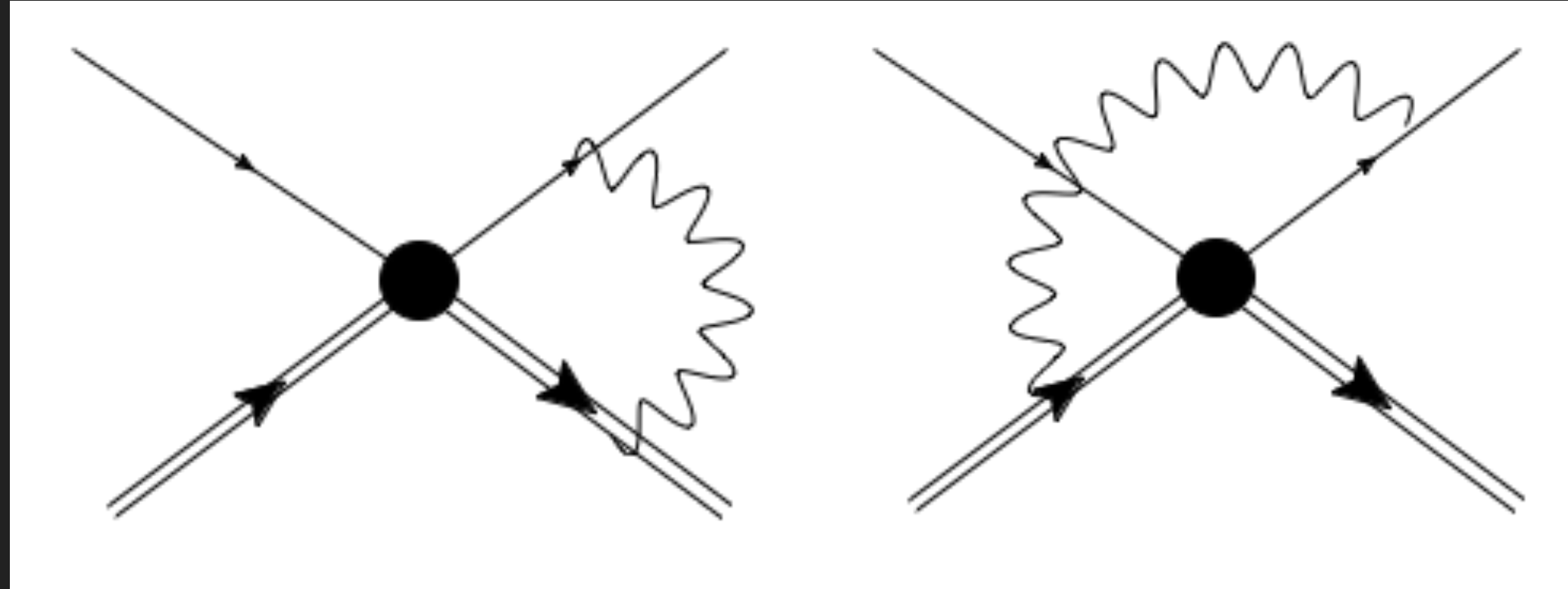


IN COLLABORATION WITH RICHARD HILL

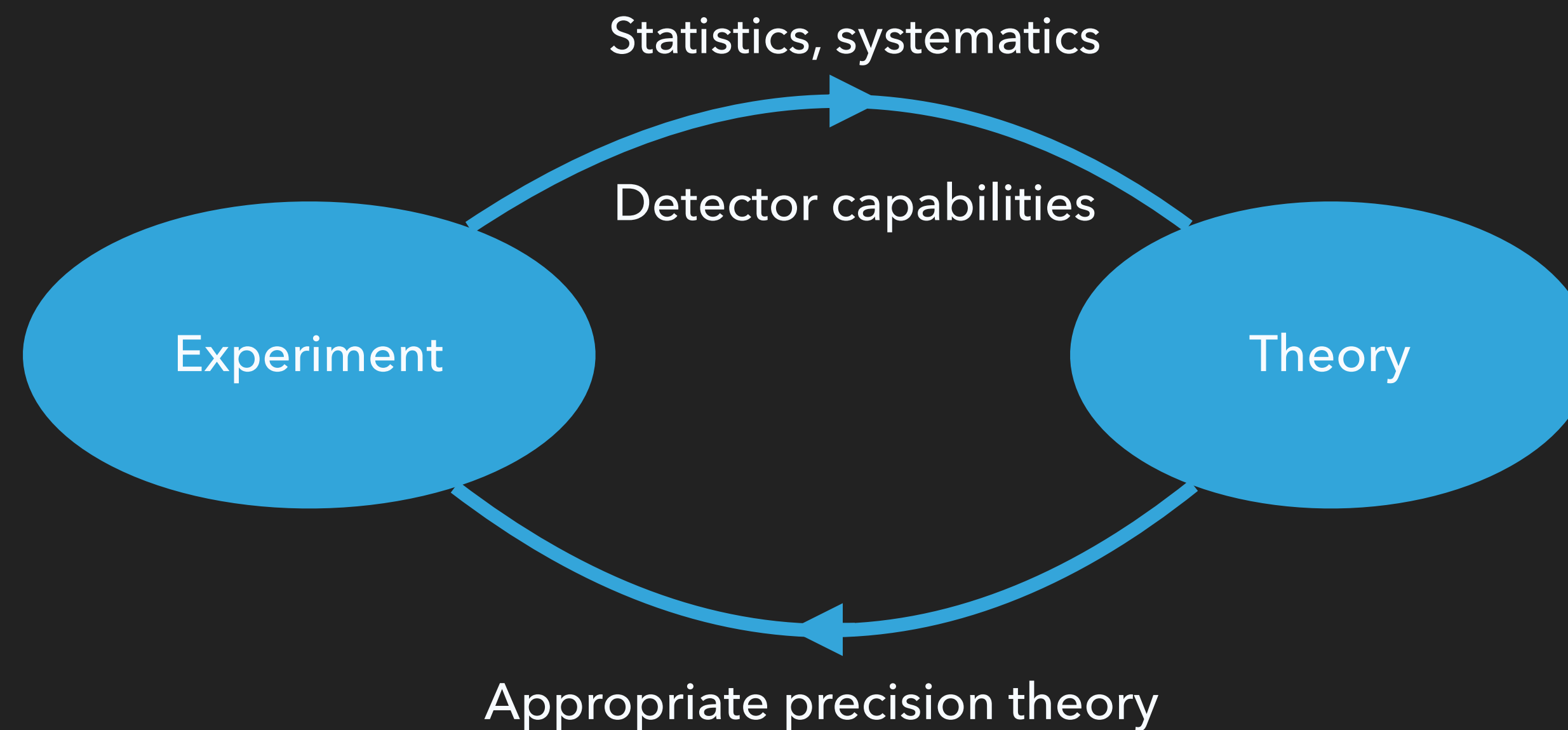


JUN 2022 | RYAN PLESTID | INCOMING NTN FELLOW

QED FACTORIZATION THEOREMS FOR SCATTERING WITH NUCLEAR TARGETS

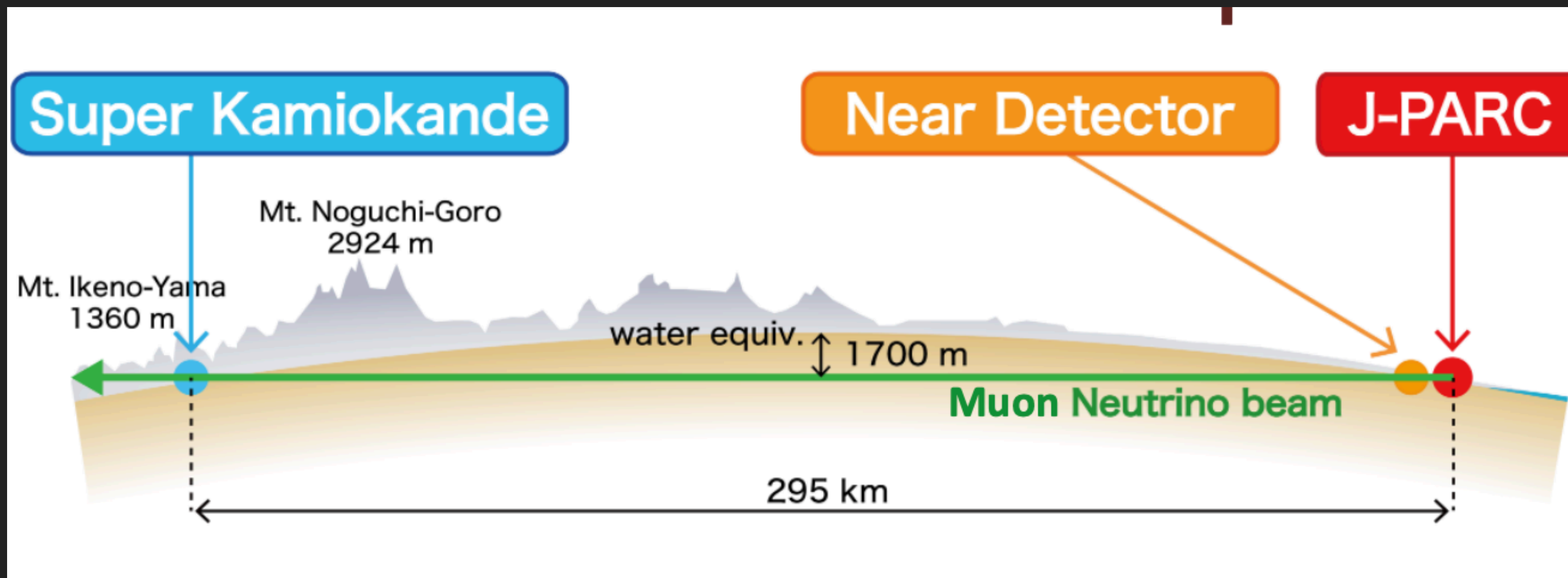
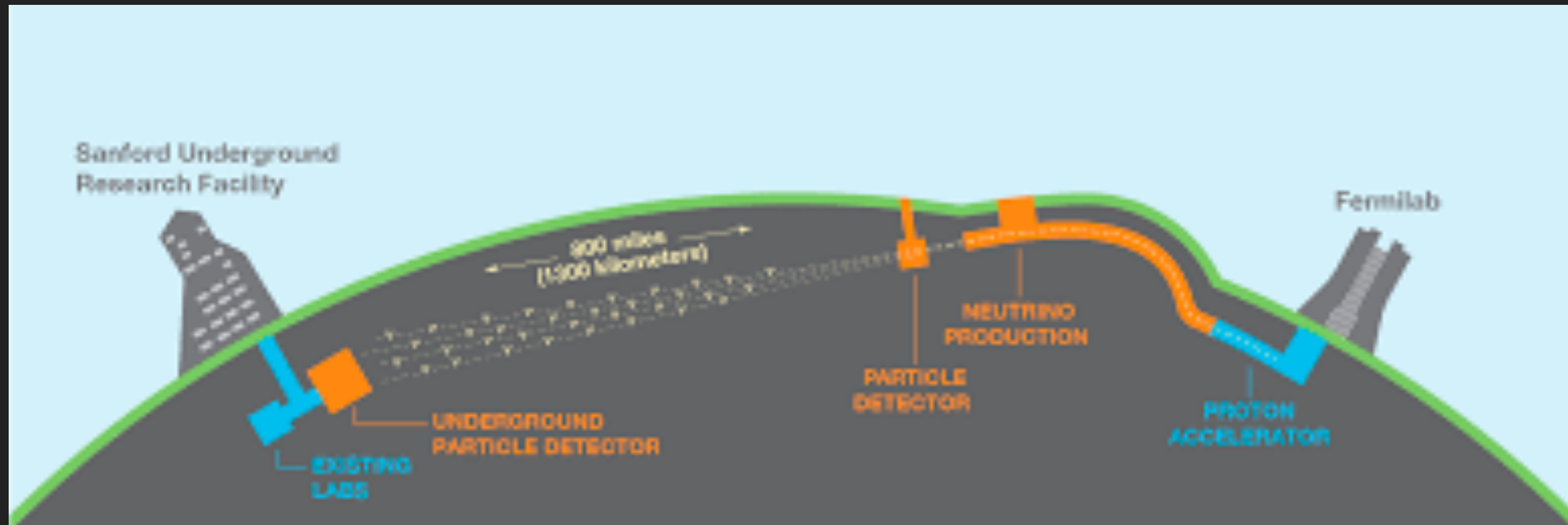
WHY SHOULD WE CARE?

- ▶ Scattering with nuclear targets is an old subject. So is there anything new to say?



1. Higher accuracy experiments demand response from theory.
2. DIS and heavy quark physics have taught us a lot about how to minimize theoretical uncertainties when describing hadronic systems.

LONG BASELINE OSCILLATIONS



QED RADIATIVE CORRECTIONS

1. Loop effects depend on lepton masses.
2. Real photon radiative depends on lepton masses.
3. Coulomb effects depend on sign of lepton charge.

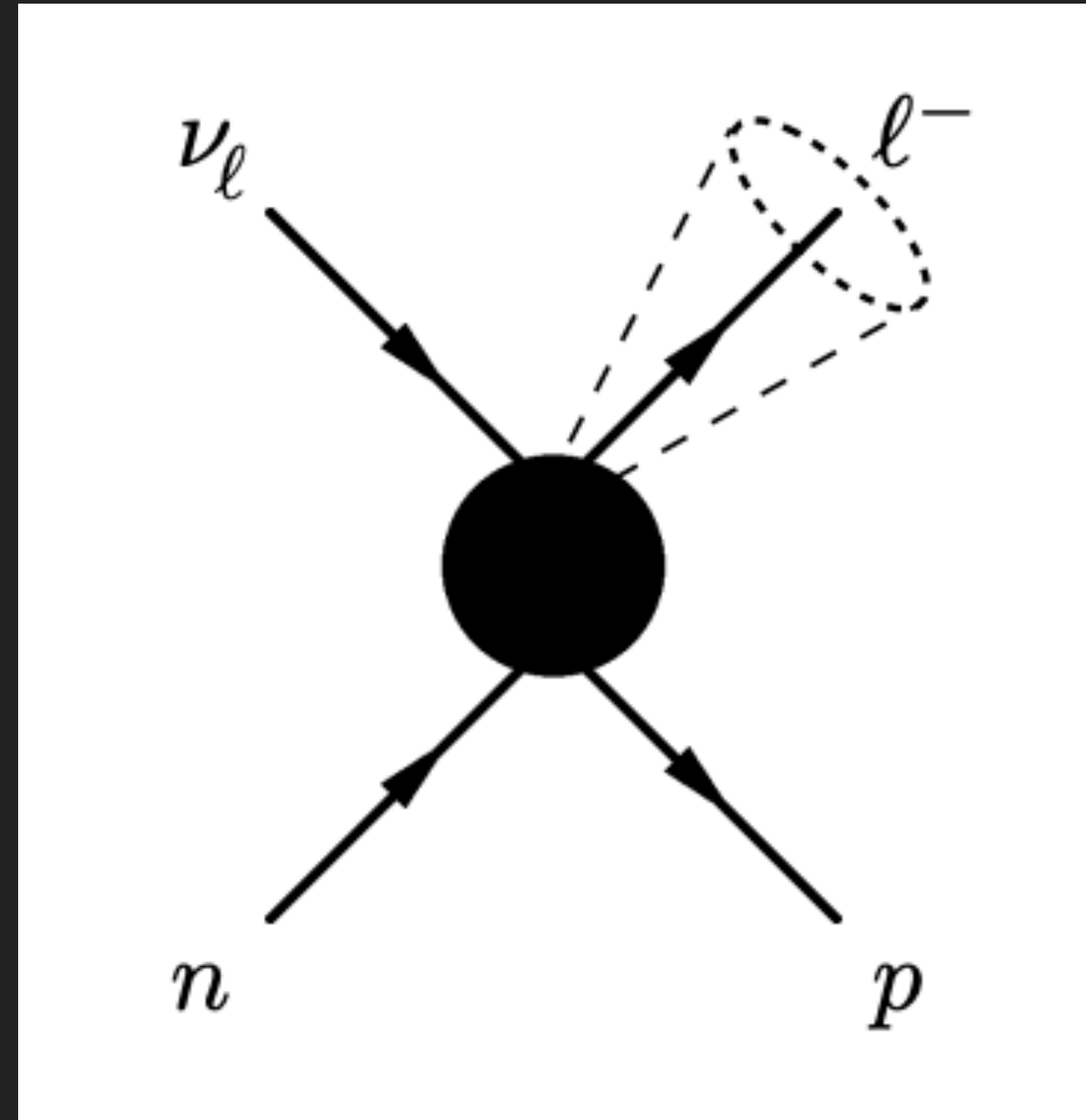


WHY FACTORIZATION THEOREMS?

- ▶ At a nominal 1% precision radiative corrections must be included.
- ▶ Photon can be soft, hard, collinear. Different scales \leftrightarrow different physics.
- ▶ "Loops and legs" make this a complicated multi-scale problem.

$$\mathcal{M} = S \otimes J \otimes H$$

- ▶ Tomalak, Chen, Hill, McFarland 2021



- ▶ arXiv:2105.07939

WHY FACTORIZATION THEOREMS?

$$\mathcal{M} = S \otimes J \otimes H$$

Soft physics.

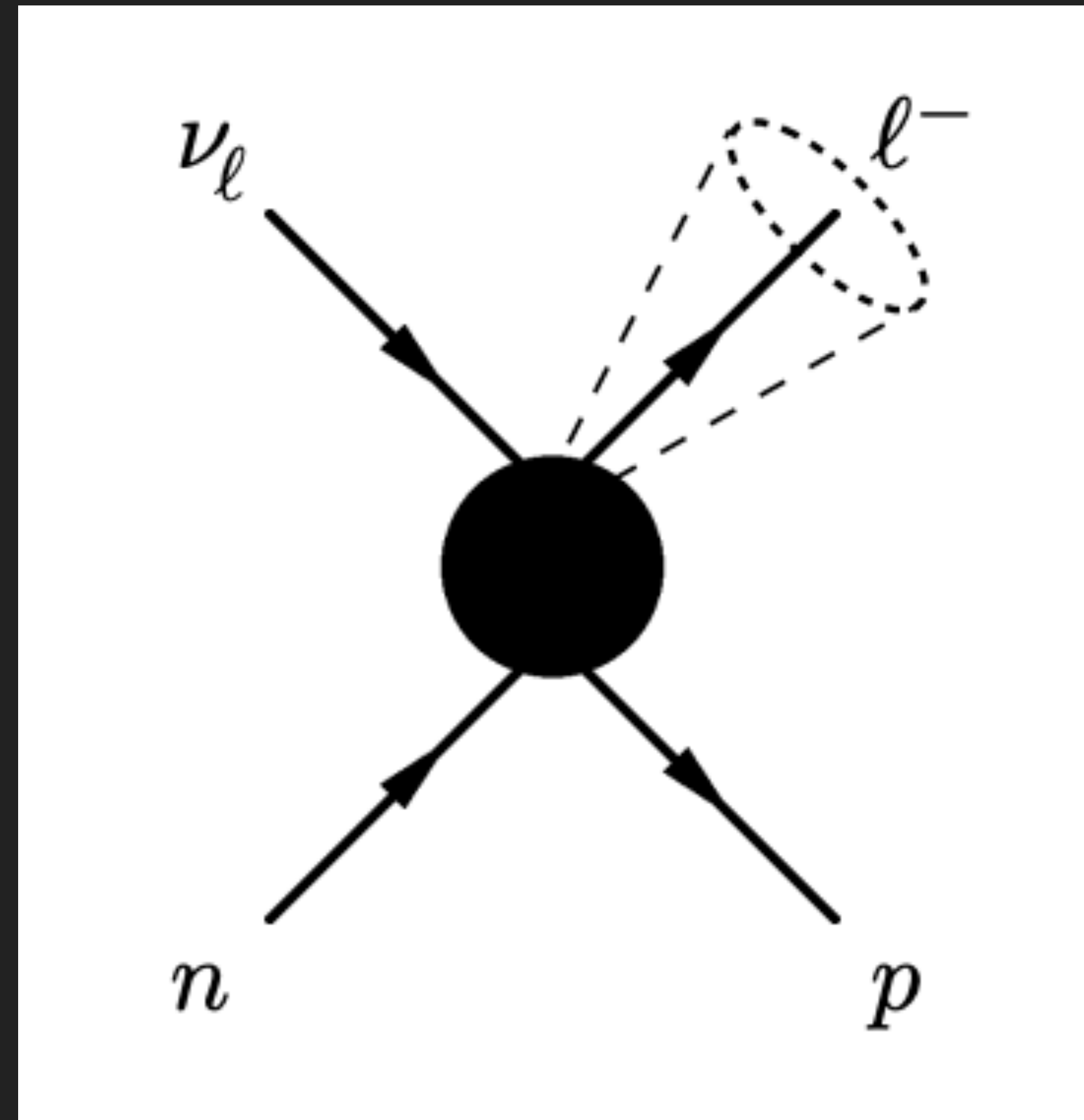
Collinear physics.

Hard physics.

- Factorization theorems let us *isolate* difficult problems from one another.

$$3 \times (\text{difficult}) \ll (\text{difficult})^3$$

- Tomalak, Chen, Hill, McFarland 2021



- arXiv:2105.07939

WHY FACTORIZATION THEOREMS?

Isolate hard nuclear physics from QED loops and legs

$$\mathcal{M} = S \otimes J \otimes H$$

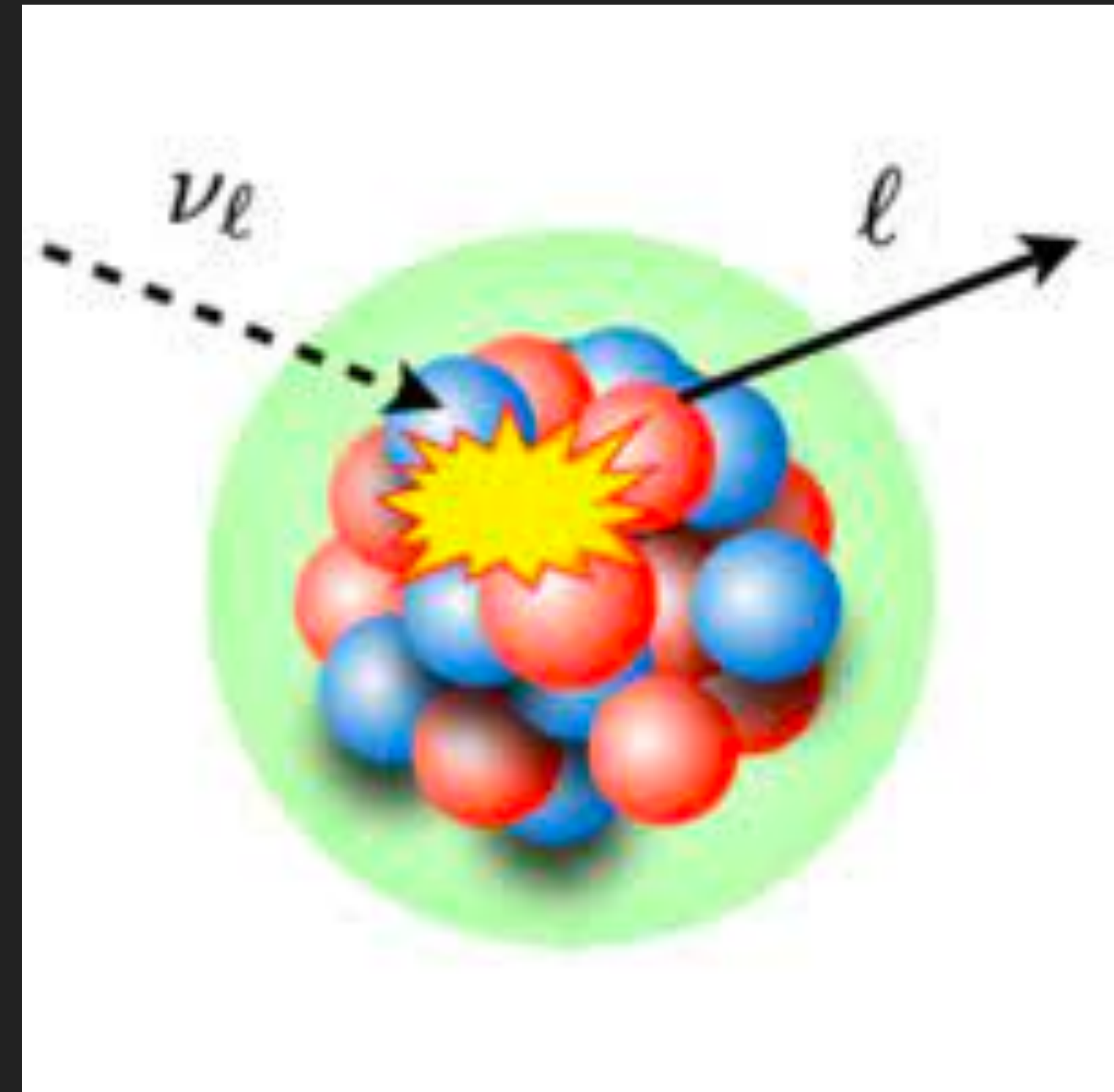
Soft physics.

Collinear physics.

Hard physics.

- Factorization theorems let us isolate difficult problems from one another.

$$3 \times (\text{difficult}) \ll (\text{difficult})^3$$



FACTORIZATION AND EFT

$$\mathcal{M} = S \otimes J \otimes H$$

Soft physics.

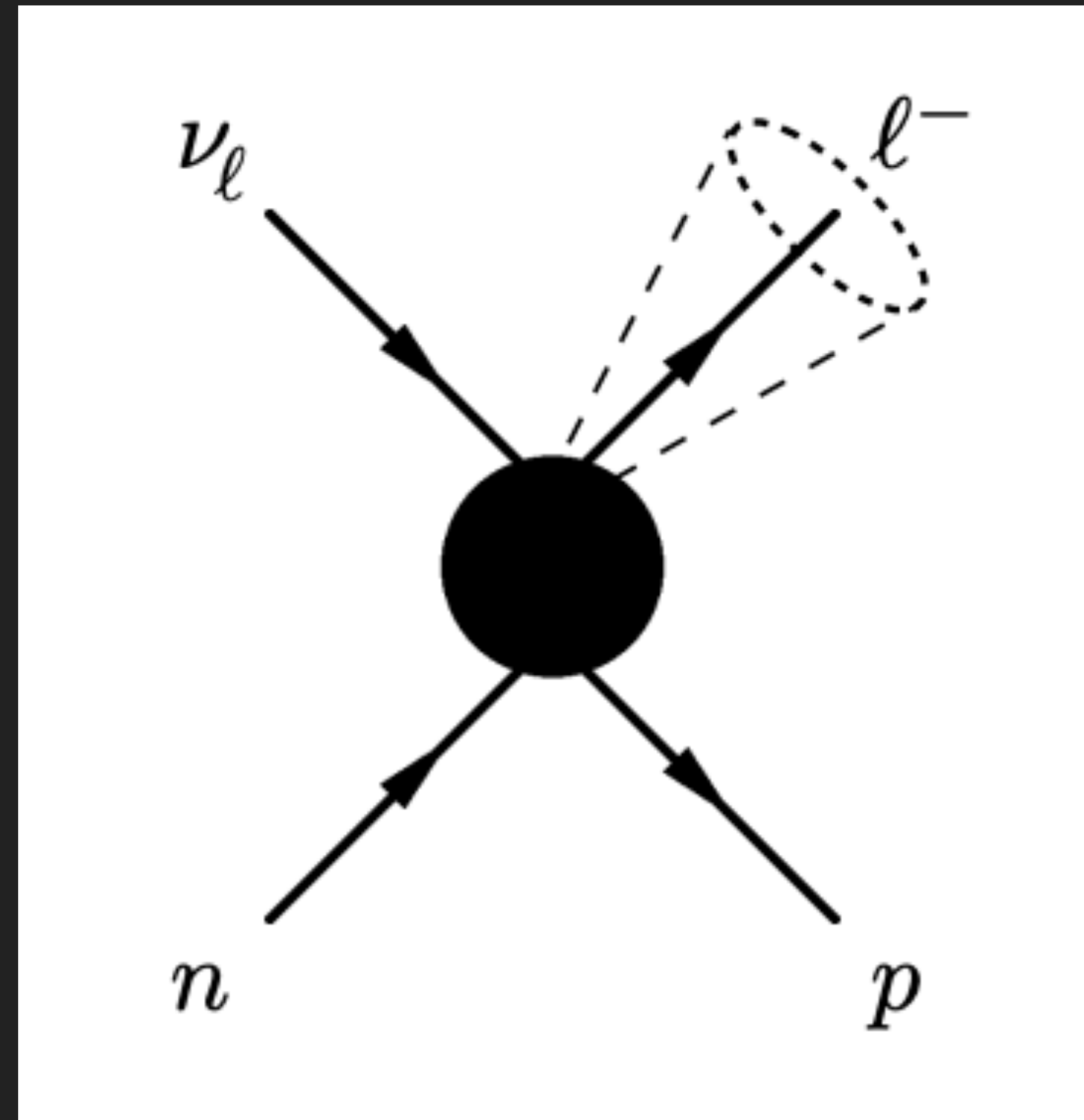
Collinear physics.

Hard physics.

- ▶ Each term in the factorization theorem corresponds to a different EFT.

Hard function \leftrightarrow Wilson Coefficient

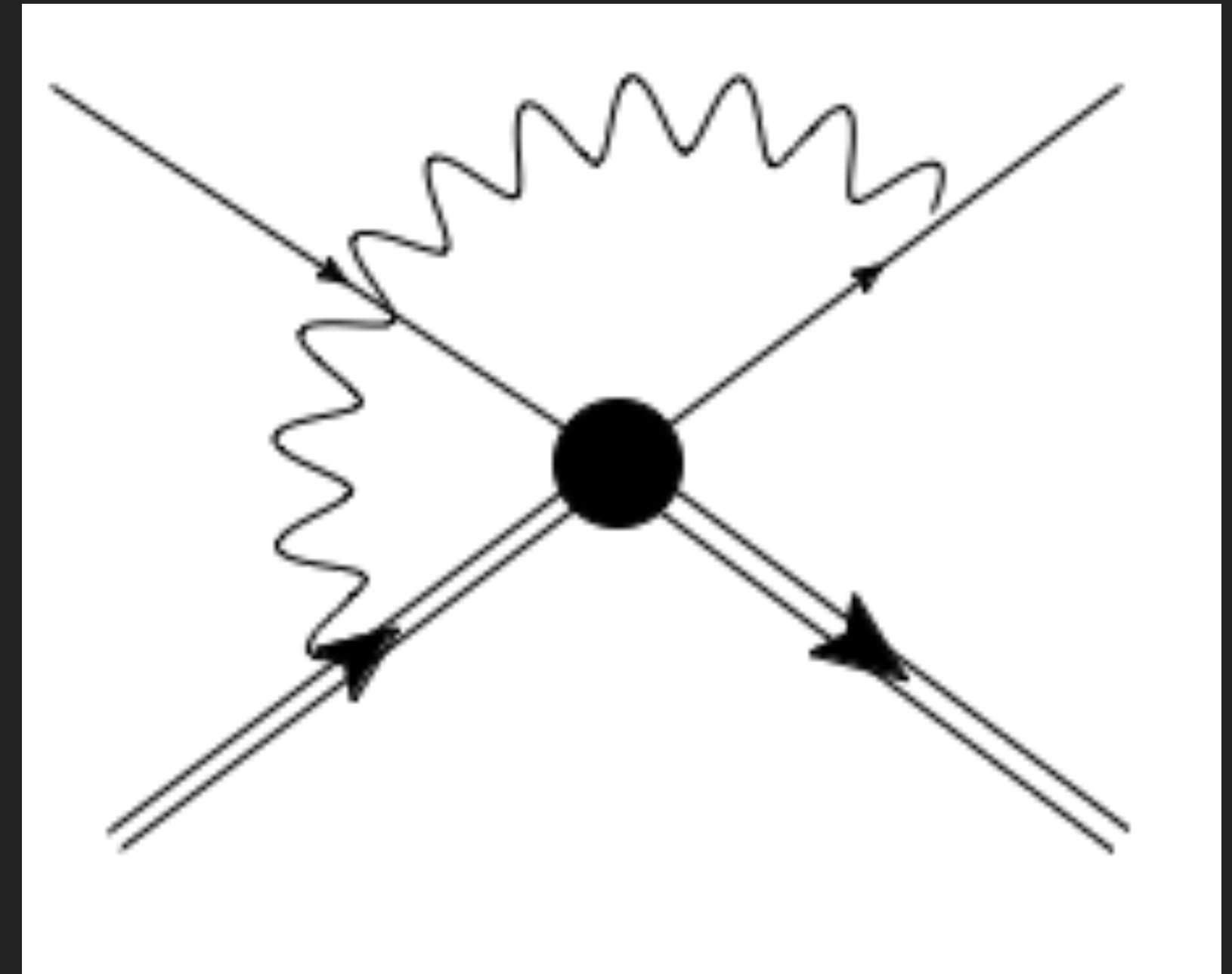
- ▶ Tomalak, Chen, Hill, McFarland 2021



- ▶ arXiv:2105.07939

WHAT IS NEW WITH NUCLEI?

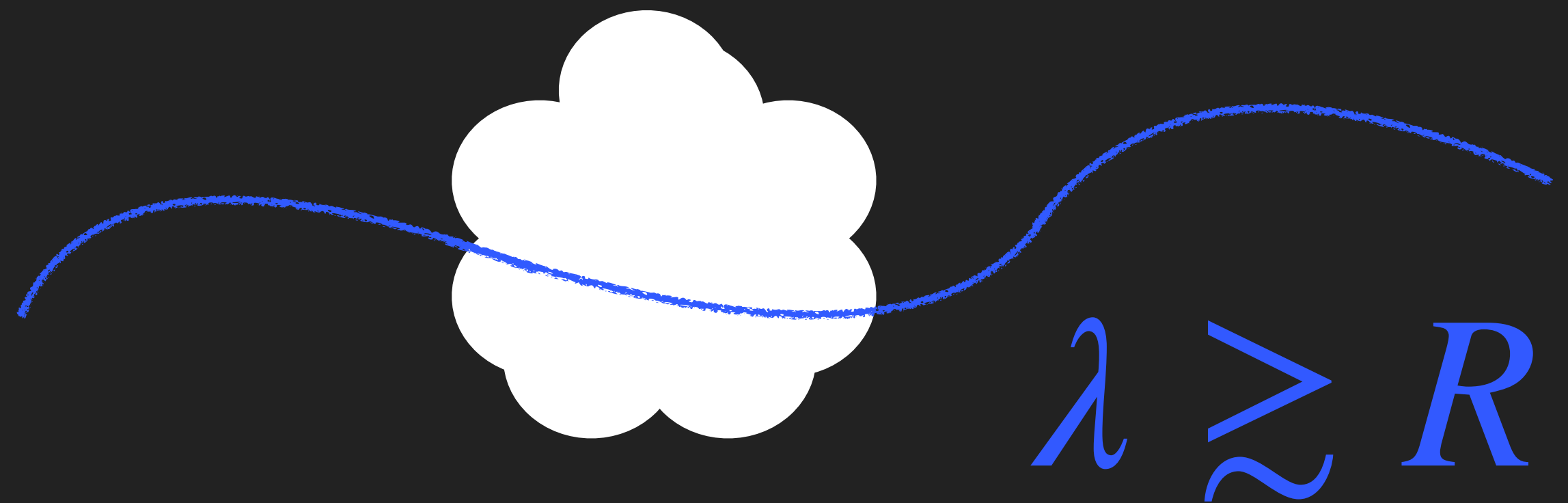
- ▶ Loops connect high energy leptons, to very heavy nuclei.
- ▶ At long wavelengths the nucleus can couple coherently to photons.
- ▶ Radiative corrections can be enhanced by the charge of the nucleus.



$$\alpha \rightarrow Z\alpha \quad \text{for} \quad \mathbf{L} \ll 1/R$$



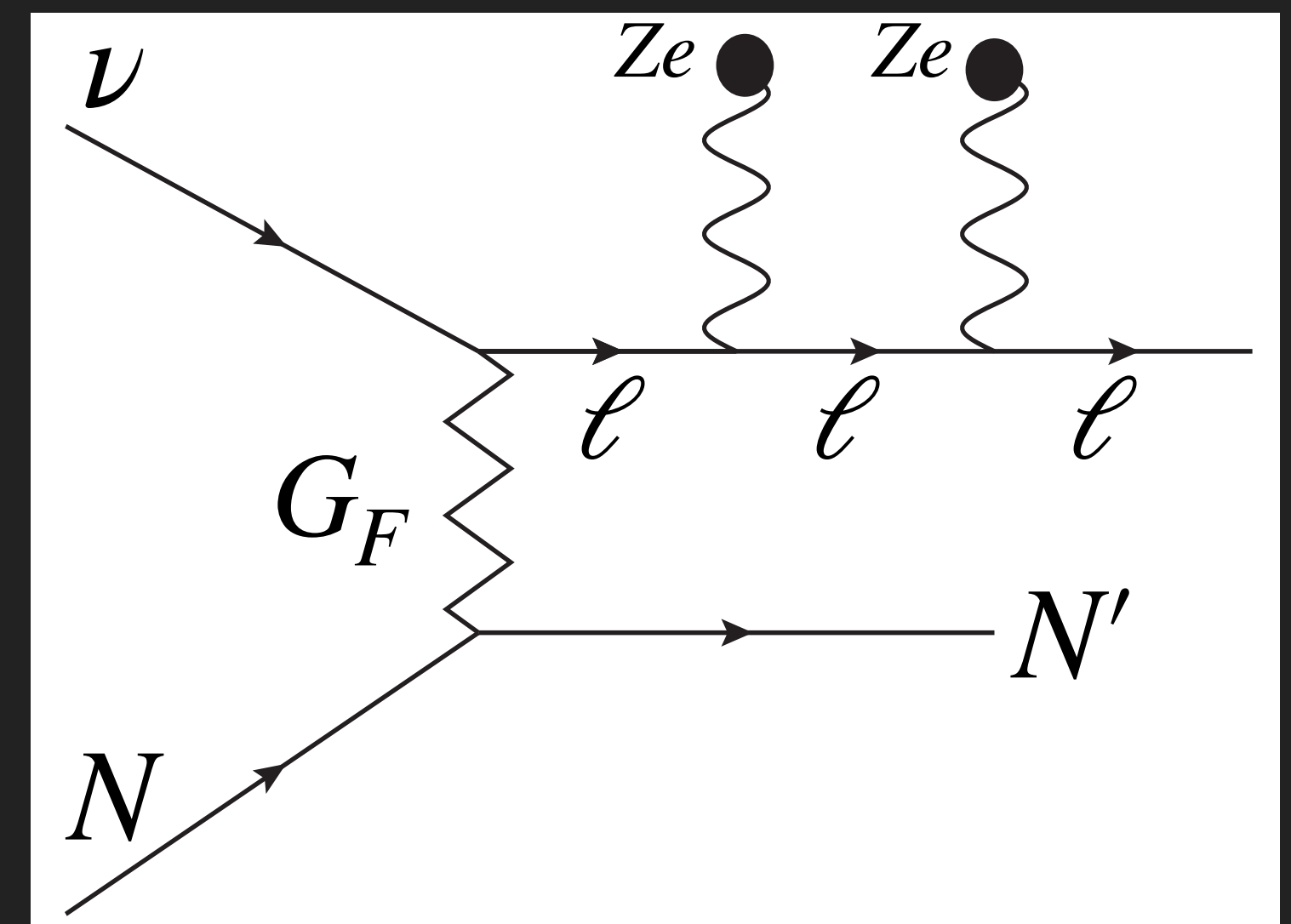
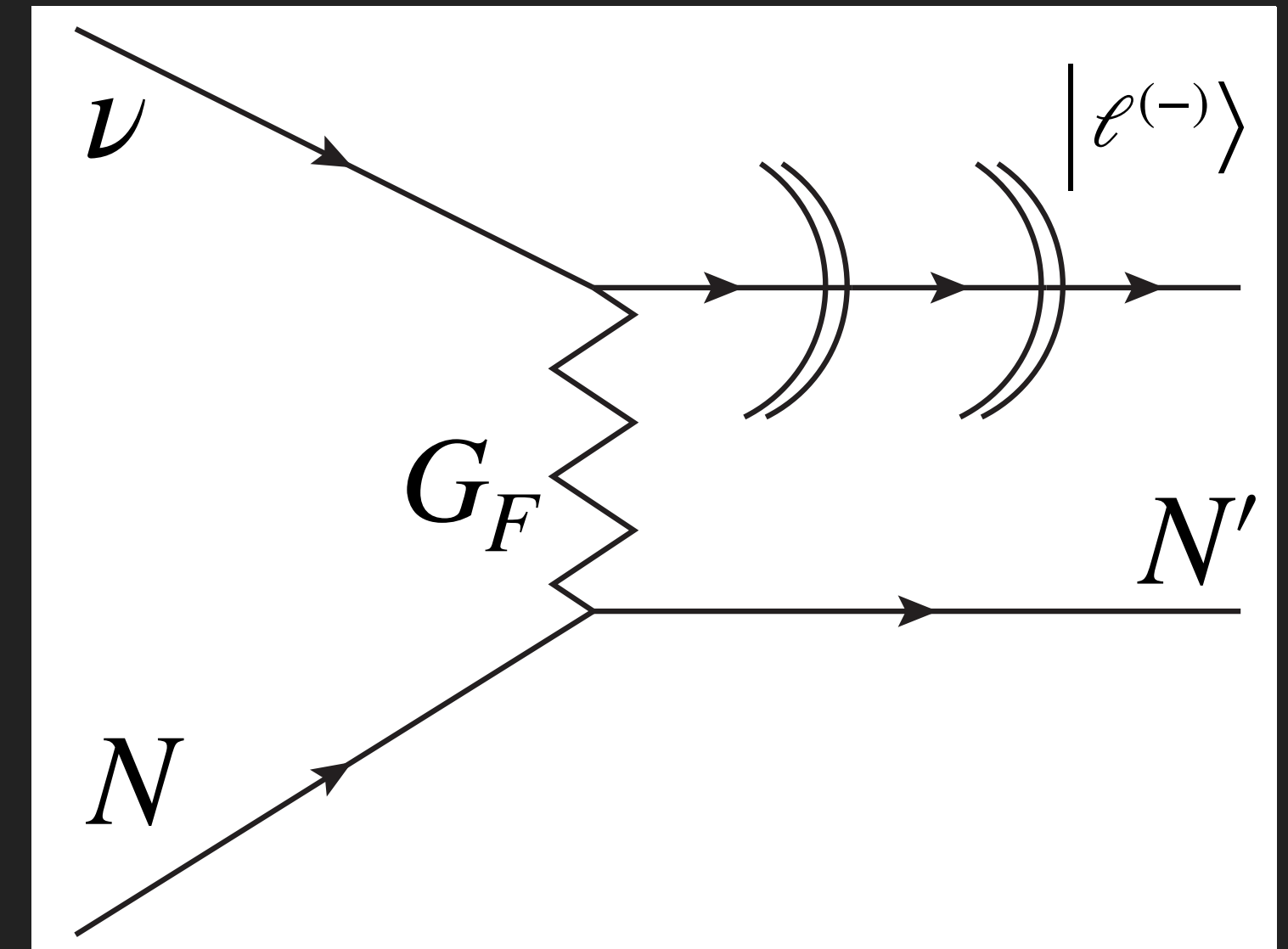
Loop momentum.



OLD PRESCRIPTIONS

- ▶ The historical approach to this problem is to treat the nucleus as a static external field.

How does this emerge from photon exchange with nuclei?



THE REST OF THIS TALK

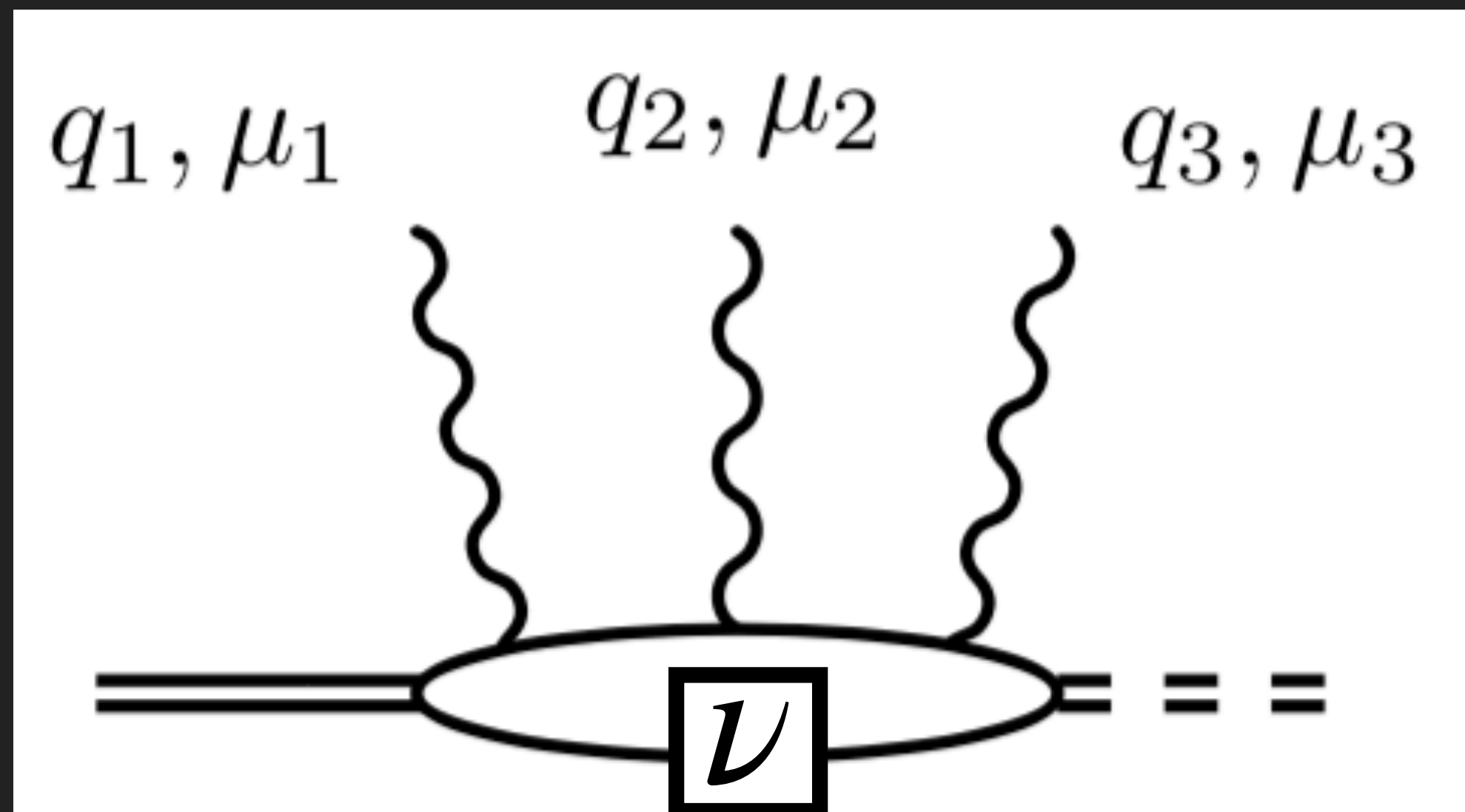
1. Crossed ladder diagrams do indeed reproduce background field (+ small corrections).
2. Comment on the structure of factorization theorems and hierarchies of scale.
3. Extracting all orders results for low momentum probes using factorization theorems.

CROSSED LADDERS AND THE STATIC LIMIT

Polology for heavy composite objects

FORMULATE DIAGRAMMATICS IN TERMS OF IN-OUT CORRELATORS

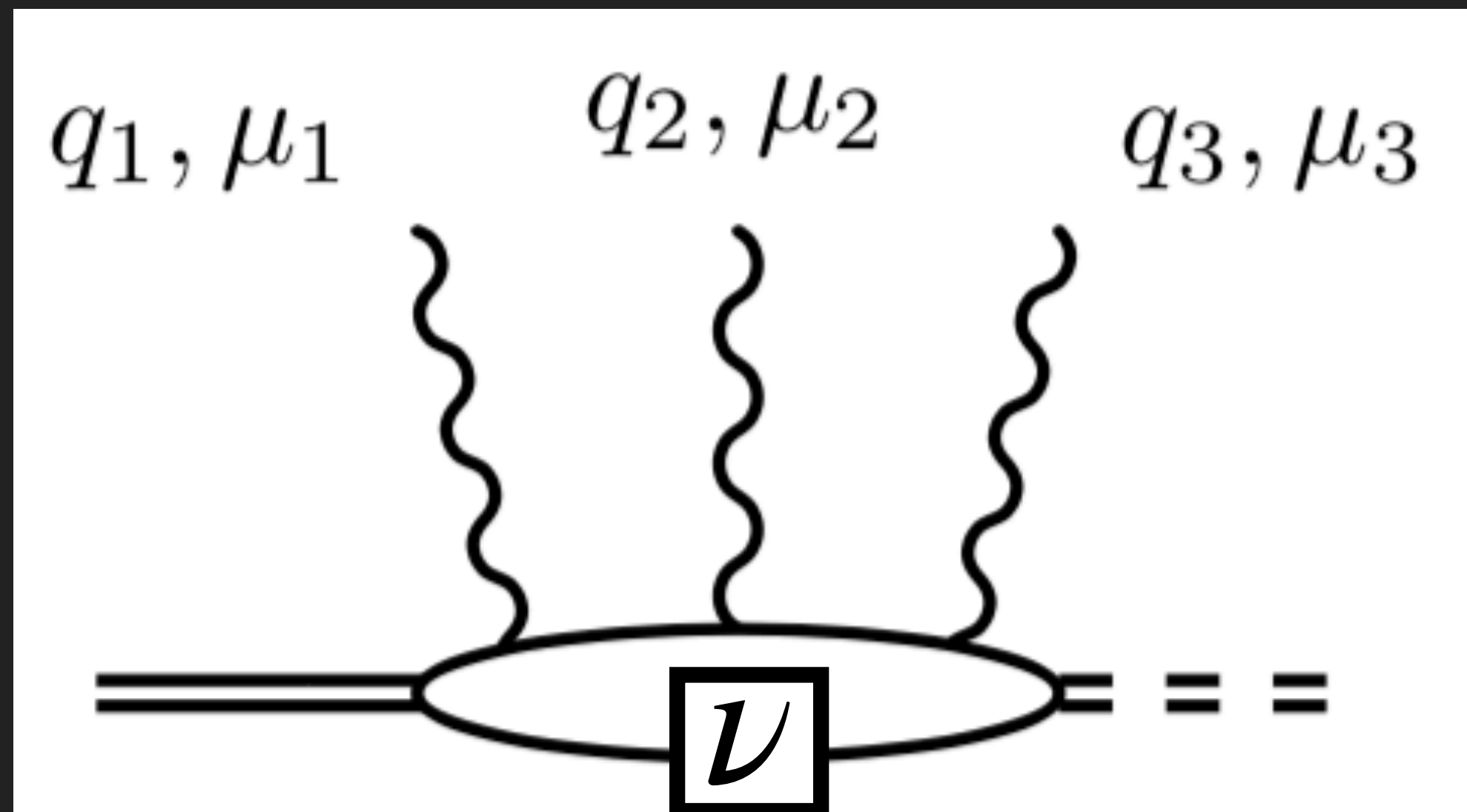
$$G_{\mu_1 \dots \mu_N ; \nu}(q_1, \dots, q_N) = \int [d^4x] e^{i \sum_i q_i \cdot x_i} \text{out} \langle B | T \{ J_{\mu_1}(x_1) \dots \mathcal{J}_\nu \dots J_{\mu_N}(x_N) \} | A \rangle_{\text{in}}$$



- ▶ Perturbation theory with “blobs”
- ▶ One hard current insertion.
- ▶ Ex. two photon exchange with doubly virtual Compton tensor.

FORMULATE DIAGRAMMATICS IN TERMS OF IN-OUT CORRELATORS

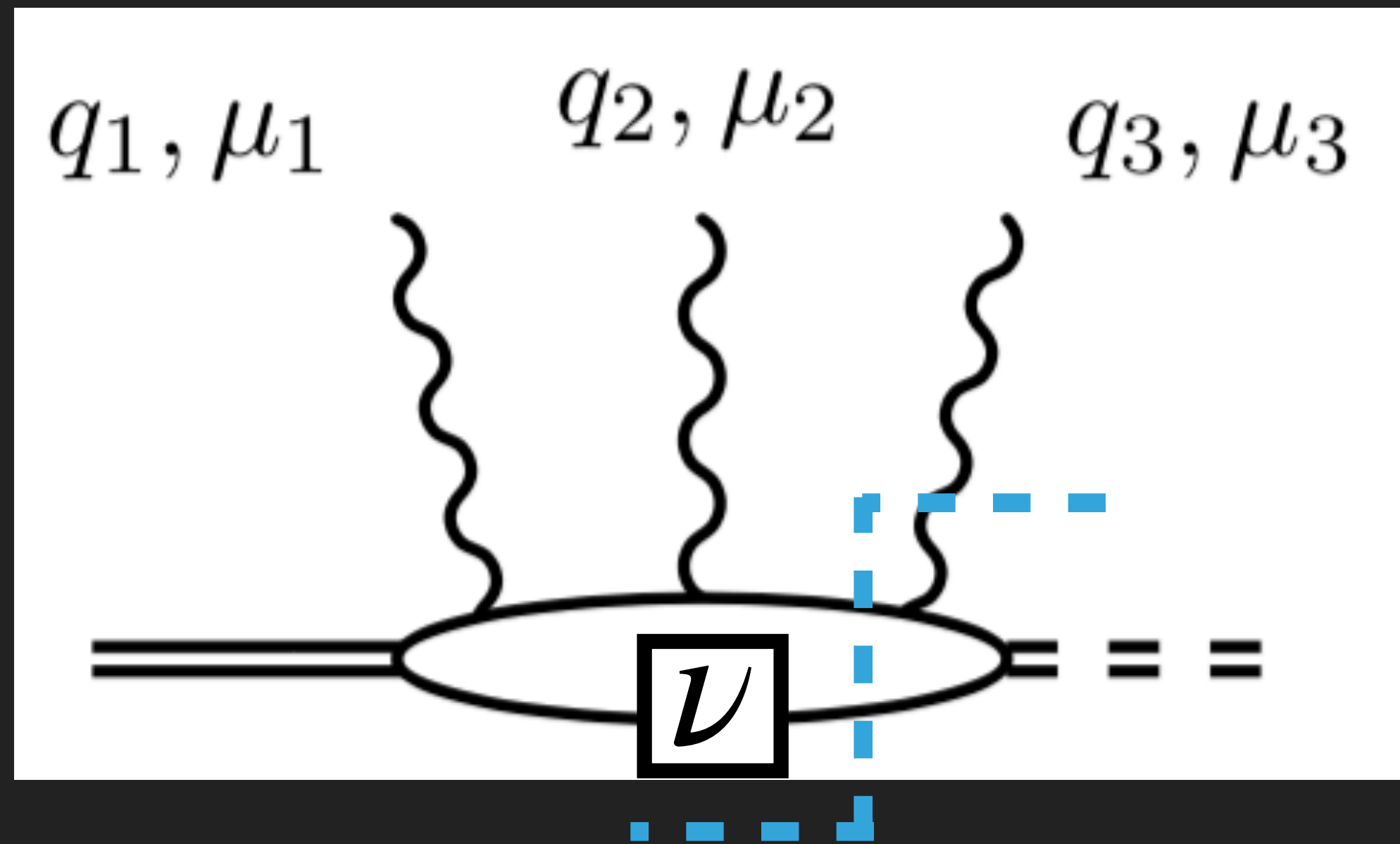
$${}_{\text{out}}\langle B | (\text{Big Blob}) | A \rangle_{\text{in}}$$



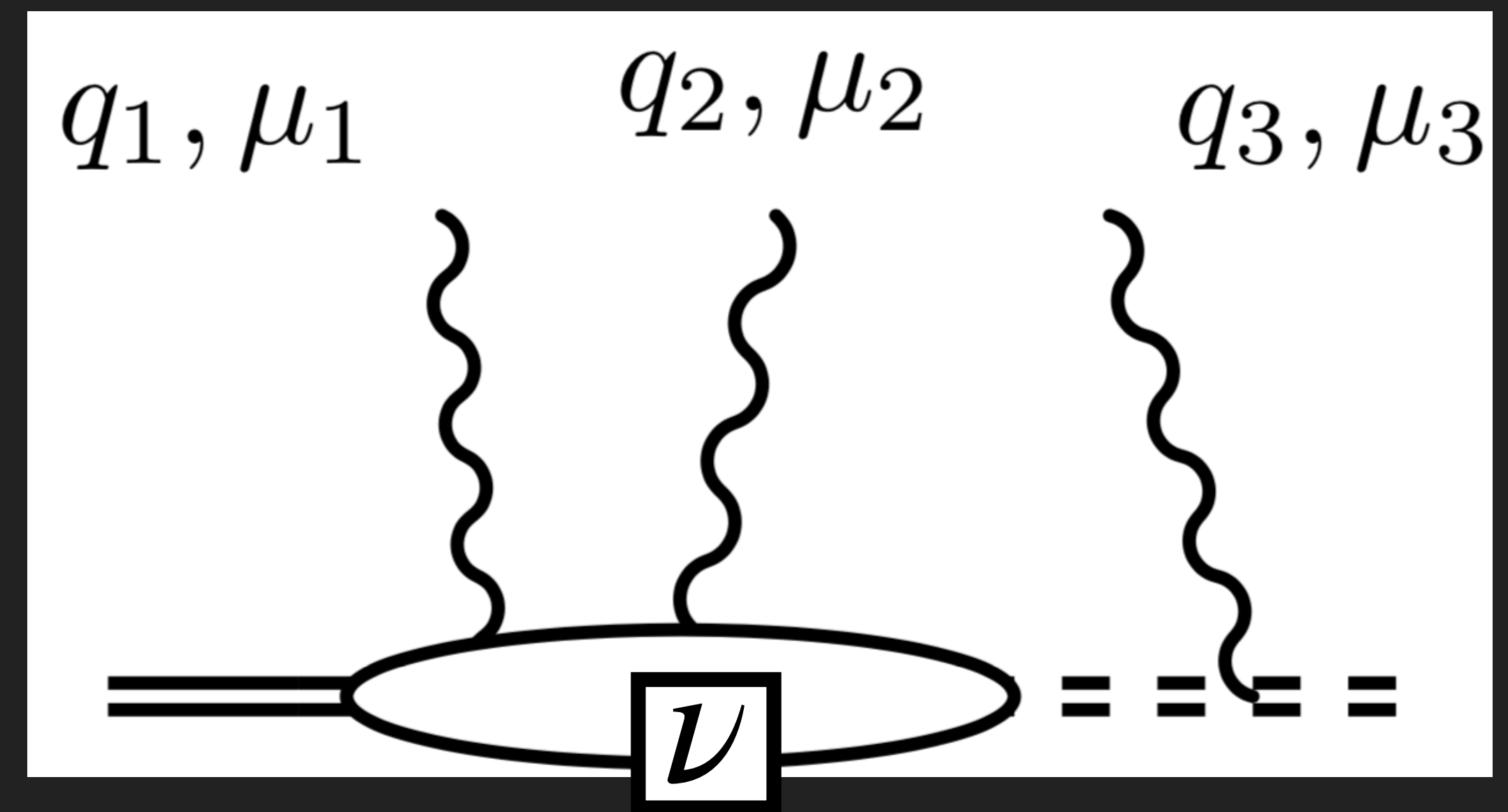
- ▶ Perturbation theory with “blobs”
- ▶ One hard current insertion.
- ▶ Ex. two photon exchange with doubly virtual Compton tensor.

POLOLOGY IN PICTURES

- ▶ Use Weinbergs "Polology" theorem to seperate out elastic poles.



=



- ▶ Non-pole terms.

- ▶ Other inelastic excitations

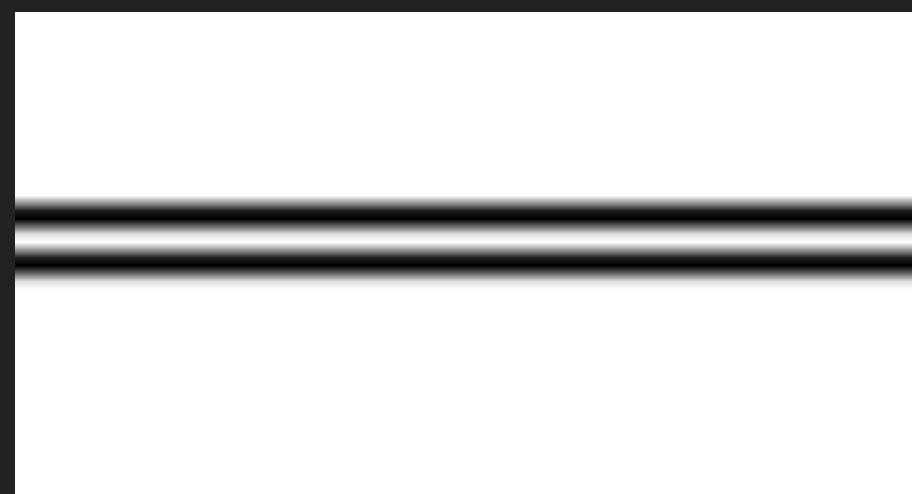


+ extra

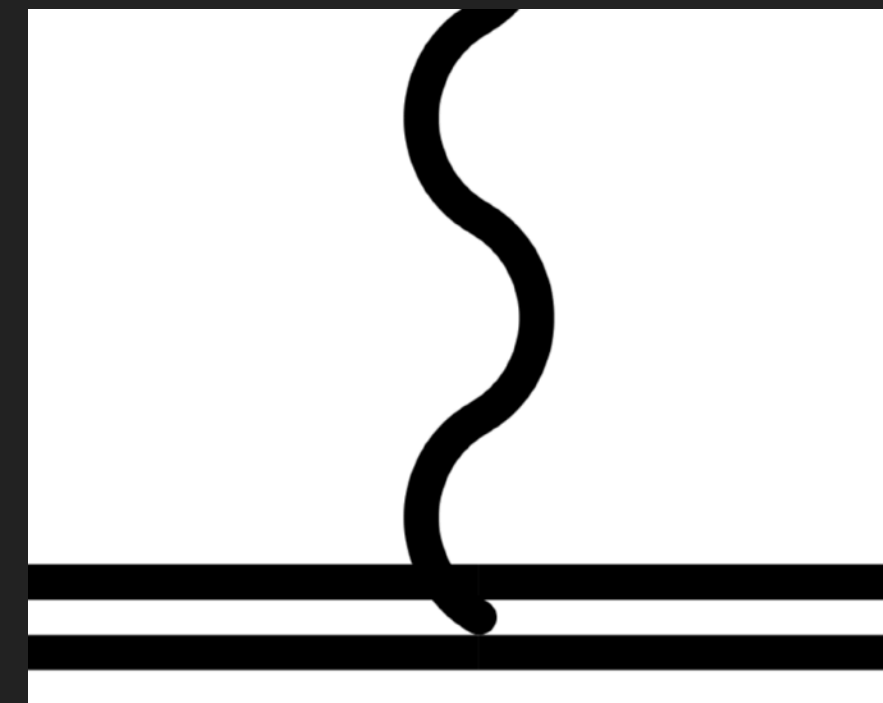
DIAGRAMATIC CONSEQUENCES

- ▶ Use eikonal propagators and couplings to photons.
- ▶ Include a charged form factor at each vertex

Note: This is gauge invariant!
(sticking in form factors often is not)



$$= \frac{i}{v \cdot q + i\epsilon}$$



$$= iZe \, v_\mu \, \underline{F_{\text{ch}}(q^2)}$$

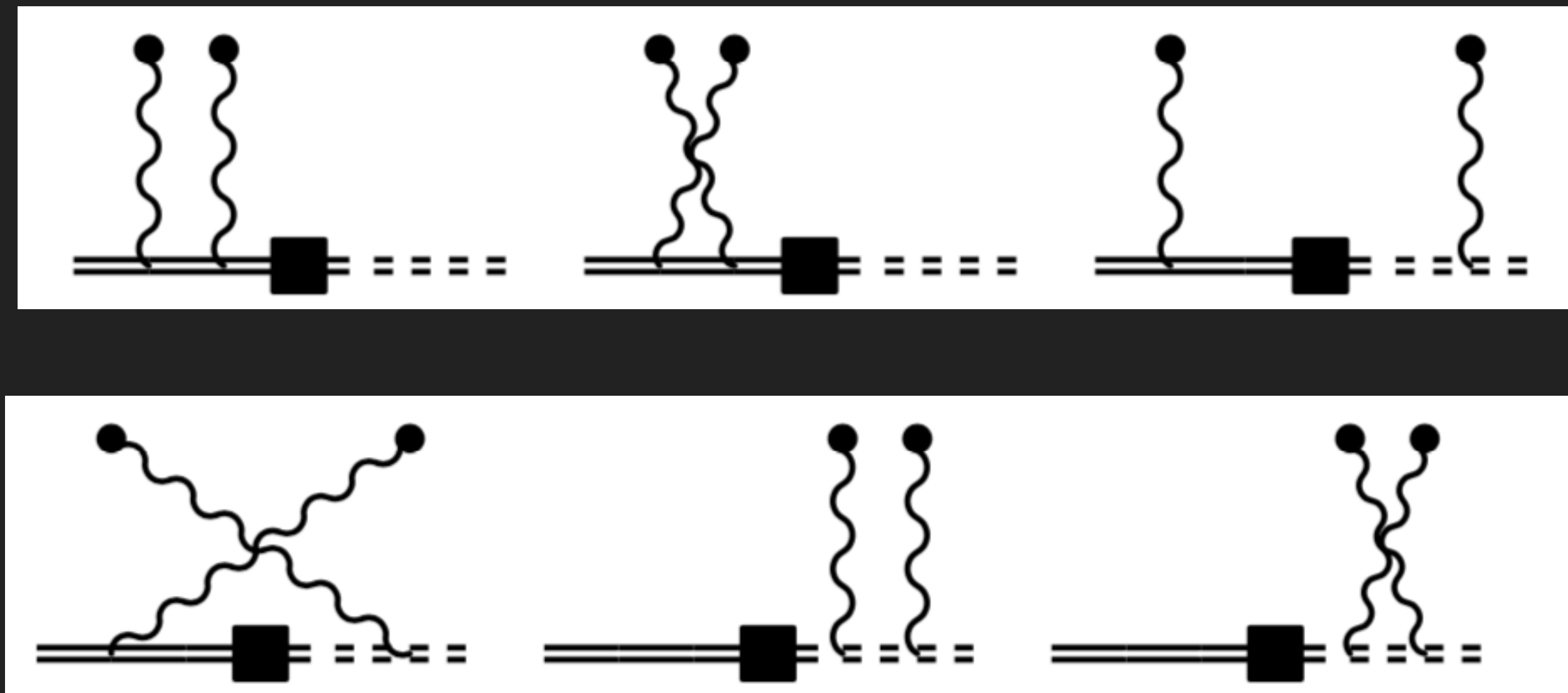
Nuclear structure inside
the loop expansion

EIKONAL FACTORS CONSTRAIN ENERGY TRANSFER

17

- Sum of all permutations of eikonal propagators gives

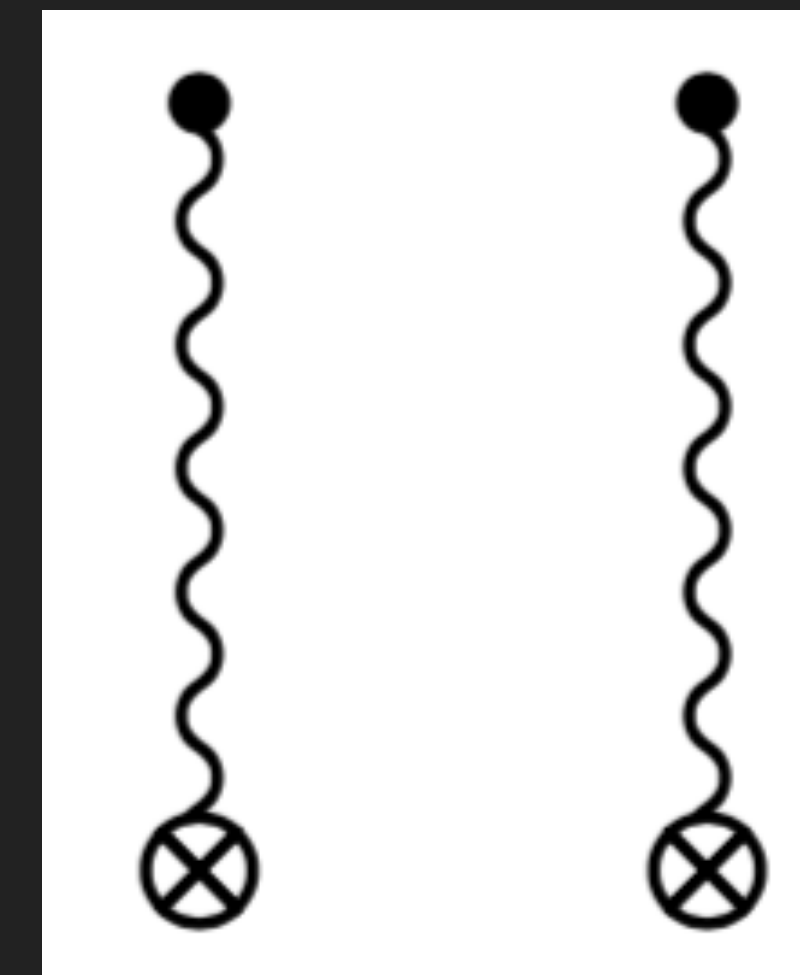
$$\sum_{ij \in \pi(1,2,3)} \frac{1}{v \cdot q_i + i\varepsilon} \frac{1}{v \cdot (q_i + q_j) + i\varepsilon} = (2\pi i) \delta(\underbrace{v \cdot q_1}_{\text{Energy transfer}}) (2\pi i) \delta(\underbrace{v \cdot q_2}_{\text{Energy transfer}})$$



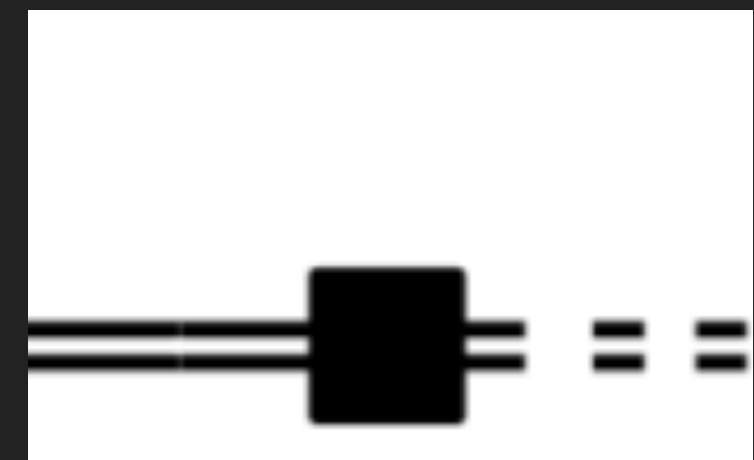
$(n+1) \times n!$



\rightarrow



$(n+1)$

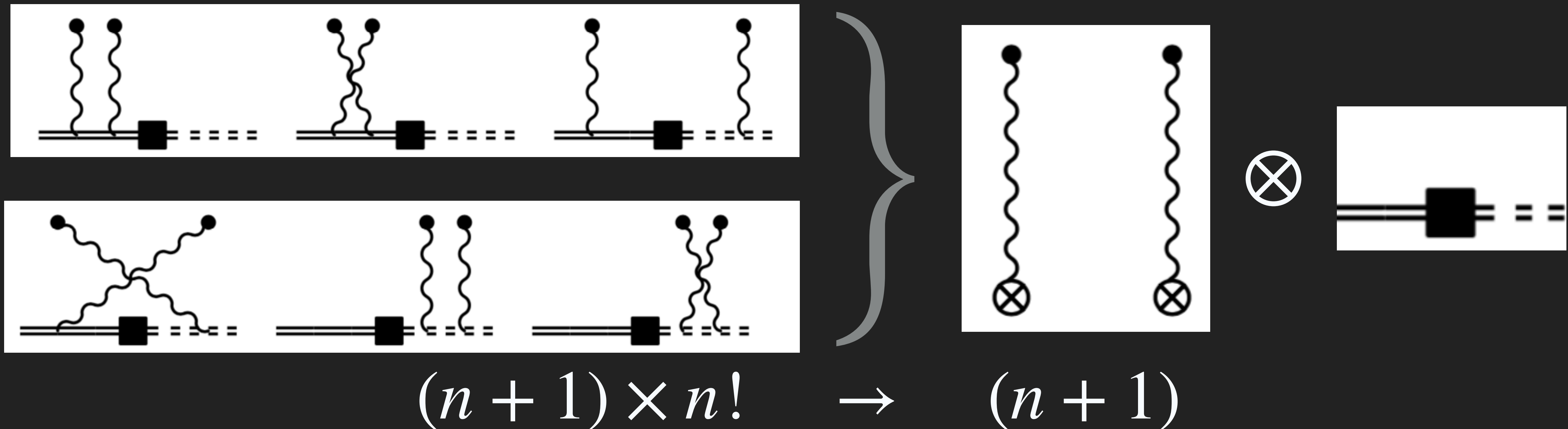


EIKONAL FACTORS CONSTRAIN ENERGY TRANSFER

18

- ▶ Sum of all permutations of eikonal propagators gives

interference of diagrams = no energy transfer



WHAT HAVE WE ASSUMED

19

- ▶ No recoil of heavy target $v_A = v_B$ *including from hard current.*
- ▶ Form factors are identical everywhere.

- ▶ More realistic scenario

$$F_A(q^2) \neq F_B(q^2)$$

$$Z_A = Z_B \pm n$$

- ▶ Can account for this with perturbation theory.

$$F_A(q^2) = F_B(q^2) + \delta F(q^2)$$

$$n \ll Z_A \approx Z_B$$

SUMMARY:

20

- ▶ External field approximation is valid up to *small corrections* controlled by:

- ▶ Nuclear asymmetry

$$\delta F_{AB}(Q^2) \sim O(\alpha)$$

- ▶ Recoil from hard vertex

$$1 - v \cdot v' = O(Q_H/M_A)$$

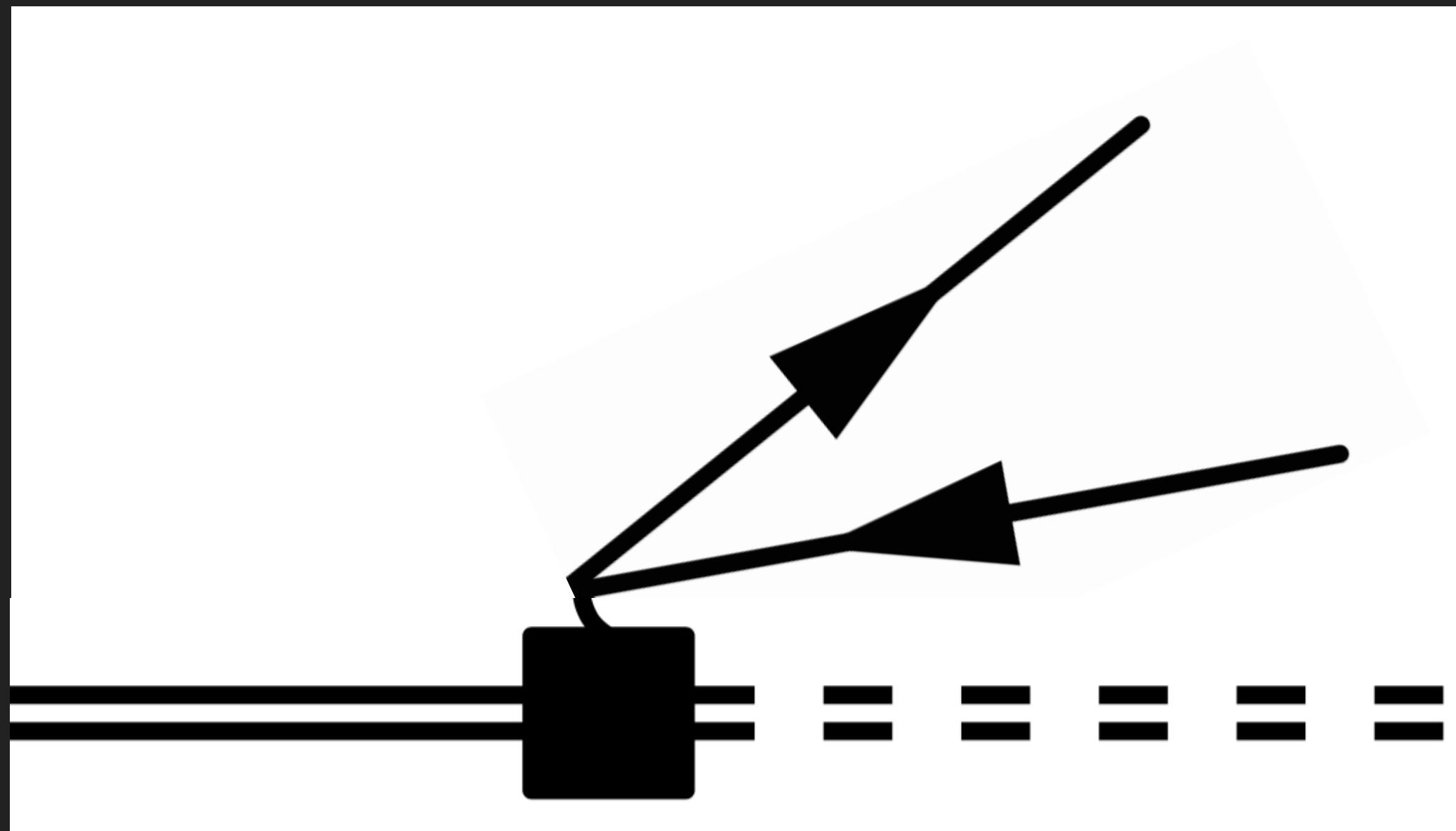
- ▶ Inelastic excitations

$$\sim O(\alpha) \quad \text{or} \quad \sim O(Q^2 R^2)$$

THE STRUCTURE OF FACTORIZATION THEOREMS WITH NUCLEI

The role nuclear structure, and hierarchies of scale

CONCRETE EXAMPLE: LEPTON PAIRS



Nuclear Mass

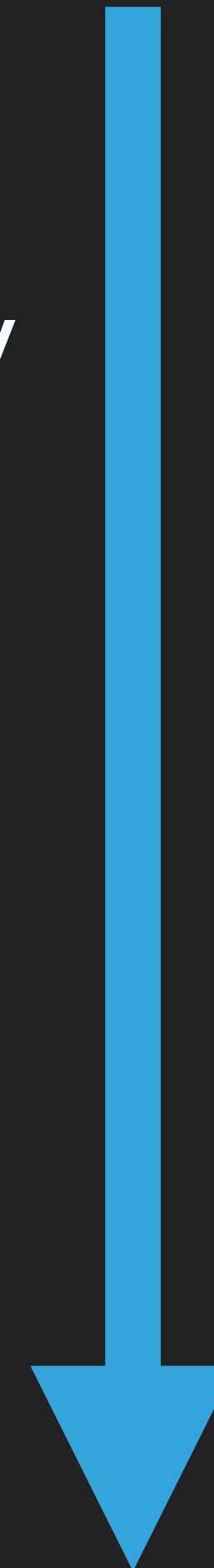
600 MeV

30 MeV

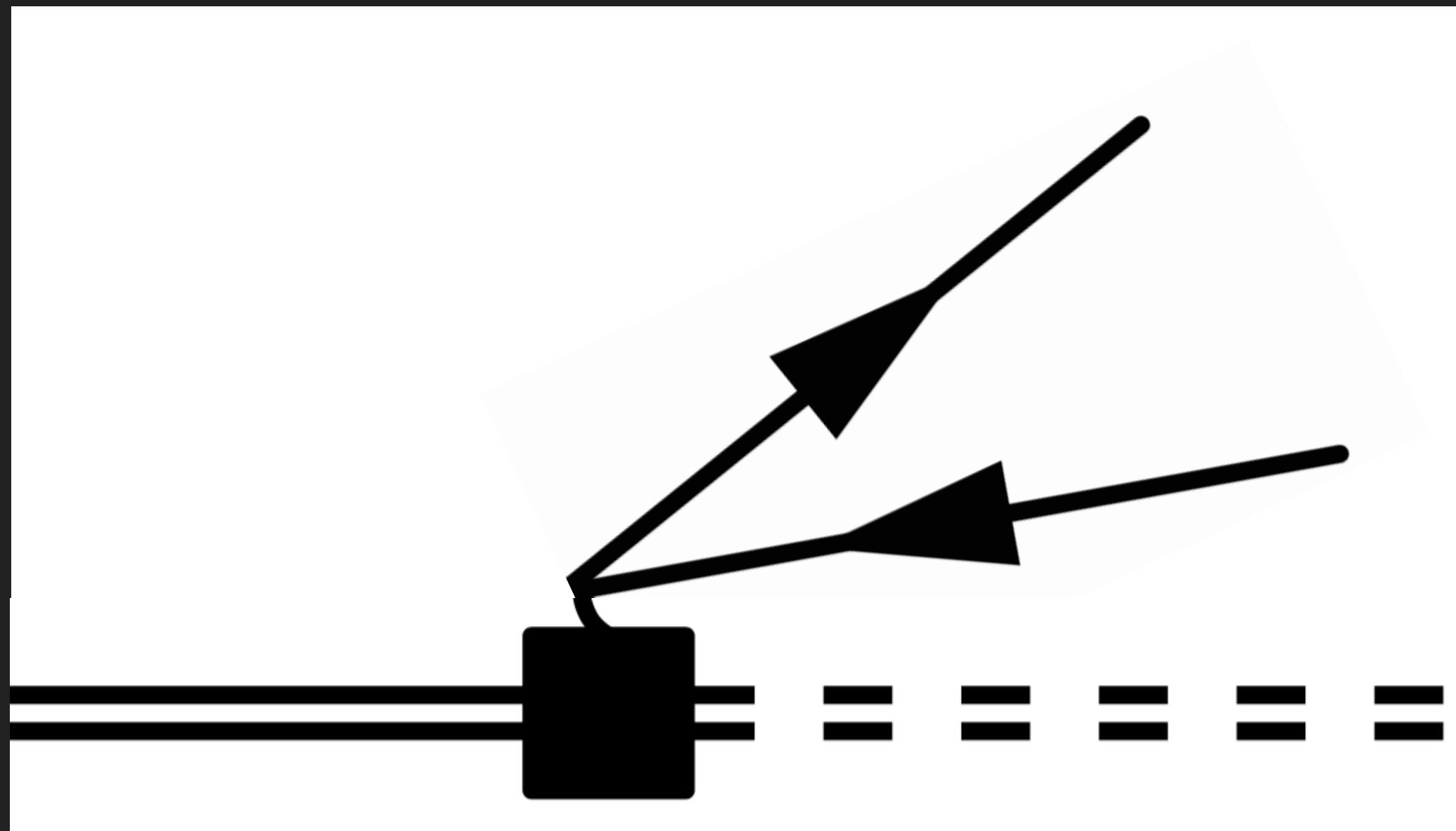
ϵ_2 $|\mathbf{p}_2|$

Lepton Energy

Nuclear
Coherence



CONCRETE EXAMPLE: LEPTON PAIRS



Nuclear Mass

30 MeV

Nuclear
Coherence

10 MeV

Lepton Energy

ϵ_2 $|\mathbf{p}_2|$

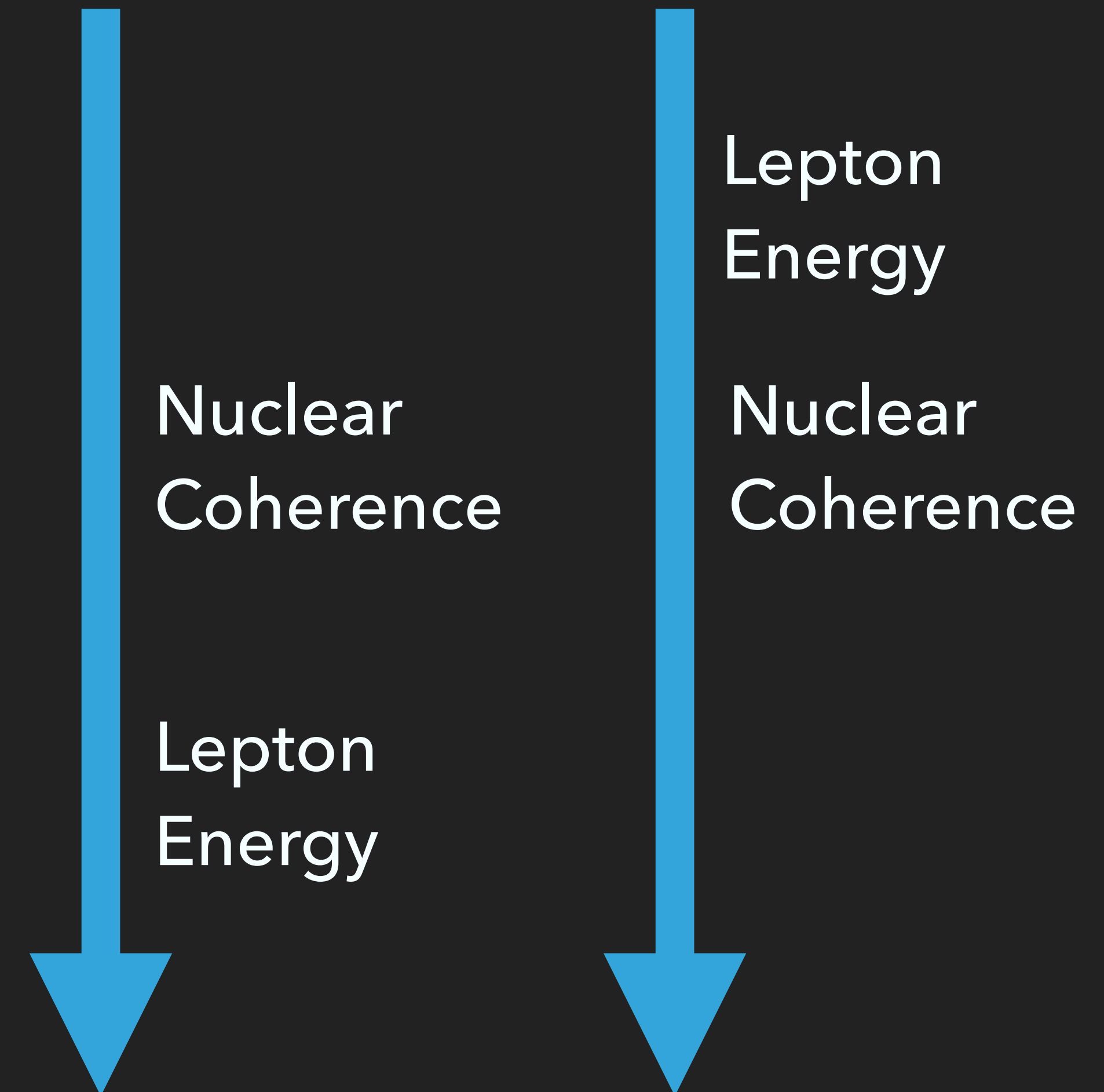
DIFFERENT HIERARCHIES OF SCALE MATTER

$$\mathcal{M} = S \otimes H$$

- ▶ To the lepton, what is hard, and what is soft is set by it's own momentum.
- ▶ When $|\mathbf{p}| \ll 1/R$ the Coulomb field can give "big kicks" to the lepton.
- ▶ When $|\mathbf{p}| \gg 1/R$ the Coulomb field can only give "little kicks" to the lepton.

Point like

Eikonal



DIFFERENT HIERARCHIES OF SCALE MATTER

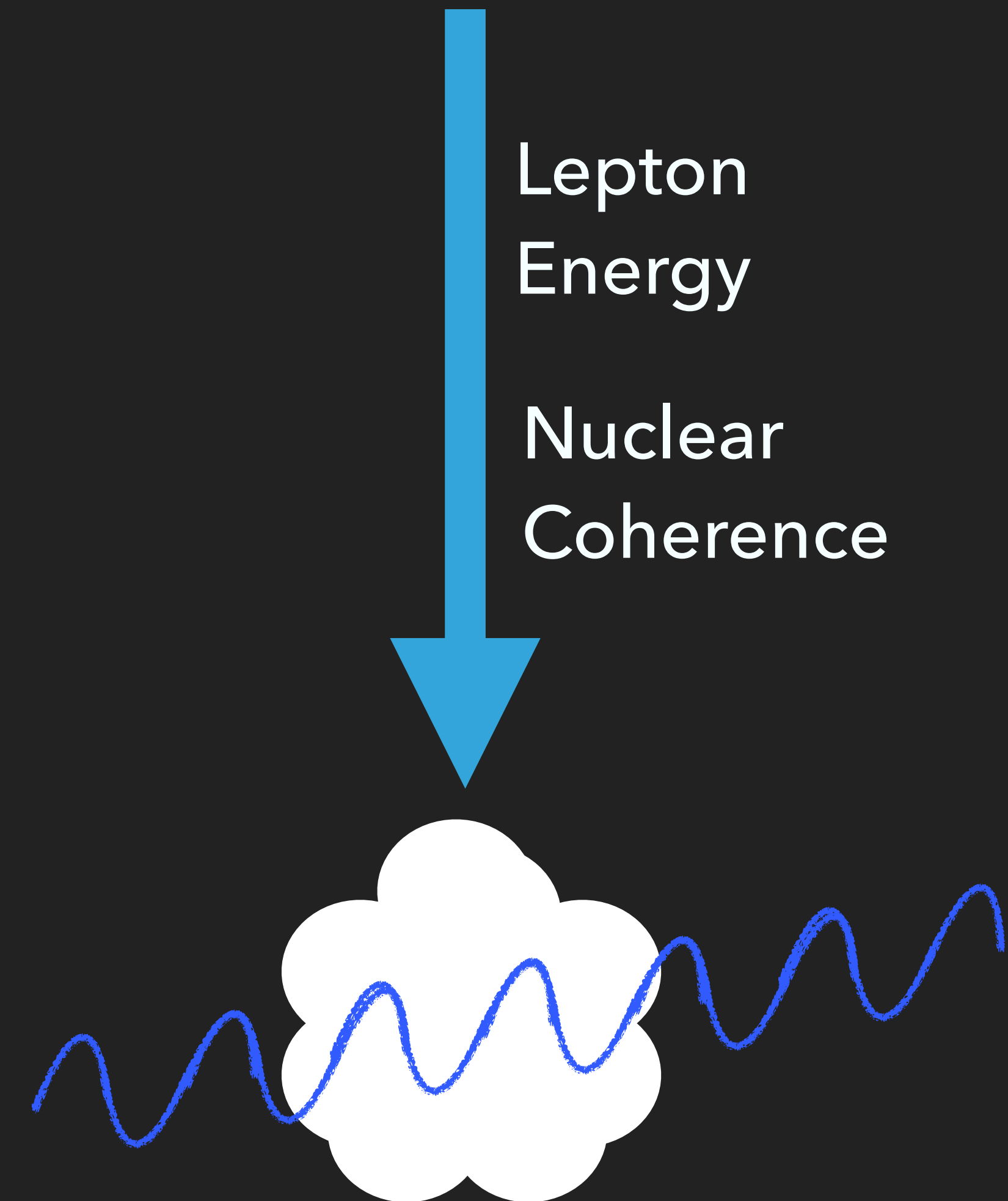
$$\mathcal{M} = \underline{S} \otimes H$$

- ▶ In the high energy limit, Coulomb corrections are embedded in the soft-function.

levant for GeV energy neutrinos.

- ▶ Ongoing work with R. Hill and O. Tomalak.

Eikonal



DIFFERENT HIERARCHIES OF SCALE MATTER

$$\mathcal{M} = S \otimes \underline{H}$$

- ▶ In the “point-like” limit, virtual momentum transfers can be comparable to the lepton’s momentum.
- ▶ Coulomb corrections actually reside inside the ***hard function***.

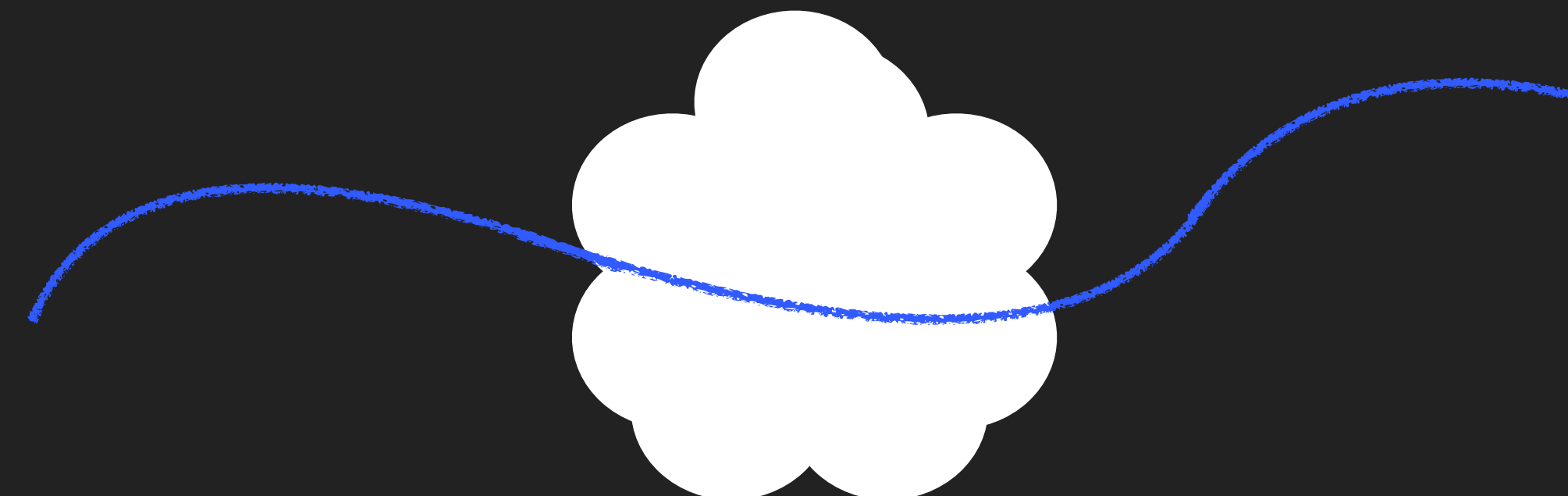


This was a surprise

Point like

Nuclear
Coherence

Lepton
Energy



ALL ORDERS COULOMB CORRECTIONS IN THE POINT-LIKE LIMIT

Wavefunctions, and factorization, for amplitudes at all-orders in $Z\alpha$

POINT-LIKE FACTORIZATION THEOREM

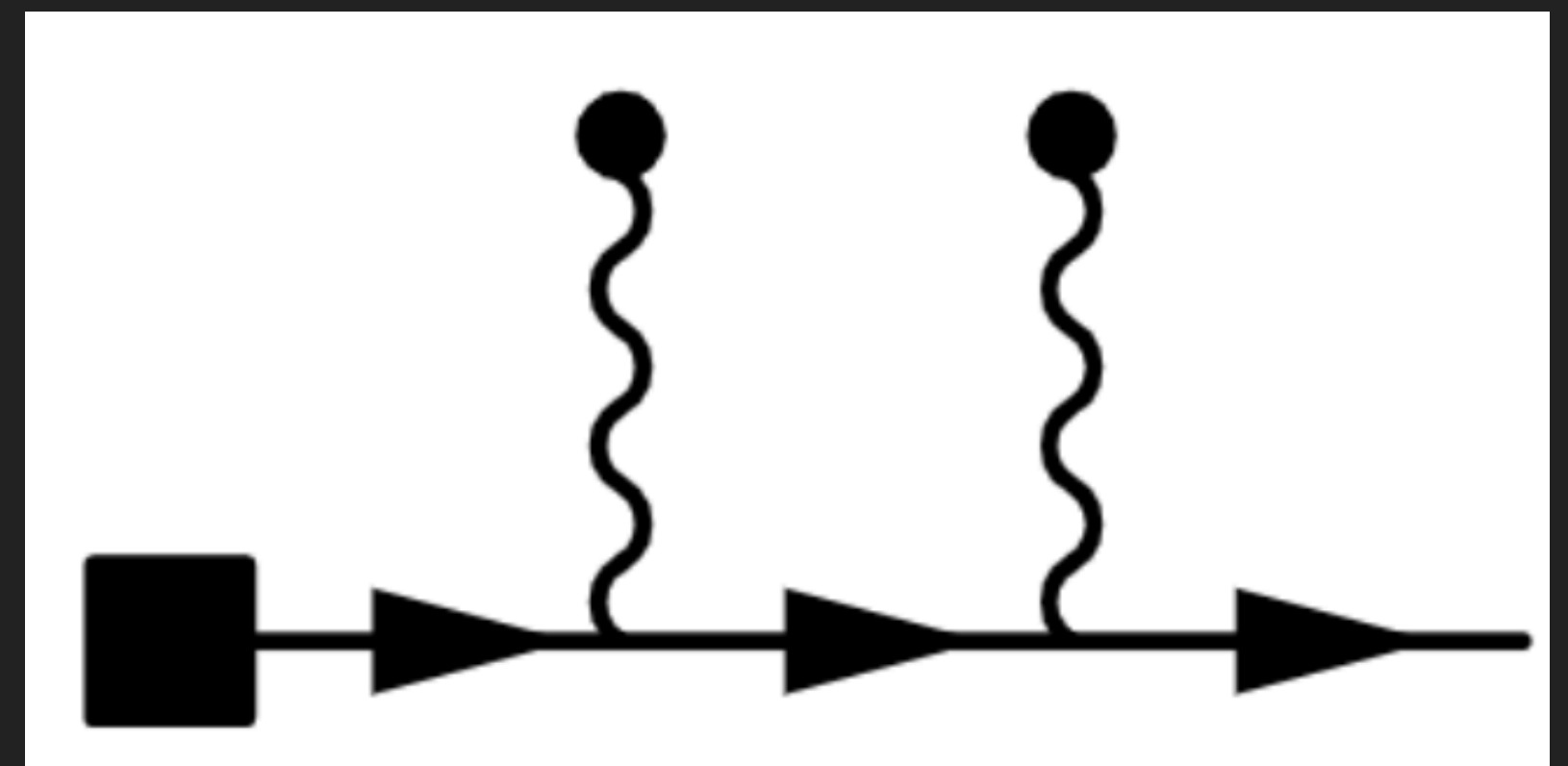
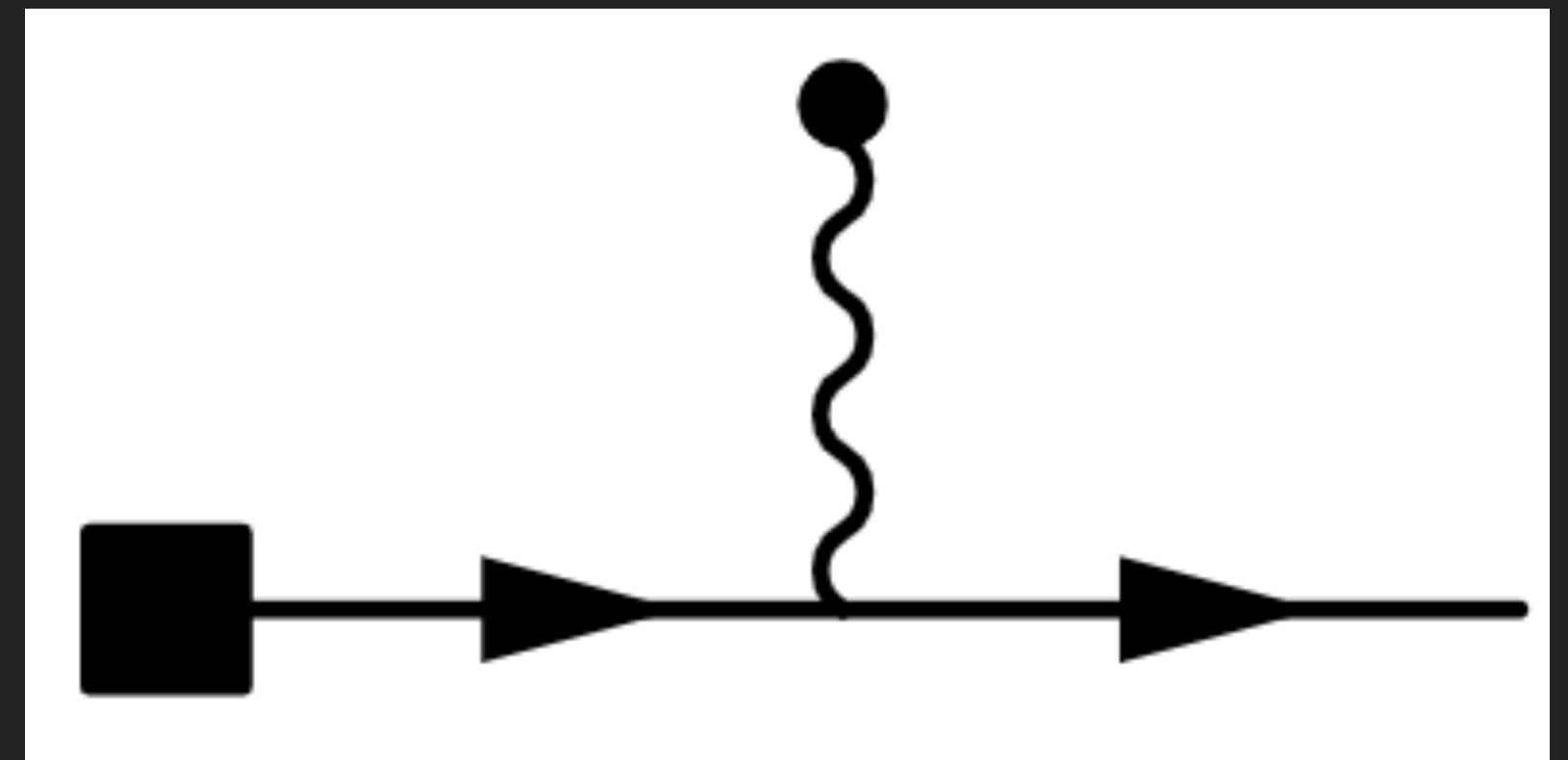
Structure dependent
corrections

$$\mathcal{M} = \mathcal{M}_S \otimes \mathcal{M}_H \otimes \mathcal{M}_{UV}$$

- ▶ Consider a charged particle produced in by a contact interaction.
- ▶ Coulomb corrections reside inside the hard function.

↑ We want to compute this

$$\mathcal{M}_0 = \bar{u}(p) \times \blacksquare$$



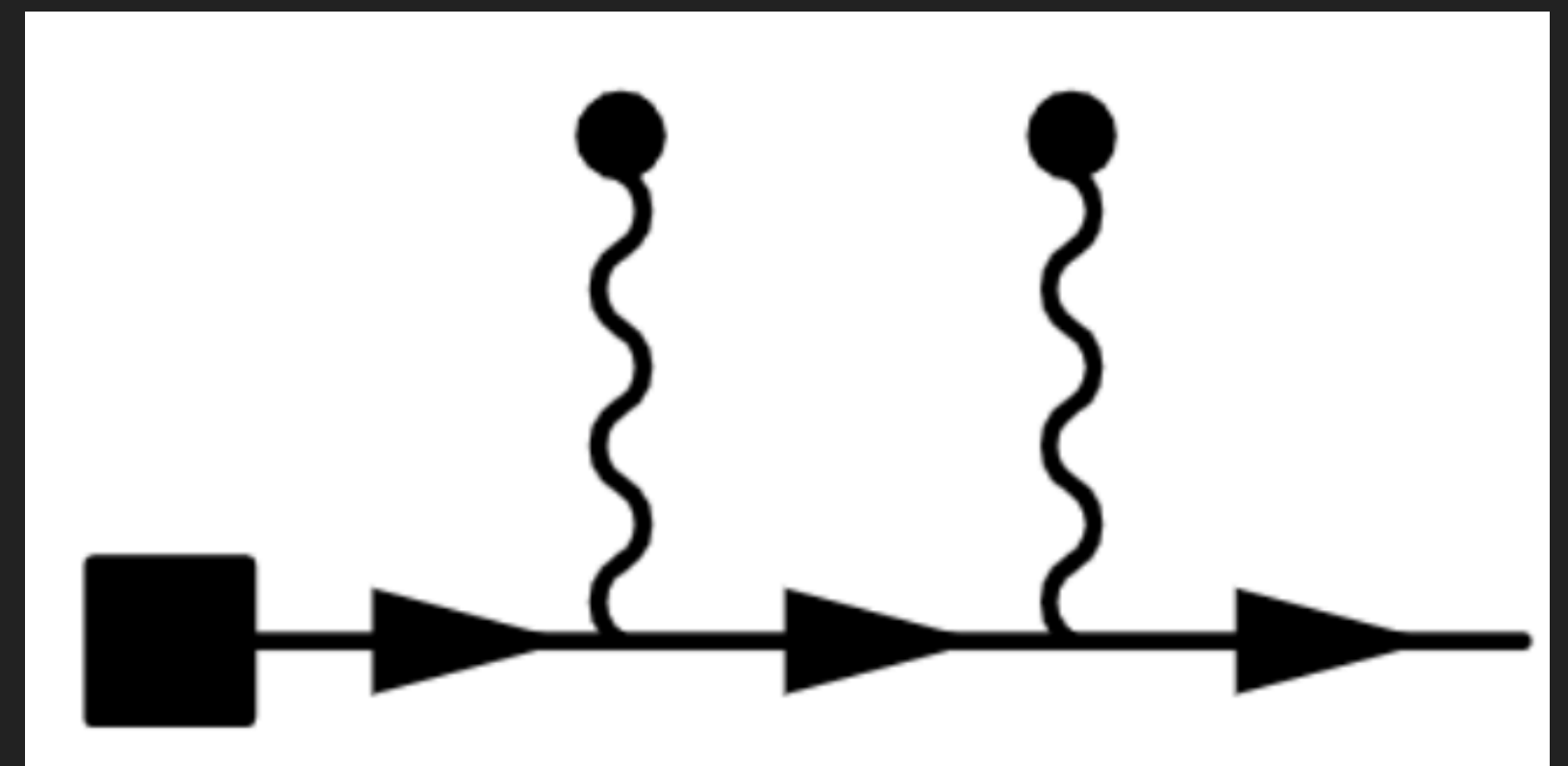
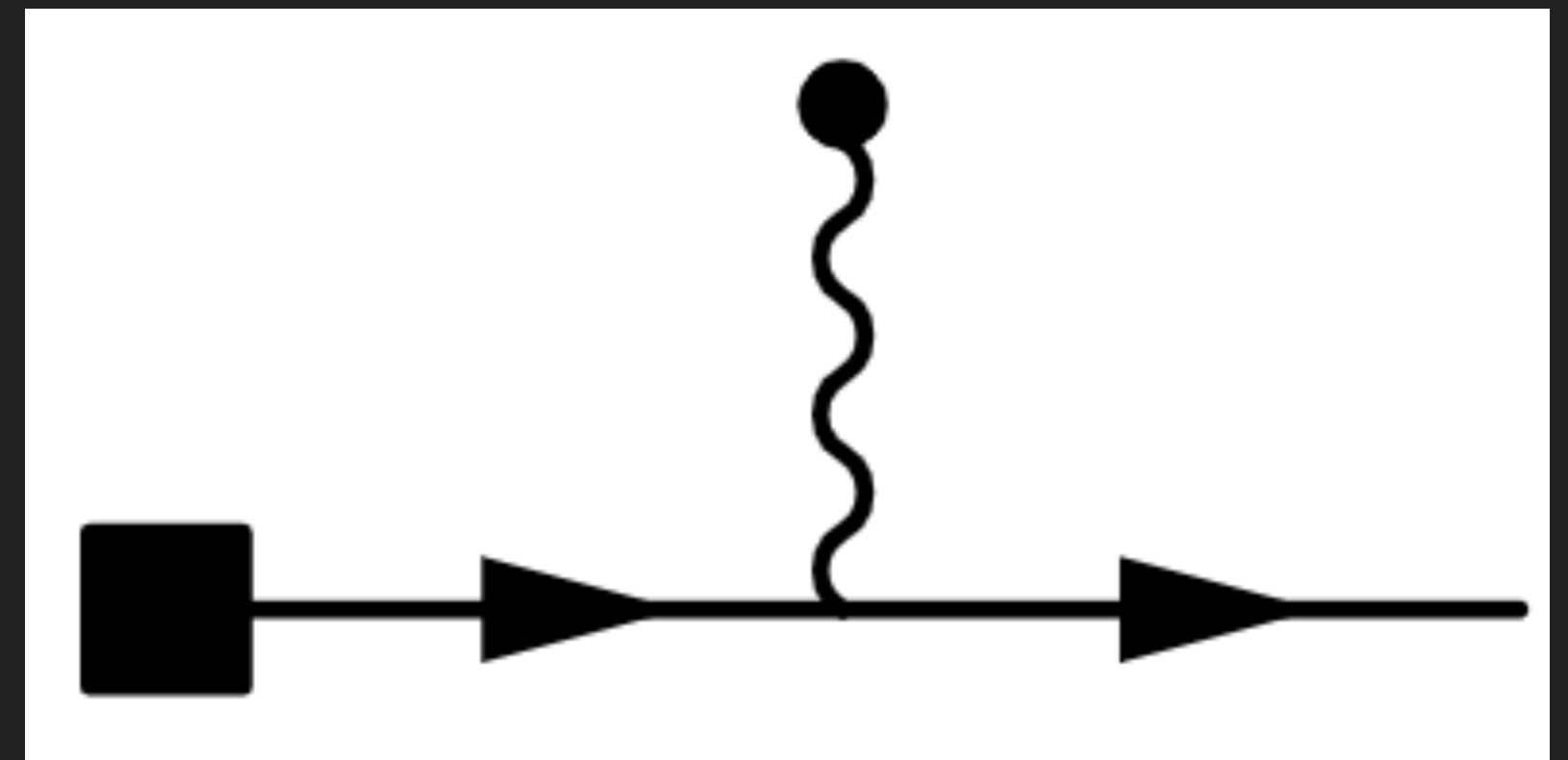
START SIMPLE: NON-RELATIVISTIC CASE

$$\mathcal{M} = \mathcal{M}_S \otimes \mathcal{M}_H$$

- ▶ This should reproduce classic Sommerfeld factor

$$\begin{aligned} S(\xi) &= \left| \psi(0) \right|^2 \\ &= \left| \Gamma(1 + i\xi) e^{-\pi\xi/2} \right|^2 \end{aligned}$$

$$\mathcal{M}_0 = \bar{u}(p) \times \blacksquare$$



START SIMPLE: NON-RELATIVISTIC CASE

$$\mathcal{M} = \mathcal{M}_S \otimes \mathcal{M}_H$$

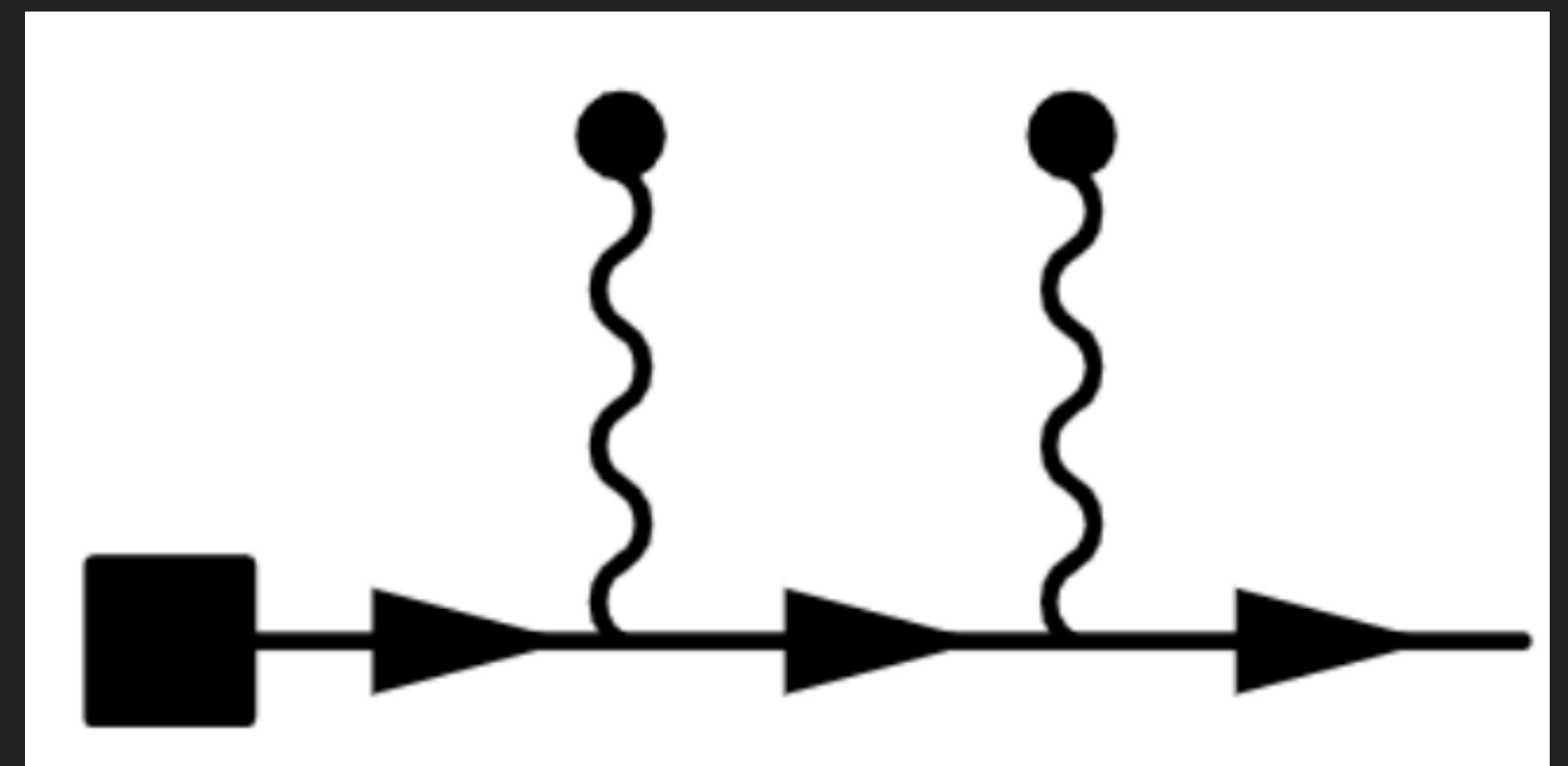
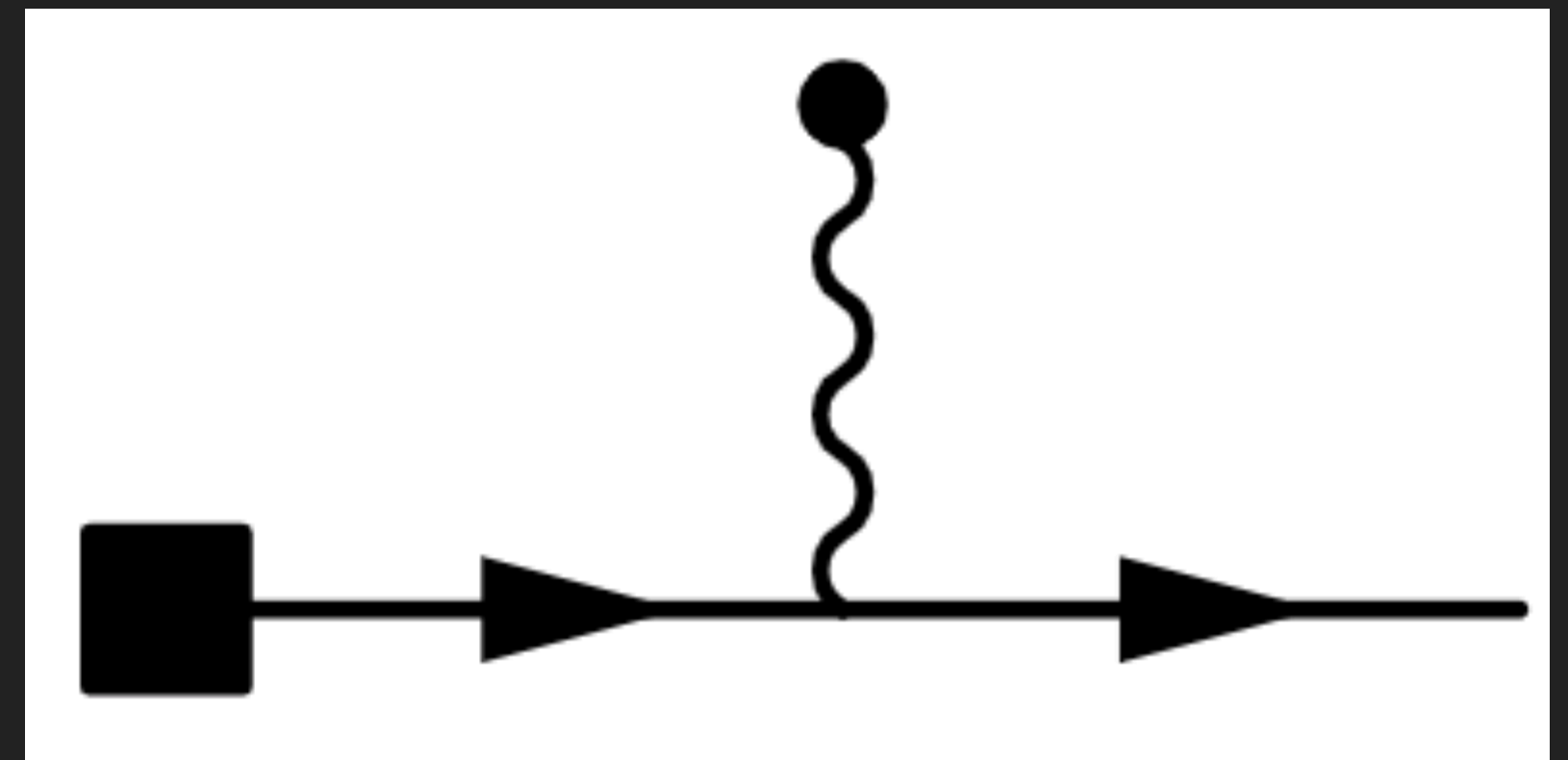
$$\psi_{\lambda,p}(0) = \mathcal{M}_S(\lambda) \mathcal{M}_H(p)$$

Solve Schrodinger

Known to all orders

$$\mathcal{M}_H(p) = \mathcal{M}_S^{-1} \psi_{\lambda,p}^\dagger(0)$$

$$\mathcal{M}_0 = \bar{u}(p) \times \blacksquare$$



START SIMPLE: NON-RELATIVISTIC CASE

- ▶ Actual result holds at amplitude level!
- ▶ Need a careful treatment of IR divergences to obtain the right answer.

All orders proof of re-summation
into an exponential



$$\mathcal{M}_S = e^{i\xi \log(\mu/\lambda)}$$

$$\mathcal{M}_H = e^{i\xi [\log(2p/\mu) + \gamma_E]} \Gamma(1 - i\xi) e^{\xi\pi/2}$$

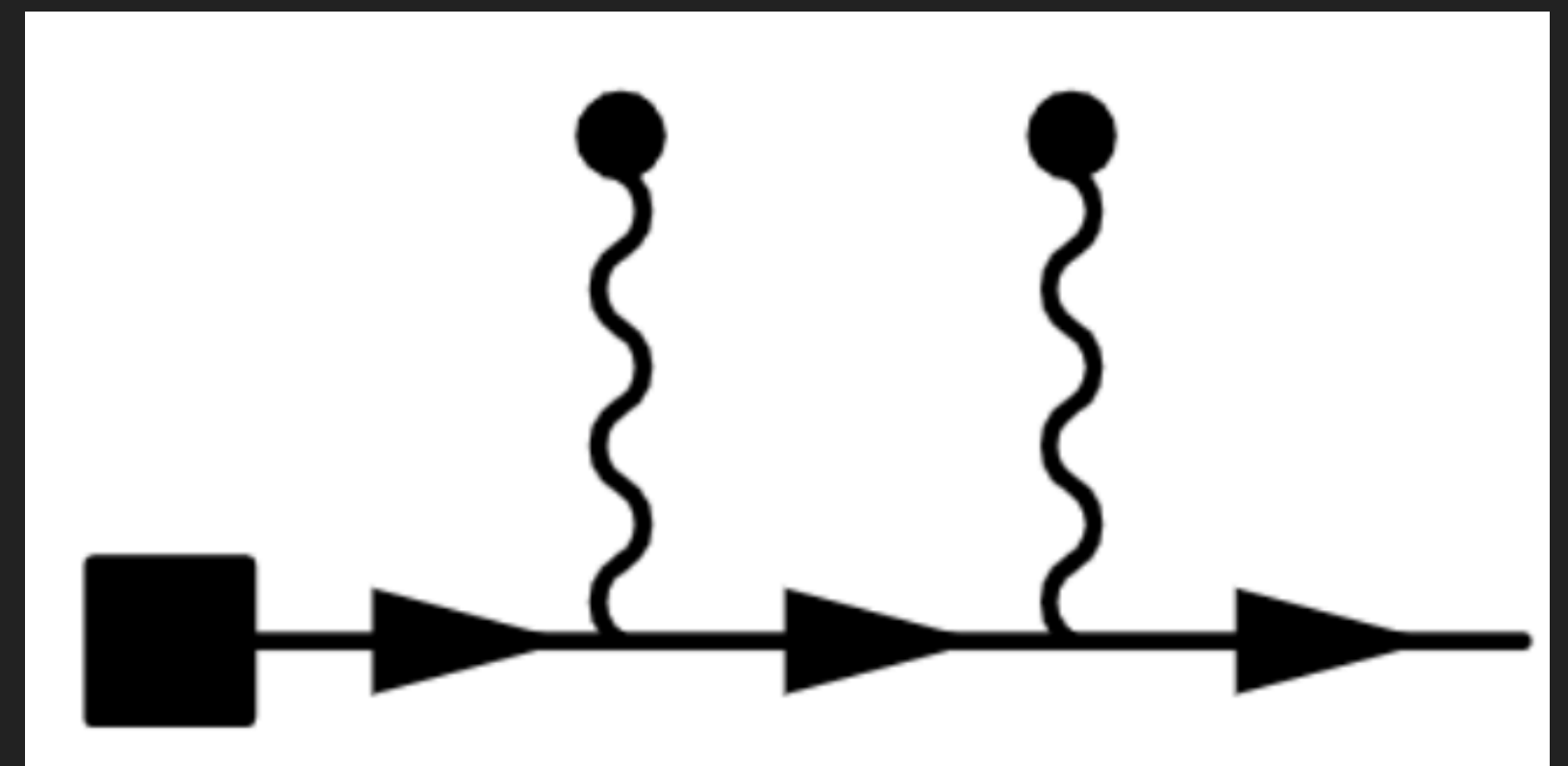
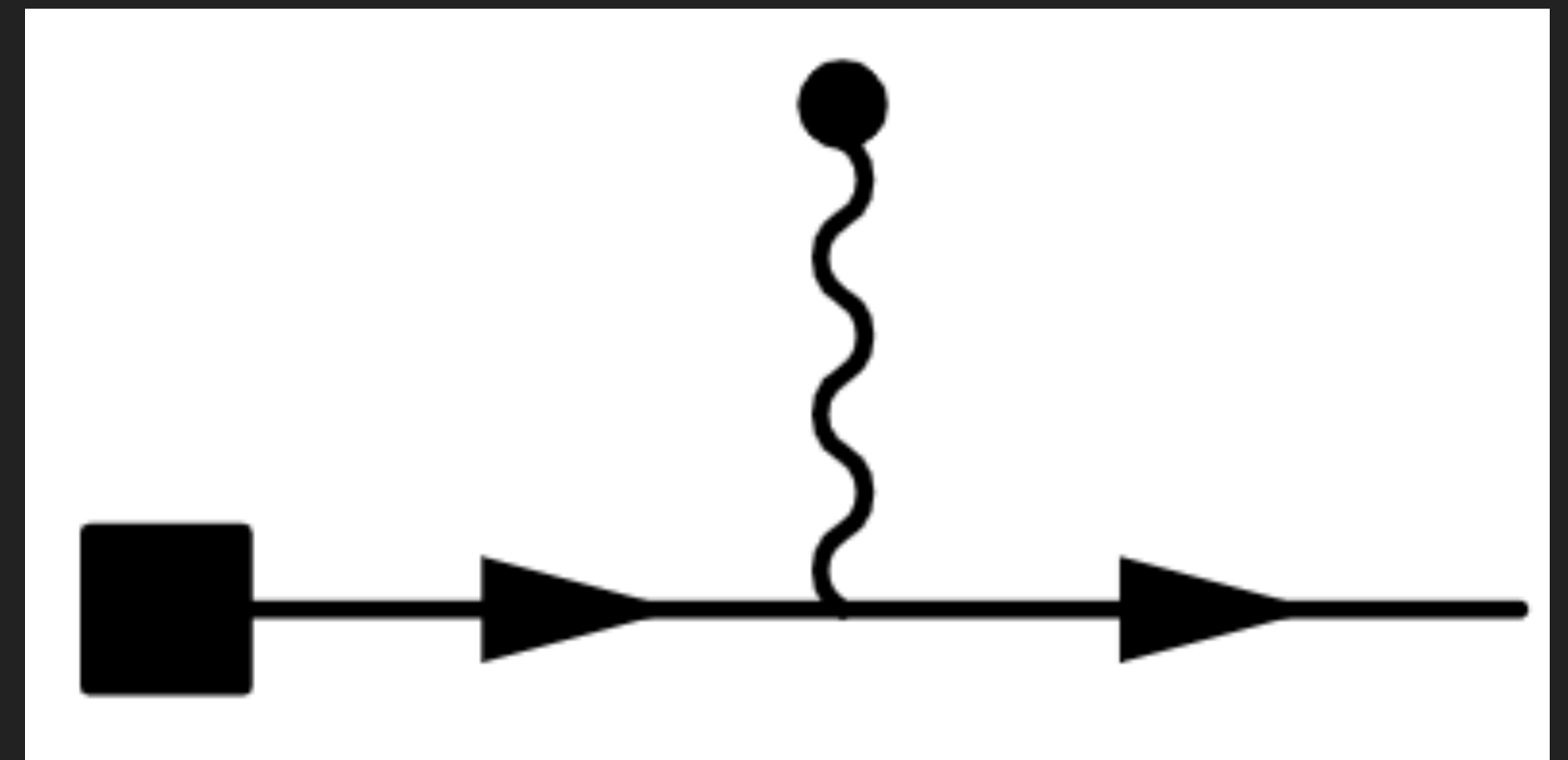
FACTORIZATION FOR RELATIVISTIC FERMIONS

$$\mathcal{M} = \mathcal{M}_S \otimes \mathcal{M}_H \otimes \mathcal{M}_{UV}$$

- ▶ Relativistic problem has new UV divergences.
- ▶ How do we extract the hard function?

Compute \mathcal{M}_{UV} to all orders

$$\mathcal{M}_0 = \bar{u}(p) \times \blacksquare$$

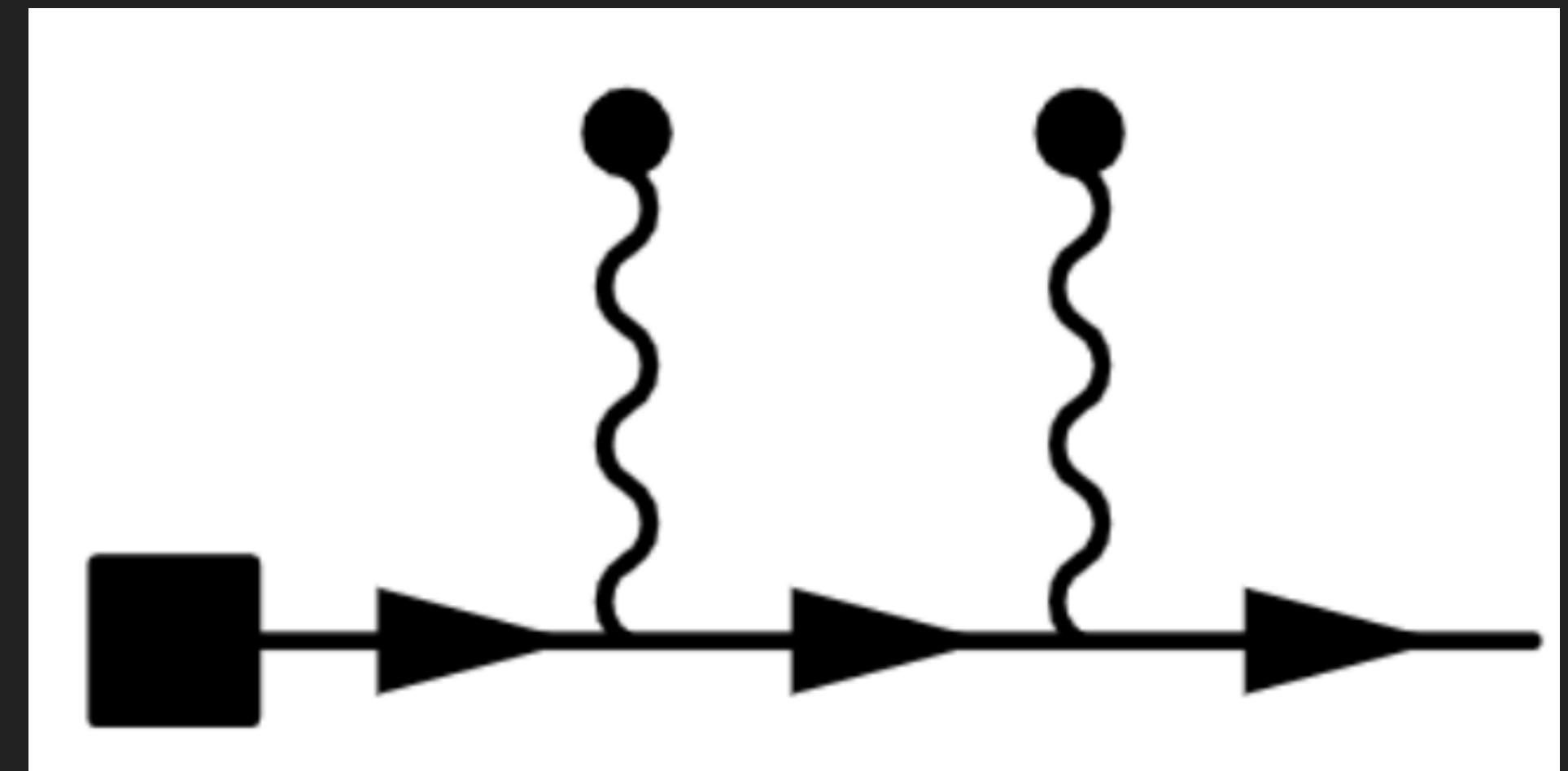


FACTORIZATION FOR RELATIVISTIC FERMIONS

$$\mathcal{M}_H(p) = \overline{\Psi}_{\lambda,p}(x) \mathcal{M}_{UV}^{-1}(x) \mathcal{M}_S^{-1}(\lambda)$$

Compute \mathcal{M}_{UV} to all orders

Obtain \mathcal{M}_H to all orders



SPECIAL PROPERTIES OF UV REGION

- ▶ UV region has novel loop properties.
- ▶ Can be computed at all orders in perturbation theory.
- ▶ The series can be summed, and converges.
- ▶ Result can be normalized in $\overline{\text{MS}}$ at all orders/non-perturbatively in $Z\alpha$

RESULT

- ▶ Fundamental result in QED at all orders.
- ▶ Universally applicable for low momentum scattering with nuclei.

CONCLUSION AND OUTLOOK

Where are we now and where are we going?

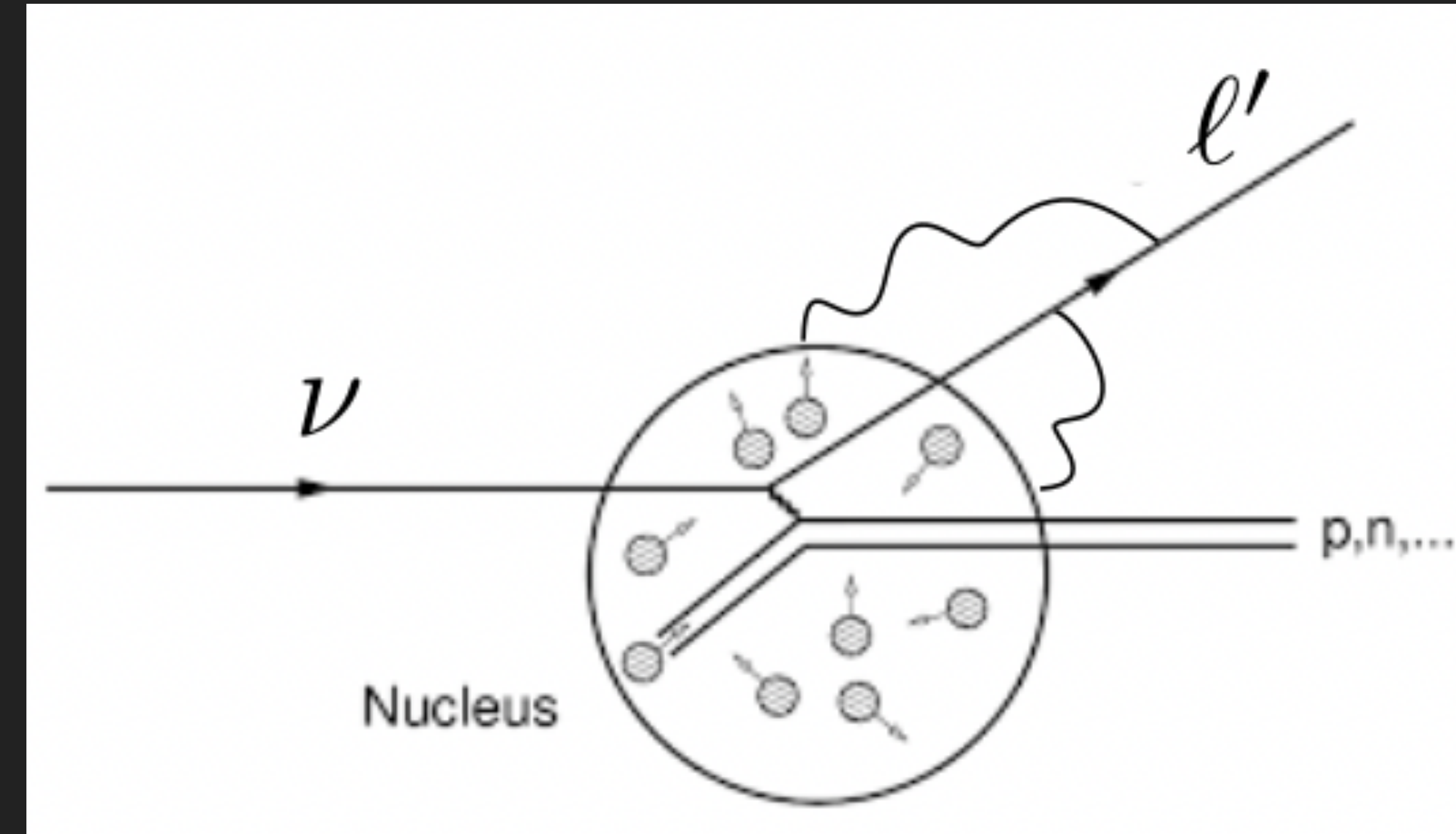
WHERE ARE WE NOW?

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- ▶ Nucleus introduces new scales.

- 1) Ultra heavy mass scale.

- 2) Low scale of nuclear coherence.



- ▶ Regions and diagrams that generate Coulomb potential understood.
- ▶ Structure of factorization theorem incorporating nuclear scale is clear.
- ▶ Factorization demonstrated to all orders in $Z\alpha$ in point-like limit.

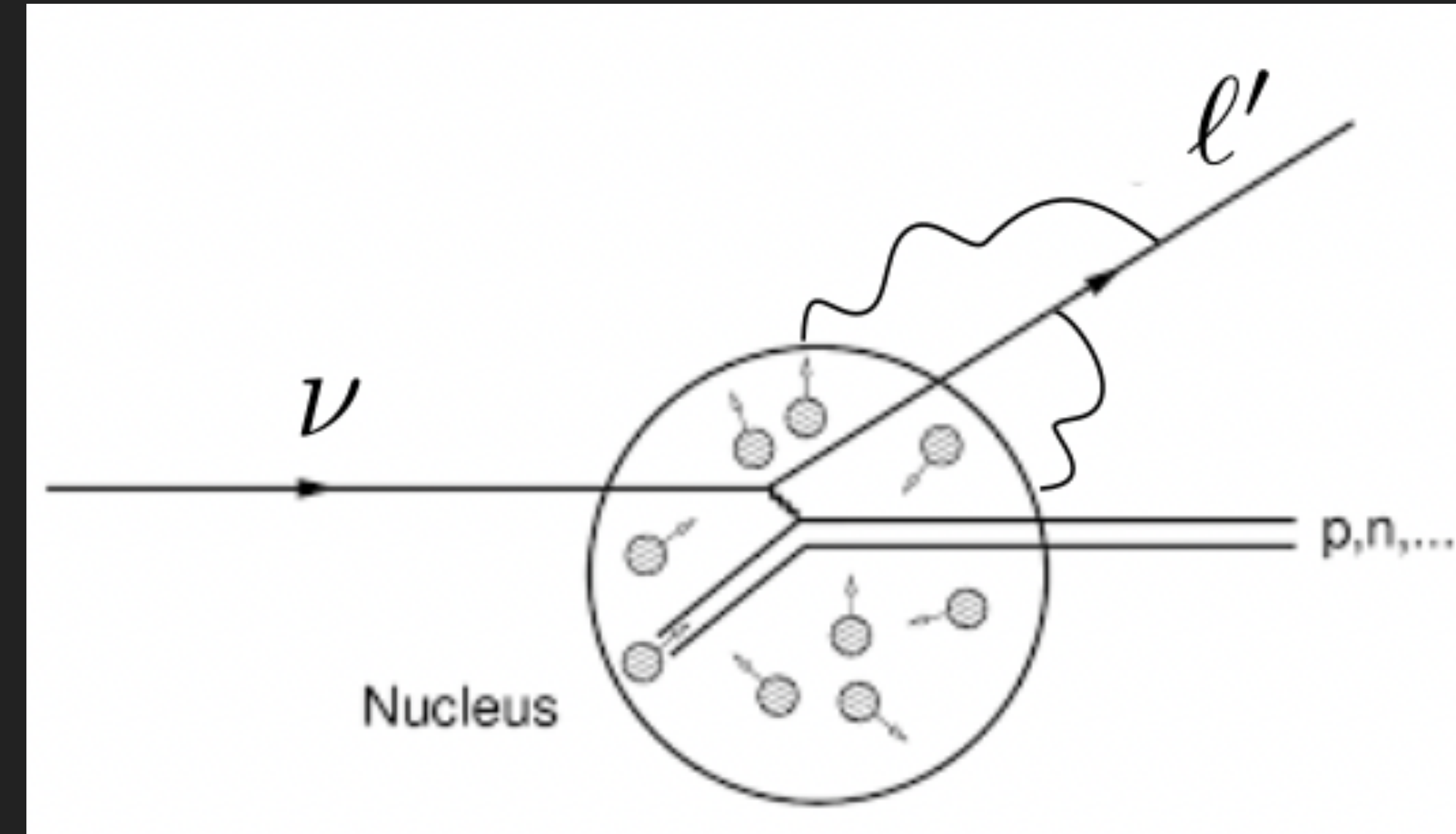
WHERE ARE WE GOING?

37

- ▶ Nucleus introduces new scales.

- 1) Ultra heavy mass scale.

- 2) Low scale of nuclear coherence.



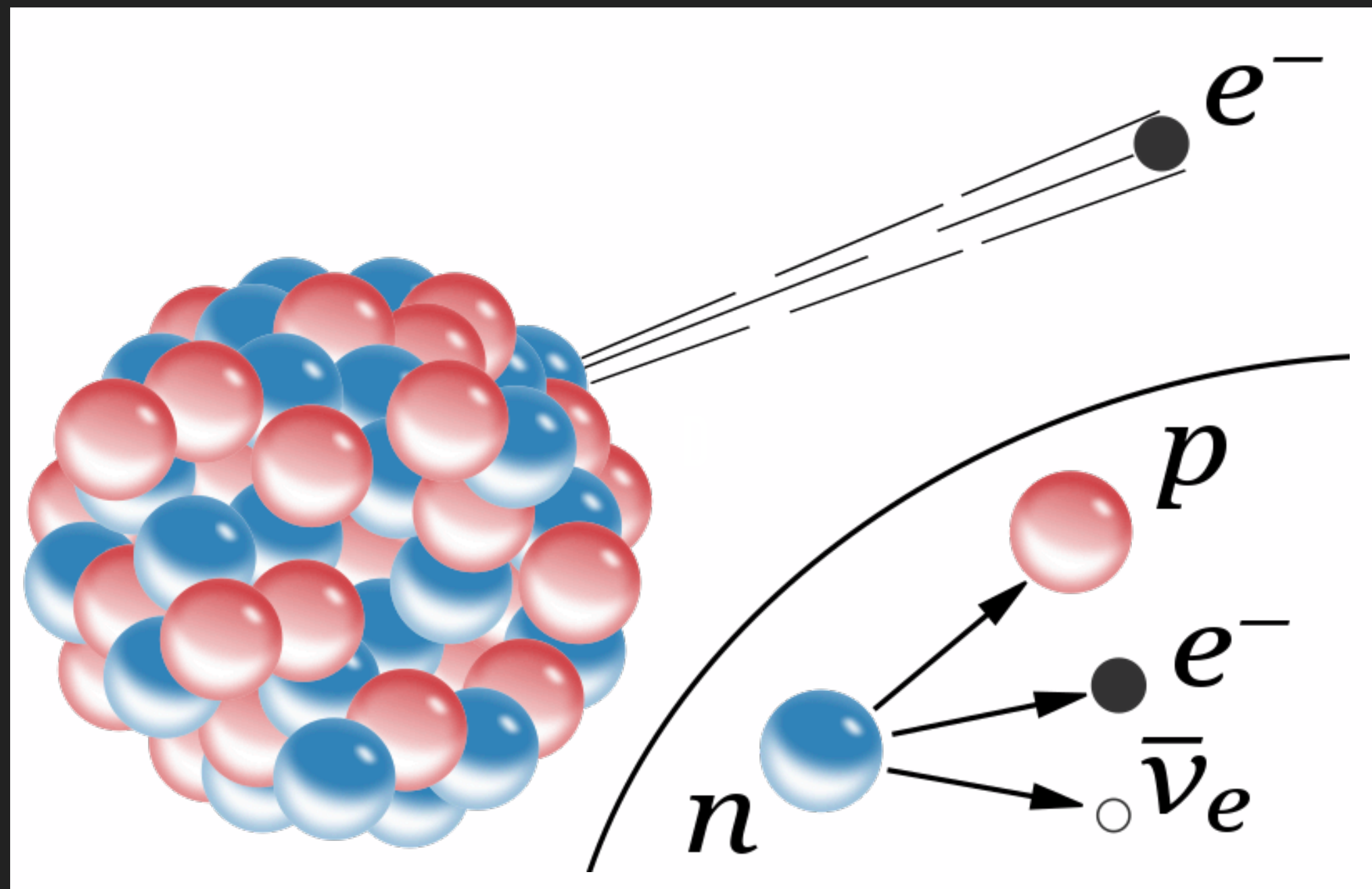
- ▶ How do distorted wave effects interface with nuclear structure? Are they independent from nucleon FSI?

- ▶ What is the best way to interface with existing tools?

BROADER IMPACT

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- ▶ Many of the issues I describe here are essential for the neutrino cross section problem (this is **our community's problem**).
- ▶ The effects are universal to any high-precision experiment with nuclear targets. e.g. muon conversion, beta decay of heavy nuclei.



- ▶ Coulomb effects due to large- Z nuclei are inherently complicated by nuclear structure.

$$A \sim 2Z \quad r_A \sim A^{1/3} r_0$$

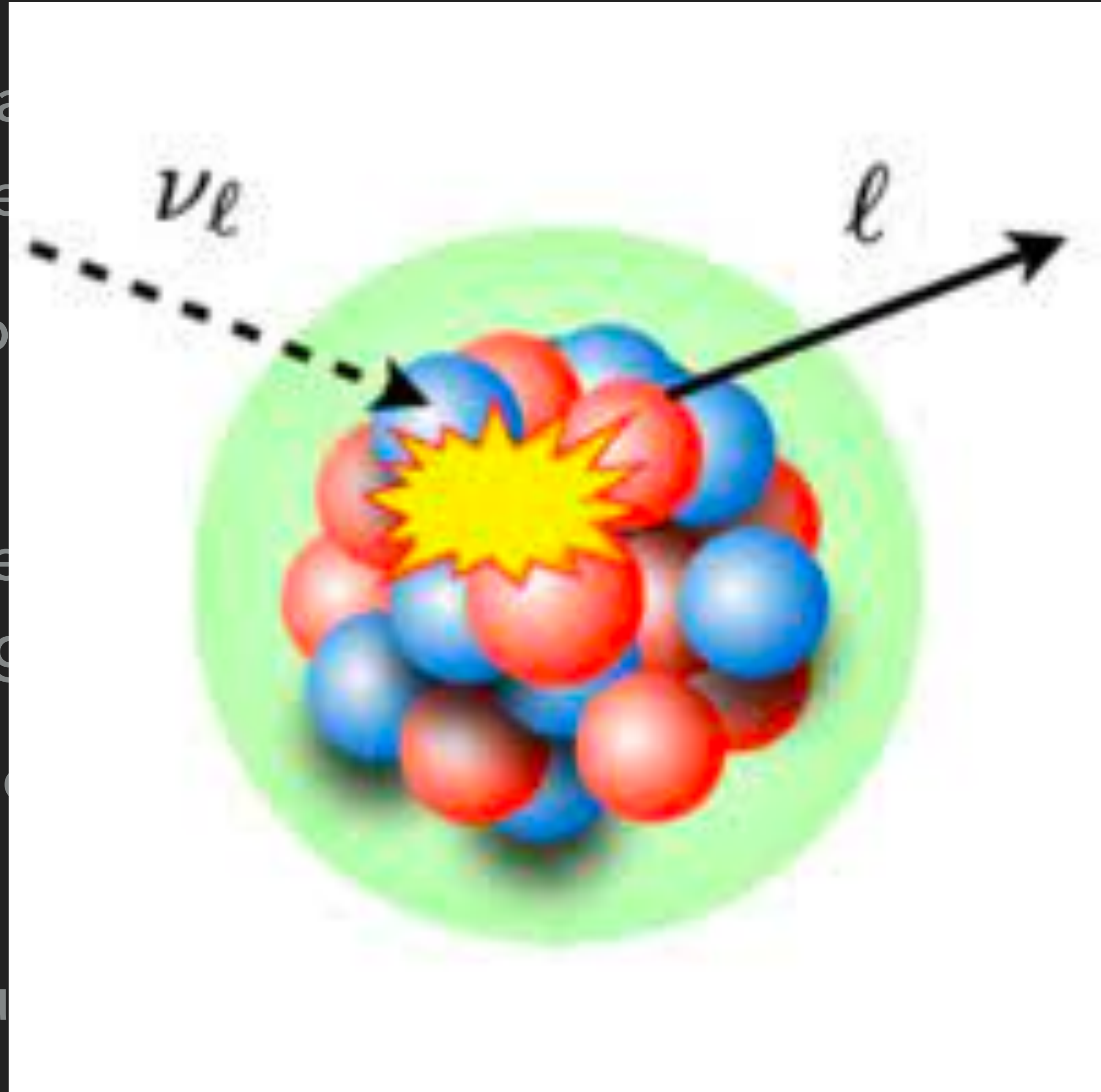
1. QED corrections are a mandatory theory input for %-level precision on cross sections (large Z , large logs).
2. Background field is an approximation, valid in the static limit up to corrections of $O(\alpha)$.
3. Nuclear radius sets new scale in factorization theorems. Qualitatively different structure depending on hierarchy of scales.
4. Factorization has been tested at all orders in perturbation theory in the point-like limit.
5. New formal structures and fundamental amplitudes computed in closed form for the first time.

CONCLUSIONS

Isolate hard nuclear physics from QED loops and legs

40

1. QED corrections are a mandatory part of the calculation of cross sections (large Z , large Q^2).
2. Background field is an approximation to the full QED corrections of $O(\alpha)$.
3. Nuclear radius sets new scale for the calculation of different structure depending on the process.
4. Factorization has been tested in the point-like limit.
5. New formal structures and full closed form for the first time.



ACKNOWLEDGEMENTS

41

- I would like to thank O. Tomalak for collaboration on related work.
- I would like to thank the Neutrino Theory Network for supporting my visit to UKY and FNAL in 2018 where these ideas were seeded.
- I would like to thank Susan Gardner for stimulating discussion on beta decay.
- I would like to thank the Fermilab theory group for their generous and welcoming hospitality over the past 3 years.
- I acknowledge support from the Intensity Frontier Fellow program at Fermilab (2020-2021).

NTN WORKSHOP | JUNE 2022

BACKUP SLIDES

LOOP EXPANSIONS FOR WAVEFUNCTIONS

CONNECTION BETWEEN DISTORTED WAVES AND LOOPS

- ▶ We have shown that perturbative expansions of HPET loops match onto perturbative expansion of external field.
- ▶ Within external field calculation how can we see correspondence with distorted wave methods?

CONNECTION BETWEEN DISTORTED WAVES AND LOOPS | NR-QM

- Consider Lipmann-Schwinger Equation for in/out states

$$|\psi_p^{(\pm)}\rangle = |\phi_p\rangle + \frac{1}{H - E_p \pm i\epsilon} V |\psi_p^{(\pm)}\rangle$$

$$|\psi_p^{(\pm)}\rangle = |\phi_p\rangle + \frac{1}{H - E_p \pm i\epsilon} V |\phi_p\rangle + \frac{1}{H - E_p \pm i\epsilon} V \frac{1}{H - E_p \pm i\epsilon} V |\phi_p\rangle + \dots$$

$$\text{Insert } 1 = \int \frac{d^3Q}{(2\pi)^3} |Q\rangle\langle Q|$$

CONNECTION BETWEEN DISTORTED WAVES AND LOOPS | NR-QM

$$|\psi_p^{(\pm)}\rangle = |\phi_p\rangle + \frac{1}{H - E_p \pm i\varepsilon} V |\phi_p\rangle + \dots$$

$$|\psi_p^{(\pm)}\rangle = |\mathbf{p}\rangle + \int \frac{d^3Q}{(2\pi)^3} \frac{1}{2\mathbf{P} \cdot \mathbf{Q} + Q^2 \pm i\varepsilon} \frac{Z\alpha}{Q^2} |\mathbf{p} + \mathbf{Q}\rangle$$

$$\langle x | \psi_p^{(\pm)} \rangle = e^{i\mathbf{p} \cdot \mathbf{x}} \left(1 + \int \frac{d^3Q}{(2\pi)^3} \frac{1}{2\mathbf{P} \cdot \mathbf{Q} + Q^2 \pm i\varepsilon} \frac{Z\alpha}{Q^2} e^{i\mathbf{Q} \cdot \mathbf{x}} + \dots \right)$$

THIS ALLOWS US TO RESUM DIAGRAMMATICS

$$\langle x | \psi_p^{(\pm)} \rangle = e^{i\mathbf{p} \cdot \mathbf{x}} \left(1 + \int \frac{d^3 Q}{(2\pi)^3} \frac{1}{2\mathbf{P} \cdot \mathbf{Q} + \mathbf{Q}^2 \pm i\epsilon} \frac{Z\alpha}{\mathbf{Q}^2} e^{i\mathbf{Q} \cdot \mathbf{x}} + \dots \right)$$

- ▶ Can use method of regions to separate out physics at different scales.
- ▶ E.g. introduce IR regulator $\mathbf{Q}^2 \rightarrow \mathbf{Q}^2 + \sigma^2$.
Two regions: hard ($Q \sim P$) and soft ($Q \sim \sigma$).
- ▶ Coulomb phase comes from soft region, RG exponentiation.
- ▶ Sommerfeld comes from IR-divergent piece in hard region.

METHOD OF REGIONS

CONSIDER A LOOP INTEGRAL WITH TWO SCALES

$$\int dQ \frac{1}{Q^2 + \lambda^2} \frac{1}{2p \cdot Q + Q^2}$$
$$= \int_{\text{soft}} dQ \frac{1}{Q^2 + \lambda^2} \frac{1}{2p \cdot Q} + \int_{\text{hard}} dQ \frac{1}{Q^2} \frac{1}{2p \cdot Q + Q^2} + \mathcal{O}\left(\frac{\lambda}{p}\right)$$

Taylor expansion.



FACTORIZATION FROM METHOD OF REGIONS

$$\mathcal{M} = \sum (Z\alpha)^n \mathcal{M}^{(n)}$$

$$\mathcal{M} = \mathcal{M}_S \mathcal{M}_H + O(\lambda/p)$$

$$\mathcal{M}_S = \sum (Z\alpha)^n \mathcal{M}_S^{(n)} \quad \mathcal{M}_H = \sum (Z\alpha)^n \mathcal{M}_H^{(n)}$$



Only one region in each sum.