Flavor Symmetries for Neutrino Mixing

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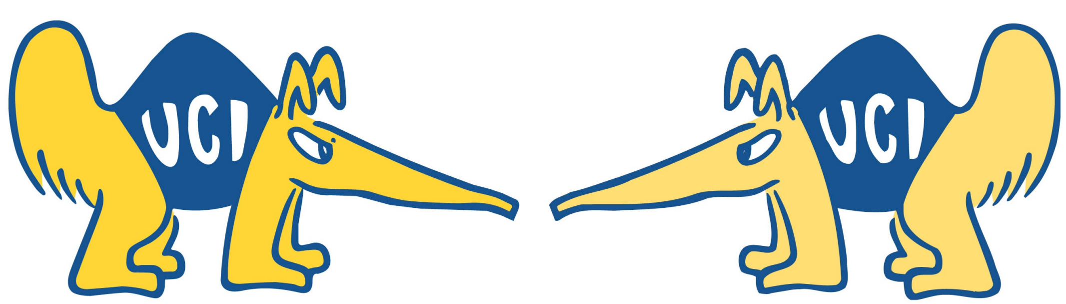
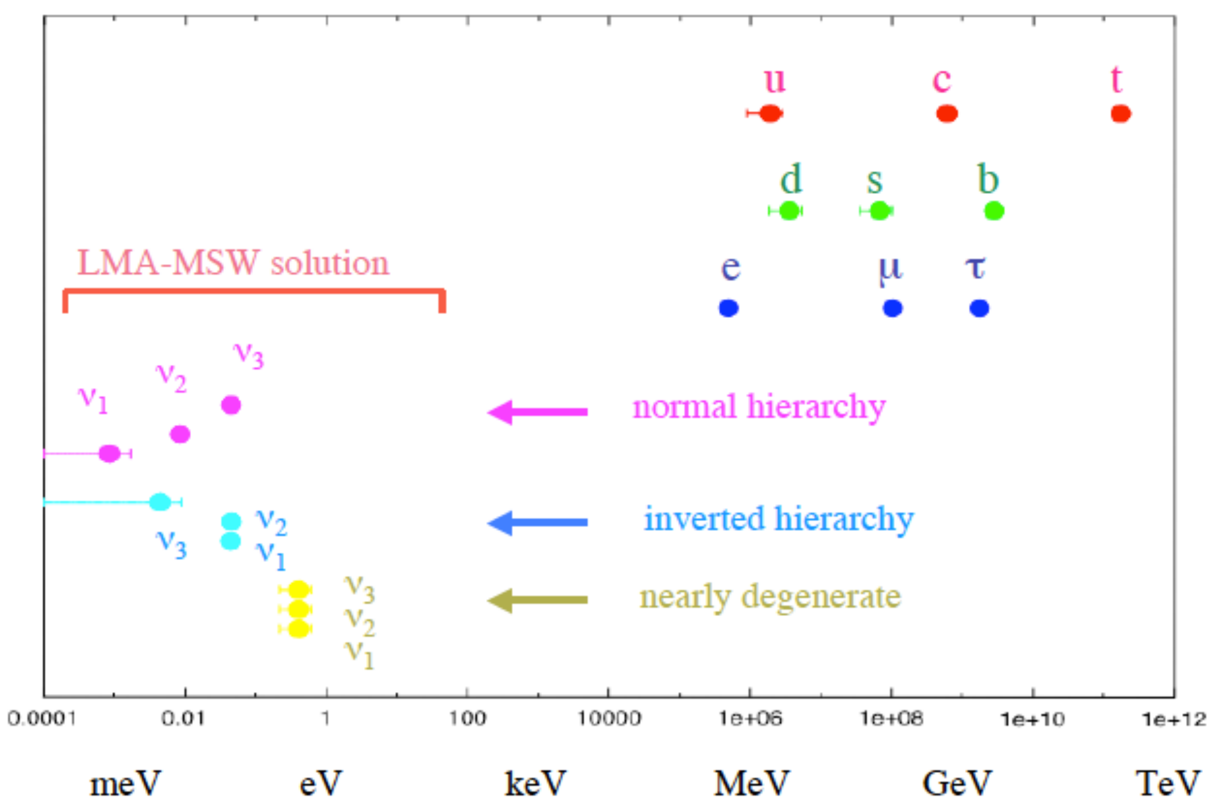


Neutrino Theory Network, Fermilab, June 22, 2022

Open Questions - Theoretical

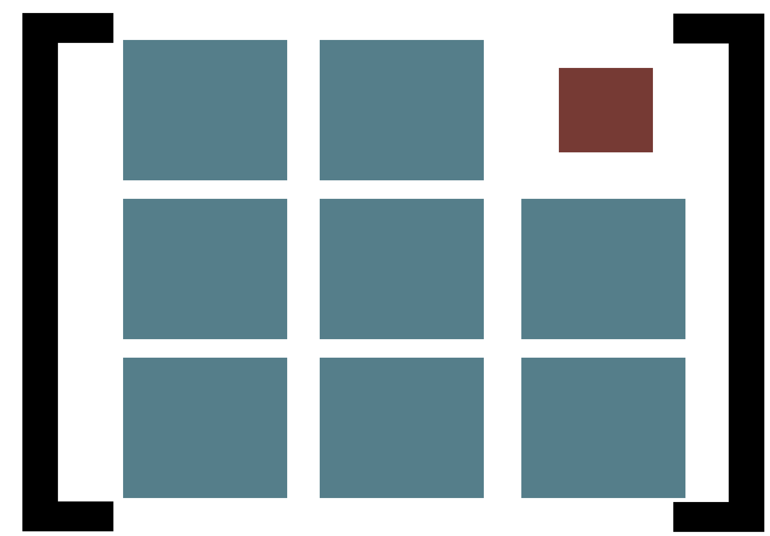
☞ **Smallness of neutrino mass:**

$$m_\nu \ll m_{e,\,u,\,d}$$

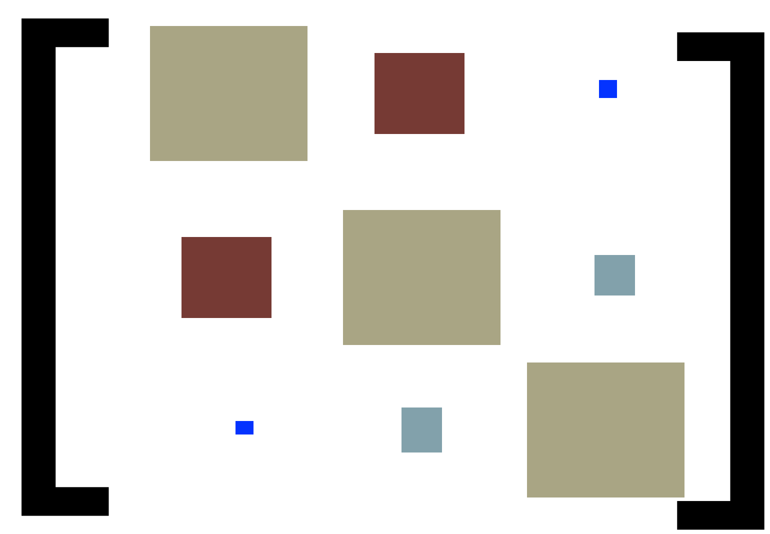


Fermion mass and hierarchy problem ➠ Dominant fraction (22 out of 28) of free parameters in SM

☞ **Flavor structure:**



leptonic mixing

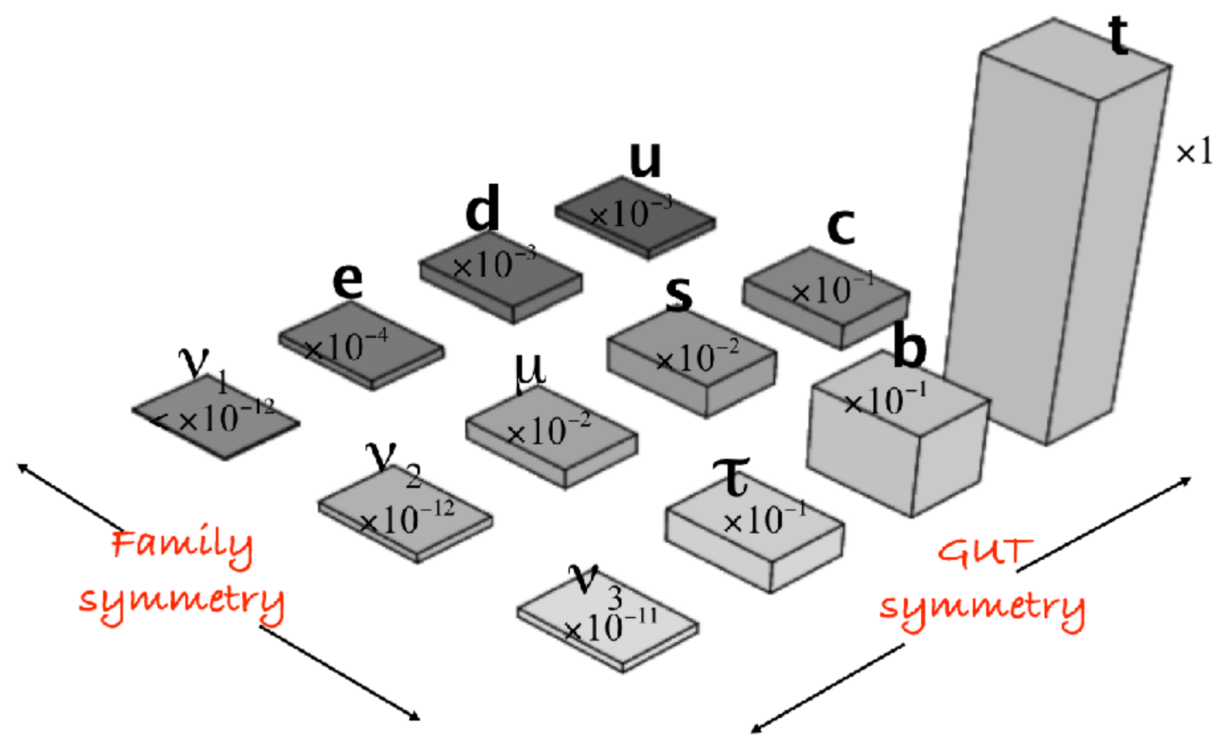


quark mixing

Flavor Structure from Symmetries

Grand Unified Theories: GUT symmetry

Quarks ↔ Leptons



[Figure Credit: King, Luhn, arXiv:1301.1340]

Family Symmetry:

e-family ↔ muon-family ↔ tau-family

Symmetry Relations

Symmetry $\Rightarrow$ relations among parameters

$\Rightarrow$ reduction in number of fundamental parameters

Symmetry Relations

Symmetry $\Rightarrow$ relations among parameters

$\Rightarrow$ reduction in number of fundamental parameters

Symmetry $\Rightarrow$ experimentally testable

correlations among physical observables

Testing Symmetry Relations ⇒ Precision

Symmetry ⇒ experimentally testable correlations among physical observables

mass hierarchy

CP phase

$0\nu\beta\beta$

cLFV

mixing angles

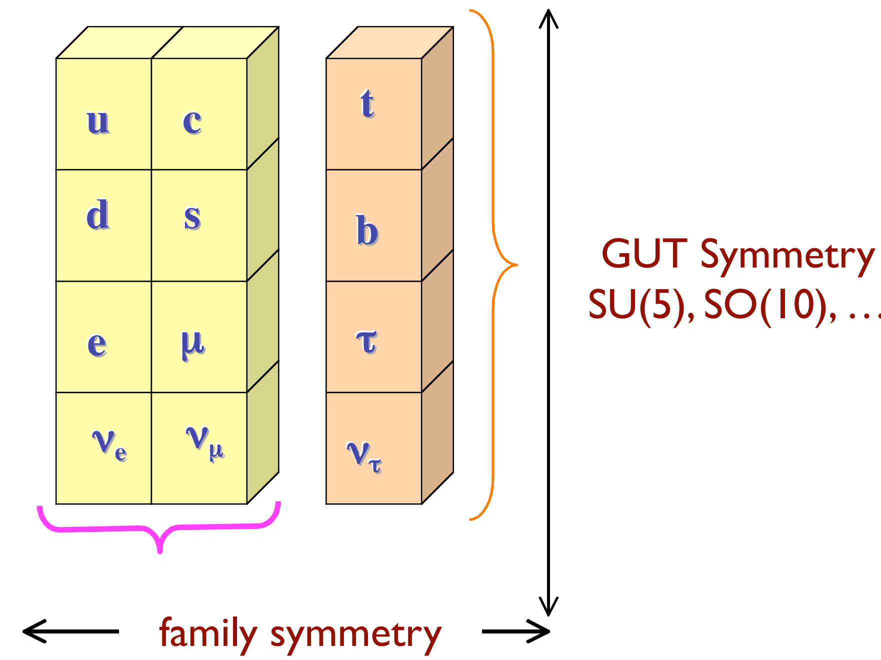
Testing correlations ⇒ Precision

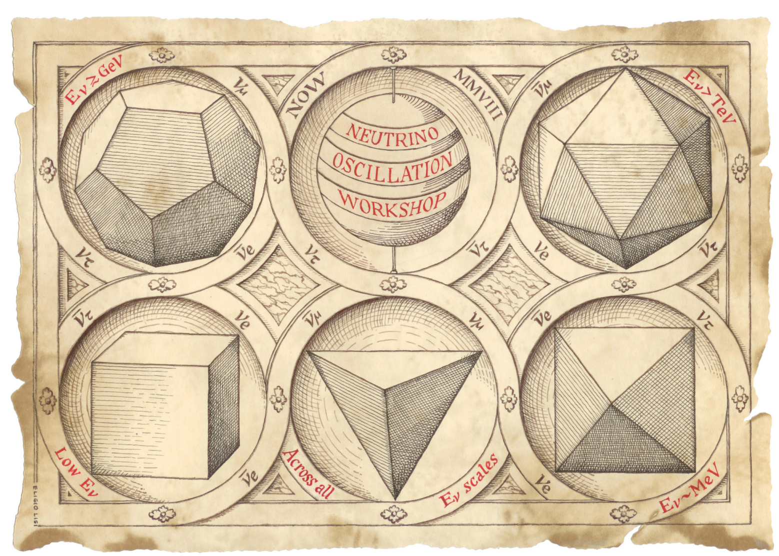
Why Should We Care?

- Understanding a wealth of data, fundamentally
- SM flavor sector: no understanding of significant fraction of SM parameters; (c.f. SM gauge sector)
- Neutrinos as window into BSM physics
  - neutrino mass generation unknown (suppression mechanism, scale)
  - Uniqueness of neutrino masses ➜ connections w/ NP frameworks
- **Neutrinos affords opportunities for new explorations**
  - New Tools
  - May address other puzzles in particle physics
    - Window into early Universe
  - UV connection

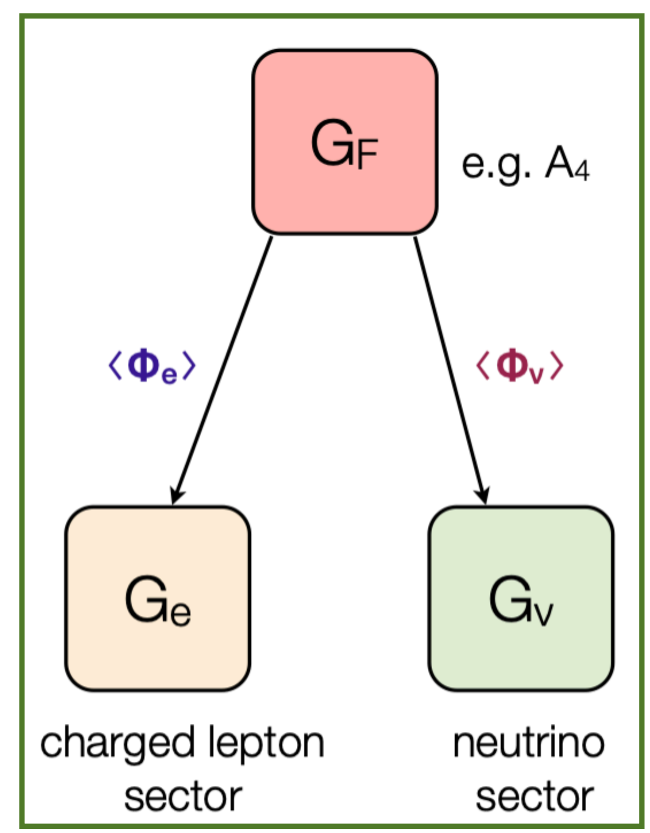
Non-Abelian Discrete Flavor Symmetries

- Large neutrino mixing motivates discrete flavor symmetries
  - A_4 (tetrahedron)
  - T' (double tetrahedron)
  - S_3 (equilateral triangle)
  - S_4 (octahedron, cube)
  - A_5 (icosahedron, dodecahedron)
  - Δ_{27}
  - Q_6
  - .....





[Eligio Lisi for NOW2008]



Tri-bimaximal Neutrino Mixing

- **Latest Global Fit (3σ)**

Esteban, Gonzalez-Garcia, Maltoni, Schwetz, Zhou (2020)

$$\sin^2\theta_{23} = 0.437\ (0.374 - 0.626)$$ $[\theta^{lep}_{23} \sim 49.2°]$

$$\sin^2\theta_{12} = 0.308\ (0.259 - 0.359)$$ $[\theta^{lep}_{12} \sim 33.4°]$

$$\sin^2\theta_{13} = 0.0234\ (0.0176 - 0.0295)$$ $[\theta^{lep}_{13} \sim 8.57°]$

- **Tri-bimaximal Mixing Pattern**

Harrison, Perkins, Scott (1999)

$$U_{TBM} = \begin{pmatrix} \sqrt{2/3} & \sqrt{1/3} & 0 \\ -\sqrt{1/6} & \sqrt{1/3} & -\sqrt{1/2} \\ -\sqrt{1/6} & \sqrt{1/3} & \sqrt{1/2} \end{pmatrix}$$

$$\sin^2\theta_{\mathrm{atm,\,TBM}} = 1/2 \qquad \sin^2\theta_{\odot,\mathrm{TBM}} = 1/3$$

$$\sin\theta_{13,\mathrm{TBM}} = 0.$$

Neutrino Mass Matrix from A4

Ma, Rajasekaran (2001); Babu, Ma, Valle (2003); Altarelli, Feruglio (2005)

- Imposing A4 flavor symmetry on the Lagrangian
- A4 spontaneously broken by flavon fields

2 free parameters

$$M_\nu = \frac{\lambda v^2}{M_x} \begin{pmatrix} 2\xi_0 + u & -\xi_0 & -\xi_0 \\ -\xi_0 & 2\xi_0 & u - \xi_0 \\ -\xi_0 & u - \xi_0 & 2\xi_0 \end{pmatrix}$$

relative strengths $\Rightarrow$ CG's

- always diagonalized by TBM matrix, independent of the two free parameters

$$U_{\mathrm{TBM}} = \begin{pmatrix} \sqrt{2/3} & 1/\sqrt{3} & 0 \\ -\sqrt{1/6} & 1/\sqrt{3} & -1/\sqrt{2} \\ -\sqrt{1/6} & 1/\sqrt{3} & 1/\sqrt{2} \end{pmatrix}$$

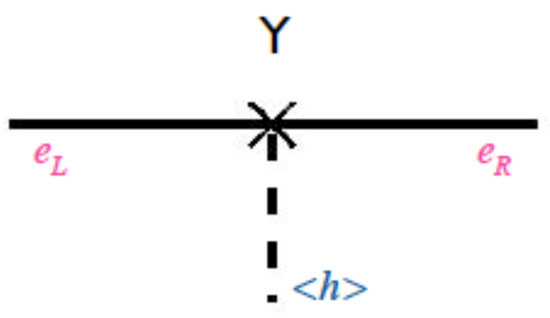
Neutrino Mixing Angles from Group Theory

Novel Origin of CP Violation

- CP violation ⇔ complex mass matrices

$$\overline{U}_{R,i}(M_u)_{ij}Q_{L,j} + \overline{Q}_{L,j}(M_u^\dagger)_{ji}U_{R,i} \xrightarrow{\mathcal{CP}} \overline{Q}_{L,j}(M_u)_{ij}U_{R,i} + \overline{U}_{R,i}(M_u)^*_{ij}Q_{L,j}$$

- Conventionally, CPV arises in two ways:
  - Explicit CP violation: complex Yukawa coupling constants Y
  - Spontaneous CP violation: complex scalar VEVs <h>



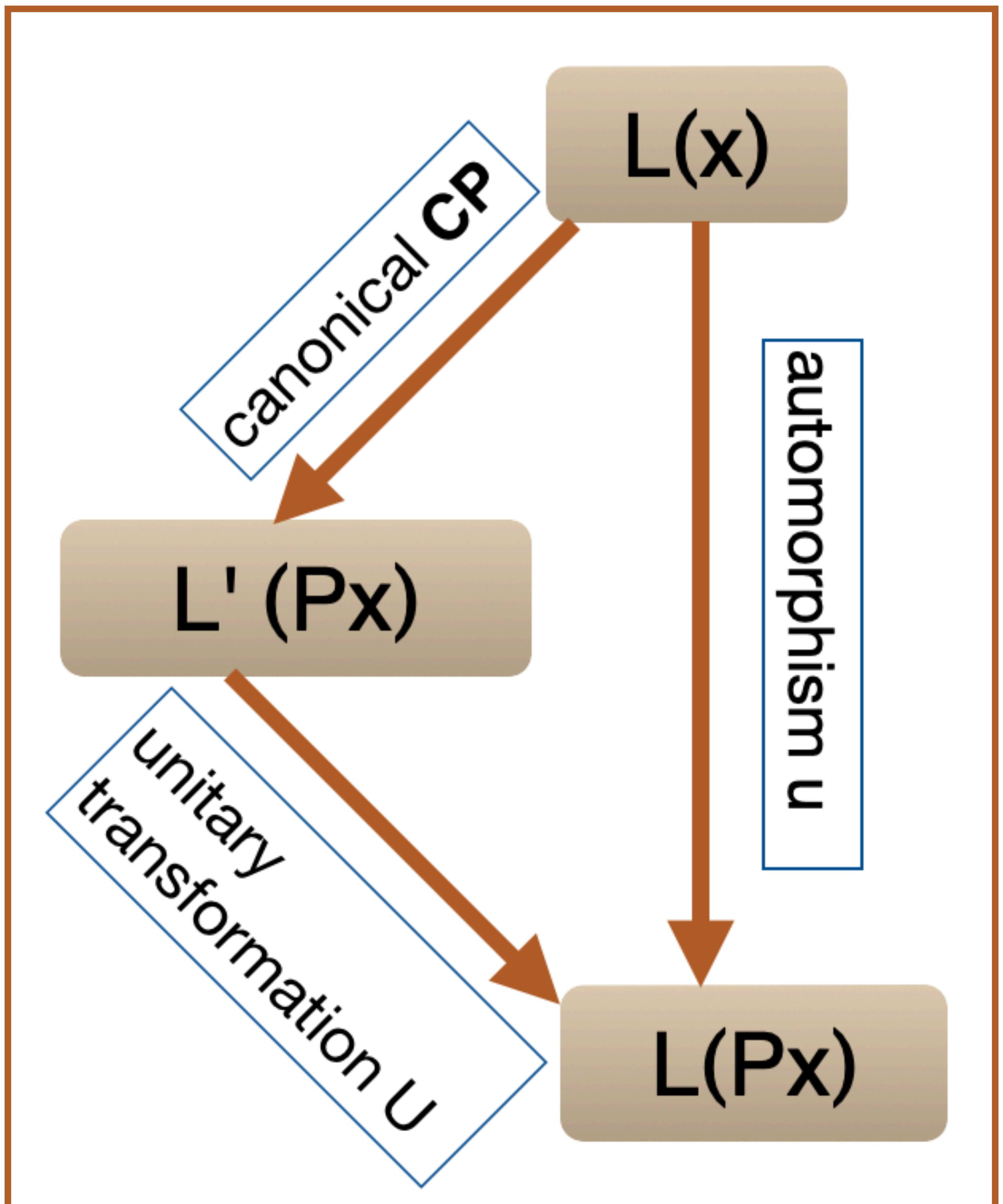
- **Complex CG coefficients in certain discrete groups ⇒ explicit CP violation**
  - CPV in quark and lepton sectors purely from complex CG coefficients

M.-C.C., K.T. Mahanthappa, Phys. Lett. B681, 444 (2009)

CG coefficients in non-Abelian discrete symmetries
➪ relative strengths and phases in entries of Yukawa matrices
➪ mixing angles and phases (and mass hierarchy)

Group Theoretical Origin of CP Violation

M-CC, Mahanthappa (2009); M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)



complex CGs ⇨ G and physical CP transformations do not always commute

Class-inverting outer automorphism

⇕

Physical CP

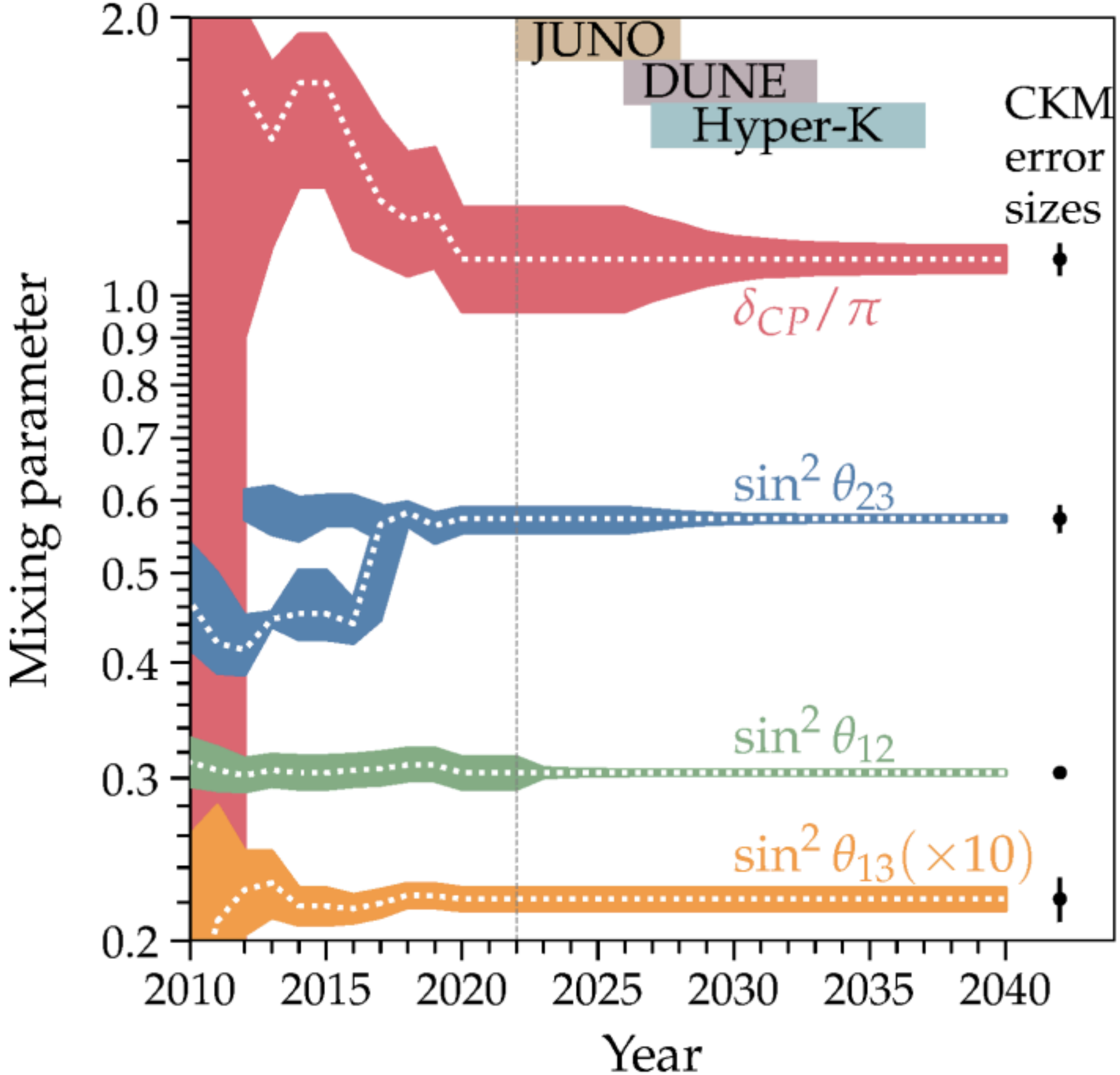
Novel Origin of CP Violation

M-CC, Mahanthappa (2009); M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

complex CGs ⇨ CP symmetry cannot be defined for certain groups

CP Violation from Group Theory!

Experimental Precision



Are precision in model predictions compatible with experimental precision?

Figure from Song, Li, Argüelles, Bustamante, Vincent (2020)

Corrections to Kinetic Terms

- Corrections to the kinetic terms induced by family symmetry breaking generically are present, should be properly included   Leurer, Nir, Seiberg (1993); Dudas, Pokorski, Savoy (1995); Dreiner, Thomeier (2003)
  - can be along different directions than RG corrections
  - dominate over RG corrections (no loop suppression, copious heavy states)
  - only subdominant for quark flavor models
  - could be sizable for neutrino mass models based on discrete family symmetries, e.g. A_4 M.-C.C, M. Fallbacher, M. Ratz, C. Staudt (2012)
    - nontrivial flavor structure can be induced
    - non-zero CP phase can be induced
    - Presence of additional undetermined parameters

Kähler Corrections

M.-C.C., Fallbacher, Ratz, Staudt (2012)

- Superpotential: holomorphic

$$\mathscr{W}_{\text{leading}} = \frac{1}{\Lambda}(\Phi_e)_{gf}\, L^g\, R^f\, H_d + \frac{1}{\Lambda\,\Lambda_\nu}(\Phi_\nu)_{gf}\, L^g\, H_u\, L^f\, H_u$$

$$\longrightarrow \mathscr{W}_{\text{eff}} = (Y_e)_{gf}\, L^g\, R^f\, H_d + \frac{1}{4}\kappa_{gf}\, L^g\, H_u\, L^f\, H_u$$

order parameter
<flavon vev> / Λ ~ θc

- Kähler potential: non-holomorphic

$$K = K_{\text{canonical}} + \Delta K$$

- Canonical Kähler potential

$$K_{\text{canonical}} \supset \left(L^f\right)^\dagger \delta_{fg}\, L^g + \left(R^f\right)^\dagger \delta_{fg}\, R^g$$

- Correction

$$\Delta K = \left(L^f\right)^\dagger (\Delta K_L)_{fg}\, L^g + \left(R^f\right)^\dagger (\Delta K_R)_{fg}\, R^g$$

- can be induced by flavon VEVs
- important for order parameter ~ θc
- can lead to non-trivial mixing

Kähler Corrections

M.-C.C., Fallbacher, Ratz, Staudt (2012)

- Consider infinitesimal change, x :

$$K \;=\; K_{\text{canonical}} + \Delta K \;=\; L^\dagger\,(1 - 2x\,P)\,L$$

- rotate to canonically normalized L′:

$$L \;\to\; L' \;=\; (1 - x\,P)\,L$$

$\Rightarrow$ corrections to neutrino mass matrix

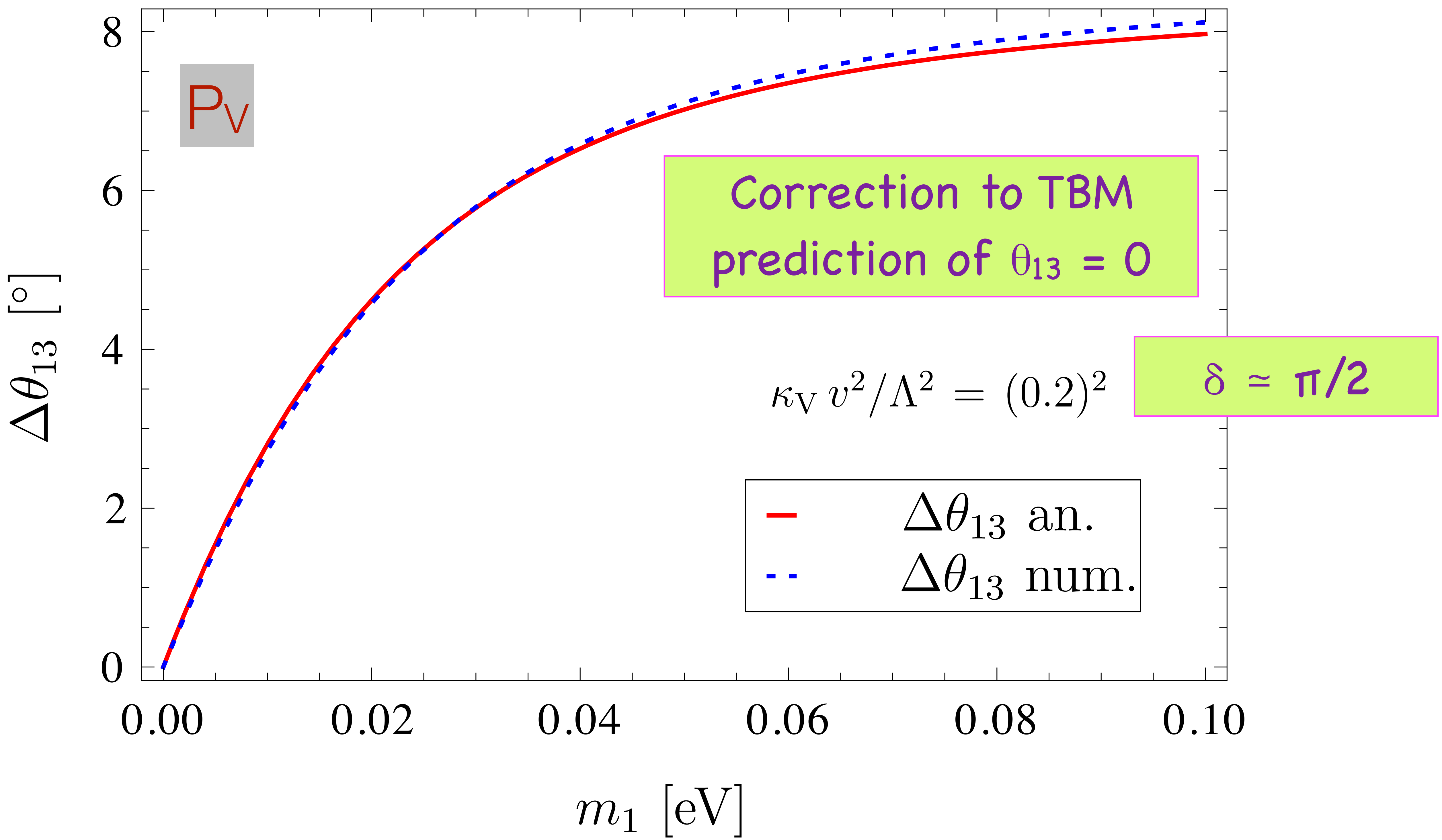
$$\begin{aligned}
\mathcal{W}_\nu &= \frac{1}{2}(L\cdot H_u)^T\kappa_\nu(L\cdot H_u) \\
&\simeq \frac{1}{2}[(\mathbb{1}+xP)L'\cdot H_u]^T\kappa_\nu[(\mathbb{1}+xP)L'\cdot H_u] \\
&\simeq \frac{1}{2}(L'\cdot H_u)^T\kappa_\nu L'\cdot H_u + x(L'\cdot H_u)^T(P^T\kappa_\nu+\kappa_\nu P)L'\cdot H_u
\end{aligned}$$

with

$$\kappa\cdot v_u^2 \;=\; 2m_\nu$$

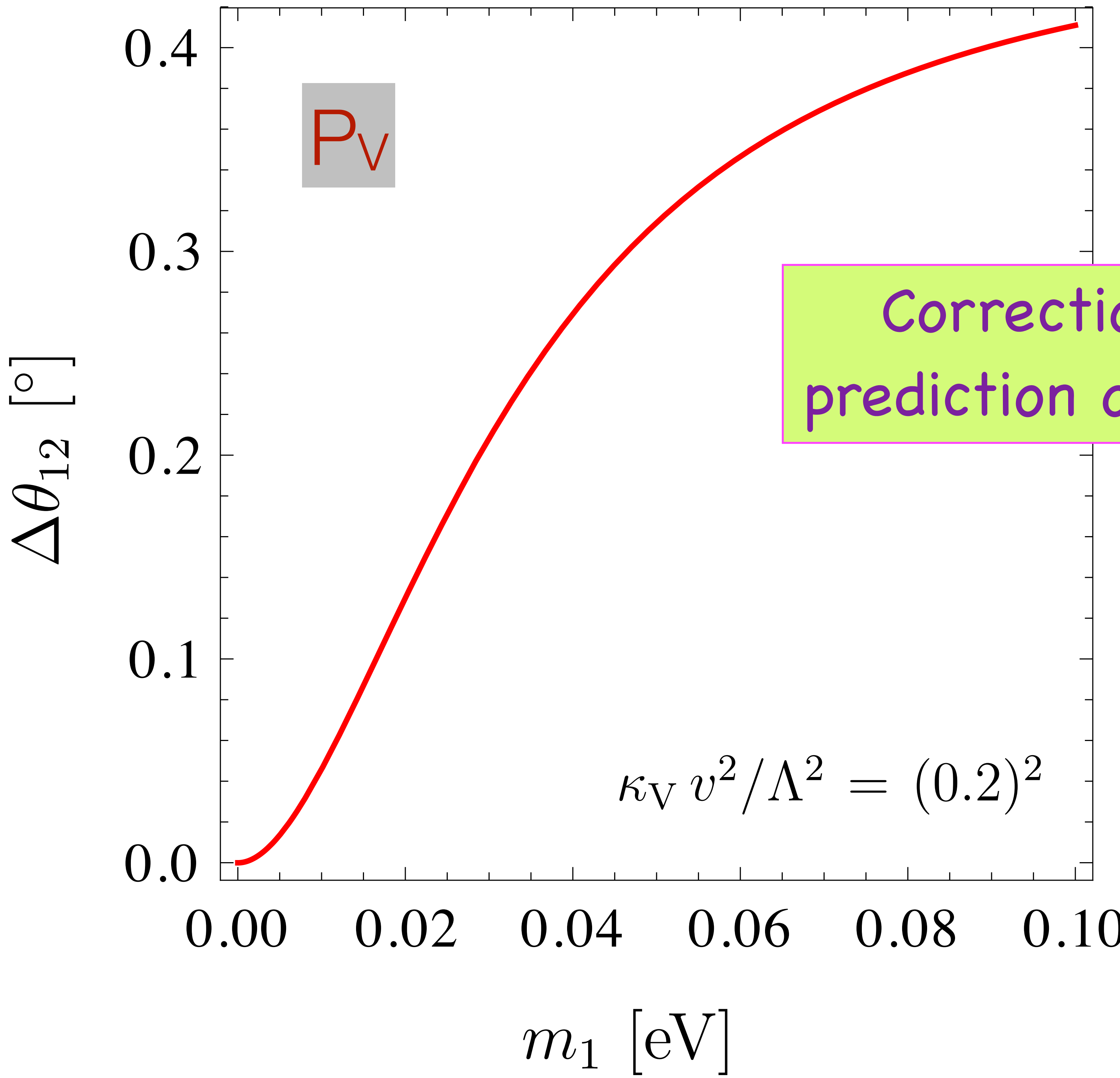
An Example: Enhanced θ_{13} in A_4

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)



Corresponding Change in θ_{12}

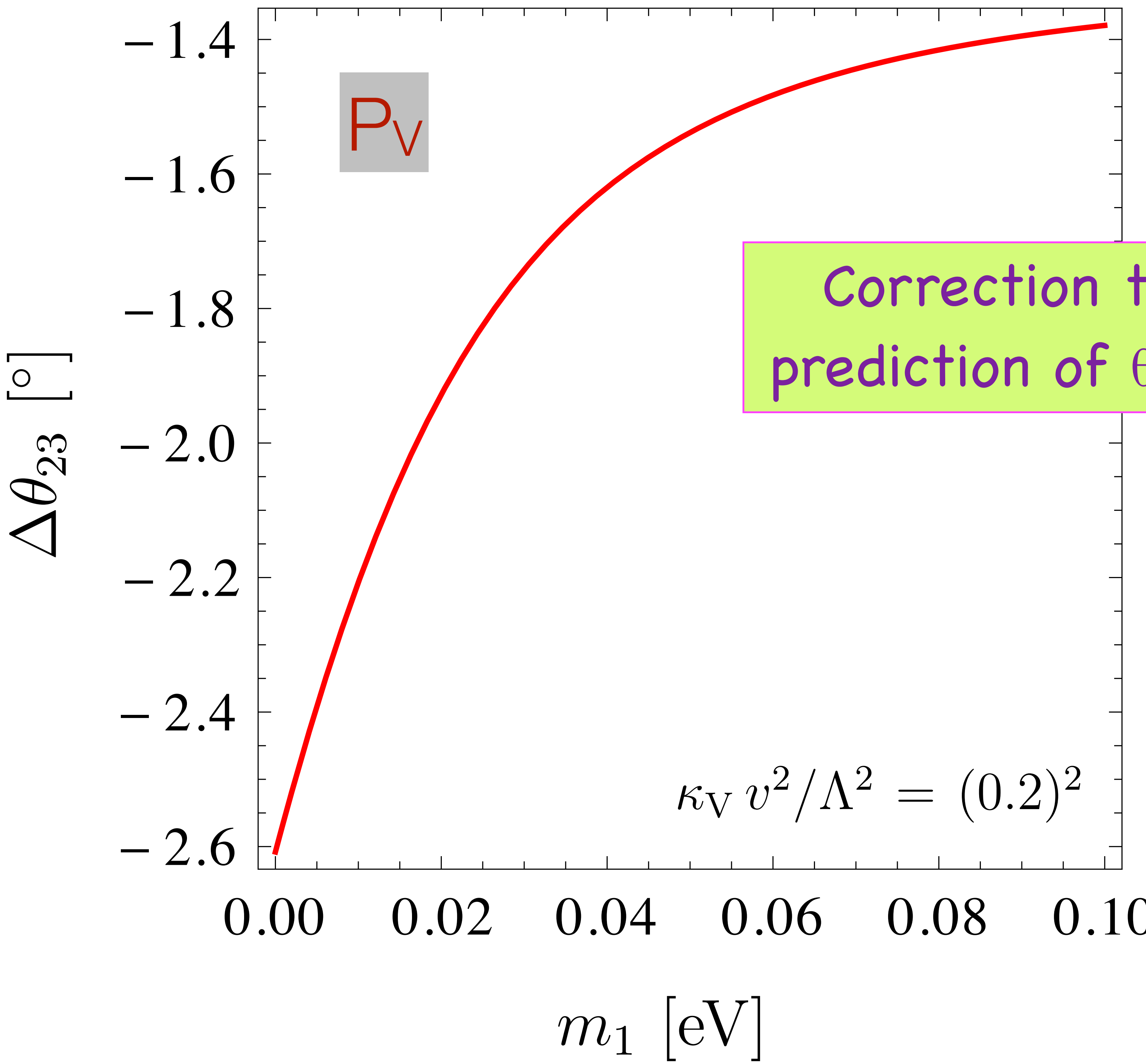
M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)



Correction to TBM prediction of $\theta_{12} = 35.3°$

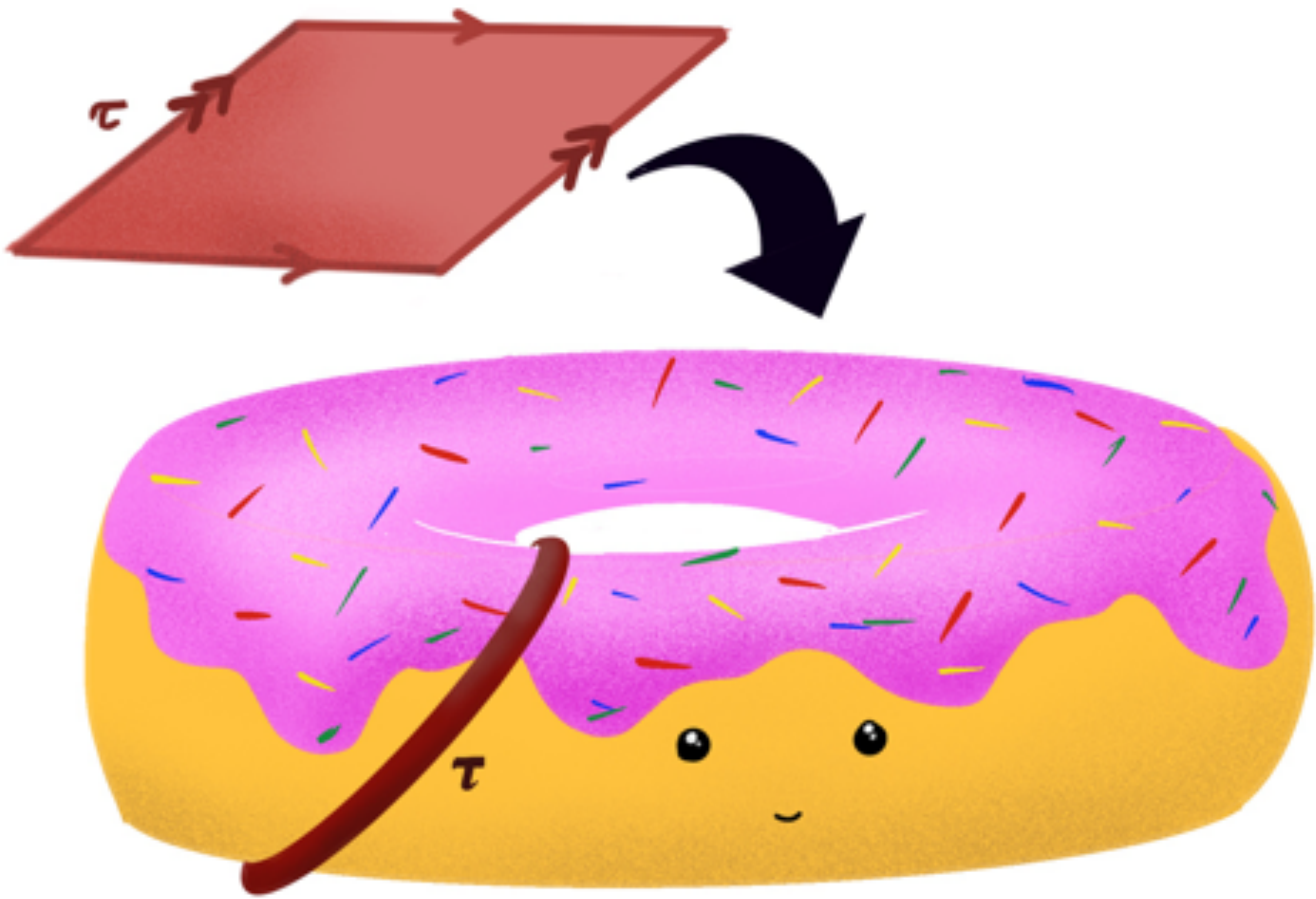
Corresponding Change in θ_{23}

M.-C.C., M. Fallbacher, M. Ratz, C. Staudt (2012)



Correction to TBM prediction of θ_{23} = 45°

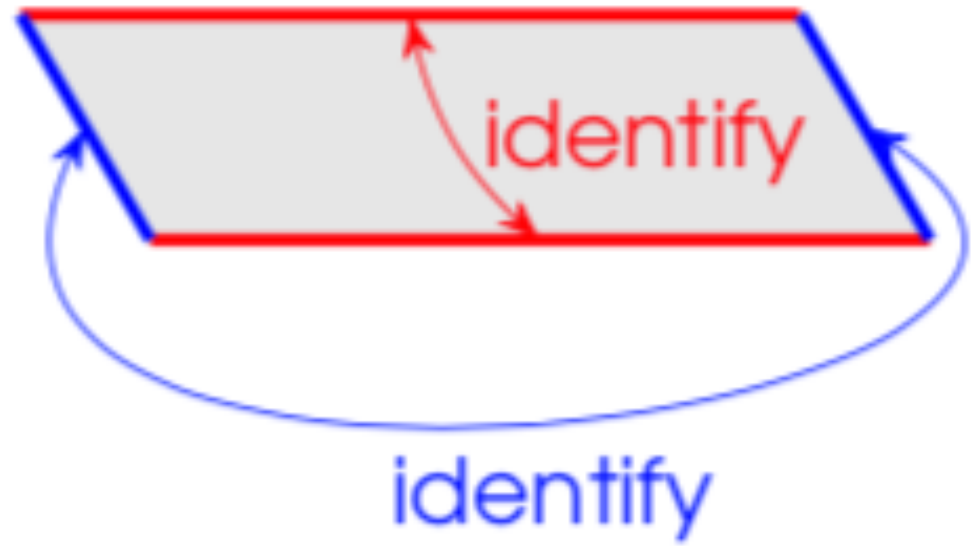
Modular Flavor Symmetries

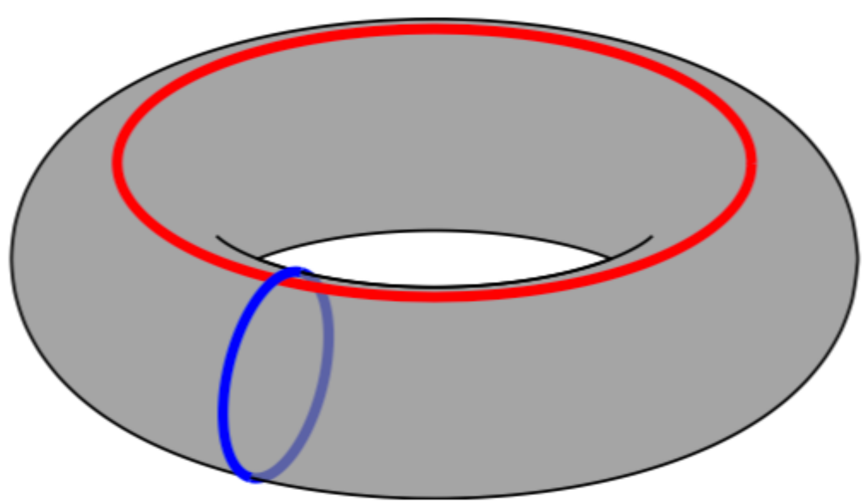


Artwork by Shreya Shukla

Donuts = TORI

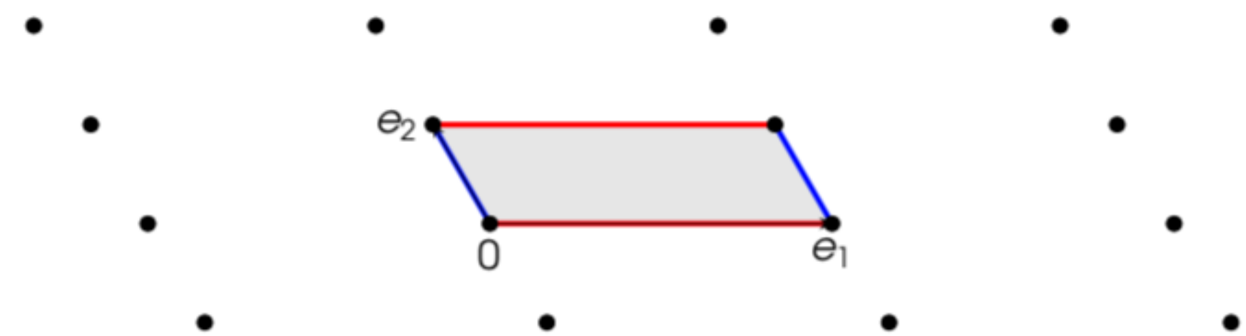
constructed
from
parallelogram





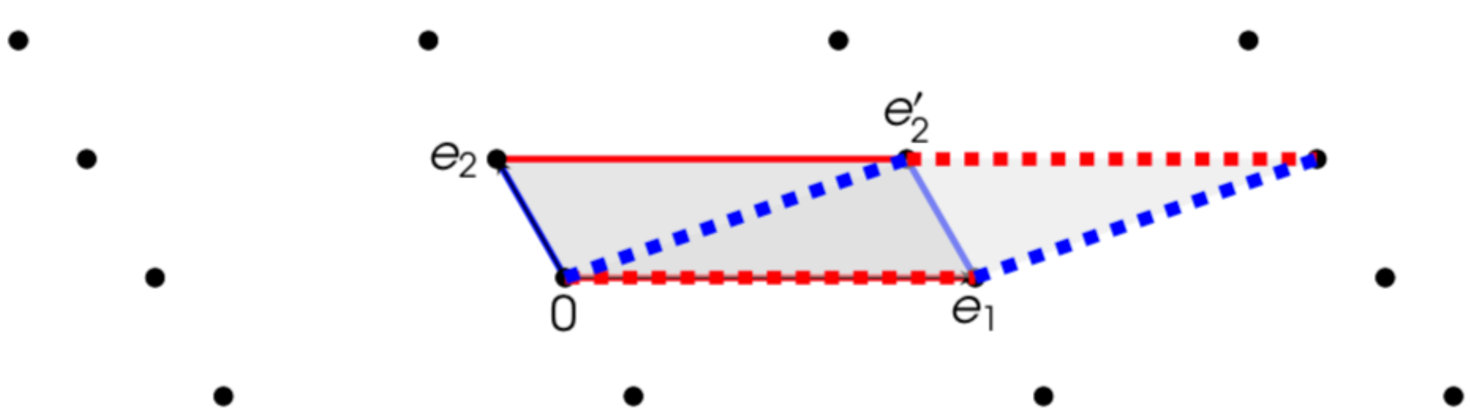
two cycles

Modular Symmetries



edges ⇒ lattice basis vectors

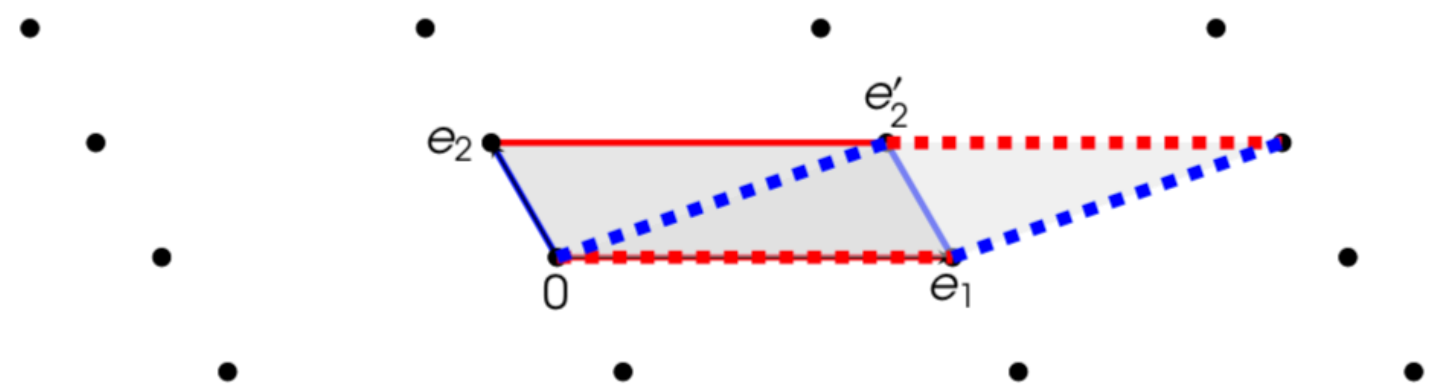
points in plane identified if differ by a lattice translation



Equivalent TORI related by Modular Symmetries

Modular Symmetries

- TORI: fundamental domain not unique



- Basis Vectors are related: $$\begin{pmatrix} e_2 \\ e_1 \end{pmatrix} \overset{\gamma}{\mapsto} \begin{pmatrix} e'_2 \\ e'_1 \end{pmatrix} = \begin{pmatrix} a & b \\ c & d \end{pmatrix} \begin{pmatrix} e_2 \\ e_1 \end{pmatrix} =: \gamma \begin{pmatrix} e_2 \\ e_1 \end{pmatrix}$$

$a, b, c, d \in \mathbb{Z}$

- Volume of fundamental domain the same $\Rightarrow$ $\det \gamma = 1$

Modular Symmetries

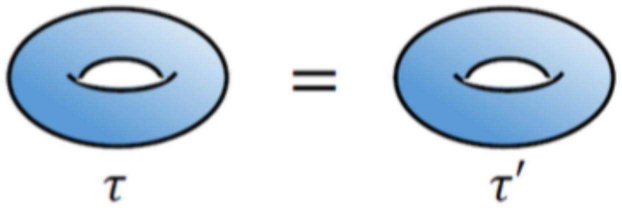
- Two basic transformations:

$$T \;:\; e_2 \mapsto e_2' = e_2 + e_1 \qquad \curvearrowright \gamma = \begin{pmatrix} 1 & 1 \\ 0 & 1 \end{pmatrix} =: T$$

$$S \;:\; e_1 \mapsto e_1' = e_2 \quad \text{and} \quad e_2 \mapsto e_2' = -e_1 \quad \curvearrowright \gamma = \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix} =: S$$

- In complex coordinates: modulus $\tau = e_2/e_1$

$$\tau \overset{S}{\longmapsto} \frac{-1}{\tau} \quad \text{and} \quad \tau \overset{T}{\longmapsto} \tau + 1$$



- S and T generate $\mathrm{SL}(2,\mathbb{Z})$ and satisfy

$$\boxed{S^2 = (S\,T)^3 = \mathbb{1}}$$

Modular Symmetries

- **Finite Modular Group** (quotient group): $\Gamma_N := \Gamma/\Gamma(N)$ where principal congruence group $\Gamma(N)$ is

$$\Gamma(N) = \left\{ \begin{pmatrix} a & b \\ c & d \end{pmatrix} \in \underbrace{\boxed{\mathrm{SL}(2,\mathbb{Z})/\mathbb{Z}_2}}_{\Gamma}\, ;\ \begin{pmatrix} a & b \\ c & d \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} \mod N \right\}$$

- Generators of the quotient group Γ_N satisfy

$$\boxed{S^2 = 1, \quad (ST)^3 = 1, \quad T^N = 1}$$

- Some examples

$$\Gamma_2 \simeq S_3, \qquad \Gamma_3 \simeq A_4, \qquad \Gamma_4 \simeq S_4, \qquad \Gamma_5 \simeq A_5$$

Modular Symmetries

Feruglio (2017)

- Imposing modular symmetry Γ on the Lagrangian:

$$\mathscr{L} \supset \sum Y_{i_1,\, i_2,\, \ldots\ldots\, i_n} \Phi_{i_1} \Phi_{i_2} \cdots \Phi_{i_n}$$

$$\tau \xrightarrow{\gamma} \gamma\tau := \frac{a\tau + b}{c\tau + d}\,,$$

$$\Phi_j \xrightarrow{\gamma} (c\tau + d)^{k_j} \boxed{\rho_{r_j}(\gamma)} \Phi_j\,, \quad \text{where } \gamma := \begin{pmatrix} a & b \\ c & d \end{pmatrix}$$

k_i : integers

representation matrix of Γ_N

- Yukawa Couplings = Modular Forms at level "N" w/ weight "k"

$$f_i(\gamma\tau) = (c\tau + d)^{-k} \boxed{[\rho_N(\gamma)]_{ij}} f_j(\tau)$$

$k = k_{i_1} + k_{i_2} + \ldots + k_{i_n}$

representation matrix of Γ_N

A Toy Modular A_4 Model

Feruglio (2017)

- Weinberg Operator $\mathscr{W}_\nu = \frac{1}{\Lambda}[(H_u \cdot L)\; Y\; (H_u \cdot L)]_1$
- Traditional A4 Flavor Symmetry
  - Yukawa Coupling Y $\rightarrow$ Flavon VEVs (A_4 triplet, 6 real parameters)

$$Y \rightarrow \langle \phi \rangle = \begin{pmatrix} a \\ b \\ c \end{pmatrix} \quad \Longrightarrow \quad m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} 2a & -c & -b \\ -c & 2b & -a \\ -b & -a & 2c \end{pmatrix}$$

- Modular A4 Flavor Symmetry
  - Yukawa Coupling Y $\rightarrow$ Modular Forms (A4 triplet, 2 real parameters)

$$Y \rightarrow \begin{pmatrix} Y_1(\tau) \\ Y_2(\tau) \\ Y_3(\tau) \end{pmatrix} \quad \Longrightarrow \quad m_\nu = \frac{v_u^2}{\Lambda}\begin{pmatrix} 2Y_1(\tau) & -Y_3(\tau) & -Y_2(\tau) \\ -Y_3(\tau) & 2Y_2(\tau) & -Y_1(\tau) \\ -Y_2(\tau) & -Y_1(\tau) & 2Y_3(\tau) \end{pmatrix}$$

Modular Forms

Feruglio (2017)

- Level (N) = 3, Weight (k) = 2, in terms of Dedekind eta-function

$$
\begin{aligned}
Y_1(\tau) &= \frac{i}{2\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}-\frac{27\eta'(3\tau)}{\eta(3\tau)}\right]\\
Y_2(\tau) &= \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\omega^2\,\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\omega\,\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right]\\
Y_3(\tau) &= \frac{-i}{\pi}\left[\frac{\eta'\left(\frac{\tau}{3}\right)}{\eta\left(\frac{\tau}{3}\right)}+\omega\,\frac{\eta'\left(\frac{\tau+1}{3}\right)}{\eta\left(\frac{\tau+1}{3}\right)}+\omega^2\,\frac{\eta'\left(\frac{\tau+2}{3}\right)}{\eta\left(\frac{\tau+2}{3}\right)}\right] \quad .
\end{aligned}
$$

$$
\eta(\tau) = q^{1/24}\prod_{n=1}^{\infty}(1-q^n) \qquad\qquad q \equiv e^{i2\pi\tau}
$$

A Toy Modular A_4 Model

Feruglio (2017)

- Input Parameters:

$$\boxed{\tau = 0.0111 + 0.9946i} \qquad v_u^2/\Lambda$$

- Predictions:

$$\frac{\Delta m^2_{sol}}{|\Delta m^2_{atm}|} = 0.0292$$

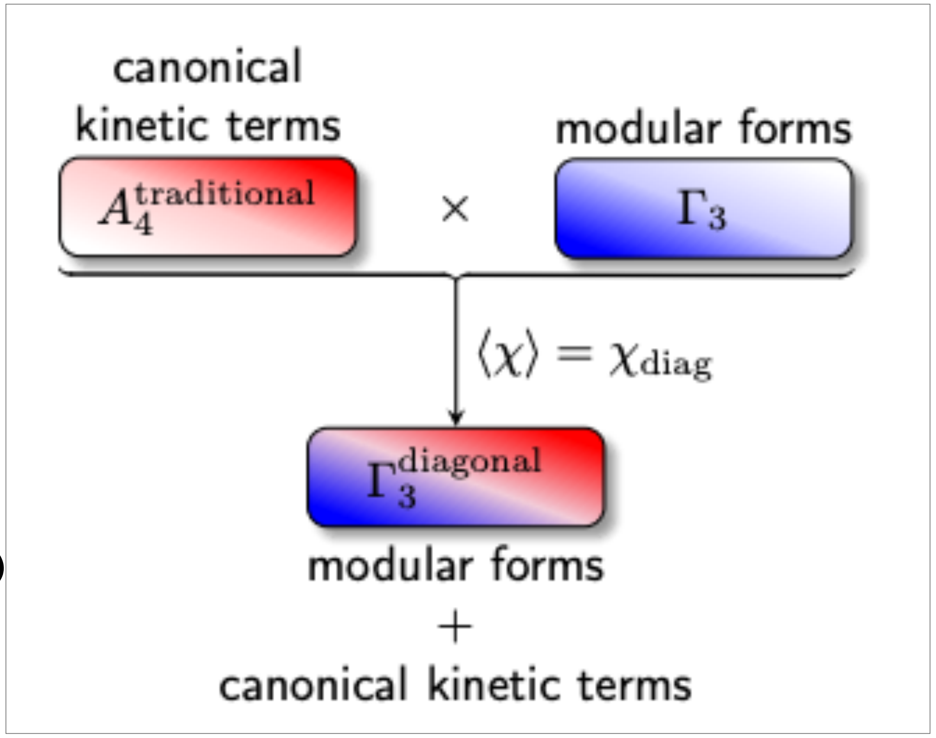
$$\sin^2\theta_{12} = 0.295 \qquad \sin^2\theta_{13} = 0.0447 \qquad \sin^2\theta_{23} = 0.651$$

$$\frac{\delta_{CP}}{\pi} = 1.55 \qquad \frac{\alpha_{21}}{\pi} = 0.22 \qquad \frac{\alpha_{31}}{\pi} = 1.80 \quad .$$

$$m_1 = 4.998 \times 10^{-2}\ eV \qquad m_2 = 5.071 \times 10^{-2}\ eV \qquad m_3 = 7.338 \times 10^{-4}\ eV$$

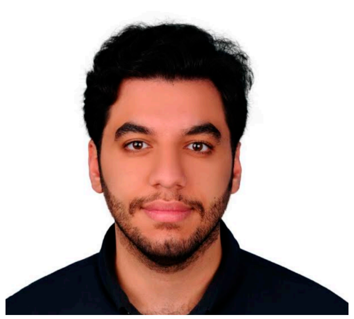
Modular Symmetries: Bottom-Up Meet Top-Down

- Bottom-Up:
  - reducing the number of parameters: in extreme case, entire neutrino mass matrix controlled by τ Feruglio (2017)
  - traditional NA flavor symmetries: corrections to kinetic terms -> sizable for NA discrete symmetries for leptons Leurer, Nir, Seiberg ('93); Dudas, Pokorski, Savoy ('95); M.-C.C, M. Fallbacher, M. Ratz, C. Staudt (2012)
    - (Quasi-eclectic) setup with modular symmetries: corrections to kinetic terms can be under control $\longrightarrow$ **reduction of theory uncertainty** MCC, Knapp-Pérez, Ramos-Hamud, Ramos-Sánchez, Ratz, Shukla (2021)



- Top-Down:
  - Modular flavor symmetries from strings e.g. Baur, Nilles, Trautner, Vaudrevange
  - Modular Symmetries from magnetized tori e.g. Almumin, MCC, Knapp-Pérez, Ramos-Sánchez, Ratz, Shukla (2021)

Acknowledgements



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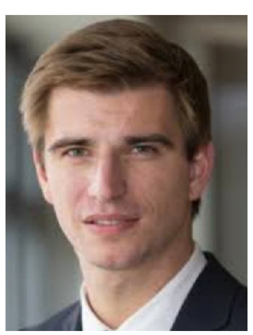

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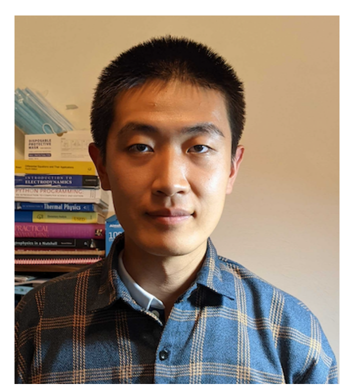

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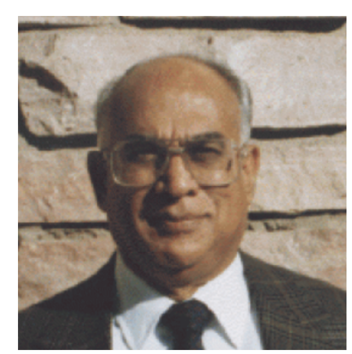

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Saúl Ramos-Sánchez (UNAM, Mexico)

Michael Ratz (UCI)

Outlook

- Fundamental origin of fermion mass & mixing patterns still unknown
  - It took decades to understand the gauge sector of SM
- Uniqueness of Neutrino masses offers exciting opportunities to explore BSM Physics

Almumin, MCC, Cheng, Knapp-Pérez, Li, Mondol, Ramos-Sánchez, Ratz, Shukla (2022)

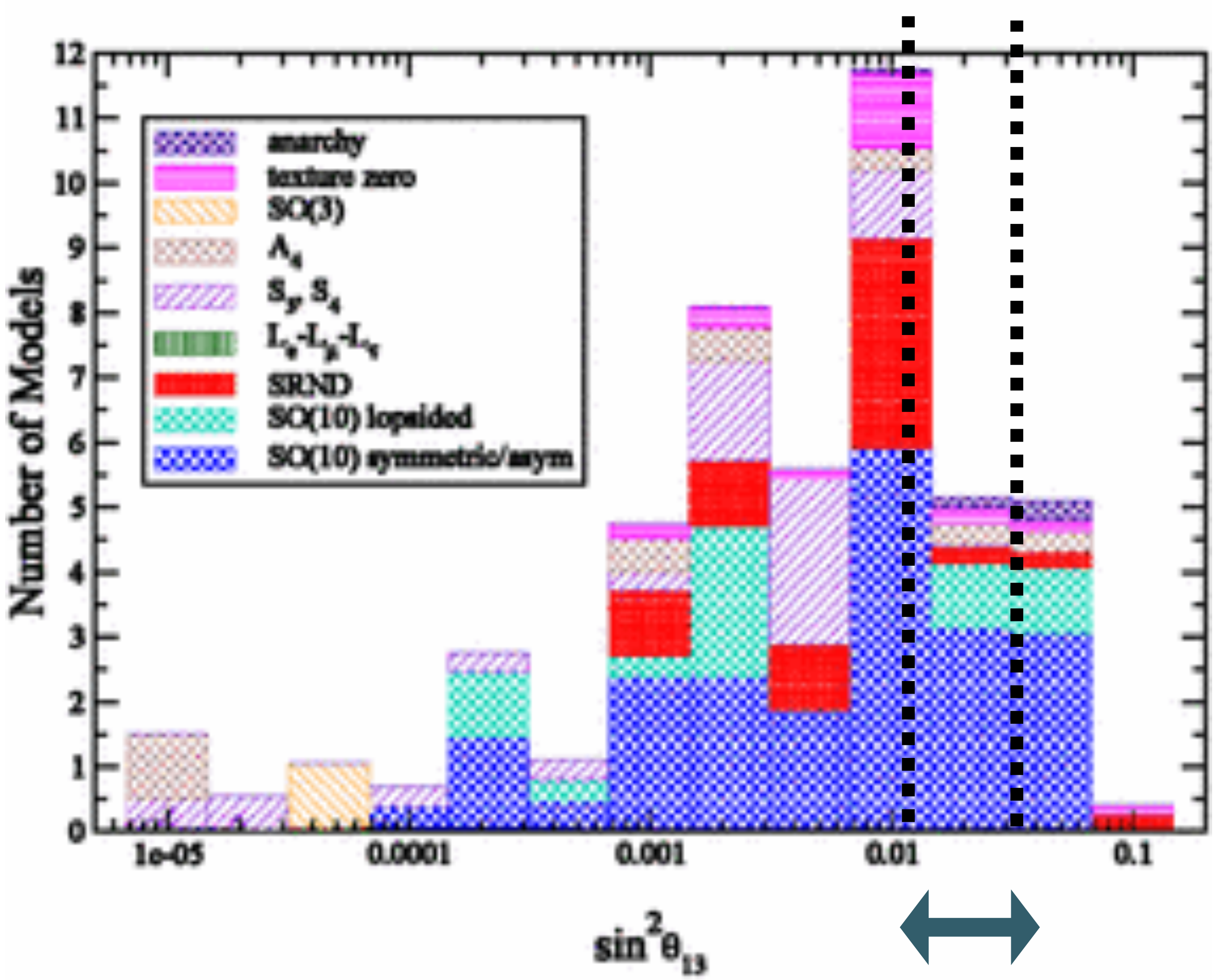
  - Many NP frameworks; addressing other puzzles
  - Early Universe (leptogenesis, non-thermal relic neutrinos)
- New Tools/insights:
  - Non-Abelian Discrete Flavor Symmetries
    - Deep connection between outer automorphisms and CP
  - Modular Flavor Symmetries - promising approach
    - Enhanced predictivity of flavor models (enhanced theory precision)
    - Possible connection to UV physics (e.g. string theories) -> promising venue toward realistic theories
- **TD-BU: diverse perspectives drive intellectual excellence**

Backup Slides

Mixing Parameters and Mass ordering

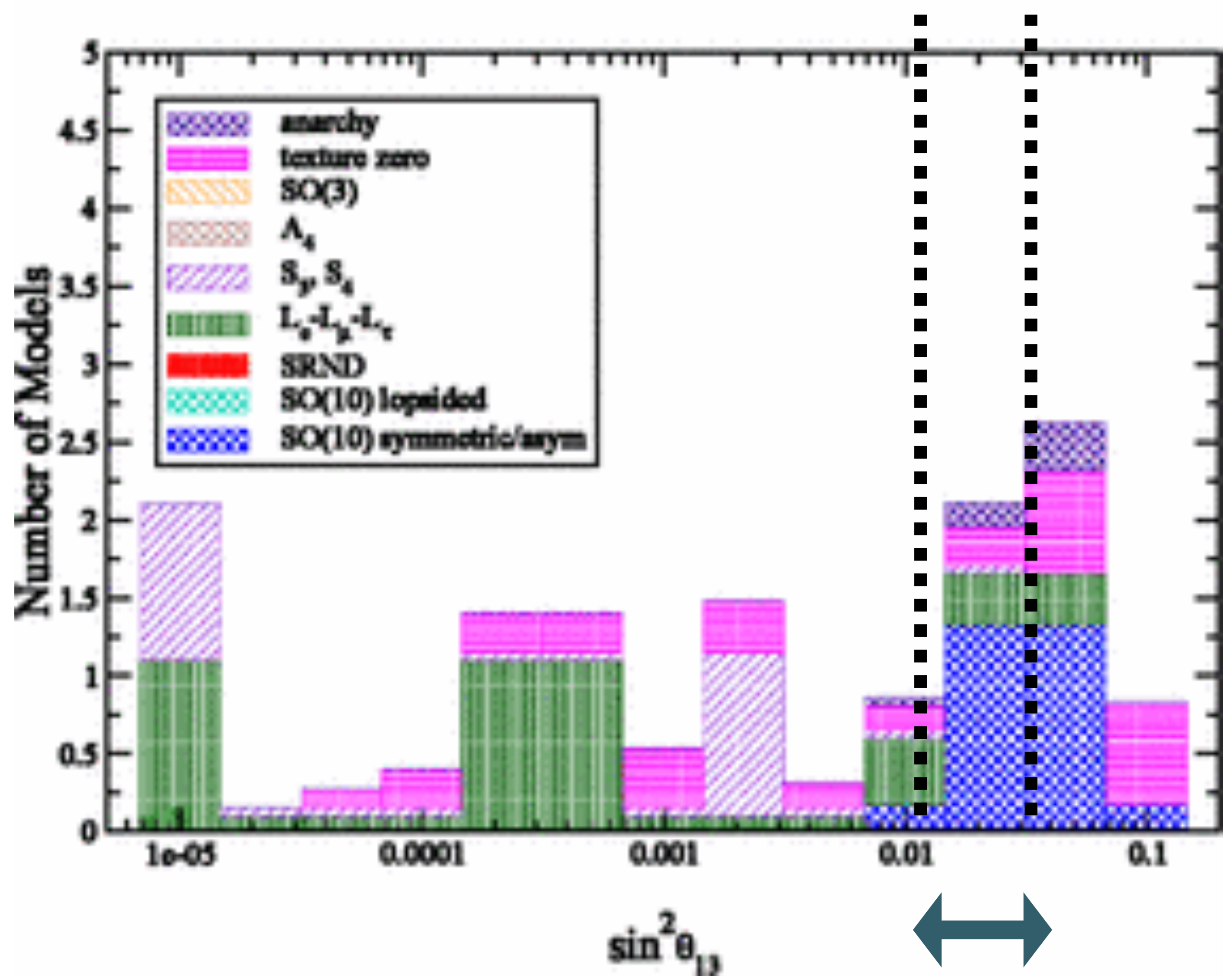
underlying symmetries

Normal Hierarchy



Daya Bay

Inverted Hierarchy



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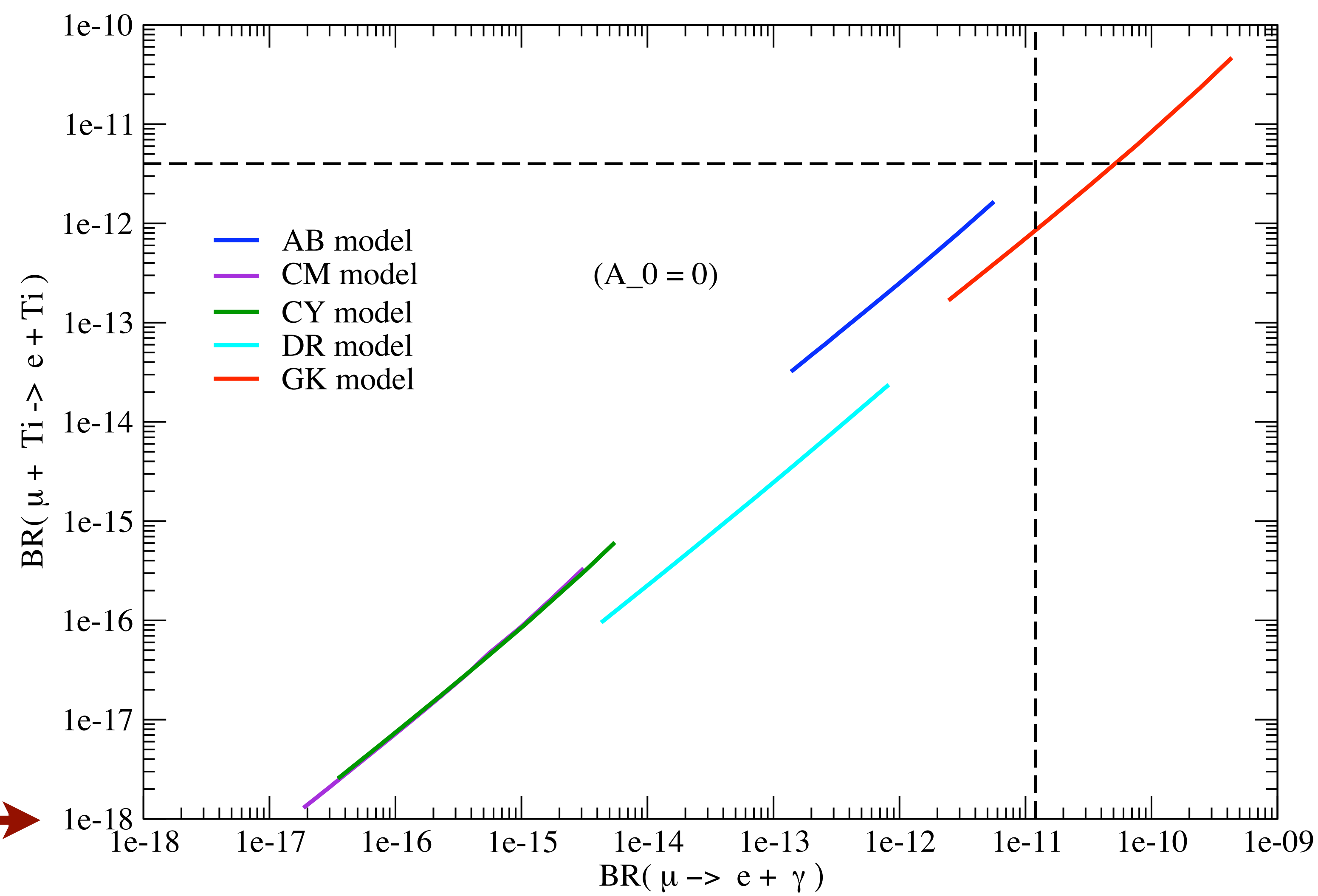
C. Albright, M.-C.C. (2006)

Correlations: cLFV Processes

- Predictions for cLFV processes in five "viable" SUSY (10) models
  - mSUGRA boundary conditions
  - Including dark matter constraints from WMAP

Albright, MCC (2008)



μ-e conversion could be powerful in distinguishing different models

sensitivity of proposed MECO-type exp

reach at MEG

Example: SU(5) Compatibility ⇒ T′ Family Symmetry

M.-C.C, K.T. Mahanthappa (2007, 2009)

- Double Tetrahedral Group T´: double covering of A4
- Symmetries ⇒ 10 parameters in Yukawa sector ⇒ 22 physical observables
- Symmetries ⇒ **correlations among quark and lepton mixing parameters**

$$\boxed{\theta_{13} \simeq \theta_c/3\sqrt{2}}$$

CG's of SU(5) & T´

no free parameters!

$$\boxed{\tan^2\theta_\odot \simeq \tan^2\theta_{\odot,TBM} + \frac{1}{2}\theta_c \cos\delta}$$

neutrino solar mixing

1/2

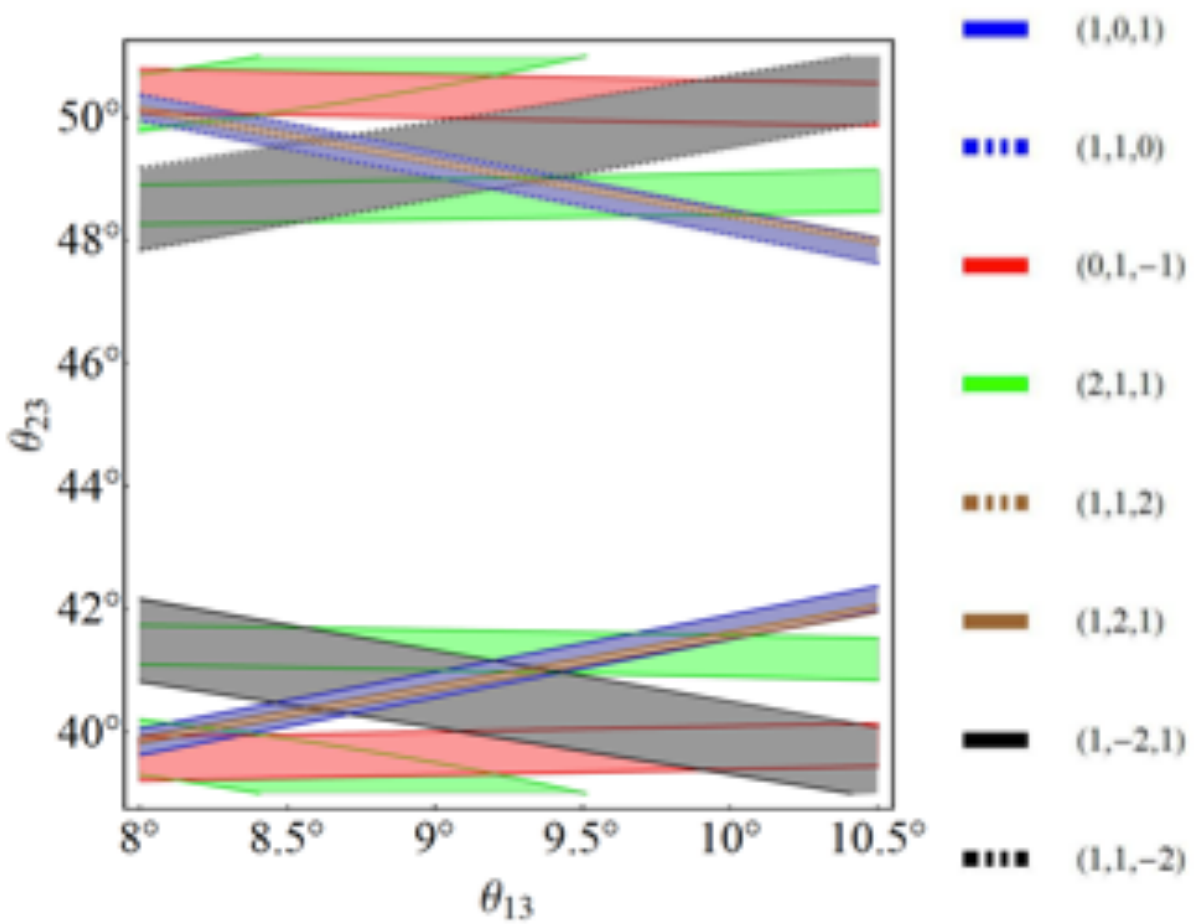
quark Cabibbo mixing

leptonic CP phase

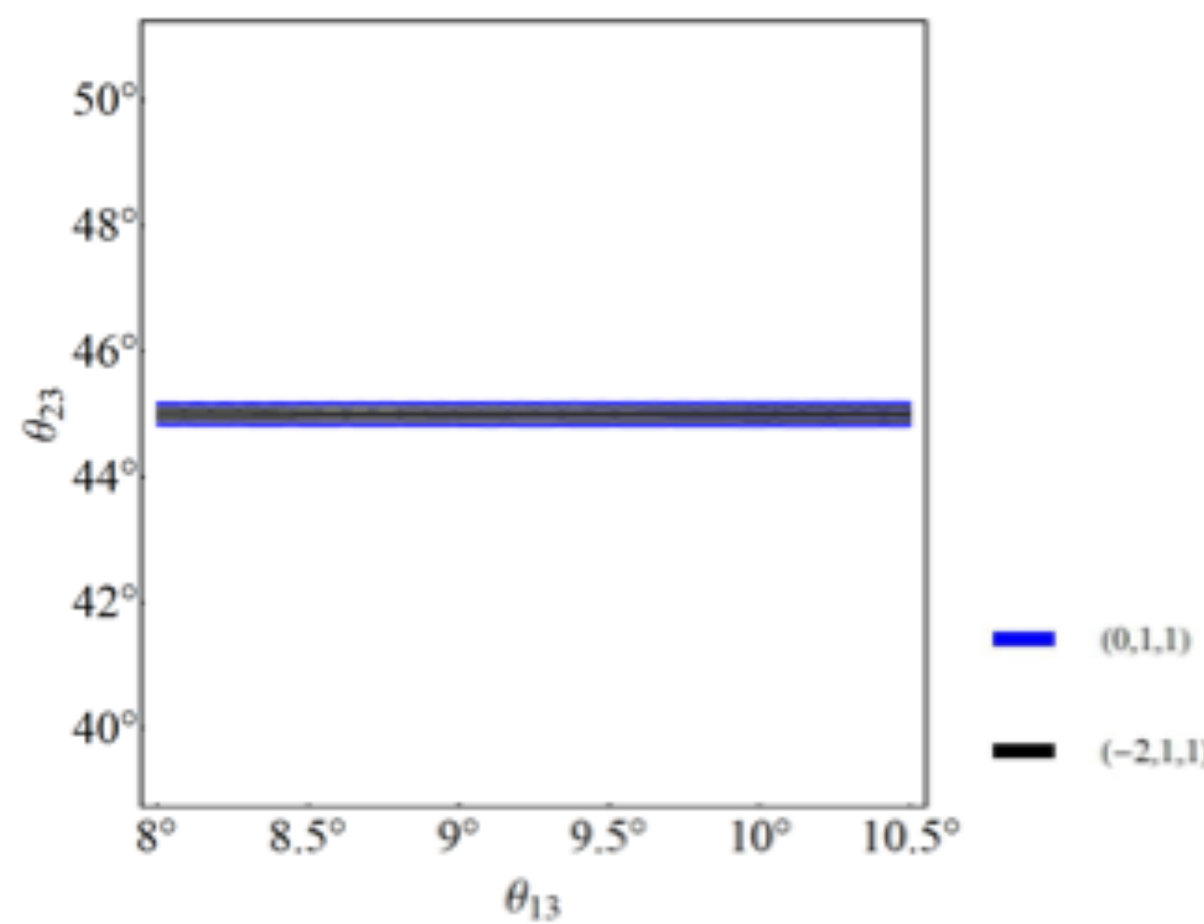
"Large" Deviations from TBM in A_4

M.-C.C, J. Huang, J. O'Bryan, A. Wijangco, F. Yu, (2012)

- Different A4 breaking patterns:



normal



inverted

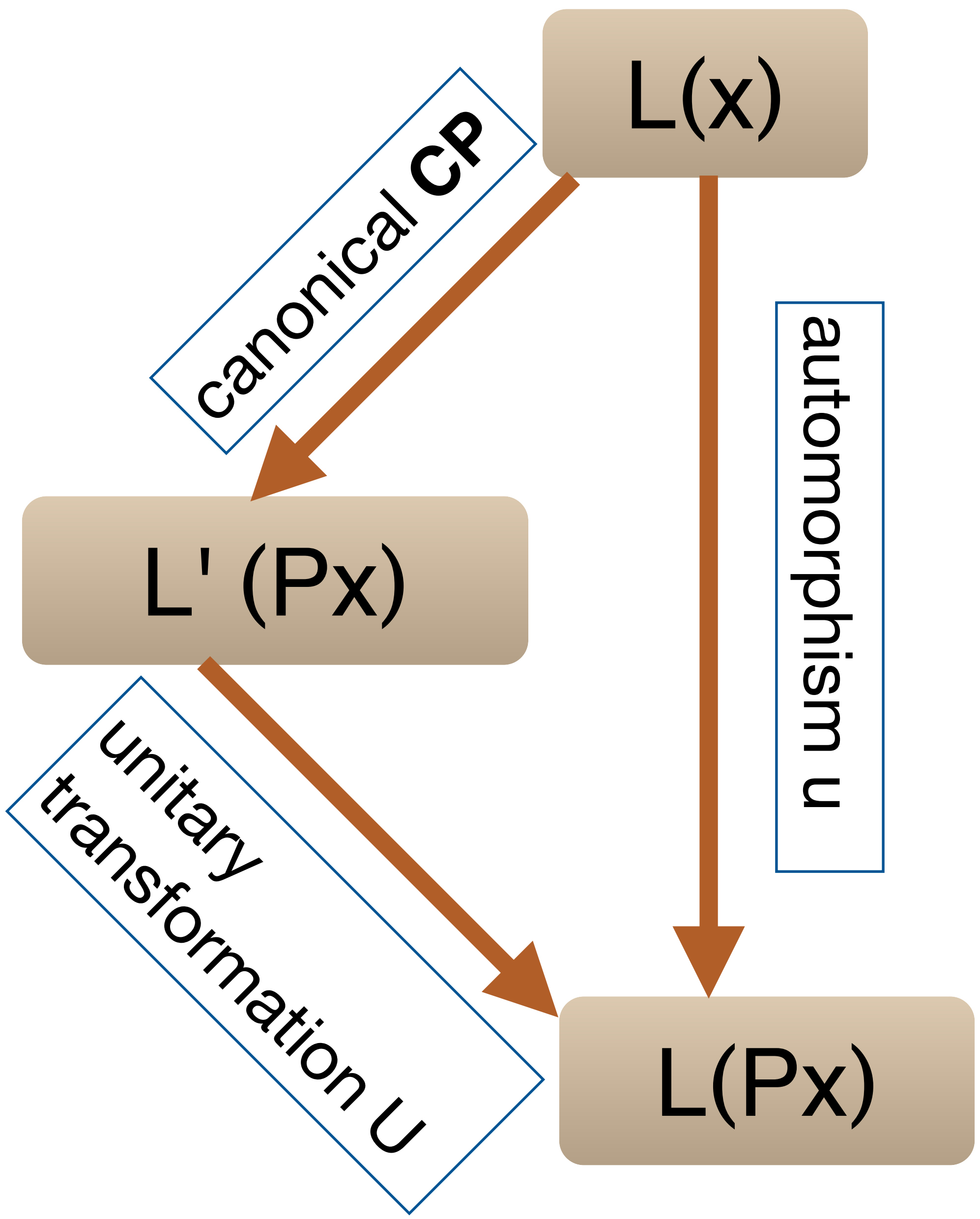
deviations correlated

non-maximal θ_{23} ⇨ normal hierarchy

mass ordering ⇨ symmetry breaking patterns

Group Theoretical Origin of CP Violation

complex CGs ➪ G and physical CP transformations do not commute



M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)

$$\Phi(x) \xmapsto{\widetilde{\mathcal{CP}}} U_{\mathrm{CP}}\, \Phi^*(\,\mathcal{P}\;x)$$

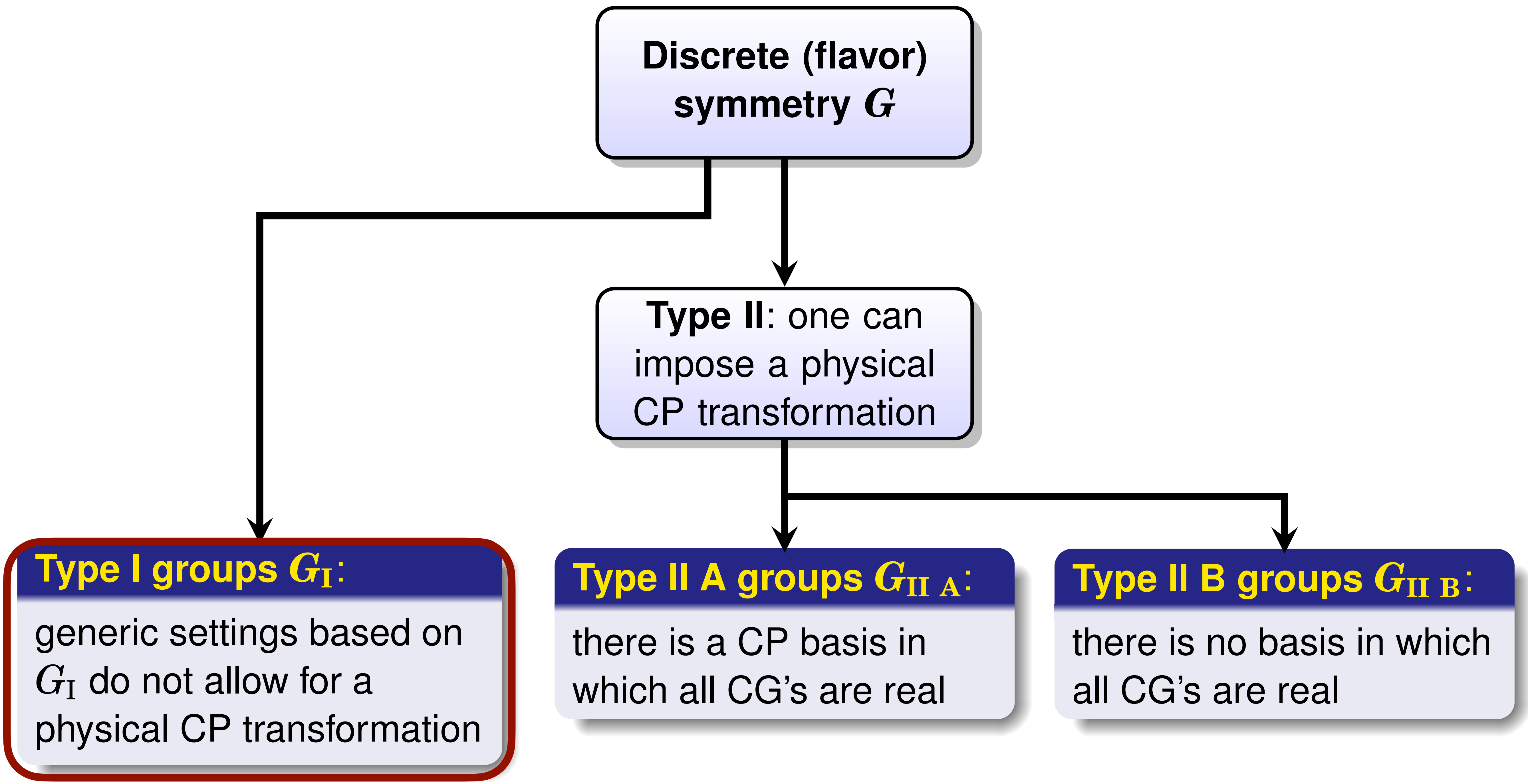
$$\rho_{\boldsymbol{r}_i}\big(u(g)\big) \;=\; U_{\boldsymbol{r}_i}\, \rho_{\boldsymbol{r}_i}(g)^*\, U_{\boldsymbol{r}_i}^\dagger \qquad \forall\, g \in G \text{ and } \forall\, i$$

u has to be a class-inverting,
involutory automorphism of G
➪ **non-existence of such automorphism in certain groups**
➪ **calculable physical CP violation in generic setting**

examples: T_7, $\Delta(27)$, …..

Group Theoretical Origin of CP Violation

M.-C.C, M. Fallbacher, K.T. Mahanthappa, M. Ratz, A. Trautner, NPB (2014)



Kähler Corrections in Modular A4 Model

M.-C.C., Ramos-Sánchez, Ratz (2019)