

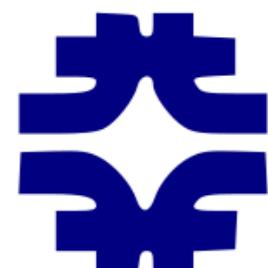
Lattice QCD for Neutrinos



Neutrino Theory Network Workshop

Fermilab

June 21, 2022



Fermilab

Precision neutrino physics

Comparisons of near + far detector neutrino fluxes can be used to precisely determine oscillation parameters, discover neutrino CP violation, ...

Relating measured final-state event rates to incoming neutrino flux requires precise knowledge of νA cross-section

$$\frac{N_{\text{near}}}{N_{\text{far}}} = \frac{\int dE_\nu \Phi_{\text{near}}(E_\nu) \sigma(E_\nu)}{\int dE_\nu \Phi_{\text{far}}(E_\nu) \sigma(E_\nu)}$$

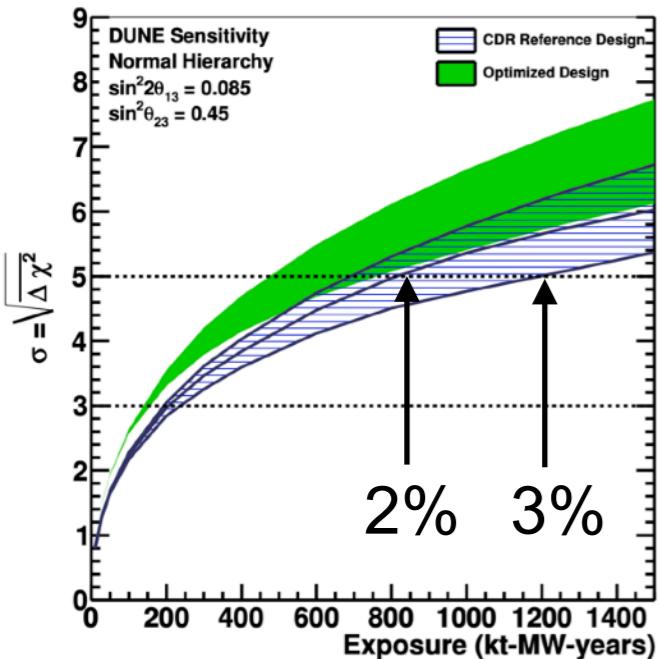
Near-detector neutrino flux

Cross-section

Far-detector flux (depends on oscillation parameters)

Experimentally measured event rates

50% CP Violation Sensitivity

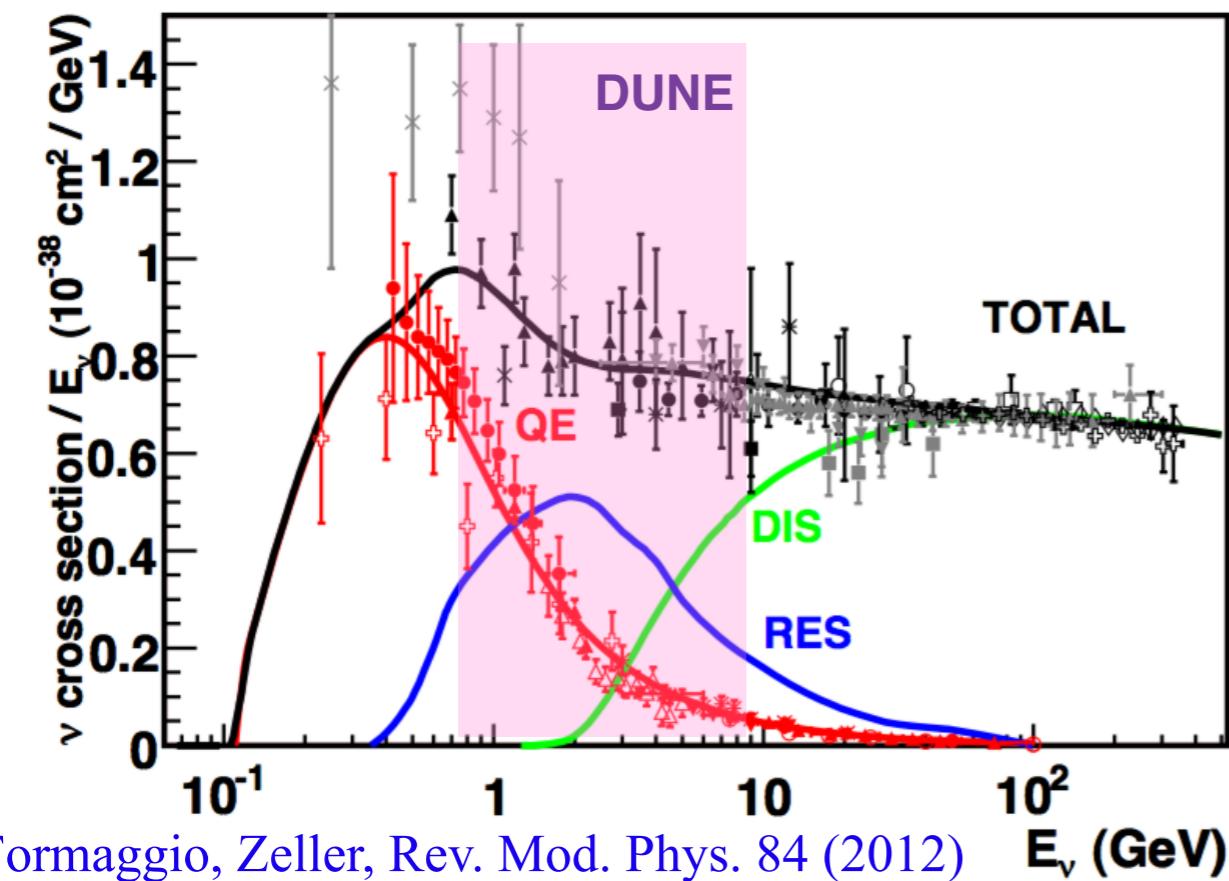


DUNE aims to have few-percent cross-section uncertainty

- 2% vs 3% cross-section uncertainty estimated to change exposure required to discover CP violation by 50%

Acciarri et al (DUNE) arXiv 1512.06148

Neutrino-nucleus scattering

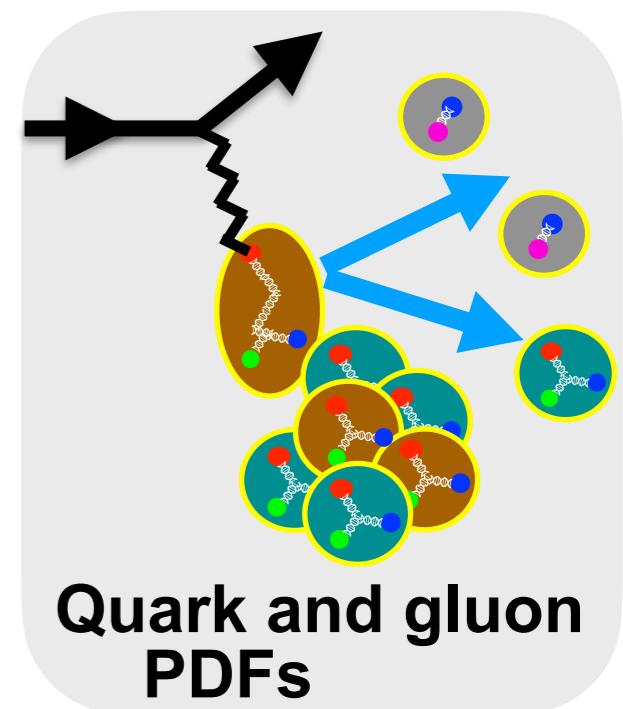
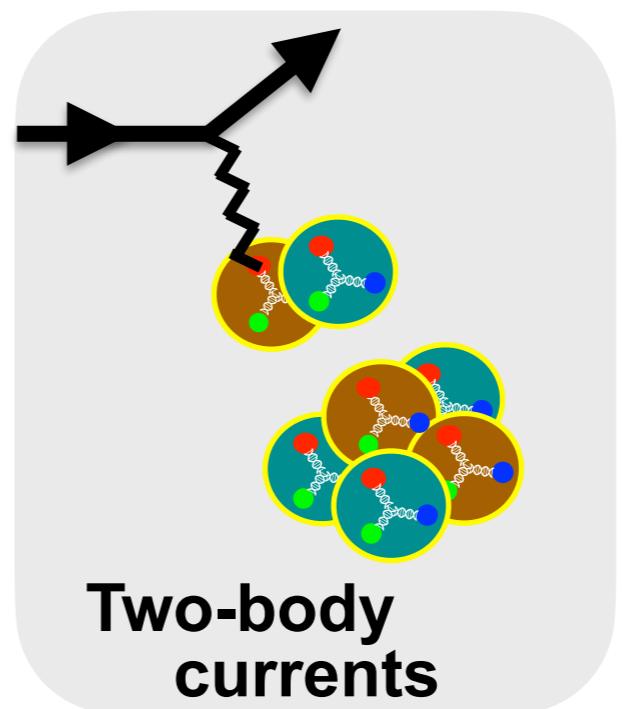
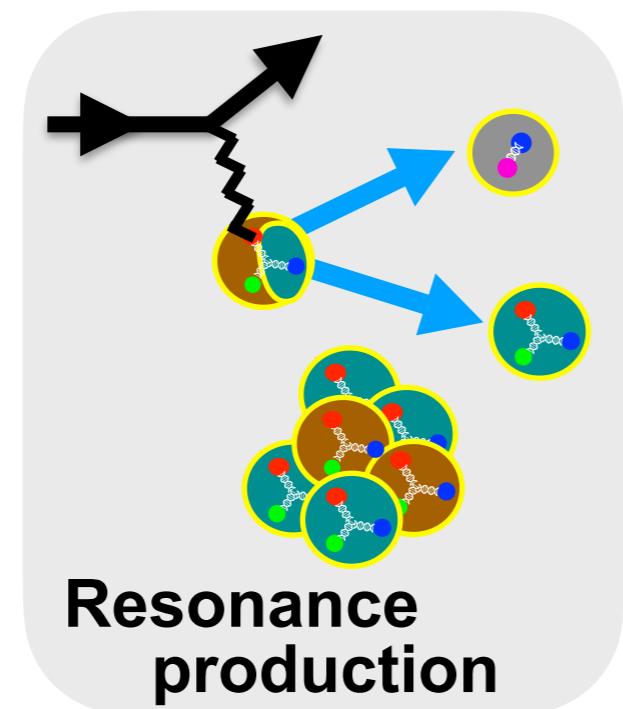
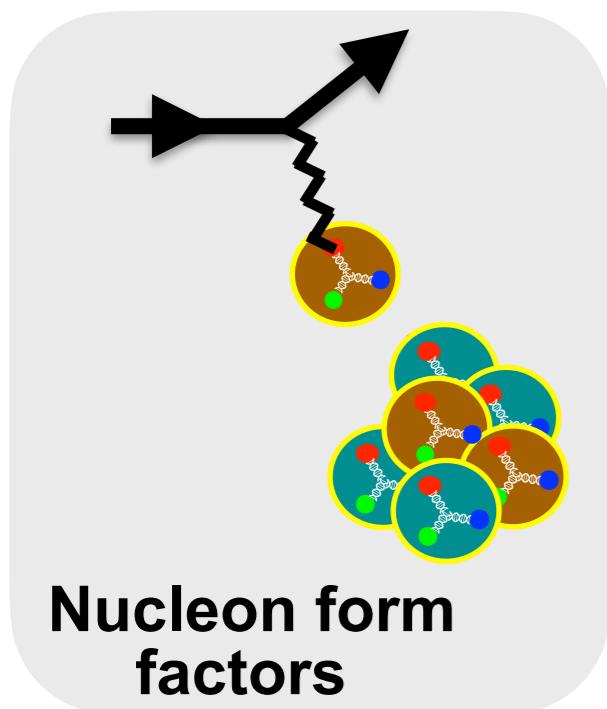


Accelerator neutrino fluxes cover a wide range of energies where different processes dominate cross-section:

- Quasi-elastic nucleon scattering
- Resonance production
- Deep inelastic scattering

Theory input required to decompose cross section into such processes and therefore predict its energy dependence

Effective theories for different energies require different inputs



Lattice QCD, EFT, and νA

LQCD can provide results for few-nucleon observables that can be matched to nuclear EFTs and models that can make predictions for larger nuclei



See Snowmass Whitepaper - Alvarez-Ruso et al “Theoretical Tools for Neutrino Scattering” arXiv:2203.09030

Results provided by LQCD and experiment are complementary

Easy for LQCD:

- Axial vs vector currents
- Isovector vs isoscalar
- Pions

Hard for LQCD:

- Large baryon number
- Real-time dynamics
- Multi-hadron states
- (Light quark masses)

Lattice QCD and νA

νA scattering amplitudes factorize into leptonic and hadronic parts

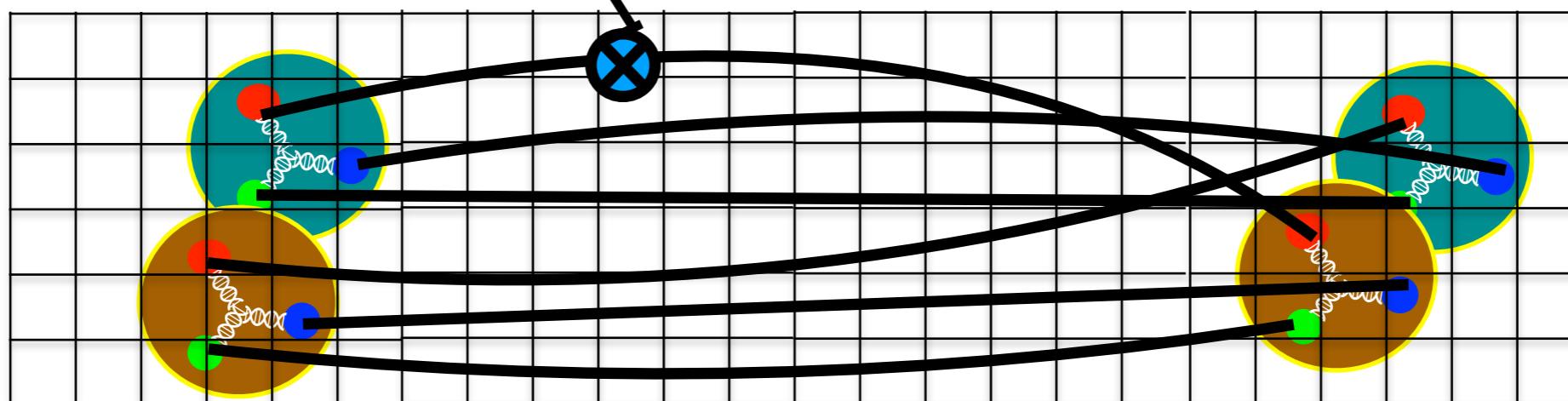
$$\mathcal{M}_{\nu A \rightarrow \ell f} \propto (\bar{u}_\ell \gamma_\mu \gamma_5 u_\nu) \langle f | \bar{q} \gamma_\mu \gamma_5 q | A \rangle + \dots$$

Generic Euclidean hadronic matrix elements calculable (in principle) using lattice QCD

$$\langle \mathcal{O} \rangle = \int \mathcal{D}U \mathcal{D}\bar{q} \mathcal{D}q e^{-S_{QCD}(U, q, \bar{q})} \mathcal{O}(U, q, \bar{q}) \approx \frac{1}{N_{\text{cfg}}} \sum_{i=1}^{N_{\text{cfg}}} \mathcal{O}(U_i)$$

Quark fields integrated out
analytically, propagators
obtained with matrix inversion
(Dirac matrix size $\sim 10^9 \times 10^9$)

Monte Carlo sample
gluon fields with
probability $\propto e^{-S}$



$\langle 0 | f$

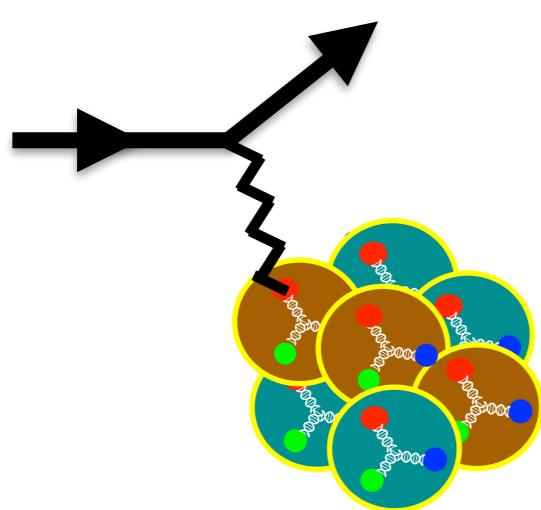
$\bar{q} \gamma_\mu \gamma_5 q$

$A^\dagger | 0 \rangle$

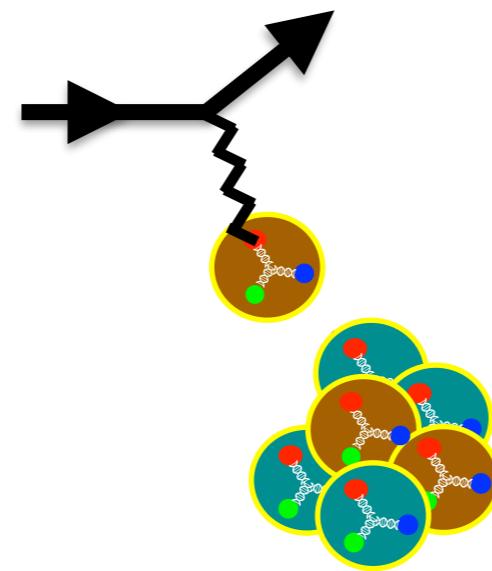
Nucleon form factors

Single-nucleon currents dominate at low energies where nucleons are effective degrees of freedom for nuclei, multi-nucleon currents provide corrections

Impulse approximation

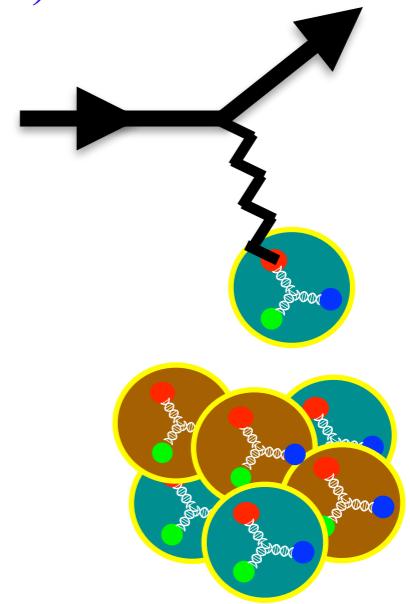


\approx



Benhar, Day, Sick, Rev. Mod. Phys. 80 (2008)

+



Nucleon vector and axial form factors are key inputs to nuclear EFTs / models

$$\langle N(p') | A_\mu(q) | N(p) \rangle = \bar{u}(p') \left[G_A(Q^2) \gamma_\mu \gamma_5 + \frac{\tilde{G}_P(Q^2)}{2M_N} q_\mu \gamma_5 \right] u(p)$$

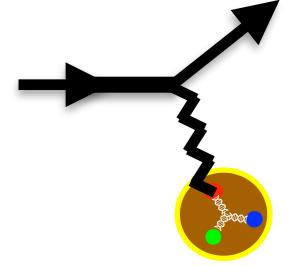
Electron scattering experiments can precisely measure vector form factors, which provides important validation

See e.g. Khachatryan et al. [CLAS and e4v] Nature 599 (2021)

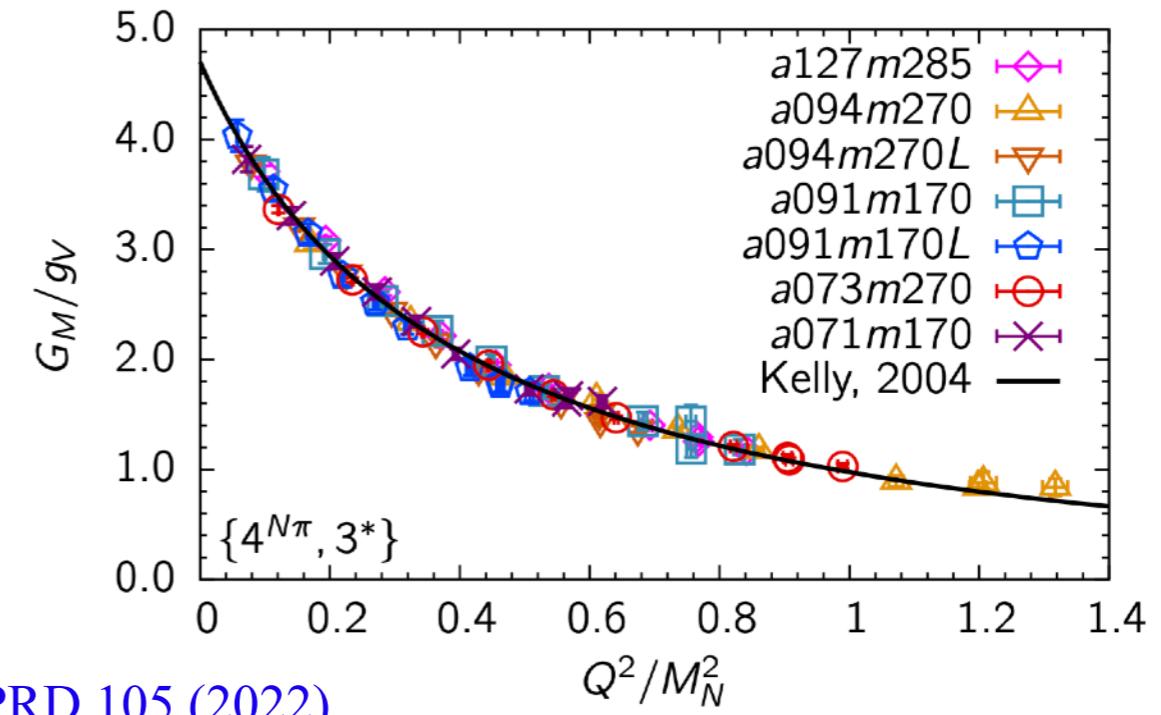
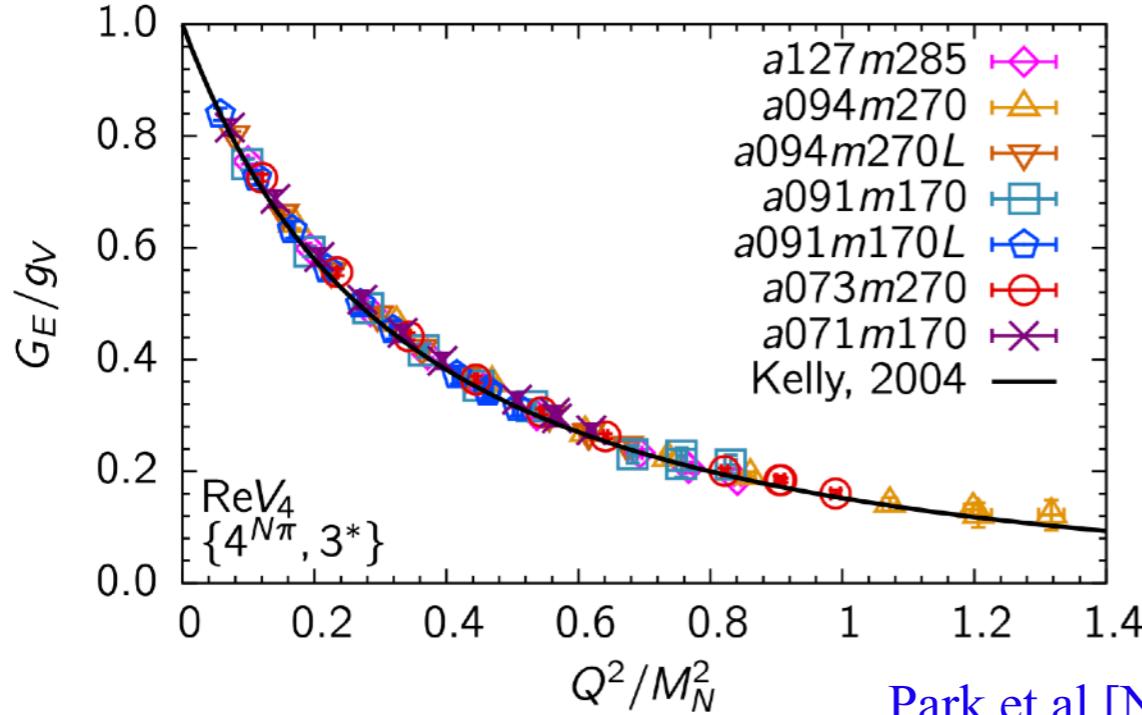
Corrections to impulse approximation can be included systematically

Review: Rocco, Front. Phys. 29 (2020)

Form factors and LQCD



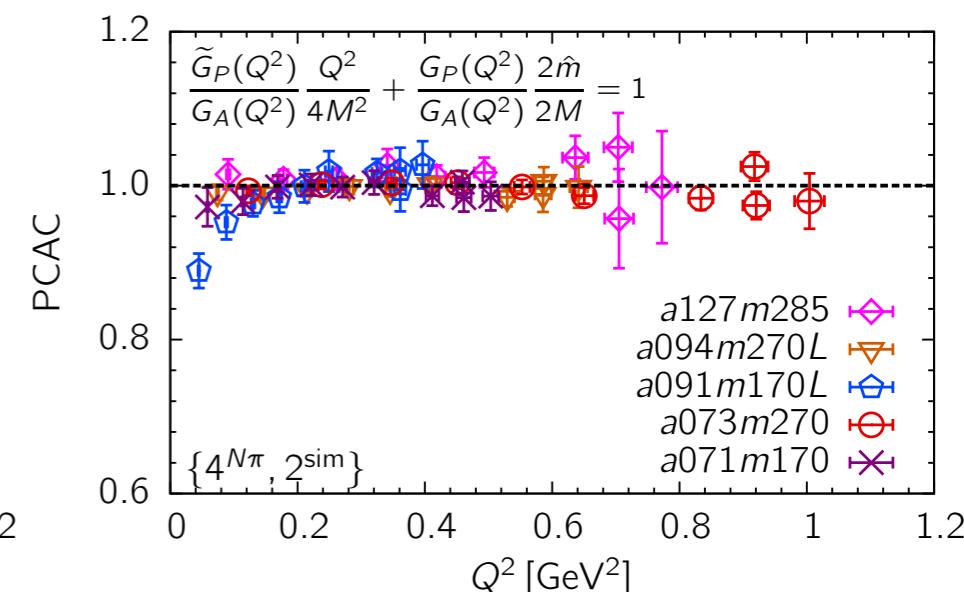
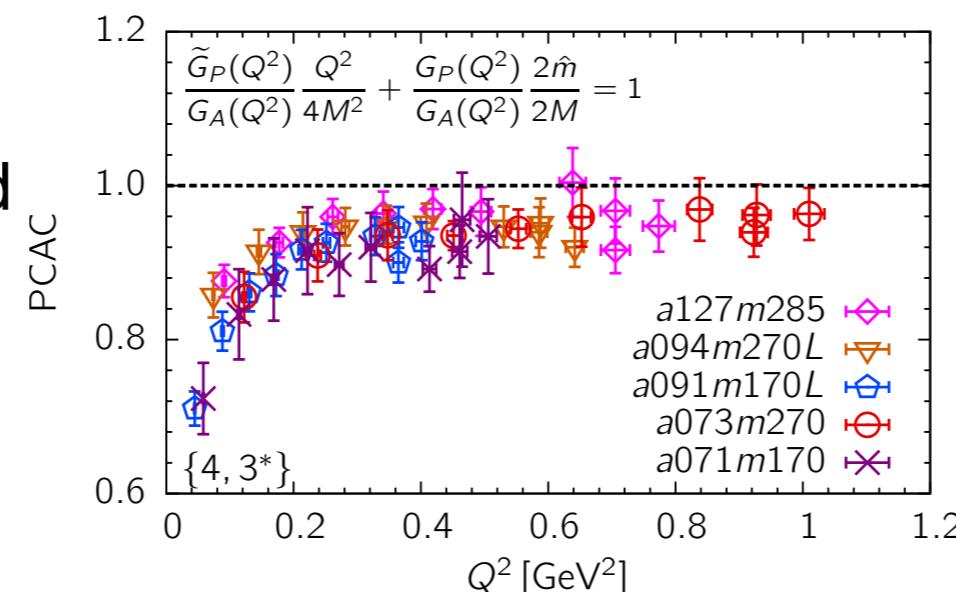
Vector and axial form factors recently calculated using nearly physical quark masses:



Park et al [NME], PRD 105 (2022)

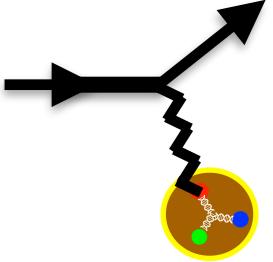
LQCD nucleon electric and magnetic form factor results agree with phenomenological parameterizations after accounting for excited-state and discretization effects

Careful treatment of $N\pi$ excited states required to reproduce consequences of axial ward identities that assume ground-state dominance

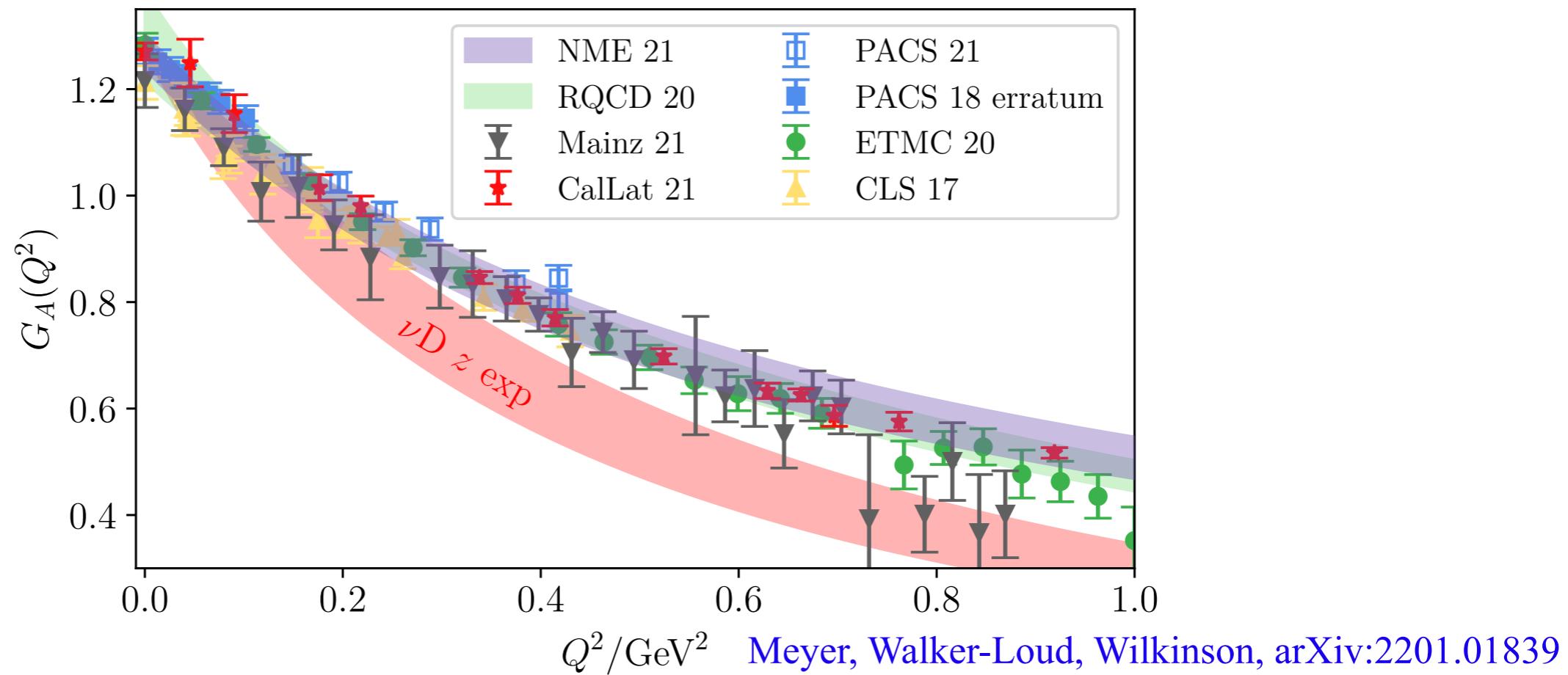


Park et al [NME], PRD 105 (2022)

Axial form factors



Recent axial form factor calculations include physical quark masses, continuum / infinite-volume extrapolations, and excited-state fits that account explicitly for $N\pi$ states



Differences between LQCD and experimental axial form factor determinations could arise from challenging LQCD systematic uncertainties (excited states, lattice spacing, ...)

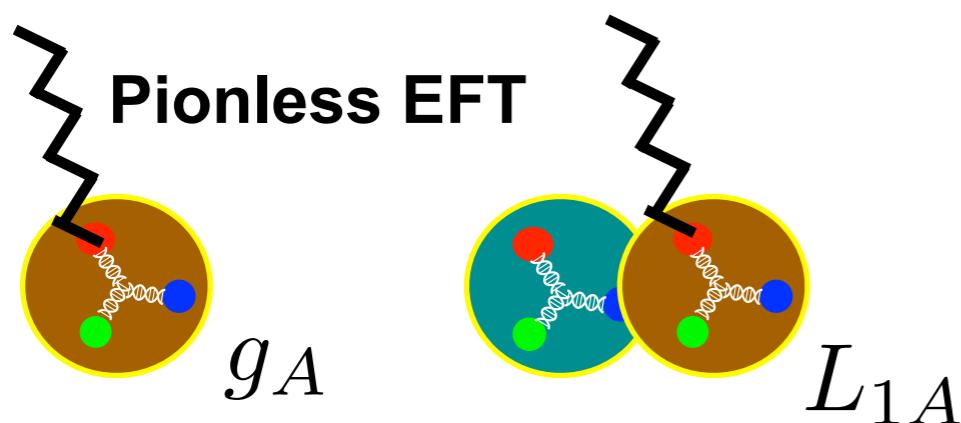
Differences could also arise from underestimated uncertainties in phenomenological form factor determinations using deuterium bubble chamber data

Axial currents in nuclei

QCD interactions make nuclei differ from collections of nucleons

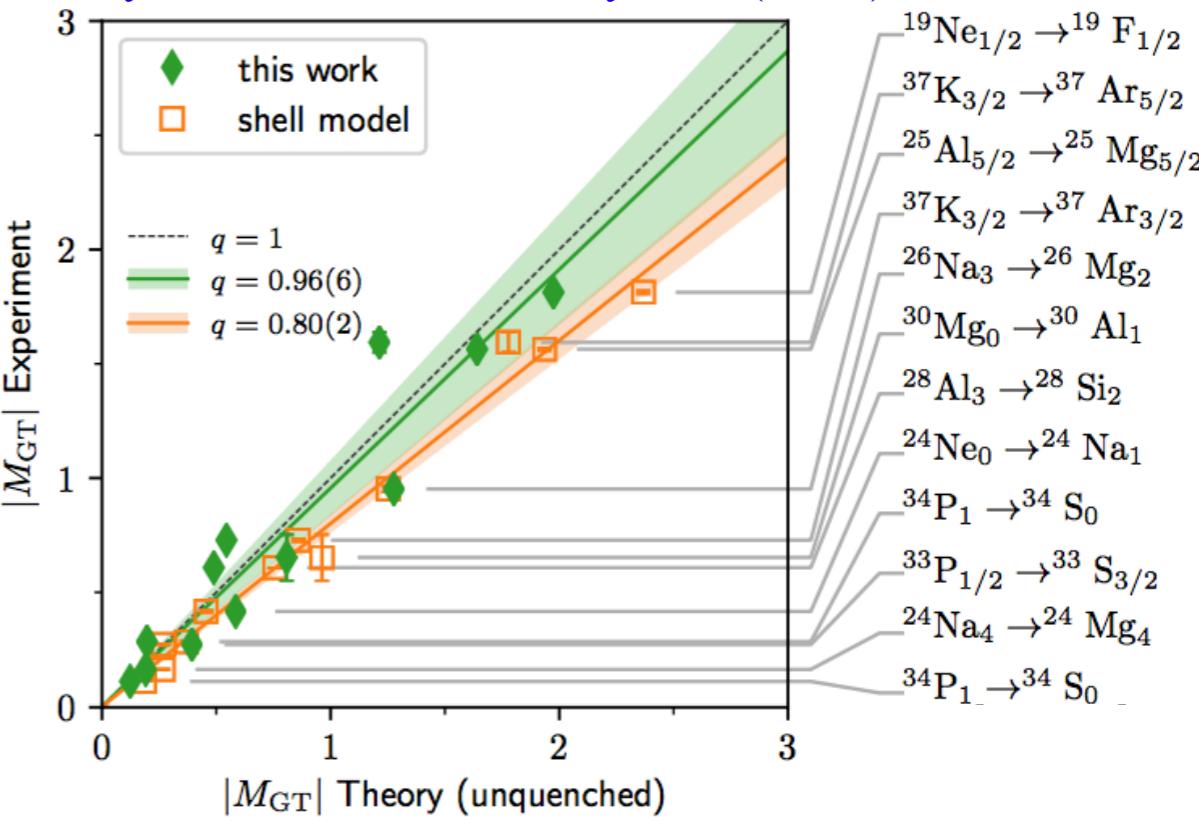
Modern nuclear theory calculations including multi-nucleon correlations and currents with e.g. chiral EFT can reproduce experiment without “quenching” g_A

Few-nucleon LQCD results can constrain two-body currents in nuclear models and EFTs



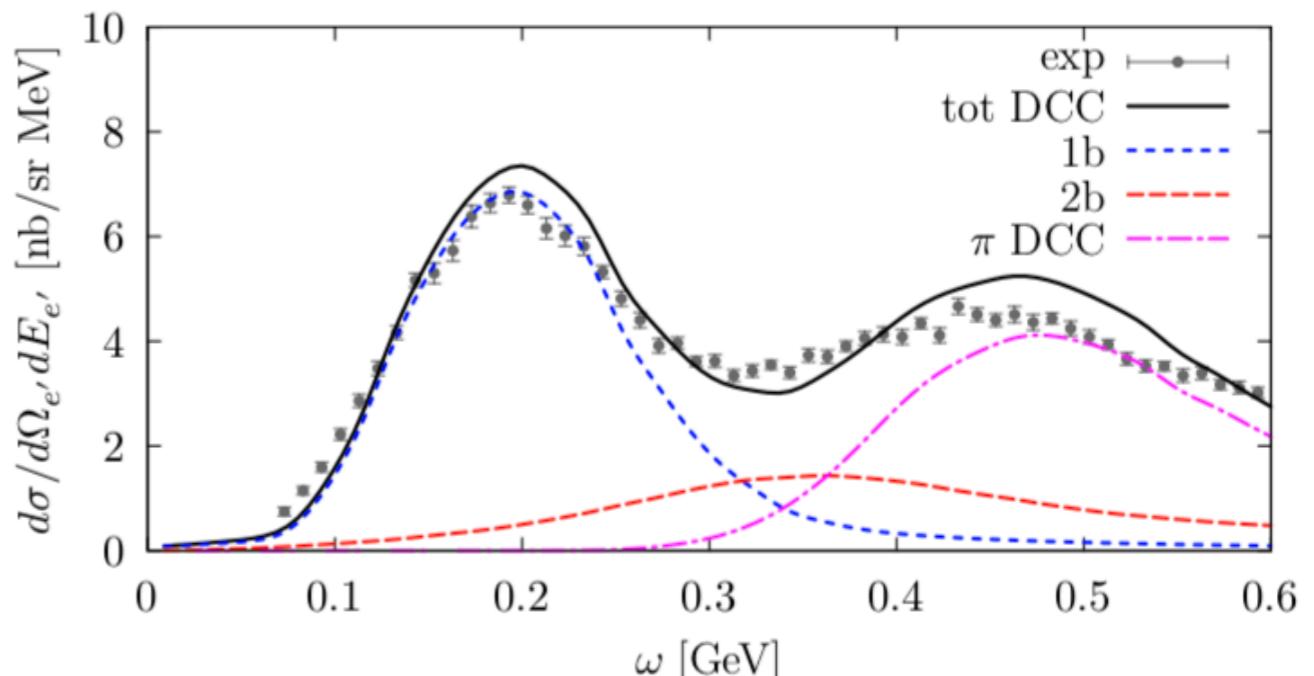
Percent-level accuracy for neutrino-nucleus cross sections will require more precise two-body current inputs

Gysbers et al, Nature Phys. 15 (2019)



Rocco, Nakamura, Lee, Lovato, PRC 100 (2019)

$E_e = 961 \text{ MeV}, \theta_e = 37.5^\circ$

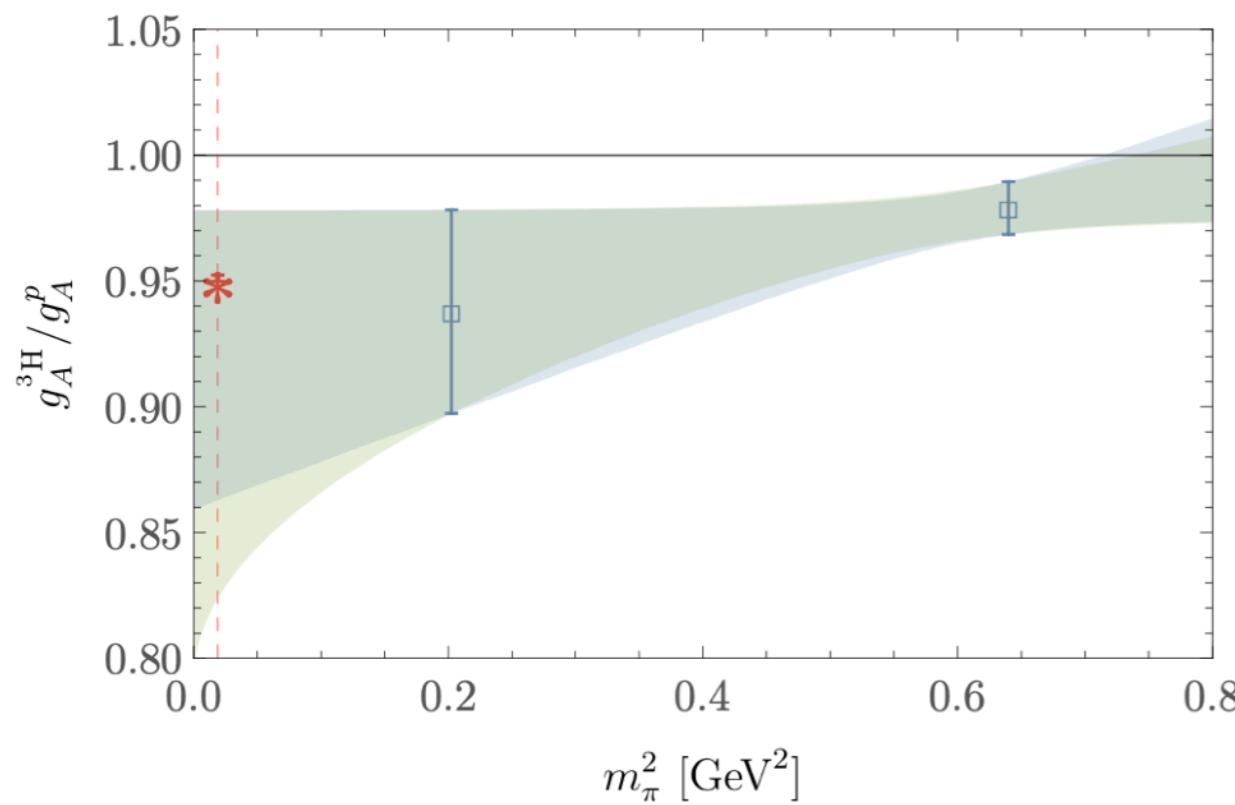
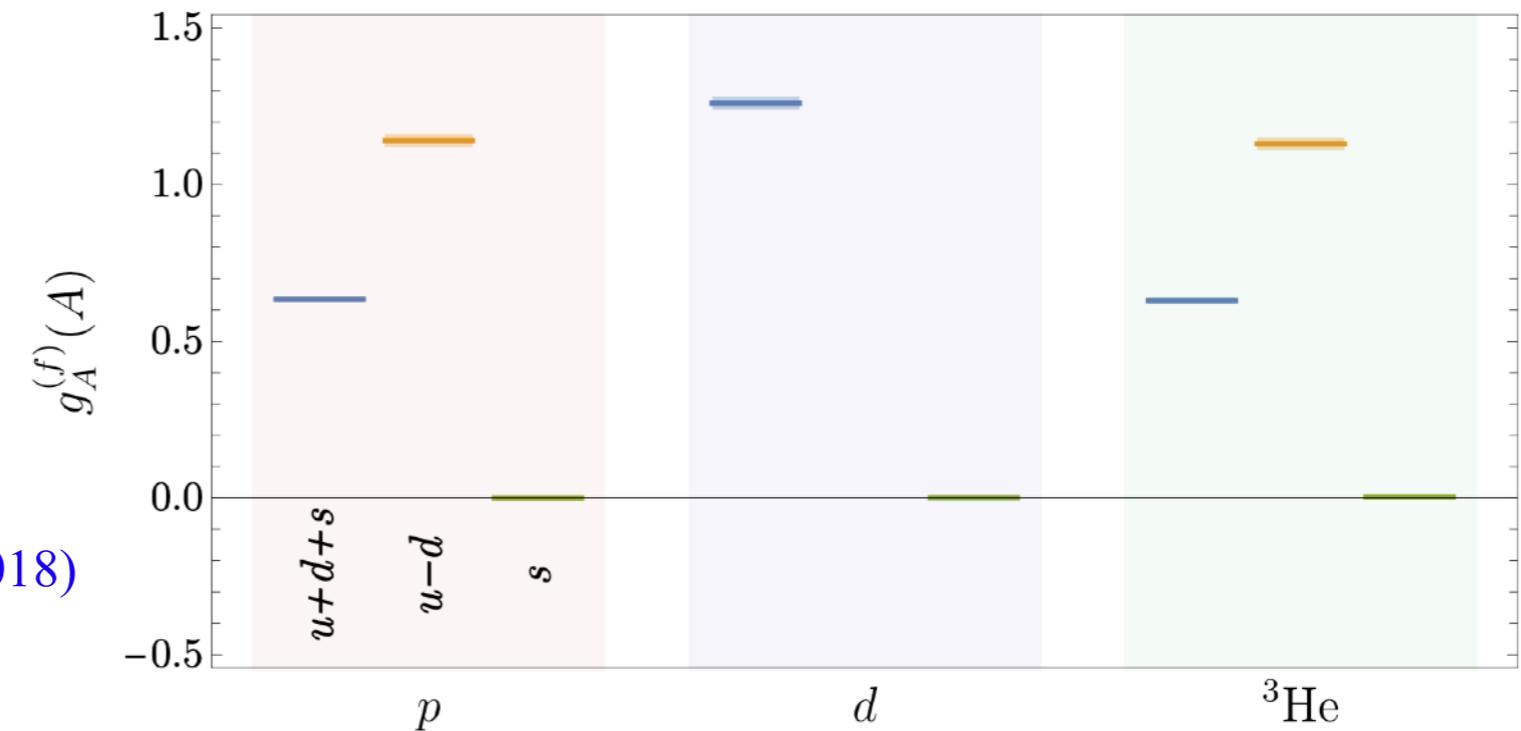


Two-body currents in LQCD



Flavor decomposition of axial matrix elements of two and three nucleon systems computed with $m_\pi = 806$ MeV

Chang, MW et al [NPLQCD], PRL 120 (2018)



Parreño, MW et al [NPLQCD] PRD 103 (2021)

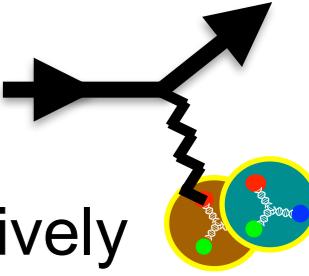
Axial current matrix element calculations with $m_\pi = 450$ MeV permit preliminary extrapolation of triton axial charge to physical point

Several systematic uncertainties remain, but encouraging agreement with experiment seen

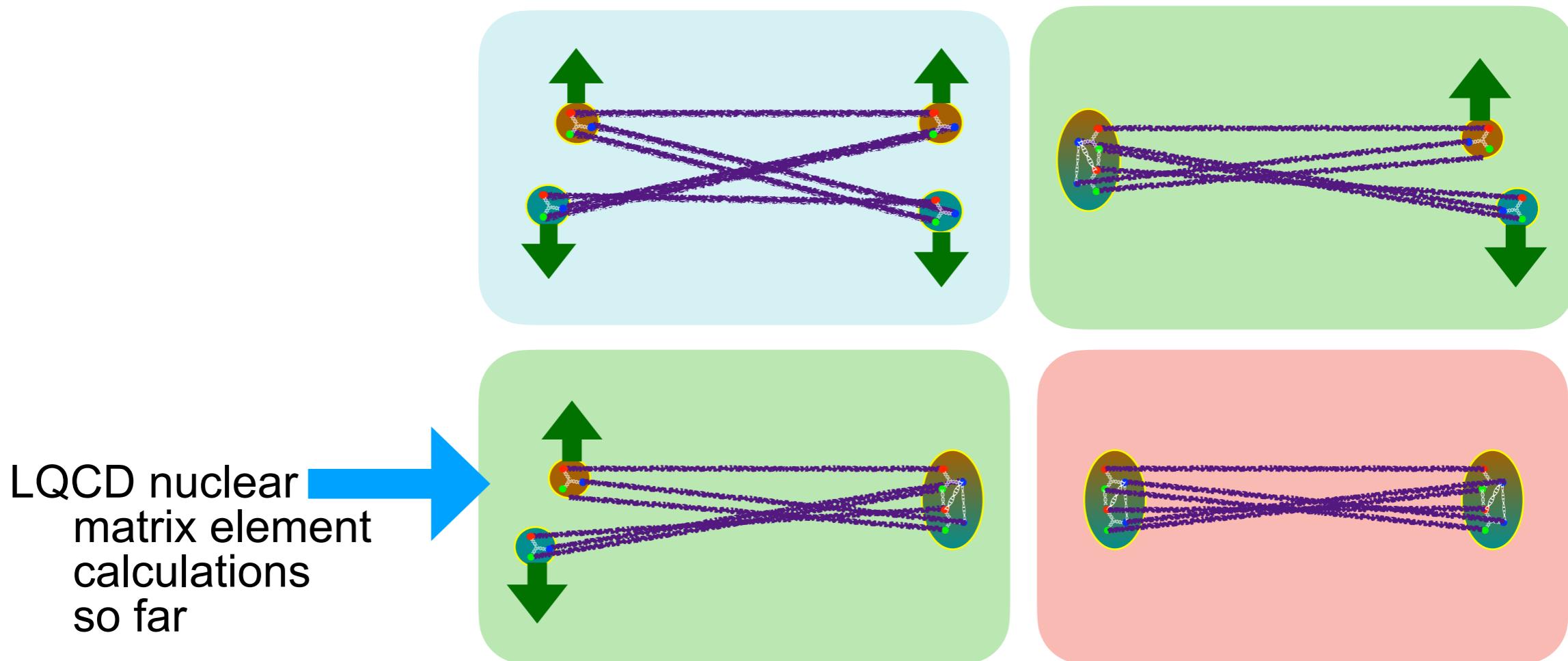
Matching to finite-volume pionless EFT used to constrain L_{1A}

Detmold and Shanahan, PRD 103 (2021)

Variational methods



Excited-state effects from unbound multi-nucleon scattering states are not effectively suppressed in computationally accessible Euclidean correlation functions



First calculations of plane-wave nucleon-nucleon (upper left) correlation functions and symmetric correlation-function matrices indicate that excited-state effects lead to large systematic uncertainties on calculated energy levels

Francis et al, PRD 99 (2019)

Green et al, PRL 127 (2021)

Hörz et al, PRC 103 (2021)

Amarasinghe, MW et al, arXiv:2108.10835

Interpolating operators

Known from $\pi\pi$ scattering studies near the ρ resonance that local and nonlocal operators can be nearly orthogonal

Dudek et al, PRD 87 (2013)

$$\bar{q}(x)\Gamma q(x)$$

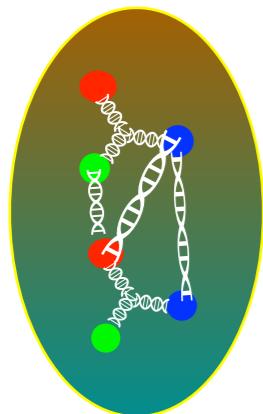
$$\pi(\vec{p}_1)\pi(\vec{p}_2)$$

Wilson et al, PRD 92 (2015)

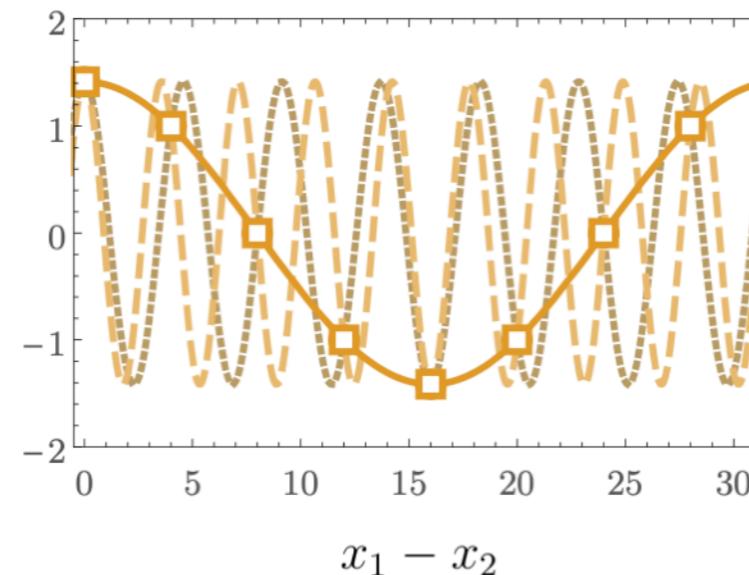
Calculations with $t \sim$ few fm neglecting one type of operator show plateau-like behavior but energy spectra with “missing levels” (compared to more complete calculations)

Wide range of two-nucleon operators explored [Amarasinghe, MW et al, arXiv:2108.10835](#)

Hexaquark:

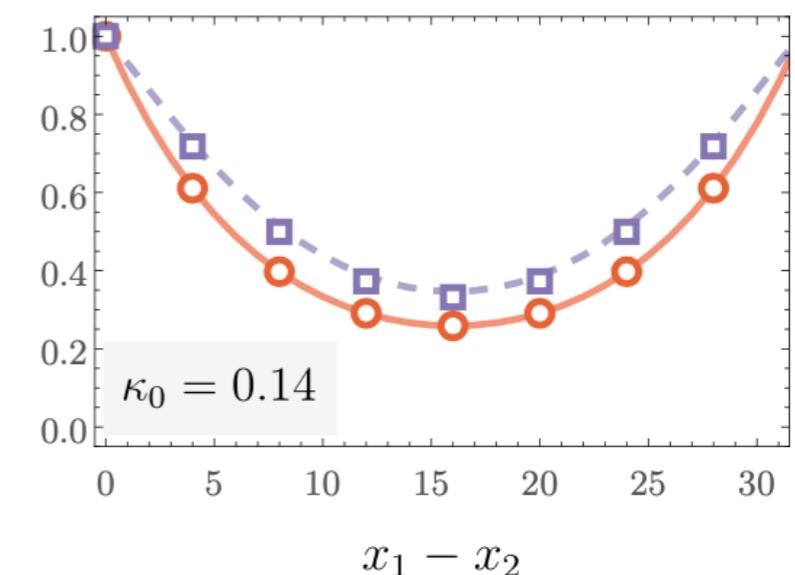


Dibaryon:



Six Gaussian smeared quarks

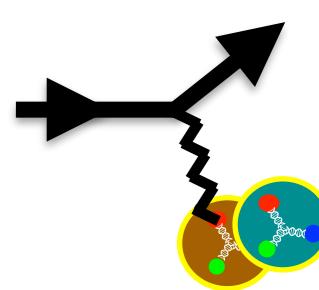
Quasi-local:



Two plane-wave baryons with relative momenta

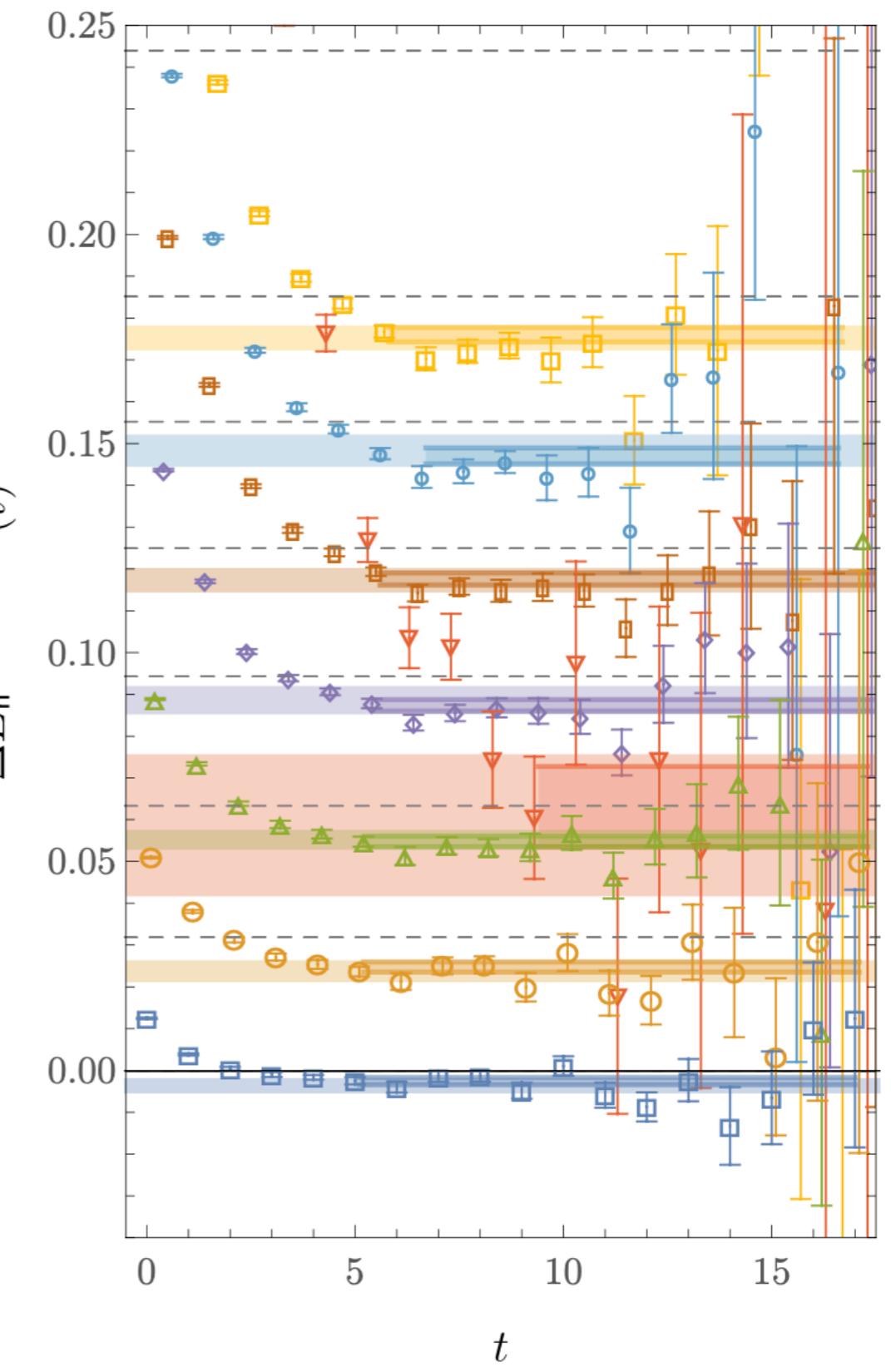
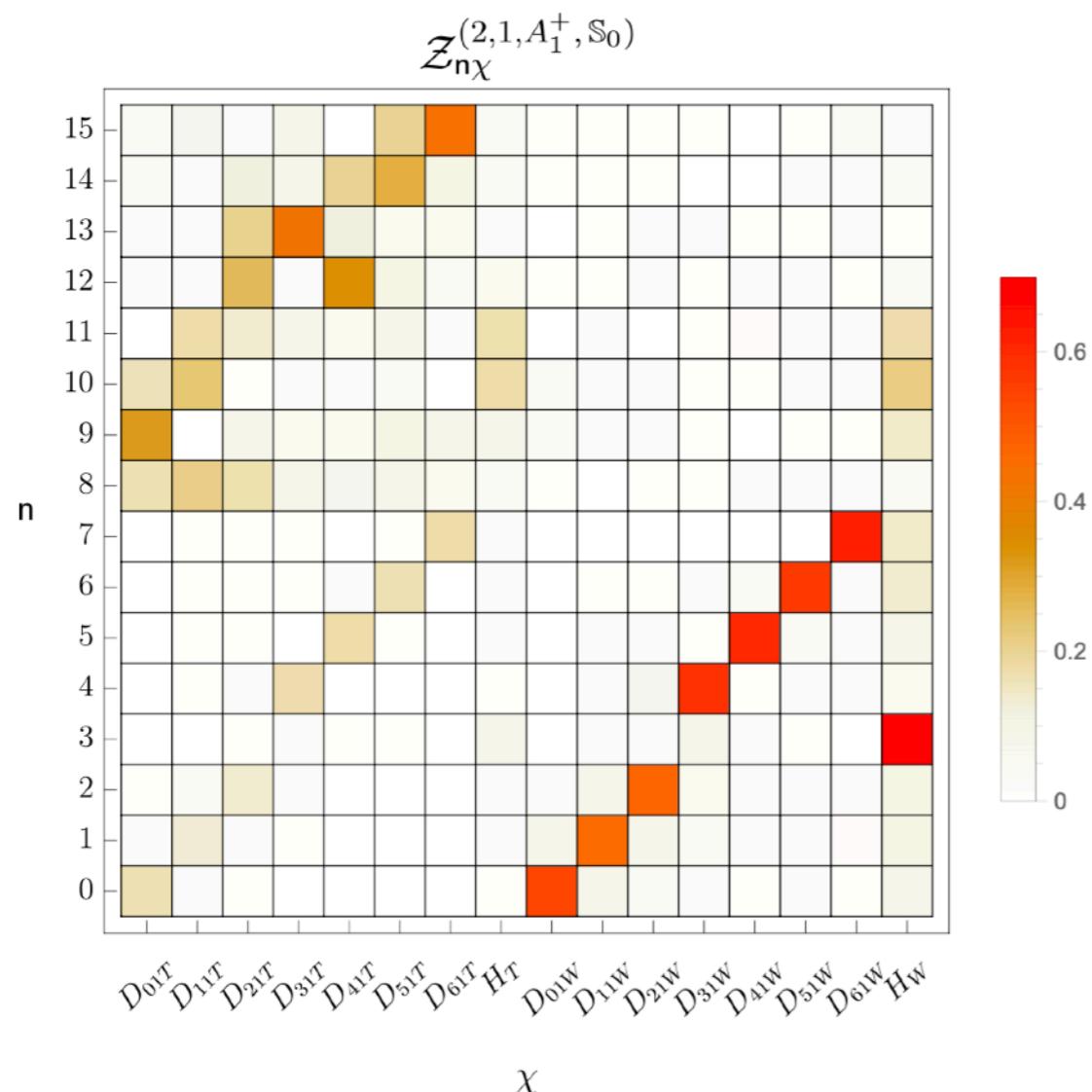
Two exponentially localized baryons

Two nucleons in a box

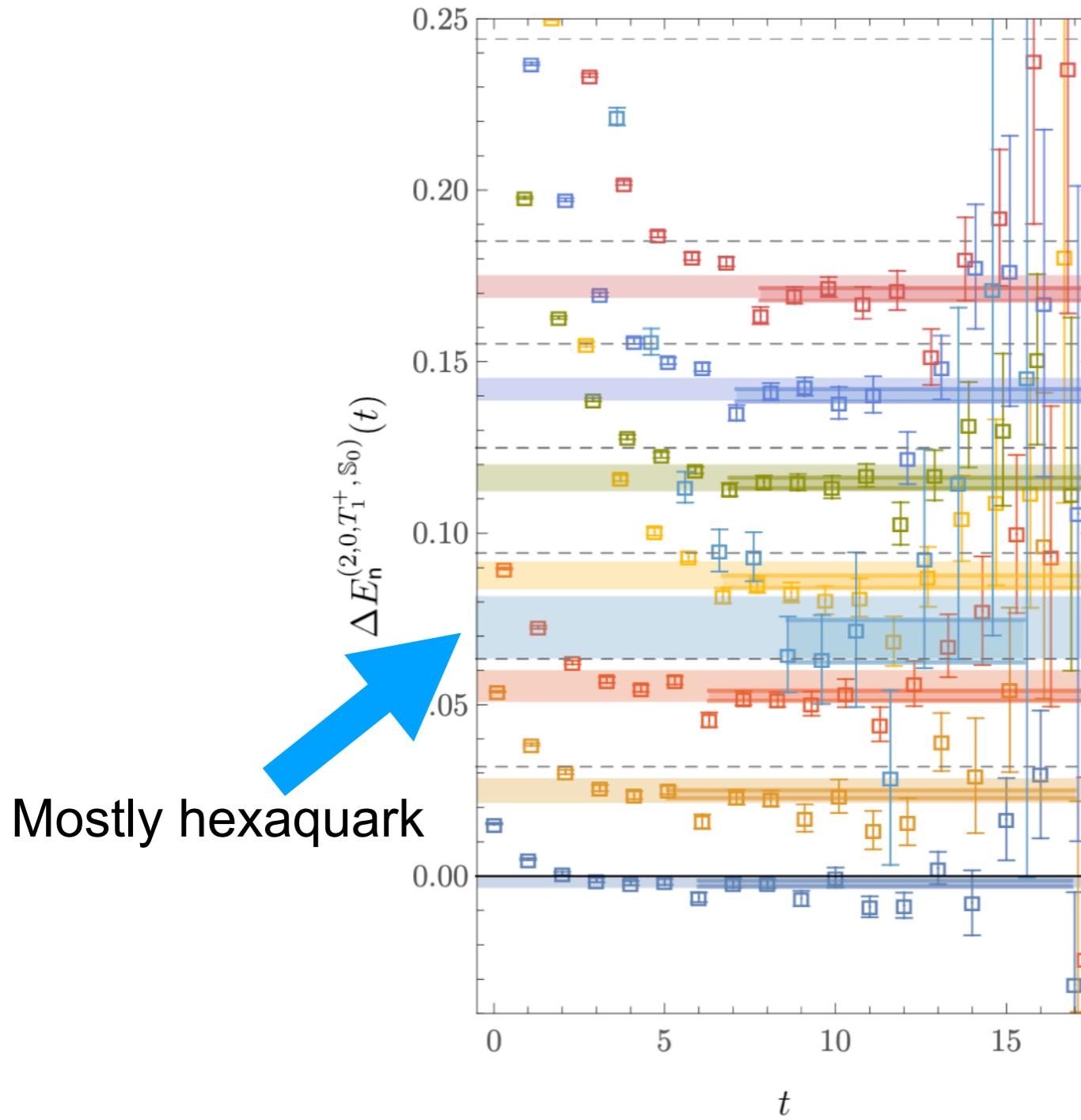
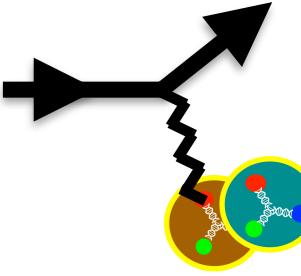


Diagonalization of correlation-function matrices can be used to remove excited-state contamination from states strongly overlapping with other operators

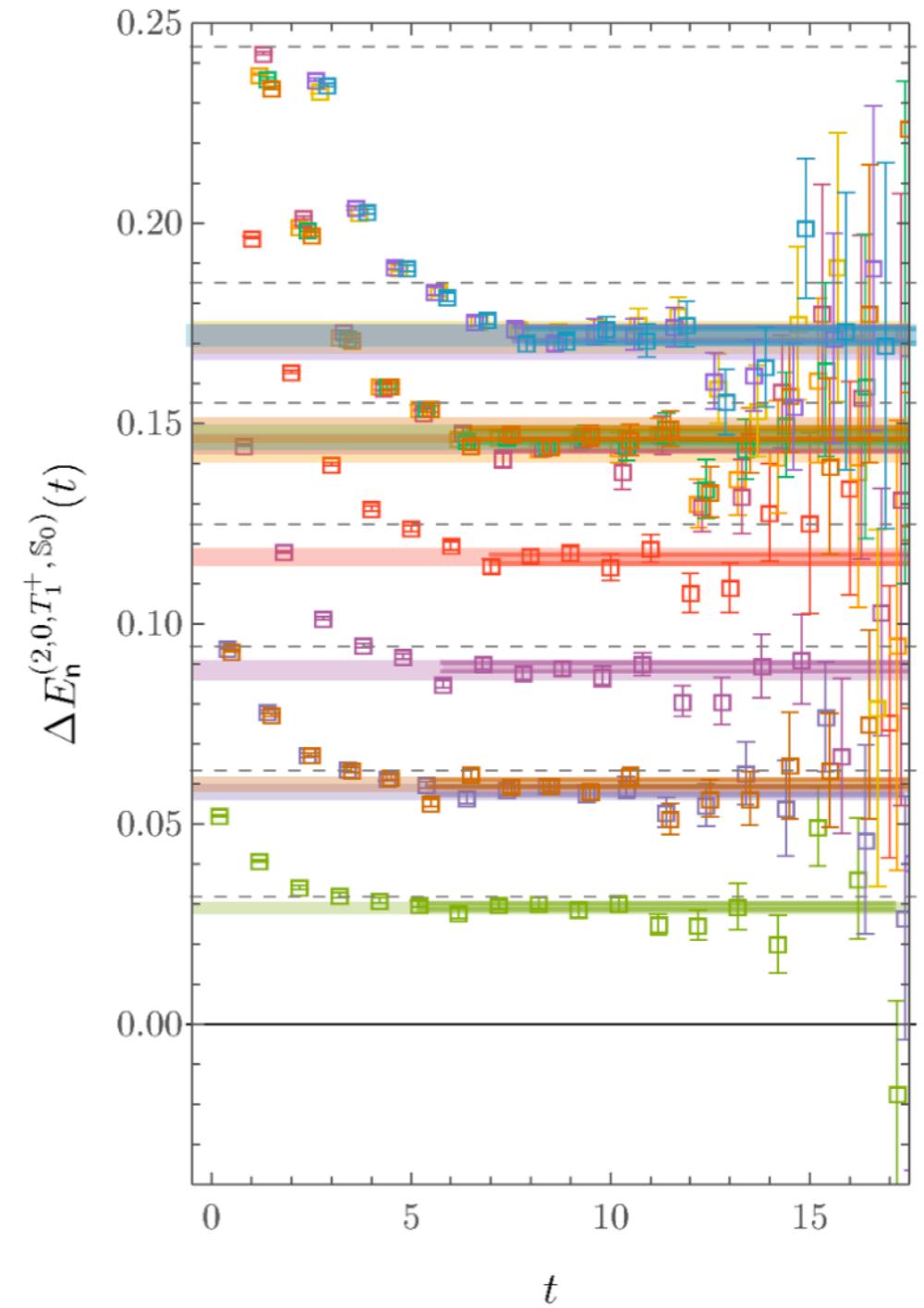
Each energy level dominantly overlaps with one operator structure, sub-dominant operators collectively 30%



Deuteron spectrum



Cubic analog of S-wave



Cubic analog of D -wave, G -wave, ...

Interpolating-operator dependence

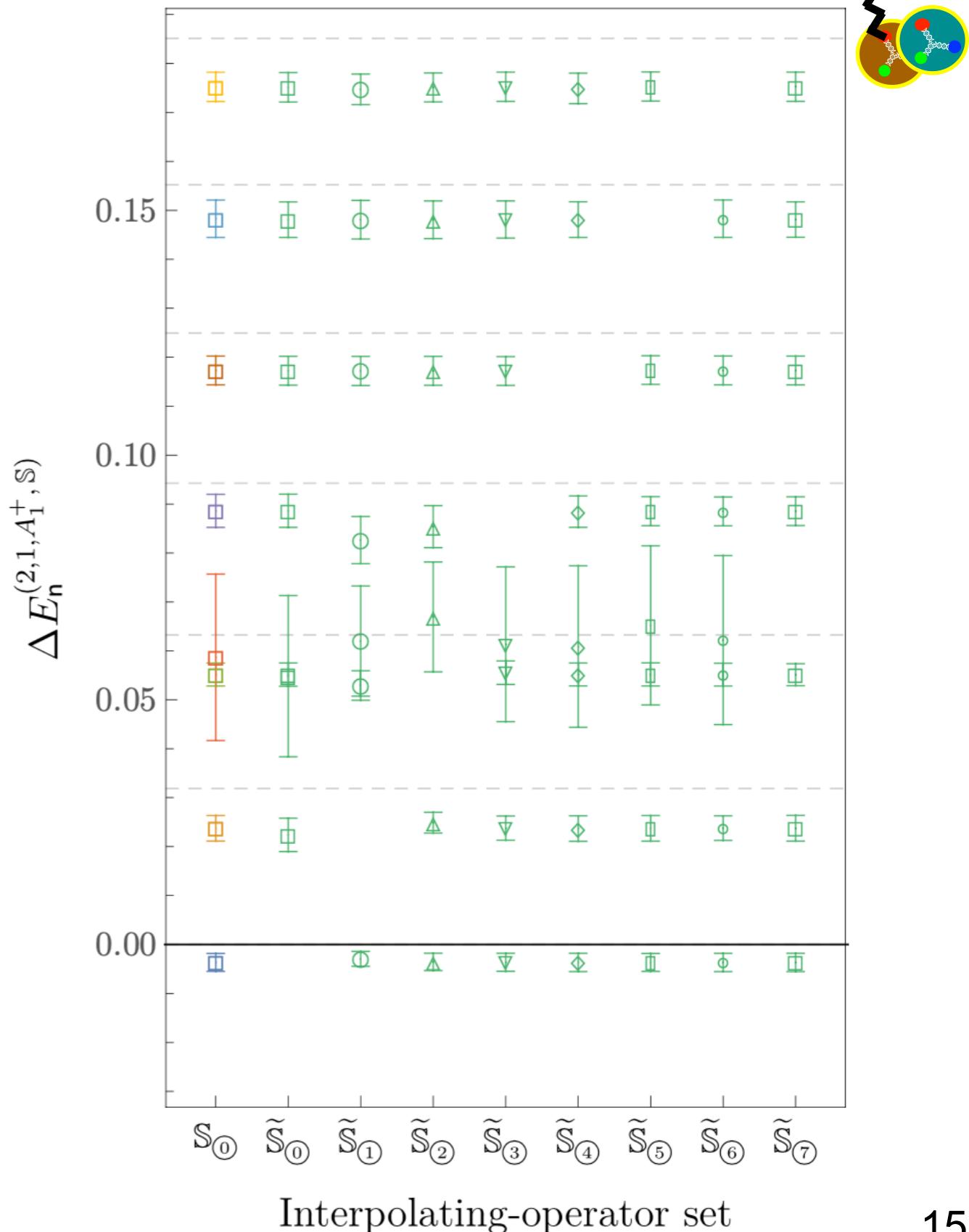
Removing the operator structure with maximum overlap on to a given energy level leads to “missing energy levels”

Even with 10s of interpolating operators, possible to “miss” ground-state

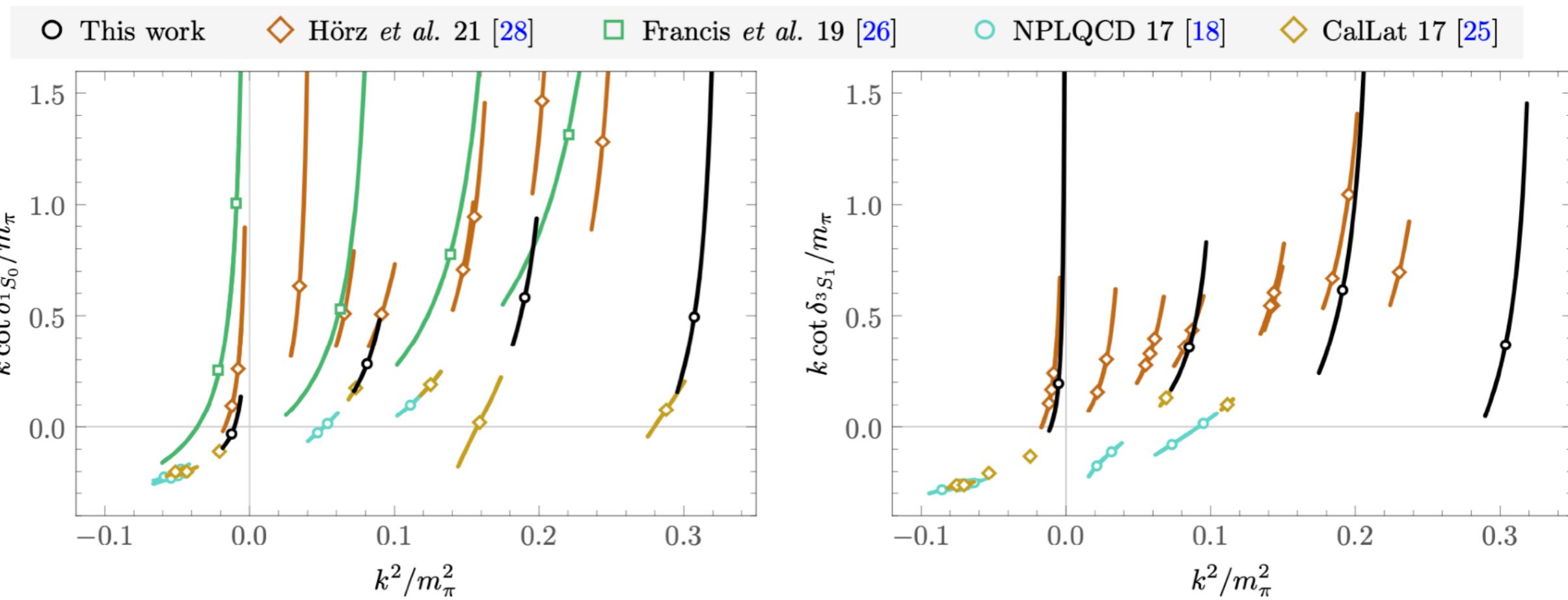
- valid lower bound on ground-state energy, but best-fit results can differ by $5+ \sigma$

Consistent with various dibaryon and hexaquark operators being approximately orthogonal

Much larger ($t \gtrsim 1/\delta \sim 5$ fm) source/sink separations would be needed to resolve spectrum using interpolating-operator set missing dominant operators



NN phase shift comparisons

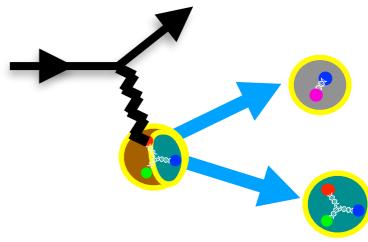


S-wave phase shift results using variational methods and symmetric dibaryon correlation functions consistent among several groups

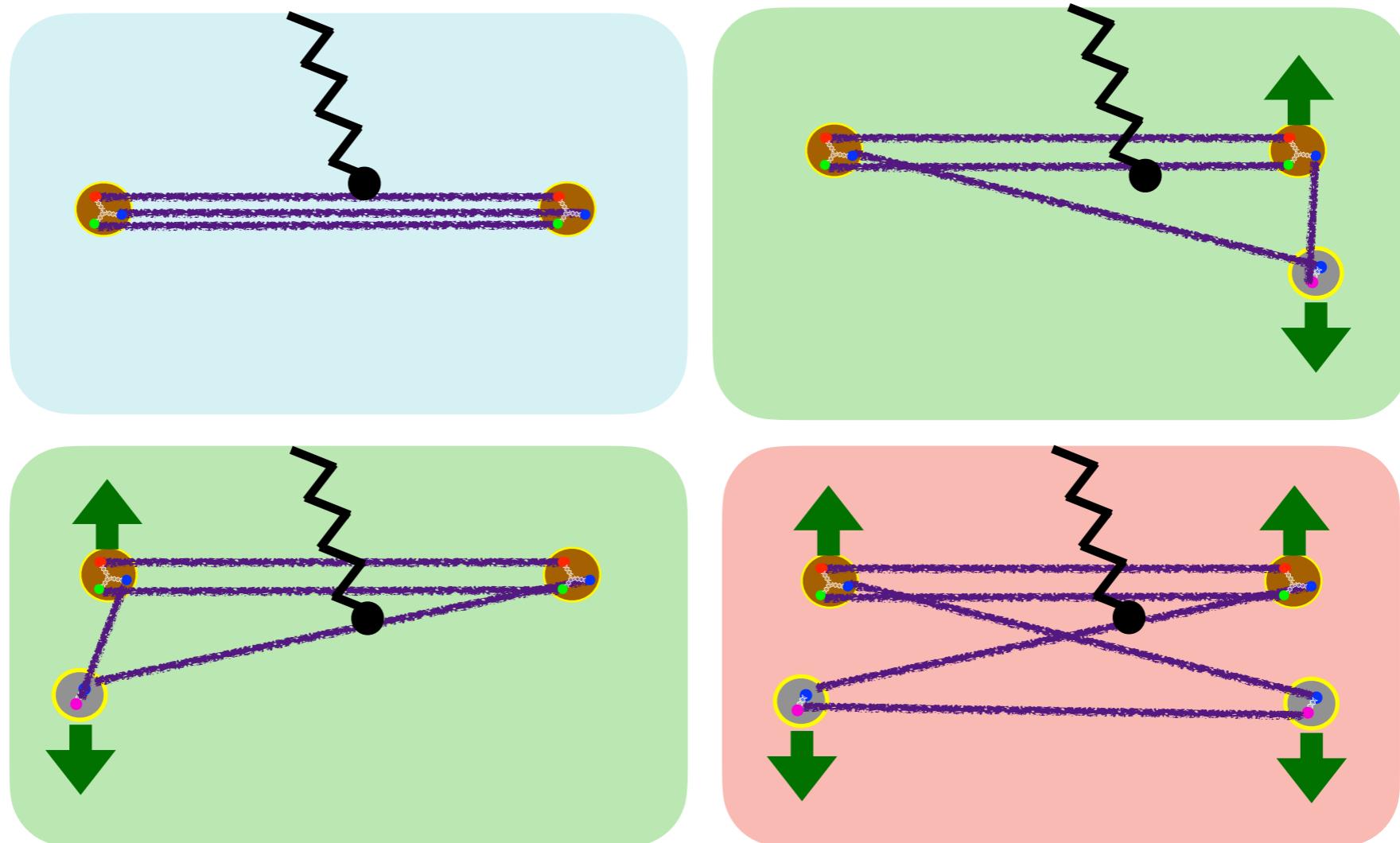
Discrepancies with previous results using dibaryon-hexaquark correlation functions on the same gauge-field ensemble from multiple groups

Further variational studies are needed to conclusively determine whether two-nucleon systems bind with heavier-than-physical quark masses

Variational methods for $N\pi$



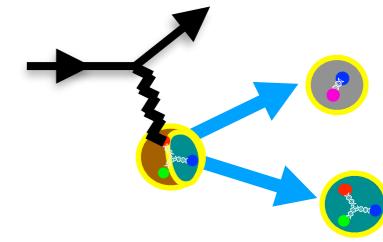
Analogous variational methods can be applied to study $N\pi$ systems



Same calculation can be used to explicitly remove excited-state contamination from elastic nucleon form factors and access pion-production amplitudes

$N \rightarrow \Delta$ transition form factors can be calculated if $I=1/2$ and $I=3/2$ operators included

$N\pi$ systems in LQCD

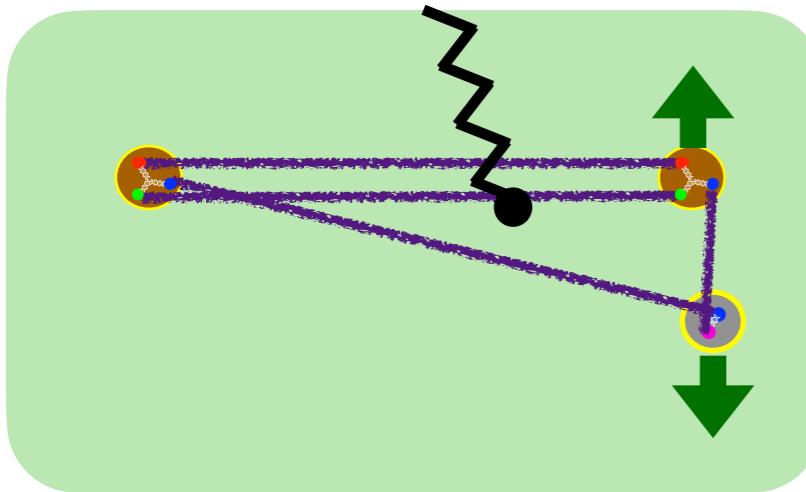


The $I=J=3/2$ sector has been explored using variational calculations including both localized $\Delta \sim qqq$ and $N\pi$ operators

Andersen, Bulava, Hörz, Morningstar, PRD 97 (2018)

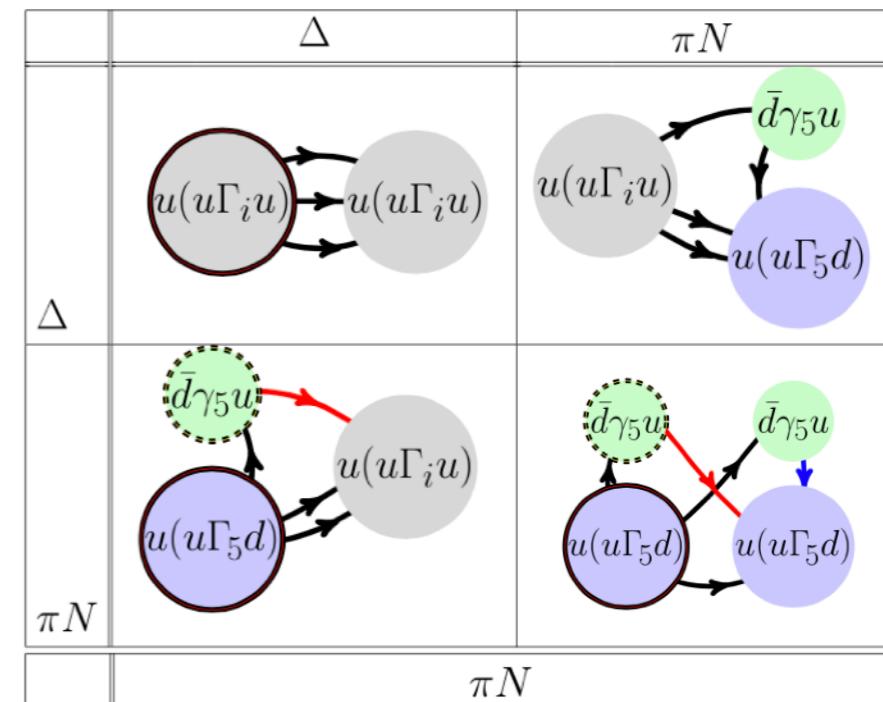
Silvi, Paul, Alexandrou, Krieg, Leskovec, Meinel, Negele, Petschlies, Pochinsky, Rendon, Syritsyn, and Todaro, PRD 23 (2021)

Finite-volume energy spectra are mapped through generalizations of Lüscher's quantization condition to constraints on P-wave $N\pi$ scattering phase shifts

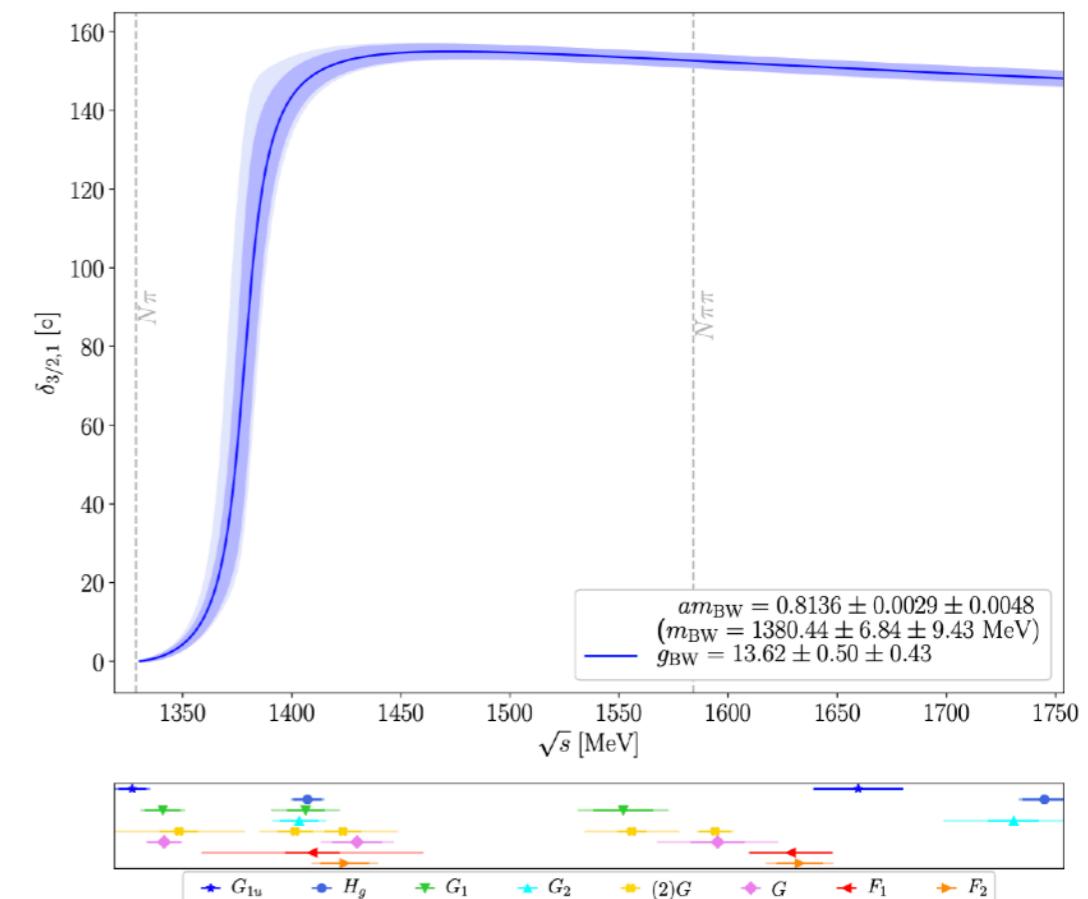


Calculations of $N \rightarrow \Delta$ transition form factors under active exploration

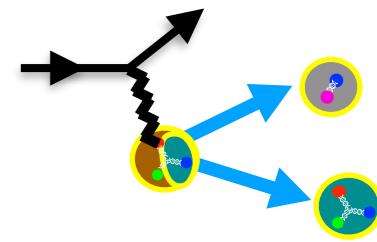
Barca, Bali, and Collins, arXiv:2110.11908 [hep-lat]



Silvi et al, PRD 23 (2021)

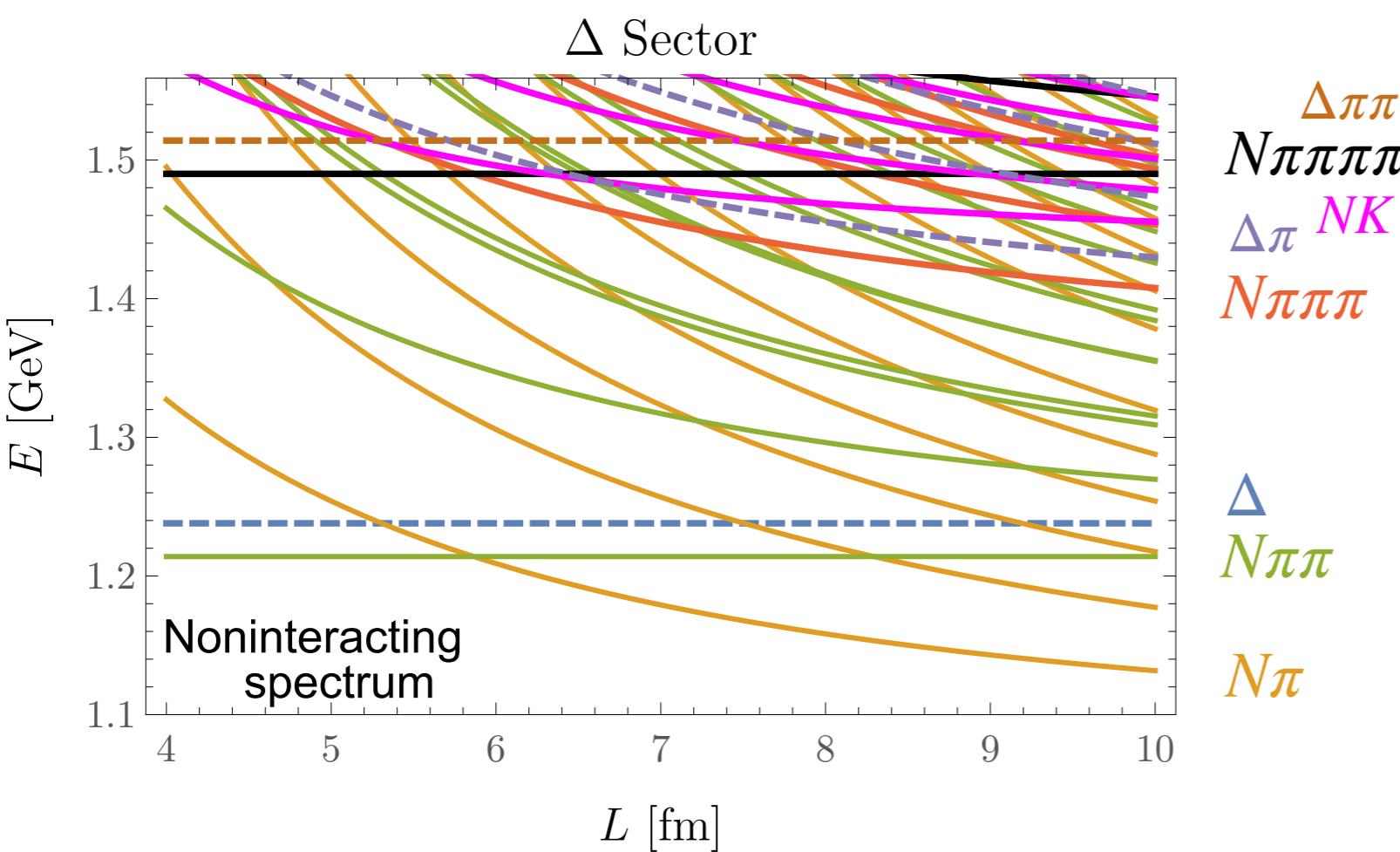


$N\pi, N\pi\pi, \dots$ systems in LQCD

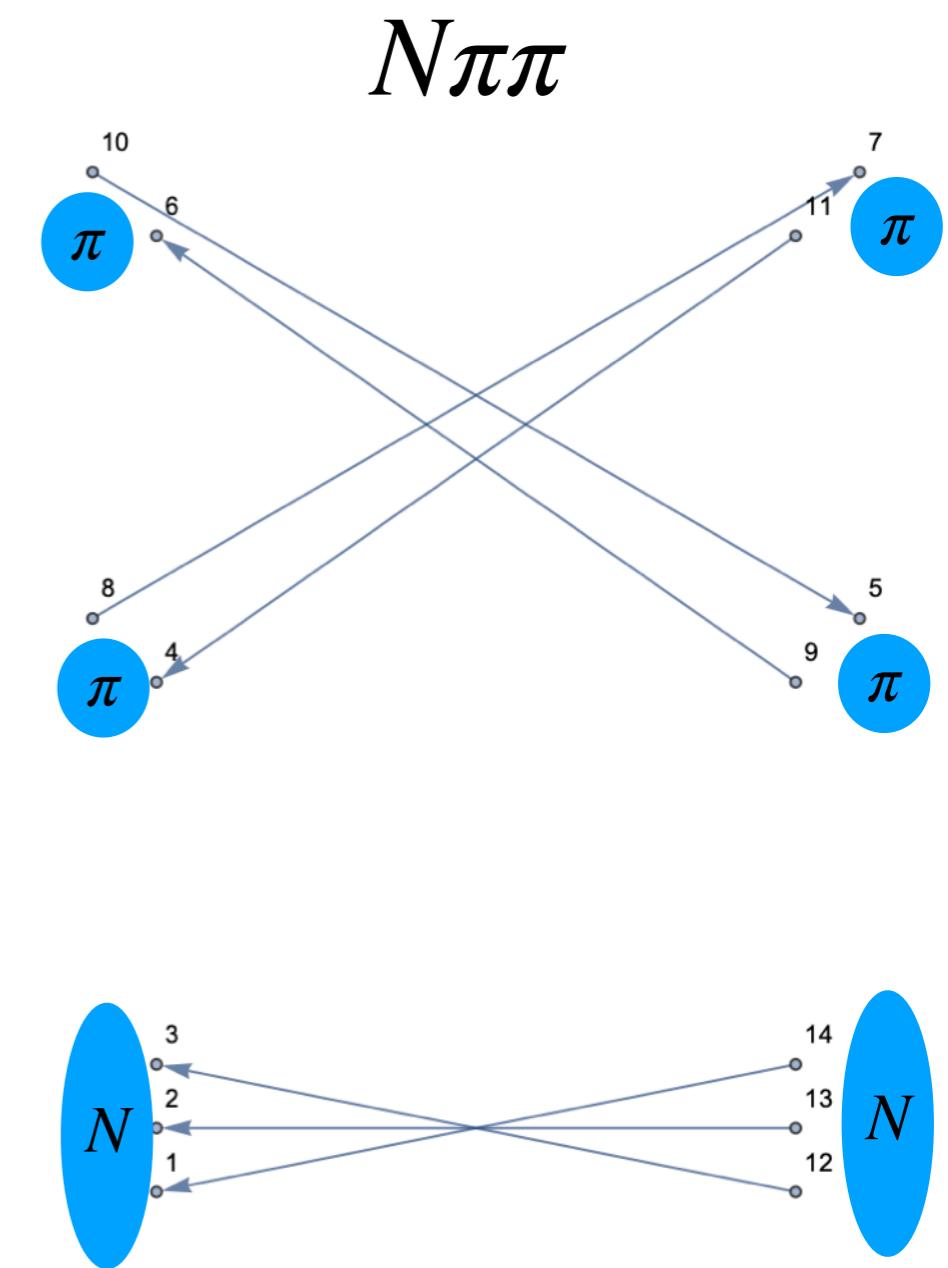


Finite-volume spectrum is complicated by meson/nucleon resonances

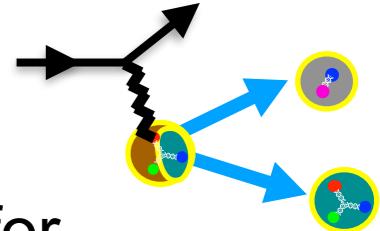
Several types of multi-hadron states expected in energy region relevant for Δ and higher nucleon resonances



Wick contractions proliferate for multi-hadron operators — similar tricks can be used as in NN



Wavefunction construction



Infinite volume wavefunctions:

- Infinitely many different orbital angular momentum states are possible for 2+ hadron states
- Explicitly constructed from products of spherical harmonics + Clebsch-Gordan

Finite cubic box wavefunctions:

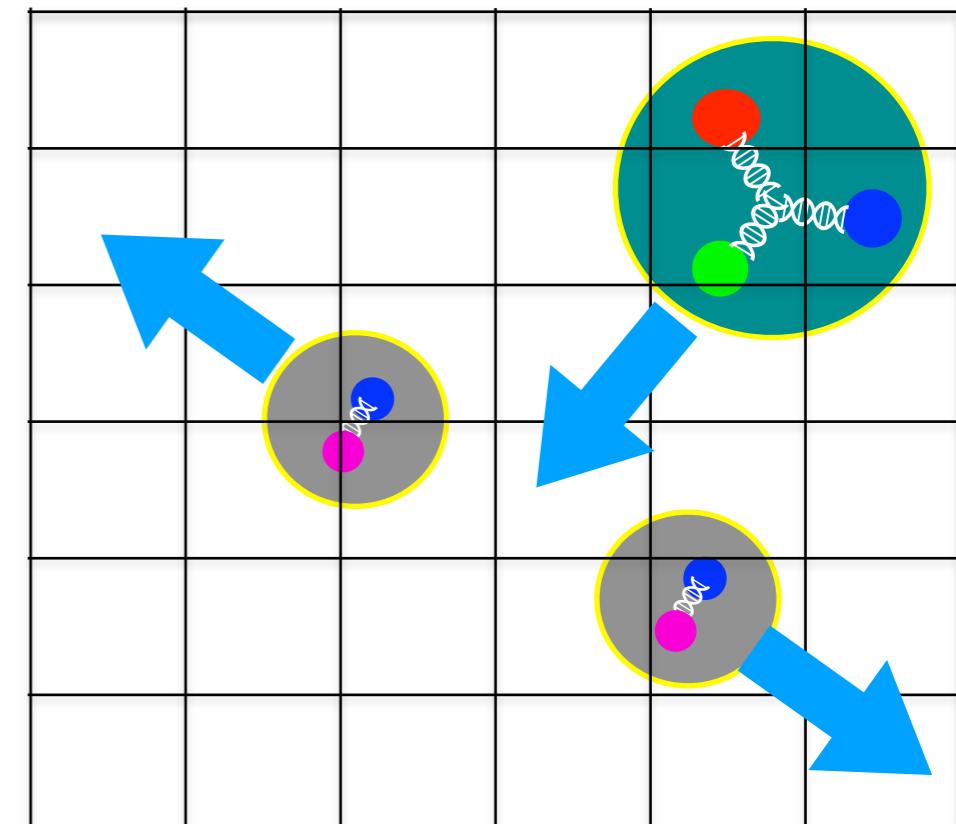
2-hadron: Luu and Savage PRD 83 (2011)

Morningstar et al, PRD 88 (2013)

N -hadron: Detmold, Jay, Kanwar, Shanahan, MW, *in preparation*

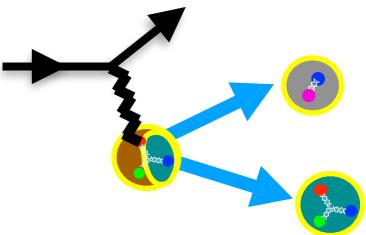
Fully classified by symmetry groups of center-of-mass and total wavefunctions (“little group” and “stabilizer group”)

- Rest frame: 7 classes of wavefunctions possible (same for all N)
- Boosted frame: 16 classes of wavefunctions possible (same for all N)



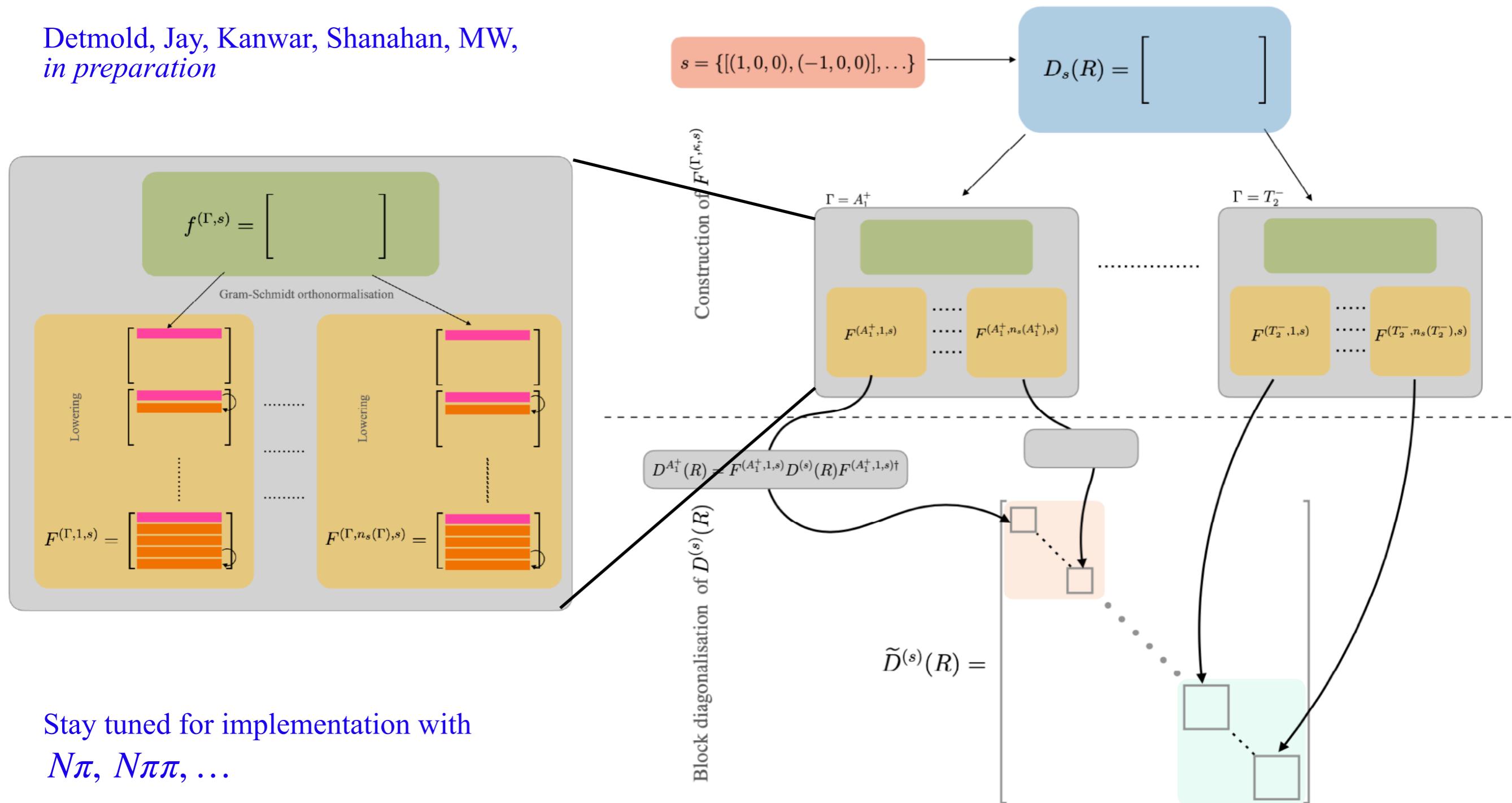
Identical particles / spin can be incorporated with subsequent group theory projections / products

Wavefunction construction



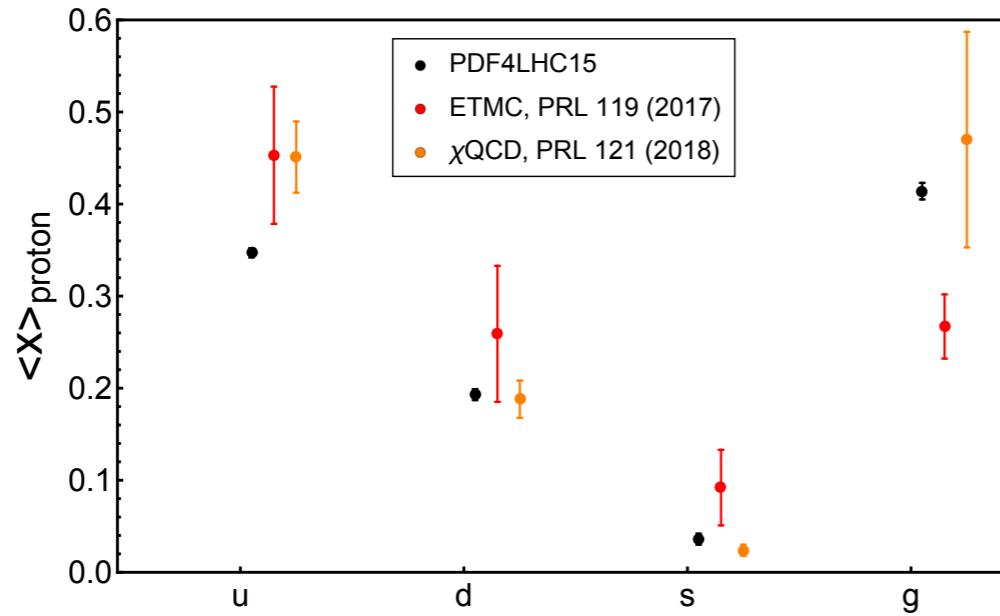
Explicit recipe constructed for projecting arbitrary N hadron wavefunctions into irreps of appropriate little group

Detmold, Jay, Kanwar, Shanahan, MW,
in preparation



DIS and LQCD

LQCD can also compute quantities relevant to neutrino DIS



$\langle x \rangle_{\text{proton}}^q$ and $\langle x \rangle_{\text{proton}}^g$ calculated by several groups

Review: Lin et al, Prog. Part. Nucl. Phys. 100 (2018)

Large momentum effective theory connects Euclidean matrix elements to light-cone PDFs

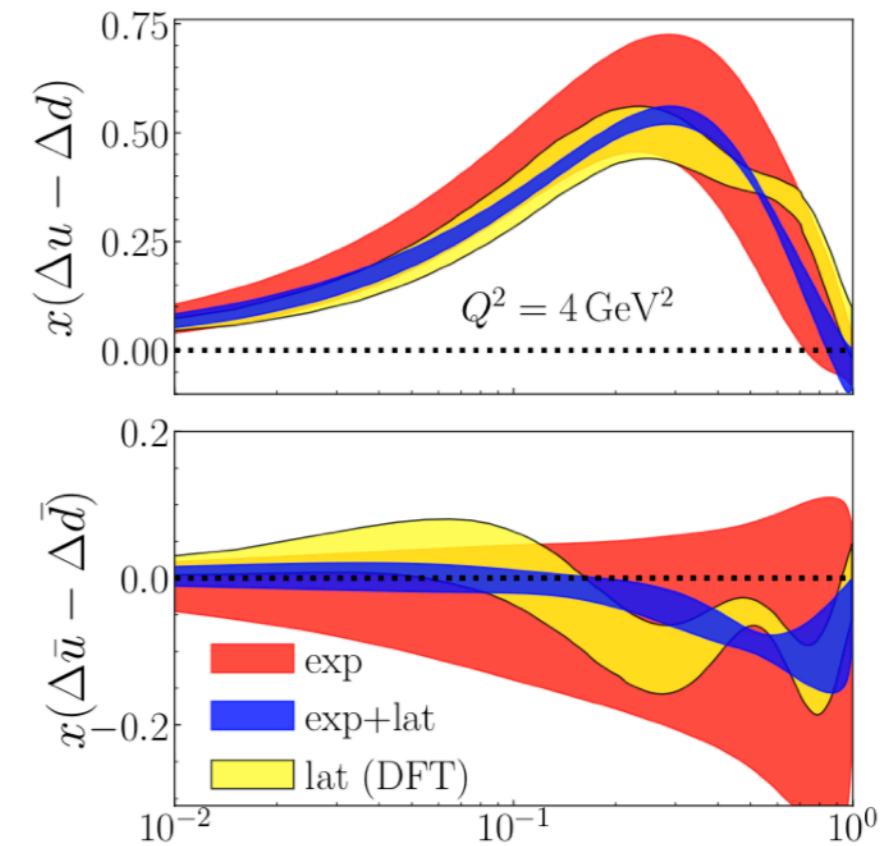
Review: Ji et al, Rev. Mod. Phys. 93 (2021)

Current LQCD results can improve global analyses of isovector polarized PDFs that are relevant for weak interactions in neutrino DIS

Chen, Cohen, Ji, Lin, Zhang, Nucl. Phys. B 911 (2016)

Alexandrou, et al, PRL 121 (2018)

Alexandrou et al, PRL 126 (2021)



Bringewatt et al [JAM], PRD 103 (2021)

Nuclear momentum fractions

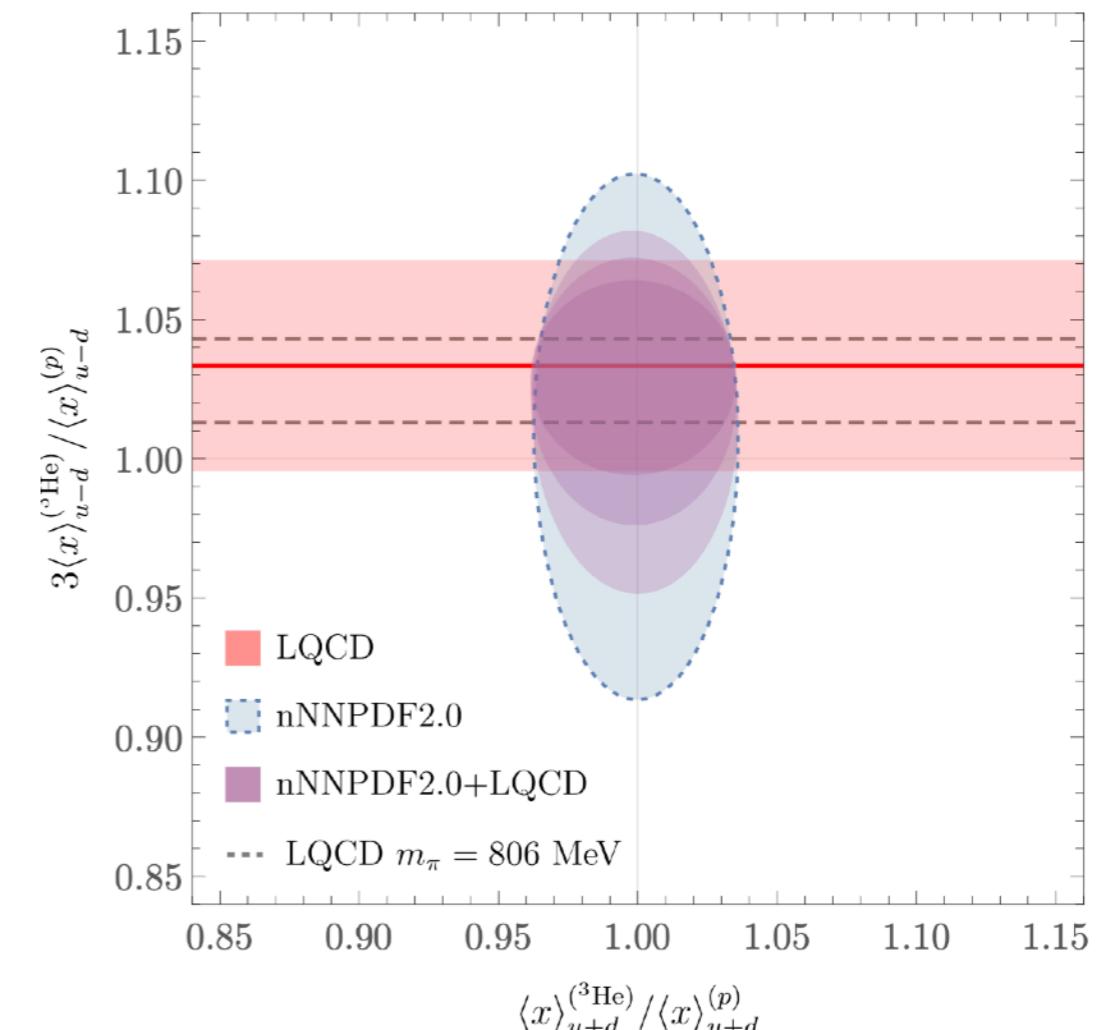
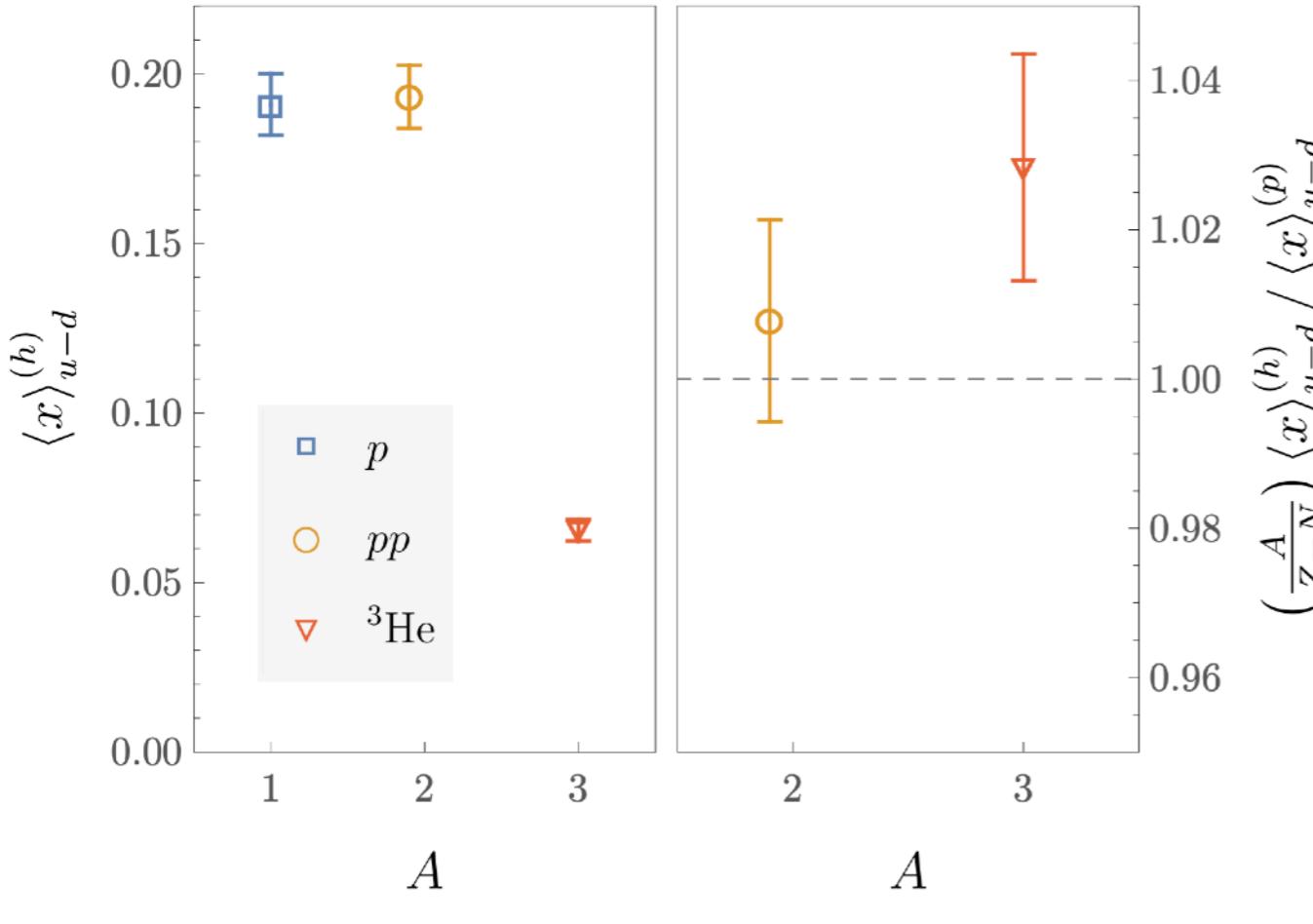
First calculations of gluon and isovector quark momentum fractions of light nuclei

Winter, MW et al [NPLQCD], PRD 96 (2017)

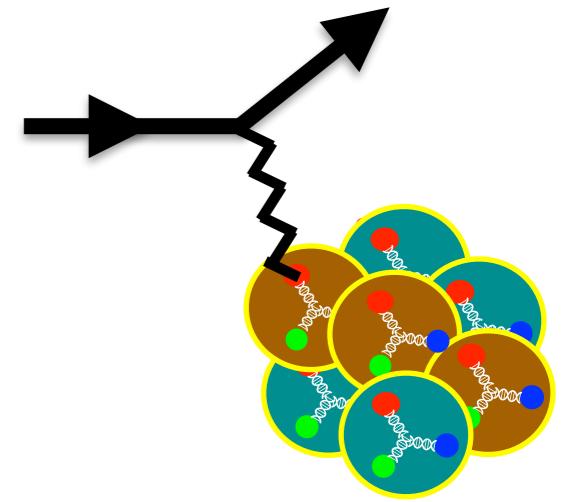
Detmold, MW et al [NPLQCD] PRL 126 (2021)

Results matched to painless EFT to determine two-body current operator relevant to isovector EMC effects

Although systematic uncertainties are not fully controlled (one lattice spacing, volume, quark mass, ...) demonstrates potential for LQCD to usefully constrain nuclear PDFs



The hadron tensor



LQCD methods for calculating exclusive cross-sections break down above multi-particle thresholds (≤ 3 hadrons is state-of-the-art)

Reviews: Briceño, Dudek, and Young, Rev. Mod. Phys. 90 (2018)

Hansen and Sharpe, Ann. Rev. Nucl. Part. Sci. 69 (2019)

How can we use LQCD to constrain higher energies, e.g. shallow inelastic scattering region?

The hadron tensor:

$$\frac{d^2\sigma}{dE'd\cos\theta} \propto L_{\mu\nu} W^{\mu\nu}$$

Lepton tensor (perturbative)

Hadron tensor
(nonperturbative QCD)

Liu and Dong, PRL 72 (1994)

Aglietti et al, Phys. Lett. B 432 (1998)

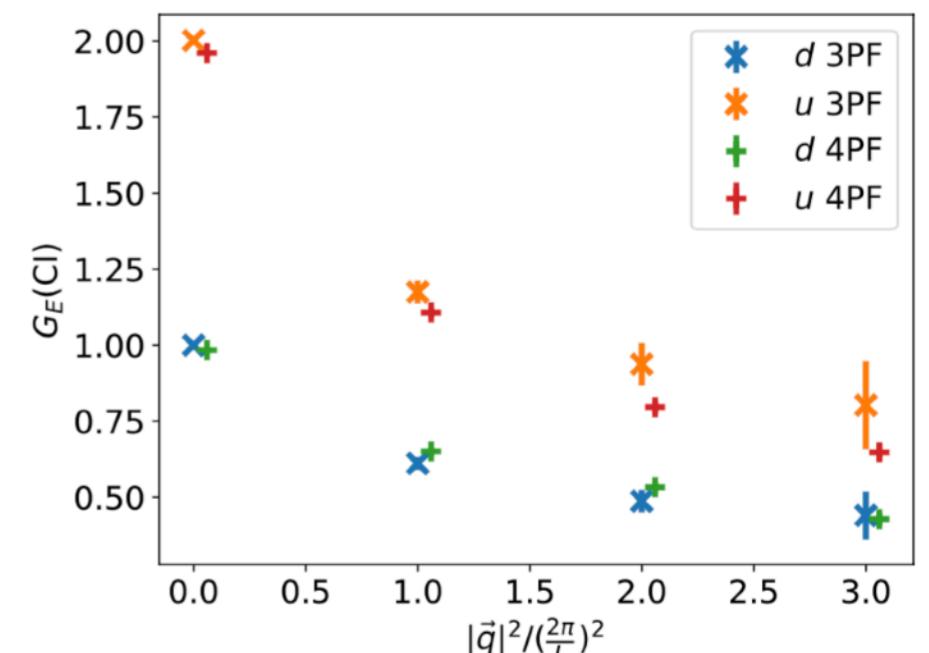
Liu, PRD 62 (2000)

LQCD hadron tensor calculations require a challenging inverse Laplace transform, calculations exploring several different methods underway

Liang, Draper, Liu, Rothkopf, and Yang [χ QCD] PRD 101 (2020)

Fukaya, Hashimoto, Kaneko, and Ohki, PRD 102 (2020)

Proof-of-principle results demonstrate consistency between nucleon hadron tensor and form factor calculations for elastic scattering kinematics



Liang, Liu, and Yang [χ QCD]
EPJ Web Conf. 175 (2018)

Questions

