How much does the MSW effect contributes to the reactor antineutrino anomaly?

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Abstract. It has been pointed out that there is a $5.7 \pm 2.3$ discrepancy between the predicted and the observed reactor antineutrino flux in very short baseline experiments. Several causes for this anomaly have been discussed, including a possible non-standard forth sterile neutrino. In order to quantify how much non-standard this anomaly really is, the standard MSW effect is reviewed. Knowing that reactor antineutrinos are produced in a dense medium (the nuclear fuel) and is usually detected in a less dense one (water, or scintillator), non-adiabatic effects are expected to happen, creating a difference between the creation and detection mixing angles.

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INTRODUCTION

Although neutrino interactions with matter are feeble (the mean free path of low energy neutrinos on lead is larger than a light-year), matter can still affect neutrinos through coherent scattering, what is called MSW effect. On Earth based neutrino sources this can be almost always neglected since it depends on the density of the surroundings, which varies between 1 and $4 \text{g/cm}^3$ in the Earth's crust. On the other hand, it is the major effect to be considered on solar neutrinos since the Sun’s core density varies from 150 to $160 \text{g/cm}^3$. But the dependence of the MSW effect on neutrinos is not just related to the concentration of matter, but also with how much it changes along the way. Abrupt changes lead to what is called Non-Adiabatic effects that have strong influences on the oscillation pattern, enhancing or suppressing the survival probability of a given neutrino flavor. This is exactly what happens with reactors antineutrinos: they are created inside the nuclear fuel rods, which are as dense as $18 \text{g/cm}^3$, and immediately leave to water. From their extreme relativistic reference frame the experienced change on the surroundings is almost discontinuous, what could in principle lead to very strong effects. Of course, no strong change on the vacuum oscillation pattern is observed, raising the questions: How much does the MSW effect contributes to the so called reactor antineutrino anomaly[1]? Does it contribute at all, and if so, could it explain the observed deficit?

THE MSW EFFECT

When propagating in the presence of regular matter, electron (anti)neutrinos experience coherent forward scattering. The hamiltonian that describes the neutrino state evolution must consider a contribution from the weak equivalent to a “potential” where each possible interaction (i.e. charged or neutral current) have its contribution. These contributions alter the electron neutrino state, keeping the coherence of the mass-flavor mixing. In some ways, this is analogous to the effects that lead to the refractive index of light in a medium and is called MSW effect[2]. The effective Schrödinger equation, in two families\(^1\), is given by:

\[
\frac{i}{\hbar} \frac{d}{dx} \begin{pmatrix} \tilde{\nu}_1 \\ \tilde{\nu}_2 \end{pmatrix} = \begin{pmatrix} m_1^2/2E & -i\frac{d\theta}{dx} \\ i\frac{d\theta}{dx} & m_2^2/2E \end{pmatrix} \begin{pmatrix} \tilde{\nu}_1 \\ \tilde{\nu}_2 \end{pmatrix}
\] (1)

where the tilde $\sim$ sign represents the presence of matter on the definition of the mass states $\nu_1$ and $\nu_2$. Changing eq.1 to the flavor base gives the relation between the vacuum mixing $\theta$ and the mixing in the presence of matter $\tilde{\theta}(x)$, given

\(^1\) To appreciate the possible effects on $\theta_{13}$, three families should be considered[3] in a future work.
by:

$$\sin^2 2\tilde{\theta} = \frac{\sin^2 2\theta}{\sqrt{(\cos 2\theta - a)^2 + \sin^2 2\theta}}$$  \hspace{1cm} (2)$$

where \(a\) gives the strength of the MSW effect:

$$a = \pm \sqrt{2} G_F E_\nu \cdot n_e (x) \frac{\Delta m^2}{\Delta \tilde{m}^2} \hspace{1cm} (3)$$

where \(G_F\) is the Fermi constant, \(E_\nu\) is the neutrino energy, \(n_e\) is the electron number density and \(\Delta m^2\) is the squared mass difference (in vacuum). The plus and minus signs correspond to neutrinos and antineutrinos, respectively. The squared mass difference is also affected by the surroundings, being modified as \(\Delta \tilde{m}^2\):

$$\Delta \tilde{m}^2 = \frac{\sin^2 2\theta}{\sin^2 2\tilde{\theta}} \Delta m^2.$$  \hspace{1cm} (4)

Equation 4 represents the change in the energy levels of the neutrino states.

**DECOMPRESSION AND LEVEL CROSSING**

The hamiltonian on Eq. 1 can be simplified for two extreme cases: constant medium density and abrupt changes, from the neutrino’s reference frame. These are represented respectively by:

$$H(i) = \frac{\Delta \tilde{m}}{2E_\nu} \sigma_z$$ \hspace{1cm} (5)$$

and

$$H(ii) = \frac{d\tilde{\theta}}{dx} \sigma_y.$$ \hspace{1cm} (6)$$

Integrating Eq. 1 with approximation \(H(i)\) (Eq.5) leads to the usual oscillation pattern, with the proper substitution: \(\theta \rightarrow \tilde{\theta}\) and \(\Delta m^2 \rightarrow \Delta \tilde{m}^2\). Our hypothesis is based on the fact that the parameters relating mass and flavor states (namely \(\tilde{\theta}\) and \(\Delta \tilde{m}^2\)) are different between the creation and the detection points, due to the difference in density. In a nuclear reactor, the antineutrinos are created inside the fuel rods and detected in water or liquid scintillators. During the neutrino history, it is possible to distinguish between two effects created by the MSW.

The first one happens just as the neutrino state leaves the denser medium (nuclear fuel) to the less dense one (water). Due to its extreme relativistic frame, this change can be viewed as discontinuous and leads to a discrete change in the neutrino state. In the mass base, integrating Eq. 1 with approximation \(H(ii)\) (Eq.6) gives:

$$\nu' = e^{i\Delta \tilde{\theta} \sigma_x} \nu$$ \hspace{1cm} (7)$$

where \(\Delta \tilde{\theta}(a \rightarrow b) = \tilde{\theta}_b - \tilde{\theta}_a\). From now on, this effect will be referred to as **decompression** (in a symbolic analogy to the effects experienced by a diver when emerging from the highly pressurized depths).

Due to the geometrical disposition of the reactor’s fuel rods, it is more than likely that the neutrino will enter and leave the denser medium several times. This effect will be referred here as **level crossing**, and is characterized by the successive and alternating application of the hamiltonians \(H(i)\) and \(H(ii)\). This leads to a relatively complex modification of the oscillation probability, which is then given by:

$$P_{ee} = \cos^4 \Delta \tilde{\theta} + \sin^2 \Delta \tilde{\theta} \sin^2 \Sigma \tilde{\theta}$$

$$+ (\cos^2 \Delta \tilde{\theta} - \cos^2 \Sigma \tilde{\theta}) \sin^2 \alpha$$

$$+ \frac{1}{4} \sin 2\Sigma \tilde{\theta} \sin^2 2\Delta \tilde{\theta} \sin 2\alpha$$ \hspace{1cm} (8)$$

where \(\Delta \tilde{\theta} = \theta - \tilde{\theta}_{fuel}\) and \(\Sigma \tilde{\theta} = \theta + \tilde{\theta}_{fuel}\). In this expression it is considered that every media in the neutrino’s way can be approximated by vacuum, except the high density fuel. The propagation parameter \(\alpha\) is given by:
\[ \alpha = \frac{\Delta m^2}{4E_{\nu}} (L_{\text{reactor}} + L_{\text{baseline}}) \]  

(9)

where \( L_{\text{reactor}} \) and \( L_{\text{baseline}} \) are the total distances traveled inside the fuel and on the vacuum respectively. The usual baselines are such that \( L_{\text{baseline}} \gg L_{\text{reactor}} \).

**DISCUSSIONS**

The phenomenology behind Eq. 8 is rich and takes some time to be appreciated. First of all, it should be noted that the propagation parameter \( \alpha \) appears as the argument of two trigonometric functions \( \sin^2 x \) and \( \sin 2x \), but both of them have the same period \( \pi \), so that the oscillation pattern remains unchanged. Another important feature of this solution is that even for \( \alpha = 0 \) (no distance traveled) the survival probability is different from 1, what is expected from the vacuum case. On the case showed here:

\[ P_{ee} \approx \cos^4 \Delta \tilde{\theta} \quad \text{when} \quad \alpha = 0 , \]

(10)

where it is considered that \( \sin^2 \Delta \tilde{\theta} \approx 0 \). Anyway, when using the know values of the fuel density, Eq. 10 gives a value that is actually indistinguishable from unit, for any energy inside the reactor spectrum. It is relevant to remark that this value grows with energy and can be appreciable for high energies (like those from neutrino factories).

The only new effect in the reach of the reactor neutrino energy seems to appear for relatively larger baselines: \( L_{\text{baseline}} \geq 100m \). Specially in the case of the present generation of \( \theta_{13} \) experiments (Double Chooz, Reno and Daya Bay), with baselines of the order of 1km, a 2% reduction of neutrino with energies \( E_{\nu} < 2\text{MeV} \) is expected due to the enhanced conversion cause by the decompression.

**CONCLUSIONS**

This is a review of the MSW effect in the light of the reactor antineutrino scenario. When applied to the production of neutrinos inside the dense nuclear fuel rod, this standard and well established (by the solar neutrino observations) effect leads to small modifications in this particular case, and can not be responsible for the reactor antineutrino anomaly[1]. Nevertheless, it is important to note that with the multiple detectors scheme, the present generation of \( \theta_{13} \) experiments could have their systematics reduced enough to observe a 2% reduction in the lower part of the spectrum, when comparing the longer 1km baseline to the shorter \( \sim 100m \) one.

**REFERENCES**