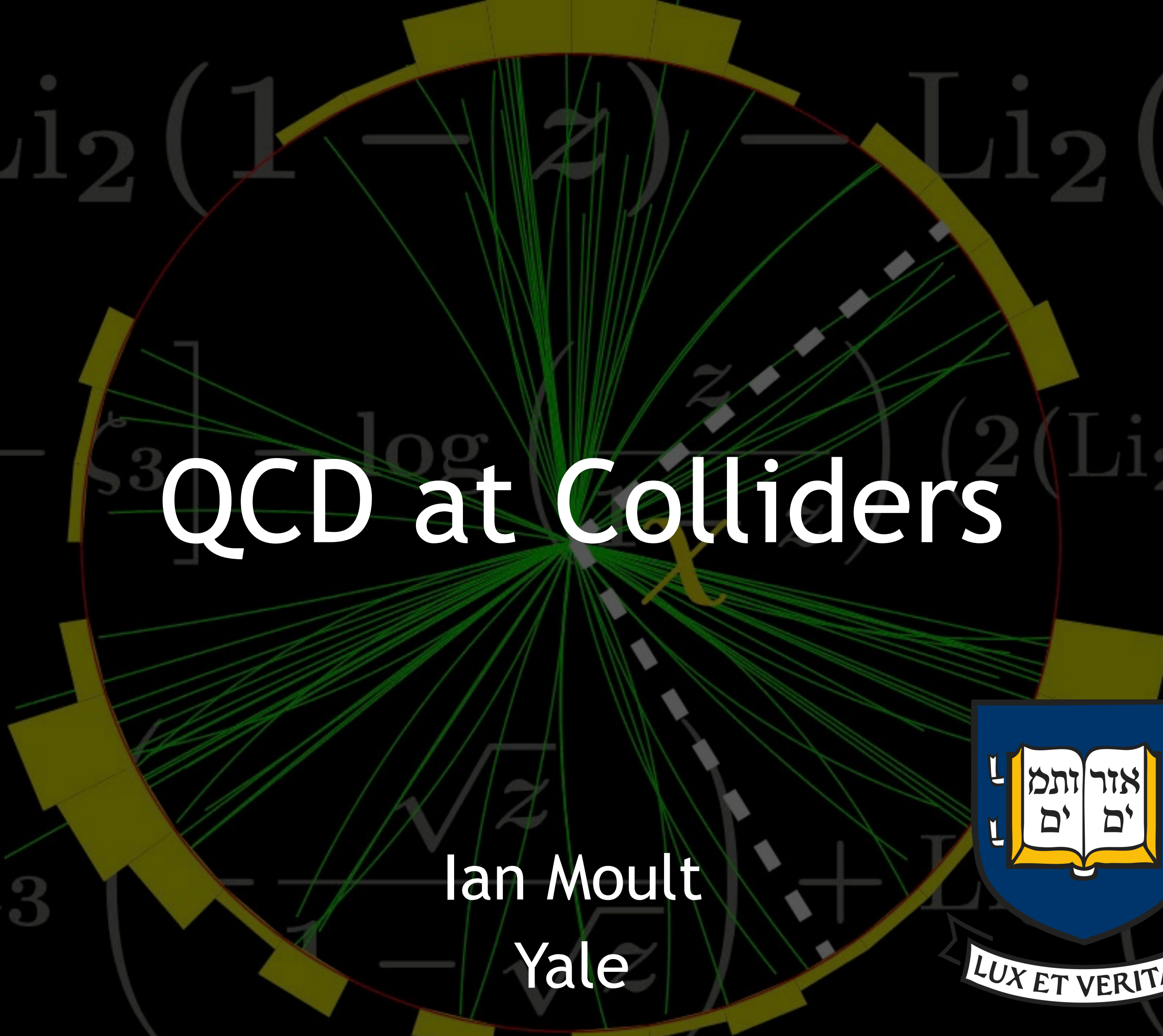
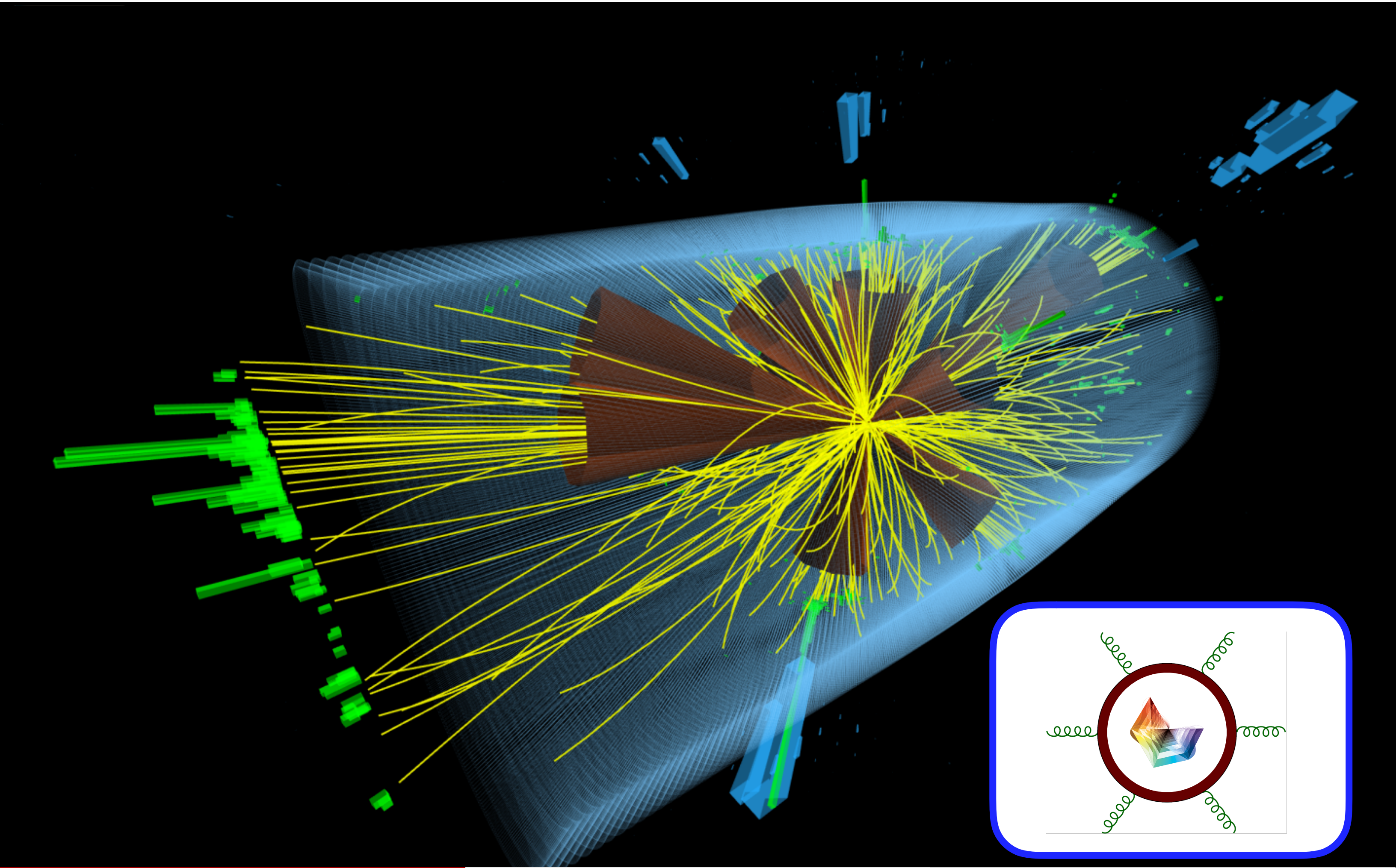


QCD at Colliders

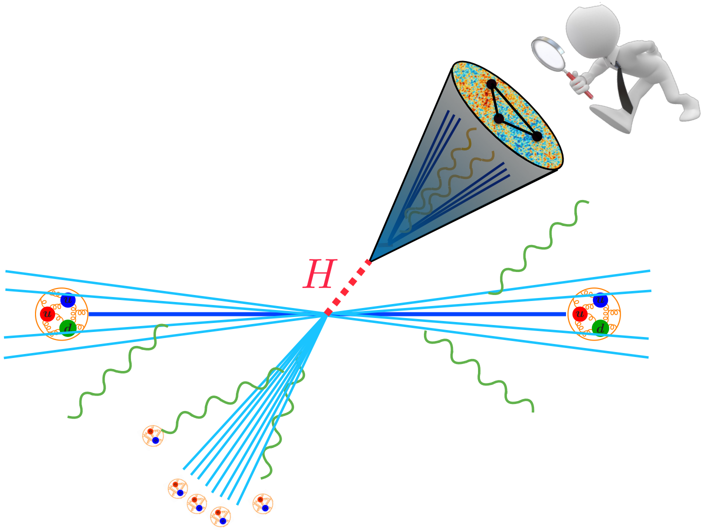
Ian Moult
Yale

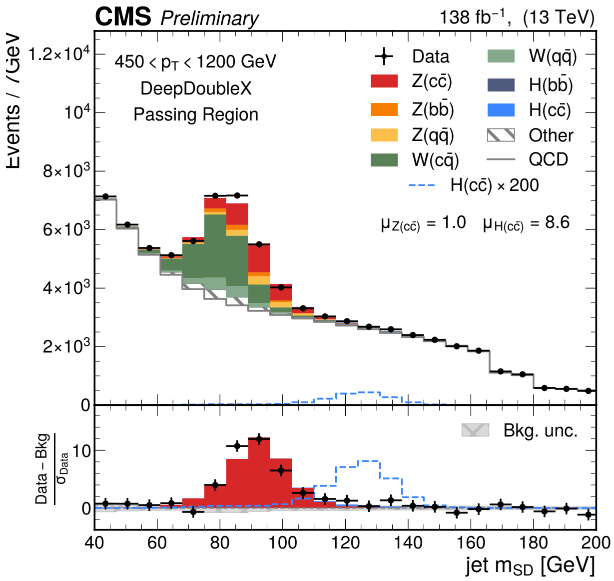
QCD at Colliders: Jets



QCD at Colliders: Jet Substructure

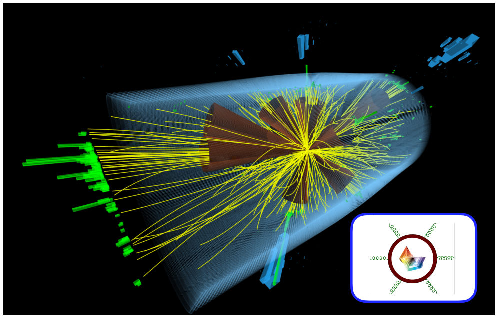
- Internal structure of jets reveals details of the microscopic collisions.

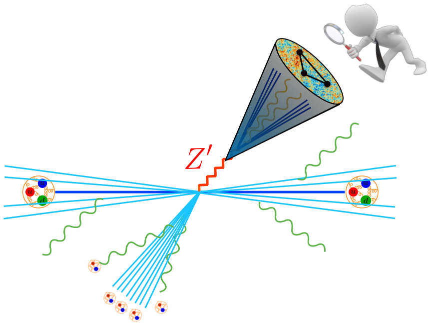




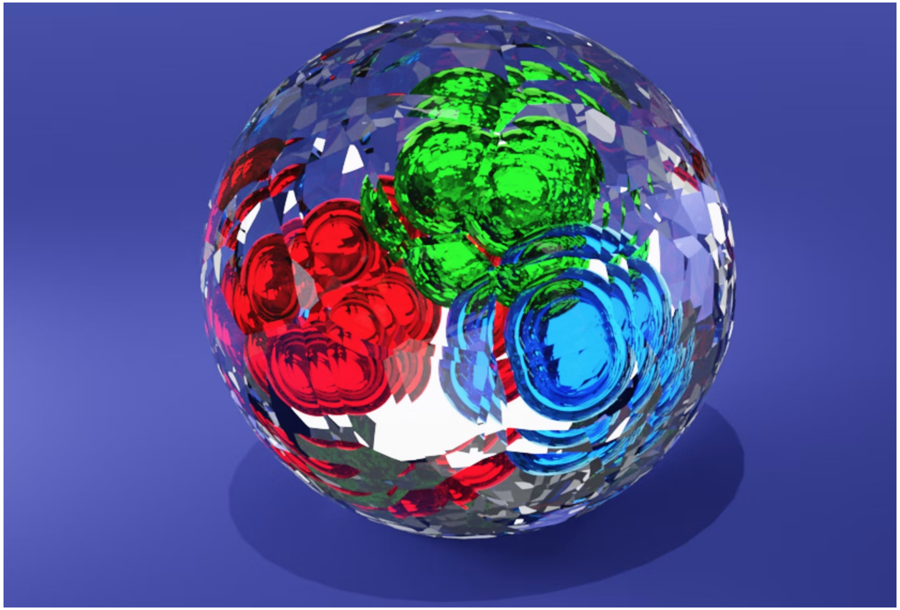
From the QCD Lagrangian to Jet Substructure

$$\mathcal{L}_{\mathsf{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + \sum_f \bar{q}_f (i \not{D} - m_f) q_f$$





The QCD Lagrangian and its Emergent Behavior



The QCD Lagrangian

- Microscopic degrees of freedom of Quantum Chromodynamics (QCD) are quarks and gluons:

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + \sum_f \bar{q}_f (i\not{D} - m_f) q_f + \frac{\theta}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} G^a_{\alpha\beta} G^a_{\mu\nu}$$

SU(3) Gauge theory with fundamental matter

$$D_\mu = \partial_\mu + i g_s T^a A^a_\mu$$

$$F^a_{\mu\nu} = \partial_\mu A^a_\nu - \partial_\nu A^a_\mu - g_s f_{abc} A^b_\mu A^c_\nu$$

$$[T^a, T^b] = i f^{abc}$$

QCD as a Classical Field Theory

- Setting the quark masses to zero, $m_f \to 0$, classical QCD is a scale invariant theory.

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + \sum_f \bar{q}_f (i \not{D} - m_f) q_f + \frac{\theta}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} G^a_{\alpha\beta} G^a_{\mu\nu}$$

$x \to \lambda x$

$\psi \to \lambda^{-3/2} x$

$A \to \lambda^{-1} x$

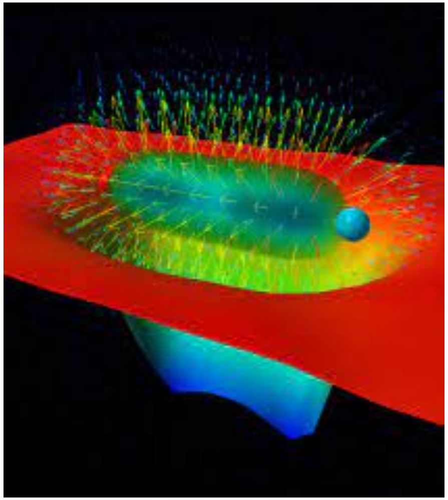
$\int d^4x \, \mathcal{L}_{QCD}$

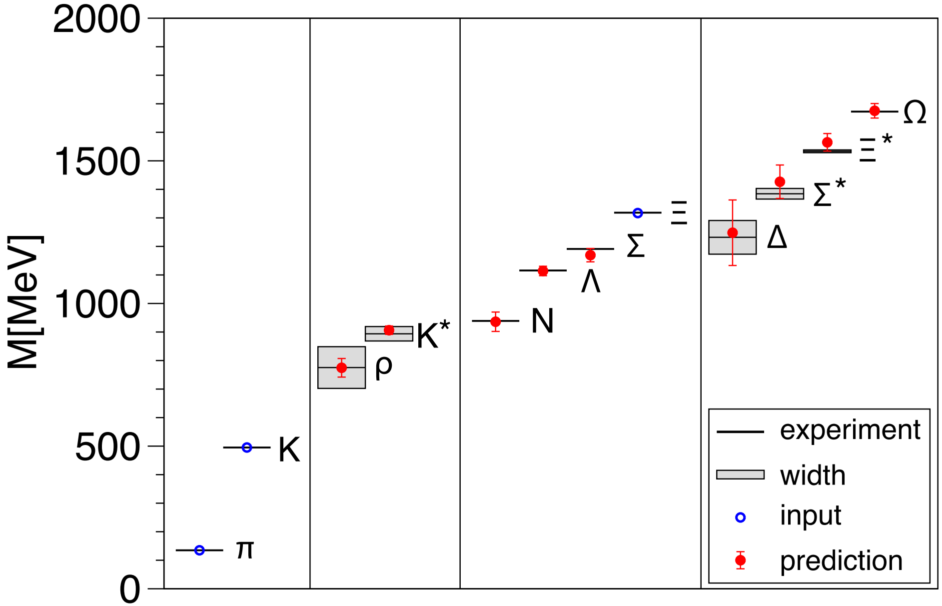
- It exhibits a spectrum of (non-linear) waves of arbitrary wave length.
- No length gap (in classical theory don't have h, but through $E = h\omega$ would correspond to a mass gap.)

QCD as a Quantum Field Theory

- At the quantum level, QCD exhibits a gapped spectra of hadronic (SU(3) neutral) states.

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4}G^a_{\mu\nu}G^{\mu\nu a} + \sum_f \bar{q}_f(i\not{D} - m_f)q_f + \frac{\theta}{16\pi^2}\epsilon^{\mu\nu\alpha\beta}G^a_{\alpha\beta}G^a_{\mu\nu}$$

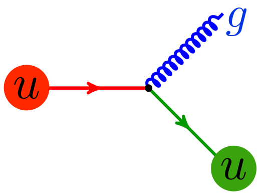




Emergent Behavior of QCD

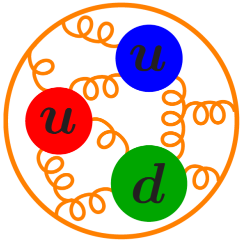
- Microscopic degrees of freedom of Quantum Chromodynamics (QCD) are quarks and gluons:

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + \sum_f \bar{q}_f (i\not{D} - m_f) q_f$$

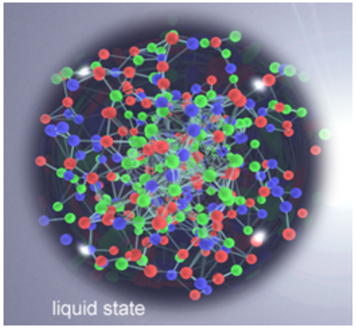


- QCD exhibits a variety of complicated emergent behavior:

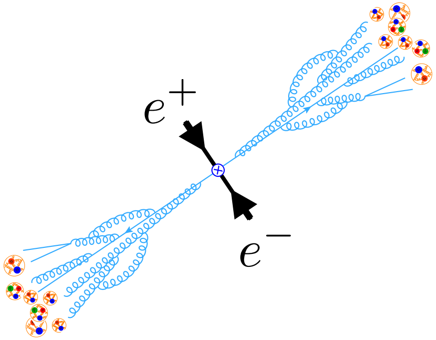
Hadrons



Quark Gluon Plasma

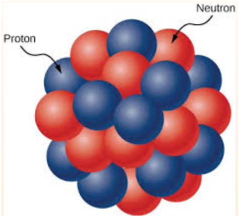


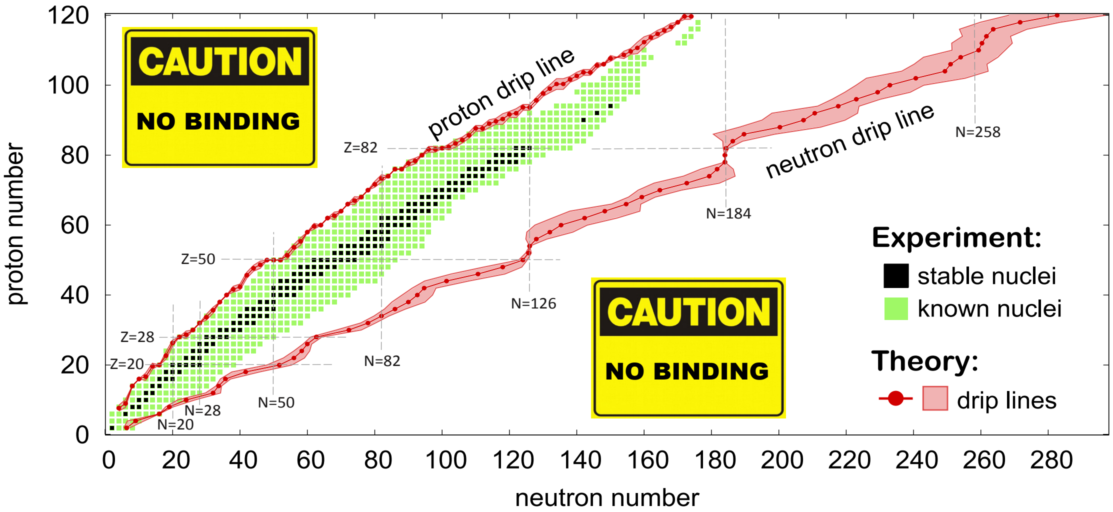
Jets

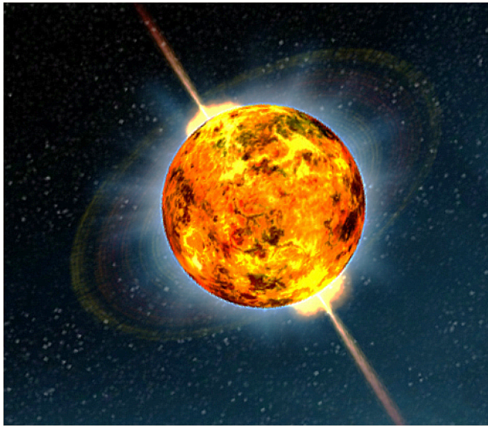


Doubly Emergent Behavior

- All of nuclear physics emerges from this simple Lagrangian.





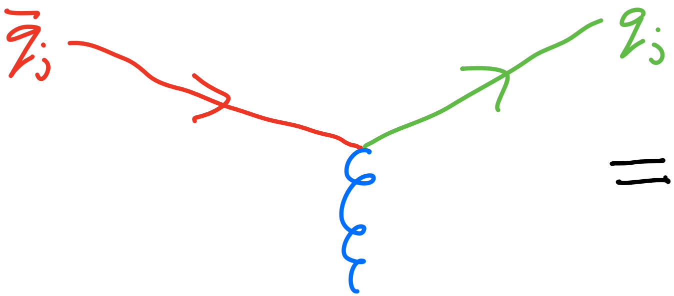


- An extremely rich theory that continues to be at the forefront of research.

QCD Feynman Rules

- Despite this wealth of complex behaviors, we can just look at the perturbative Feynman rules and see where we can get:

$$\mathcal{L}_{\text{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + \sum_f \bar{q}_f (i \not{D} - m_f) q_f + \frac{\theta}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} G^a_{\alpha\beta} G^a_{\mu\nu}$$

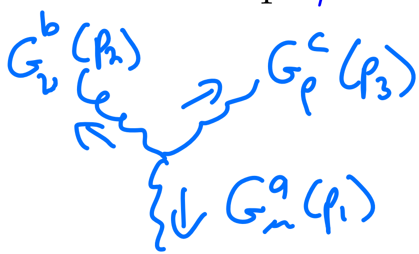


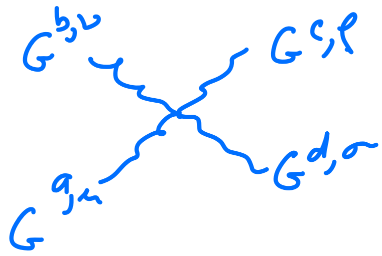
$$= -i g_s T^a \gamma_\mu$$

QCD Feynman Rules

- Despite this wealth of complex behaviors, we can just look at the perturbative Feynman rules and see where we can get:

$$\mathcal{L}_{\mathrm{QCD}} = -\frac{1}{4} G^a_{\mu\nu} G^{\mu\nu a} + \sum_f \bar{q}_f (i\not{D} - m_f) q_f + \frac{\theta}{16\pi^2} \epsilon^{\mu\nu\alpha\beta} G^a_{\alpha\beta} G^a_{\mu\nu}$$

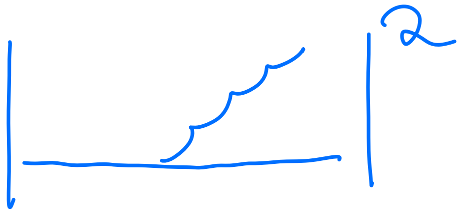


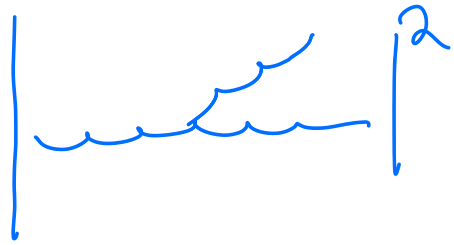


$$g_s f^{abc}\left[(p_1-p_2)_\rho g_{\mu\nu} + (p_2-p_3)_\mu g_{\nu\rho} + (p_3-p_1)_\nu g_{\rho\mu}\right]$$

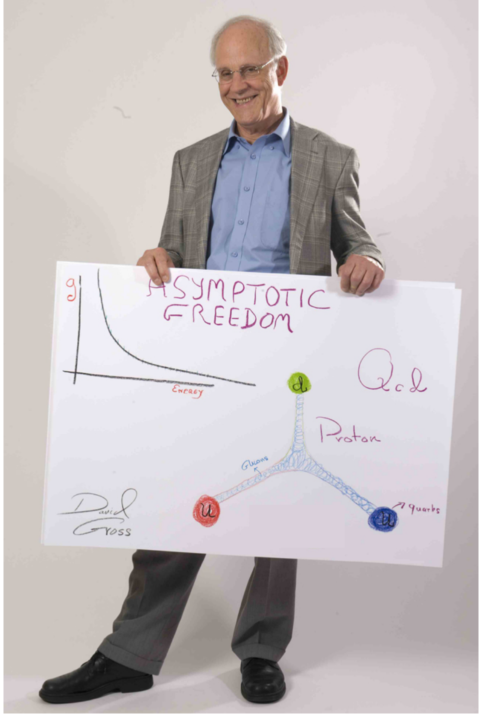
$$ig_s^2\left[f^{eac}f^{ebd}(g_{\mu\nu}g_{\rho\sigma}-g_{\mu\sigma}g_{\nu\rho}) + f^{ead}f^{ebc}(g_{\mu\nu}g_{\rho\sigma}-g_{\mu\rho}g_{\nu\sigma}) + f^{eab}f^{ecd}(g_{\mu\rho}g_{\nu\sigma}-g_{\mu\sigma}g_{\nu\rho})\right]$$

Example: Gluon Emission





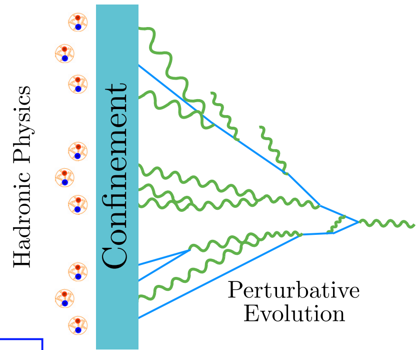
The QCD Beta Function



The QCD Beta Function

- For determining the structure of collider physics, the single most important feature of QCD is the beta function.

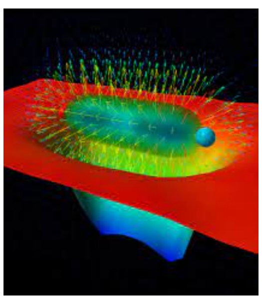
$$\alpha_s = \frac{g_s^2}{4\pi}$$

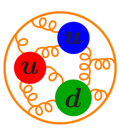


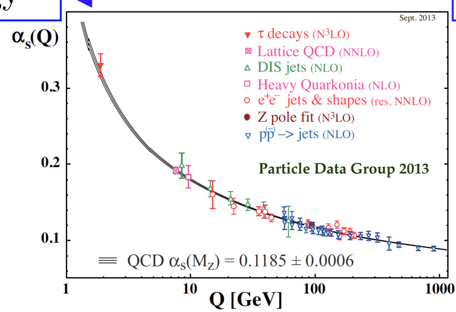
Long Time Scale
Low Energy

$\longleftarrow \alpha_s(Q)$

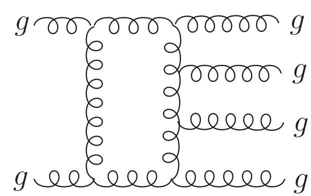
Short Time Scale
High Energy





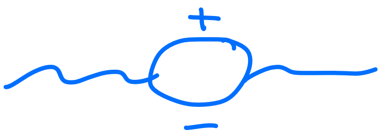


$$\hat{O} = \alpha_s + \alpha_s^2 + \cdots$$

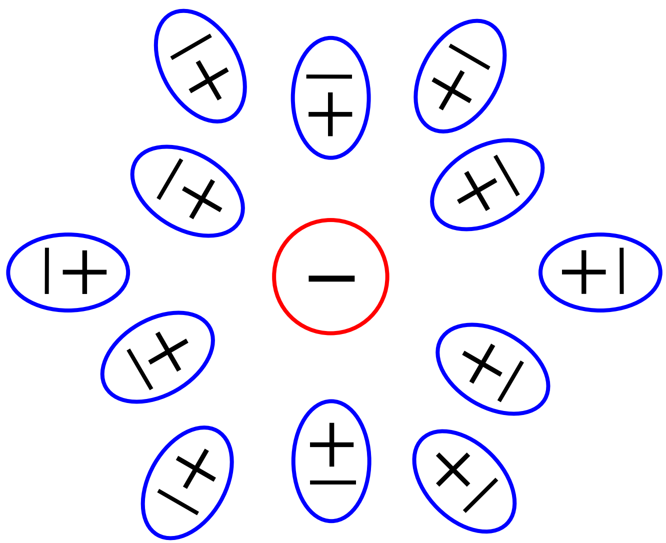


The QED Beta Function

- Recall that in QED, fluctuating $q\bar{q}$ pairs screen electric charge.



$$\mu \frac{d}{d\mu} \alpha_e = \beta_{\mathsf{QED}}(\alpha_e) \geq 0$$

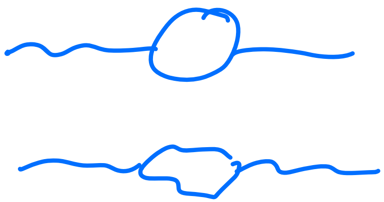


- Electromagnetic charge becomes weaker at long distances.

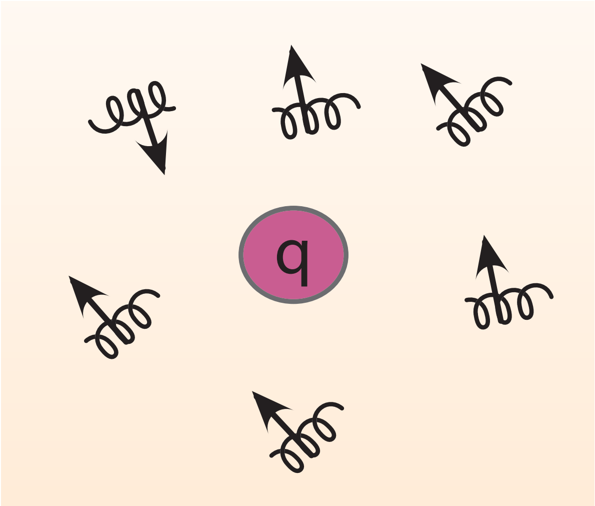
The QCD Beta Function

- Interacting gluons anti-screen:

$$\mu \frac{d}{d\mu} \alpha_s = \beta(\alpha_s)$$



$$\beta(\alpha_s) = -2\alpha_s \left(\frac{\alpha_s}{4\pi}\right) \left(\frac{11}{3} C_A - \frac{4}{3} T_F n_f\right) \leq 0$$



- This is unique to non-abelian gauge theories.

Dimensional Transmutation

- Solving this equation leads to the form of the running coupling:

$$\alpha_s(\mu) = \frac{\alpha_s(Q)}{1+\alpha_s(Q) b_0 \log(\mu/Q)}$$

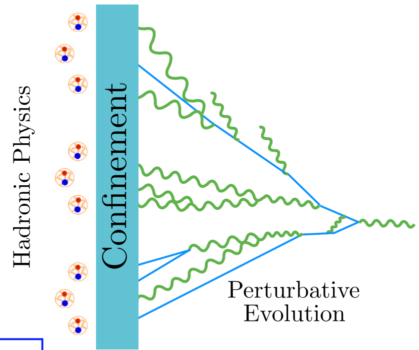
- This necessitates the introduction of a scale. Take it to be the scale, Λ, at which the coupling becomes strong.

$$\alpha_s(\mu) = \frac{1}{b_0 \log(\mu/\Lambda)}$$

- This is referred to as Dimensional Transmutation: a mass scale has appeared in quantizing the scale invariant theory.

The QCD Beta Function

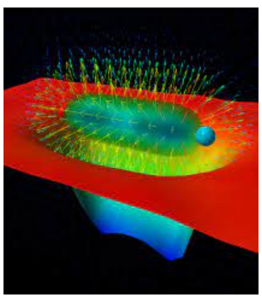
- Asymptotic freedom allows us to compute perturbatively in terms of quark/gluon degrees of freedom at high energies.

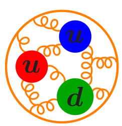


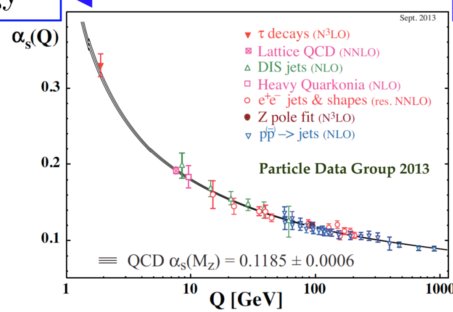
Long Time Scale
Low Energy

$\alpha_s(Q)$

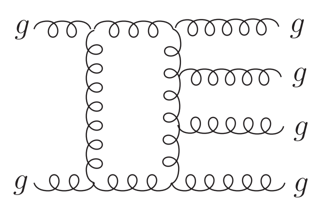
Short Time Scale
High Energy



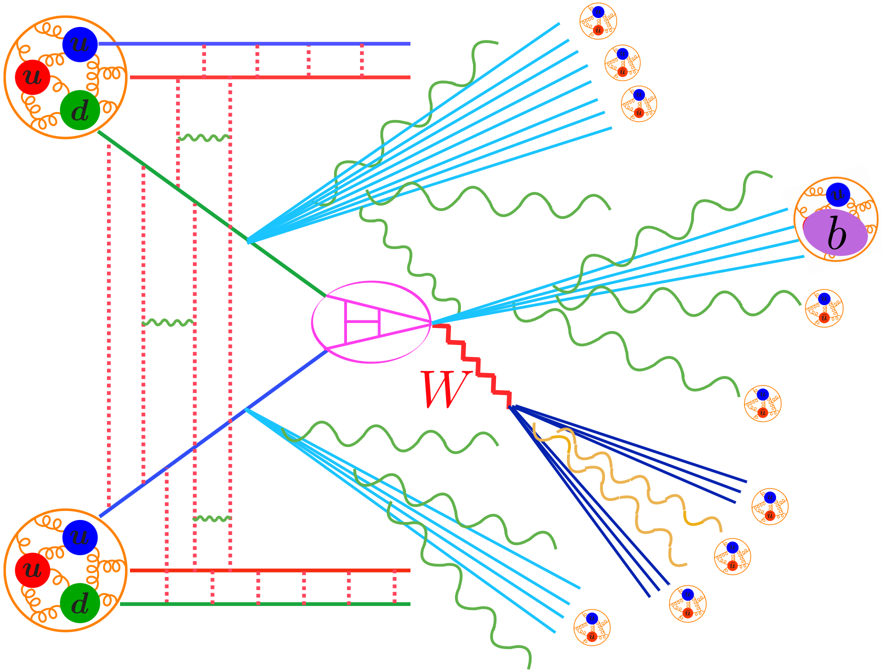




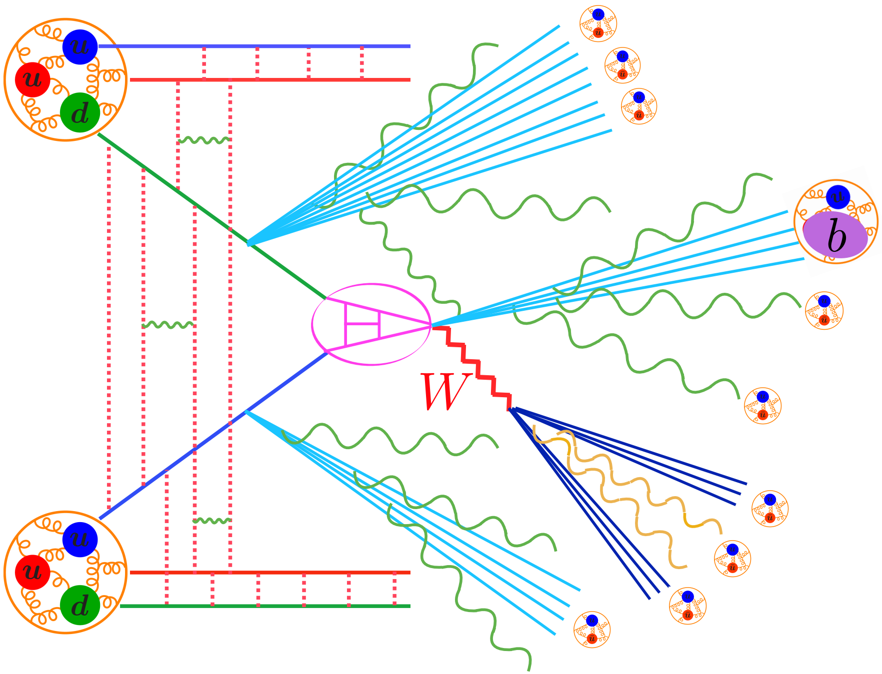
$$\hat{O} = \alpha_s + \alpha_s^2 + \cdots$$



- However, experimentalists detect, and collide hadrons, not quarks/gluons.

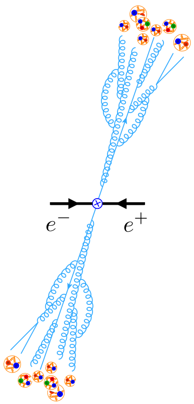


Factorization and the Anatomy of a Collider Event

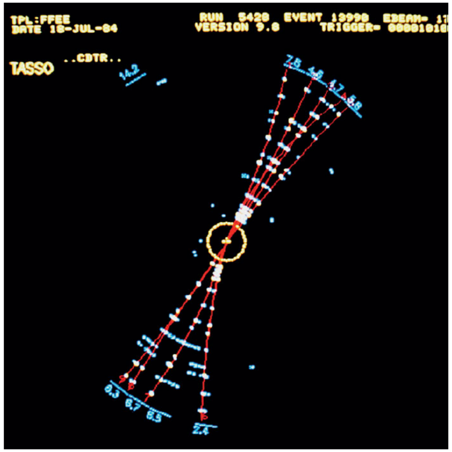


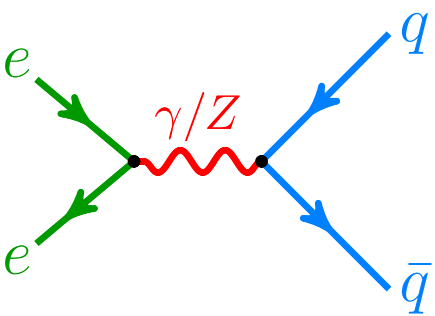
Jets at Colliders

- Jets play a central role in colliders as proxies for quarks and gluons
  $\Longrightarrow$ long distance manifestation of the microscopic interaction.

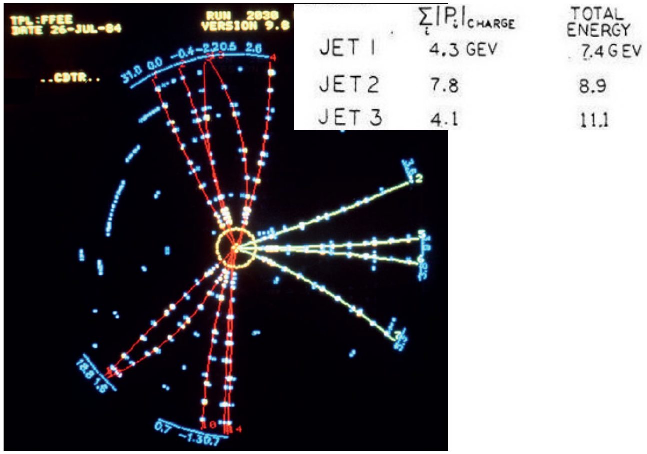


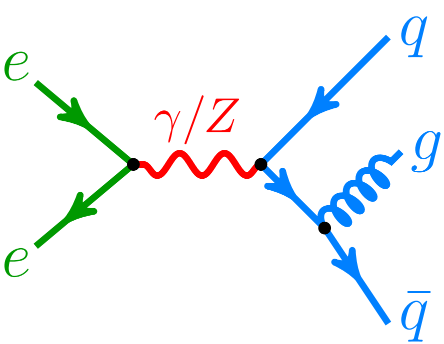
2-Jet Event





3-Jet Event

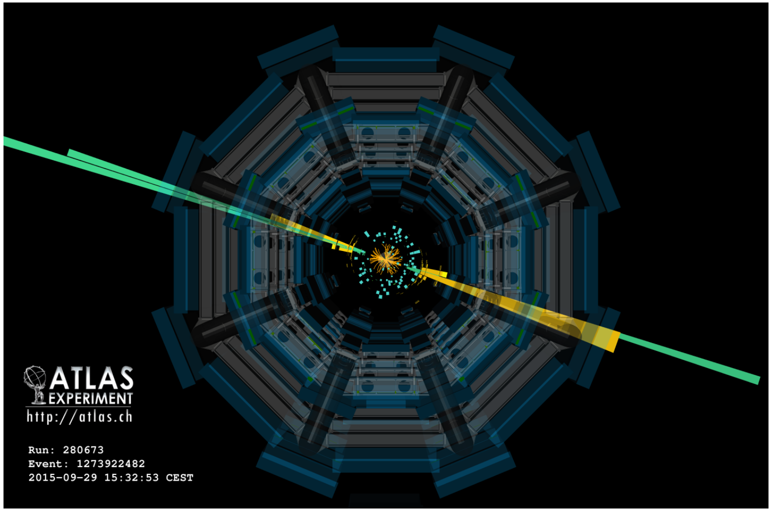




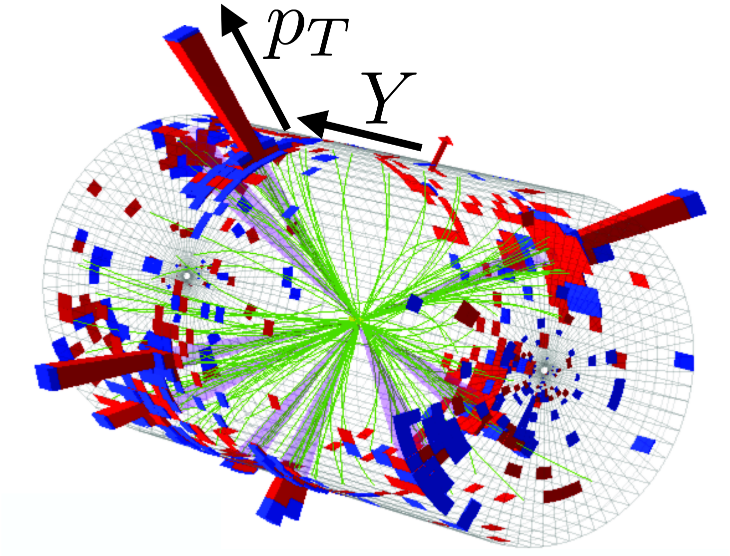
Jets at the Large Hadron Collider

- This continues at the LHC.

$p_T = 3.2$ TeV



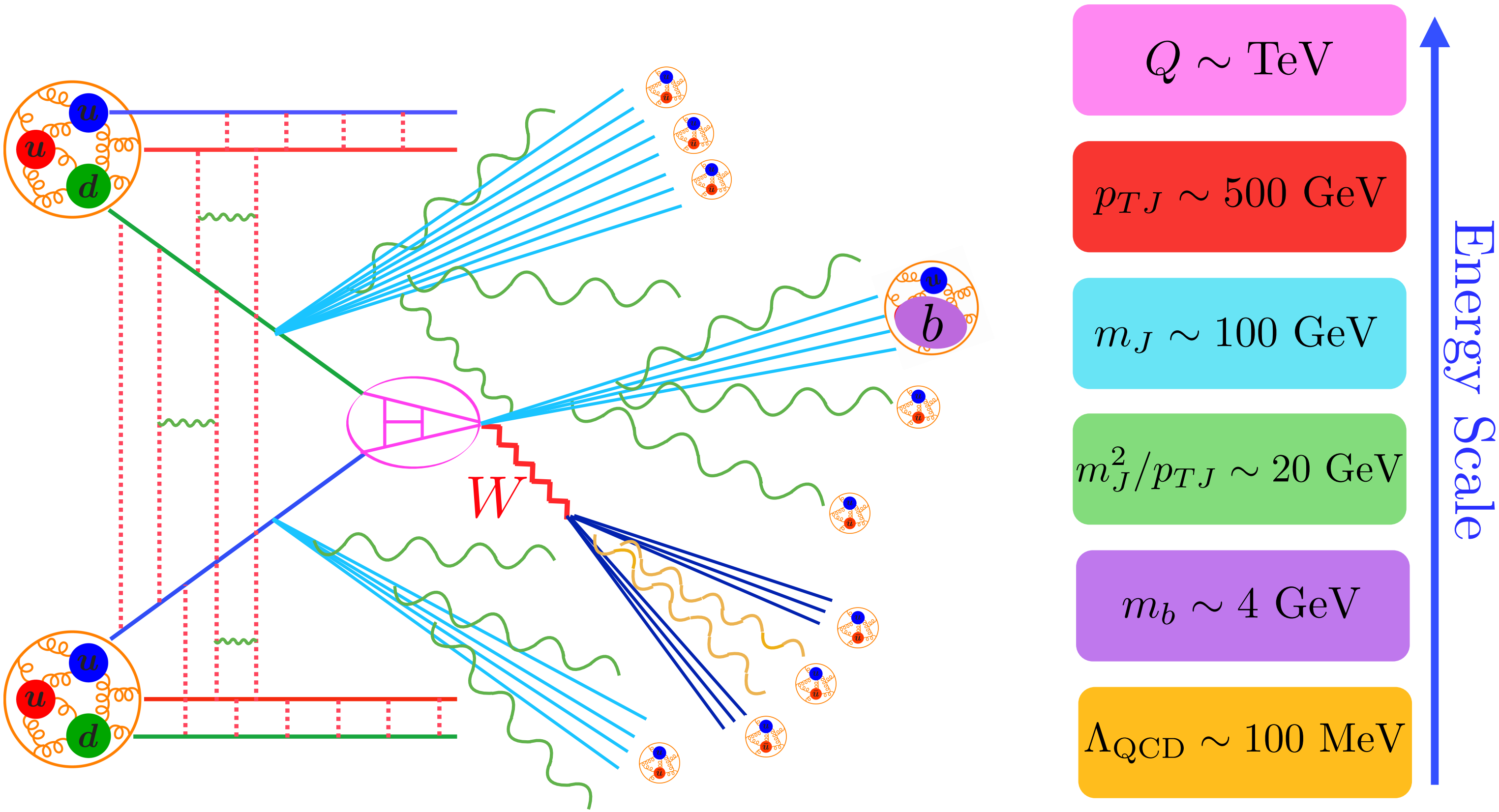
9 Jet Event



- How can we make this connection precise: Factorization!

Factorization

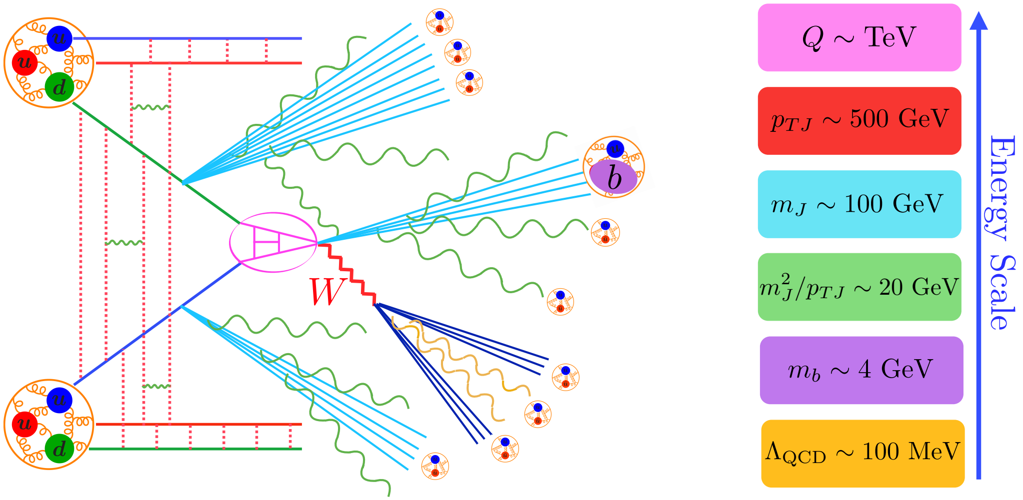
- A collision involves interactions at many hierarchical energy scales.



Factorization

- Tractable due to factorization:

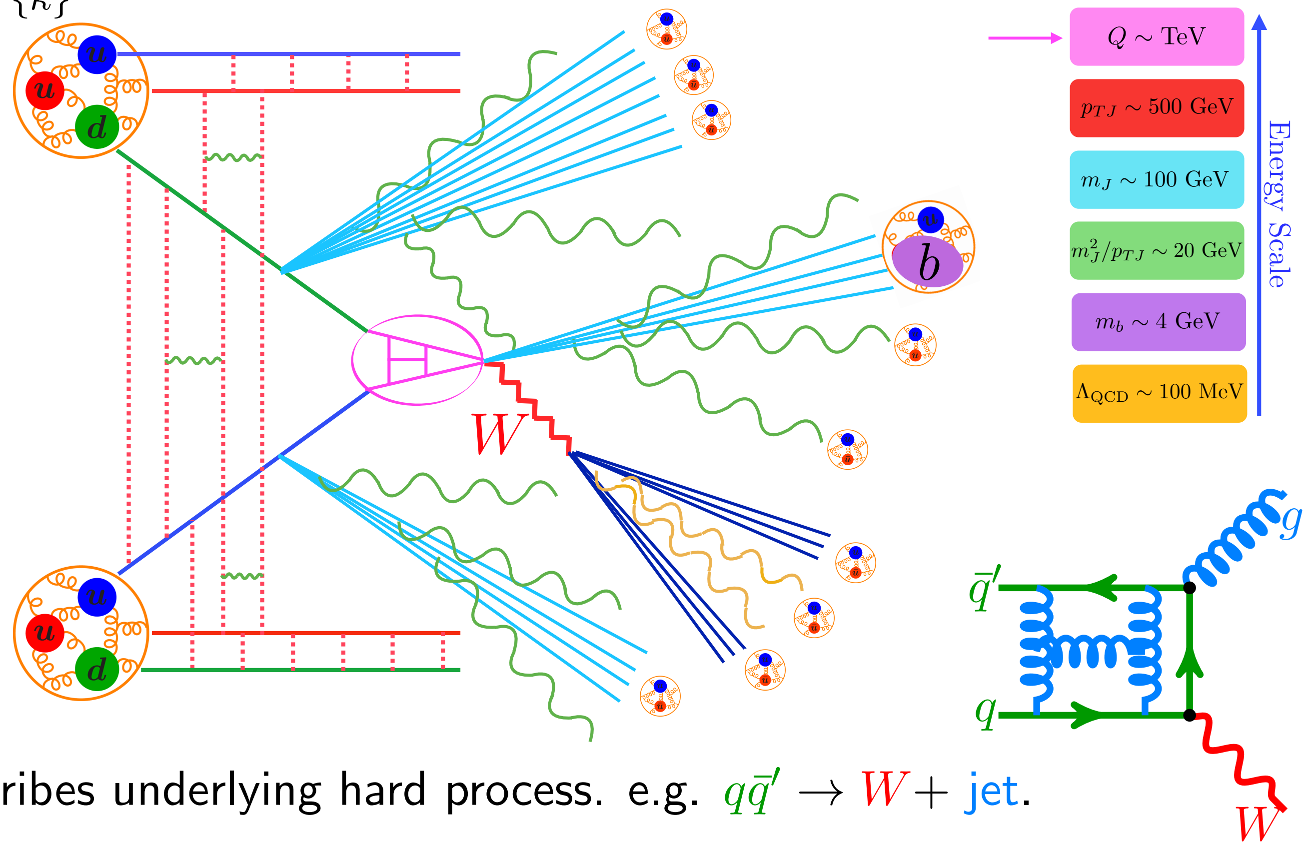
$$\frac{d\sigma}{d\mathcal{M}_1\cdots} = \sum_{\{\kappa\}} \text{tr} H_\kappa J_{\kappa_i} \otimes \cdots \otimes J_{\kappa_j} \otimes f_{p/i} f_{p/j} \otimes f_{k\to H} \otimes \cdots \otimes f_{l \to H}$$



- $\frac{d\sigma}{d\mathcal{M}_1\cdots}$ written as a convolution of single scale objects.

Factorization

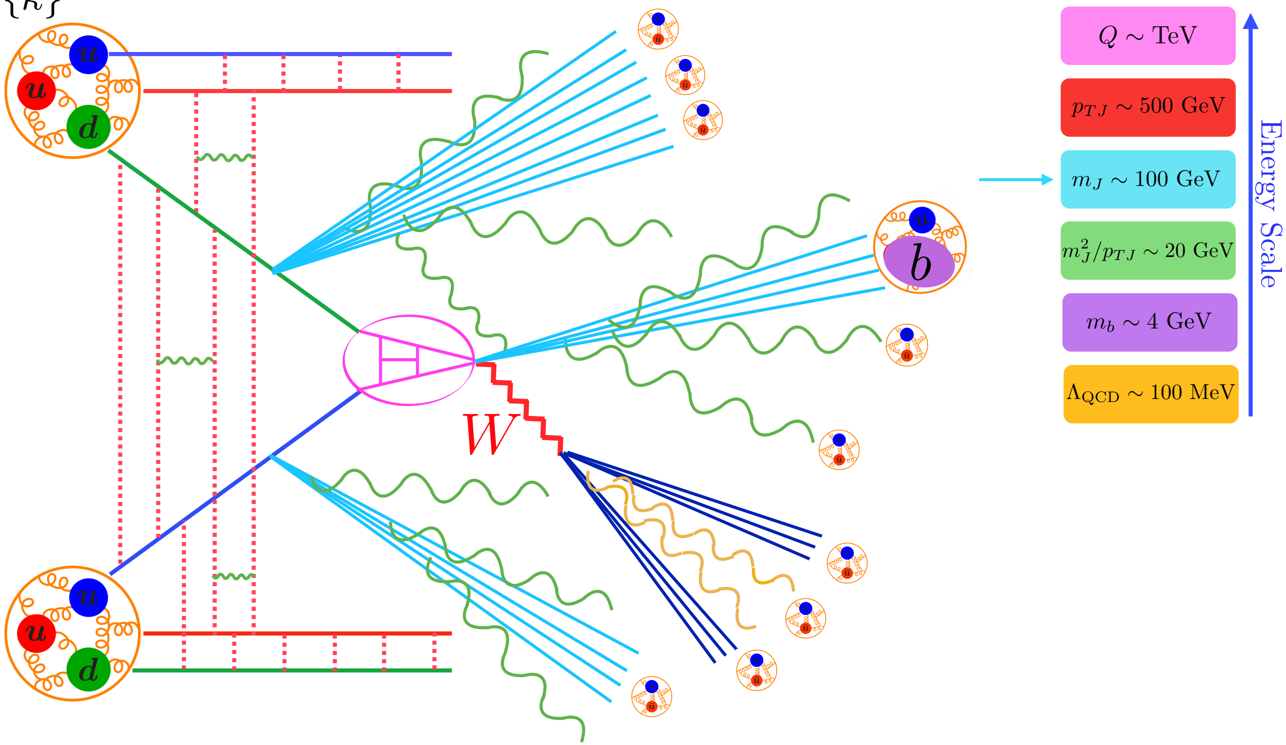
$$\frac{d\sigma}{d\mathcal{M}_1\cdots} = \sum_{\{\kappa\}} \mathrm{tr} H_\kappa J_{\kappa_i} \otimes \cdots \otimes J_{\kappa_j} \otimes f_{p/i} f_{p/j} \otimes f_{k\to H} \otimes \cdots \otimes f_{l\to H}$$



- H_κ: Describes underlying hard process. e.g. $q\bar{q}' \to W +$ jet.

Factorization

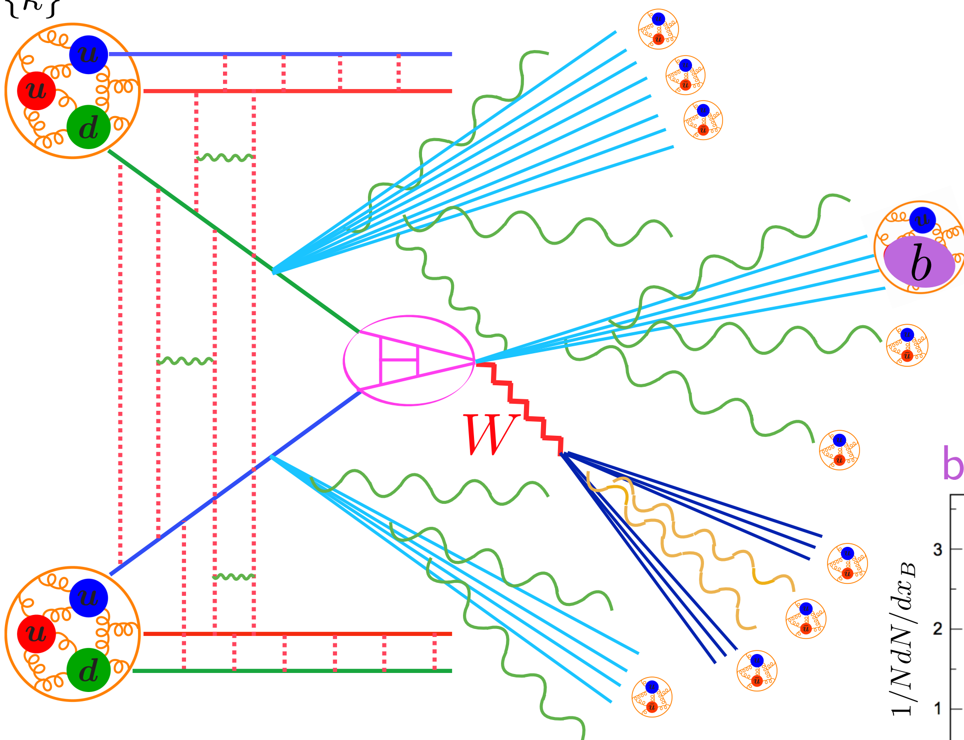
$$\frac{d\sigma}{d\mathcal{M}_1\cdots} = \sum_{\{\kappa\}} \mathrm{tr} H_\kappa J_{\kappa_i} \otimes \cdots \otimes J_{\kappa_j} \otimes f_{p/i} f_{p/j} \otimes f_{k\to H} \otimes \cdots \otimes f_{l\to H}$$

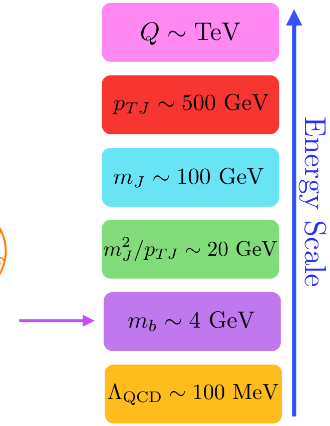


- $J_{\kappa_i} \times \cdots \times J_{\kappa_j}$: Describe dynamics of jets.

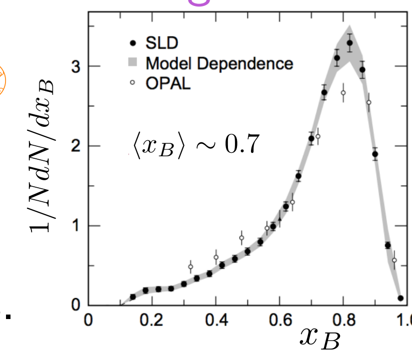
Factorization

$$\frac{d\sigma}{d\mathcal{M}_1\cdots} = \sum_{\{\kappa\}} \mathrm{tr} H_\kappa J_{\kappa_i} \otimes \cdots \otimes J_{\kappa_j} \otimes f_{p/i} f_{p/j} \otimes f_{k\to H} \otimes \cdots \otimes f_{l\to H}$$





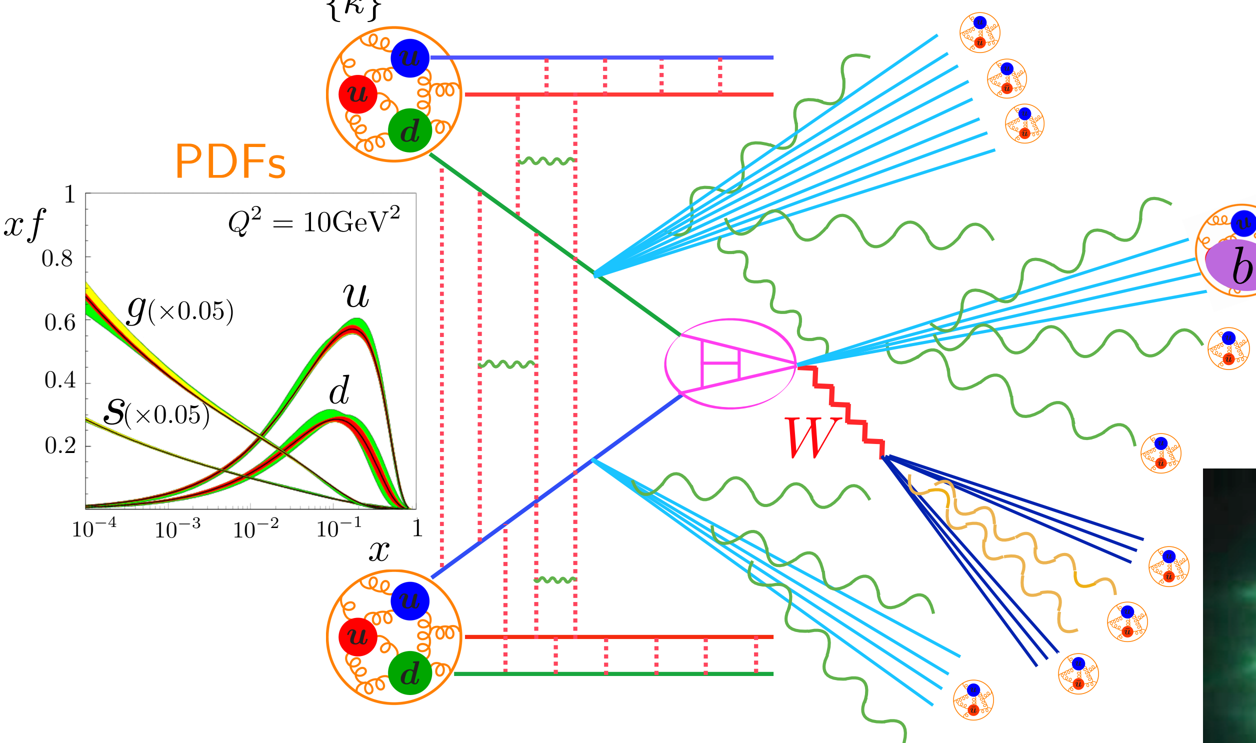
b-Fragmentation

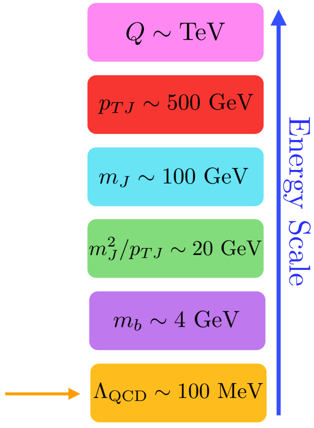


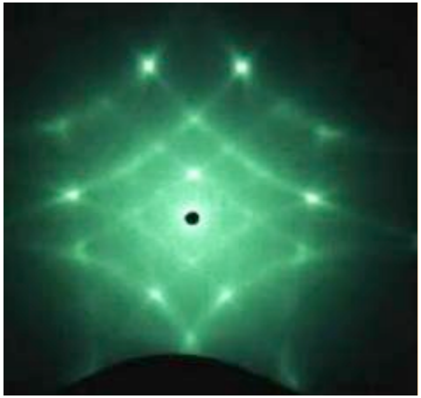
- $f_{k\to H}$: Describe fragmentation to identified hadrons.

Factorization

$$\frac{d\sigma}{d\mathcal{M}_1\cdots} = \sum_{\{\kappa\}} \mathrm{tr}\, H_\kappa J_{\kappa_i} \otimes \cdots \otimes J_{\kappa_j} \otimes f_{p/i} f_{p/j} \otimes f_{k\to H} \otimes \cdots \otimes f_{l\to H}$$

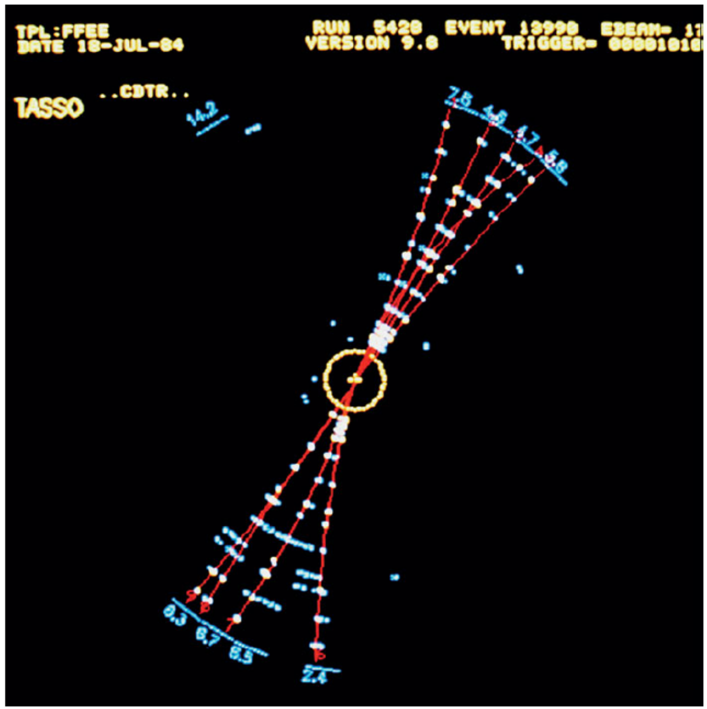






- $f_{p/i}, f_{p/j}$: PDFs describing incoming protons.

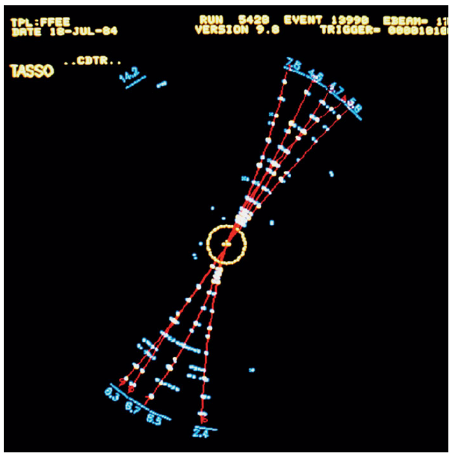
Jet Cross Sections



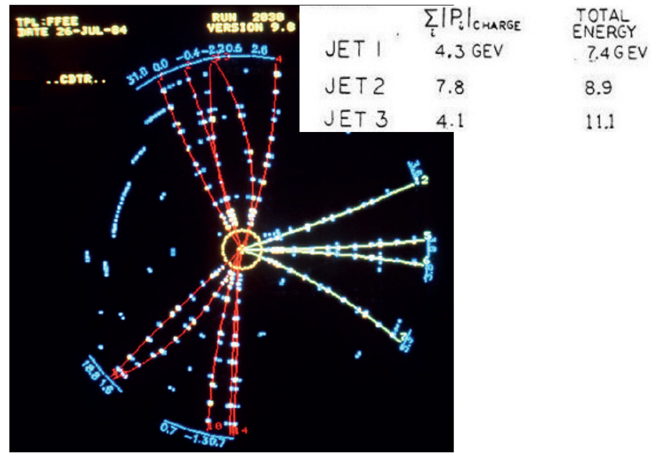
Jets Cross Sections

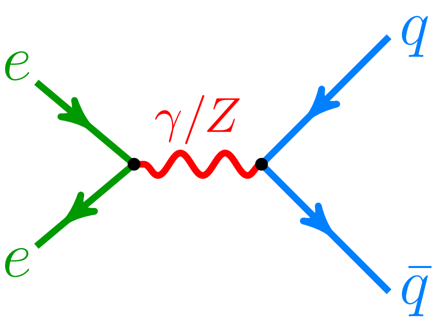
- Factorization allows robust separation of physics at different scales.
- How can we ask questions that we can answer in perturbation theory?

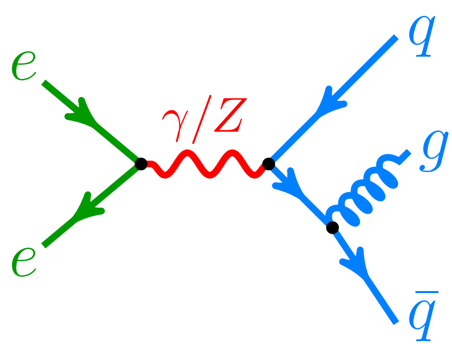
2-Jet Event



3-Jet Event

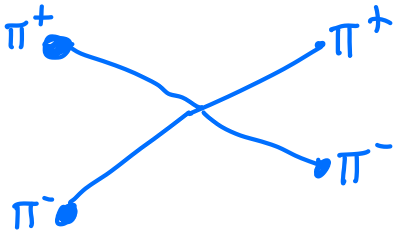


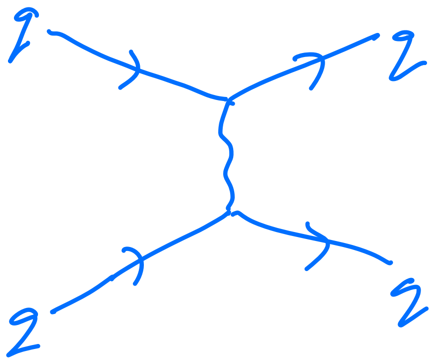




Jets Cross Sections

- In standard QFT courses, one learns how to compute scattering cross sections from the underlying Feynman rules.
- This is made more complicated since quarks and gluons are not asymptotic states.





Sterman-Weinberg

- The understanding of this from first principles field theory was first spelled out by Sterman and Weinberg

Jets from Quantum Chromodynamics

George Sterman
Institute for Theoretical Physics, State University of New York at Stony Brook, Stony Brook, New York 11790

and

Steven Weinberg
Lyman Laboratory of Physics, Harvard University, Cambridge, Massachusetts 02138

(Received 26 July 1977)

The properties of hadronic jets in e^+e^- annihilation are examined in quantum chromodynamics, without using the assumptions of the parton model. We find that two-jet events dominate the cross section at high energy, and have the experimentally observed angular distribution. Estimates are given for the jet angular radius and its energy dependence. We argue that the detailed results of perturbation theory for production of arbitrary numbers of quarks and gluons can be reinterpreted in quantum chromodynamics as predictions for the production of jets.

The observation[1] of hadronic jets in e^+e^- annihilation provides one of the most striking confirmations of the parton picture.[2] In particular, the distribution of events in the angle θ between the jet axis and the e^+-e^- beam line is observed to be very close to the form $1+\cos^2\theta$ that would be expected for the production of a pair of relativistic charged pointlike particles of spin $\frac{1}{2}$. We shall argue here that the existence, angular distribution, and some aspects of the structure of these jets follow as consequences of the perturbation expansion[3] of quantum chromodynamics[4] (QCD), without assuming the parton picture (in particular, the transverse-momentum cutoff) in advance. Thus, the observed features of jets provide evidence for an underlying asymptotically free gauge field theory with elementary spin-$\frac{1}{2}$ quarks. We also wish here to demonstrate a general approach, which may be applicable to a wide range of high-energy phenomena.

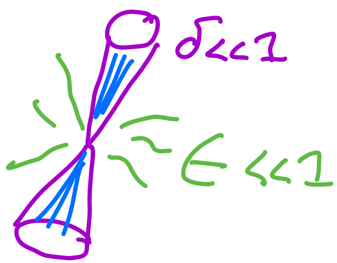
Infrared and Collinear Safety

- ## Sterman-Weinberg Jets and IRC Safety

However, by performing various sums over states, it is possible to define a wide range of cross sections which are free of $m \to 0$ singularities. To learn what they are, we observe that quantum field theories of massless particles have always been found (in the absence of superrenormalizable couplings) to be physically sensible, i.e., that any cross section which would actually be measurable in such a massless theory is free of infrared divergences in each order of perturbation theory.[5] Hence in the real world with $m \neq 0$, any sort of partial cross section which would be measurable for $m = 0$ is expected to be free of singularities in m as $m \to 0$, and can therefore be calculated perturbatively[3] in QCD for $E \to \infty$.

For instance, the cross section for production of a definite number of particles does have singularities for $m \to 0$, because for $m = 0$ we could not expect to be able to tell the difference between one particle or several particles moving in the same direction. At the opposite extreme, the total cross section for $e^+e^- \to$ hadrons would clearly be measurable even for zero quark mass, and hence must be free of singularities in m (to

To study jets, we consider the partial cross section $\sigma(E, \theta, \Omega, \epsilon, \delta)$ for e^+e^- hadron production events, in which all but a fraction $\epsilon \ll 1$ of the total e^+e^- energy E is emitted within some pair of oppositely directed cones of half-angle $\delta \ll 1$, lying within two fixed cones of solid angle Ω (with $\pi\delta^2 \ll \Omega \ll 1$) at an angle θ to the e^+e^- beam line. We expect this to be measurable for $m = 0$, because the only quarks or gluons which are likely to be diffracted or radiated away from a calorimeter at θ have very long wavelength, and so carry negligible energy. Thus σ should be free of mass singularities for $m \to 0$, and calculable



As expected, each separate contribution is singular for $\mu \to 0$, but cancellations[8] occur in the sum, and the final result is free of mass singularities:

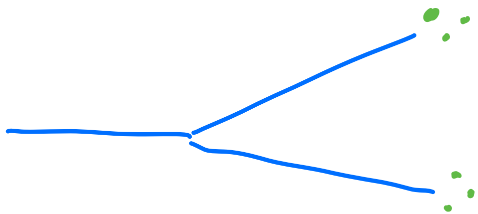
$$\sigma(E, \theta, \Omega, \epsilon, \delta) = (d\sigma/d\Omega)_0\, \Omega[1 - (g_E{}^2/3\pi^2)(3\ln\delta + 4\ln\delta\ln 2\epsilon + \pi^2/3 - \tfrac{5}{2})]. \quad (6)$$

Infrared and Collinear Safety

- Infrared and Collinear Safety is a criterion to guarantee minimal sensitivity to the hadronization process, and finite results in perturbation theory:

Collinear Safety: insensitivity to a particle splitting collinearly

Soft Safety: insensitivity to a soft emission.



⇒ set by perturbative physics!

Jet Charge

- As an application, is the charge of a jet IRC safe?

It might be thought that the partial jet cross section $\sigma(E, \theta, \Omega, \epsilon, \delta)$ should be measurable for massless theories, and hence free of mass singularities in the limit of zero mass, even if we specify the charge in each jet. If this were the case (and if there are no failures of perturbation theory[3] in QCD when jet charges are measured) then our calculation would not account for real jets with integer total charge, since to order $g_E{}^2$ it is only possible to produce jets of third-integral charge.[12] However, direct calculation to order $g_E{}^4$ shows that the cross section for final states with a definite value for the charge emitted in a given solid angle will have singularities in the limit that the quark mass vanishes. As far as we can tell, the reason that cross sections for the emission of massless particles with a definite total charge into a definite solid angle cannot be

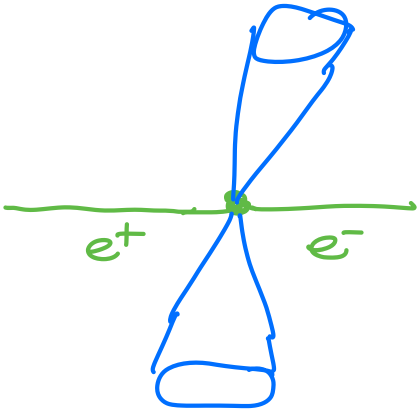
[13]This was independently suggested to one of us (S.W.) by E. Witten.

Angular Distribution

- Jets are rigorously proxies for the underlying matrix elements of quarks and gluons!

We can also conclude from Eq. (6) that the two-jet events have just the same $1 + \cos^2\theta$ angular distribution as in the Born approximation for e^+e^-

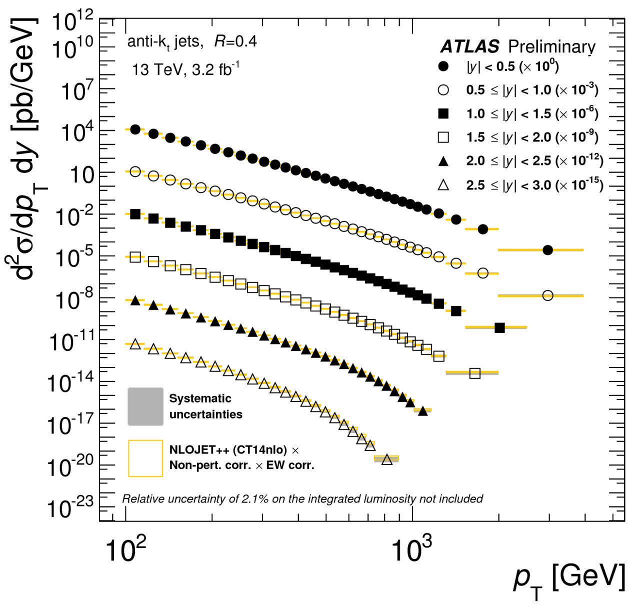
The methods of this paper can be applied to any field theory, not just QCD. However only in an asymptotically free field theory like QCD can we deduce the simple behavior which seems to be observed experimentally: a total cross section dominated at high energy by two-jet events, with an angular distribution characteristic of the lowest-order production of elementary particles.

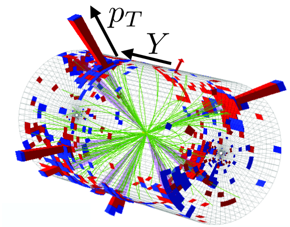


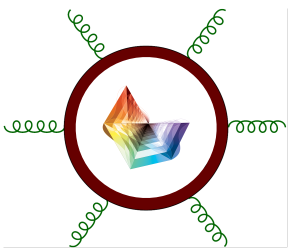
Jets at the LHC

- Obtaining a precise description of jet cross sections has been a significant driver of theory developments in Quantum Field Theory.

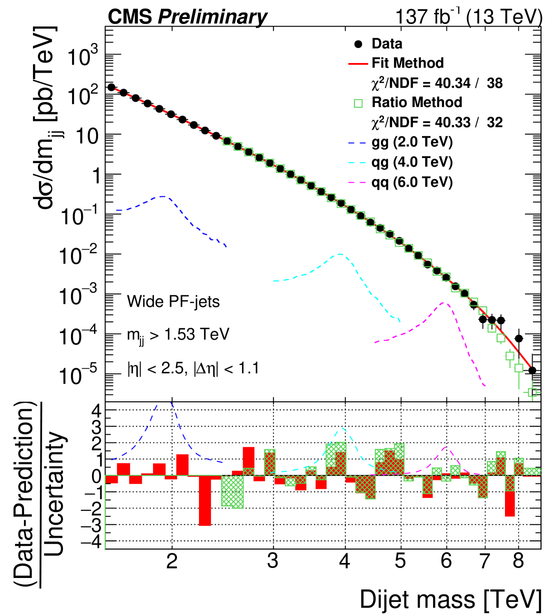
Jet Kinematic Distributions





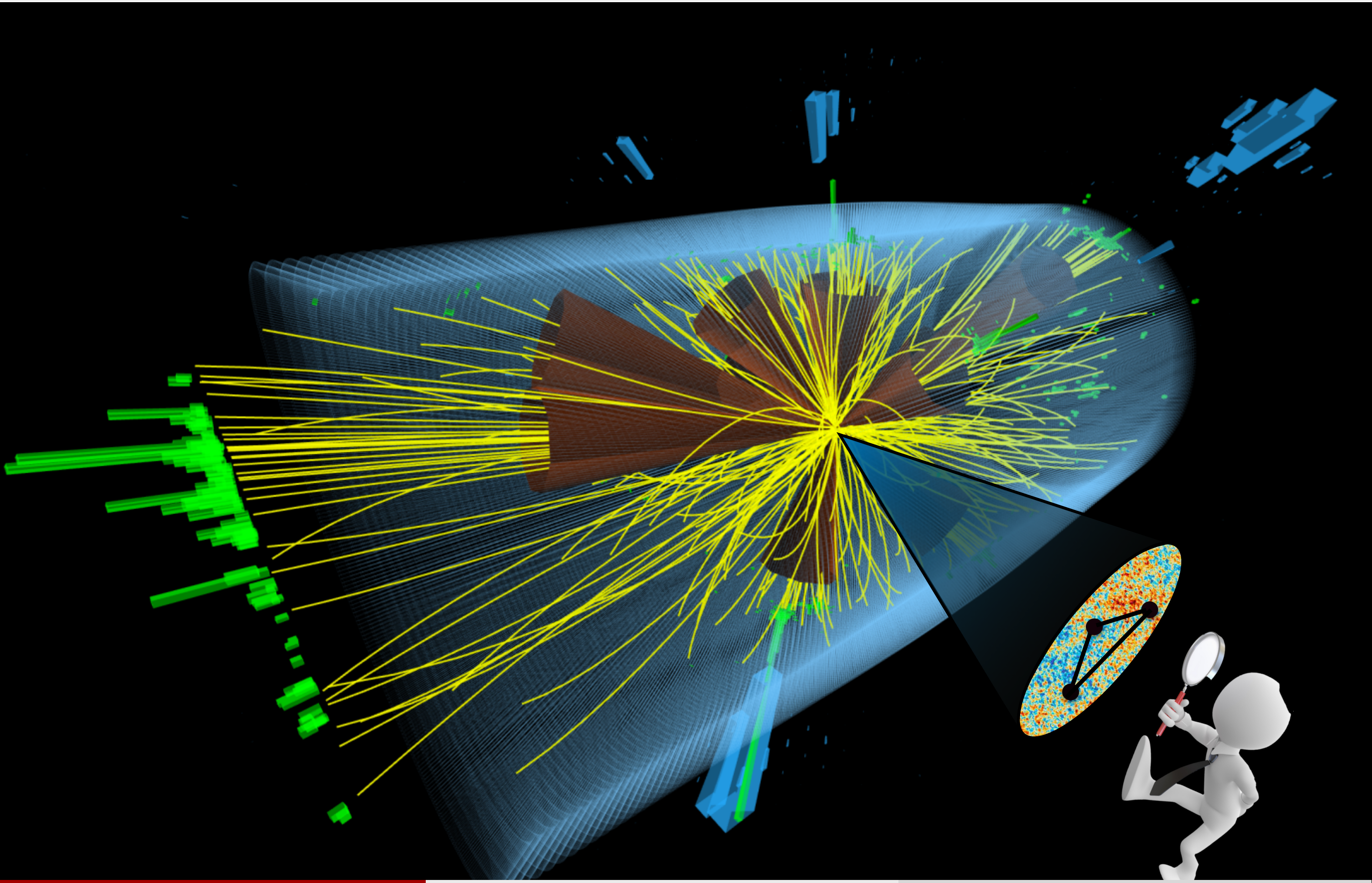


Dijet Mass



- Enables precision tests of QCD and searches for new physics.

Next Lecture: Jet Substructure!



Thanks!