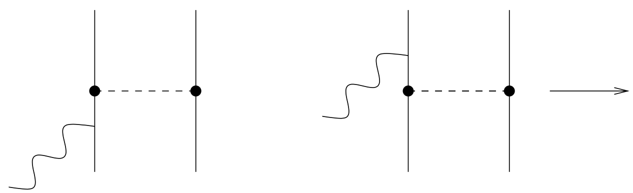
Isoscalar and Isovector Magnetic Form Factors of ^{3}He/^{3}H



- Isoscalar two-body current contributions small
- Leading isovector two-body currents from OPE

EM Charge Operators

Leading two-body charge operator derived from analysis of the virtual pion photoproduction amplitudes:



$$\text{diagram (a)} = v_{ij}^{\pi} \frac{1}{E_i - E} \frac{F_1^S + F_1^V \tau_{i,z}}{2} \rightarrow \text{included in IA}$$

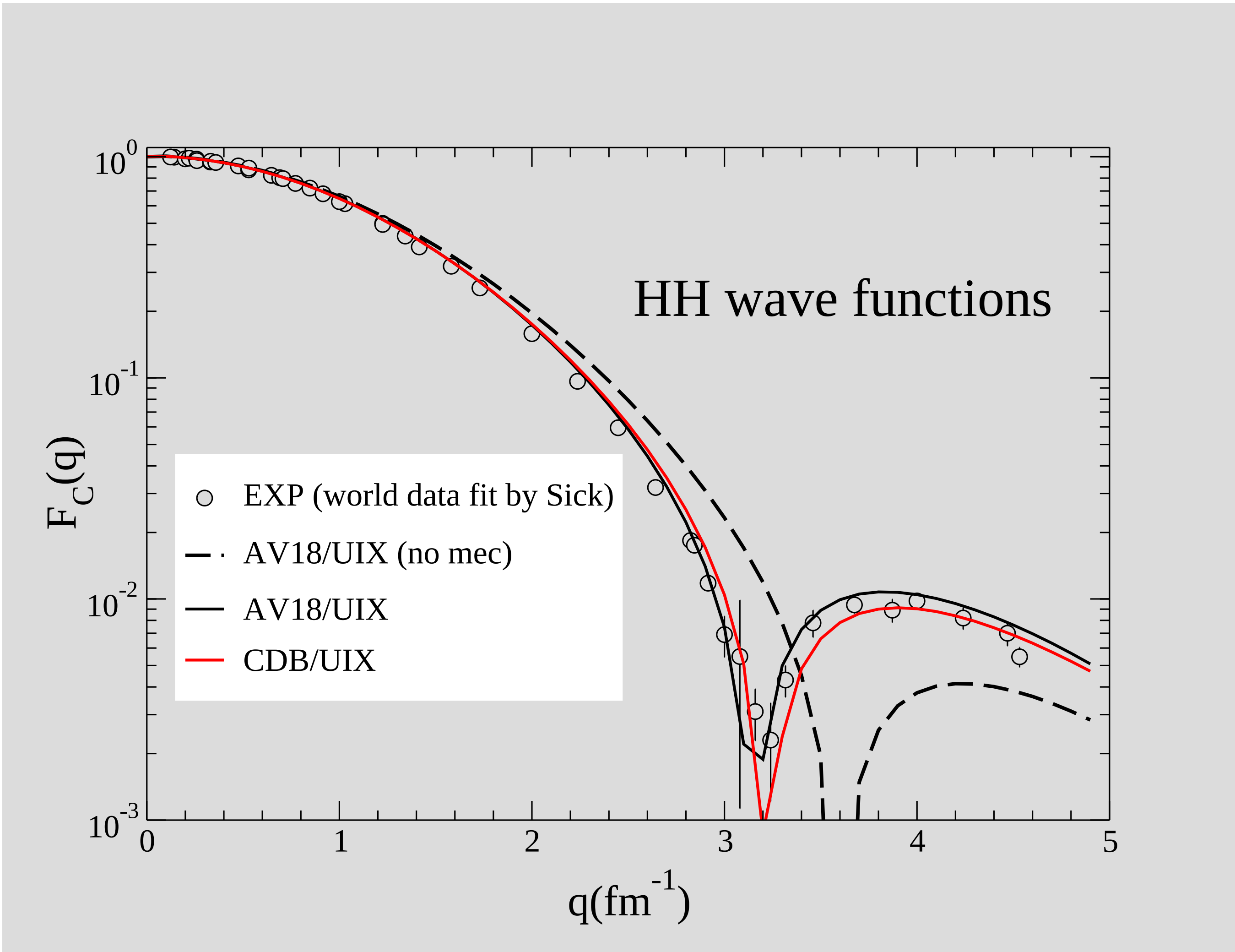
$$+ \frac{f^2}{4\, m\, m_\pi^2} \frac{\boldsymbol{\sigma}_i \cdot \mathbf{q}\, \boldsymbol{\sigma}_j \mathbf{k}_j}{k_j^2 + m_\pi^2} \boldsymbol{\tau}_i \cdot \boldsymbol{\tau}_j \frac{F_1^S + F_1^V \tau_{i,z}}{2} + \mathcal{O}(E_i - E)$$

- Essential for predicting the charge f.f.'s of ^{2}H, ^{3}H, ^{3}He, and ^{4}He
- Additional (small) contributions from vector exchanges as well as transition mechanisms like $\rho\pi\gamma$ and $\omega\pi\gamma$

EM observables in A=2–9 nuclei well reproduced: μ's and $M1$ widths, elastic and inelastic f.f.'s, inclusive response functions, . . .

^{4}He Charge Form Factor

Viviani *et al.* (2007)



The χEFT approach

Weinberg, PLB**251**, 288 (1990); NPB**363**, 3 (1991); PLB**295**, 114 (1992)

- χEFT exploits the χ-symmetry exhibited by QCD to restrict the form of π interactions with other π's, and with N's, Δ's, ...
- The pion couples by powers of its momentum Q, and $\mathcal{L}_{\rm eff}$ can be systematically expanded in powers of Q/Λ_χ ($\Lambda_\chi \simeq 1$ GeV)

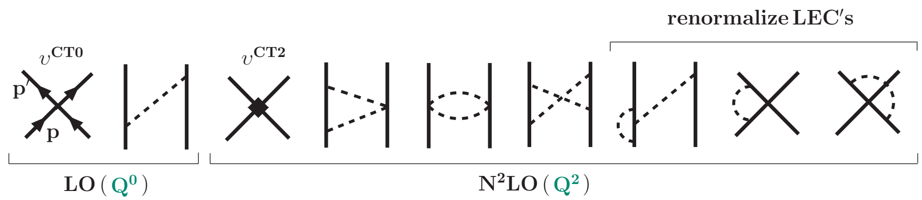
$$\mathcal{L}_{\rm eff} = \mathcal{L}^{(0)} + \mathcal{L}^{(1)} + \mathcal{L}^{(2)} + ...$$

- χEFT allows for a perturbative treatment in terms of a Q–as opposed to a coupling constant–expansion
- The unknown coefficients in this expansion–the LEC's–are fixed by comparison with experimental data
- Nuclear χEFT provides a practical calculational scheme, capable (in principle) of systematic improvement

Nuclear Interactions and EM Currents in χEFT

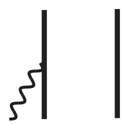
Pastore et al. (2009–2011)

NN potential:

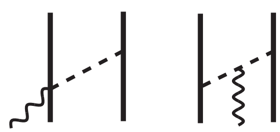


and consistent EM currents:

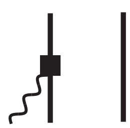
LO : eQ^{-2}



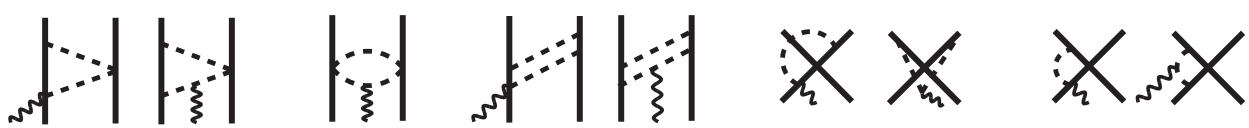
NLO : eQ^{-1}



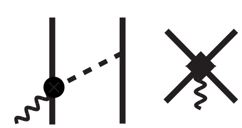
N^2LO : eQ^0



N^3LO : eQ



unknown LEC's →

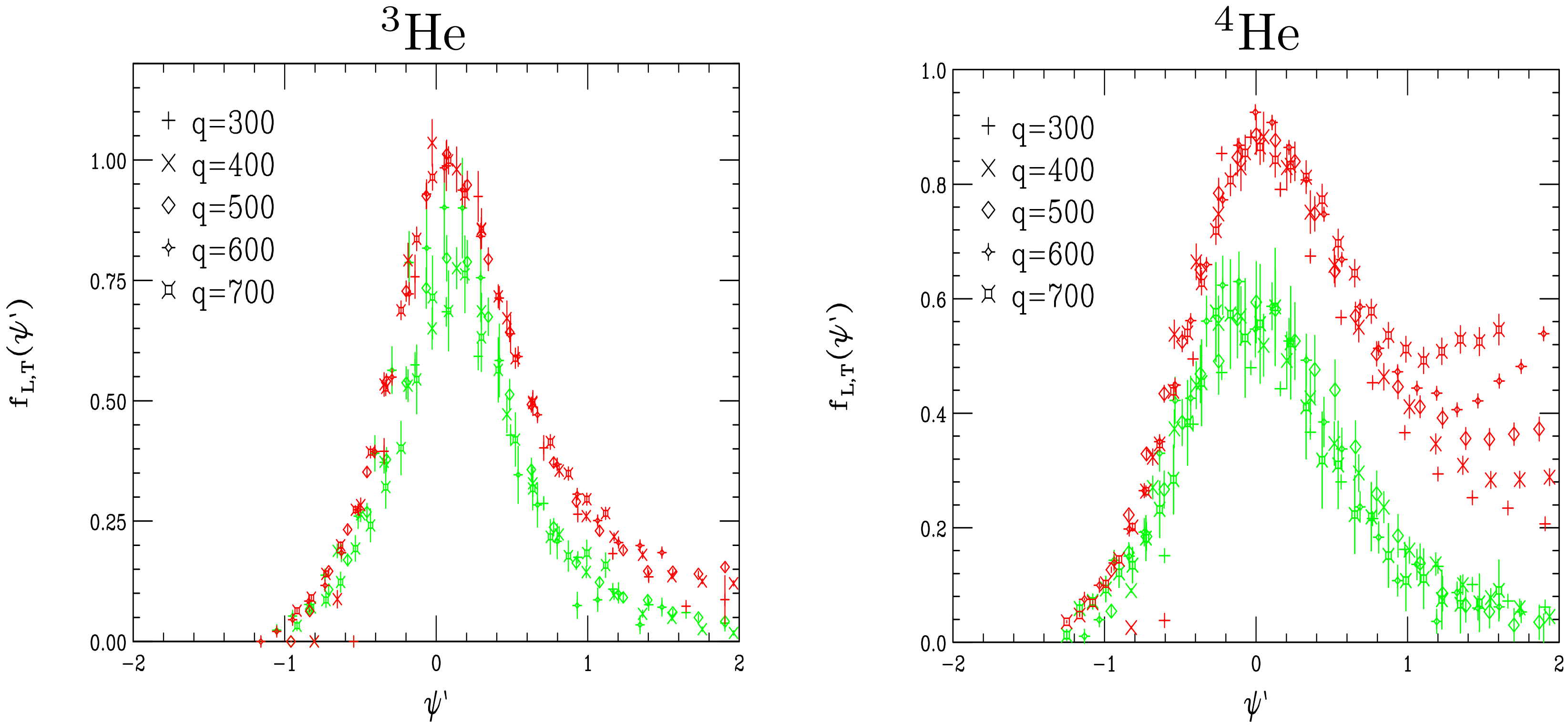


Inclusive (e, e') Scattering and MEC

- Experimental evidence
- Theoretical analysis via:
  1. Sum rules
  2. Explicit calculations of response functions
- Large MEC contributions to R_T

(e, e') Inclusive Response: Scaling Analysis

Donnelly and Sick (1999)



- Scaling variables: $\psi' \simeq y/k_F$ and $f_{L,T} = k_F\, R_{L,T}/G_{L,T}$
- Data at variance with PWIA expectation that $f_L \simeq f_T$
- Excess strength, especially for ^{4}He, in transverse response

Approaches to (e, e') Inclusive Scattering (IS)

Two response functions characterize (e, e') IS

$$R_\alpha(q,\omega) = \sum_{f \neq 0} \delta(\omega + E_0 - E_f) \mid \langle f \mid O_\alpha(\mathbf{q}) \mid 0 \rangle \mid^2 \qquad \alpha = L, T$$

require knowledge of continuum states: hard to calculate for $A \geq 3$

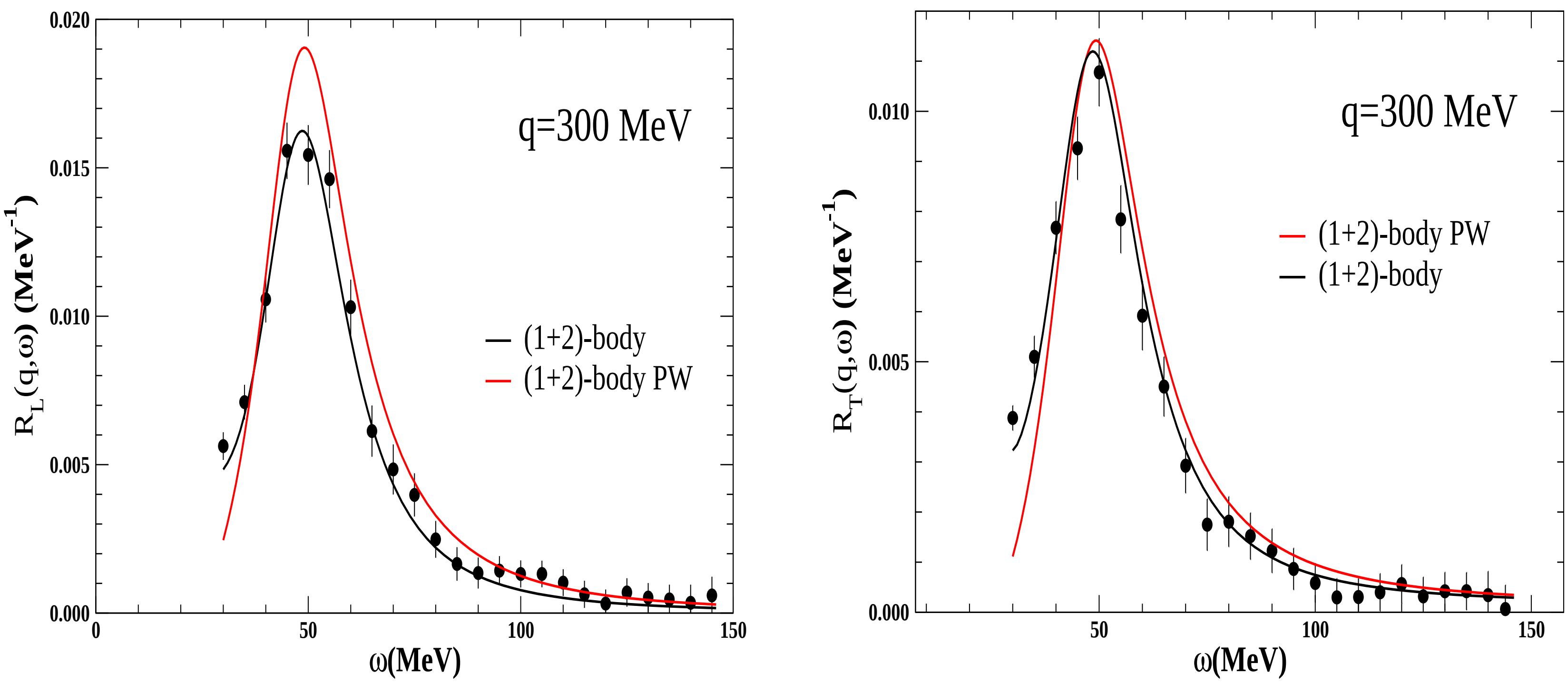
- Sum rules: integral properties of response functions
- Integral transform techniques

$$E(q,\tau) = \int_0^\infty \mathrm{d}\omega \, K(\tau,\omega) \, R(q,\omega)$$

  and suitable choice of kernels (i.e., Laplace or Lorentz) allows use of closure over $\mid f \rangle$, thus avoiding need of explicitly calculating nuclear excitation spectrum

- While in principle exact, both these approaches have drawbacks

^{2}H Longitudinal and Transverse Response Functions



Plane-wave versus fully interacting final states

Sum Rules

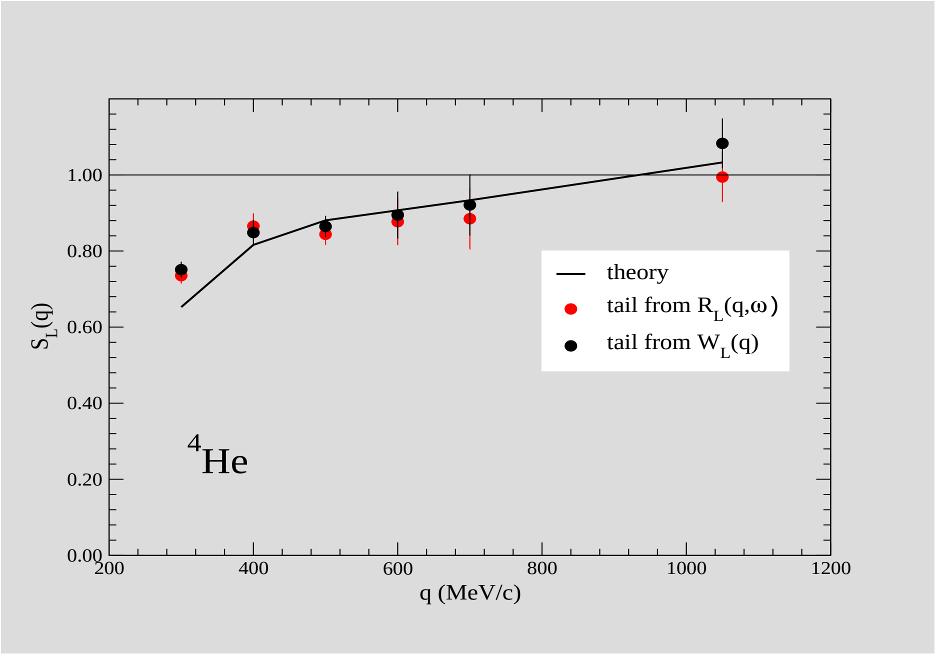
Schiavilla *et al.* (1989); Carlson *et al.* (2002–2003)

$$\begin{aligned} S_\alpha(q) &= C_\alpha \int_{\omega_{\rm th}^+}^\infty {\rm d}\omega \, \frac{R_\alpha(q,\omega)}{G_{Ep}^2(q,\omega)} \\ &= C_\alpha \left[\langle 0 |\, O_\alpha^\dagger(\mathbf{q})\, O_\alpha(\mathbf{q}) \,| 0 \rangle - |\, \langle 0 |\, O_\alpha(\mathbf{q}) \,| 0 \rangle \,|^2 \right] \end{aligned}$$

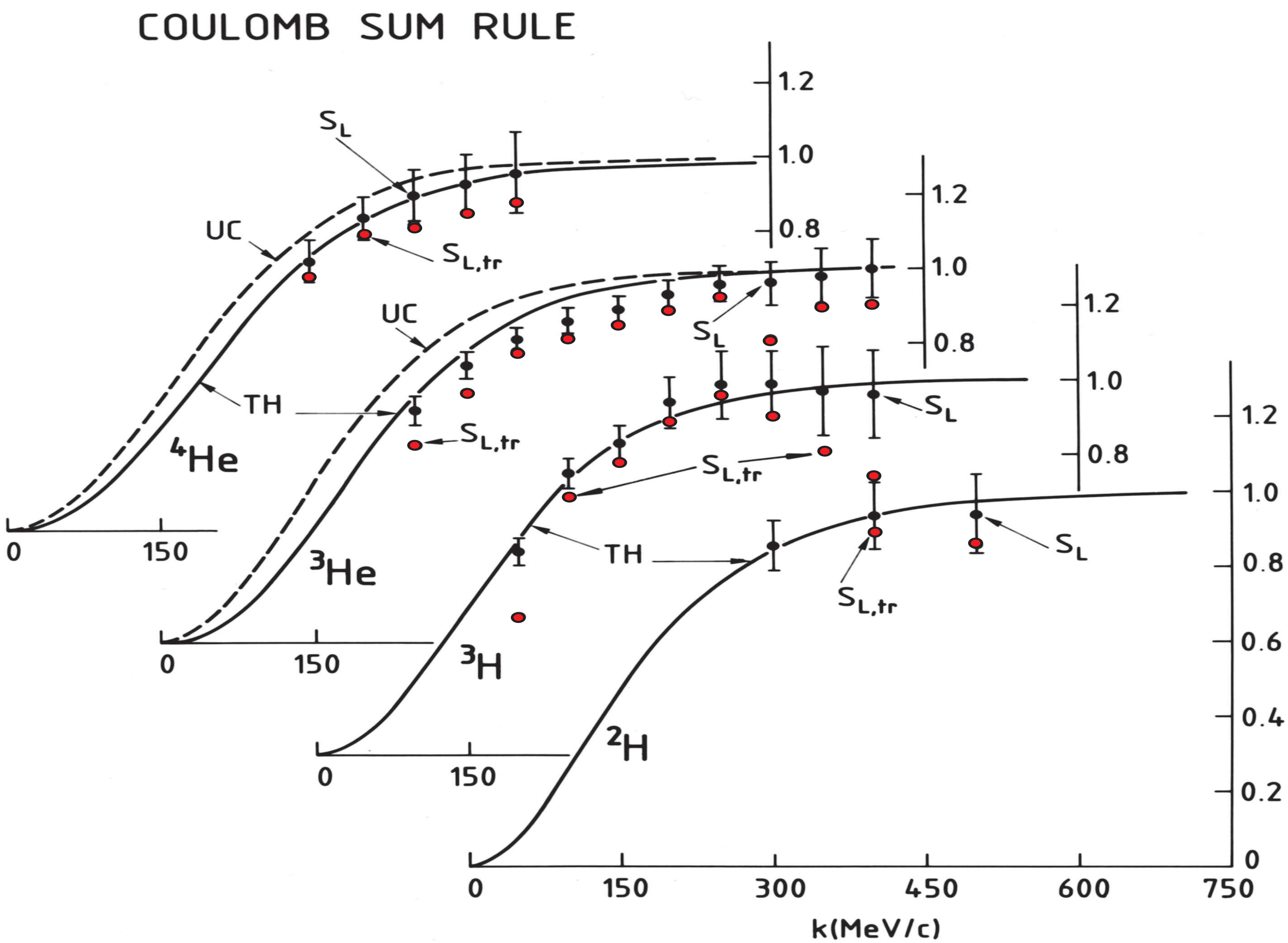
- $O_\alpha(\mathbf{q}) = \rho(\mathbf{q})$ or $\mathbf{j}_\perp(\mathbf{q})$ for $\alpha = L$ or T (divided by G_{Ep})
- C_α are normalization factors so as $S_\alpha(q \to \infty) = 1$ when only one-body are retained in ρ and $\mathbf{j}_\perp$
- $S_\alpha(q)$ only depend on ground state and can be calculated exactly with quantum Monte Carlo (QMC) methods
- Direct comparison between theory and experiment problematic:
  1. $R_\alpha(q,\omega)$ measured by (e,e') up to $\omega_{\rm max} \le q$
  2. Present theory ignores explicit pion production mechanisms, crucial in the Δ-peak region of R_T

The ^{4}He Coulomb Sum Rule

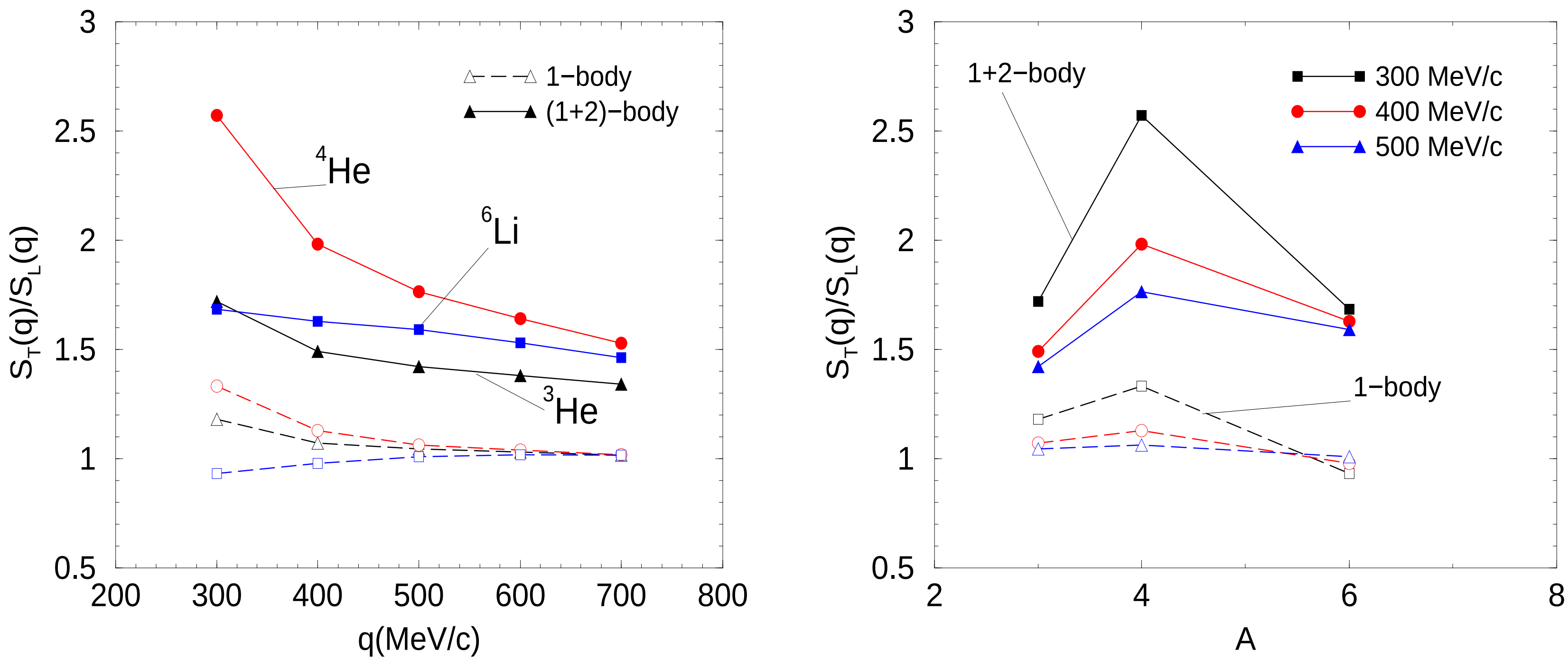
- RC/MEC (small) contributions to $S_L(q)$ tend to cancel out
- Theory and experiment in agreement when using free G_{Ep}



$$W_L(q) = \frac{1}{Z}\int_{\omega_{\rm th}^+}^{\infty} \mathrm{d}\omega\, \omega\, \frac{R_L(q,\omega)}{G_{Ep}^2(q,\omega)} = \frac{1}{2Z}\langle 0|\left[\rho^\dagger(\mathbf{q})\,,\left[H\,,\,\rho(\mathbf{q})\right]\right]|0\rangle$$

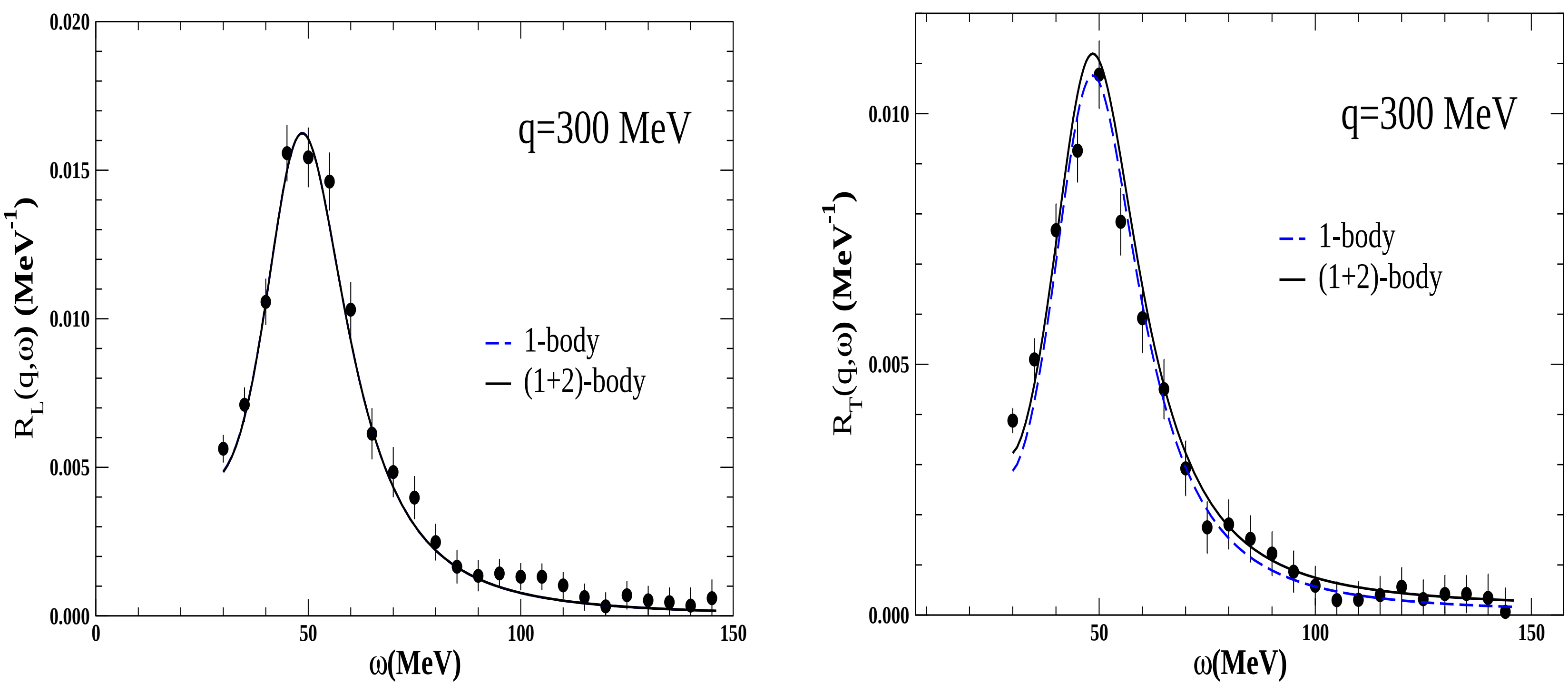

COULOMB SUM RULE
1.2
1.0
0.8
S_L
UC
S_L,tr
TH
4He
0
150
1.2
1.0
0.8
S_L
UC
S_L,tr
TH
3He
0
150
1.2
1.0
0.8
S_L
S_L,tr
TH
3H
0
150
1.2
1.0
0.8
0.6
0.4
0.2
0
S_L
S_L,tr
2H
0
150
300
450
600
750
k(MeV/c)

Excess Transverse Strength



- How much of the excess transverse strength $\Delta S_T = S_T - S_T^{1b}$ is in the quasi-elastic peak region?
- Can we understand the A-dependence of ΔS_T?

^{2}H Longitudinal and Transverse Response Functions



Most of transverse strength is in the tail:

$$C_T \int_{\omega_{\rm th}^+}^{\omega_{\rm max}} {\rm d}\omega \, \frac{R_T(q,\omega)}{G_{Ep}^2(q,\omega)} = 0.96 \;\; \text{versus} \;\; S_T^{\rm 1b+2b} = 1.13 \;\; (S_T^{\rm 1b} = 0.90)$$

A-Systematics of ΔS_T

Carlson *et al.* (2002)

Excess transverse strength from 2-body currents due to pn pairs

$$\Delta S_T{}^{A}(q) \simeq C_T \int_0^\infty \mathrm{d}x \, \mathrm{tr} \left[F(x;q) \, \rho^{A}(x;pn) \right]_{\sigma\tau}$$

F=matrix in two-nucleon $\sigma\tau$-space depending on $\mathbf{j}_{\perp,ij}$

ρ^A=A-dependent two-nucleon density matrix in $\sigma\tau$-space

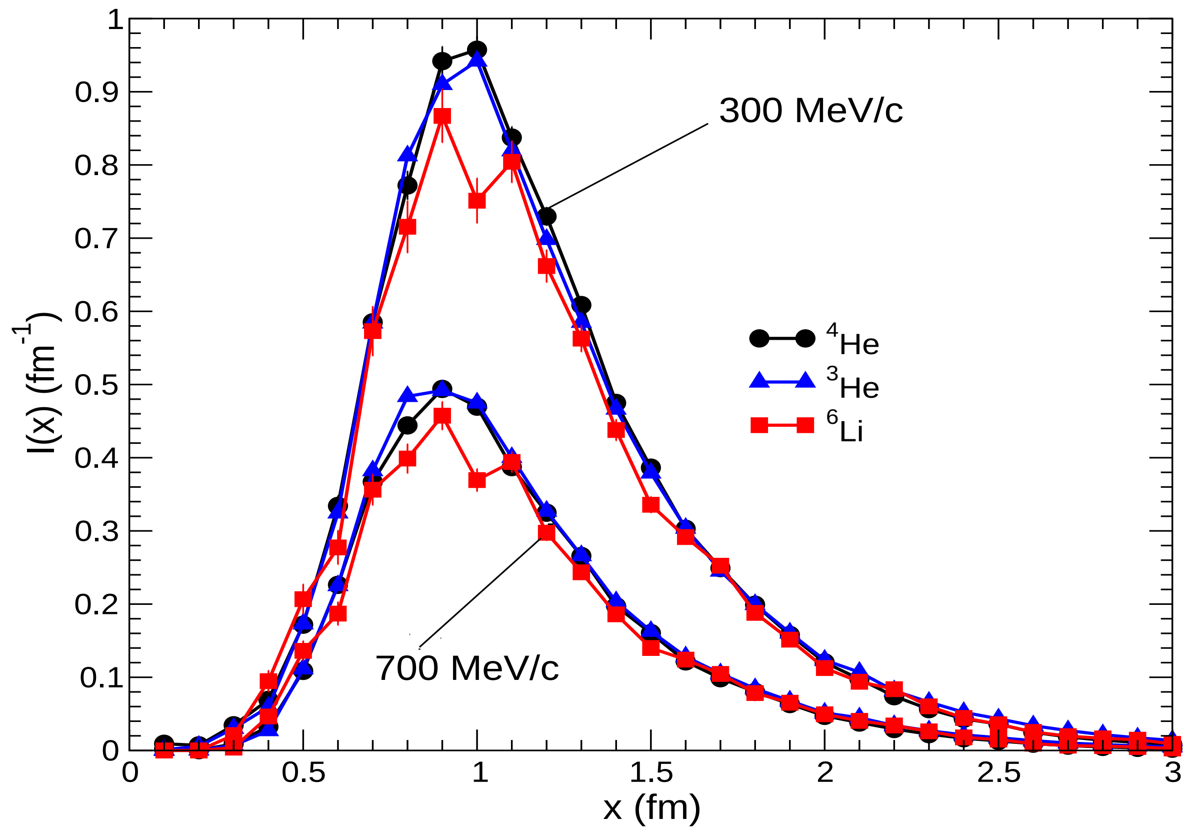
- ρ^A affected by central and tensor correlations
- Scaling property

$$\rho^{A}(x;pn,T=0) \simeq R_A \, \rho^{d}(x)$$

and similarly for $T=1$ pn pairs with $\rho^d \to \rho^{qb}$

	^{3}He	^{4}He	^{6}Li
R_A	2.0	4.7	6.3

A-Scaling Property



$$\int_0^\infty \mathrm{d}x\, I(x) = \Delta S_T \propto R_A / \left(Z\, \mu_p^2 + N\, \mu_n^2\right)$$

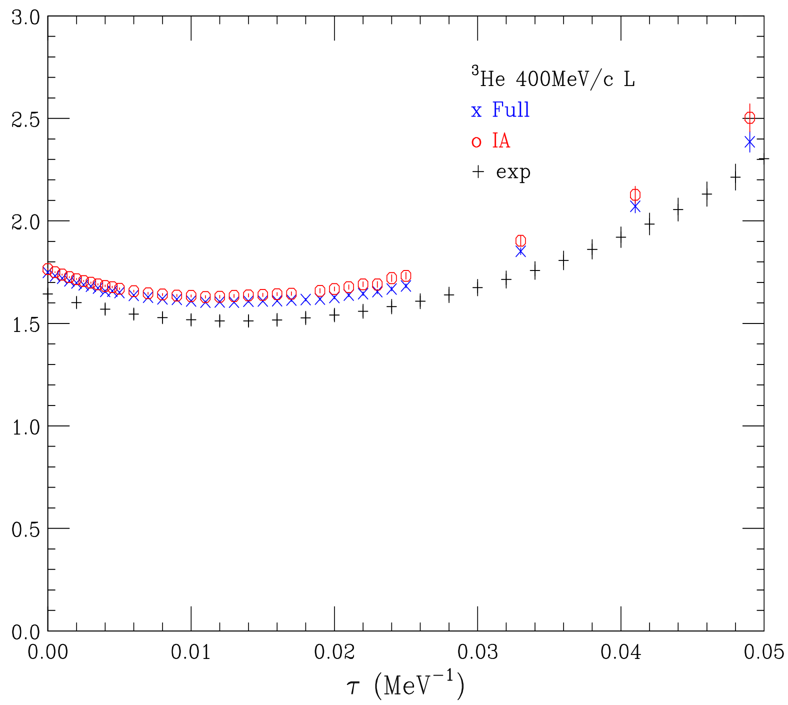
Euclidean Response Functions

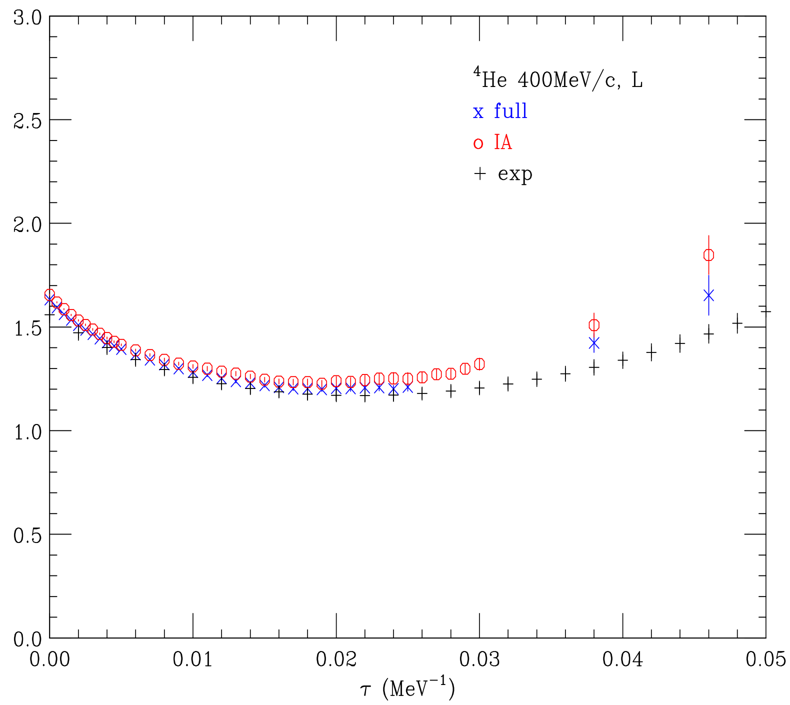
Carlson and Schiavilla (1992,1994)

$$\widetilde{E}_\alpha(q,\tau) = \int_{\omega_{\rm th}^+}^{\infty} {\rm d}\omega\, {\rm e}^{-\tau(\omega-E_0)}\, \frac{R_\alpha(q,\omega)}{G_{Ep}^2(q,\omega)}$$
$$= \langle 0|\, O_\alpha^\dagger(\mathbf{q}){\rm e}^{-\tau(H-E_0)} O_\alpha(\mathbf{q})\, |0\rangle - \text{(elastic term)}$$

- ${\rm e}^{-\tau(H-E_0)}$ evaluated stochastically with QMC
- No approximations made, exact
- At $\tau=0$, $\widetilde{E}_\alpha(q;0) \propto S_\alpha(q)$; as τ increases, $\widetilde{E}_\alpha(q;\tau)$ is more and more sensitive to strength in quasi-elastic region
- Inversion of $\widetilde{E}_\alpha(q;\tau)$ is a numerically ill-posed problem; Laplace-transform data instead

^{3}He and ^{4}He Longitudinal Euclidean Response Functions

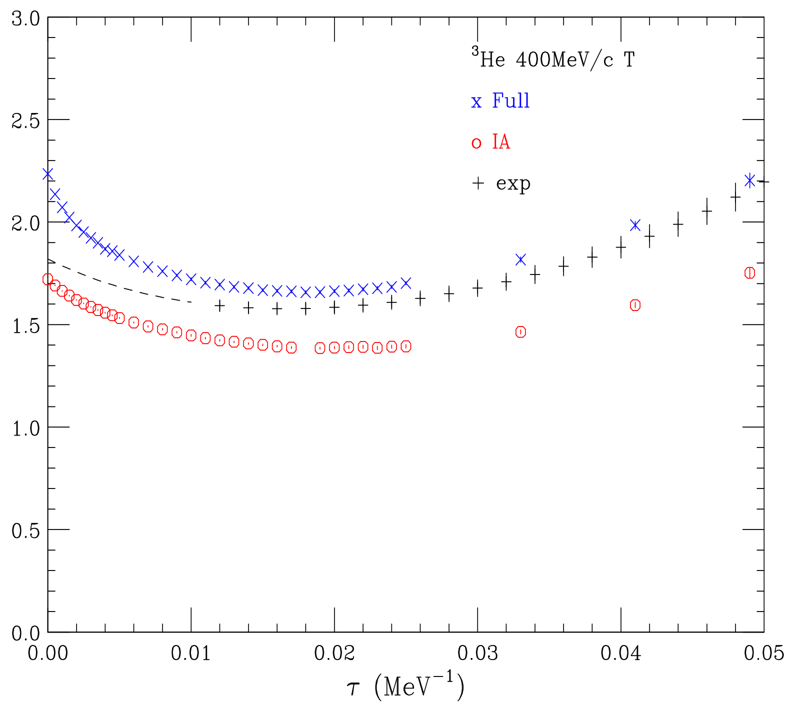


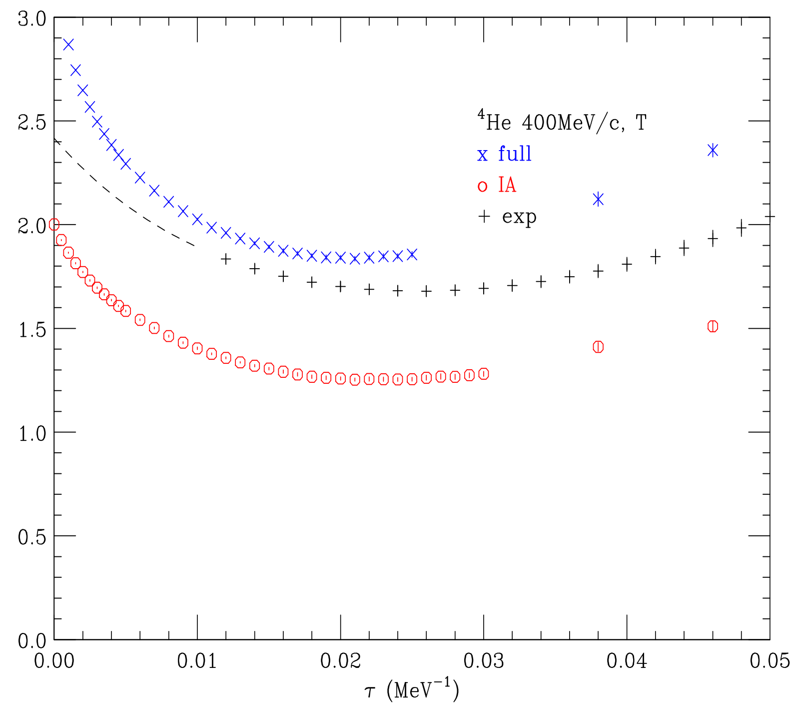


$$E_\alpha(q,\tau) = \exp\left[\tau\, q^2/(2\,m)\right]\ \widetilde{E}_\alpha(q,\tau)$$

and $E_L(q,\tau) \rightarrow Z$ for a collection of protons initially at rest

^{3}He and ^{4}He Transverse Euclidean Response Functions





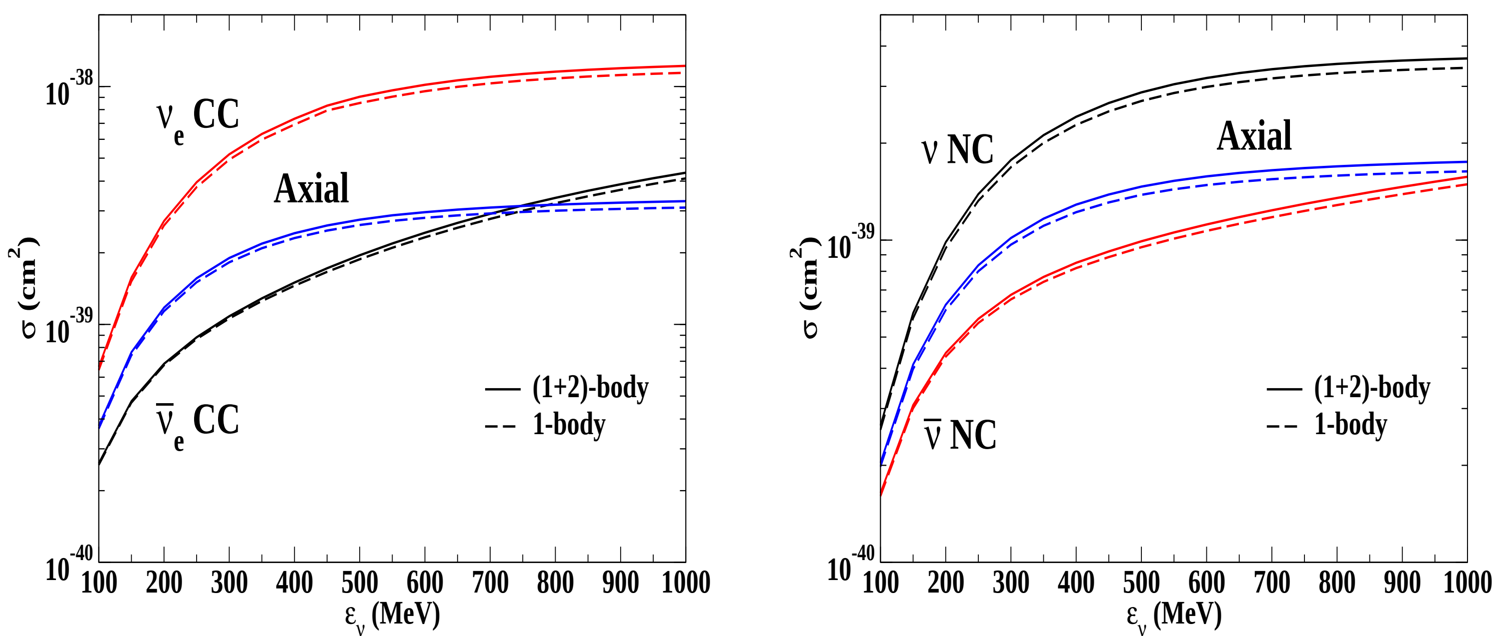
- Excess strength in quasielastic region ($\tau > 0.01$ MeV^{-1})
- Larger in $A = 4$ than in $A = 3$, as already inferred from S_T

Summary

- Exact QMC calculations of sum rules and Euclidean responses in light nuclei, based on realistic interactions and currents
- Large enhancement due to MEC of transverse sum rules
- Euclidean response calculations show that this enhancement may be as large 20–30% in the quasi-elastic peak region
- Implications for the excess of measured cross sections relative to theory seen in weak CC processes from ^{12}C at MiniBooNE ?

ν-Deuteron Scattering up to GeV Energy

Shen *et al.* (2012)



$$j^\mu_{NC} = -2\sin^2\theta_W\, j^\mu_{\gamma,S} + (1-2\sin^2\theta_W)\, j^\mu_{\gamma,z} + j^{\mu 5}_z$$

$$j^\mu_{CC} = j^\mu_\pm + j^{\mu 5}_\pm \qquad j_\pm = j_x \pm i\, j_y \qquad \left[\, T_a \,,\, j^\mu_{\gamma,z} \,\right] = i\,\epsilon_{azb}\, j^\mu_b$$

j^μ_{CC} reproduces well known weak transitions in $A \leq 7$ nuclei and μ-capture rates in d and ^{3}He [Schiavilla and Wiringa (2002); Marcucci *et al.* (2012)]

Outlook

- Inclusive ν scattering characterized by 5 response functions
- Sum rules of these (CC and NC) weak responses are being computed with QMC in $A \leq 12$
- QMC calculations of Euclidean responses will follow
- Determine MEC contributions in quasi-elastic ν-A scattering