Astrophysical neutrinos and their entanglement

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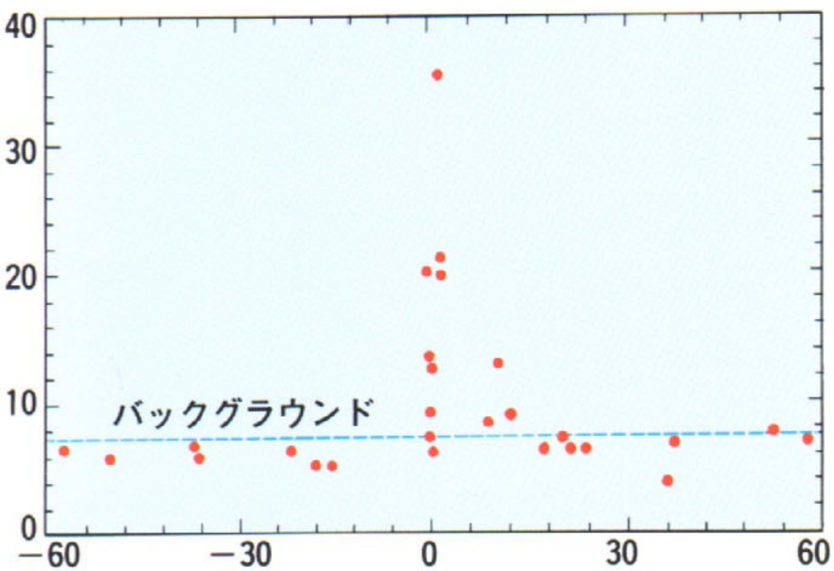
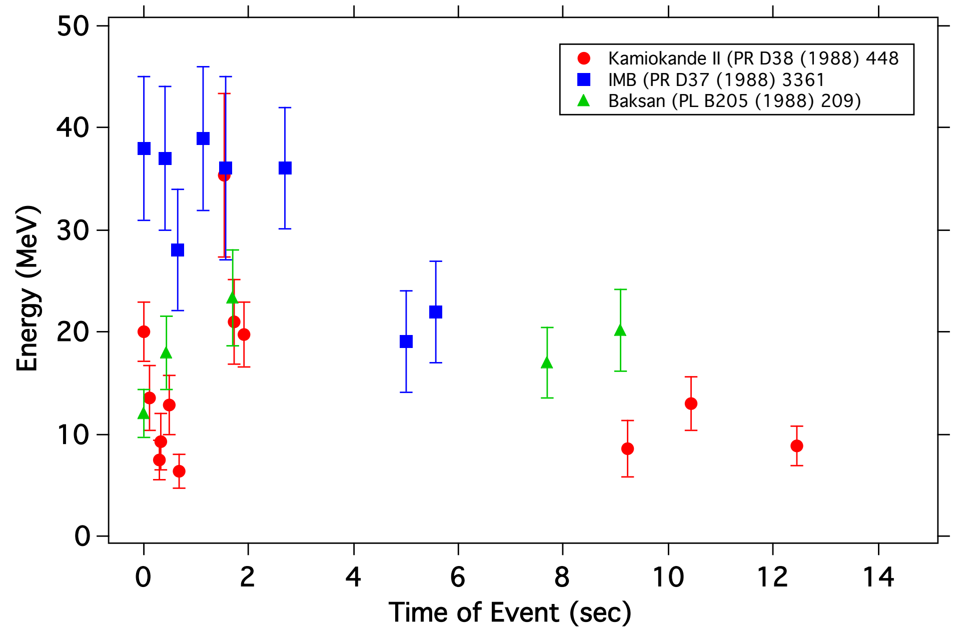
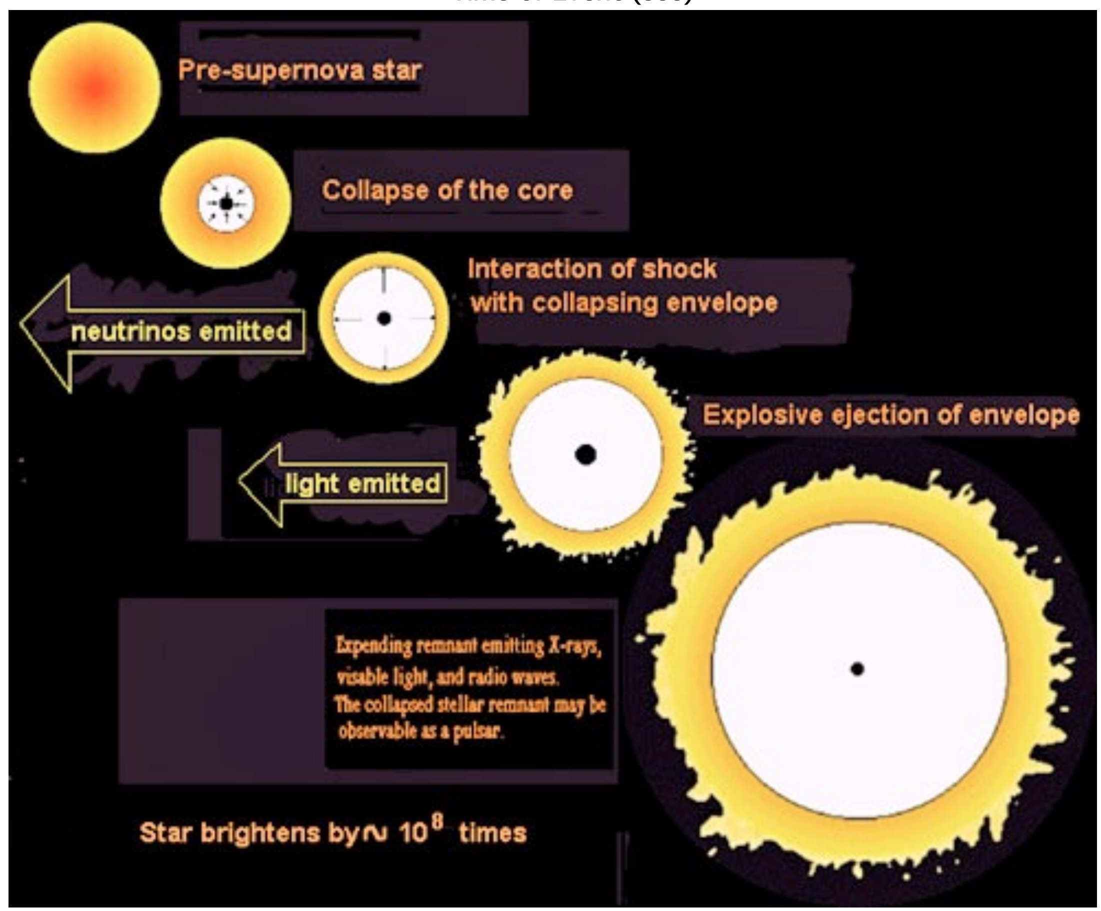
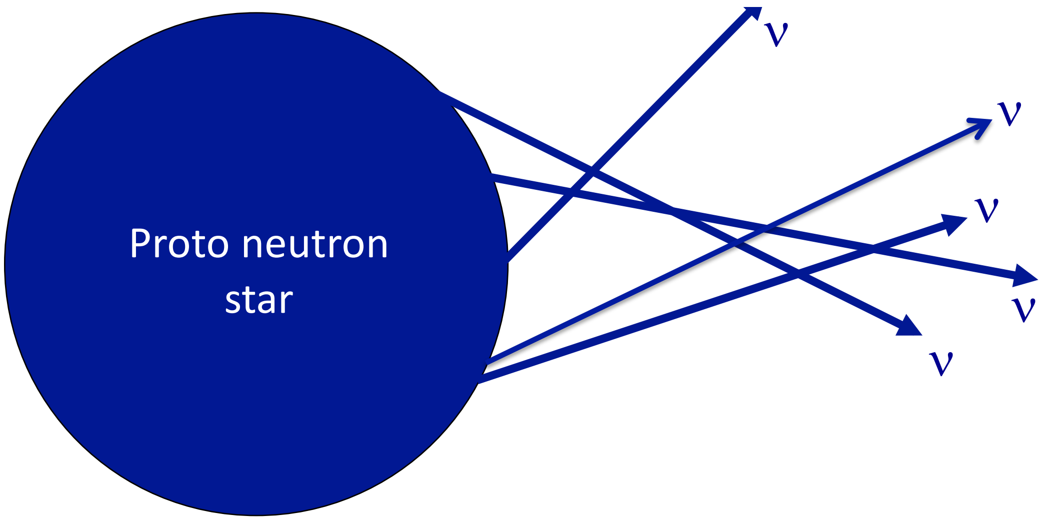
Neutrinos from core-collapse supernovae 1987A







- $M_{prog} \gtrsim 8\ M_{sun} \Rightarrow \Delta E \approx 10^{53}$ ergs $\approx 10^{59}$ MeV
- 99% of the energy is carried away by neutrinos and antineutrinos with $10 \lesssim E_\nu \lesssim 30$ MeV $\Rightarrow 10^{58}$ neutrinos

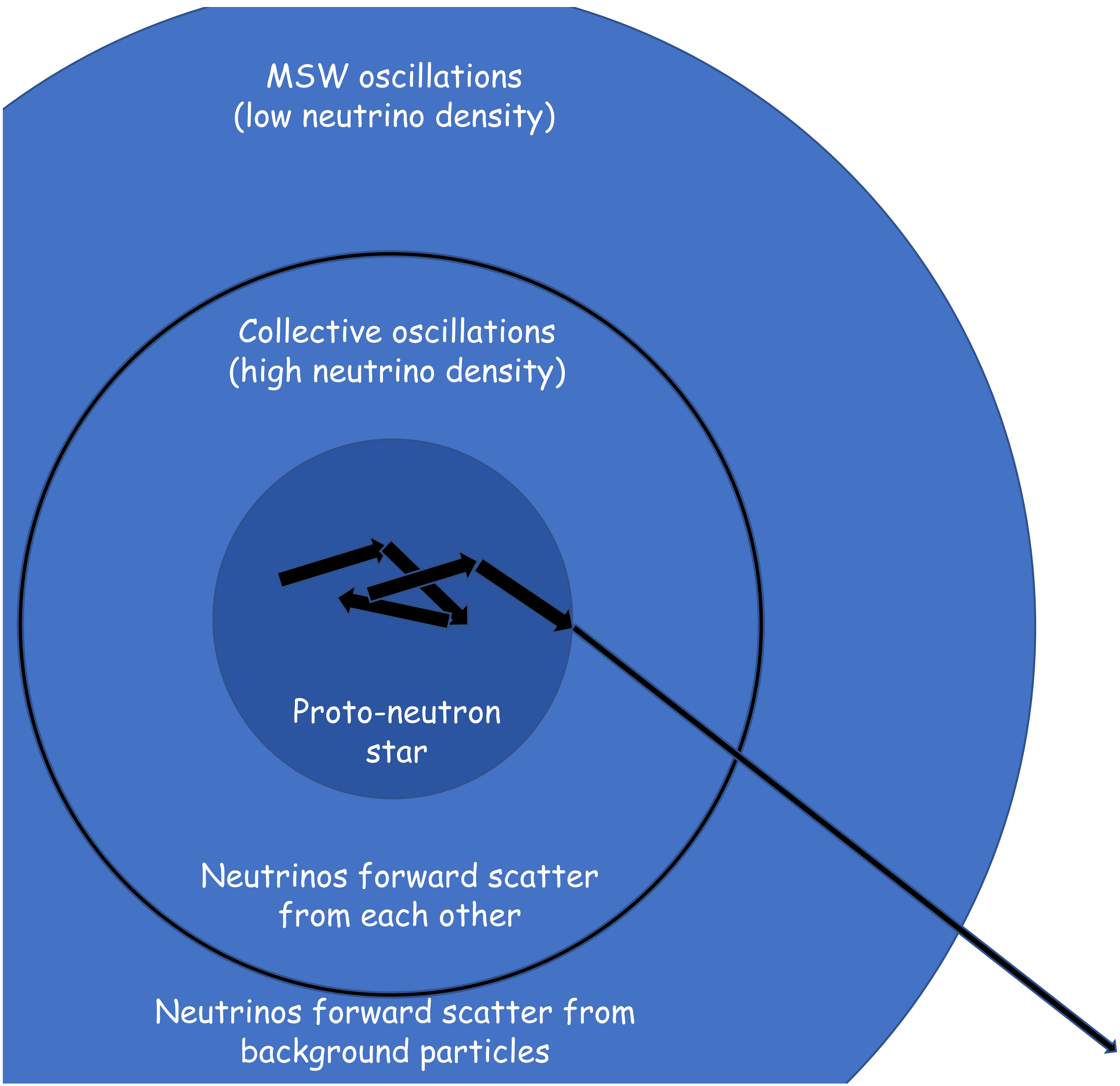


Energy released in a core-collapse SN: $\Delta E \approx 10^{53}$ ergs $\approx 10^{59}$ MeV
99% of this energy is carried away by neutrinos and antineutrinos!
~ 10^{58} Neutrinos!
This necessitates including the effects of $\nu\nu$ interactions!

$$H = \underbrace{\sum a^{\dagger} a}_{\substack{\nu \text{ oscillations} \\ \text{MSW effect}}} + \underbrace{\sum (1-\cos\varphi) a^{\dagger} a^{\dagger} a a}_{\text{neutrino-neutrino interactions}}$$

The second term makes the physics of a neutrino gas in a core-collapse supernova a very interesting many-body problem, driven by weak interactions.

Neutrino-neutrino interactions lead to novel collective and emergent effects, such as conserved quantities and interesting features in the neutrino energy spectra (spectral "swaps" or "splits").


MSW oscillations
(low neutrino density)
Collective oscillations
(high neutrino density)
Proto-neutron
star
Neutrinos forward scatter
from each other
Neutrinos forward scatter from
background particles

$$\frac{\partial \rho}{\partial t} = -i[H, \rho] + C(\rho)$$

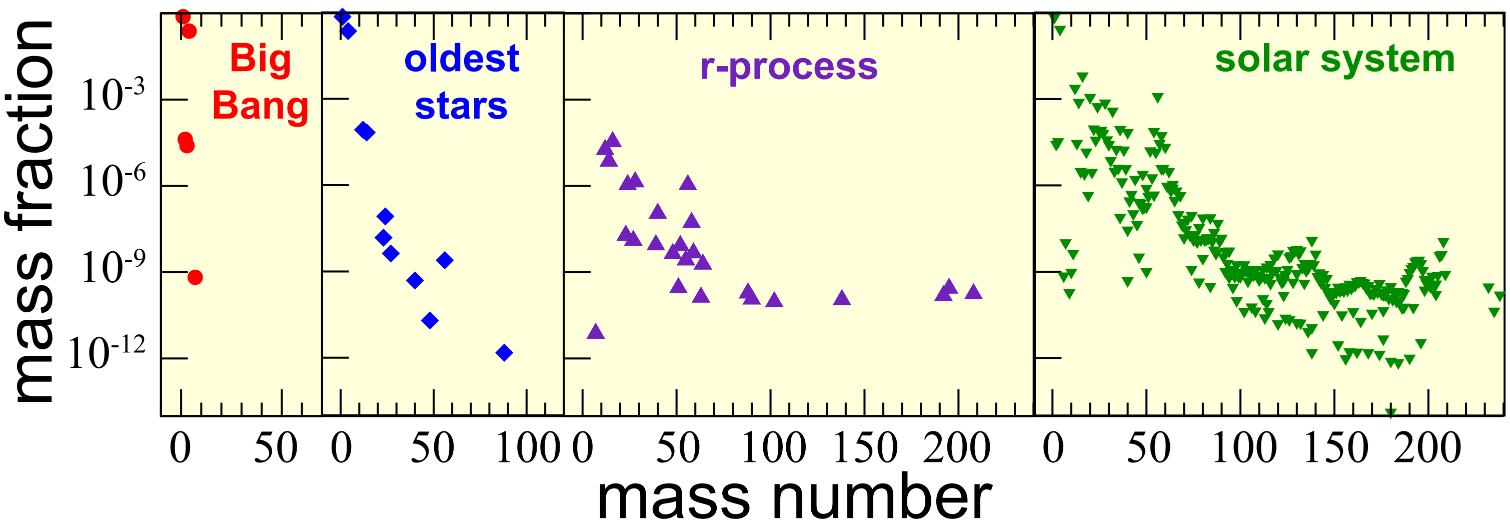
H = neutrino mixing

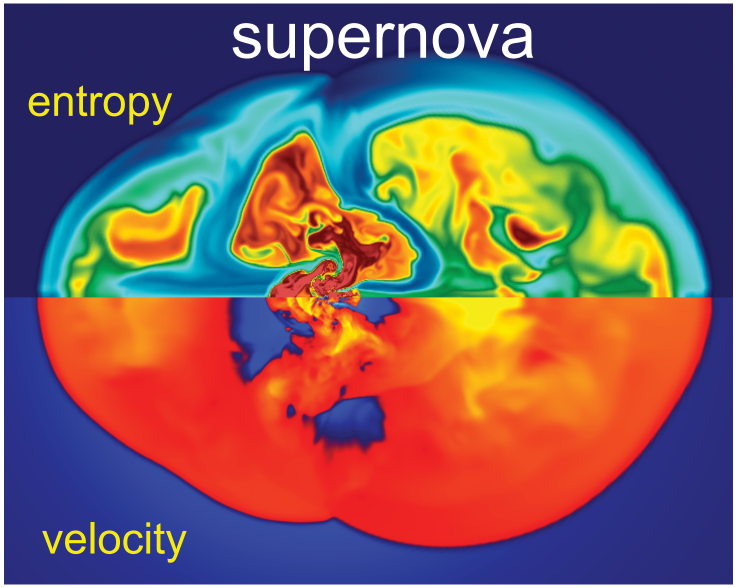
+ forward scattering of neutrinos off other background particles (MSW)

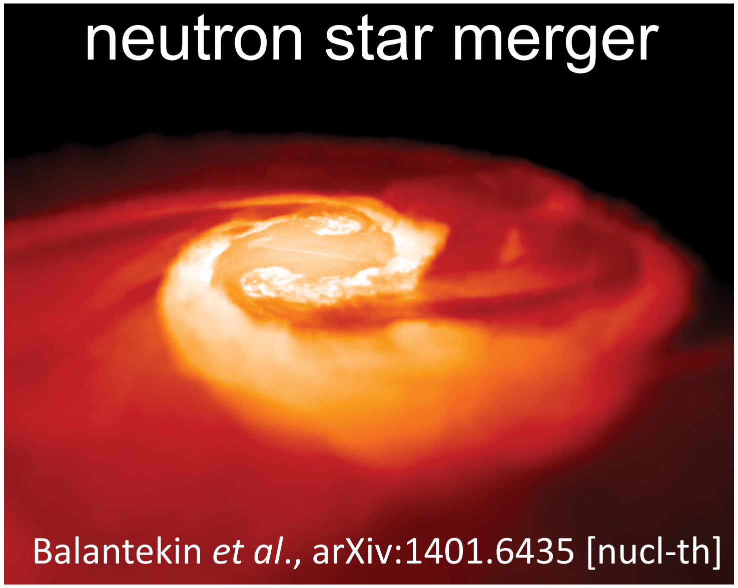
+ forward scattering of neutrinos off each other

C = collisions

The origin of elements





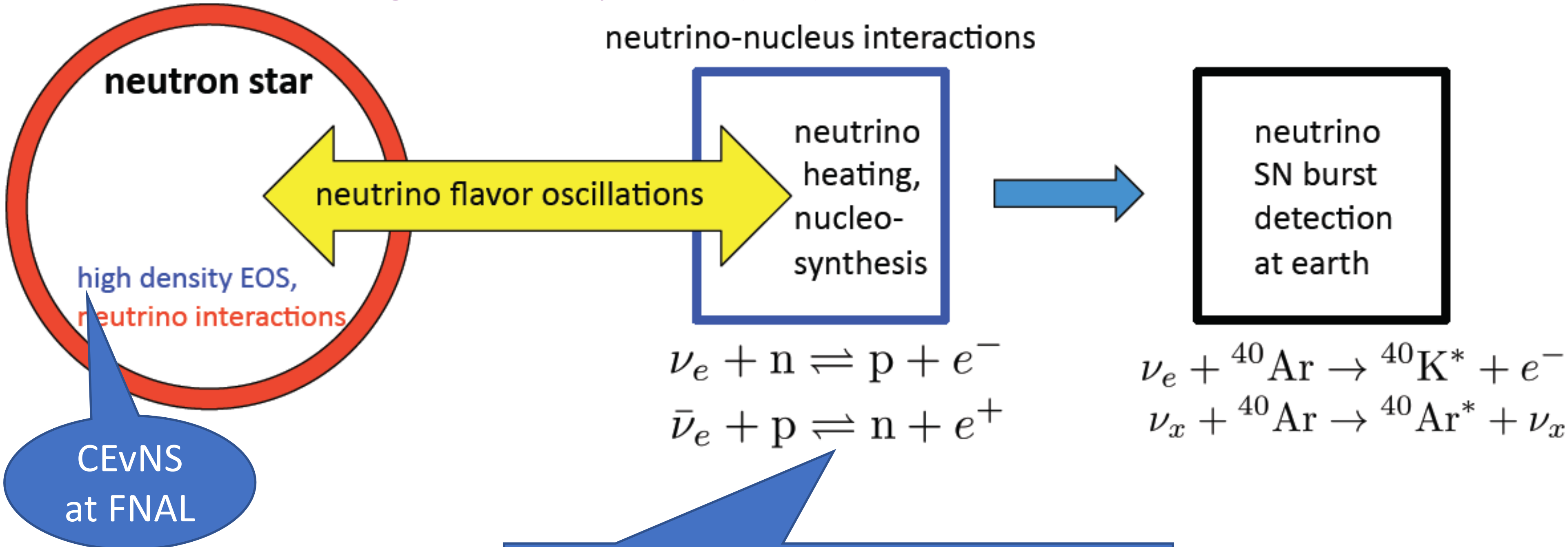


Neutrinos not only play a crucial role in the dynamics of these sites, but they also control the value of the electron fraction, the parameter determining the yields of the r-process.

Possible sites for the r-process

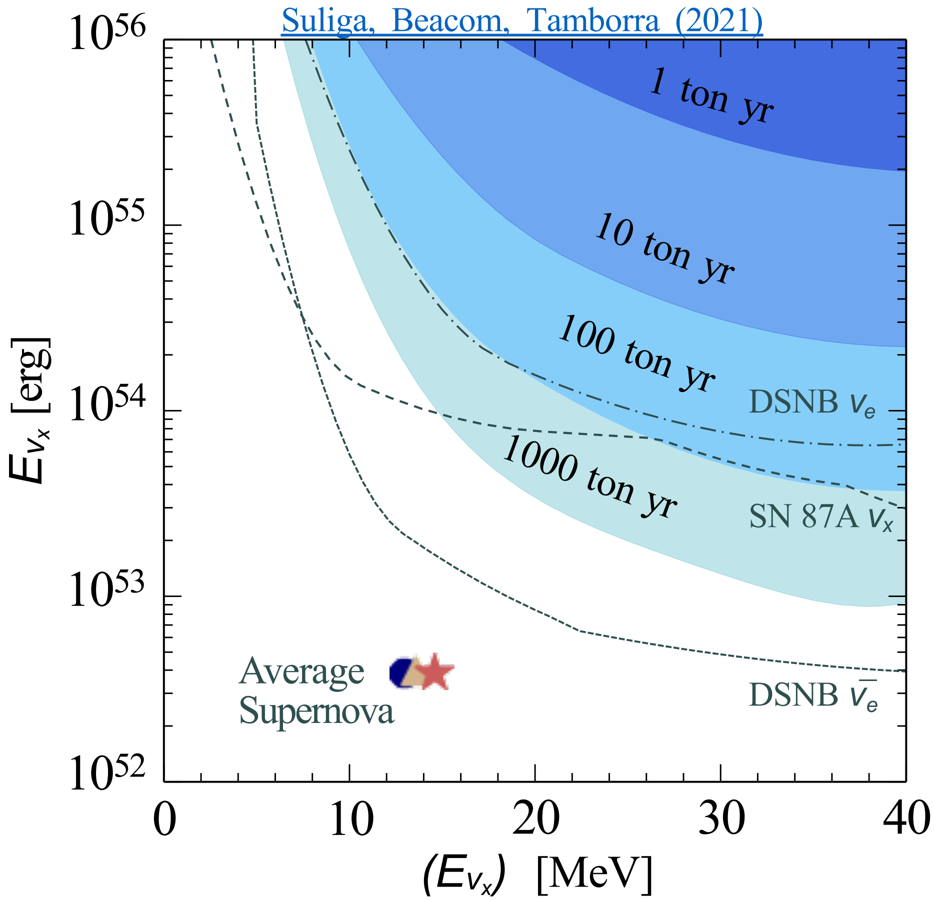
Understanding a core-collapse supernova requires answers to a variety of questions some of which need to be answered, both theoretically and experimentally.

Balantekin and Fuller, Prog. Part. Nucl. Phys. **71** 162 (2013)



Neutron-to-proton ratio depends on relative intensities of electron neutrinos and electron antineutrinos, which in turn depend on neutrino oscillations

Diffuse supernova neutrino background (DSNB)



- $\bar{\nu}_e$: soon to be detected by SK + Gd, JUNO
- ν_e: posssibly detectable by DUNE
- ν_x: CEνNS detectors can improve the existing limits to almost $\bar{\nu}_e$ level

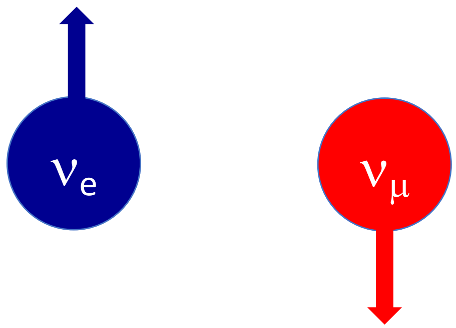
See Suliga's talk

Detection of all flavors required to

- rule out potential non-standard scenarios
- bring us closer to understaning the supernova physics

Guseinov (1967), Totani et al. (2009), Ando, Sato (2004), Lunardini (2009), Beacom (2010), Horiuchi et al. (2011), Lunardini, Tamborra (2012), Møller, Suliga, Tamborra, Denton (2018), Nakazato et al. (2018), Kresse et al. (2020) ...

Neutrino flavor isospin



$$\hat{J}_+ = a_e^\dagger a_\mu \qquad \hat{J}_- = a_\mu^\dagger a_e$$

$$\hat{J}_0 = \frac{1}{2}\left(a_e^\dagger a_e - a_\mu^\dagger a_\mu\right)$$

These operators can be written in either mass or flavor basis

Free neutrinos (only mixing)

$$\hat{H} = \frac{m_1^2}{2E} a_1^\dagger a_1 + \frac{m_2^2}{2E} a_2^\dagger a_2 + (\cdot\cdot)\hat{1}$$

$$= \frac{\delta m^2}{4E}\cos 2\theta\left(-2\hat{J}_0\right) + \frac{\delta m^2}{4E}\sin 2\theta\left(\hat{J}_+ + \hat{J}_-\right) + (\cdot\cdot)'\,\hat{1}$$

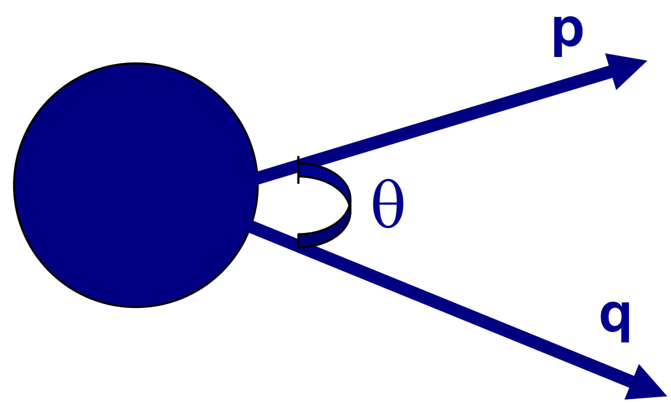
Interacting with background electrons

$$\hat{H} = \left[\frac{\delta m^2}{4E}\cos 2\theta - \frac{1}{\sqrt{2}} G_F N_e\right]\left(-2\hat{J}_0\right) + \frac{\delta m^2}{4E}\sin 2\theta\left(\hat{J}_+ + \hat{J}_-\right) + (\cdot\cdot)''\,\hat{1}$$

Neutrino-Neutrino Interactions

Smirnov, Fuller, Qian, Pantaleone, Sawyer
McKellar, Friedland, Lunardini, Raffelt,
Duan, Balantekin, Volpe, Kajino, Pehlivan …

$$\hat{H}_{\nu\nu} = \frac{\sqrt{2}G_F}{V}\int dp\,dq\left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p\cdot\vec{\mathbf{J}}_q$$



This term makes the physics of a neutrino gas in a core-collapse supernova a genuine many-body problem

$$\hat{H} = \int dp\left(\frac{\delta m^2}{2E}\vec{\mathbf{B}}\cdot\vec{\mathbf{J}}_p - \sqrt{2}G_F N_e \mathbf{J}_p^0\right) + \frac{\sqrt{2}G_F}{V}\int dp\,dq\left(1-\cos\theta_{pq}\right)\vec{\mathbf{J}}_p\cdot\vec{\mathbf{J}}_q$$

$$\vec{\mathbf{B}} = \left(\sin 2\theta,\ 0, -\cos 2\theta\right)$$

Neutrino-neutrino interactions lead to novel collective and emergent effects, such as conserved quantities and interesting features in the neutrino energy spectra (spectral “swaps” or “splits”).

Including antineutrinos

$$H = H_\nu + H_{\bar{\nu}} + H_{\nu\nu} + H_{\bar{\nu}\bar{\nu}} + H_{\nu\bar{\nu}}$$

Requires introduction of a second set of SU(2) algebras!

Including three flavors

Requires introduction of SU(3) algebras.

Both extensions are straightforward, but tedious!
Balantekin and Pehlivan, J. Phys. G **34**, 1783 (2007).

This problem is "exactly solvable" in the single-angle approximation

$$H = \sum_{p} \frac{\delta m^2}{2p} \hat{B} \cdot \vec{J}_p + \frac{\sqrt{2} G_F}{V} \sum_{\mathbf{p},\mathbf{q}} (1 - \cos\vartheta_{\mathbf{pq}}) \, \vec{J}_{\mathbf{p}} \cdot \vec{J}_{\mathbf{q}}$$

$$\Downarrow$$

$$H = \sum_{p} \omega_p \vec{B} \cdot \vec{J}_p + \mu(r) \vec{J} \cdot \vec{J}$$

Single-angle approximation Hamiltonian:

$$H = \sum_p \frac{\delta m^2}{2p} J_p^0 + 2\mu \sum_{\substack{p,\, q \\ p \neq q}} \mathbf{J}_p \bullet \mathbf{J}_q$$

Eigenstates:

$$\left| x_i \right\rangle = \prod_{i=1}^{N} \sum_k \frac{J_k^\dagger}{\left(\delta m^2 / 2k\right) - x_i} \left| 0 \right\rangle$$

$$\underbrace{-\frac{1}{2\mu} - \sum_k \frac{j_k}{\left(\delta m^2 / 2k\right) - x_i} = \sum_{j \neq i} \frac{1}{x_i - x_j}}_{\text{Bethe ansatz equations}}$$

$$\mu = \frac{G_F}{\sqrt{2}V} \left\langle 1 - \cos\Theta \right\rangle$$

Invariants:

$$h_p = J_p^0 + 2\mu \sum_{\substack{p,\, q \\ p \neq q}} \frac{\mathbf{J}_p \bullet \mathbf{J}_q}{\delta m^2 \left(\frac{1}{p} - \frac{1}{q} \right)}$$

Pehlivan, ABB, Kajino, & Yoshida
Phys. Rev. D 84, 065008 (2011)

Two of the adiabatic eigenstates of this equation are easy to find in the single-angle approximation:

$$H = \sum_p \omega_p \vec{B} \cdot \vec{J}_p + \mu(r) \vec{J} \cdot \vec{J}$$

$$|j, +j\rangle = |N/2, N/2\rangle = |\nu_1, \ldots, \nu_1\rangle$$

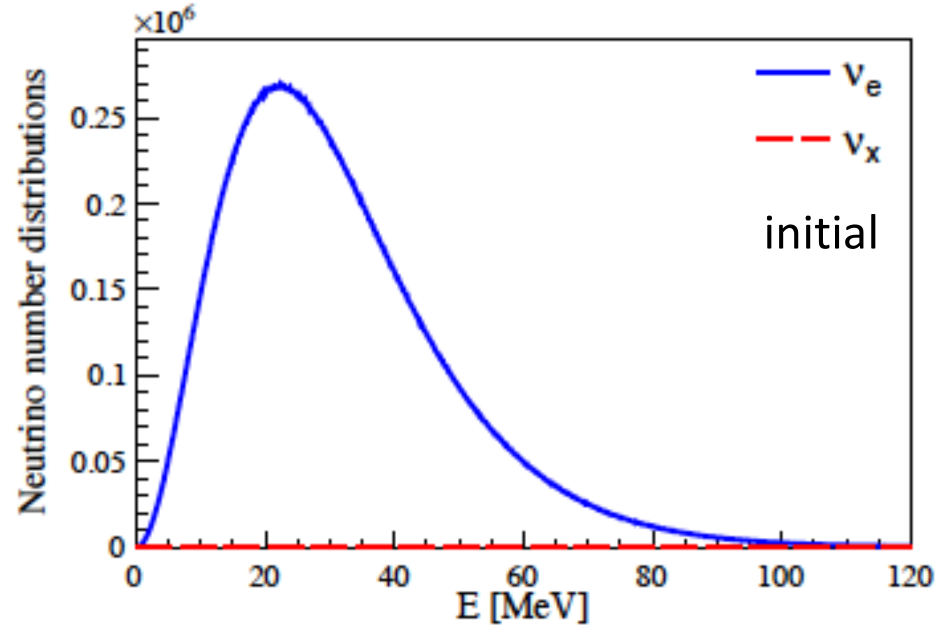
$$|j, -j\rangle = |N/2, -N/2\rangle = |\nu_2, \ldots, \nu_2\rangle$$

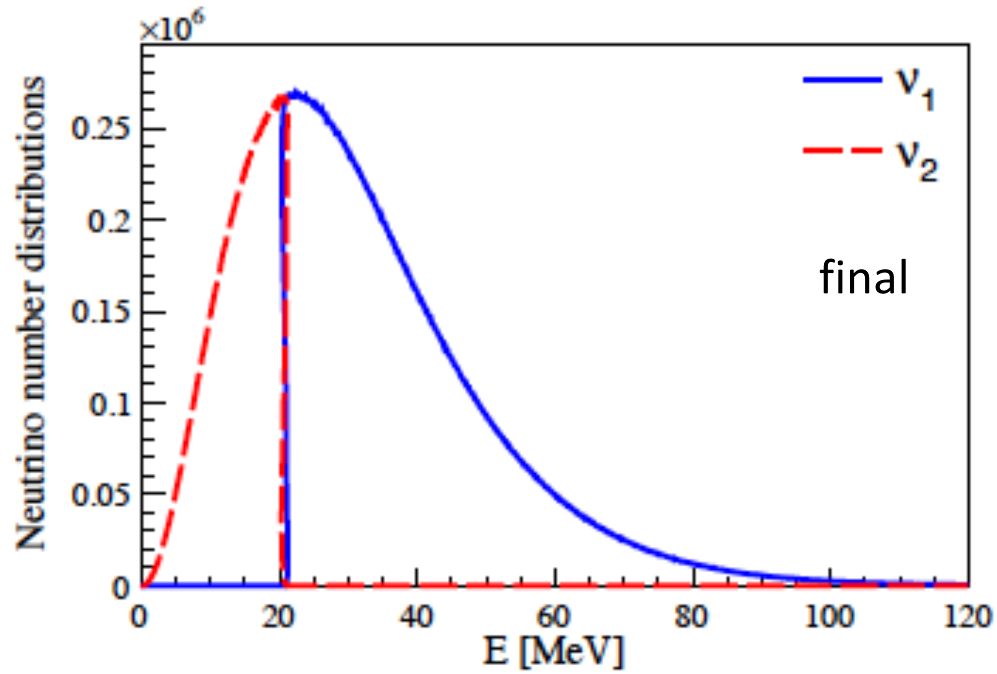
$$E_{\pm N/2} = \mp \sum_p \omega_p \frac{N_p}{2} + \mu \frac{N}{2} \left(\frac{N}{2} + 1 \right)$$

To find the others will take a lot more work

Away from the mean-field: Adiabatic solution of the *exact* many-body Hamiltonian for extremal states

Adiabatic evolution of an initial thermal distribution (T = 10 MeV) of electron neutrinos. 10^8 neutrinos distributed over 1200 energy bins with solar neutrino parameters and normal hierarchy.





Birol, Pehlivan, Balantekin, Kajino
arXiv:1805.11767
PRD98 (2018) 083002

A system of N particles each of which can occupy k states (k = number of flavors)

Exact Solution → **Mean-field approximation**

Entangled and unentangled states → Only unentangled states

Dimension of Hilbert space: k^N

Dimension of the diagonalizing space: kN

von Neumann entropy $S = - \mathrm{Tr}\,(\rho \log \rho)$

	Pure State	Mixed State
Density matrix	$\rho^2 = \rho$	$\rho^2 \neq \rho$
Entropy	$S = 0$	$S \neq 0$

Pick one of the neutrinos and introduce the reduced density matrix for this neutrino (with label "b")

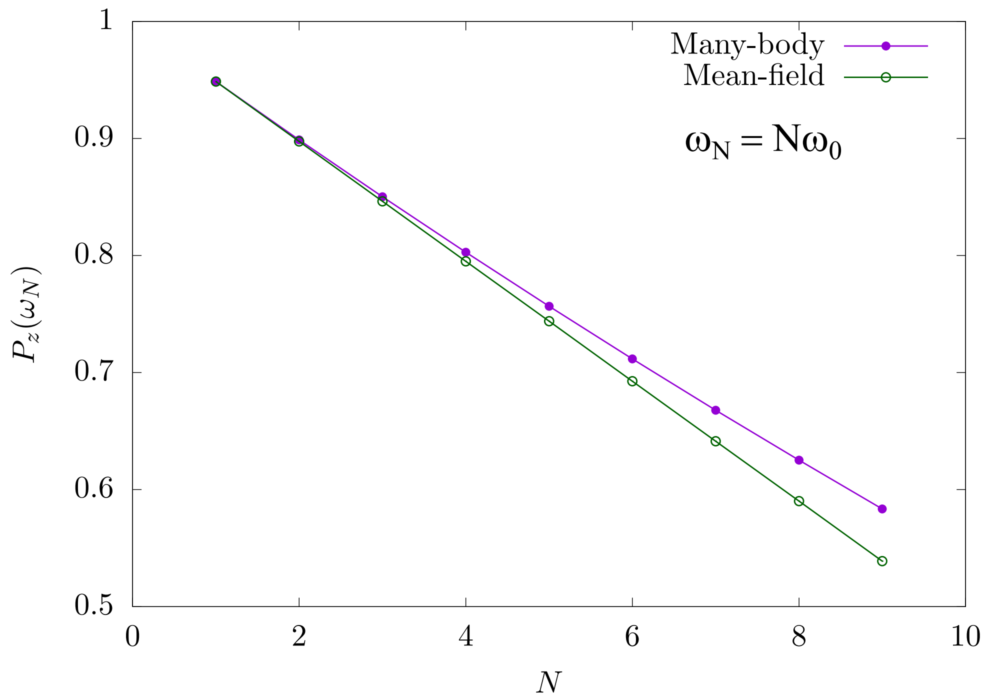
$$\boxed{\tilde{\rho} = \rho_b = \sum_{a,c,d,\ldots} \langle \nu_a, \nu_c, \nu_d, \cdots | \rho | \nu_a, \nu_c, \nu_d, \cdots \rangle}$$

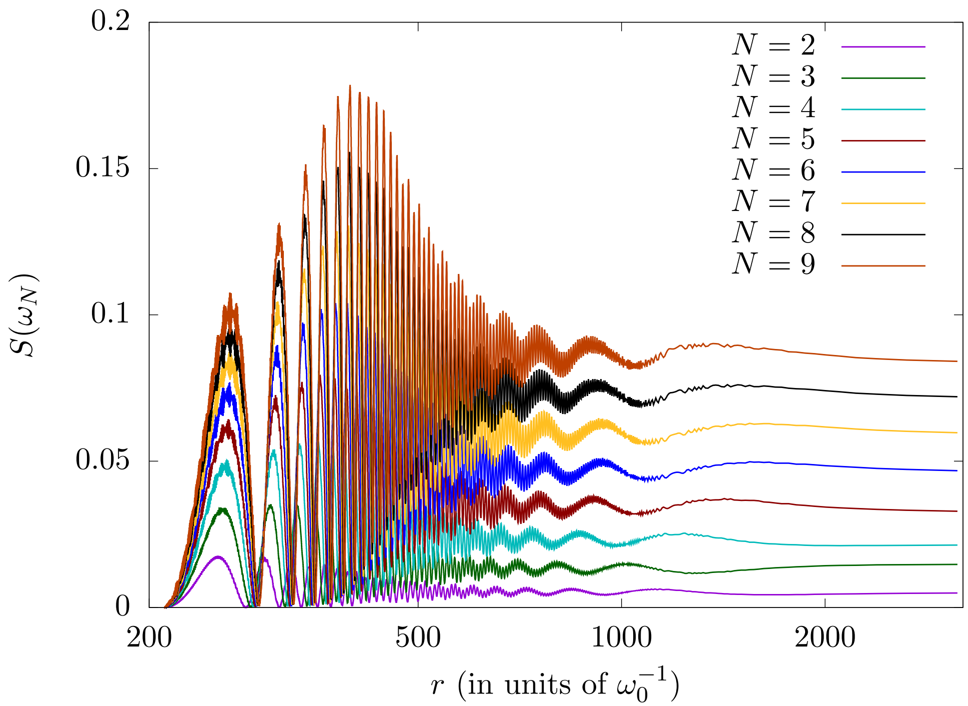
Entanglement entropy

$$S = -\mathrm{Tr}\,(\tilde{\rho}\ \log \tilde{\rho})$$

$$\tilde{\rho} = \frac{1}{2}(\mathbb{I} + \vec{\sigma} \cdot \vec{P})$$

$$S = -\frac{1-|\vec{P}|}{2} \log\left(\frac{1-|\vec{P}|}{2}\right) - \frac{1+|\vec{P}|}{2} \log\left(\frac{1+|\vec{P}|}{2}\right)$$



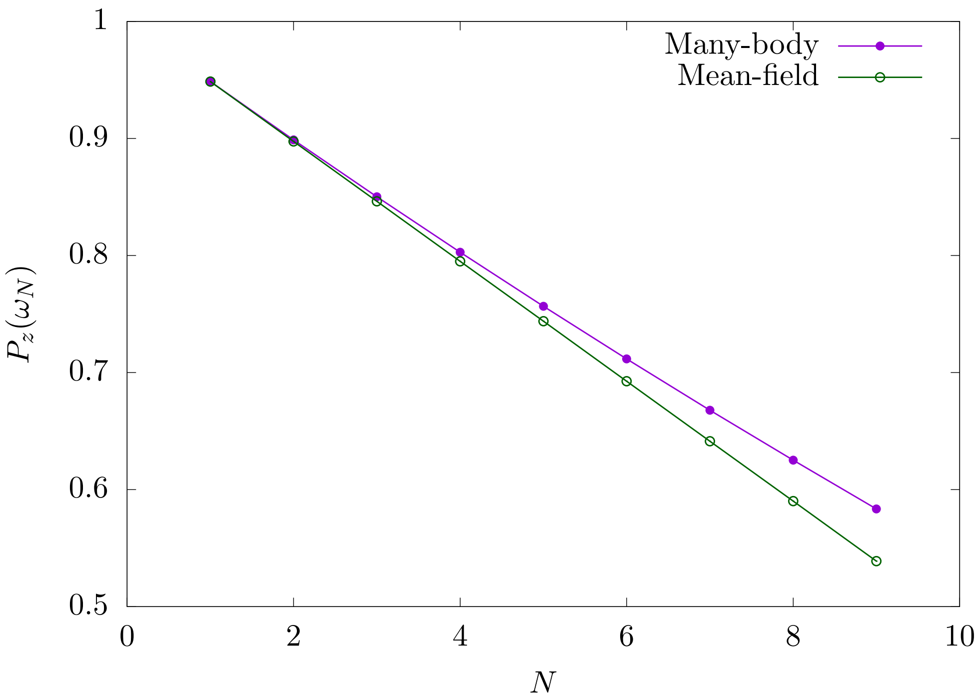


Initial state:
all electron neutrinos

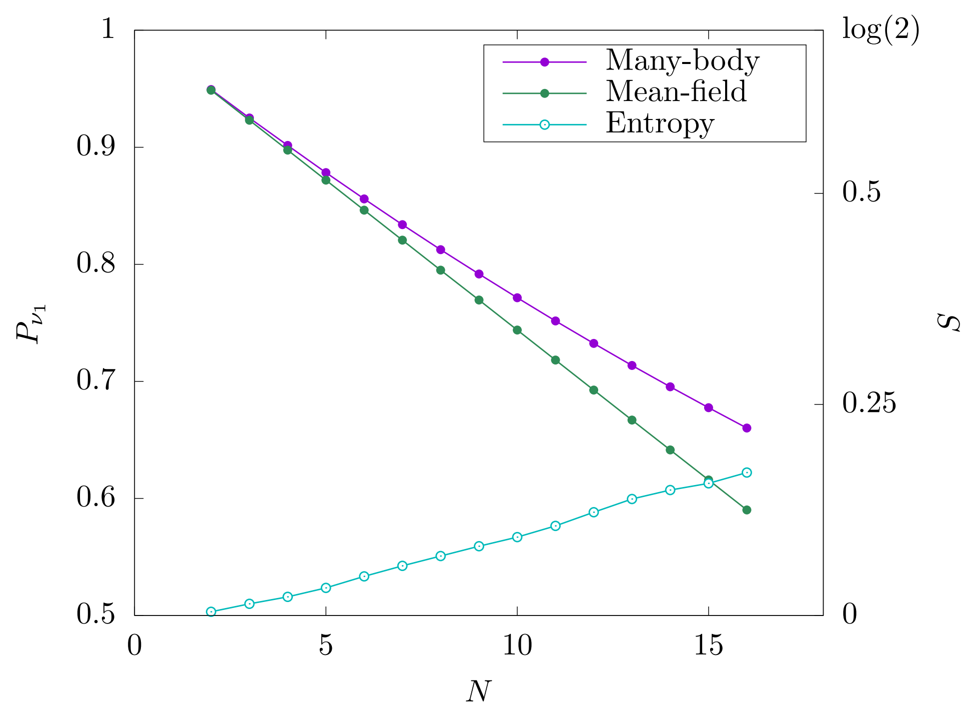
Note: S = 0 for mean-field approximation

Cervia, Patwardhan, Balantekin, Coppersmith, Johnson, arXiv:1908.03511
PRD, **100**, 083001 (2019)

- Bethe ansatz method has numerical instabilities for larger values of N. However, it is very valuable since it leads to the identification of conserved quantities.
- For this reason, we also explored the use of Runge Kutta and tensor network techniques. This was both to check Bethe ansatz results for N less than 10 and to explore the case with N larger than 10.



Cervia, Patwardhan, Balantekin,
Coppersmith, Johnson,
arXiv:1908.03511
PRD **100**, 083001 (2019)



Patwardhan, Cervia, Balantekin,
arXiv:2109.08995
PRD **104**, 123035 (2021)

Mean Field: $\rho = \rho_1 \otimes \rho_2 \otimes \cdots \otimes \rho_N$

$$\omega_A = \frac{\delta m^2}{2E_A}$$

$\mathbf{P} = \mathrm{Tr}\,(\rho \mathbf{J})$

Mean-field evolution

$$\frac{\partial}{\partial t}\mathsf{P}^{(A)} = (\omega_A \mathcal{B} + \mu \mathsf{P}) \times \mathsf{P}^{(A)}$$

$$\mathsf{P} = \sum_A \mathsf{P}^{(A)}.$$

$$\frac{\partial}{\partial t}\mathsf{P} = \mathcal{B} \times \left(\sum_A \omega_A \mathsf{P}^{(A)} \right)$$

$\mathcal{B} \cdot \mathsf{P}$ is a constant of motion.

$$\frac{\partial}{\partial t}\mathsf{P}^{(A)} = (\omega_A \mathcal{B} + \mu \mathsf{P}) \times \mathsf{P}^{(A)}$$

$$\mathsf{P} = \sum_A \mathsf{P}^{(A)}.$$

$$\mathsf{P}^{(A)} = \alpha_A \mathcal{B} + \beta_A \mathsf{P} + \gamma_A (\mathcal{B} \times \mathsf{P}),$$

$$\sum_A \alpha_A = 0, \quad \sum_A \beta_A = 1, \quad \sum_A \gamma_A = 0.$$

If initially all N neutrinos have the same flavor, then in the mass basis would be $\alpha_0 = 0, \beta_0 = 1/N$, and $\gamma_0 = 0$.

$$\frac{\partial}{\partial t}\mathsf{P} = \left(\sum_A \beta_A \omega_A\right)(\mathcal{B} \times \mathsf{P}) + \left(\sum_A \gamma_A \omega_A\right)[(\mathcal{B} \cdot \mathsf{P})\mathcal{B} - \mathsf{P}]$$

Adopt for the mass basis and define $\Gamma = \left(\sum_A \gamma_A \omega_A\right)$. Unless Γ is positive the solutions for P_x and P_y exponentially grow.

$$P_{x,y} = \Pi_{x,y} \exp\left(-\int \Gamma(t)dt\right)$$

$$\frac{\partial}{\partial t}\Pi_x = \left(\sum_A \beta_A \omega_A\right)\Pi_y, \quad \frac{\partial}{\partial t}\Pi_y = -\left(\sum_A \beta_A \omega_A\right)\Pi_x.$$

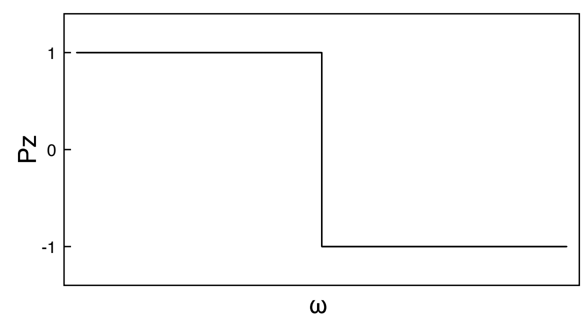
$$P_{x,y} = \Pi_{x,y} \exp\left(- \int \Gamma(t) dt \right)$$

$$\frac{\partial}{\partial t}\Pi_x = \left(\sum_A \beta_A \omega_A \right) \Pi_y, \quad \frac{\partial}{\partial t}\Pi_y = - \left(\sum_A \beta_A \omega_A \right) \Pi_x.$$

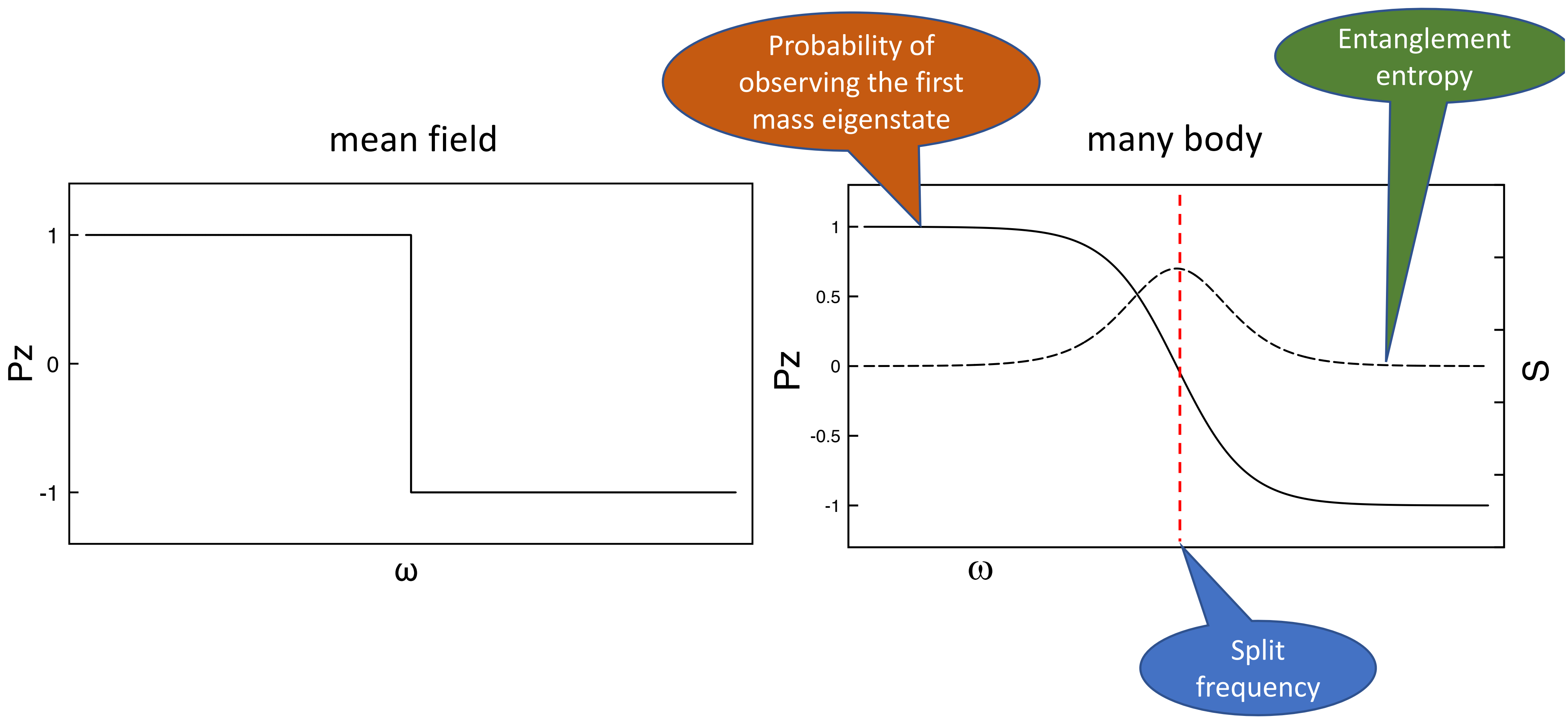
In the mean-field approximation Π_x and Π_y precess around $\mathcal{B}$ with a time-dependent frequency (through the time-dependence of β_As). Then P_x and P_y also precess similarly while decaying due to the exponential terms. Hence asymptotically P_x and P_y tend to be very small. Then x and y components of each $\mathrm{P}^{(A)}$ are asymptotically very small. Since $|\mathrm{P}^{(A)}|^2 = 1$ for uncorrelated neutrinos, it then follows that

$$\left(\mathrm{P}_z^{(A)}\right)^2 \sim 1$$

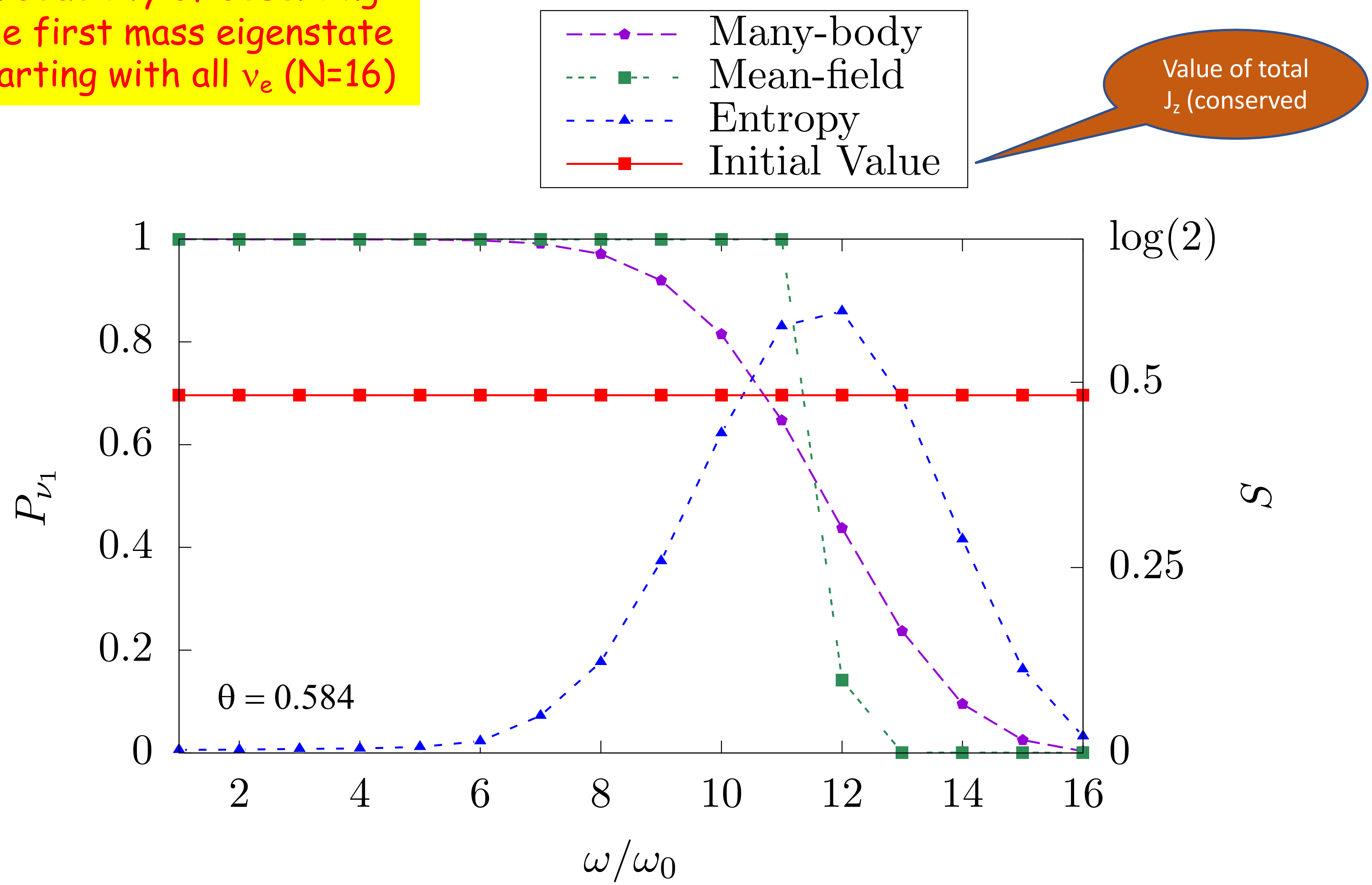
asymptotically. Consequently allowed asymptotic values of $\mathrm{P}_z^{(A)}$ are $\sim \pm 1$. Since the constant of motion $\sum_A \mathrm{P}_z^{(A)}$ (in the mass basis) is fixed by the initial conditions, some of the final $\mathrm{P}_z^{(A)}$ values will be $+1$ and some of them will be -1. This is the "spectral split" phenomenon. Depending on the initial conditions, there may exist one or more spectral splits.



We find that the presence of spectral splits is a good proxy for deviations from the mean-field results

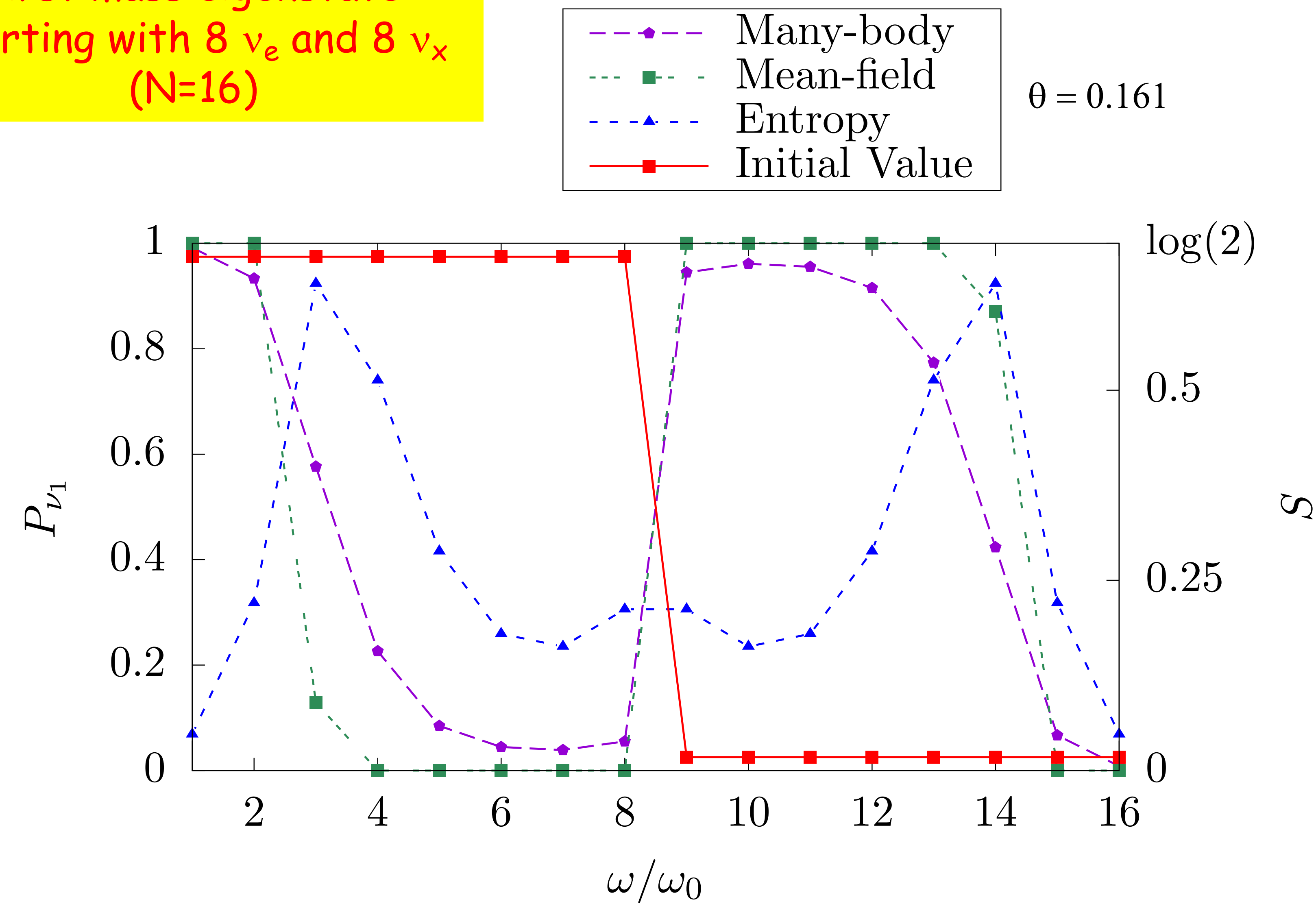


Probability of observing the first mass eigenstate starting with all ν_e (N=16)



Patwardhan, Cervia, Balantekin, arXiv:2109.08995
Phys. Rev. D **104**, 123035 (2021)

Probability of observing the first mass eigenstate starting with 8 ν_e and 8 ν_x (N=16)

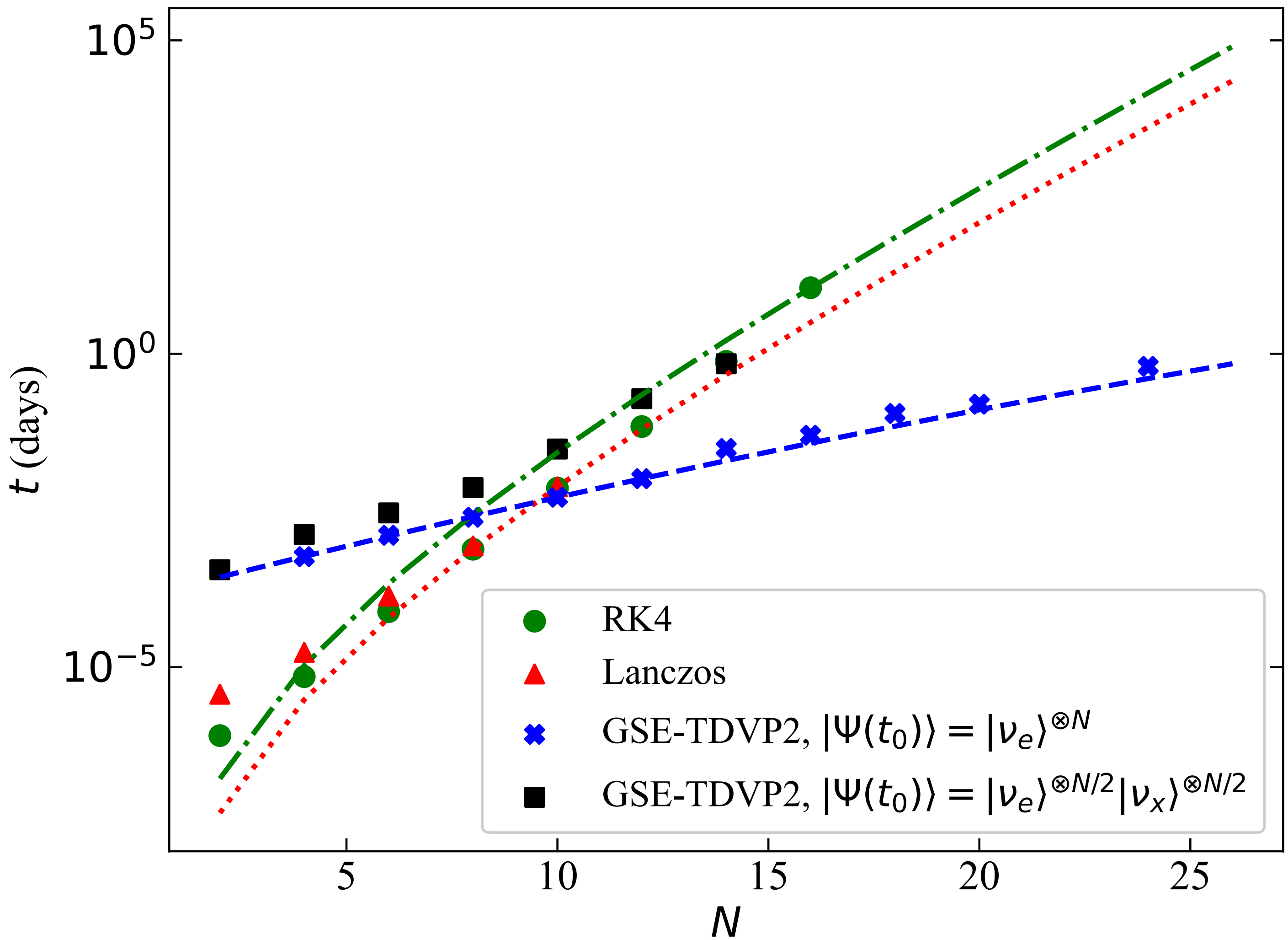


Patwardhan, Cervia, Balantekin, arXiv:2109.08995

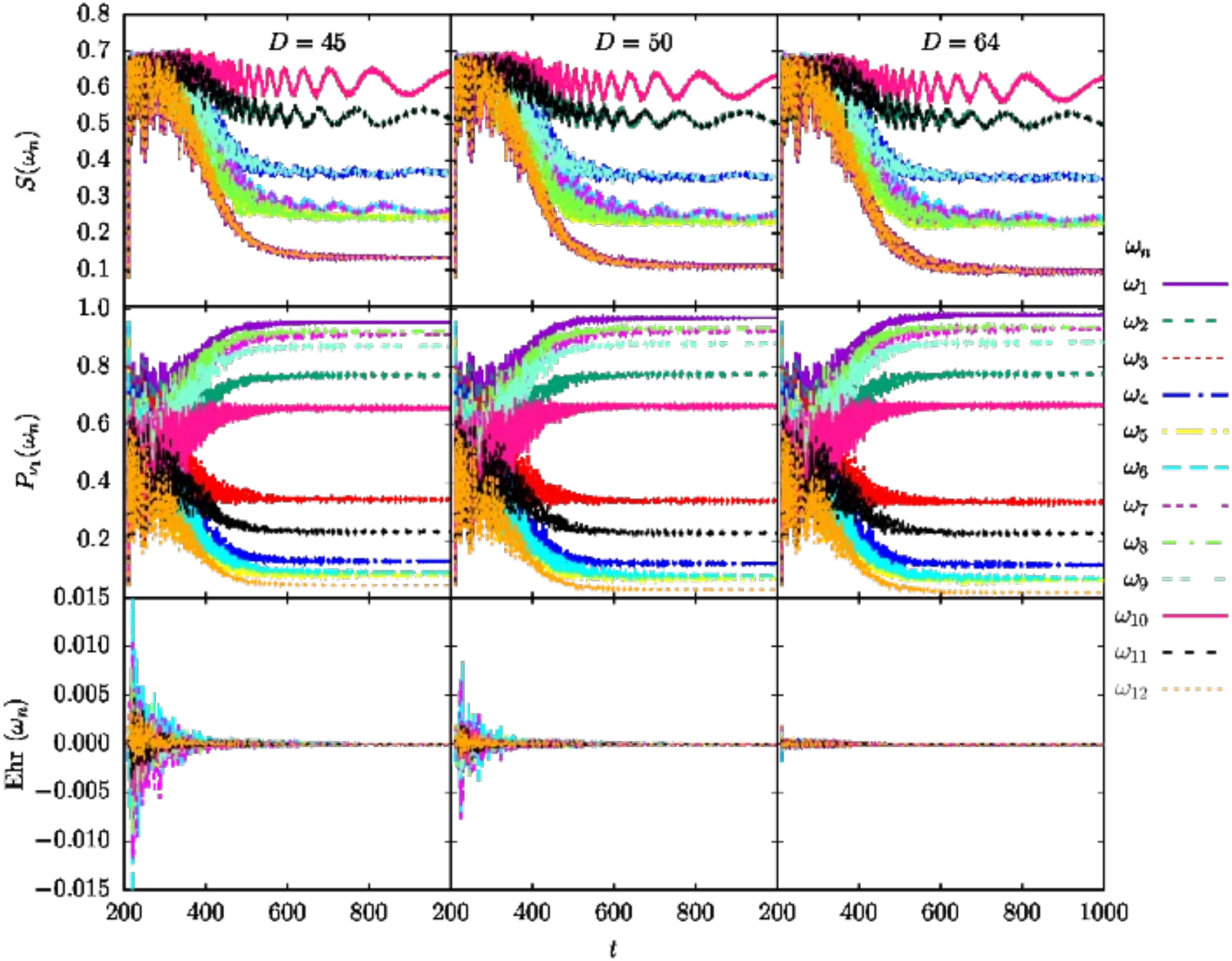
Where do we go from here?

- Explore the efficacy of tensor methods utilizing invariants obtained in the Bethe ansatz approach. ✅
  Cervia, Siwach, Patwardhan, Balantekin, Coppersmith, Johnson, arXiv: 2202.01865
- Use tensor methods to explore scaling behavior (Can you get away with smaller bond dimensions?)
  Cervia, Siwach, Patwardhan, Balantekin, Coppersmith, Johnson
- Explore the impact of using many-body solution instead of the mean-field solution in calculating element synthesis (especially r- and rp-process).
  X. Wang, Patwardhan, Cervia, Surman, Balantekin

Computation times:



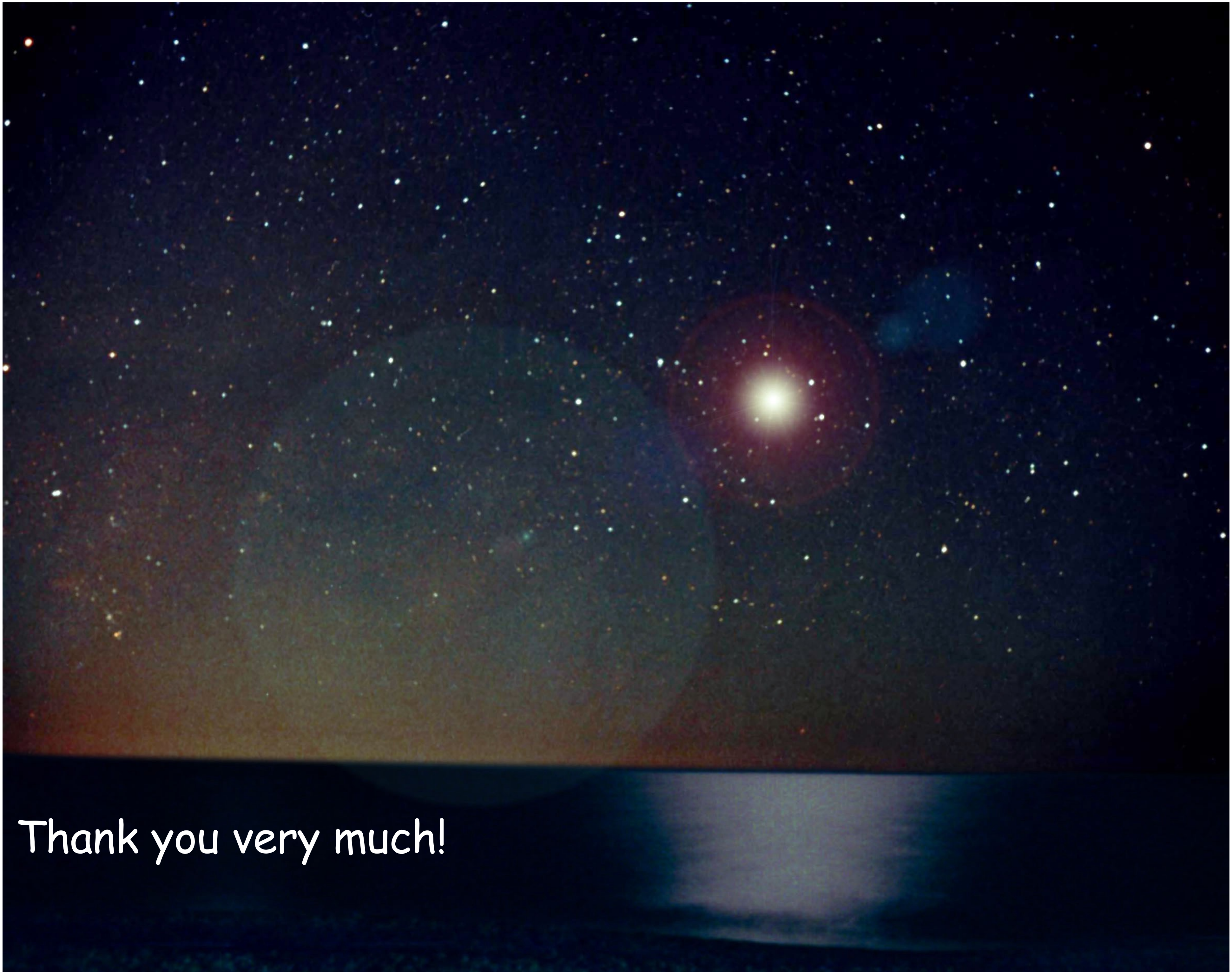
Cervia, Siwach, Patwardhan, Balantekin, Coppersmith, Johnson, arXiv:2202.01865



Time evolution for 12 neutrinos (initially six ν_e and six ν_x). D is the bond dimension. The largest possible value of D is $2^6=64$.

CONCLUSIONS

- Calculations performed using the mean-field approximation have revealed a lot of interesting physics about collective behavior of neutrinos in astrophysical environments. Here we have explored possible scenarios where further interesting features can arise by going beyond this approximation.
- We found that the deviation of the adiabatic many-body results from the mean field results is largest for neutrinos with energies around the spectral split energies. In our single-angle calculations we observe a broadening of the spectral split region. This broadening does not appear in single-angle mean-field calculations and seems to be larger than that was observed in multi-angle mean-field calculations (or with BSM physics).
- This suggests hybrid calculations may be efficient: many-body calculations near the spectral split and mean-field elsewhere.
- There is a strong dependence on the initial conditions.


Thank you very much!