Impact of systematic uncertainties for the neutrino parameter measurement in superbeam experiments

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Main motivation of this work
comparing the performances of different nuclear models for physically interesting neutrino observables

here I propose a simplified analysis of the T2K data based on
BUT using the new appearance data shown in ICHEP

- fitting the T2K data in appearance for $\theta_{13}$ and $\delta$
  - reproducing the T2K data for the $\nu_\mu \rightarrow \nu_e$ oscillation
  - the effect of using different cross sections

- fitting the T2K data in disappearance
- assuming $X 10$ statistics in appearance
\( \nu \) flavour conversion has been confirmed in many experiments

\[
U = R_{23}(\theta_{23})R_{13}(\theta_{13}, \delta)R_{12}(\theta_{12})
\]

The neutrino oscillation probability (in matter)

\[
P_{\alpha\beta} = |A_{\alpha\beta}|^2 = \sum_{i,j} \tilde{U}_{\alpha i}^* \tilde{U}_{\beta i} \tilde{U}_{\alpha j}^* \tilde{U}_{\beta j} \exp \left( i \frac{\tilde{m}_j^2 - \tilde{m}_i^2}{2E} L \right)
\]

\( E \) is the neutrino energy, \( L \) is the baseline length, \( \tilde{m}_i \) and \( \tilde{U}_{\beta j} \) are the mass of the \( i \)th neutrino mass eigenstate and the mixing matrix in matter

- Usual assumption: \( U \) is a \( 3 \times 3 \) unitary mixing matrix
- three angles \( \theta_{ij} \) and one CP phase \( \delta \)

\[ \downarrow \]

the standard framework implies 7 parameters to describe \( \nu \) oscillation in matter
Great interest on $\theta_{13}$ and $\delta$

The appearance neutrino oscillation probability ($\alpha \neq \beta$)

$$P_{\nu_\mu \to \nu_e} = s_{23}^2 \sin^2 2\theta_{13} \sin^2 (\Delta_{atm} L) + c_{23}^2 \sin^2 2\theta_{12} \sin^2 (\Delta_{sol} L) + \tilde{J} \cos (\delta_{CP} + \Delta_{atm} L) (\Delta_{sol} L) \sin (2 \Delta_{atm} L)$$

Many future experiments will look for a precise measurement of $\theta_{13}$.
Large $\theta_{13}$ means good chance to reveal the CP violation in the leptonic sector.

One needs to control:
- flux composition
- detector response
- nuclear cross sections
The \( \nu \)-nucleus cross sections \((\nu A \rightarrow \mu X)\)

- **FG** = Fermi Gas \(\text{R. A. Smith, E. J. Moniz, Nucl. Phys. B43 (1972) 605}\)
- **SF** = Spectral Function \(\text{O. Benhar et al., Phys. Rev. D 72 (2005) 053005}\)
- **RMF** = Relativistic mean field \(\text{J. M. Udias et al., Phys. Rev. C 64, 024614 (2001)}\)
- **RPA** = Random Phase Approximation
  \(\text{M. Martini et al., Phys. Rev. C80, 065501 (2009)}\) \(\rightarrow\) from now on: the MECM model
Useful tools

- **GloBES**, to simulate the T2K experiment

- **MonteCUBES**, to fit the experimental data

**caveat:**
we use an energy resolution function to "mimick" the relation between the true and reconstructed neutrino energy
but see for a detailed discussion:
statistics is too small to draw definite conclusions but the exercise may serve to illustrate how to use ”real” data to study $\nu - N$ cross sections

**STRATEGY**

- we first used the software GLoBES to reproduce the official T2K analysis (cross sections are based on Fermi Gas)

1. cross section normalization with the $\nu_\mu$ inclusive CC at the ND
   (in the energy range $[0, 5]$ GeV, $3.01 \times 10^{20}$ POT)
   we have to reproduce $\sim 1.6 \times 10^4$ $\nu_\mu$ inclusive CC

![Graph](image-url)
Playing with the T2K appearance data

2 computation of the expected events at the far detector and compare with the T2K MonteCarlo estimates (in the energy range \([0.1, 1.25] \text{ GeV}\))

![Graph showing expected events vs energy]


<table>
<thead>
<tr>
<th>channel</th>
<th>bin 1</th>
<th>bin 2</th>
<th>bin 3</th>
<th>bin 4</th>
<th>bin 5</th>
<th>total</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>exp data</strong></td>
<td>0</td>
<td>4</td>
<td>3</td>
<td>3</td>
<td>1</td>
<td>11</td>
</tr>
<tr>
<td><strong>MC estimates</strong></td>
<td>(\nu_\mu \to \nu_e)</td>
<td>1.00</td>
<td>2.15</td>
<td>3.7</td>
<td>1.45</td>
<td>0.35</td>
</tr>
<tr>
<td>(\nu_e \to \nu_e)</td>
<td>0.10</td>
<td>0.35</td>
<td>0.40</td>
<td>0.35</td>
<td>0.30</td>
<td>1.50</td>
</tr>
<tr>
<td>NC</td>
<td>0.10</td>
<td>0.50</td>
<td>0.30</td>
<td>0.20</td>
<td>0.15</td>
<td>1.25</td>
</tr>
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</table>

the comparison allows to "mimick" the experimental efficiencies \(\varepsilon_i\) bin-by-bin

it turns out that \(\varepsilon \sim 0.3\)
Playing with the T2K appearance data

we performed a very simple $\chi^2$ analysis

$$\chi^2 = \frac{(N_{com} - N_D)^2}{\sigma_D^2 + N_{NC} + N_{\nu_e} + S}$$

- $S = (S_D N_D)^2 + (S_{NC} N_{NC})^2 + (S_D N_{\nu_e})^2$
- $N_{com}, N_D$ are the computed number of oscillated events and the data, respectively
- $N_{NC}, N_{\nu_e}$ are the event rates for NC and $\nu_e$ contamination, respectively
- $\sigma_D$ is the bin uncertainties on the data: $(0, 2, 1.5, 1.5, 0.5)$
- $S_D = 0.07$ and $S_{NC} = 0.3$ are systematic errors on the (data,$\nu_e$) and NC events

best fit ($\chi^2_{min} = 3.74$): $\sin^2(2\theta_{13}) = 0.089$ $\delta_{CP} = 0.22$

obviously, good agreement with the official T2K results
Playing with the T2K appearance data

- for a different model, we repeat the previous steps using the same $\varepsilon_i$
- redo the analysis for the MECM model

**Table:**

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<th>Exp Result</th>
<th>MECM</th>
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<tr>
<td>$\nu_\mu \rightarrow \nu_e$</td>
<td>8.65</td>
<td>11.08</td>
</tr>
<tr>
<td>$\nu_e \rightarrow \nu_e$</td>
<td>1.5</td>
<td>1.97</td>
</tr>
<tr>
<td>NC</td>
<td>1.25</td>
<td>1.25*</td>
</tr>
</tbody>
</table>

- total rates for $\sin^2 2\theta_{13} = 0.1$
Playing with the T2K results

Playing with the T2K appearance data

- total rates for $\sin^2 2\theta_{13} = 0.1$
- larger signal, must be compensated by smaller $\theta'_{13}$

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$\chi^2_{min} = 3.65$

$\sin^2(2\theta_{13}) = 0.065$

$\delta_{CP} = 0.14$
Playing with the T2K results

Playing with the T2K appearance data

- comparing FG and MECM models

- showing the $\chi^2 - \chi_{min}^2$ function for 1 dof ($\delta_{CP} = 0$, good for both models)

\[
\sin^2 2\theta_{13}^{MECM} = 0.081(0.047 - 0.049)
\]
\[
\sin^2 2\theta_{13}^{FG} = 0.114(0.060 - 0.063)
\]

- results are clearly compatible at 1σ
now the disappearance data

The disappearance neutrino oscillation probability \( (\alpha = \beta) \)

\[
P(\nu_\mu \rightarrow \nu_\mu) = 1 - \sin^2 2\theta_{23} \sin^2 (\Delta_{atm} L)
\]

  - The T2K collaboration collected 31 data events, grouped in 13 energy bins
  - the sample extends up to 6 GeV and it is mainly given by \( \nu_\mu \) CCQE, \( \nu_\mu \) CC non-QE, \( \nu_e \) CC and NC.
  - we normalized the FG cross section to the total rates: 17.3, 9.2, 1.8 and <0.1 events for \( \nu_\mu \) CCQE, \( \nu_\mu \) CC non-QE, NC and \( \nu_e \) CC, respectively.
  - we have adopted a conservative 15% normalization error and energy calibration error at the level of \( 10^{-3} \) for both signal and background.
now the disappearance data

<table>
<thead>
<tr>
<th></th>
<th>best fit ((\sin^2 2\theta_{23}, \Delta m^2_{23}))</th>
<th>(\sin^2 2\theta_{23})-range</th>
<th>(\Delta m^2_{23})-range</th>
</tr>
</thead>
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<tr>
<td><strong>FG</strong></td>
<td>((0.99, 2.56))</td>
<td>(&gt; 0.86)</td>
<td>((2.22-2.90))</td>
</tr>
<tr>
<td><strong>MECM</strong></td>
<td>((1.00, 2.62))</td>
<td>(&gt; 0.91)</td>
<td>((2.31-2.93))</td>
</tr>
</tbody>
</table>
we assume the same energy distribution for the appearance channel and multiply the total $\nu_e$ by a factor of 10

\[
\sin^2 2\theta_{13}^{FG} = 0.108^{+0.024}_{-0.028} \quad \delta \sim 111^\circ \\
\sin^2 \theta_{13}^{MECM} = 0.078^{+0.019}_{-0.018} \quad \delta \sim 131^\circ
\]
Summary

- we played a bit with the T2K data, comparing the results for $\theta_{13}$ and $\delta_{CP}$ obtained with the FG and MECM models
  - idea: give an estimate of the systematic effects encoded in the knowledge of the $\nu$-N cross section (rough estimate)

|          | $|\Delta \theta_{13}|/\theta_{13}^{FG}$ | $|\Delta \theta_{23}|/\theta_{23}^{FG}$ | $|\Delta \Delta m_{23}|/(\Delta m_{23}^{2})^{FG}$ |
|----------|--------------------------------------|--------------------------------------|--------------------------------------|
| $X \ 1$  | 30%                                  | 6.0%                                 | 2.3%                                 |
| $X \ 10$ | 28%                                  | 4.6%                                 | 1.5%                                 |

- $\Delta \delta_{CP}/\delta_{CP}^{FG} \sim 15\%$
Backup slides
The Random Phase Approximation (RPA)

\[
\frac{d^2 \sigma_{IA}}{d\Omega dE_l} \propto \sum_i K_i R_i
\]

- \(K_i\) = kinematical factors
- \(R_i\) = response functions,

\[
R(\omega, q) = -\frac{\mathcal{V}}{\pi} \text{Im}[\Pi(\omega, q, q)].
\]

To lowest order the QE cross section is given by the terms in \(R_{NN}\) \([R_{NN}^{NN}\) (isovector interaction), \(R_{\sigma\tau}^{NN}\) (isospin spin-transverse interaction)]

Lowest-order contribution from \(R_{NN}, R_{N\Delta}\) and \(R_{\Delta\Delta}\).

Wiggly lines represent the external probe, solid lines correspond to the propagation of a nucleon (or a hole), double lines to the propagation of a \(\Delta\) and dashed lines to an effective interaction between nucleons and/or \(\Delta\)s.

Dotted lines show which particles are placed on-shell.
The Relativistic Fermi Gas Model

- many MonteCarlo codes (GENIE, NuWro, Neut, Nuance) use some version of the Fermi model
  - target nucleons are moving (Fermi motion) subject to a nuclear potential (binding energy)
  - the ejected nucleon does not interact with other nucleons (Plane Wave Impulse Approximation)
  - Pauli blocking reduces the available phase space for scattered particle
- in terms of Spectral Function:

\[
P_{RFGM} = \left( \frac{6\pi^2 A}{p_F^3} \right) \theta(p_F - \vec{p})\delta(E_{\vec{p}} - E_B + E)
\]

where

- \( p_F \) = Fermi momentum \((225\) MeV for Oxygen) 
- \( E_B \) = average binding energy \((25\) MeV for Oxygen) 
- \( E \) = removal energy
Before and after normalization

![Graph showing normalization results before and after normalization.](attachment:image.png)