Theory of QED radiative corrections to neutrino scattering at accelerator energies

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based on
2105.07939,
2204.11379

with O. Tomalak, Q. Chen, K. McFarland, C. Wret

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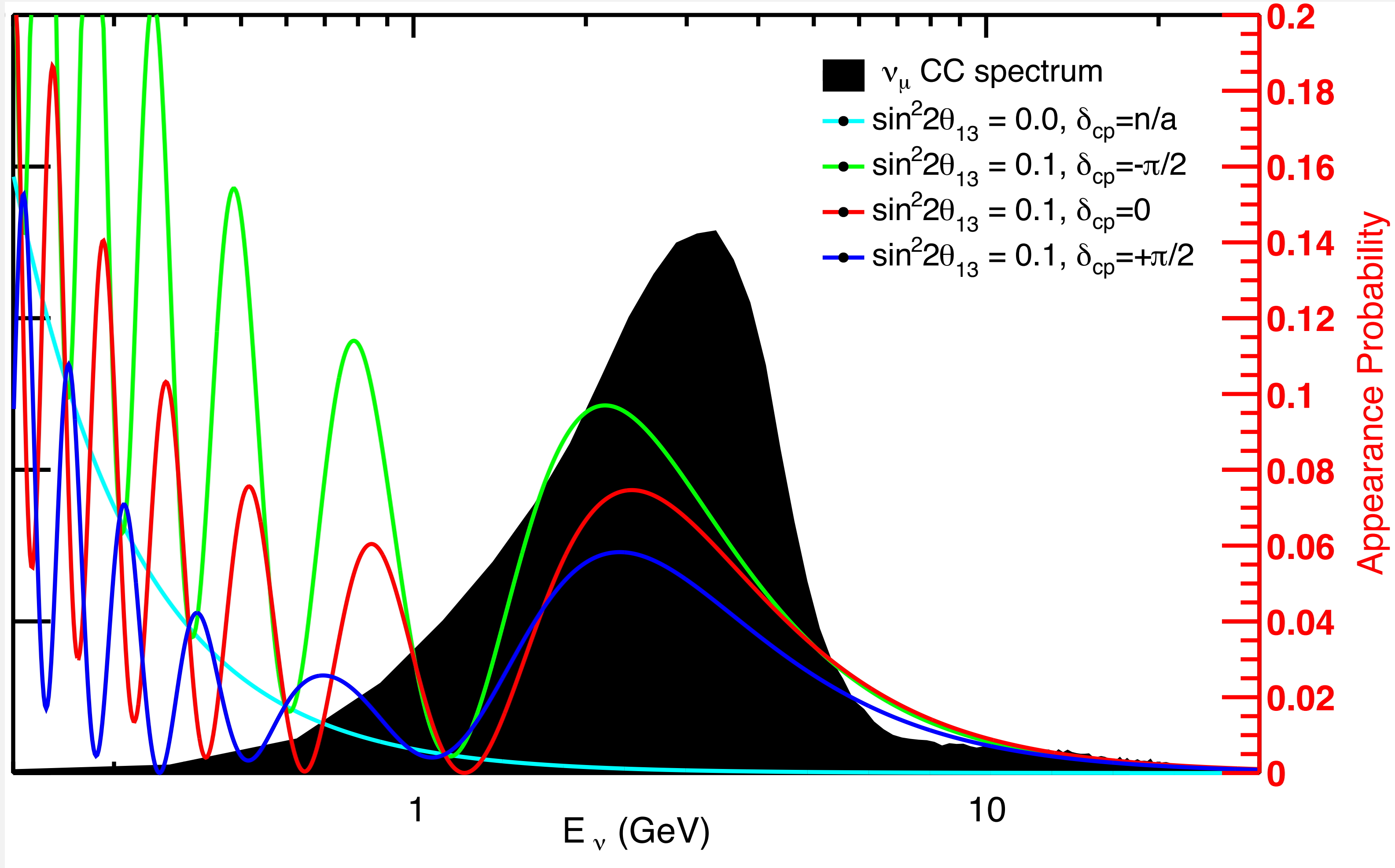
Outline

- Motivation
- Experimental context
- Theory
- Results
- Discussion

ν_e appearance from a ν_μ beam

neutrino oscillation experiment is **simple in conception**:



LBNE, 1307.7335

but **difficult in practice:** rely on theory to determine cross sections: e.g. $\sigma(\nu_e)/\sigma(\nu_\mu)$ to a precision of 1%

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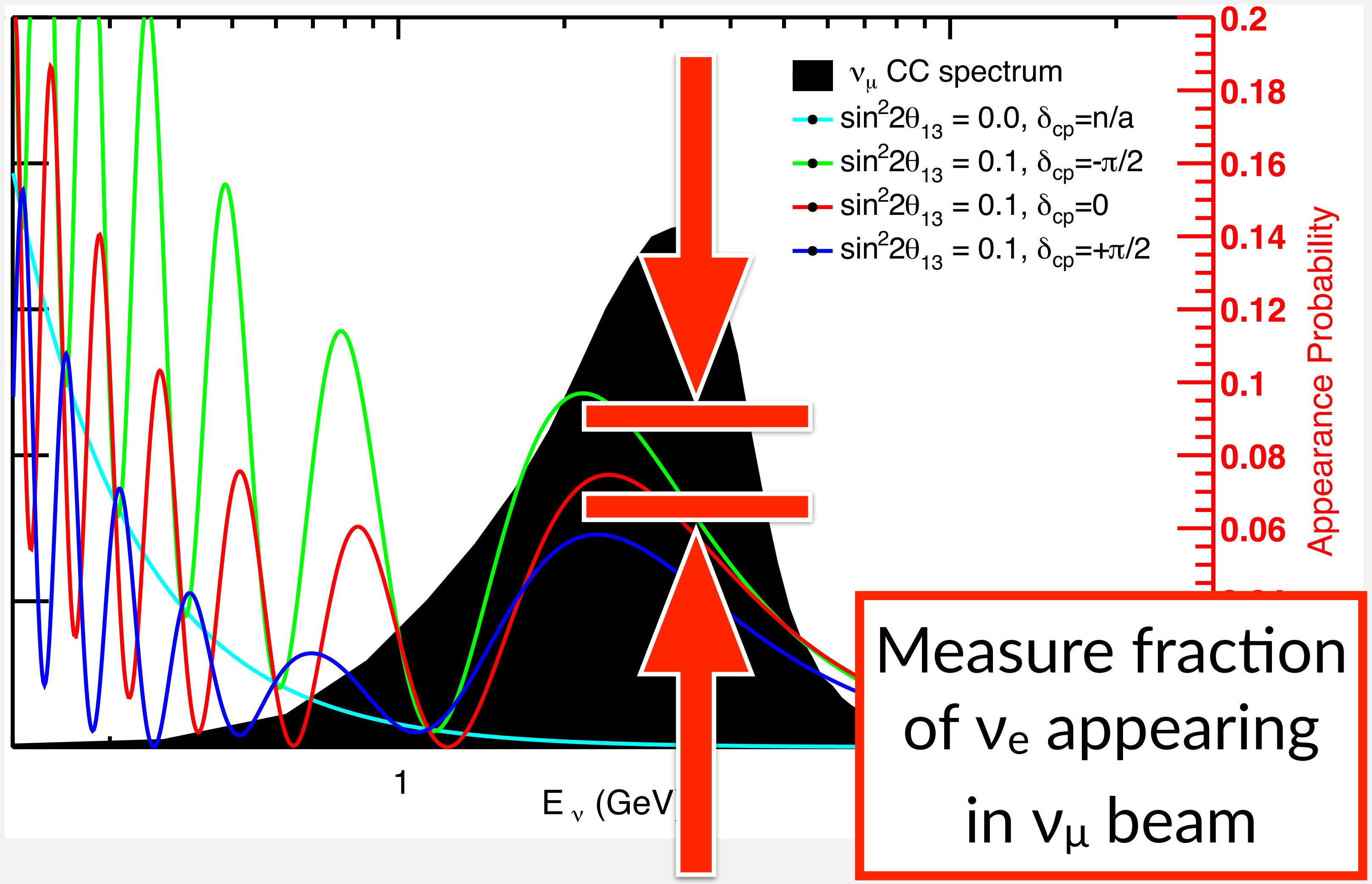

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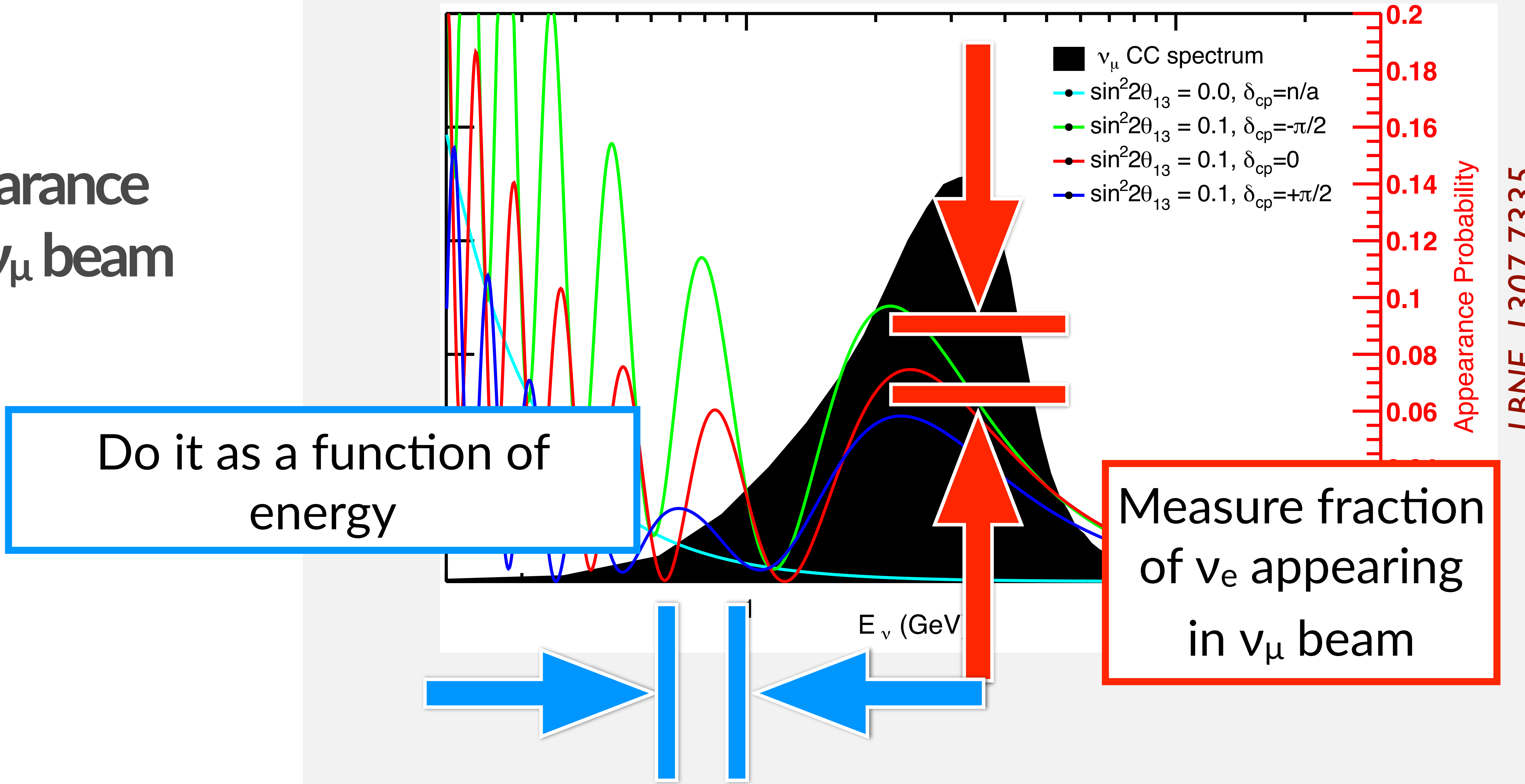

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Radiative correction program

Related topics

- four Fermi starting point

  1911.01493 (Hill and Tomalak)

- neutrino scattering on electrons

  1907.03379 (Tomalak and Hill)

- coherent neutrino-nucleus scattering

  2011.05960 (Tomalak, Machado, Pandey, Plestid)

- Coulomb corrections

  see talk by R. Plestid this afternoon

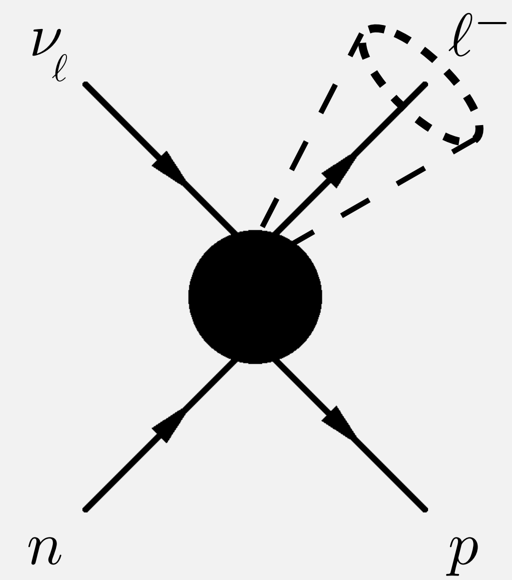
- neutrino scattering on nucleons (this talk)

  2105.07939 (Tomalak, Chen, Hill, McFarland)

  2204.11379 (Tomalak, Chen, Hill, McFarland, Wret)

"RJH acknowledges support from the Neutrino Theory Network at Fermilab during the early stages of this work"

Small parameters



QED radiative corrections are suppressed by α_{QED}, but enhanced by large logarithms:

$\log(m_\ell^2/E_\nu^2)$ — **electron/muon mass**

$\log(\Delta\theta^2)$ — **jet angular size**

$\log(\Delta E/E_\nu)$ — **soft photon threshold**

Radiative corrections depend on hadronic structure, but in factorized manner:

$$\sigma = S(\Delta E)J(\Delta\theta, m_\ell)H(\Lambda)$$

Small parameters

- make use of small parameters:

$$\frac{m_\mu^2}{\Lambda_{\rm hard}^2} \approx 0.01 \qquad (\Delta\theta)^2 \approx 0.03 \qquad \frac{\Delta E}{\Lambda_{\rm hard}} \approx 0.02$$

$$(\Delta E = 20\,\mathrm{MeV}\,, \quad \Delta\theta = 10^\circ\,, \quad \Lambda_{\rm hard} = 1\,\mathrm{GeV})$$

- map into (soft-collinear) effective theory with percent-level expansion parameter

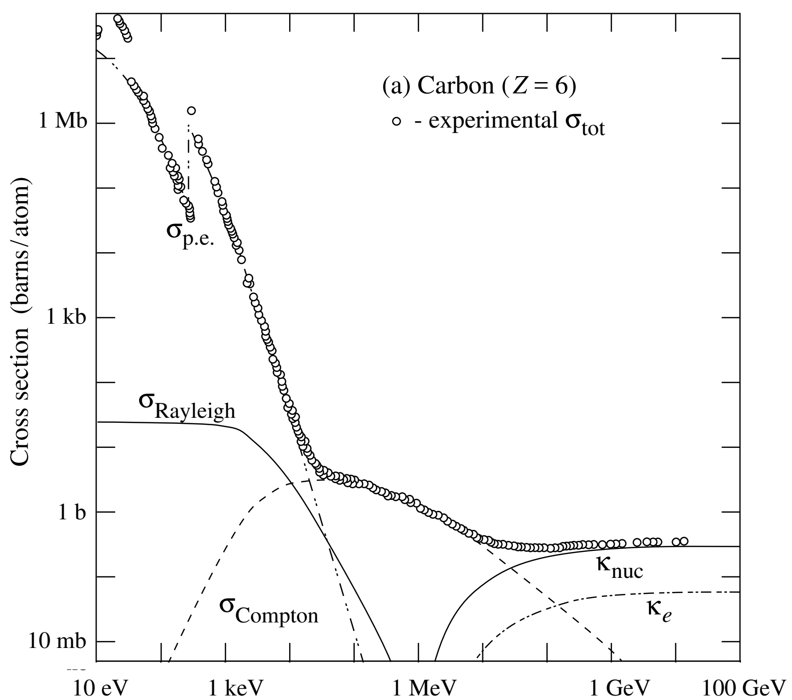
cf. factorization applications for pp collisions or heavy mesons decays: in the present case

low-energy=calculable

high-energy=hadronic/nonperturbative

muon mass is included at tree level, neglected corrections begin at percent times order alpha (negligible at DUNE precision)

Experimental context



Hubbell et al. (NIST database)

Consider

- Exclusive observables: tree level process plus soft or soft+collinear radiation

- Inclusive observables: also hard non-collinear photon radiation

soft photon threshold:

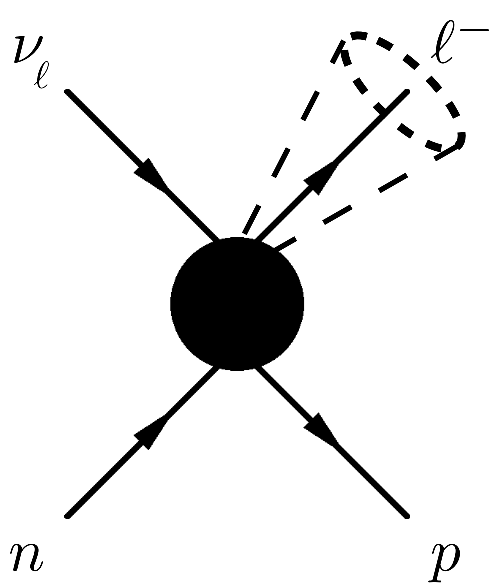
photons with energy smaller than ΔE are unseen by the detector.

identify ΔE as transition point from Compton scattering to e+e- pair production as dominant contribution to total photon cross section

$$\Delta E \approx 30\,\mathrm{MeV}\,, \quad 25\,\mathrm{MeV}\,, \quad 12\,\mathrm{MeV}$$

polystyrene scintillator **water** **argon**

Experimental context



Consider

- Exclusive observables: tree level process plus soft or soft+collinear radiation

- Inclusive observables: also hard non-collinear photon radiation

jet angular size (electron flavor):

photons within angle $\Delta\theta$ of electron are indistinguishable from electron-initiated shower

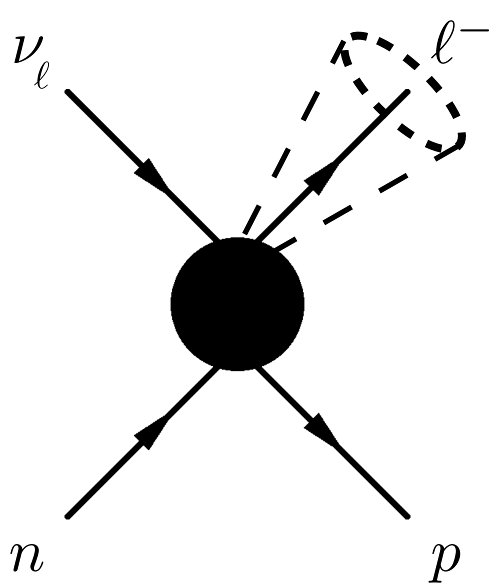
identify $\Delta\theta$ as cone size formed by jet radius (Molière radius) and jet length (length of mean shower maximum)

$$\Delta\theta \approx 9^\circ, \quad 10^\circ, \quad 16^\circ$$ (500 MeV primary electron in polystyrene scintillator, water, liquid argon)

$\Delta\theta$ decreases logarithmically with primary electron energy (~factor two smaller at 3 GeV vs 500 MeV)

$\Delta\theta \approx 5^\circ$ (NOvA), $\Delta\theta \approx 10^\circ$ (T2K,Hyper-K, DUNE), $\Delta\theta \approx 20^\circ$ (SBN)

Experimental context



Consider

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- Inclusive observables: also hard non-collinear photon radiation

jet angular size (muon flavor):

in tracking target detector (e.g. segmented scintillator, gaseous tracker in magnetic field, liquid argon TPC), energy of muon determined by range or curvature: collinear photons unobserved

in W.Ch. detector, photons contribute to reconstructed muon energy if angle is consistent with multiple scattering of muon

$\Delta\theta \approx 2^\circ$ weakly depends on material (polystyrene scintillator, water, liquid argon), and energy

since not much radiation is contained within 2 degrees, $\Delta\theta \approx 0$ is a good (not necessary) approximation in all detectors

Experimental context

Consider

- Exclusive observables: tree level process plus soft or soft+collinear radiation

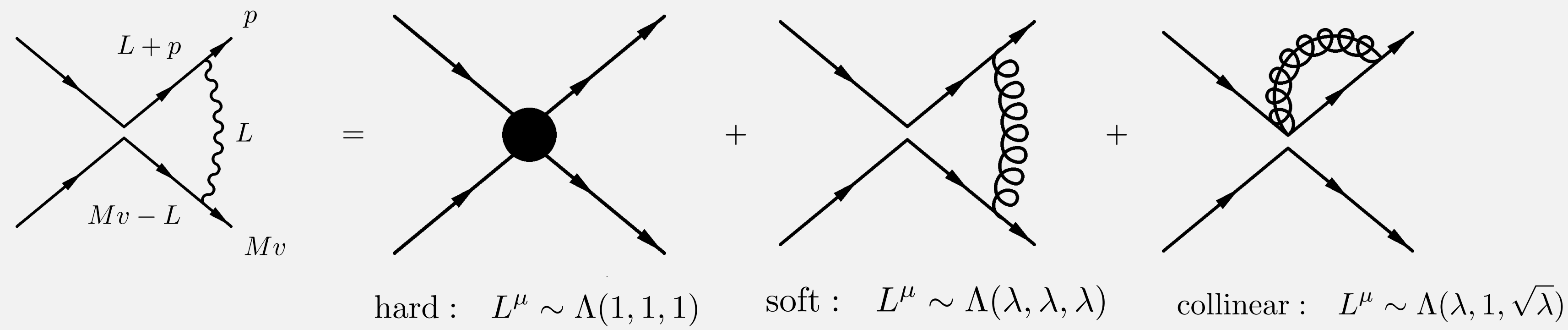
- Inclusive observables: also hard non-collinear photon radiation

experiment may not veto hard non-collinear photons

- retain definitions of electron/muon jets with $E_\gamma \lesssim \Delta E, \theta_\gamma \lesssim \Delta\theta$

- include hard non-collinear photons in cross section

Theory



static limit $m_\ell^2 \ll E_\nu^2 \ll \Lambda^2 \sim M_N^2$

$$\mathcal{L}_{\text{eff}} = -\sqrt{2}\mathrm{G_F} V_{ud}\, \bar{\ell}\gamma^\mu \mathrm{P_L}\nu_\ell\, \bar{h}_v^{(p)}\gamma_\mu\left[c_V + c_A\gamma_5\right]h_v^{(n)} + \text{h.c.}$$

$$\delta_\mathrm{H} = -\frac{1}{4}\ln^2\frac{4E_\nu^2}{\mu^2} + \frac{1}{2}\ln\frac{4E_\nu^2}{\mu^2} - 1 + \frac{19\pi^2}{24}$$

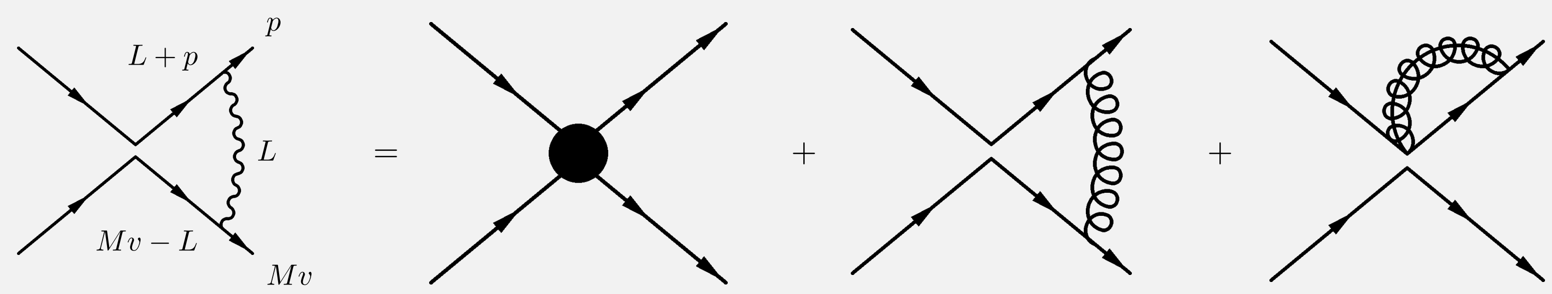
$$\delta_\mathrm{J} = \frac{1}{4}\ln^2\frac{\mu^2}{m_\ell^2} + \frac{1}{4}\ln\frac{\mu^2}{m_\ell^2} + 1 + \frac{\pi^2}{24}$$

$$\delta_\mathrm{S} = \left(1 - \ln\frac{2E_\nu}{m_\ell}\right)\left(\ln\frac{\mu^2}{(\Delta E)^2} + \ln\frac{2E_\nu}{m_\ell}\right) + 1 - \frac{\pi^2}{6}$$

$$\lambda \sim \frac{m_\ell^2}{\Lambda^2} \sim (\Delta\theta)^2 \sim \frac{\Delta E}{\Lambda}$$

resum large logs by choosing $\mu_S^2 \sim \lambda^2\Lambda^2$ (soft), $\mu_J^2 \sim \lambda\Lambda^2$ (jet), $\mu_H^2 \sim \Lambda^2$ (hard)

Theory



beyond the static limit, hard region becomes nonperturbative function

general factorization theorem:

$$\frac{d\sigma}{dQ^2} \propto H\left(\frac{E_\nu}{M}, \frac{Q^2}{M^2}, \frac{\mu}{M}\right)\left[J\left(\frac{\mu}{m_\ell}\right) R\left(\frac{\mu}{m_\ell}, v_\ell\cdot v_p\right) S\left(\frac{\mu}{\Delta E}, v_\ell\cdot v_p, v\cdot v_\ell, v\cdot v_p\right)\right.$$
$$\left.+\int_0^{1-\frac{\Delta E}{E_\ell^{\rm tree}}} dx\, j\left(\frac{\mu}{m_\ell}, x, v\cdot v_\ell \Delta\theta\right) R\left(\frac{\mu}{m_\ell}, x v_\ell\cdot v_p\right) S\left(\frac{\mu}{\Delta E}, x\, v_\ell\cdot v_p, x\, v\cdot v_\ell, v\cdot v_p\right)\right]$$

in second term for real collinear radiation, $x = E_\ell / E_\ell^{\rm tree}$ is fraction of jet energy carried by charged lepton

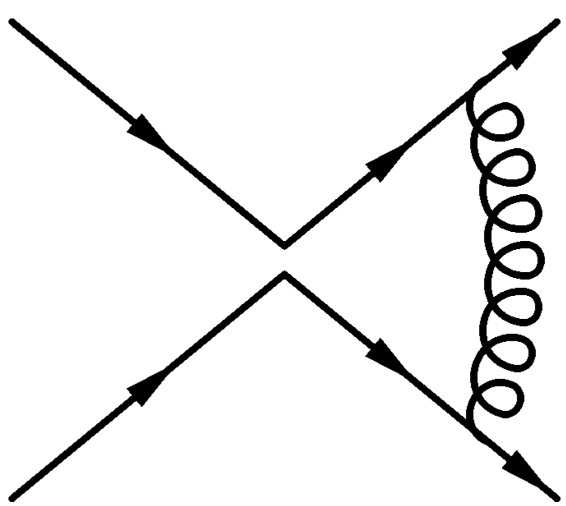
Theory

$$\frac{d\sigma}{dQ^2} \propto H\left(\frac{E_\nu}{M}, \frac{Q^2}{M^2}, \frac{\mu}{M}\right)\left[J\left(\frac{\mu}{m_\ell}\right) R\left(\frac{\mu}{m_\ell}, v_\ell \cdot v_p\right) S\left(\frac{\mu}{\Delta E}, v_\ell \cdot v_p, v \cdot v_\ell, v \cdot v_p\right)\right.$$
$$\left. + \int_0^{1-\frac{\Delta E}{E_\ell^{\mathrm{tree}}}} dx\, j\left(\frac{\mu}{m_\ell}, x, v \cdot v_\ell, \Delta\theta\right) R\left(\frac{\mu}{m_\ell}, x v_\ell \cdot v_p\right) S\left(\frac{\mu}{\Delta E}, x v_\ell \cdot v_p, x\, v \cdot v_\ell, v \cdot v_p\right)\right]$$

soft function universal, depending on electric charge and four-velocity of external particles

for proton at rest, reduces to static limit result. At one loop:

$$S\left(\frac{\mu}{\Delta E}, \frac{E_\ell}{m_\ell}, \frac{E_\ell}{m_\ell}, 1\right) \underset{E_\ell \gg m_\ell}{\longrightarrow} 1 + \frac{\alpha}{\pi}\left[-\ln^2 \frac{2E_\ell}{m_\ell} + \ln \frac{2E_\ell}{m_\ell} + 2\left(1 - \ln \frac{2E_\ell}{m_\ell}\right) \ln \frac{\mu}{2\Delta E} + 1 - \frac{\pi^2}{6}\right]$$



Higher loop orders are related to universal anomalous dimensions and are systematically included

$$\frac{d\sigma}{dQ^2} \propto H\left(\frac{E_\nu}{M}, \frac{Q^2}{M^2}, \frac{\mu}{M}\right)\left[\boxed{J\left(\frac{\mu}{m_\ell}\right) R\left(\frac{\mu}{m_\ell}, v_\ell \cdot v_p\right)} S\left(\frac{\mu}{\Delta E}, v_\ell \cdot v_p, v \cdot v_\ell, v \cdot v_p\right)\right.$$
$$\left. + \int_0^{1-\frac{\Delta E}{E_\ell^{\mathrm{tree}}}} dx\, j\left(\frac{\mu}{m_\ell}, x, v \cdot v_\ell \Delta\theta\right) R\left(\frac{\mu}{m_\ell}, x v_\ell \cdot v_p\right) S\left(\frac{\mu}{\Delta E}, x\, v_\ell \cdot v_p, x\, v \cdot v_\ell, v \cdot v_p\right)\right]$$

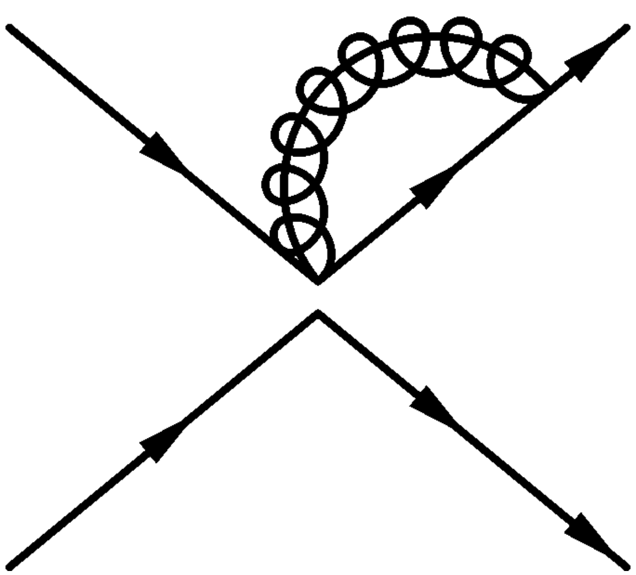
Theory

virtual jet function (J) indep. of hadronic structure and identical to the static limit result.

At one loop:

$$J\left(\frac{\mu}{m_\ell}\right) = 1 + \frac{\alpha}{4\pi}\left(\ln^2 \frac{\mu^2}{m_\ell^2} + \ln \frac{\mu^2}{m_\ell^2} + 4 + \frac{\pi^2}{6}\right)$$

Higher orders can be systematically included



Remainder (R) function starts at two loop order, translates between MS-bar for QED with and without the charged lepton

Theory

$$\frac{d\sigma}{dQ^2} \propto H\left(\frac{E_\nu}{M}, \frac{Q^2}{M^2}, \frac{\mu}{M}\right)\left[J\left(\frac{\mu}{m_\ell}\right) R\left(\frac{\mu}{m_\ell}, v_\ell \cdot v_p\right) S\left(\frac{\mu}{\Delta E}, v_\ell \cdot v_p, v \cdot v_\ell, v \cdot v_p\right)\right.$$
$$\left. + \int_0^{1-\frac{\Delta E}{E_\ell^{\rm tree}}} dx\, \boxed{j\left(\frac{\mu}{m_\ell}, x, v \cdot v_\ell \Delta\theta\right)} R\left(\frac{\mu}{m_\ell}, x v_\ell \cdot v_p\right) S\left(\frac{\mu}{\Delta E}, x\, v_\ell \cdot v_p, x\, v \cdot v_\ell, v \cdot v_p\right)\right]$$

real radiation jet function (j) also indep. of hadronic structure

$$j\left(\frac{\mu}{m_\ell}, x, \eta\right) = \frac{\alpha}{\pi}\left[\frac{1}{2}\frac{1+x^2}{1-x}\ln(1+x^2\eta^2) - \frac{x}{1-x}\frac{x^2\eta^2}{1+x^2\eta^2}\right] \qquad \eta = \Delta\theta E_\ell / m_\ell$$

small-mass limit (electron)

$$\int_0^{1-\Delta E/E_\ell^{\rm tree}} \mathrm{d}x\, j\left(\frac{\mu}{m_\ell}, x, \eta\right) \xrightarrow[\eta \gg 1]{} \frac{\alpha}{\pi}\left[(2\ln\eta - 1)\ln\frac{E_\ell^{\rm tree}}{\Delta E} - \frac{3}{2}\ln\eta - \frac{\pi^2}{3} + \frac{9}{4} + \frac{\pi}{2\eta} + \mathcal{O}\left(\frac{1}{\eta^2}\right)\right]$$

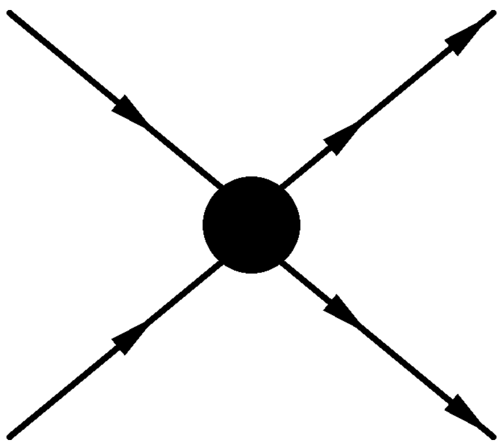
combined with J and S, replaces $m_e \to \Delta\theta$ as collinear regulator

small-angle limit (muon)

$$\int_0^{1-\Delta E/E_\ell^{\rm tree}} \mathrm{d}x\, j\left(\frac{\mu}{m_\ell}, x, \eta\right) \xrightarrow[\eta \ll 1]{} \frac{\alpha}{\pi}\left[\frac{\eta^2}{24} + \left(\ln\frac{E_\ell^{\rm tree}}{\Delta E} - \frac{23}{10}\right)\frac{\eta^4}{2} + \mathcal{O}\left(\eta^6\right)\right]$$

vanishes smoothly as $\Delta\theta \to 0$

Theory



$$\frac{d\sigma}{dQ^2} \propto \boxed{H\left(\frac{E_\nu}{M}, \frac{Q^2}{M^2}, \frac{\mu}{M}\right)} \Bigg[J\left(\frac{\mu}{m_\ell}\right) R\left(\frac{\mu}{m_\ell}, v_\ell \cdot v_p\right) S\left(\frac{\mu}{\Delta E}, v_\ell \cdot v_p, v \cdot v_\ell, v \cdot v_p\right) + \int_0^{1-\frac{\Delta E}{E_\ell^{\mathrm{tree}}}} dx\, j\left(\frac{\mu}{m_\ell}, x, v \cdot v_\ell \Delta\theta\right) R\left(\frac{\mu}{m_\ell}, x v_\ell \cdot v_p\right) S\left(\frac{\mu}{\Delta E}, x\, v_\ell \cdot v_p, x\, v \cdot v_\ell, v \cdot v_p\right) \Bigg]$$

hard function is nonperturbative but universal for electron and muon flavor

Four invariant amplitudes at leading power

$$T_{\nu_\ell n \to \ell^- p} = \sqrt{2} \mathrm{G_F} V_{ud}\, \bar{\ell}^- \gamma^\mu \mathrm{P_L} \nu_\ell\, \bar{p} \left(f_1 \gamma_\mu + f_2 \frac{i\sigma_{\mu\rho} q^\rho}{2M} + f_A \gamma_\mu \gamma_5 - f_A^3 \frac{K_\mu}{M} \gamma_5 \right) n$$

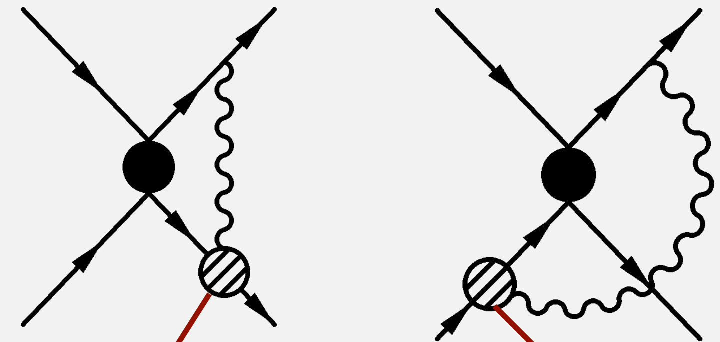
Define conventional separation into Born and non-Born pieces ($\overline{\mathrm{MS}}$ definition of form factors $F_{V1}(Q^2)$, etc.)

$$f_1(\nu, Q^2) = \sqrt{Z_\ell Z_h^{(p)}} \left(F_{V1}(Q^2) + f_1^v(\nu, Q^2) \right)$$

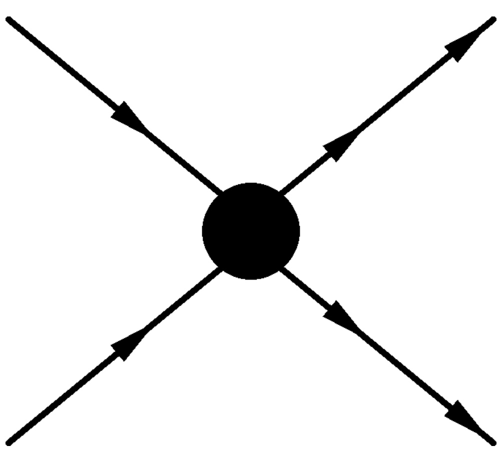
Theory

$$\frac{d\sigma}{dQ^2} \propto \boxed{H\left(\frac{E_\nu}{M},\frac{Q^2}{M^2},\frac{\mu}{M}\right)} \Big[J\left(\frac{\mu}{m_\ell}\right) R\left(\frac{\mu}{m_\ell}, v_\ell\cdot v_p\right) S\left(\frac{\mu}{\Delta E}, v_\ell\cdot v_p, v\cdot v_\ell, v\cdot v_p\right) + \int_0^{1-\frac{\Delta E}{E_\ell^{\mathrm{tree}}}} dx\, j\left(\frac{\mu}{m_\ell}, x, v\cdot v_\ell \Delta\theta\right) R\left(\frac{\mu}{m_\ell}, x v_\ell\cdot v_p\right) S\left(\frac{\mu}{\Delta E}, x\, v_\ell\cdot v_p, x\, v\cdot v_\ell, v\cdot v_p\right)\Big]$$

use a simple Sticking In Form Factors (SIFF) model with conservative uncertainties for the non-Born piece



$$T^v_{\nu_\ell n\to\ell^- p} = e^2 \int \frac{\mathrm{d}^dL}{(2\pi)^d} \bar\ell \gamma_\mu \frac{-\not p' - \not L}{(L+p')^2 - m_\ell^2}\gamma^\sigma \mathrm{P_L} \nu_\ell \Pi^{\mu\nu}(L)\,\bar p \left(\Gamma^p_\nu \frac{\not k' - \not L + M}{(L-k')^2 - M^2}\Gamma_\sigma + \Gamma_\sigma \frac{\not k + \not L + M}{(L+k)^2-M^2}\Gamma^n_\nu\right) n$$

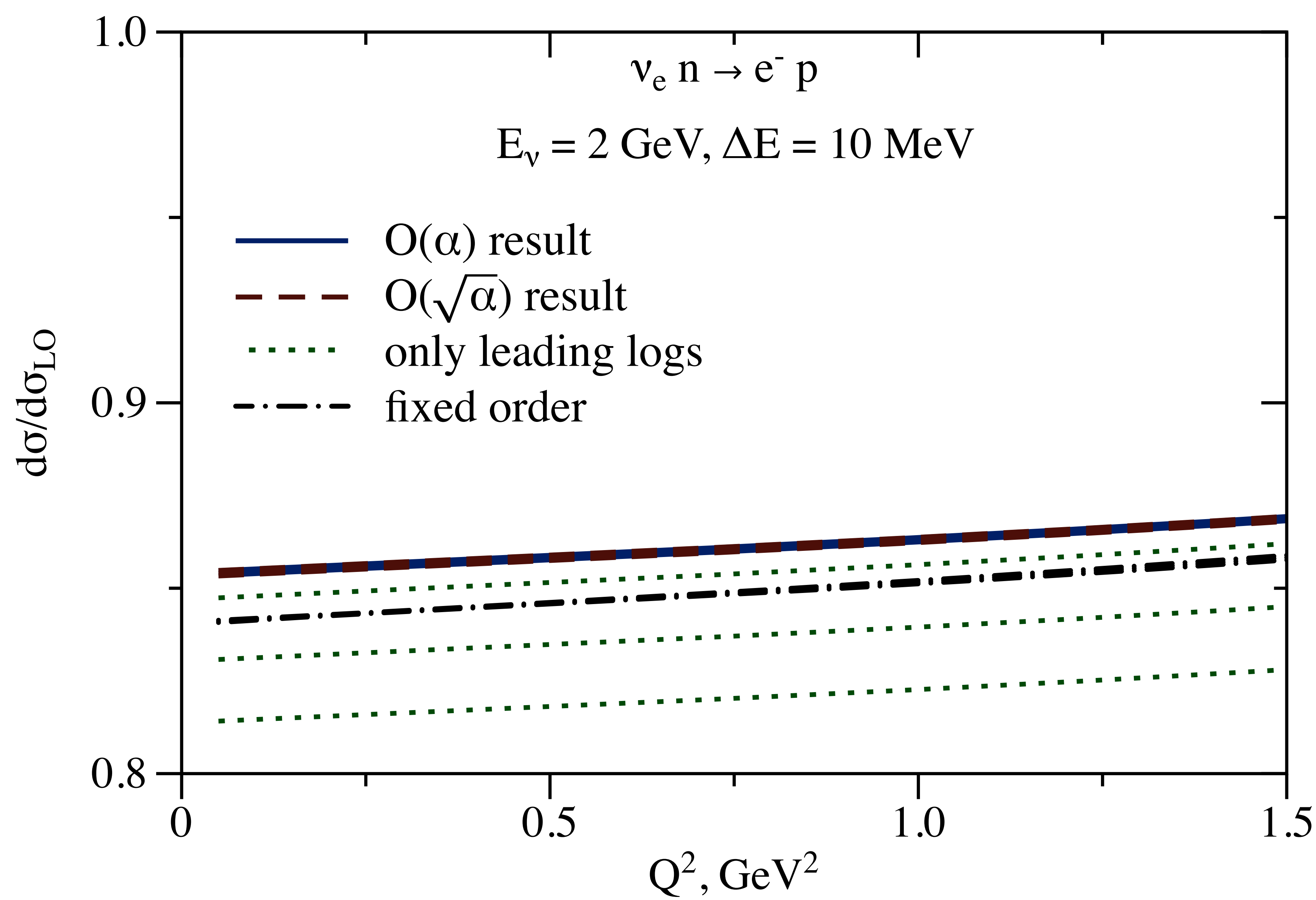


- similar to standard ansatz in electron-proton scattering analyses.

- uncertainties: vary form factors within experimental bounds, ansatz for neglected inelastic states

Results

Perturbative uncertainty is controlled

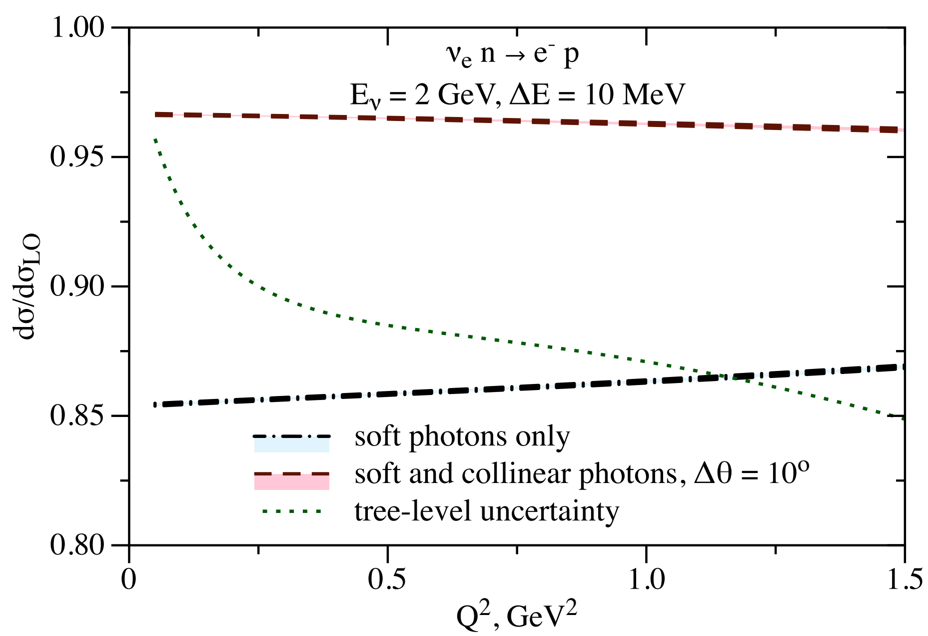


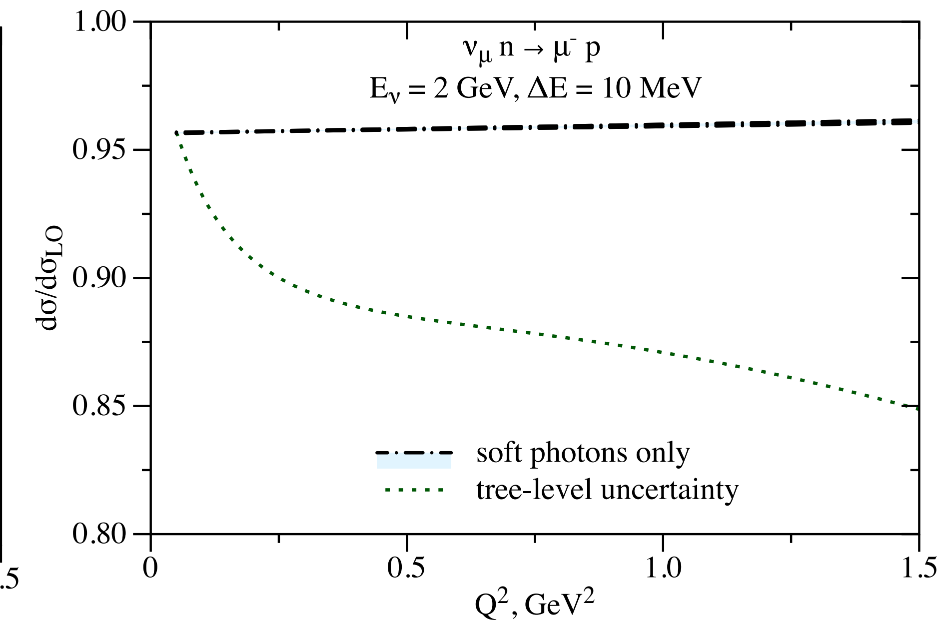
- counting $\alpha \log^2 \lambda = \mathcal{O}(1)$, resume through $\mathcal{O}(\alpha)$,

- perturbative uncertainty estimated by renormalization scale variation

Results

Corrections are large and depend on flavor and on experimental parameters





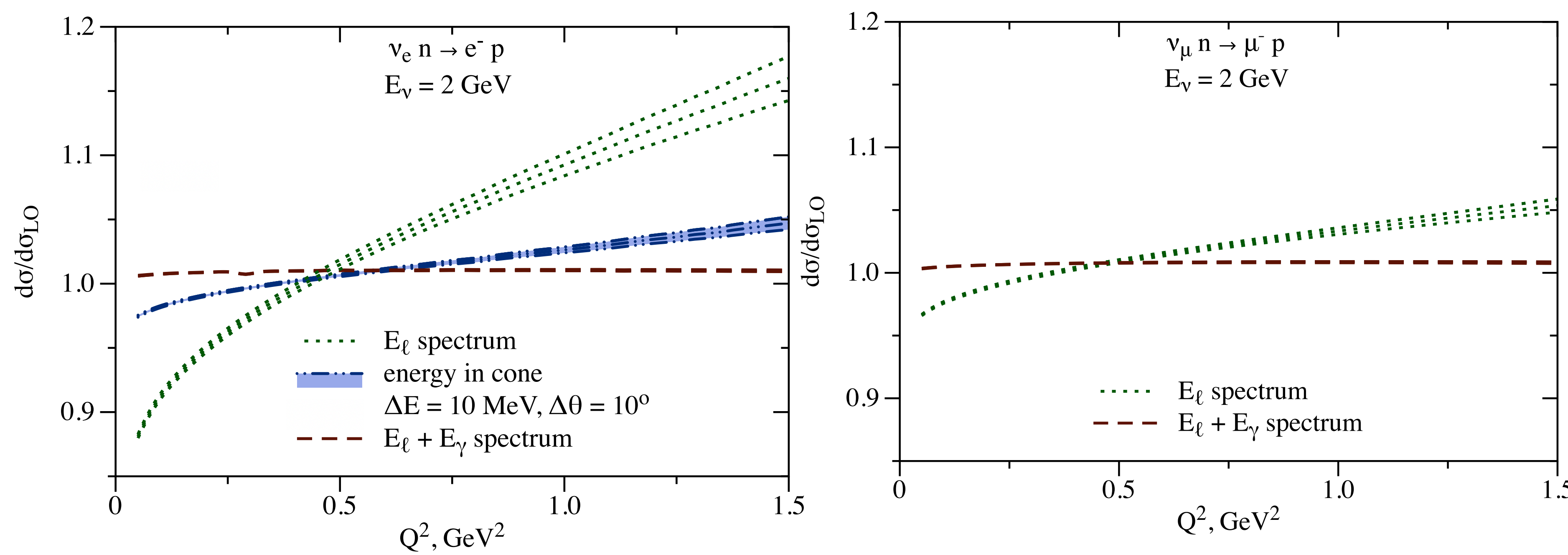
note:

- directly comparable "soft" curves for electron and muon very different

- similarity of "soft and collinear" curve for electron and "soft" for muon results from coincidence involving detector-dependent $\Delta\theta$ for electron and m_μ for muon

Corrections are large and depend on flavor and on experimental parameters

Results



similar for inclusive cross sections:

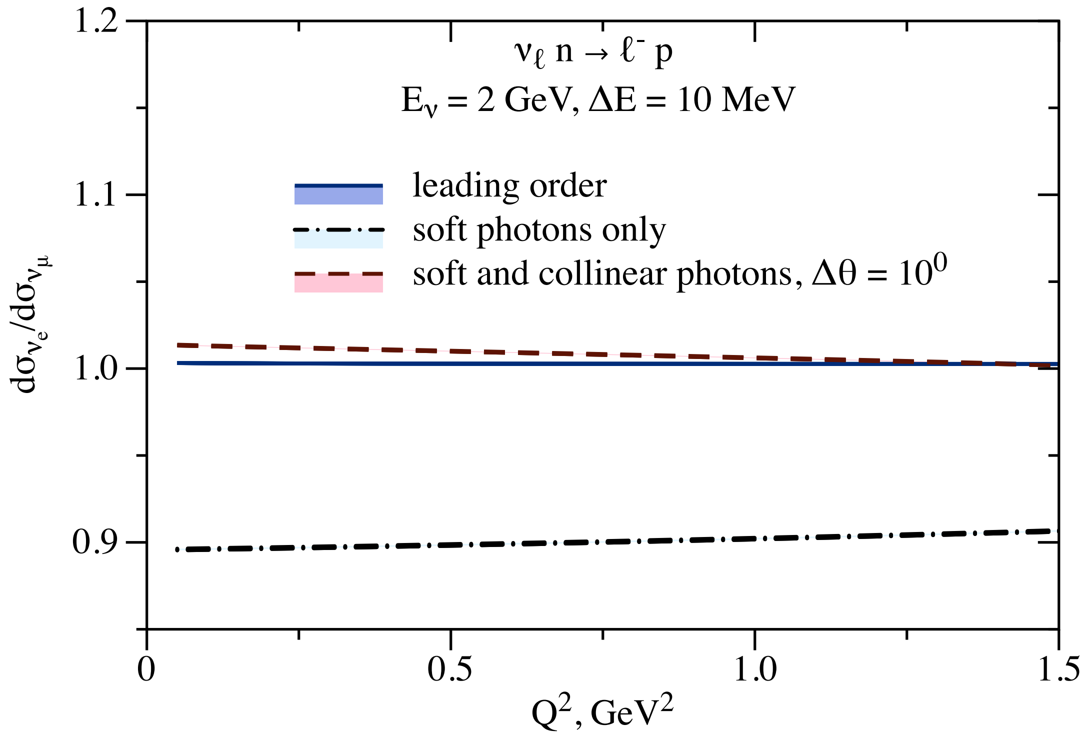
- directly comparable “soft” curves for electron and muon very different

- similarity of “soft and collinear” curve for electron and “soft” for muon results from coincidence involving detector-dependent $\Delta\theta$ for electron and m_μ for muon

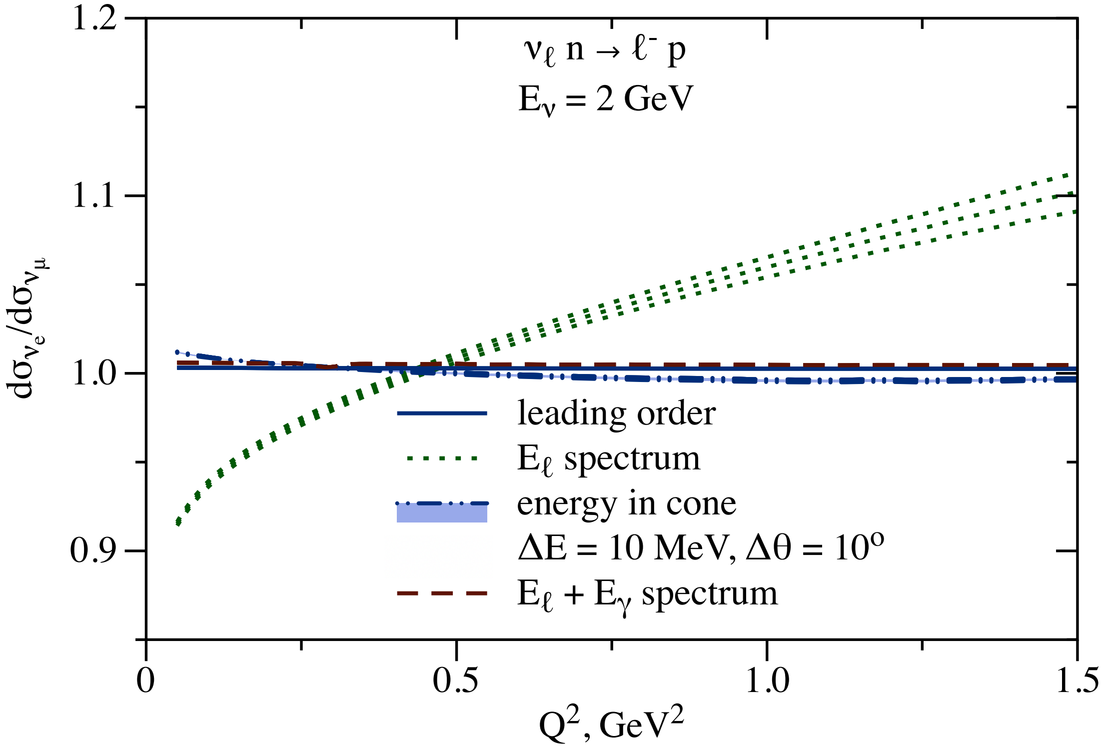
Results

Hadronic uncertainty cancels in flavor ratio

exclusive:



inclusive:



Results

Ratio of total cross sections precisely predicted

Collinear singularity (KLN) theorem at $m_\ell \to 0$:

$$\sigma(m_\ell) = A + B_0 \frac{m_\ell^2}{\Lambda^2} + B_1 \frac{m_\ell^2}{\Lambda^2} \ln \frac{m_\ell^2}{\Lambda^2} + \dots$$

$$\Longrightarrow \quad \frac{\sigma(m_\mu)}{\sigma(m_e)} = 1 + \mathcal{B}_0 \frac{m_\mu^2}{\Lambda^2} + \mathcal{B}_1 \frac{m_\mu^2}{\Lambda^2} \ln \frac{m_\mu^2}{\Lambda^2} + \mathcal{O}\left(\frac{m_e^2}{\Lambda^2}, \alpha^2 \frac{m_\mu^2}{\Lambda^2} \ln^2 \frac{m_\mu}{\Lambda}, \frac{m_\mu^4}{\Lambda^4} \right)$$

$\mathcal{B}_0 = B_0/A$, $\mathcal{B}_1 = B_1/A$

Explicit evaluation in our hadronic model:

$$\mathcal{B}_0(E_\nu = 2\,\mathrm{GeV}) = -0.28 + \mathcal{O}(\alpha, \epsilon_{\mathrm{nuc}}) \qquad \Lambda \equiv 1\,\mathrm{GeV}$$

$$\mathcal{B}_1(E_\nu = 2\,\mathrm{GeV}) = \mathcal{O}(\alpha, \epsilon_{\mathrm{nuc}})$$

	E_ν, GeV		$\left(\frac{\sigma_e}{\sigma_\mu} - 1\right)_{\mathrm{LO}}$, %	$\frac{\sigma_e}{\sigma_\mu} - 1$, %
T2K/HyperK	0.6	ν	2.47 ± 0.06	$2.84 \pm 0.06 \pm 0.37$
		$\bar{\nu}$	2.04 ± 0.08	$1.84 \pm 0.08 \pm 0.20$
NOvA/DUNE	2.0	ν	0.322 ± 0.006	$0.54 \pm 0.01 \pm 0.22$
		$\bar{\nu}$	0.394 ± 0.003	$0.20 \pm 0.01 \pm 0.19$

Results

Nuclear corrections are small in ratio of total cross sections

$$\frac{\sigma(m_\mu)}{\sigma(m_e)} = 1 + \mathcal{B}_0 \frac{m_\mu^2}{\Lambda^2} + \mathcal{B}_1 \frac{m_\mu^2}{\Lambda^2} \ln \frac{m_\mu^2}{\Lambda^2} + \mathcal{O}\left(\frac{m_e^2}{\Lambda^2}, \alpha^2 \frac{m_\mu^2}{\Lambda^2} \ln^2 \frac{m_\mu}{\Lambda}, \frac{m_\mu^4}{\Lambda^4} \right)$$

$$\mathcal{B}_0(E_\nu = 2\,\mathrm{GeV}) = -0.28 + \mathcal{O}(\alpha, \epsilon_{\mathrm{nuc}})$$

$$\Lambda \equiv 1\,\mathrm{GeV}$$

$$\mathcal{B}_1(E_\nu = 2\,\mathrm{GeV}) = \mathcal{O}(\alpha, \epsilon_{\mathrm{nuc}})$$

$$\epsilon_{\mathrm{nuc}} \sim \epsilon_b/\Lambda \sim k_F^2/\Lambda^2$$

e.g. in RFG nuclear model

$$\mathcal{B}_0(E_\nu = 2\,\mathrm{GeV}) = -0.28 \to -0.32$$

	E_ν, GeV		$\left(\frac{\sigma_e}{\sigma_\mu} - 1\right)_{\mathrm{LO}}$, %	$\frac{\sigma_e}{\sigma_\mu} - 1$, %
T2K/HyperK	0.6	ν	2.47 ± 0.06	$2.84 \pm 0.06 \pm 0.37$
		$\bar{\nu}$	2.04 ± 0.08	$1.84 \pm 0.08 \pm 0.20$
NOvA/DUNE	2.0	ν	0.322 ± 0.006	$0.54 \pm 0.01 \pm 0.22$
		$\bar{\nu}$	0.394 ± 0.003	$0.20 \pm 0.01 \pm 0.19$

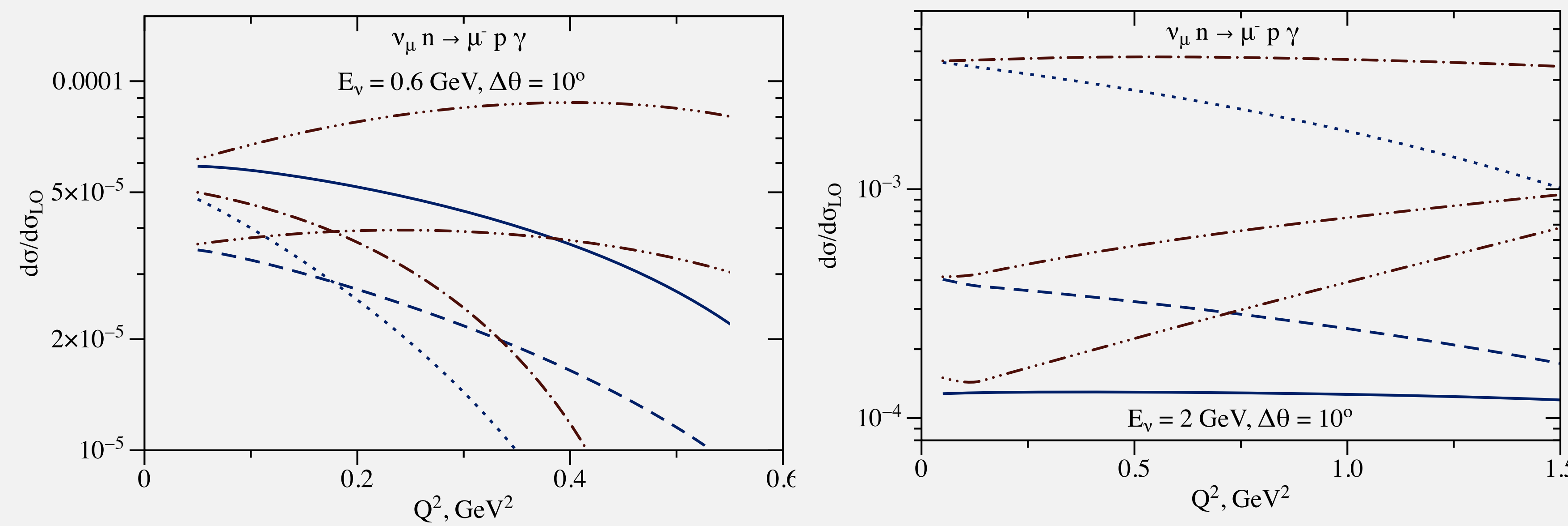
Nuclear corrections enter as $(m_\mu^2/\Lambda^2) \times \epsilon_{\mathrm{nuc}}$ and are contained in the nucleon-level error budget

Photon backgrounds can be systematically studied. Example: muon mis-identification

Results

Can also address questions beyond signal cross sections

E.g., how often does a hard photon accompany a muon and cause it to look like an electron at T2K/HyperK?



default model
default model, $E_\gamma > E_\mu - m_\mu$
default model, $E_\gamma > 200$ MeV
local model
local model, $E_\gamma > E_\mu - m_\mu$
local model, $E_\gamma > 200$ MeV

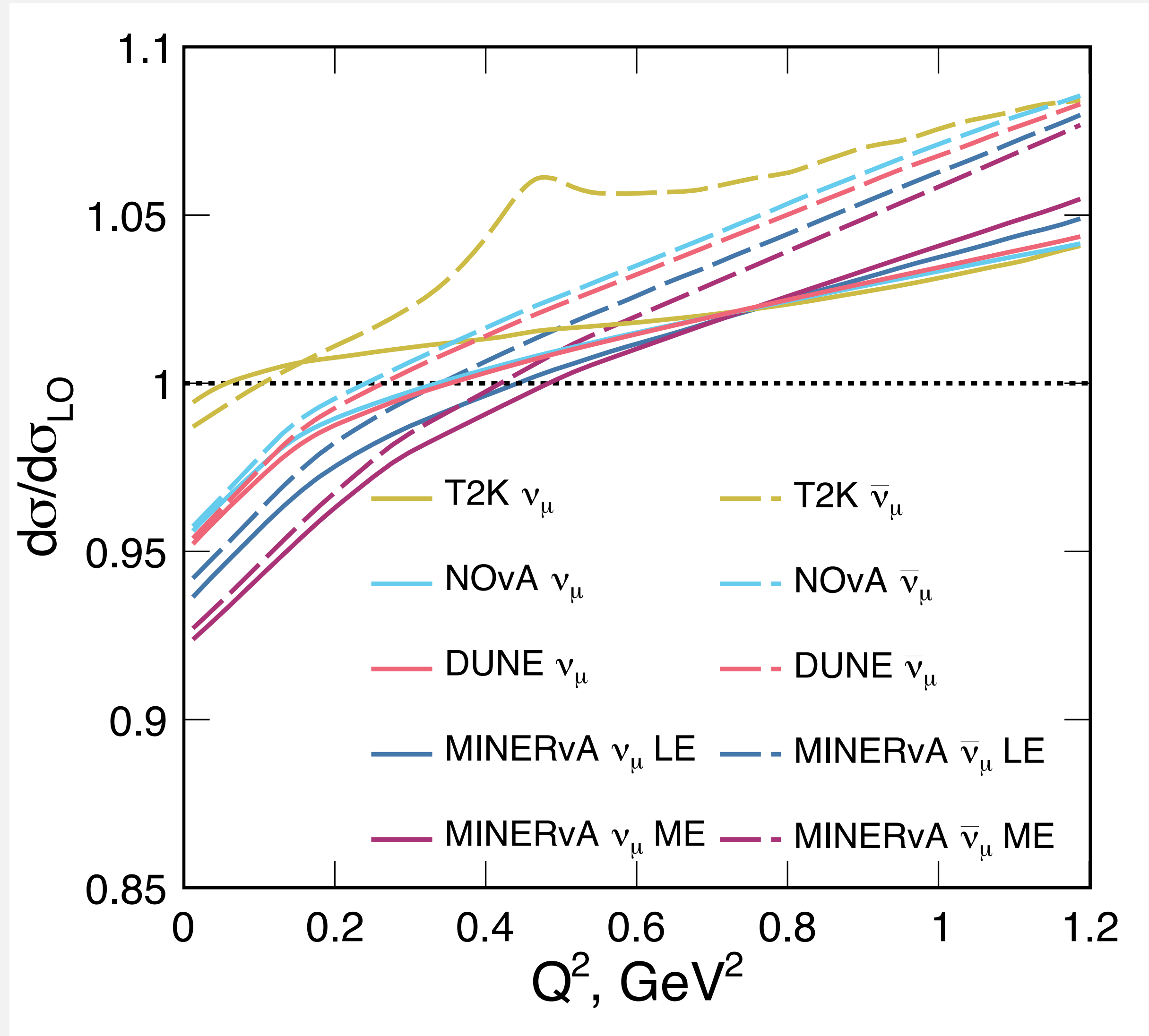
define "equivalent electron" to have energy of photon plus muon kinetic energy

require the equivalent electron shower length to be longer than muon range

order 10^{-4}. Larger effects when lower-energy collinear photons are considered

Results

Comparison to data

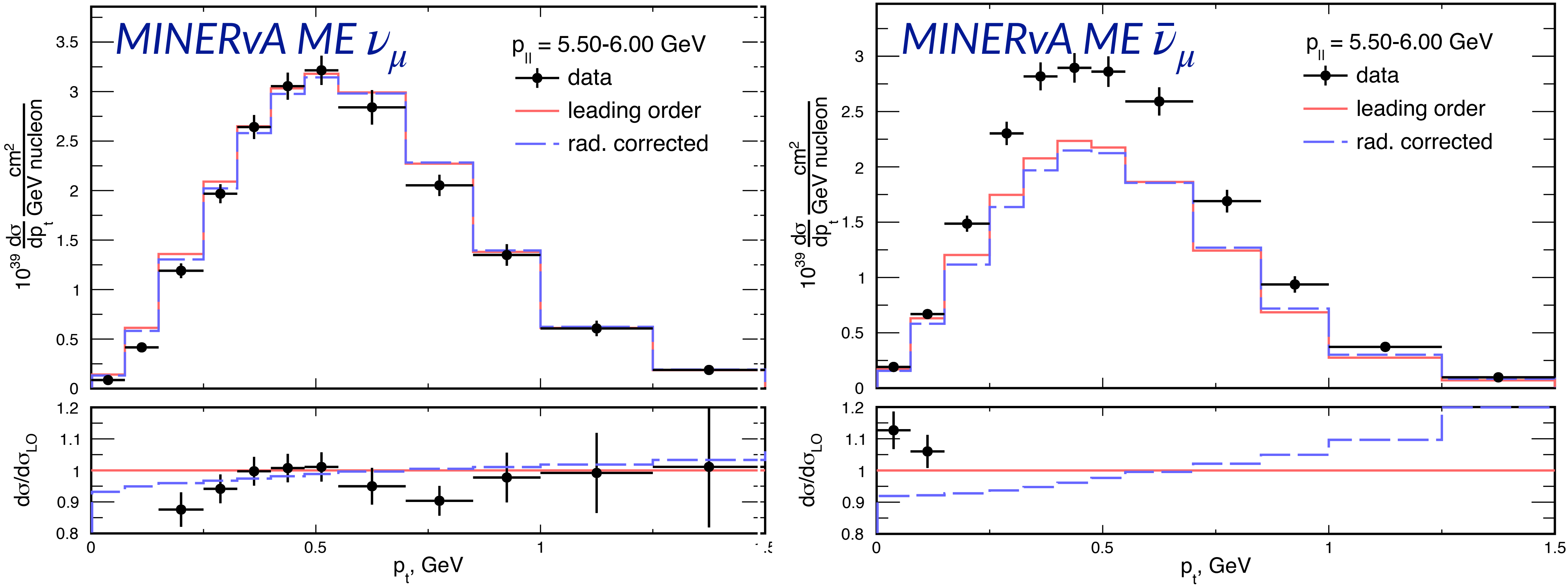


- Current data on (anti)neutrino interactions do not have the precision to validate or challenge our precise calculations of $\sigma(e)/\sigma(\mu)$ because of the sparse data on electron-neutrino and antineutrino scattering
- Corrections to muon flavor cross sections are of order current experimental precision

Comparison to data

Results

1912.09890   *Bashyal et al. 2021,2022*



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Summary

- perturbative uncertainty is controlled
- corrections are large and depend on flavor and on experimental parameters
- hadronic uncertainty cancels in flavor ratio
- nuclear corrections are small in flavor ratio of total cross sections
- photon backgrounds can be systematically studied, e.g. muon mis-id from collinear photon
- framework in place for comparison and application to data

Summary

- QED radiative corrections are an important theoretical input to oscillation experiments

  - electron flavor cross section not determined by high-statistics muon flavor data. *Happily this ratio is relatively insensitive to hadronic and nuclear uncertainties*

- radiative corrections involve cancellations that are impacted by analysis choices and detector corrections.

  - cancellation in flavor ratio between functions of $\Delta\theta$ (electron) and m_μ (muon)

- precision cross sections should provide explicit definitions of electron and muon observables

Acknowledgments

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