Systematic muon capture rates in PQRPA

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Background

The muon capture processes have been used to scrutinize the nuclear structure models, since they provide a testing ground for wave functions and, indirectly, for the interactions that generate them. Several studies were performed by employing the random phase approximation (RPA) For example in [2], the total muon capture rates for a large number of nuclei with $6 < Z < 94$ have been evaluated, the authors claimed that an important benchmark was obtained by introducing the pairing correlations. They have done this ad-hoc by multiplying the one-body transition matrix elements by the BCS occupation probabilities. However, we know that the quasiparticle RPA (QRPA) formalism is a full self-consistent procedure to describe consistently both i) short-range particle-particle (pp) pairing correlations, and ii) long-range particle-hole (ph), correlations handled with RPA. Quite recently, the relativistic QRPA (RQRPA) [2] was applied in the calculation of total muon capture rates on a large set of nuclei from ^{12}C to ^{244}Pu, for which experimental values are available.
In this work we performed a systematic study of the inclusive muon capture rates for the nuclei ^{12}C, ^{20}Ne, ^{32}Mg, ^{28}Si, ^{40}Ar, ^{52}Cr, ^{54}Cr, ^{56}Fe, and ^{58}Ni using the Projected Random Quase-particle Phase Approximation (PQRPA) as nuclear model, because it is the only RPA model that treats the Pauli Principle correctly. The calculation were performed using the QRAP code [4] to evaluate semileptonic processes using QRPA and PQRPA as nuclear models.

μ-capture rates formalism

- **Muon Capture Rate:** For the muon capture process $\mu^- + (Z,A) \rightarrow (Z-1,A) + \nu_\mu$, and for a final state J_f, the muon capture rate reads

$$\Lambda(J_f) = \frac{E_\nu^2}{2\pi}|\phi_{1S}|^2 \mathcal{T}_{MC}(J_f), \tag{1}$$

where ϕ_{1S} is the muonic bound state wave function evaluated at the origin, and $E_\nu = \mathrm{m}_\mu - (\mathrm{M}_n - \mathrm{M}_p) - E_B^\mu - E_f + E_i$, where E_B^μ is the binding energy of the muon in the $1S$ orbit.

- **Transition probability**:

$$\mathcal{T}_{MC}(|\mathbf{k}|, J_f) = \frac{4\pi G^2}{2J_i+1}\sum_{\mathsf{J}}\left[|\langle J_f||\mathsf{O}_{\emptyset,\mathsf{J}} - \mathsf{O}_{0,\mathsf{J}}||J_i\rangle|^2 + 2|\langle J_f||\mathsf{O}_{-1,\mathsf{J}}||J_i\rangle|^2\right], \tag{2}$$

- **Nuclear Matrix Elements**:

For natural parity states, with $\pi = (-)^J$, i.e., $J^\pi = 0^+, 1^-, 2^+, 3^-, \cdots$:

$$\mathsf{O}_{\emptyset\mathsf{J}} - \mathsf{O}_{0,\mathsf{J}} = g_{\mathrm{V}}\frac{\mathrm{m}_\mu - \Delta E_{\mathrm{Coul}} - E_B}{E_\nu}\mathcal{M}_{\mathsf{J}}^V, \quad \mathsf{O}_{-1\mathsf{J}} = -(g_{\mathrm{A}} + \overline{g}_{\mathrm{W}})\mathcal{M}_{-1\mathsf{J}}^{A,I} + g_V\mathcal{M}_{-1\mathsf{J}}^{V,R},$$

For unnatural parity states, with $\pi = (-)^{J+1}$, i.e., $J^\pi = 0^-, 1^+, 2^-, 3^+, \cdots$:

$$\mathsf{O}_{\emptyset\mathsf{J}} - \mathsf{O}_{0,\mathsf{J}} = g_A\mathcal{M}_{\mathsf{J}}^A + (g_{\mathrm{A}} + \overline{g}_{\mathrm{A}} - \overline{g}_{\mathrm{P}})\mathcal{M}_{0\mathsf{J}}^A, \quad \mathsf{O}_{-1\mathsf{J}} = -(g_{\mathrm{A}} + \overline{g}_{\mathrm{W}})\mathcal{M}_{-1\mathsf{J}}^{A,R} - g_V\mathcal{M}_{-1\mathsf{J}}^{V,I}. \tag{3}$$

where $G = (3.04545 \pm 0.00006)\times 10^{-12}$ is the Fermi coupling constant (in natural units) and $g_V = -g_A = 1$: vector and axial-vector effective coupling constants. The other effective coupling constants g are given in Ref.[3]. The CVC relates the vector-current pieces of the operator $O_\alpha \equiv (\mathbf{O}, iO_\emptyset) = J_\alpha e^{-i\mathbf{k}\cdot\mathbf{r}}$, as $\mathbf{k}\cdot\mathbf{O}^V \equiv \kappa O_0^V = \tilde{k}_\emptyset O_\emptyset^V$, with $\tilde{k}_\emptyset \equiv k_\emptyset - \Delta E_{\mathrm{Coul}} + \Delta\mathrm{M}$, where $\Delta E_{\mathrm{Coul}} \cong \frac{6e^2Z}{5R} \cong 1.45ZA^{-1/3}$ MeV, is the Coulomb energy difference between the initial and final nuclei, $\Delta\mathrm{M} = \mathrm{M}_n - \mathrm{M}_p = 1.29$ MeV: neutron-proton mass difference. The consequence of the CVC relation is the substitution: $g_V\mathcal{M}_{0\mathsf{J}}^V - \overline{g}_{\mathrm{V}}\mathcal{M}_{\mathsf{J}}^V \rightarrow \frac{\tilde{k}_\emptyset}{\kappa}g_V\mathcal{M}_{\mathsf{J}}^V = g_{\mathrm{V}}\frac{\mathrm{m}_\mu - \Delta E_{\mathrm{Coul}} - E_B}{E_\nu}\mathcal{M}_{\mathsf{J}}^V$, employed to obtain (3) with $\tilde{k}_\emptyset = \mathrm{m}_\mu - \Delta E_{\mathrm{Coul}} - E_B$ and $\kappa = E_\nu$ for natural parity states. The Nuclear Operators read

$$\mathcal{M}_{\mathsf{J}}^V = j_{\mathsf{J}}(\rho)Y_{\mathsf{J}}(\hat{\mathbf{r}}), \quad \mathcal{M}_{m\mathsf{J}}^V = \mathrm{M}^{-1}\sum_{\mathsf{L}\geq 0} i^{\mathsf{J}-\mathsf{L}-1}F_{\mathsf{LJ}m}j_{\mathsf{L}}(\rho)[Y_{\mathsf{L}}(\hat{\mathbf{r}})\otimes\boldsymbol{\nabla}]_{\mathsf{J}}, \quad \mathcal{M}_{\mathsf{J}}^A = \mathrm{M}^{-1}j_{\mathsf{J}}(\rho)Y_{\mathsf{J}}(\hat{\mathbf{r}})(\boldsymbol{\sigma}\cdot\boldsymbol{\nabla}), \quad \mathcal{M}_{m\mathsf{J}}^A = \sum_{\mathsf{L}\geq 0} i^{\mathsf{J}-\mathsf{L}-1}F_{\mathsf{LJ}m}j_{\mathsf{L}}(\rho)\,[Y_{\mathsf{L}}(\hat{\mathbf{r}})\otimes\boldsymbol{\sigma}]_{\mathsf{J}}$$

where $F_{\mathsf{LJ}m} = (-)^{1+m}(1, -m\mathsf{J}m|\mathsf{L}0)$, M is the nucleon mass, and $\rho = |\mathbf{k}|r$.

Numerical results

For the set of nuclei were adopted the single-particle energies of the self-consistent calculation performed by Marketin *et al.* [2]. The BCS or PBCS equations were solved in these s.p. spaces adjusting the parameters $v_s^{pair}(p)$ and $v_s^{pair}(n)$ with the procedure of Ref. [5]. The QRPA and PQRPA calculations were performed using a residual δ-force residual interaction with variable particle-particle (pp) channel coupling $t = v_t^{pp}/v_s^{pair}$. The parameters for the particle-hole (ph) channel coupling are $v_s^{ph} = 27$ and $v_t^{ph} = 64$ (in MeV fm^3). These values were fitted to ^{48}Ca from a systematic study of the GT resonances [6]. The t-parameter corresponding to the particle-particle (pp) channel coupling, responsible for the known collapse effect of QRPA, is setting to $t = 0$, as the more reasonable value after several tests. The results for the Inclusive Muon Capture Rates (IMCR) are shown in next table and Fig. 1.

Nuclei	CVC-off	CVC-on	CVC-on+	Exp.
^{12}C	3.8	4.3	4.2	3.9
^{20}Ne	20.9	23.0	22.4	20.4
^{24}Mg	42.4	45.1	43.9	48.4
^{28}Si	88.0	93.0	90.5	87.1
^{40}Ar	121.6	125.0	121.0	135.5
^{52}Cr	348.8	354.9	342.0	345.2
^{54}Cr	277.6	284.5	274.0	305.7
^{56}Fe	414.4	422.5	405.5	441.1
^{58}Ni	673.8	689.2	662.9	611.0
^{60}Ni	588.6	604.0	580.9	556.0
χ^2	16.5	20.6	15.6	

FIG. 0: IMCR, Λ_{inc}, without CVC hypothesis (CVC-off); with CVC including (CVC-on+) and not (CVC-on) the second term in (2.18).

Model	J_n^π	1_1^+	2_1^+	2_1^-	1_1^-	Λ_{inc}
PQRPA	E	0.00	0.43	6.33	6.83	
	Λ	8.80	0.20	0.60	0.85	37
PQRPA [3]	E	0.00	0.50	2.82	3.31	
	Λ	6.50	0.16	0.18	0.51	40
SM [7]	E	0.00	0.76	1.49	1.99	
	Λ	6.0	0.25	0.22	1.86	
RPA [8]	Λ	25.4 (22.8)	$\leq 10^{-3}$	0.04 (0.02)	0.22 (0.74)	
Exp. [9]	E	0.00	0.95	1.67	2.62	
	Λ	6.00 ± 0.40	0.21 ± 0.10	0.18 ± 0.10	0.62 ± 0.20	38 ± 1

FIG. 1: Λ (10^3 s^{-1}) in excited states of ^{12}B.

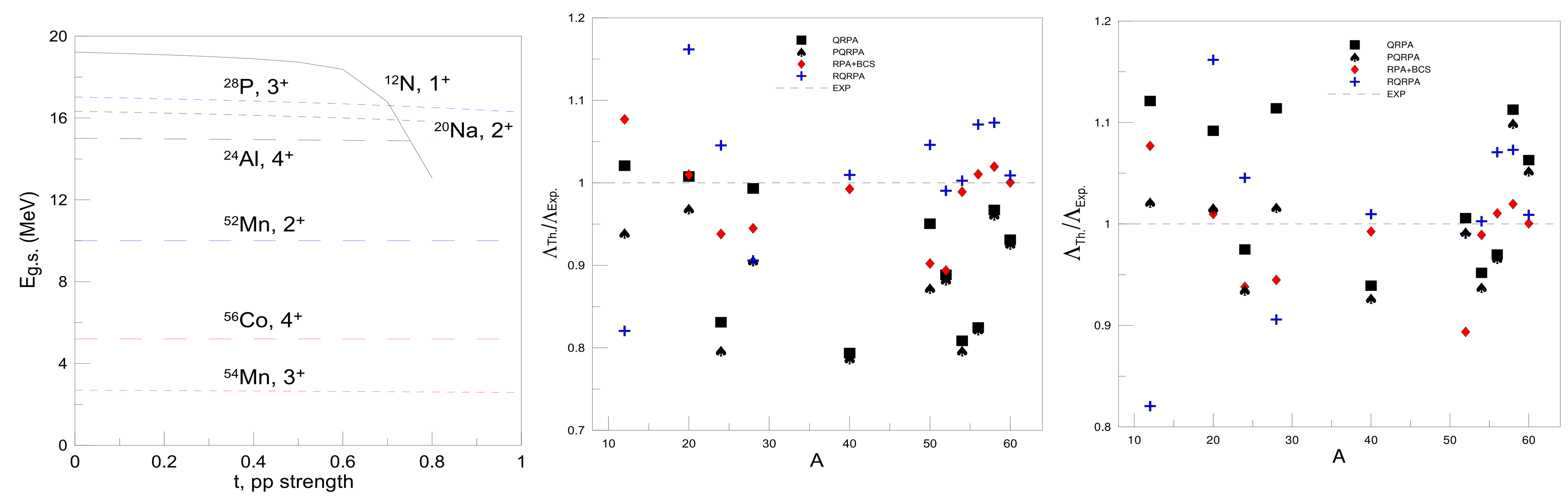

FIG. 1: *Left*: Energies for $Z+1$ nuclei as t-function in QRPA; *Central*: Theoretical/experimental quotient IMCR for $t = 0$, $g_{\mathrm{A}} = -1$, $\chi^2 = 44.6$ in PQRPA ; *Right*: Same for $t = 0$, $g_{\mathrm{A}} = -1.135$, $\chi^2 = 24.3$ in PQRPA.

An agrement for the ground state energies in $\beta^\pm$-decay and ϵ electron capture data nuclei is obtained when the parameter of pp-channel t is totally switched off, i.e, $t = 0$. These values are sketched in left panel of Fig.1. Only for ^{12}C, this g.s. energy shows a notable variation for higher t values. This effect is understood because the $J^\pi = 1^+$ g.s. in ^{12}N and ^{12}B are strongly dependent of t-value by the known QRPA collapse. The above mentioned behavior is washed in the other nuclei with $J^\pi \neq 1^+$, as we can shown in left panel of Fig. 1.

A numerical function $\chi^2 = \sum_i \frac{[\Lambda_{Th}(i) - \Lambda_{Exp}(i)]^2}{\Lambda_{Exp}(i)}$ is taking into account the deviation from theoretical to experimental values of IMCR. The theoretical results of the IMCR within the PQRPA have been compared with those obtained in other works using the models of RPA+BCS [1] and RQRPA (relativistic QRPA) [2]. This leads to a modification of the axial coupling constant $g_{\mathrm{A}} = 1$ to $g_{\mathrm{A}} = 1.135$, resulting in one better agreement with the experimental data. The influence of the CVC (Conserved Vector Current) in the muon capture rates for the presented nuclei was explicitly verified for the first time in the literature. This showed to be more significant in lighter nuclei, still more when the Coulomb term of muon-nucleus interaction is disrespected. These results are shown in the table Fig. 0, where IMCR were evaluated with/without CVC hypothesis.
A final comparison was carried through inclusive capture and exclusive muon capture rates in ^{12}C showing that the PQRPA present a good experimental agreement for the inclusive capture, but not for the exclusive one. We do not dispose of other muon capture exclusive calculations in other models to exhaustive our study.

Conclusions

We reckon that the comparison between theory and data for the inclusive muon capture is not a fully satisfactory test on the nuclear model that is used. The exclusive muon transitions are more robust for such a purpose. Therefore, it would be necessary more experimental data for the exclusive capture rates in other nuclei, beyond ^{12}C, to test if a nuclear model is satisfactory [4].

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