NOTES ON COMPUTING THE VLENF EXPOSURE

CHRISTOPHER D. TUNNELL

1. Introduction

The sensitivity of the VLENF is driven by its neutrino exposure. The physics the VLENF is trying to pursue strongly depends on numerous parameters of how the accelerator will perform. For the purpose of this note, Fermilab is the assumed site. By choosing pessimistic numbers, we can feel confident that the accelerator performance will not surprise us in a bad way.

The parameter of interest for the VLENF is the useful muon decays, call it N_μ. These are muons that decay in the decay ring directed at the far detector. We can define N_mu in terms of various accelerator parameters as follows:

$$N_\mu = (\mathrm{POT}) \times (\pi \text{ per POT}) \times \epsilon_{\mathrm{collection}} \times \epsilon_{\mathrm{injection}} \times (\mu \text{ per } \pi) \times A_{\mathrm{dynamic}} \times \Omega \tag{1}$$

where (POT) is the number of protons on target, $\epsilon_{\mathrm{collection}}$ is the collection efficiency, $\epsilon_{\mathrm{injection}}$ is the injection efficiency, (μ per π) is the chance that an injected pion results in a muon within the acceptance, A_{dynamic} is probability that a muon within the aperture is within dynamic aperture.

2. POT

For the booster, there are 5×10^{12} protons per pulse, the repetition rate is 15 Hz, and the livetime per calendar year is 2×10^{17} seconds [1]. This results in 1.5×10^{21} protons per year at 8 GeV. You will never do better than this at either 8 GeV or 60 GeV since the main injector is downstream of the booster.

Now let's do another calculation that is very similar to the above. For 60 GeV, the MI has to ramp and do what sounds like the merging of bunches, so its repetition rate is only actually 12 Hz [1], which gives us by the same logic above 6×10^{13} protons per second at 60 GeV.

Let's say we wanted to build/use/steal a 100 kilowatt target station. We know that $P = N \times E$ where P is the power, N is the number of particles per second, and E is the energy of the beam. The beam energy is 60 GeV which means that we would get 10^{13} Hz for this setup. Multiplying by a Fermilab year of 2×10^{17} seconds, then that's 2×10^{20} POT per Fermiyear. It would take 5 years of running for 10^{21}.

This number seems a little long, but doable. We do not need all 10^{21} to get a $>> 5\sigma$ measurement of sterile neutrinos but 10^{21} is mainly used because it's the amount of beam other proposals are assuming.

3. (π per POT)

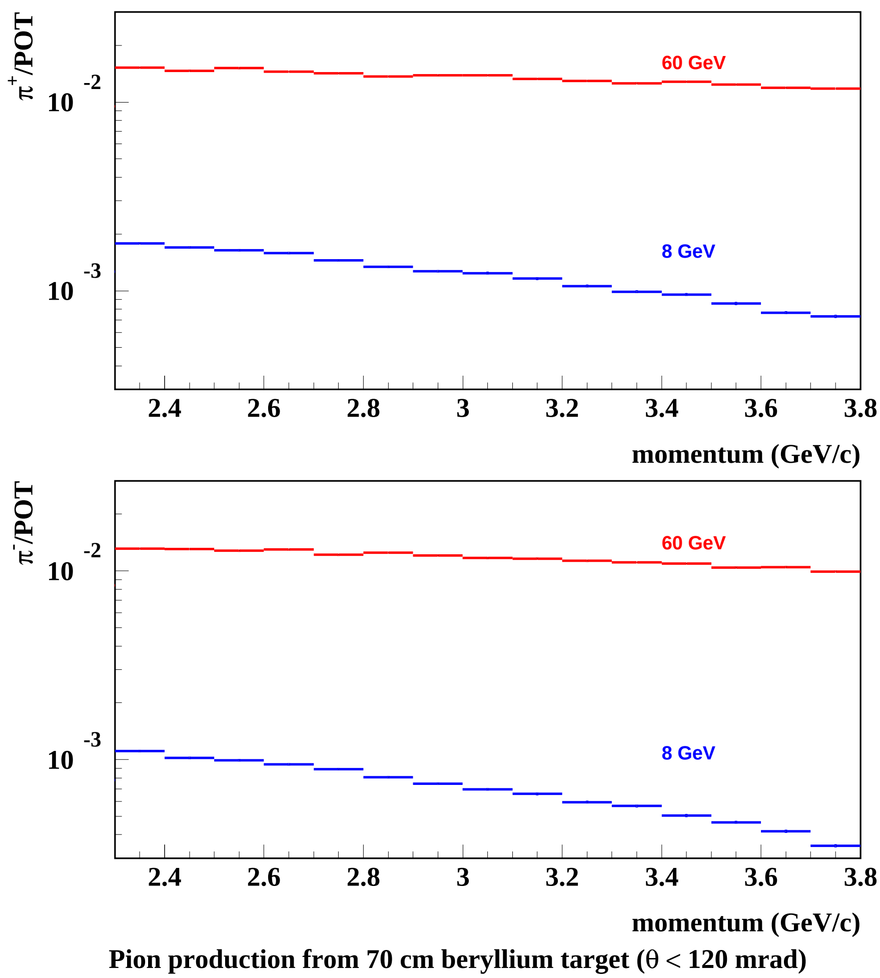

Figure 1. Pion production [2].

Per proton that is shot into the target, there is the question of the pion multiplicity in the forward direction. This has been simulated with, I assume, Mars? The forward direction in this case is defined to be angles less than 0.12. The results of this analysis can be seen in Fig. 1 where each bin is 100 MeV/c.

Without much loss of generality, let's assume that there is a decay straight before injection so we can use the work in [3] for injection and [4] for the FFAG lattice. The energy spreads within the injection schemes and the longitudinal dynamic apertures are $> 15\%$.

Let's be very conservative and say we only have a 10% spread with the initial pions. We can then integrate the differential rate in Fig. 1. The initial pions are at roughly 3 GeV since then the more backward decaying muons have energies around 2 GeV. In the figure, one can see that per bin the differential rate is about $1.6 * 10^{-2}$ per 100 MeV bin. For a 10% energy spread, we want 6 bins, which means an integrated rate of 0.1 pions per POT.

4. $\epsilon_{\text{COLLECTION}}$

We assume a conservative 10% loss of pions during the collection phase, so the efficiency is 90%.

5. (μ PER π)

The geometry of the ring we're considering has a 108 m straight section. The $c\tau\gamma$ of a 3 GeV pion is 210 meters. The probability of a pion decaying in a 108 meter straight is 40%. So at very most we'll get 40% of the pions giving us a muon.

There's also a factor to take into account the decay kinematics and accelerator aperture since mainly backward decaying muons are transported through the lattice. The number for the probability that a pion decay results in a muon with the aperture of the accelerator is assumed to be 20% [5].

Thus the combined probability that a decaying pion results in a useful muon is 8%.

6. $\epsilon_{\text{INJECTION}}$

The injection efficiency is assumed to be 90%.

7. A_{DYNAMIC}

90% of muons with the dynamic aperture of the FFAG are assumed to be retained for the majority of their lifetime.

8. Ω

The length of an individual straight is 108 meters. There are two straights. The arg length 100 meters. Thus the circumference is 2×108 m $+ 100$ m $= 316$ m. The ratio of the length of the straight that points at the detector over the circumference is 34%. This means that $\sim 34\%$ of muons that decay in the decay ring will be boosted towards the far detector.

9. Conlusion

$$N_\mu = (\text{POT}) \times (\pi \text{ per POT}) \times \epsilon_{\text{collection}} \times \epsilon_{\text{injection}} \times (\mu \text{ per } \pi) \times A_{\text{dynamic}} \times \Omega \tag{2}$$
$$= 10^{21} \times 0.1 \times 0.9 \times 0.9 \times 0.08 \times 0.9 \times 0.34 \tag{3}$$
$$= 2 \times 10^{18} \text{ good muon decays} \tag{4}$$

References

[1] Milorad Popovic. Personal communication, 2012.
[2] Sergei Striganov. Personal communication, 2012.
[3] Jaroslaw Pasternak. https://indico.fnal.gov/getFile.py/access?contribId=1&resId=1&materialId=slides&confId=5309, 2012.
[4] Akira Sato. https://indico.fnal.gov/getFile.py/access?contribId=2&resId=0&materialId=slides&confId=5309, 2012.
[5] David Neuffer. https://indico.fnal.gov/getFile.py/access?contribId=1&resId=0&materialId=slides&confId=5058, 2012.