z-expansion parametrization of nucleon vector form factors and GENIE

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Kaushik Borah

with R. J. Hill, G. Lee, O. Tomalak

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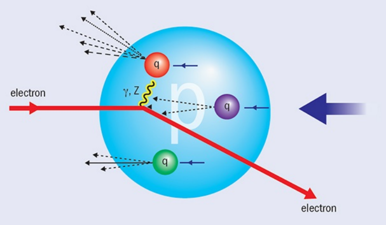
Goal of the talk

- We present the proton and neutron vector form factors, uncertainties and correlations in a convenient parametric form that is model independent and optimized for $Q^2 \lesssim$ few GeV^2.
- The form factors are determined from a global fit to electron scattering data and precise charge radii measurements.
- Including high-precision data of A1@MAMI, charged current quasielastic cross sections change by 3-5 %
- Motivation for new measurement of the proton magnetic form factor

Outline

Nucleon structure and its contribution to neutrino and other precision experiments



- Form factors provides the measure of charge and magnetic-moment distributions inside the nucleons.
- Nucleon form factors are important input for neutrino experiments (DUNE and Hyper-K) as well as atomic spectroscopy.
- Discrepancy in the measurement of form factors and charge radii impacts the measurements in neutrino experiments and atomic spectroscopy.
- We present the proton and neutron vector form factors, uncertainties and correlations in a convenient parametric form that is model independent and optimized for $Q^2 \lesssim$ few GeV^2.

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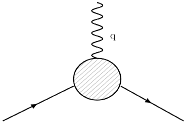
Nucleon Form Factors in Scattering - I

- The nucleon electromagnetic current is expressed in terms of *Dirac* (F_1) and *Pauli* (F_2) form factors,



$$\Gamma^\mu(q^2) = F_1(q^2)\gamma^\mu + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2(q^2)$$

- F_1 and F_2 can be written in terms of *Sachs* electric and magnetic form factors G_E and G_M,

$$F_1 = \frac{G_E + \tau G_M}{1+\tau}, \quad F_2 = \frac{G_E - G_M}{1+\tau}, \quad \tau = -\frac{q^2}{4M^2}$$

- The scattering cross section of a relativistic electron off a recoiling point-like nucleus is given by the *Mott* formula.

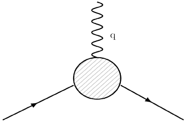
- Structure-dependent part is expressed in terms of Sachs electric and magnetic form factors. Cross section is given by the *Rosenbluth* formula

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\text{Mott}} \frac{1}{1+\tau}\left\{G_E^2 + \frac{\tau}{\epsilon}G_M^2\right\}, \quad \frac{1}{\epsilon} = 1 + 2(2+\tau)\tan^2\frac{\theta}{2}$$

- **NOTE** We can also fit our curve to the isospin rotated basis components isoscalar ($F^S_{1,2} = F^p_{1,2} + F^n_{1,2}$) and isovector ($F^V_{1,2} = F^p_{1,2} - F^n_{1,2}$) form factors. This is important for the application of nucleon form factors in neutrino experiment.

Nucleon Form Factors in Scattering - II

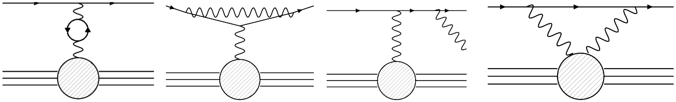
- The form factors are defined from the matrix element of one-photon exchange.



$$\Gamma^\mu(q^2) = F_1(q^2)\gamma^\mu + \frac{i}{2M}\sigma^{\mu\nu}q_\nu F_2(q^2)$$

- To extract them with a percent precision or better, standard QED radiative corrections and modern calculations of structure-dependent two-photon exchange are included.

$$d\sigma_{\text{expt}} = d\sigma_{\text{Born}}(1 + \delta_{\text{RC}})$$



Functional Forms for Form Factors

$$\frac{d\sigma}{d\Omega} = \left(\frac{d\sigma}{d\Omega}\right)_{\mathrm{Mott}} \frac{1}{1+\tau}\left\{G_E^2 + \frac{\tau}{\epsilon} G_M^2\right\}$$

- In the past, a few simple functional forms for G_E and G_M were used by truncating the expansion at some finite k_{max},

$$G_{pol}(q^2) = \sum_{k=0}^{k_{\max}} a_k (q^2)^k, \quad \text{polynomials [Simonetal.(1980), Rosenfelder(2000)]}$$

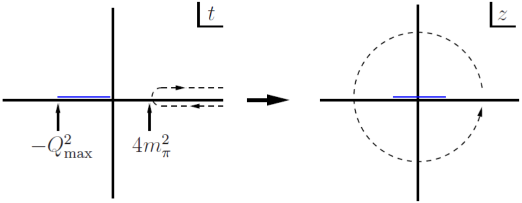
$$G_{invpol}(q^2) = \frac{1}{\sum_{k=0}^{k_{\max}} a_k (q^2)^k}, \quad \text{inverse polynomials [Arrington(2003)]}$$

$$G_{cf}(q^2) = \frac{1}{a_0 + a_1 \frac{q^2}{1 + a_2 \frac{q^2}{1+\cdots}}}, \quad \text{continued fractions [Sick(2003)]}$$

- In 2010, Hill & Paz showed that the above functional forms exhibit pathological behavior with increasing k_{max}

The Bounded z Expansion

- According to QCD constraint, nucleon form factors must be analytic in $t \equiv q^2 \equiv -Q^2$ outside of a time-like cut starting at $t_{\rm cut} = 4m_\pi^2$, the two-pion production threshold ($t_{\rm cut} = 9m_\pi^2$ for isoscalar combinations). [Hill & Paz (2010)]



- A conformal map gives a small expansion variable t_0 in kinematic region of scattering experiments that lies on the negative real axis. It is represented by the blue line for a set of data with maximum momentum transfer $Q^2_{\rm max}$.

$$z(q^2) = \frac{\sqrt{t_{\rm cut} - q^2} - \sqrt{t_{\rm cut} - t_0}}{\sqrt{t_{\rm cut} - q^2} + \sqrt{t_{\rm cut} - t_0}}$$

$$G_E = \sum_{k=0}^{k_{\rm max}} a_k [z(q^2)]^k, \quad G_M = \sum_{k=0}^{k_{\rm max}} b_k [z(q^2)]^k$$

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The Sum Rule

- Perturbative QCD requires that the form factors must fall off faster than $1/Q^3$ in the large Q^2 limit
$$Q^n G(-Q^2)\Big|_{Q^2\longrightarrow\infty} \longrightarrow 0$$
Therefore,
$$\frac{d^n G}{dz^n}\Big|_{z\longrightarrow 1} \longrightarrow 0, \quad n = 0, 1, 2, 3.$$
- In order to implement the above constraints on a form factor we can enforce the following four sum rules [Lee, Arrington, Hill (2015)]
$$\sum_{k=n}^{k_{\max}} k(k-1)\cdots(k-n+1)a_k = 0, \quad n = 0, 1, 2, 3.$$
- We choose $k_{\max} = 8$ and estimate fitting uncertainty as a difference to $k_{\max} \rightarrow k_{\max} + 1$. Four parameters (e.g., $a_1,\ a_2,\ a_3,\ a_4$) are determined by fitting to data.

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νN CCQE Cross Section

- Neutrino-nucleon charged-current quasielastic cross section is expressed in terms of form factors as [Llewellyn-Smith (1972)]

$$\frac{d\sigma}{dQ^2}(Q^2, E_\nu) = \frac{G_F^2 |V_{ud}|^2}{8\pi} \frac{M^2}{E_\nu^2} \left[A(q^2) \frac{m_l^2 - q^2}{M^2} - B(q^2) \frac{s-u}{M^2} + C(q^2) \left(\frac{s-u}{M^2} \right)^2 \right]$$

- The functions A, B and C depend on the nucleon isovector form factors $F_{1,2}^V = F_{1,2}^p - F_{1,2}^n$, axial form factor F_A and pseudoscalar form factor F_P

$$A(q^2) = 2\tau(F_1^V + F_2^V)^2 - (1+\tau)\left\{ (F_1^V)^2 + \tau(F_2^V)^2 - (F_A)^2 \right\} - r_l^2 \left\{ (F_1^V + F_2^V)^2 + (F_A + 2F_P)^2 - 4(1+\tau)F_P^2 \right\}$$

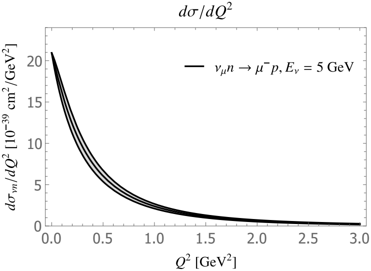
$$B(q^2) = 4\tau F_A (F_1^V + F_2^V)$$

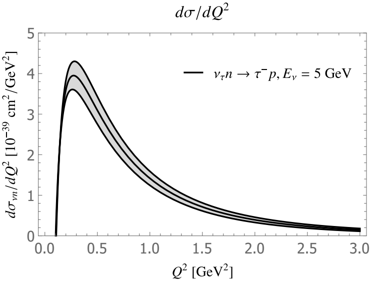
$$C(q^2) = \frac{1}{4}\left\{ (F_1^V)^2 + \tau(F_2^V)^2 + (F_A)^2 \right\}$$

- Hence the nucleon electric and magnetic form factors are important input for the neutrino cross section.

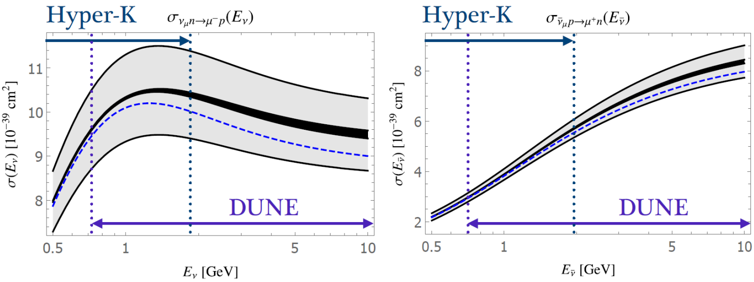
Relevant kinematics for DUNE and HYPER-K

- Electron and muon neutrino cross sections are sensitive to $Q^2 \lesssim 1\ \mathrm{GeV}^2$ while tau neutrino requires larger Q^2.
- Two isospin-decomposed fits with data below $Q^2 < 1\ \mathrm{GeV}^2$ and $Q^2 < 3\ \mathrm{GeV}^2$ are performed.



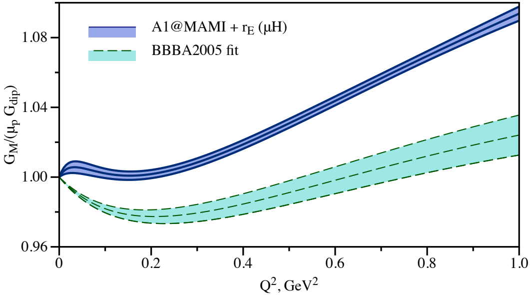


νN CCQE Cross Section Results



- **Dark band** : uncertainty of vector form factor.
- **Light band** : uncertainty of axial form factor.
- **Blue line** : BBBA2005 fit (currently-used fit) of electromagnetic form factors.
- CCQE cross section differs by 3–5% compared to currently-used form factor models (BBBA2005) when the vector form factors are constrained by recent high-statistics electron–proton scattering data from A1@MAMI.

Proton Magnetic Form Factors including A1@MAMI data



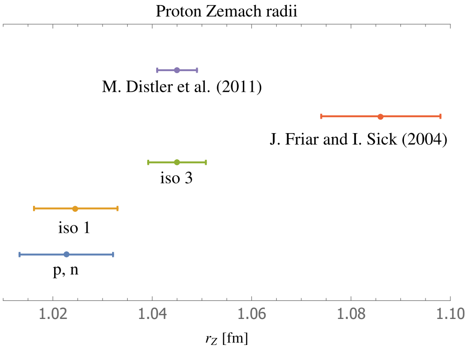
- G_M^p from A1@MAMI is significantly different to previous result.

Hyperfine Splitting and Zemach Radius

- Two-photon exchange provides the leading finite-size correction to hyperfine splitting.
- The dominant piece of two-photon exchange is given by $\Delta E_Z = -2\alpha m_r E_F r_Z$
- The Zemach radius r_Z is calculated as

$$r_Z = -\frac{4}{\pi}\int_0^\infty \frac{dQ}{Q^2}\left[\frac{G_M(Q^2)G_E(Q^2) - G_M(0)G_E(0)}{G_M(0)}\right]$$

- Proton Zemach radii are compared below,



- The Zemach radius is sensitive to charge and magnetic radii, and both electric and magnetic form factors.

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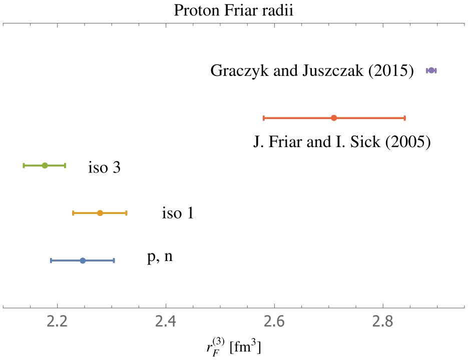
1S-2S transition in hydrogen

- Two-photon exchange provides a sizeable theoretical uncertainty to S energy levels.
- The bulk of correction is given by the Friar radius $r_F^{(3)}$ which is calculated as,

$$r_F^{(3)} = \frac{48}{\pi} \int_0^\infty \frac{dQ}{Q^2} \left[\frac{G_E^2(Q^2) - G_E^2(0) - 2Q^2 G_E'(0)}{Q^2} \right]$$

- Proton Friar radii are compared below,



- The Friar radius is sensitive to charge form factor and radius.

Conclusion and GENIE

- Including data of A1@MAMI Collaboration, CCQE cross sections shift by 3-5 % triggered by proton magnetic form factor.
- Form factor fit from relevant kinematical region is presented in a convenient form for applications in neutrino event generators, GENIE.
- Results of axial form factor from Phys. Rev. D 93, 113015 (2016) [A. S. Meyer, et al.] are in GENIE.
- Future goal is to implement similar systematic analysis of our updated vector form factors in GENIE.