BERNHARD MISTLBERGER

THE HIGGS BOSON

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THE HIGGS BOSON
AFTER DISCOVERY

THE HIGGS AFTER DISCOVERY

- The Higgs mechanism in the Standard Model allowed us with very few assumptions to unify the electroweak force and generate fundamental mass.

- The SM Higgs mechanism may be the simplest one compatible with experiment - many more working theories have been proposed!
    - Which Higgs boson did we find?
    - Are there more?
    - Is the Higgs composite?

- We want to figure out if the Higgs boson we discovered really is the one predicted by the Standard Model and if all its consequences are realized in nature.

THE HIGGS AFTER DISCOVERY – THE BIGGER PICTURE

- The coupling and mass of the Higgs boson introduce the electroweak scale into our universe. Why does it take this value?

$$\lambda = \frac{m_h^2}{2v^2} = 0.13$$

- The hope to unify with gravity:

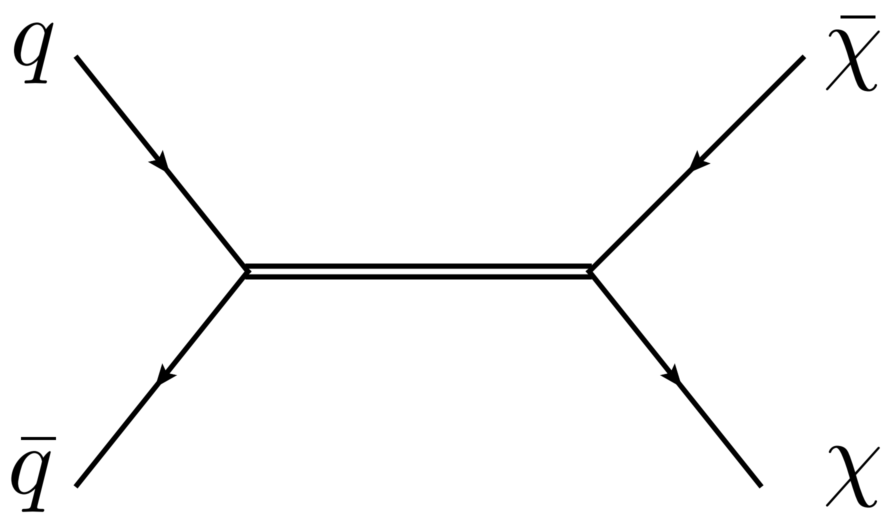
$$V_{\text{grav}} = -\frac{Gm_h^2}{r} \qquad V_{\text{Higgs}} = -\frac{\lambda^2}{r}e^{-rm_h}$$

Why do the “couplings” differ so much? **HIERARCHY PROBLEM**

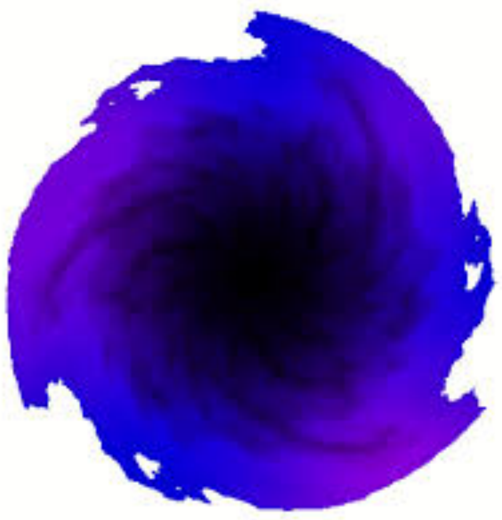
$$\frac{Gm_h^2}{\lambda^2} \sim 10^{-33}$$

THE HIGGS AFTER DISCOVERY – THE BIGGER PICTURE

- **Dark Matter!** We know it exists but we don't know what it is!
  The Higgs couples to particles with mass - maybe also dark matter?



HIGGS PORTAL



- CP violation is derived from Higgs boson Yukawa couplings.
  We don't have enough of it to explain why we have more matter than anti-matter in the universe.

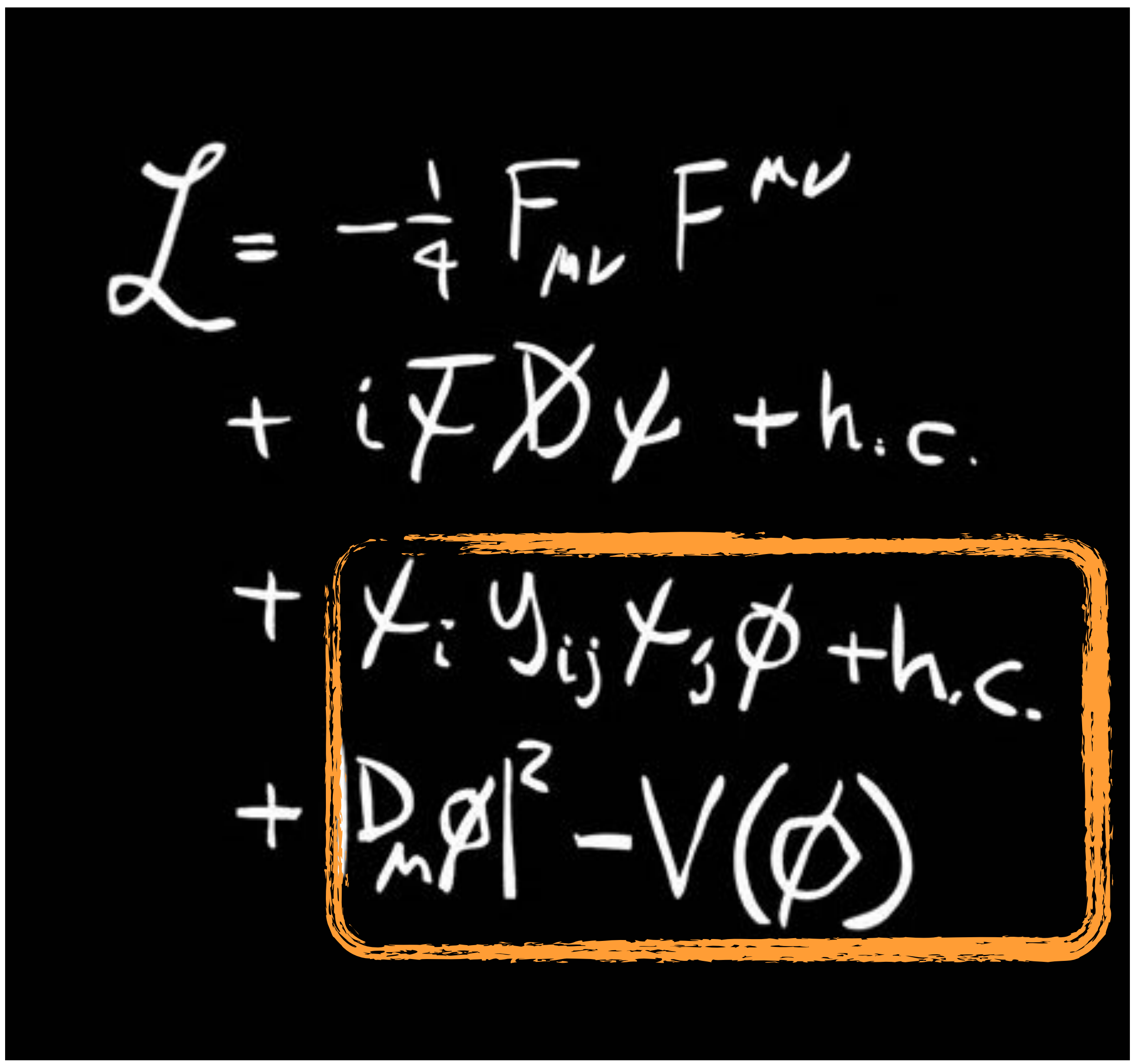
- Quarks and leptons have very different masses - why?

- Could the Higgs be driving inflation in the early universe?

THE HIGGS AFTER DISCOVERY – THE BIGGER PICTURE

- We have some pretty foundational gaps in our understanding of the universe!

Neutrino masses, matter-anti-matter asymmetry, inflation, hierarchy of masses, gravity, dark matter, dark energy, ….

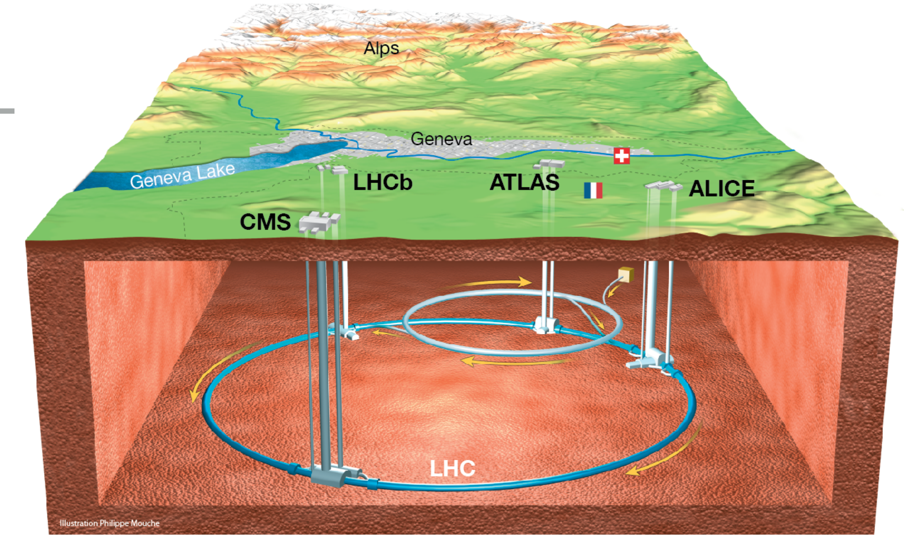


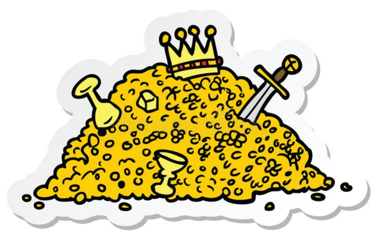
- With the LHC and the discovery of the Higgs we gained for the first time a window to study a huge part of the fundamental interactions in a controlled laboratory setting.
- Exploring this sector of interactions will get us closer to answers about the most pressing questions of physics of our time.

THE LHC – NOW

- **We are still in the early stages of LHC physics!**

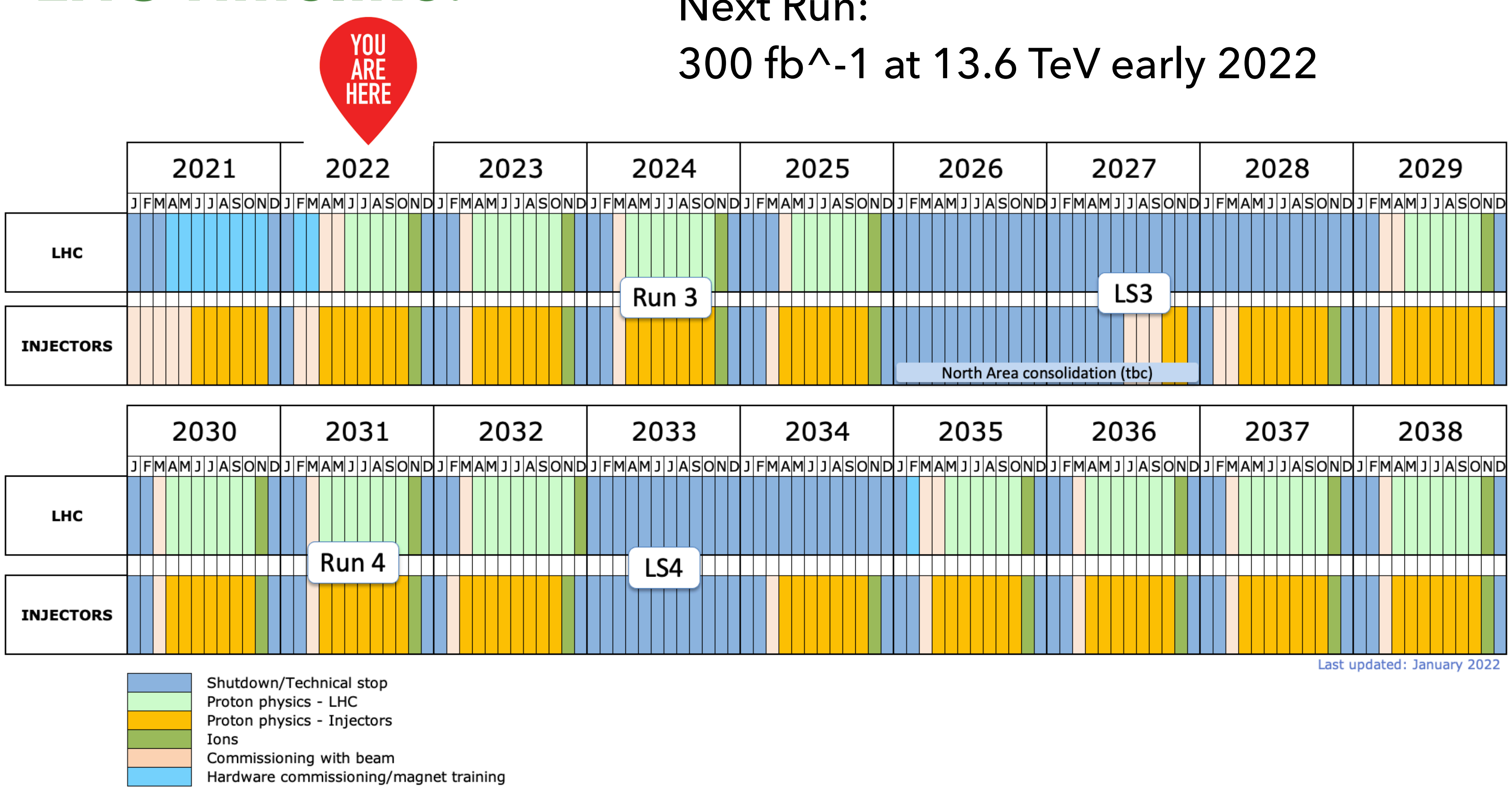
  Today: up to 139 fb^-1



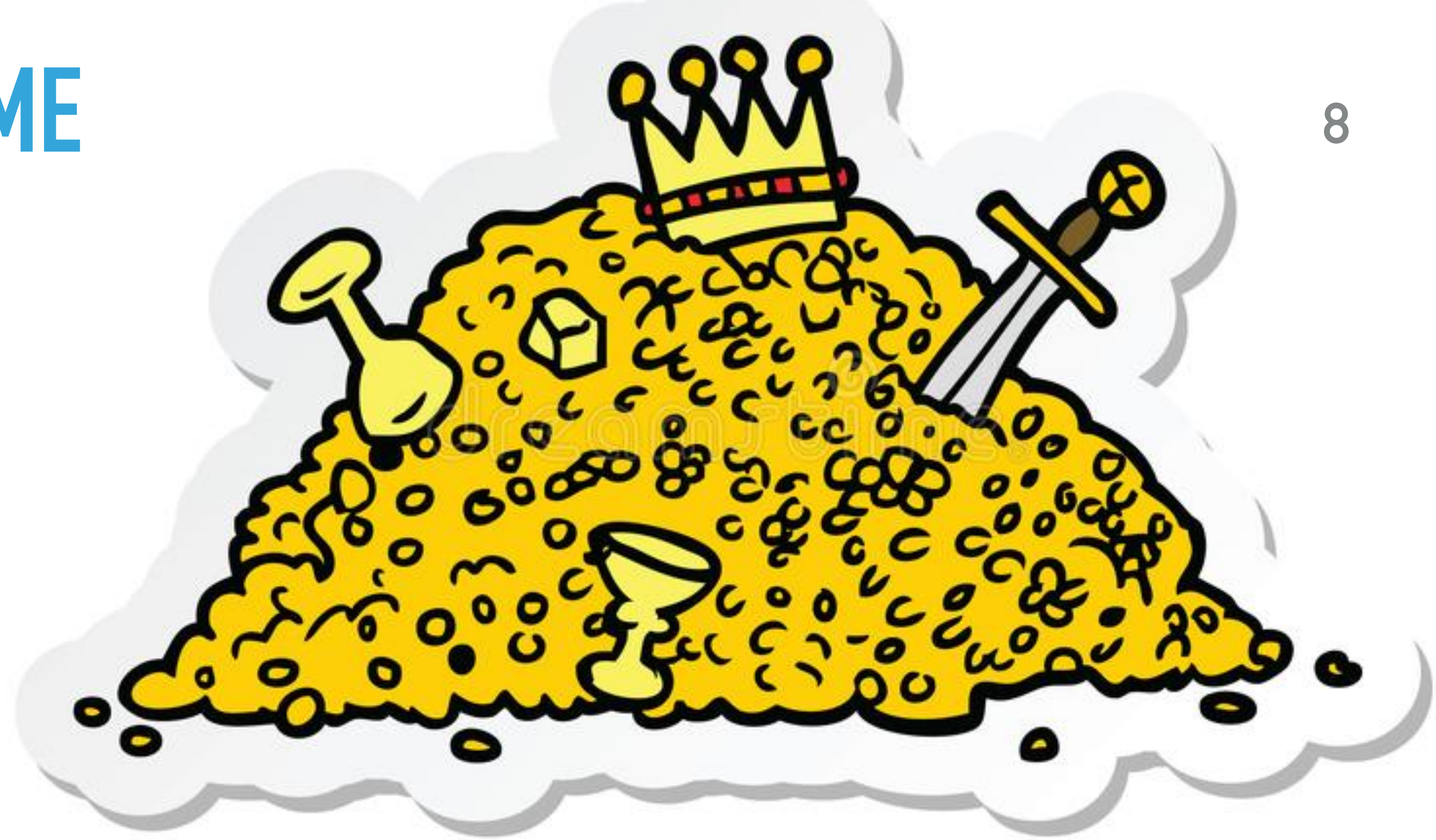


LHC Timeline:

Next Run:
300 fb^-1 at 13.6 TeV early 2022

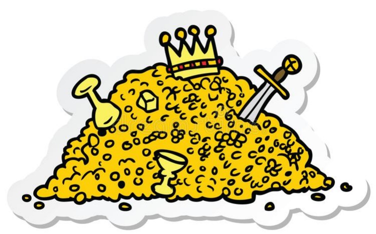


THE LHC – THINGS TO COME



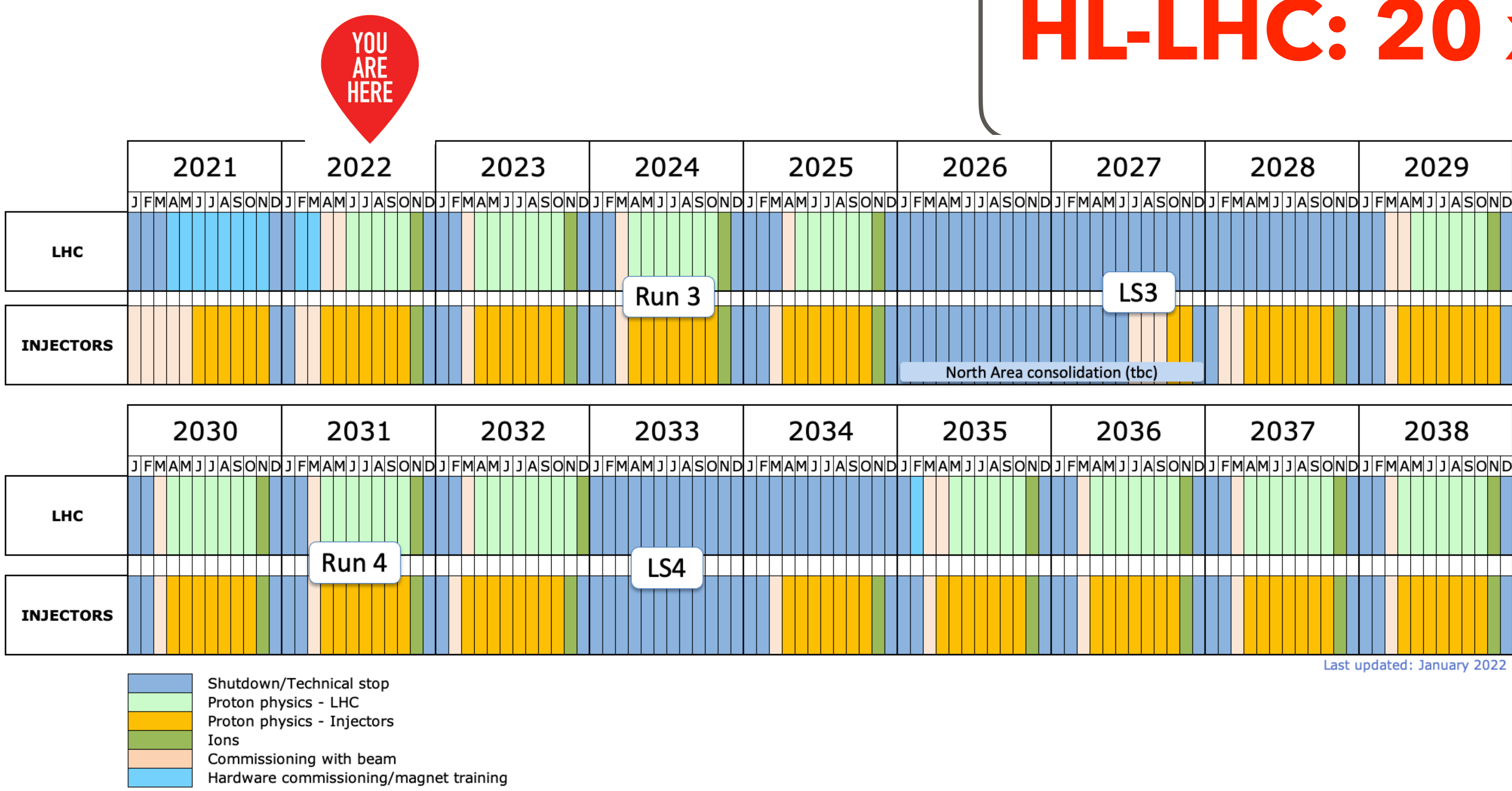
- **We are still in the early stages of LHC physics!**

Today: up to 139 fb^-1



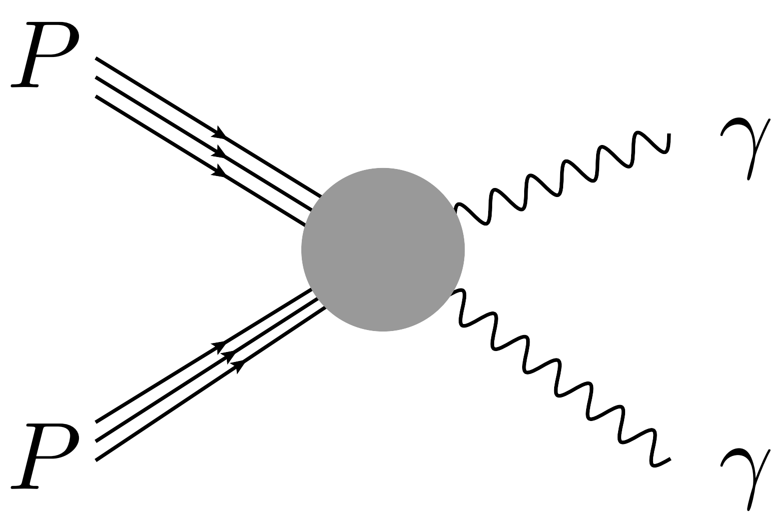
LHC Timeline:

HL-LHC: 20 x current data



OBSERVING THE HIGGS BOSON

- **We look for the low energy output of proton collisions: Photons, electrons, muons, mesons, baryons.**

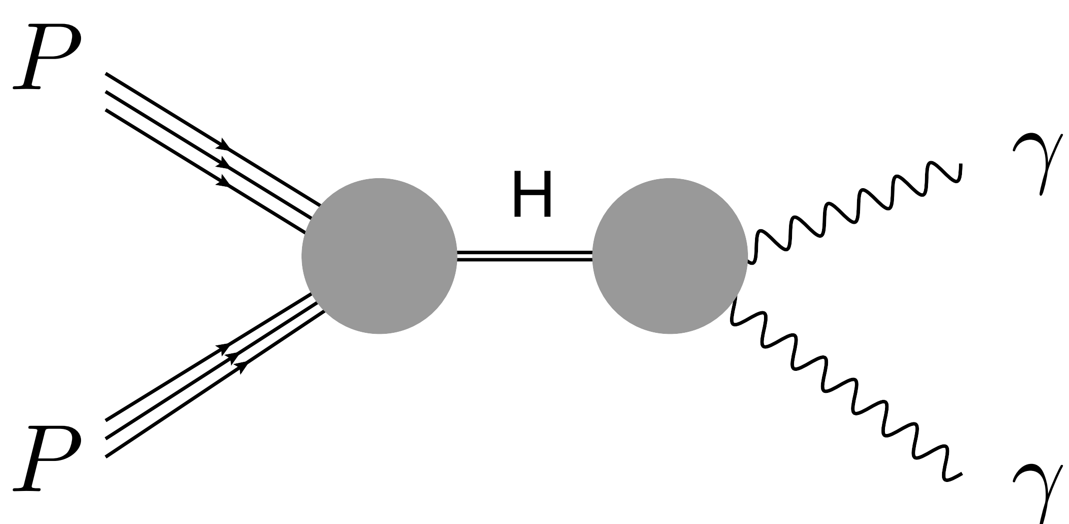


- **The rates and distributions of the final states tell us what happened in the interaction at very short time scales.**

$\sigma_{P\,P\to\gamma\gamma}$ is a function of couplings and masses and changes dependent on the property of the propagating degrees of freedom.

OBSERVING THE HIGGS BOSON

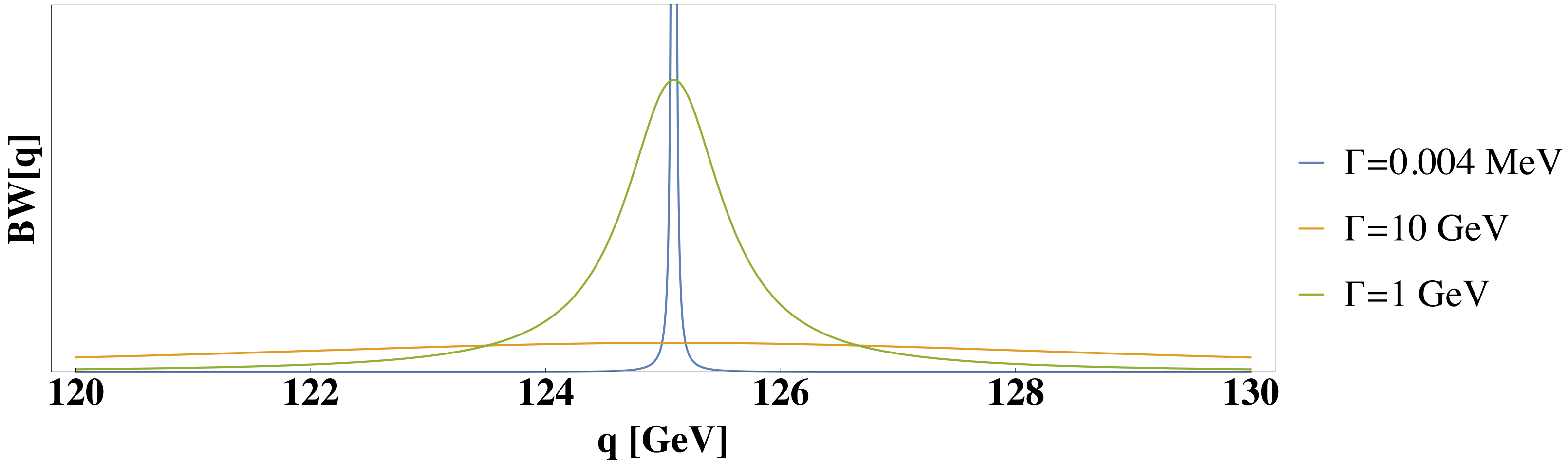
- **In the SM we can factorize the production and decay process to very good approximation!**

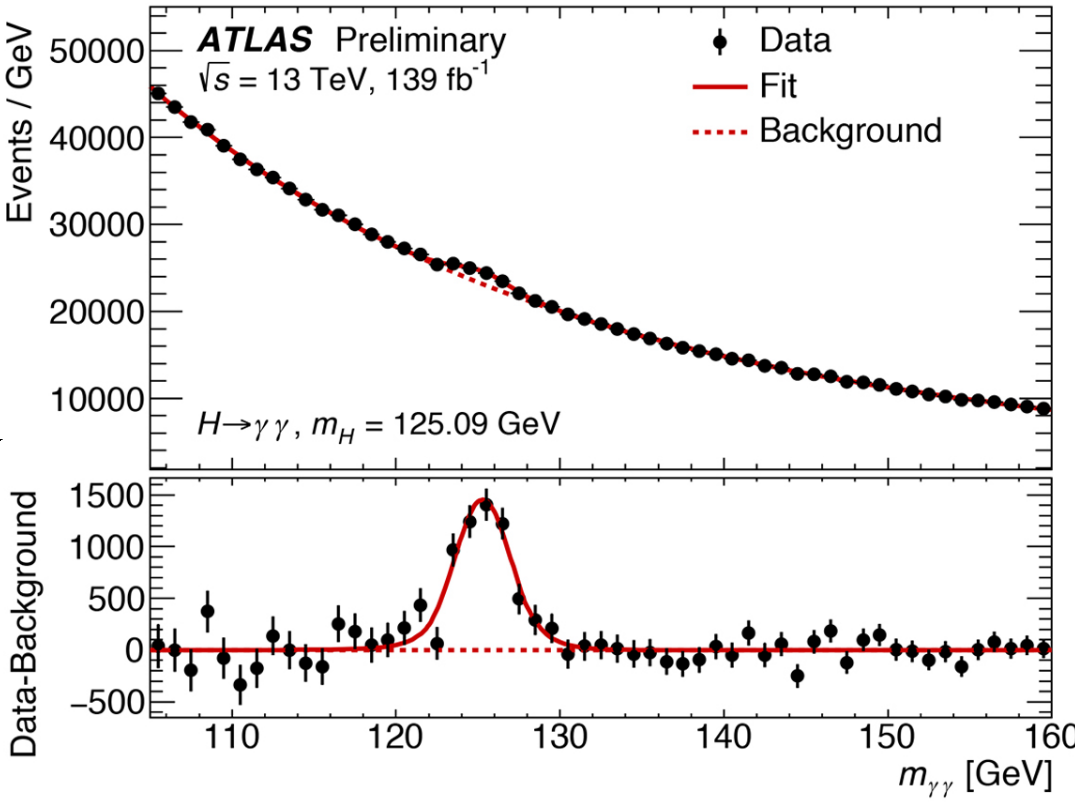


$$\sim \frac{1}{(q^2 - m_h^2)^2 + \Gamma_h^2 m_h^2}$$

- **Width of the Higgs is very narrow!**

$$\Gamma \sim 4\text{MeV} \qquad \tau \sim 10^{-22} s$$





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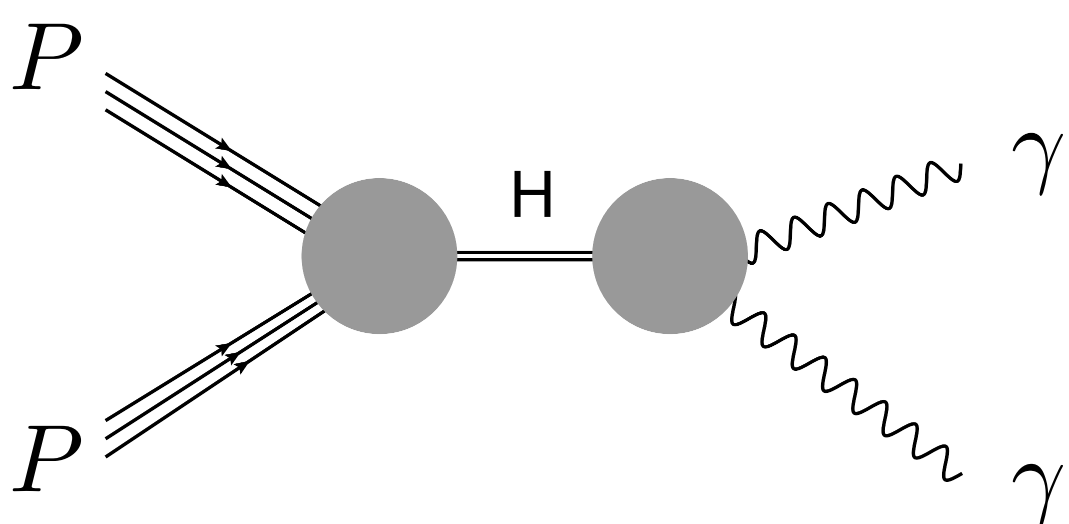


$$\sim \frac{1}{(q^2 - m_h^2)^2 + \Gamma_h^2 m_h^2}$$

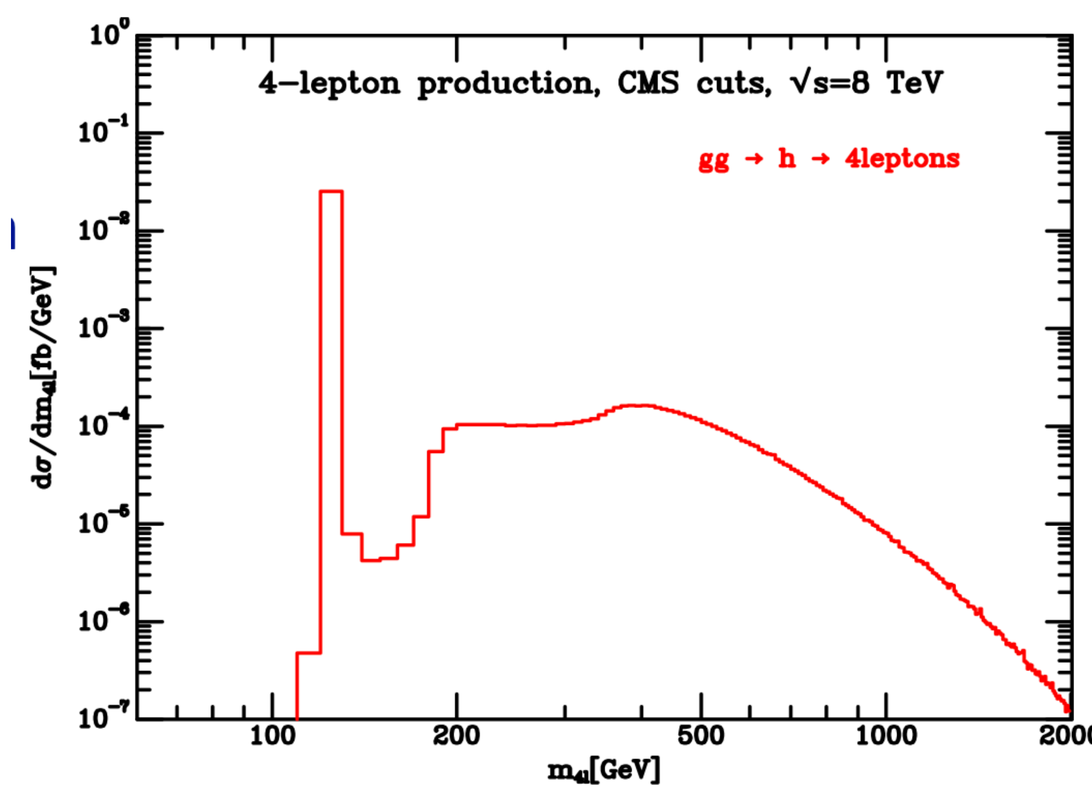
$$(\sigma \cdot \mathrm{BR})(i \to \mathrm{H} \to f) = \frac{\sigma_i \cdot \Gamma_f}{\Gamma_H}$$

- **Width of the Higgs is very narrow!**

$$\Gamma \sim 4\mathrm{MeV} \qquad \tau \sim 10^{-22} s$$

How well does it work: Example H ->ZZ ->4 leptons

$\sim 15\%$ of the cross section off peak



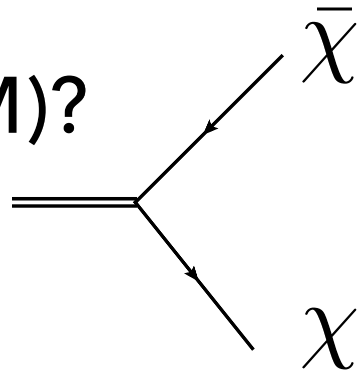
The lifetime of the Higgs

$$\tau = \frac{1}{\Gamma_{\rm tot.}}$$

- In the SM, we can compute it:

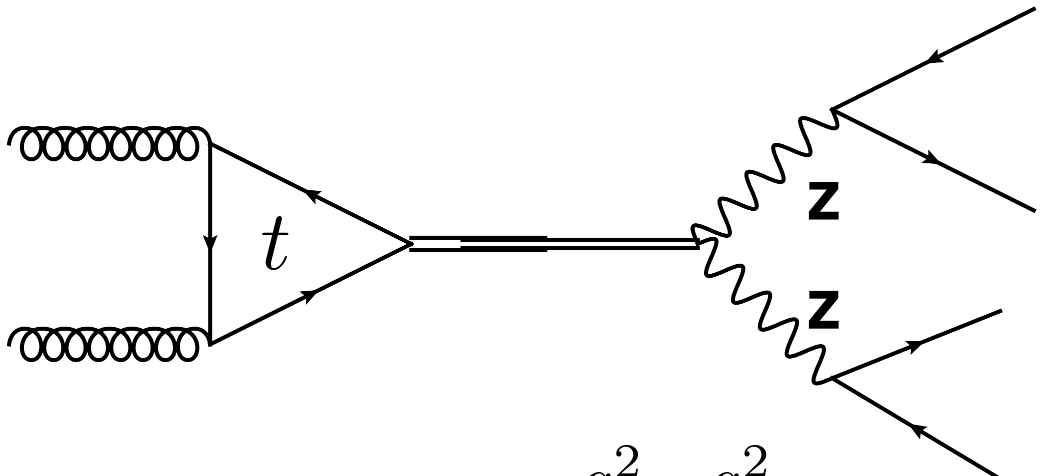
$$\Gamma_{\rm tot.} = \Gamma_{h\to b\bar{b}} + \Gamma_{h\to WW} + \Gamma_{h\to gg} + \Gamma_{h\to ZZ} + \ldots$$

- We find: $\Gamma_{\rm tot.} \sim 4\rm MeV$ $\tau \sim 10^{-22} s$
- Experimentally this is very hard to measure!
  We would need to have an energy resolution on our photons and muons at the MeV scale while they have ~ 50 GeV of momentum!
- What if the Higgs is decaying to unknown things (DM)?



The lifetime of the Higgs

- Use the large off-shell cross section in H->4l!



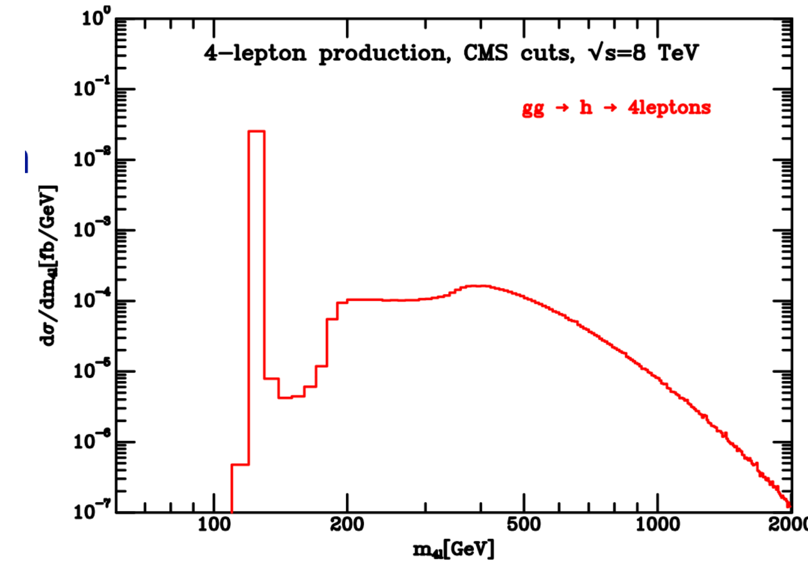
$$\sigma_{PP\to h\to 4\mu} \sim \frac{g_{hgg}^2 g_{hZZ}^2}{(q^2 - m_h^2)^2 - \Gamma_{\rm tot.}^2 m_h^2}$$

$$q^2 = m_h^2 \qquad \frac{g_{hgg}^2 g_{hZZ}^2}{\Gamma_{\rm tot.}^2 m_h^2}$$

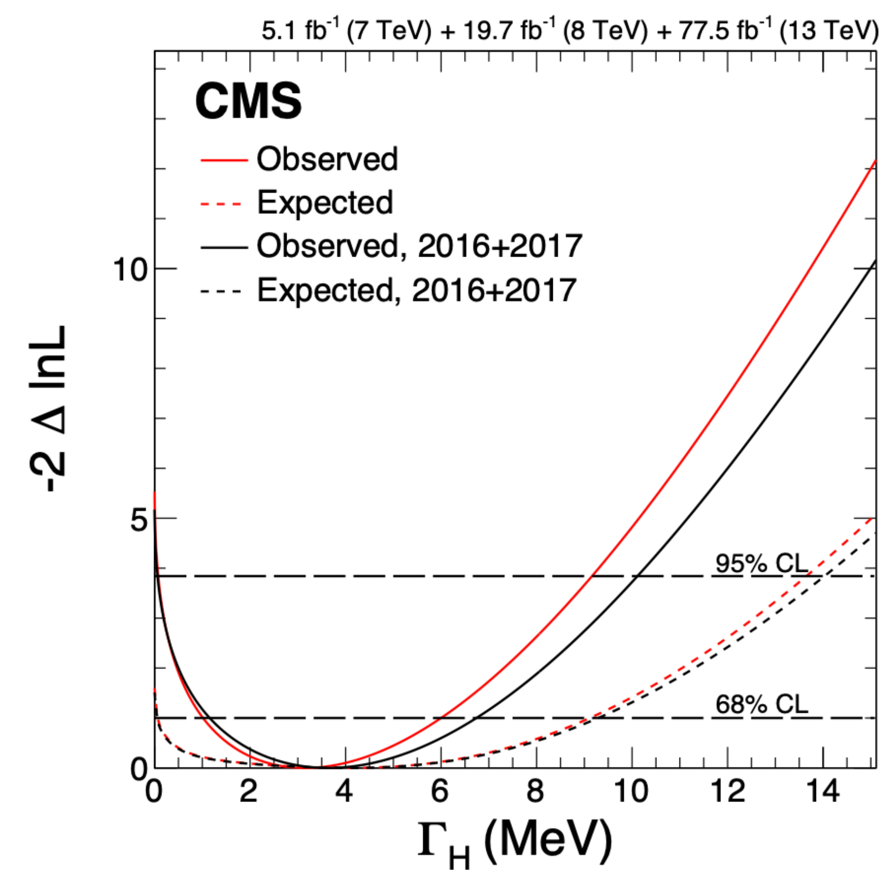
$$q^2 >> m_h^2 \qquad \frac{g_{hgg}^2 g_{hZZ}^2}{(q^2 - m_h^2)^2}$$

Ratio: $$\frac{\Gamma_{\rm tot.}^2 m_h^2}{(q^2 - m_h^2)^2}$$

arXiv:1307.4935



$$\Gamma_{\rm tot.}^{\rm CMS} = 3.2^{+2.8}_{-2.2}\,\text{MeV}$$



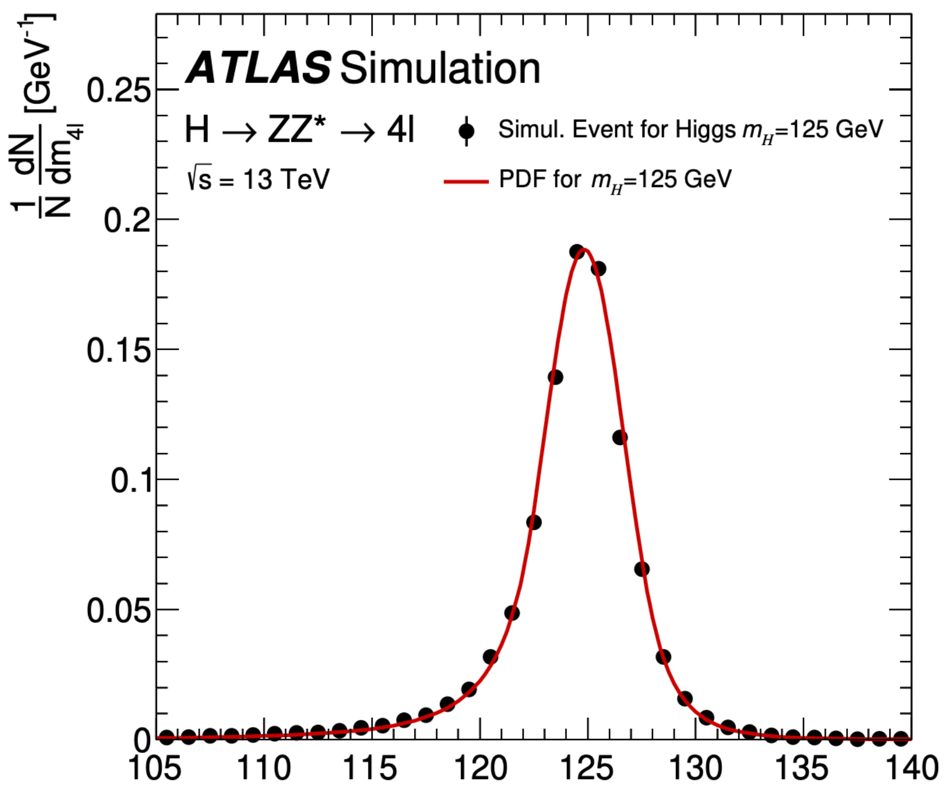
THE HIGGS BOSON MASS

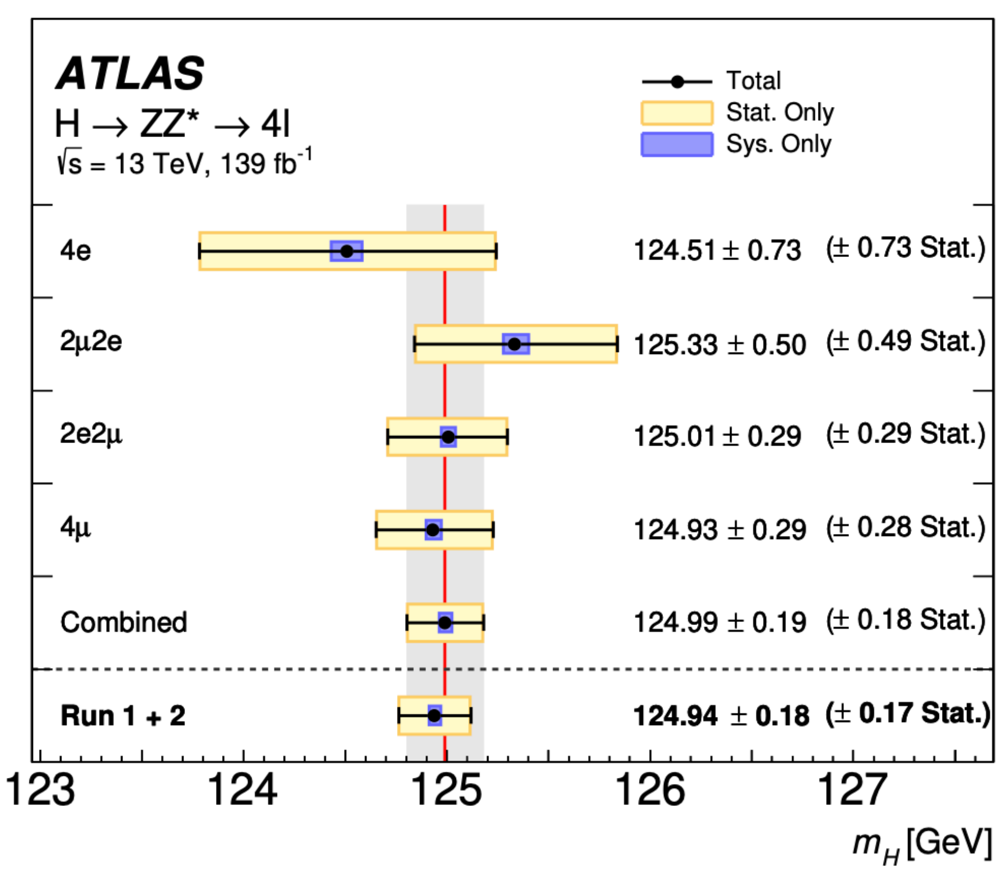
- Width of the peak due to experimental resolution.
- Extract mass from fit to peak in

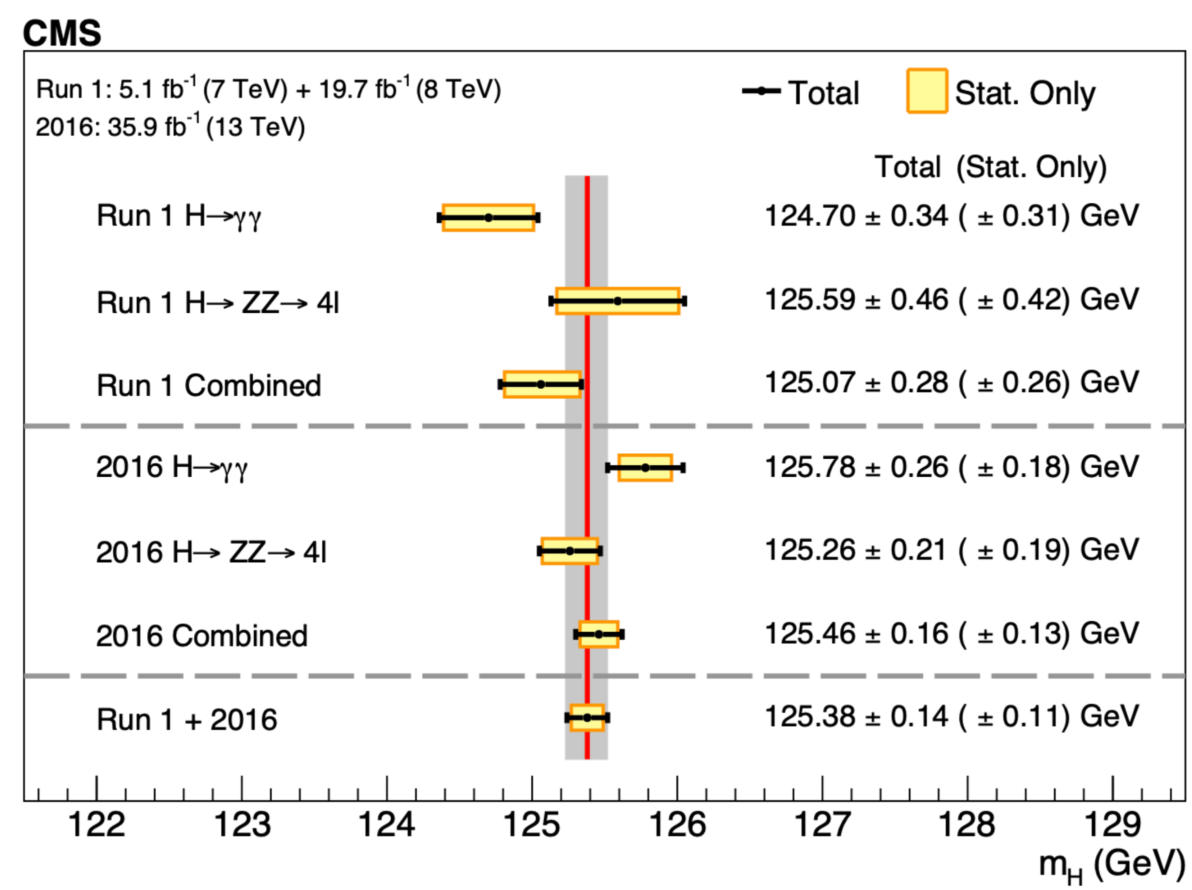
$$H \to \gamma\gamma \qquad H \to ZZ \to 4\mu$$

- A precision observable!

$$m_H = 124.94 \text{ GeV} \ \pm 0.15\%$$

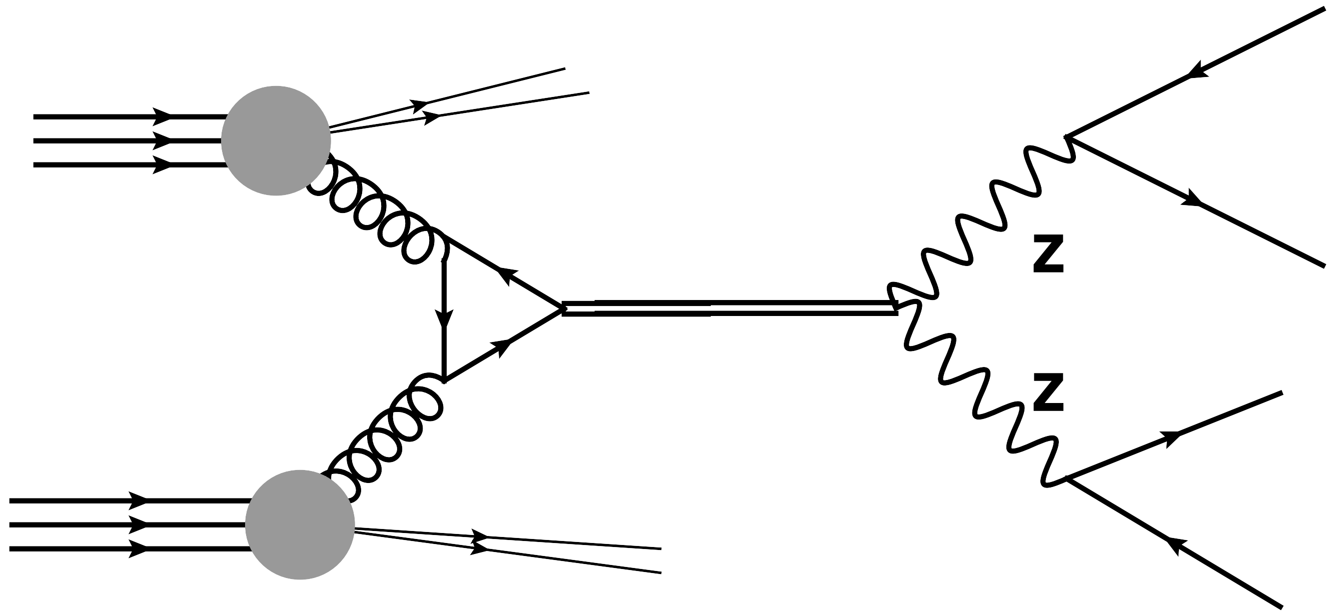






HIGGS BOSON COUPLINGS

We measure production times decay!



$$\sim Y_t^2 g_{Zh}^2$$

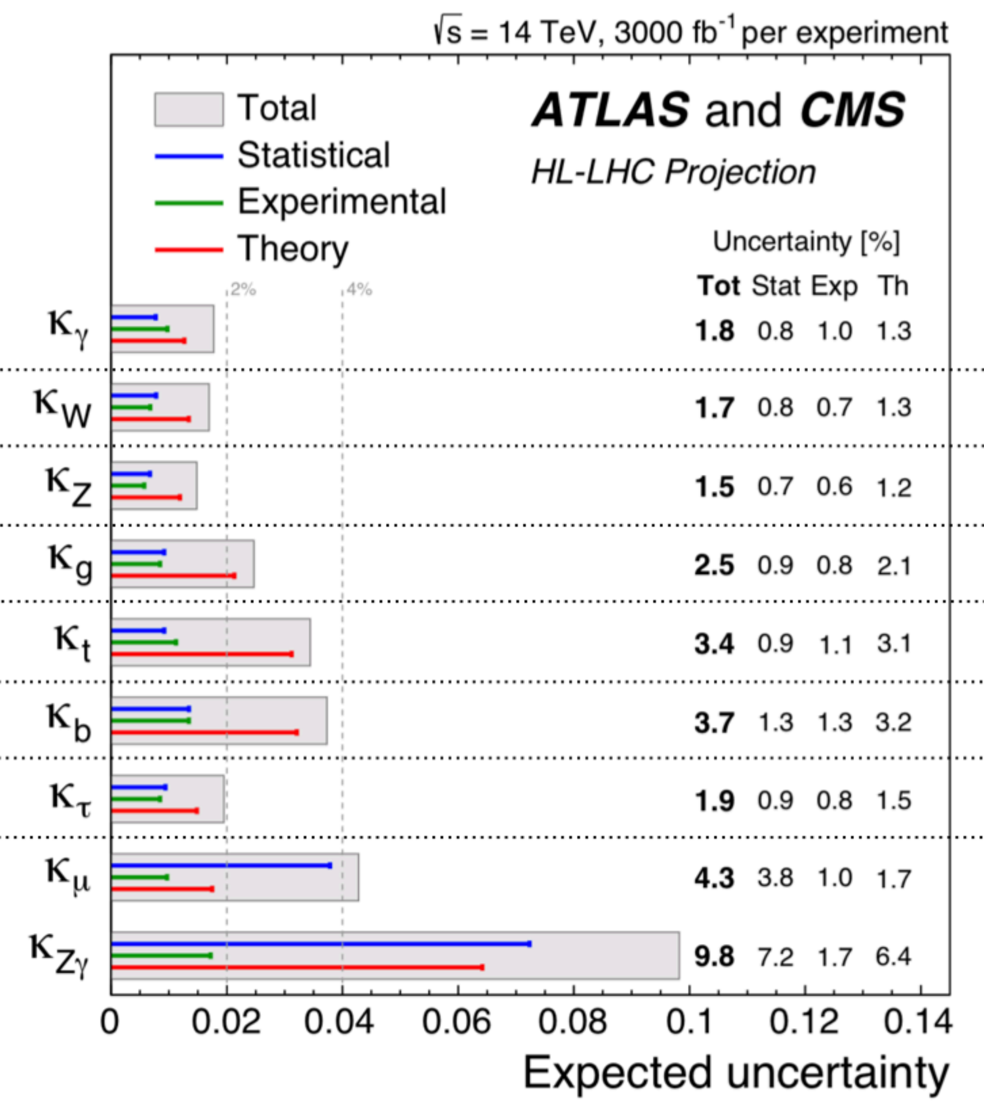
- For computations we can separate production and decay - not in observation.
- A single measurement will not allow us to determine specific coupling constants of the Higgs boson.
- Measure many different combinations!
- A convenient parametrization:

$$(\sigma \cdot \mathrm{BR})(i \to \mathrm{H} \to f) = \frac{\sigma_i \cdot \Gamma_f}{\Gamma_H}$$

$$(\sigma \cdot \mathrm{BR})(i \to \mathrm{H} \to f) = \frac{\sigma_i^{SM} \kappa_i^2 \cdot \Gamma_f^{SM} \kappa_f^2}{\Gamma_H^{SM} \kappa_H^2}$$

Kappa - Framework

	ATLAS - CMS Run 1 combination	ATLAS Run 2 (NEW Higgs 2021)	CMS Run 2	Current precision
κ_γ	13%	1.04 ± 0.06	$1.01^{+0.09}_{-0.14}$	6%
κ_W	11%	1.06 ± 0.06	$-1.11^{+0.14}_{-0.09}$	6%
κ_Z	11%	0.99 ± 0.06	0.96 ± 0.07	6%
κ_g	14%	$0.92^{+0.07}_{-0.06}$	$1.16^{+0.12}_{-0.11}$	7%
κ_t	30%	0.92 ± 0.10	1.01 ± 0.11	11%
κ_b	26%	0.87 ± 0.11	$1.18^{+0.19}_{-0.27}$	11%
κ_τ	15%	0.92 ± 0.07	0.94 ± 0.12	8%
	JHEP 08 (2016) 045	ATLAS-CONF-2021-53	CMS-PAS-HIG-19-005	



Measurements here assume no BSM in Higgs width

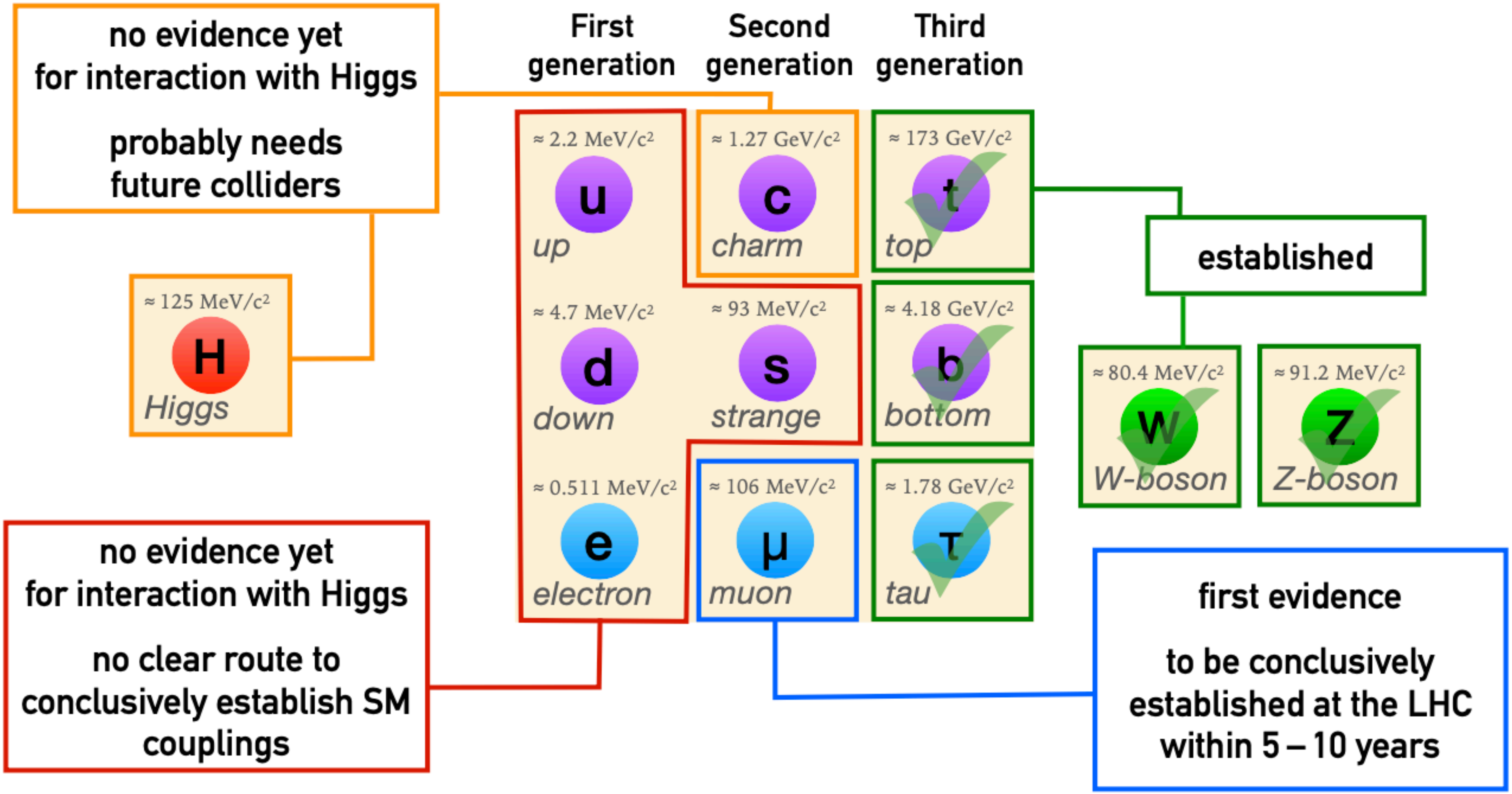
Still 25 times more data and reduction of a factor of 3 uncertainty!

TH uncertainty dominant!

Slide by Marumi Kado @ Higgs2021

HIGGS BOSON COUPLINGS

Where we stand:

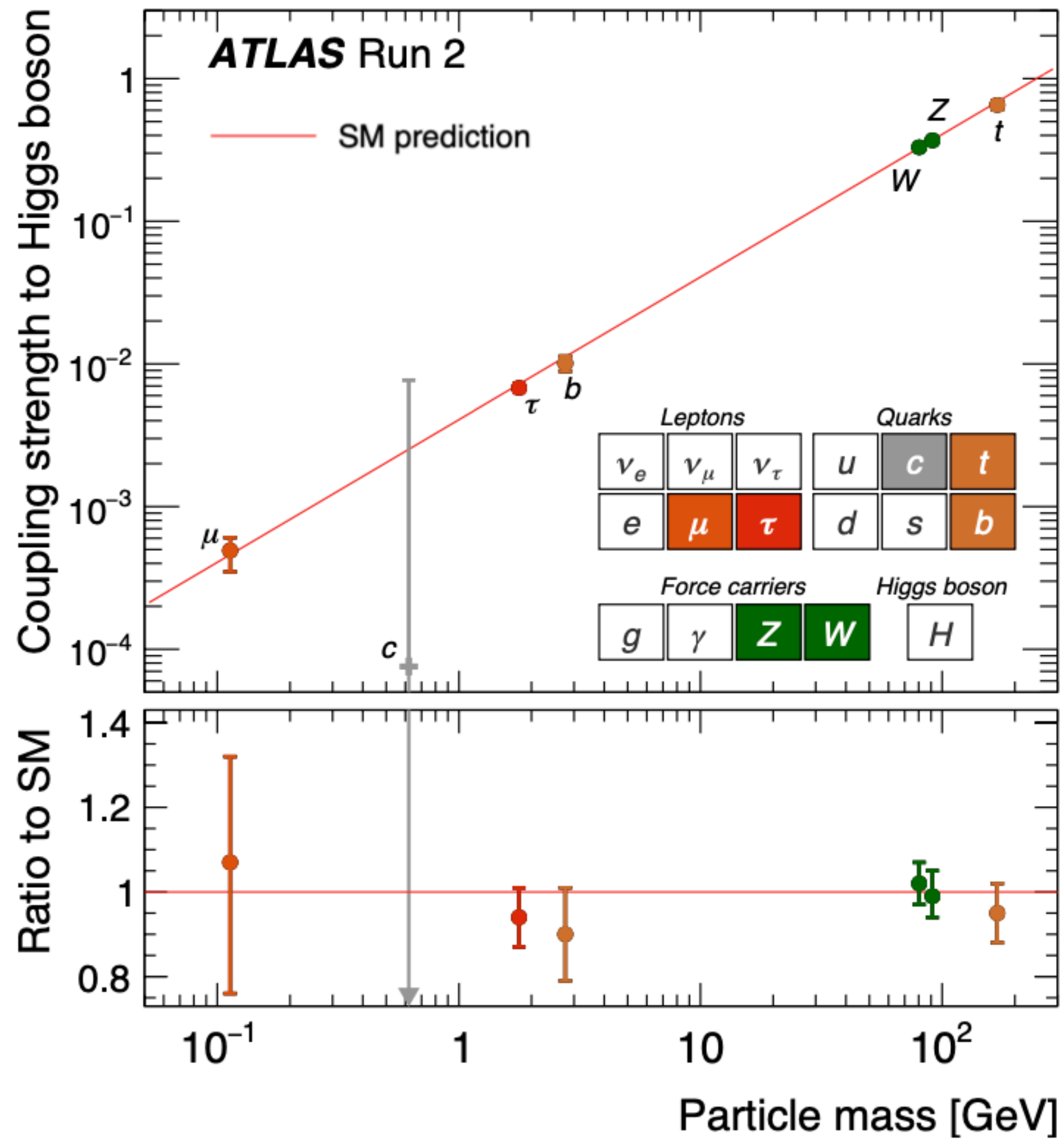


Slide from [arXiv:2207.00478v1] [Salam,Wang,Zanderighi]

HIGGS BOSON COUPLINGS

Coupling ~ Mass

- Testing a key prediction of the SM: Coupling proportional to the mass!
- Experimentally true over 3 orders of magnitude.

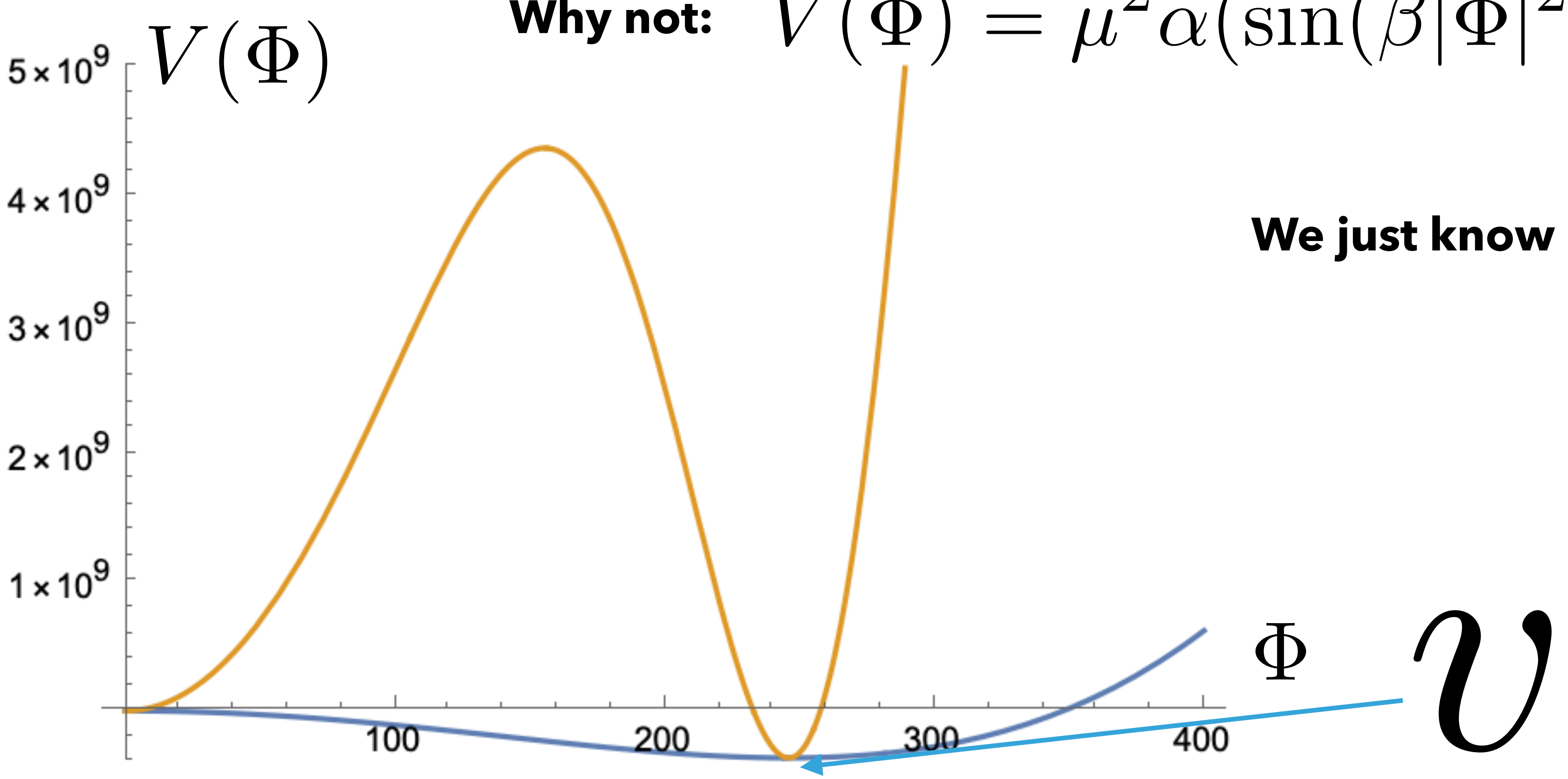


Measuring the potential of the Higgs

- In principle, we know the potential of the SM $m_h = \sqrt{2\lambda}v = 125\text{GeV}$
- The potential in the SM is the simplest solution compatible with (gauge) symmetry.

$$V(\Phi) = -\mu^2(\Phi^\dagger\Phi) + \lambda(\Phi^\dagger\Phi)^2$$

Why not: $V(\Phi) = \mu^2\alpha(\sin(\beta|\Phi|^2)) + \gamma|\Phi|^4$

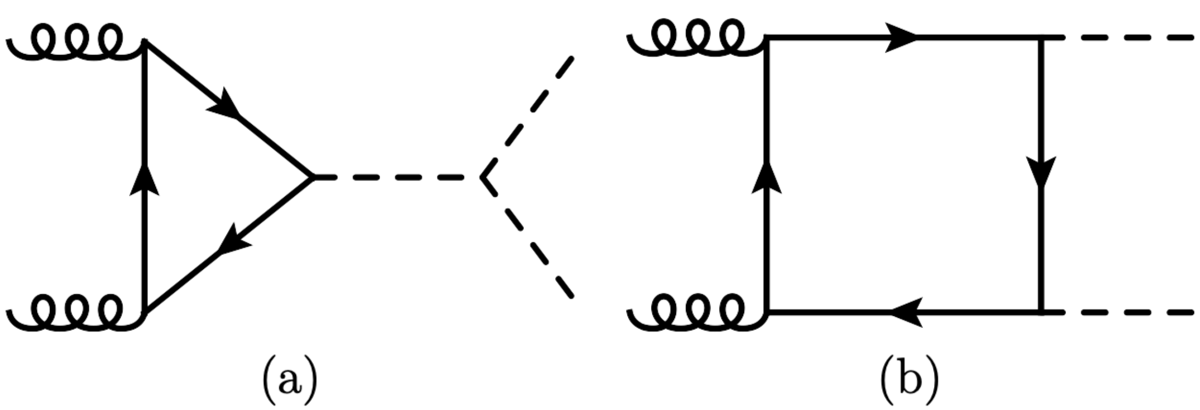


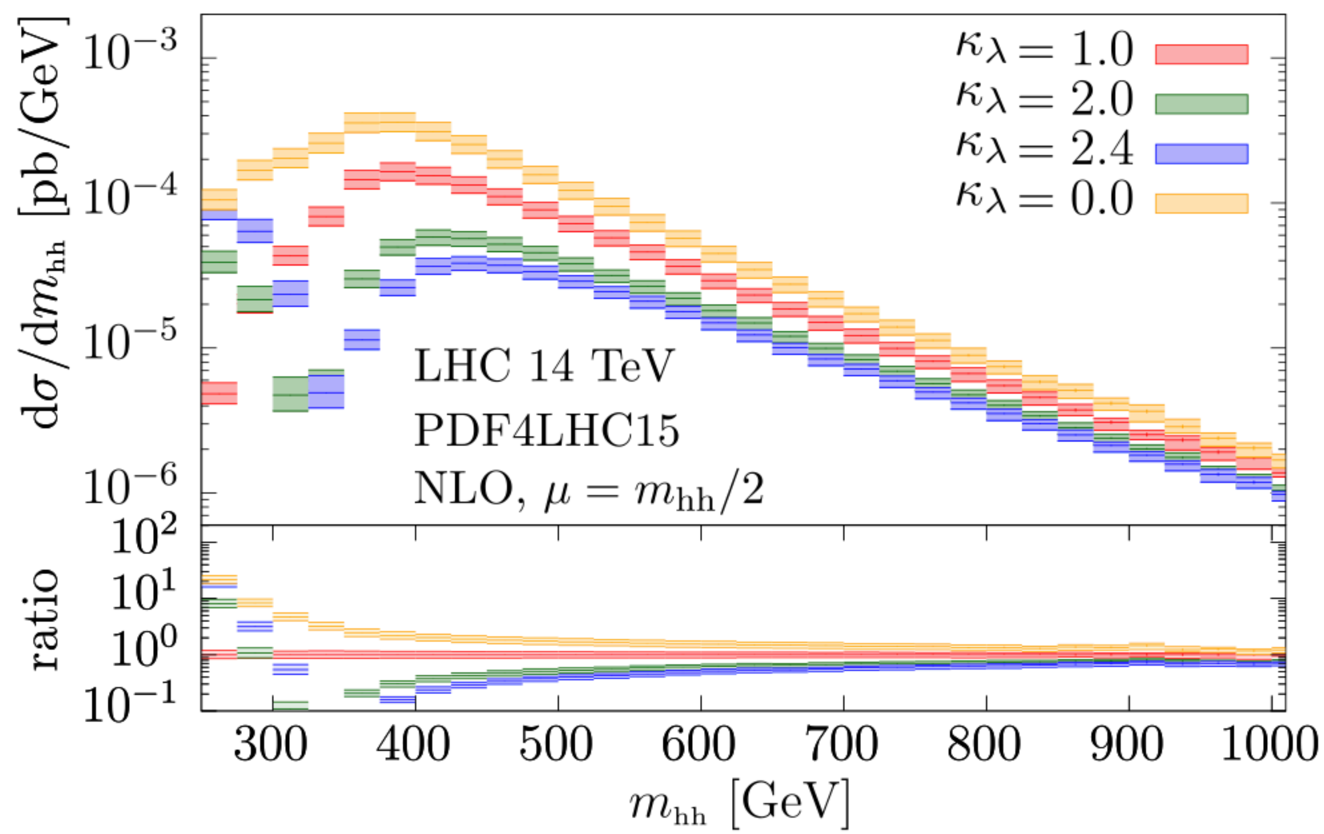
We just know about the VEV and the mass.

HIGGS BOSON SELF COUPLING

Measuring the potential of the Higgs

- Produce 2 Higgs bosons and measure the self coupling!





- Two contributions - Interfering destructively.
- Notoriously hard to compute and to measure!

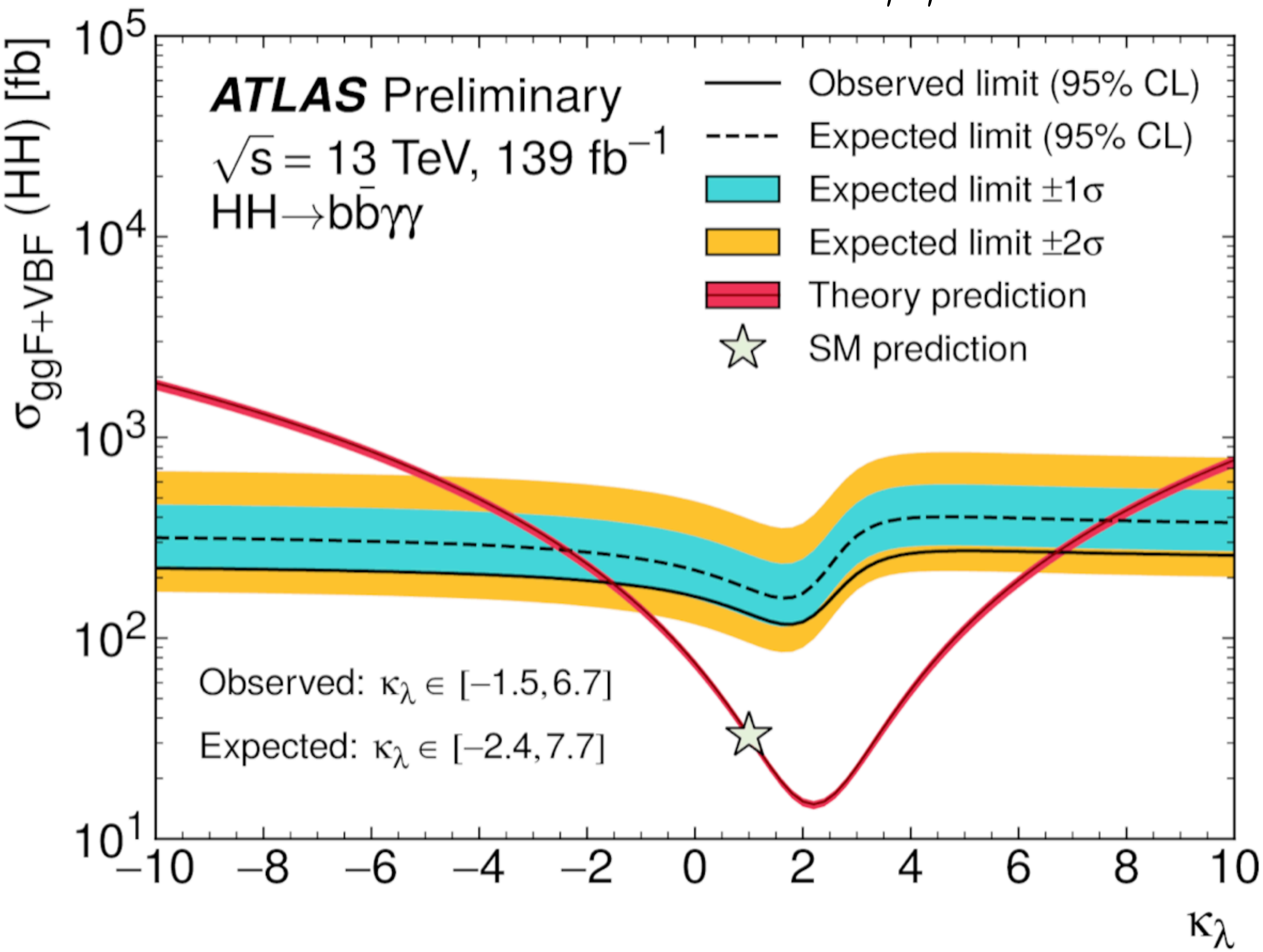
$\sigma_{PP\to HH} = 27.84\text{fb} \pm 12\%$ Compare: $\sigma_{P\to H\to\gamma\gamma} = 110\text{fb}$

- Look at bottom quark decays of the Higgs - harder to see but larger branching ratio.

Measuring the potential of the Higgs

- Produce 2 Higgs bosons and measure the self coupling!

$$PP \rightarrow HH \rightarrow b\bar{b}\gamma\gamma$$

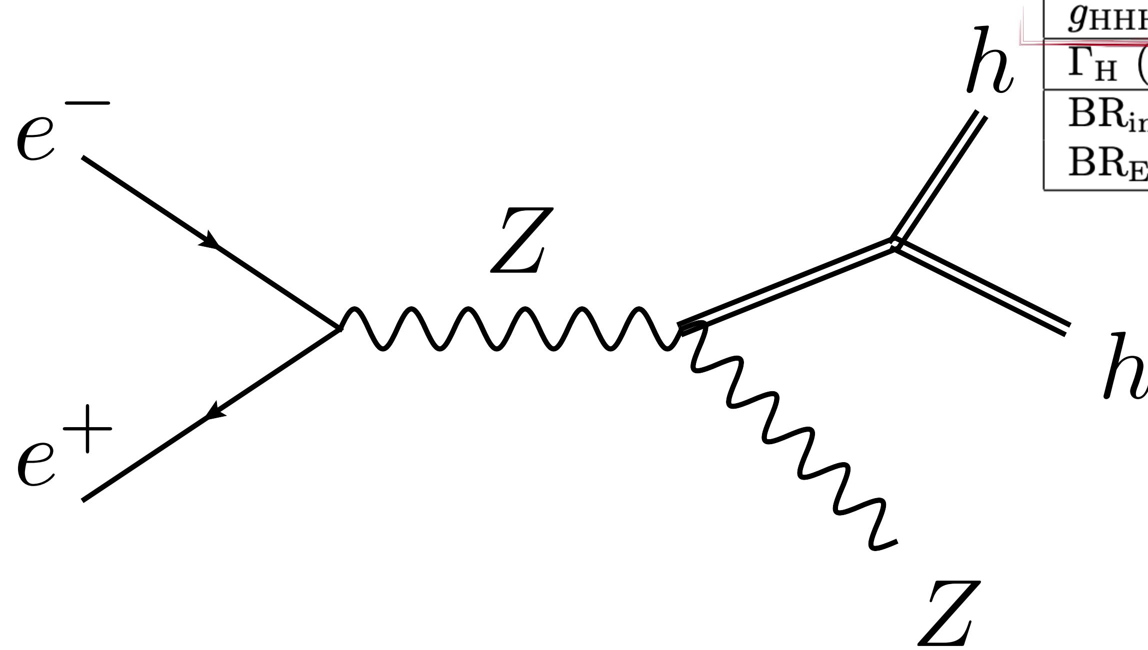


Measuring the potential of the Higgs

- Future colliders:

- Future machine colliding electrons and positrons.
- Measuring the Higgs potential is one of *the* motivations to build new machines!

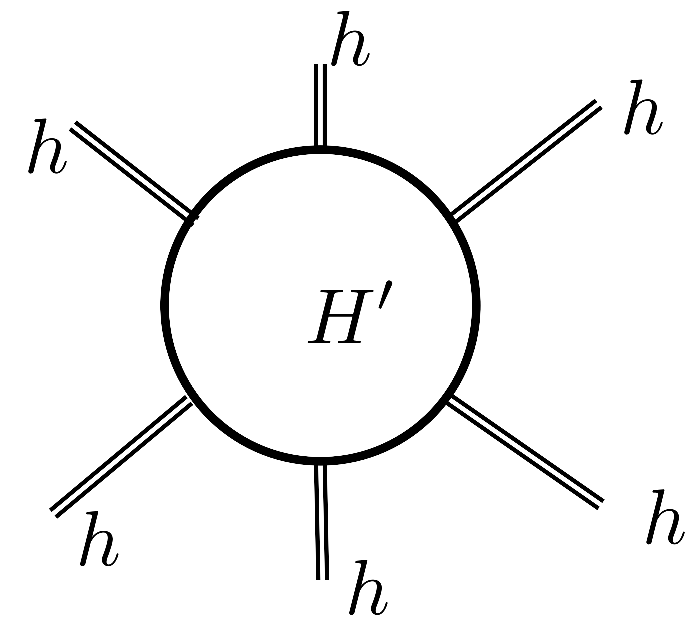
Collider	HL-LHC	ILC_{250}	$CLIC_{380}$	$CEPC_{240}$	FCC-$ee_{240\to365}$
Lumi (ab^{-1})	3	2	1	5.6	5 + 0.2 + 1.5
Years		11.5 [5]	8	7	3 + 1 + 4
g_{HZZ} (%)	1.5 / 3.6	0.29 / 0.47	0.44 / 0.66	0.18 / 0.52	**0.17 / 0.26**
g_{HWW} (%)	1.7 / 3.2	1.1 / 0.48	0.75 / 0.65	0.95 / 0.51	**0.41 / 0.27**
g_{Hbb} (%)	3.7 / 5.1	1.2 / 0.83	1.2 / 1.0	0.92 / 0.67	**0.64 / 0.56**
g_{Hcc} (%)	SM / SM	2.0 / 1.8	4.1 / 4.0	2.0 / 1.9	**1.3 / 1.3**
g_{Hgg} (%)	2.5 / 2.2	1.4 / 1.1	1.5 / 1.3	1.1 / 0.79	**0.89 / 0.82**
$g_{H\tau\tau}$ (%)	1.9 / 3.5	1.1 / 0.85	1.4 / 1.3	1.0 / 0.70	**0.66 / 0.57**
$g_{H\mu\mu}$ (%)	4.3 / 5.5	4.2 / 4.1	4.4 / 4.3	3.9 / 3.8	**3.9 / 3.8**
$g_{H\gamma\gamma}$ (%)	1.8 / 3.7	1.3 / 1.3	1.5 / 1.4	1.2 / 1.2	**1.2 / 1.2**
$g_{HZ\gamma}$ (%)	11. / 11.	11. / 10.	11. / 9.8	6.3 / 6.3	**10. / 9.4**
g_{Htt} (%)	3.4 / 2.9	2.7 / 2.6	2.7 / 2.7	2.6 / 2.6	**2.6 / 2.6**
g_{HHH} (%)	50. / 52.	28. / 49.	45. / 50.	17. / 49.	**19. / 34.**
Γ_H (%)	SM	2.4	2.6	1.9	**1.2**
BR_{inv} (%)	1.9	0.26	0.63	0.27	**0.19**
BR_{EXO} (%)	SM (0.0)	1.8	2.7	1.1	**1.0**



- Measurement of the trilinear Higgs coupling to 20-30%
- Impressive potential for other Higgs couplings!

EFFECTIVE FIELD THEORY

Parametrizing what we don't know



- Consider a very heavy scalar particle coupling to the Higgs Boson.

$$m_{H'} >> v$$

$$m_{H'}^6 \int d^4k \prod_i^6 \frac{1}{(k+p_i)^2 - m_{H'}^2} \sim m_{H'}^6 \int d^4k \frac{1}{((k)^2 - m_{H'}^2)^6} + \mathcal{O}(p_i) = \frac{i}{320\pi^2 m_{H'}^2}$$

- Very heavy mass shrinks interactions to a point.
- Generates effective coupling of 6 Higgs bosons!

$$\mathcal{L}_{H'} =\sim \frac{i}{320\pi^2 m_{H'}^2} h^6 \in \frac{\lambda_6}{\Lambda^2} (\Phi^\dagger \Phi)^3$$

Parametrizing what we don't know

- We know the mass dimension of our familiar fields.

$$[\Phi] = 1$$

- Write down every product of fields of dimension higher than 4.

$$\sum_i \frac{g_i}{\Lambda^2} O_i(\{A, Z, W, h, g, e^-, u, d, \dots\})$$

- Make sure that the resulting Lagrangian is gauge invariant under all SM symmetries. (This relates some of the newly introduced couplings.).
- Make sure there are no redundancies (equations of motion).

→ Dim=6 SMEFT

Dimension 6 Standard Model Effective Field Theory

Parametrizing what we don't know

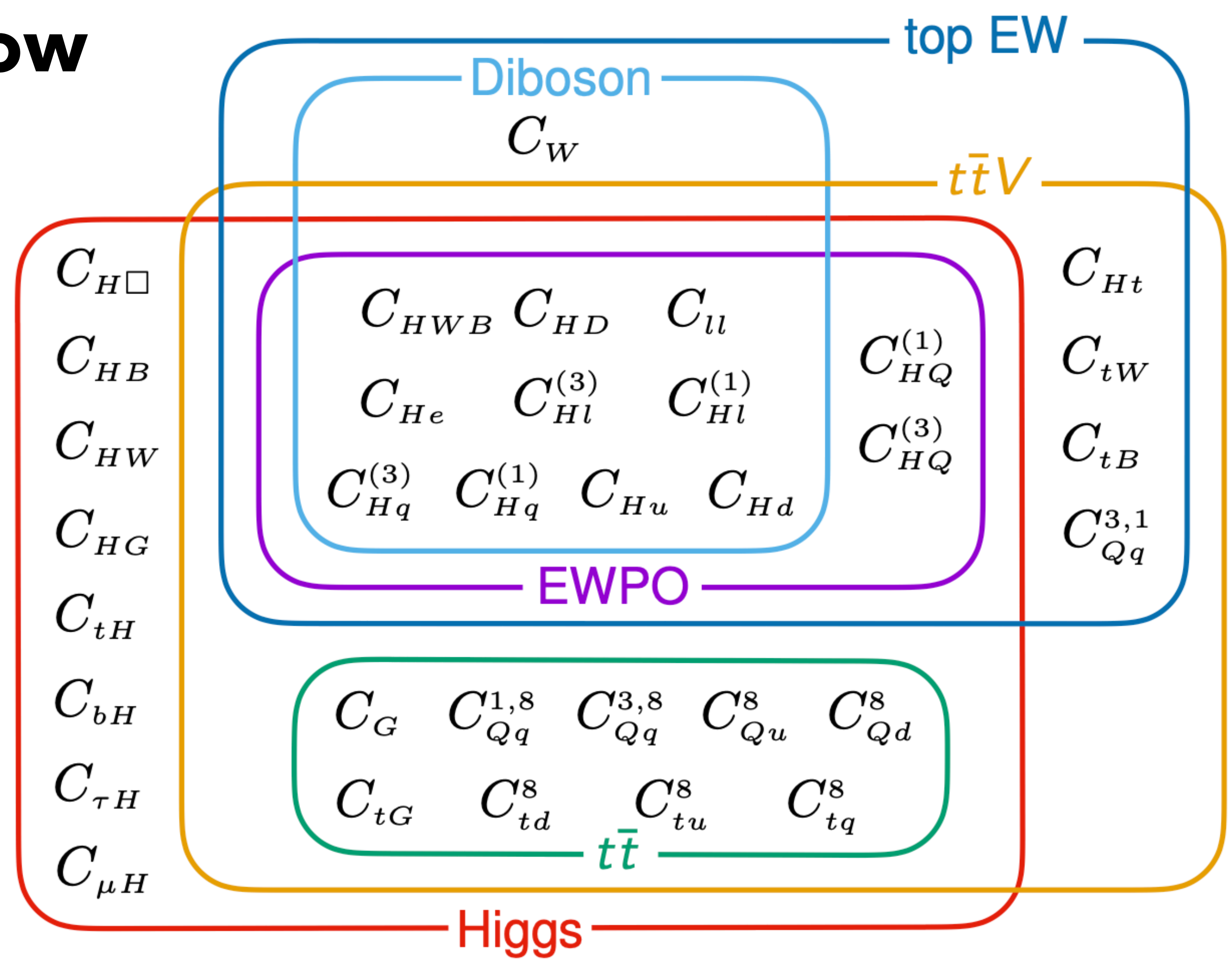
- Assuming we see only our known SM fields directly, SMEFT provides a way of parametrizing low energy effective implications of physics at a high mass scale.
- The framework is well defined from a QFT point of view.
- We can calculate! We see for any observable what the implications are.

- Many operators! 2499
- 59 if we choose only flavor diagonal operators.

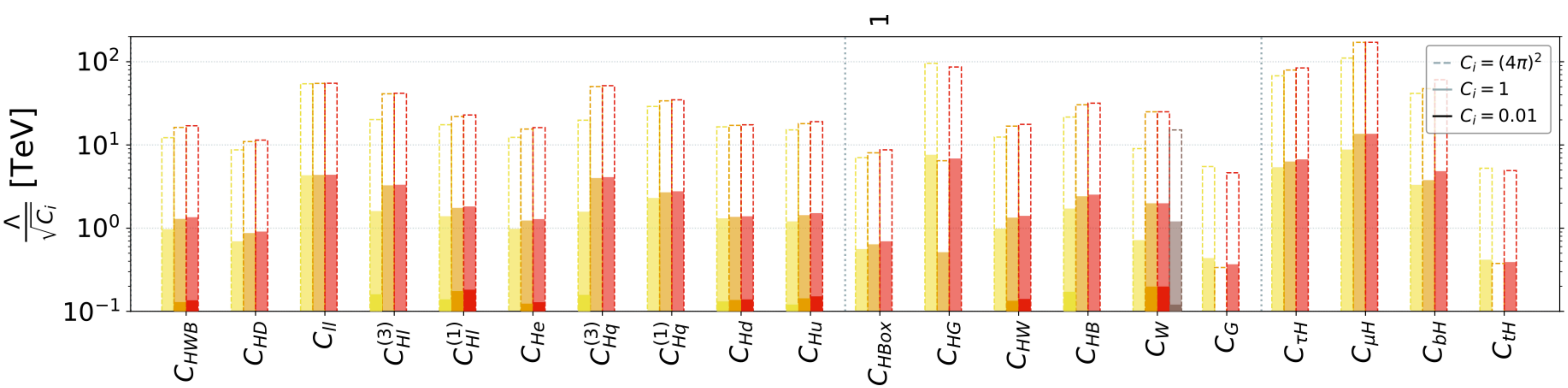
X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				
$X^2\varphi^2$		$\psi^2 X\varphi$		$\psi^2\varphi^2 D$	
$Q_{\varphi G}$	$\varphi^\dagger \varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu} e_r) \tau^I \varphi W_{\mu\nu}^I$	$Q_{\varphi l}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar l_p \gamma^\mu l_r)$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger \varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma^{\mu\nu} e_r) \varphi B_{\mu\nu}$	$Q_{\varphi l}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar l_p \tau^I \gamma^\mu l_r)$
$Q_{\varphi W}$	$\varphi^\dagger \varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu} T^A u_r) \widetilde\varphi\, G_{\mu\nu}^A$	$Q_{\varphi e}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar e_p \gamma^\mu e_r)$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger \varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu} u_r) \tau^I \widetilde\varphi\, W_{\mu\nu}^I$	$Q_{\varphi q}^{(1)}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar q_p \gamma^\mu q_r)$
$Q_{\varphi B}$	$\varphi^\dagger \varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma^{\mu\nu} u_r) \widetilde\varphi\, B_{\mu\nu}$	$Q_{\varphi q}^{(3)}$	$(\varphi^\dagger i\overleftrightarrow{D}^I_\mu \varphi)(\bar q_p \tau^I \gamma^\mu q_r)$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger \varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu} T^A d_r) \varphi\, G_{\mu\nu}^A$	$Q_{\varphi u}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar u_p \gamma^\mu u_r)$
$Q_{\varphi WB}$	$\varphi^\dagger \tau^I \varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu} d_r) \tau^I \varphi\, W_{\mu\nu}^I$	$Q_{\varphi d}$	$(\varphi^\dagger i\overleftrightarrow{D}_\mu \varphi)(\bar d_p \gamma^\mu d_r)$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger \tau^I \varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma^{\mu\nu} d_r) \varphi\, B_{\mu\nu}$	$Q_{\varphi ud}$	$i(\widetilde\varphi^\dagger D_\mu \varphi)(\bar u_p \gamma^\mu d_r)$

Parametrizing what we don't know

- Include in global fit of many observables to see if there are deviations for some operators.
- A daunting task!
- We can also pick subsets of observables - this limits at lowest orders the number of operators involved.



[arXiv:2012.02779](https://arxiv.org/abs/2012.02779)

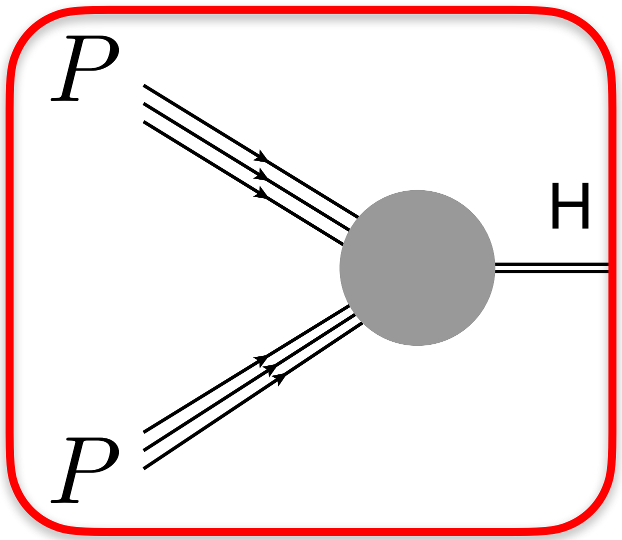


PRECISION QFT
FOR HIGGS BOSON PHYSICS

OBSERVING THE HIGGS BOSON

- **Production Cross Section of Higgs Bosons**

Production



- **How many Higgs Bosons do we produce at the LHC?**
- **Important for the extraction of coupling constants.**
- **Consistency check of SM**

$$\mu = \frac{\sigma_{\mathrm{obs.}}}{\sigma_{\mathrm{SM}}}$$

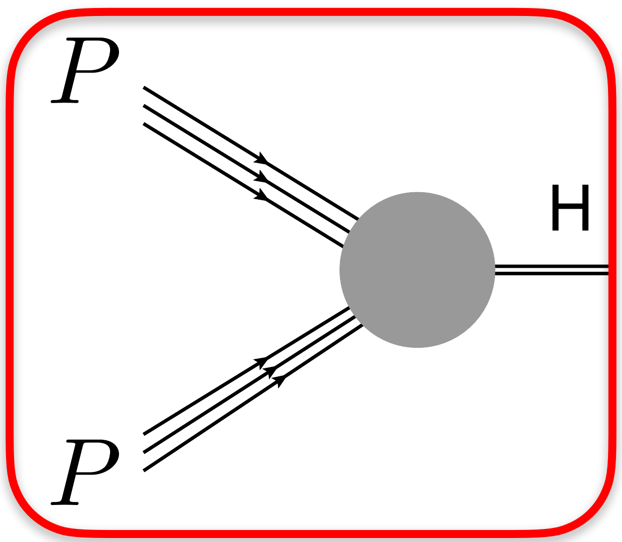
$$\mu = 1.06 \pm 0.07 = 1.06 \pm 0.04(\mathrm{stat}) \pm 0.03(\mathrm{exp.})^{+0.05}_{-0.04}(\mathrm{sig.\ th.}) \pm 0.02(\mathrm{bkg.\ th.})$$ ATLAS

$$\mu = 1.02^{+0.07}_{-0.06} = 1.02 \pm 0.04(\mathrm{stat}) \pm 0.04(\mathrm{exp.}) \pm 0.04(\mathrm{sig.{+}bkg.\ th.})$$ CMS

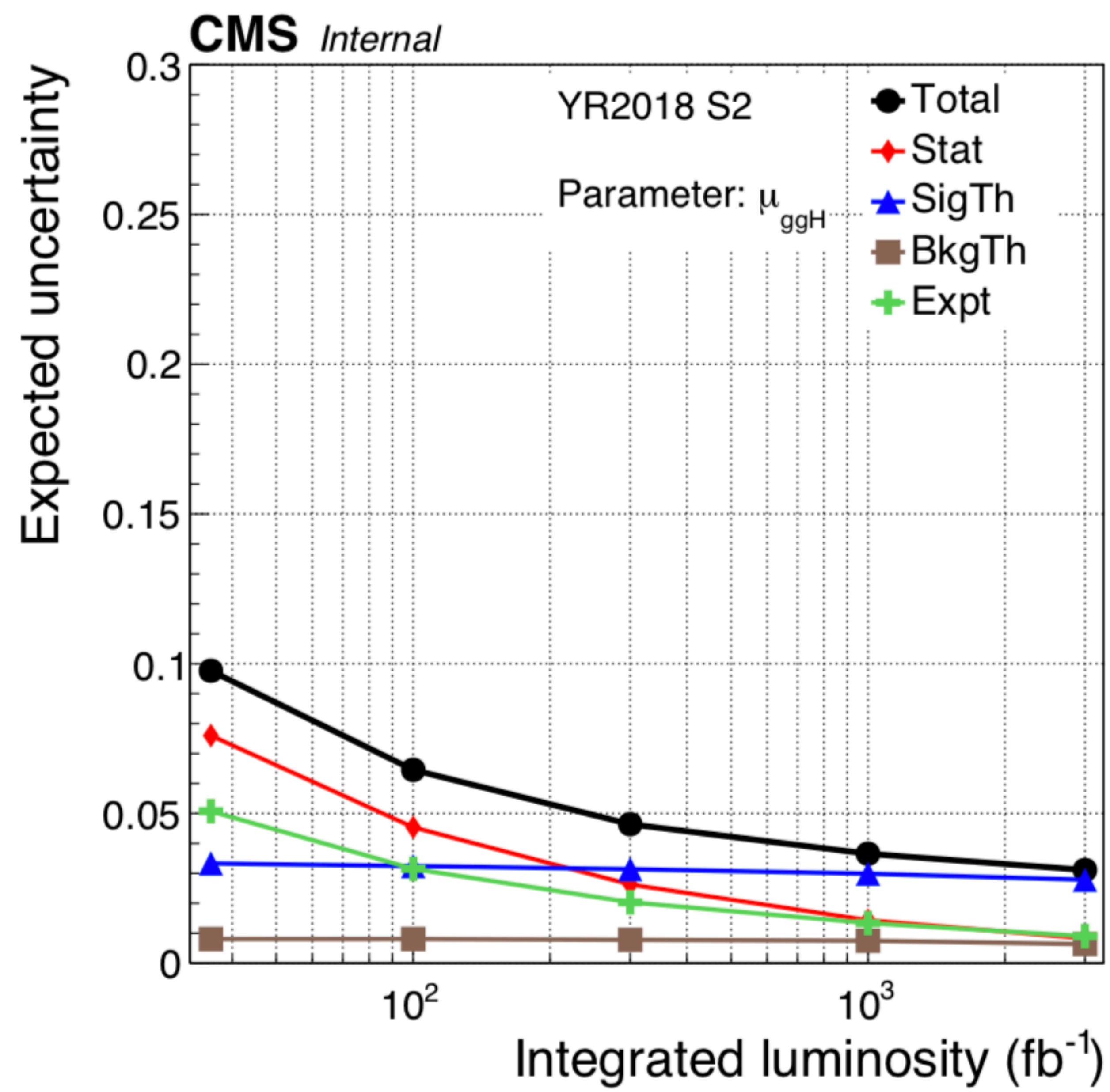
Agreement with measurements today at the level of ~ 7 %.

Production Cross Section of Higgs Bosons

Production

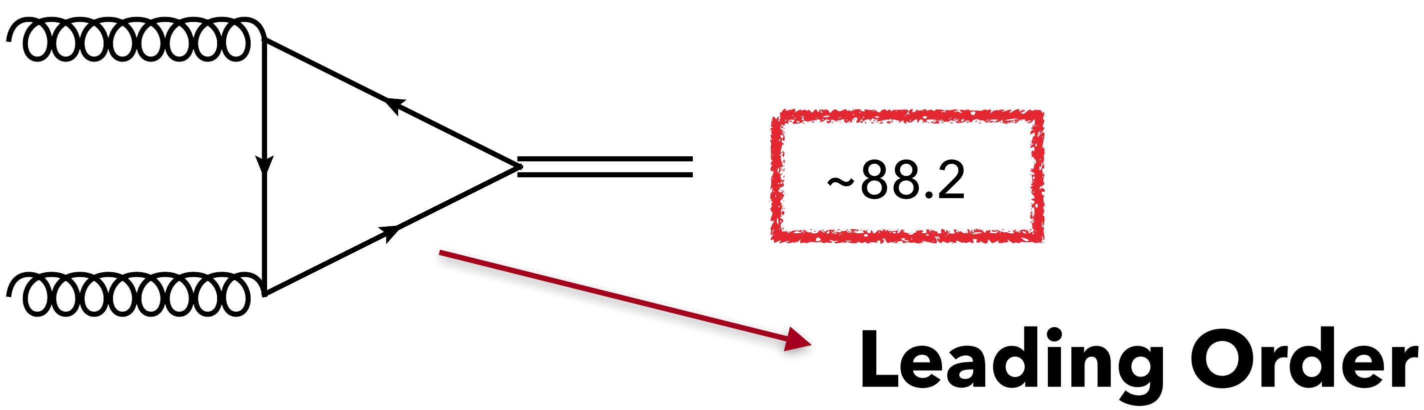


$$\mu = \frac{\sigma_{\text{obs.}}}{\sigma_{\text{SM}}}$$



- How will precision on Higgs observables develop over the next decade?
- Reaching precision targets of percent level phenomenology is a huge challenge!

PRECISION VIA PERTURBATIVE QFT



- **How do we make our predictions precise: Perturbation Theory! Expand in couplings:**

$$\alpha_S \sim 0.118$$

$$\alpha_{\text{em.}} \sim 0.0073$$

Rule of thumb: $\alpha_{\text{em.}} \sim \alpha_S^2$

- **How precise do we want to be?**

LO **NLO** **NNLO** **N3LO**

$$\hat{\sigma} = \hat{\sigma}^{(0)} + \alpha_S^1 \hat{\sigma}^{(1)} + \alpha_S^2 \hat{\sigma}^{(2)} + \alpha_S^3 \hat{\sigma}^{(3)} + \dots$$

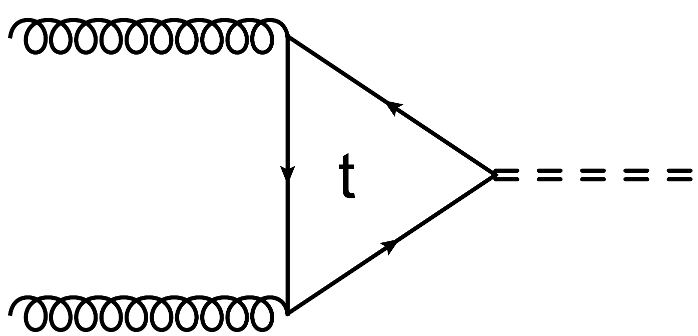
PRECISION VIA PERTURBATIVE QFT

Leading Order - Loop Induced

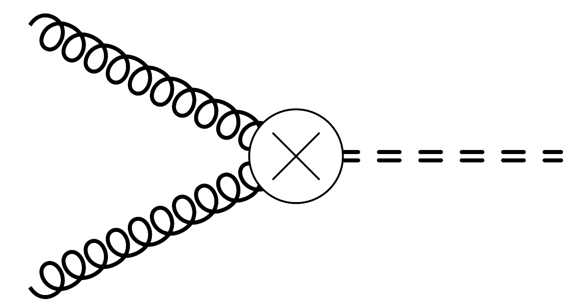
- Adding quantum corrections is **hard**!

Exploit hierarchy of masses

$$m_t >> m_H$$



$$m_t \to \infty$$



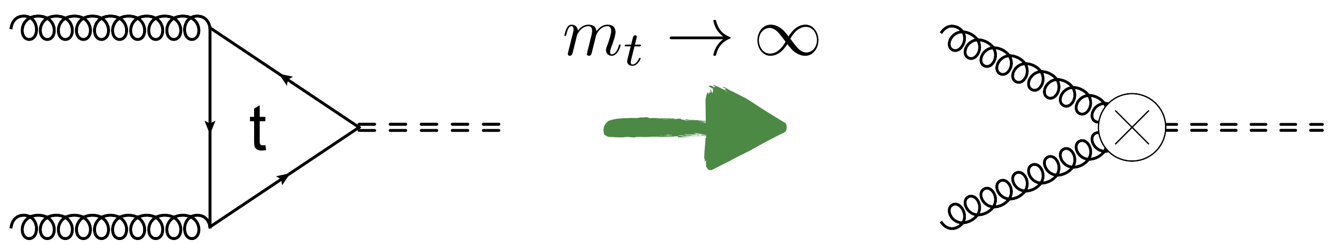
- Two ways to think about it:

1. Create an EFT by integrating out the top quark

$$\mathcal{L}_{\mathrm{EFT}} \sim \frac{h}{v} C_t G_{\mu\nu} G^{\mu\nu}$$

2. Expand in powers of $\quad \dfrac{m_h^2}{4m_t^2} \sim 0.13$

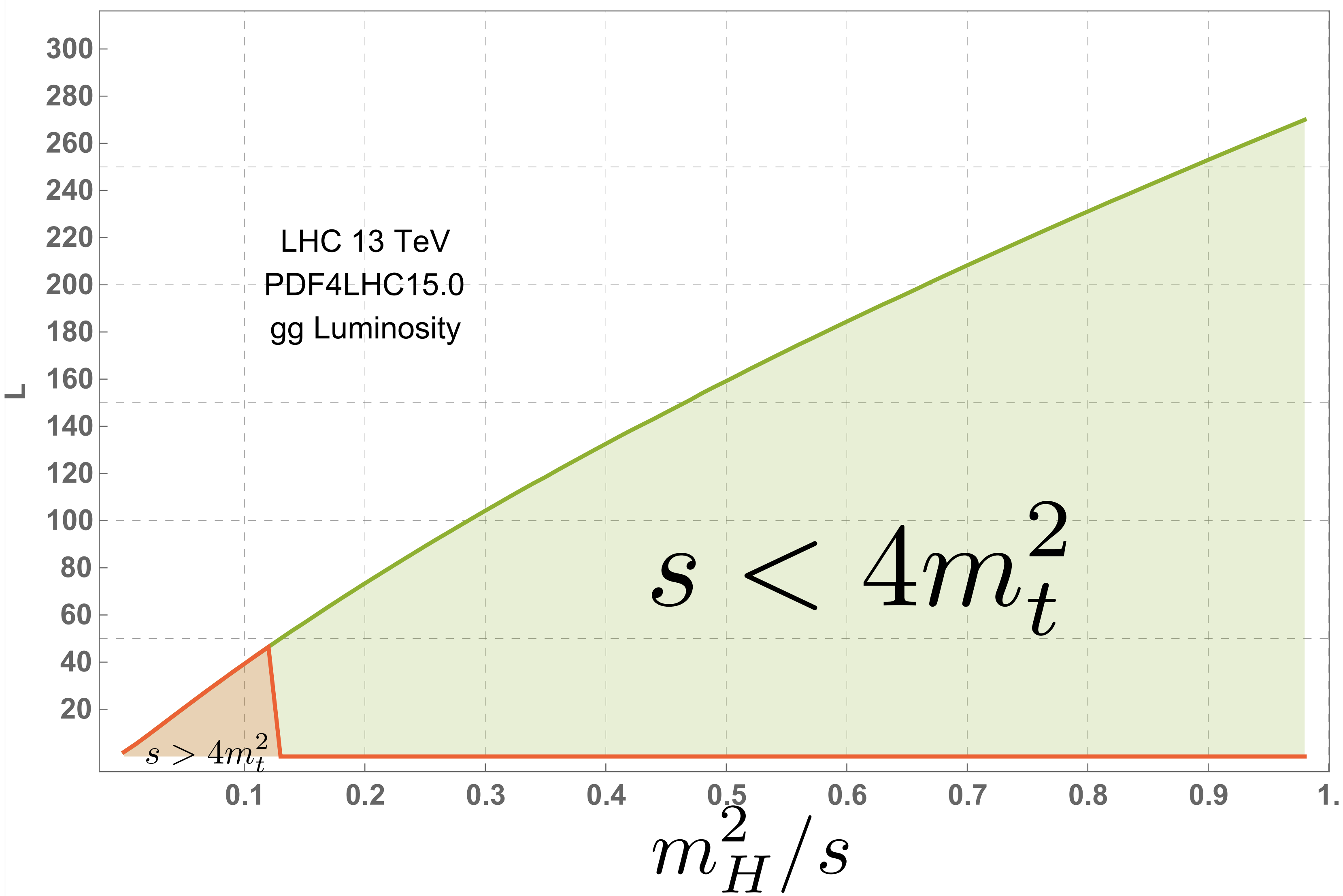
PRECISION VIA PERTURBATIVE QFT



How well does it work?

How often is the available energy in a scattering process $\sqrt{s}$ too small to produce a top quark pair?

- Quite good approximation!
- **We know the UV theory!**
- We can correct for it by including top quark mass dependence to some degree!



PRECISION VIA PERTURBATIVE QFT

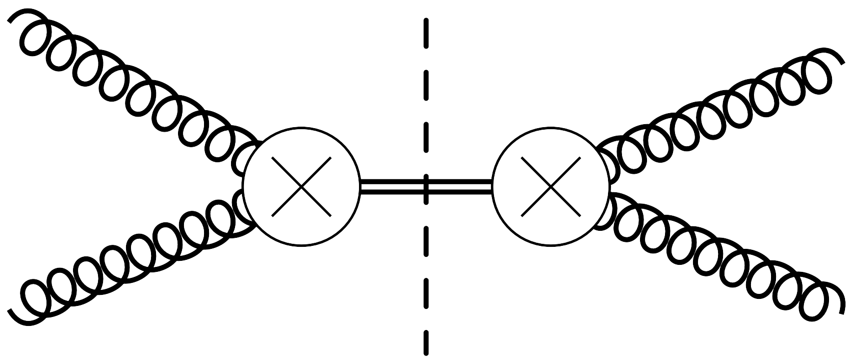
- **What precision is required for phenomenology?**
  Very precise.
  Experimental uncertainty now: 7 % and in the future: 2.5 %

- **How close to the experimental outcome should the prediction be?**
  Idealized observable. Inclusive cross section - no cuts on final state products. In the end we predict a number.

- **What methods are suitable for the problem at hand?**
  - **Automated Numerical Calculation:** Low to medium precision, easy to use. NLO corrections fully automatic, nothing at NNLO automatic.
  - **Tailored Numerical Calculation:** Medium to high precision, suitable for intricate observables, mostly slow evaluation time.
  - **Analytic calculations:** Tailored for certain observables, fast evaluation, applicable to problem with few parameters.

LO

$\sigma^{(0)}$

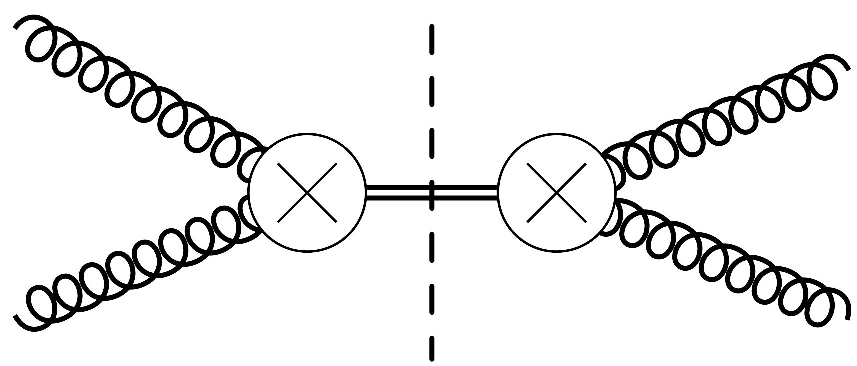


1 Diagram

GLUON FUSION

LO

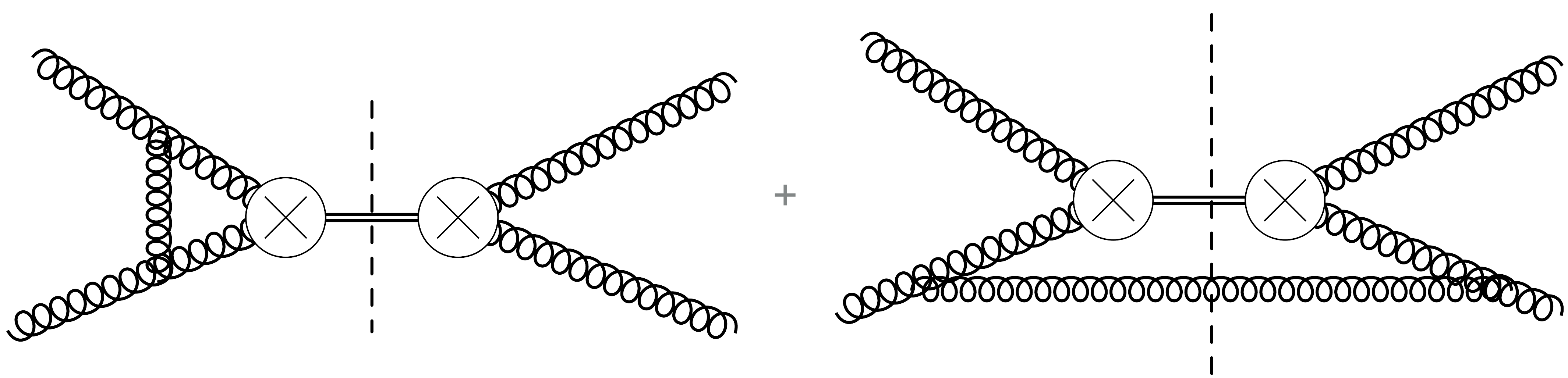
$\sigma^{(0)}$



1 Diagram

NLO

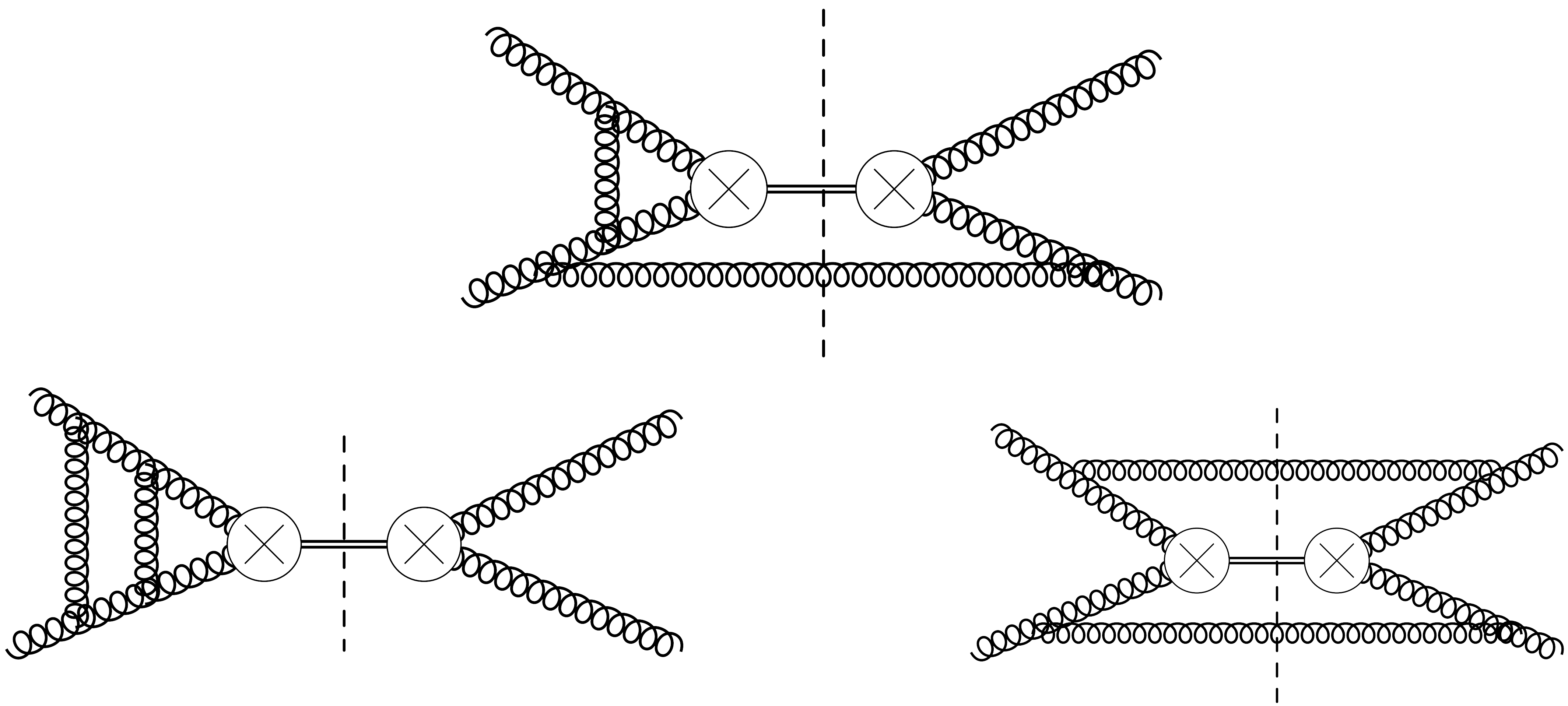
$+\alpha_S^1 \sigma^{(1)}$



~10 Diagrams

NNLO

$$+\alpha_S^2 \sigma^{(2)}$$

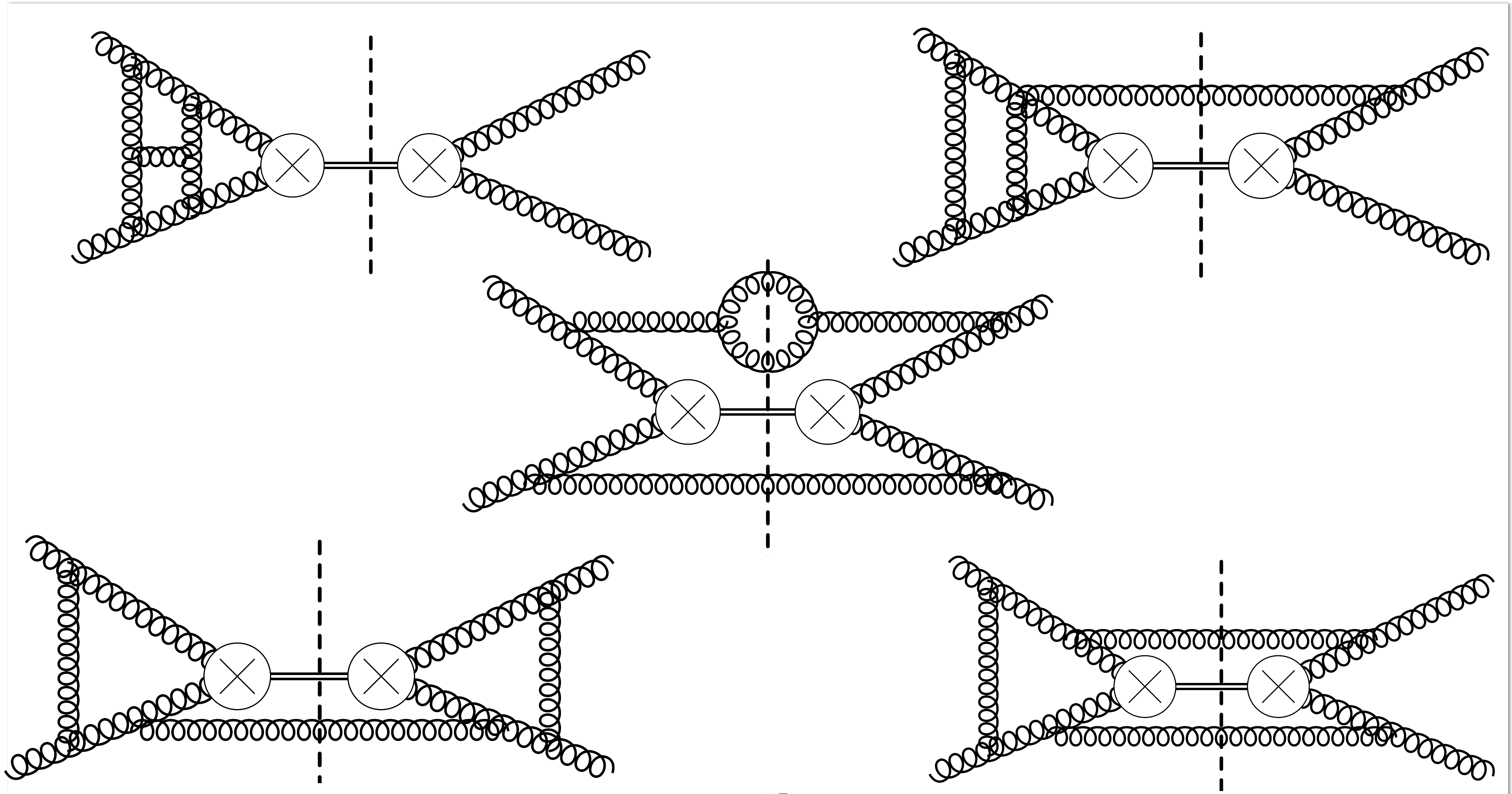


~1000 Diagrams

GLUON FUSION

$+\alpha_S^3\sigma^{(3)}$

N3LO

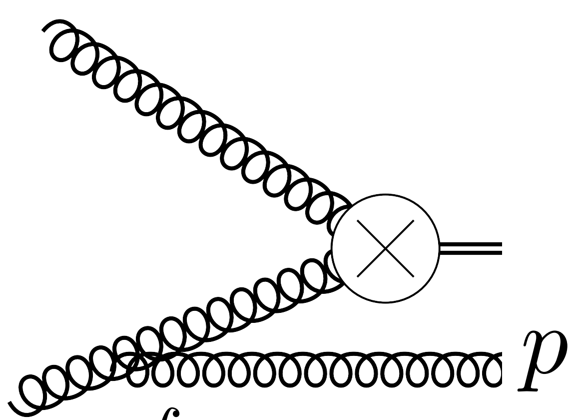


100 000 diagrams

GLUON FUSION

Feynman diagrams lead to Feynman integrals

Order	Integrals
0	1
1	~100
2	~50000
3	517531178



$$\int d^d p \delta_+(p^2)$$

- Integrand: scalar products of virtual and real particle particle momenta.
- We are interested only in the Higgs: Integrate over all other momenta in the cross section!
- Individual diagrams are UV and IR divergent: Analytic computation in dimensional regularization.

$$d = 4 - 2\epsilon$$

- There is a plethora of tools, techniques and research on how to compute scattering cross sections!

Example: Feynman integrals are related to each other!

$$c_1 I_1 + c_2 I_2 + \cdots = 0$$

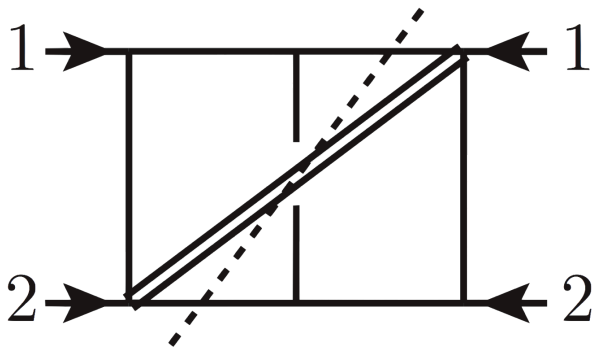
Order	Integrals	After Relations
0	1	1
1	~100	2
2	~50000	27
3	517531178	1027

- Large reduction in complexity!
- High degree of automatization and efficient computational tools required!
- Still need to compute the remaining integrals!

Integration-By-Part

$$0 = \int d^d k \frac{\partial}{\partial k^\mu} \left[k^\mu \frac{1}{k^2 - m^2} \cdots \right]$$

Example: Computing Integrals



$$= -\frac{6\epsilon\,\Gamma(6-6\epsilon)}{\Gamma(1-\epsilon)^4\Gamma(1-6\epsilon)} \int_{-i\infty}^{+i\infty} \frac{dz_2 dz_3 dz_4}{(2\pi i)^3} \Gamma(-z_2)\,\Gamma(-z_3)\,\Gamma(-z_4)$$
$$\times\, \Gamma(z_3+1)\,\Gamma(z_2-2\epsilon)\,\Gamma(-z_2-z_4)\,\Gamma(z_2+z_4+1)\,\Gamma(-\epsilon-z_3)\,\Gamma(z_3-\epsilon)$$
$$\times\, \frac{\Gamma(-2\epsilon+z_2-z_3)\,\Gamma(-\epsilon-z_4)\,\Gamma(z_4-\epsilon)}{\Gamma(-2\epsilon+z_2+1)\,\Gamma(-2\epsilon-z_3-z_4)}\,.$$

- Analytic computation helps shed light on mathematical aspects of QFT: We can study the functions that appear, analytic properties, features that re-occur at different orders …
- For example: Multiple Poly Logarithms
  Special class of functions spanning an array of scattering amplitudes and cross sections and ubiquitous in QFT and String Theory. Definition:

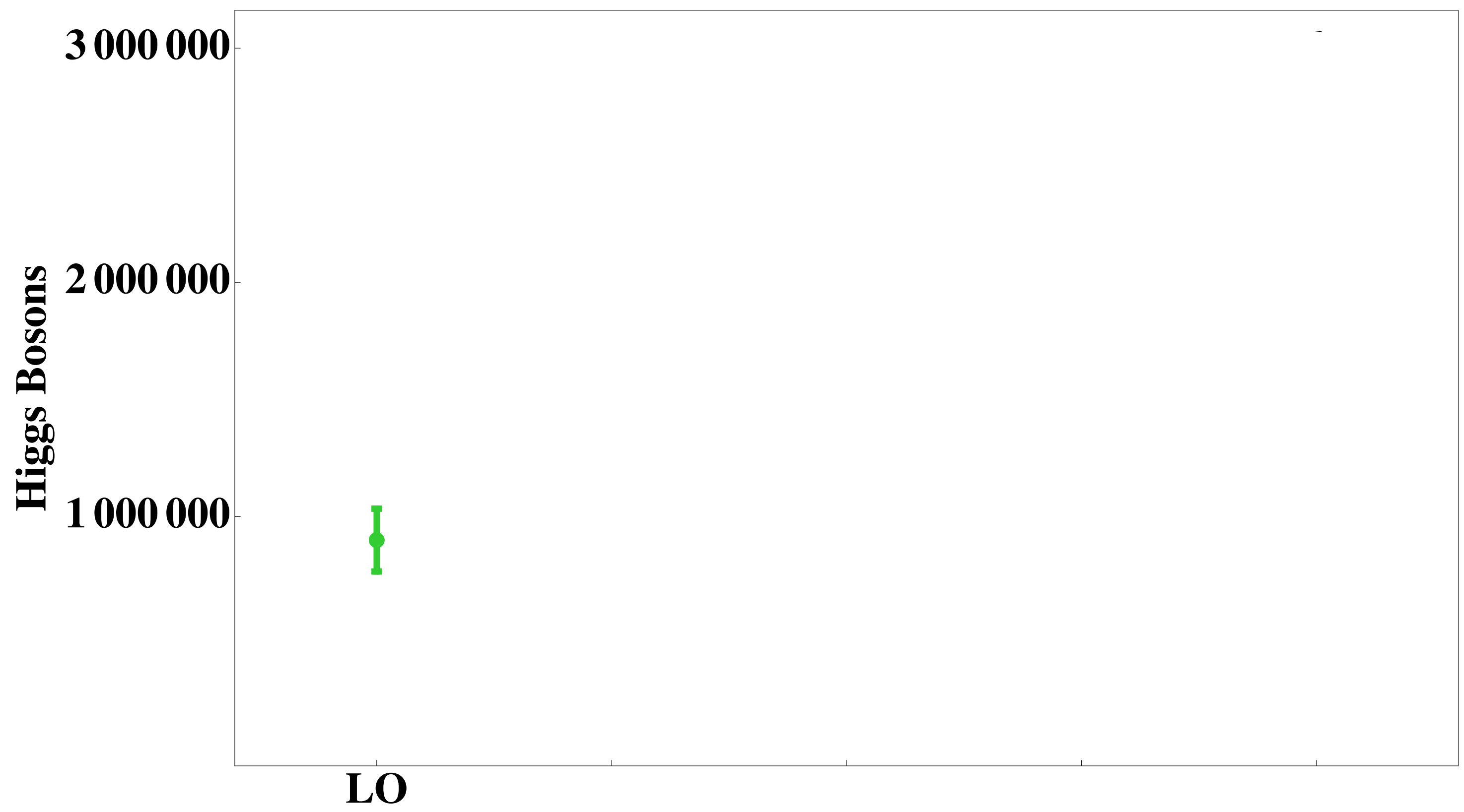
$$G(a_n,\ldots,a_1;x) \;=\; \int_0^x dt\,\frac{1}{t-a_n}G(a_{n-1},\ldots,a_1;t)$$

PRODUCING HIGGS BOSONS – THEORY

@ 13 TeV in 2018

$$\sigma = \sigma^{(0)}$$

900 000 Higgs Bosons

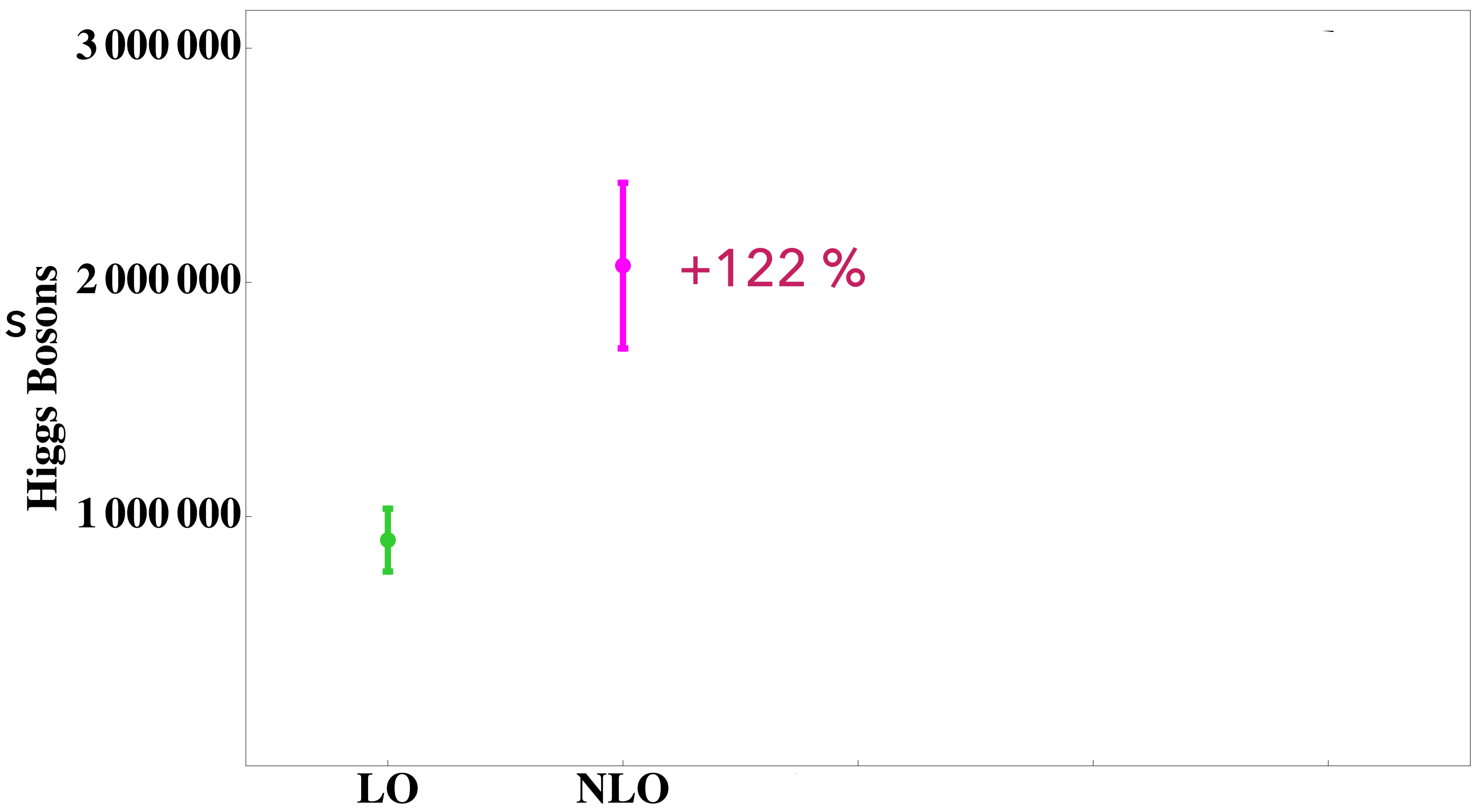


PRODUCING HIGGS BOSONS – THEORY

@ 13 TeV in 2018

$$\sigma = \sigma^{(0)}$$ 900 000 Higgs Bosons

$$+\alpha_S^1 \sigma^{(1)}$$ +1 100 000 Higgs Bosons



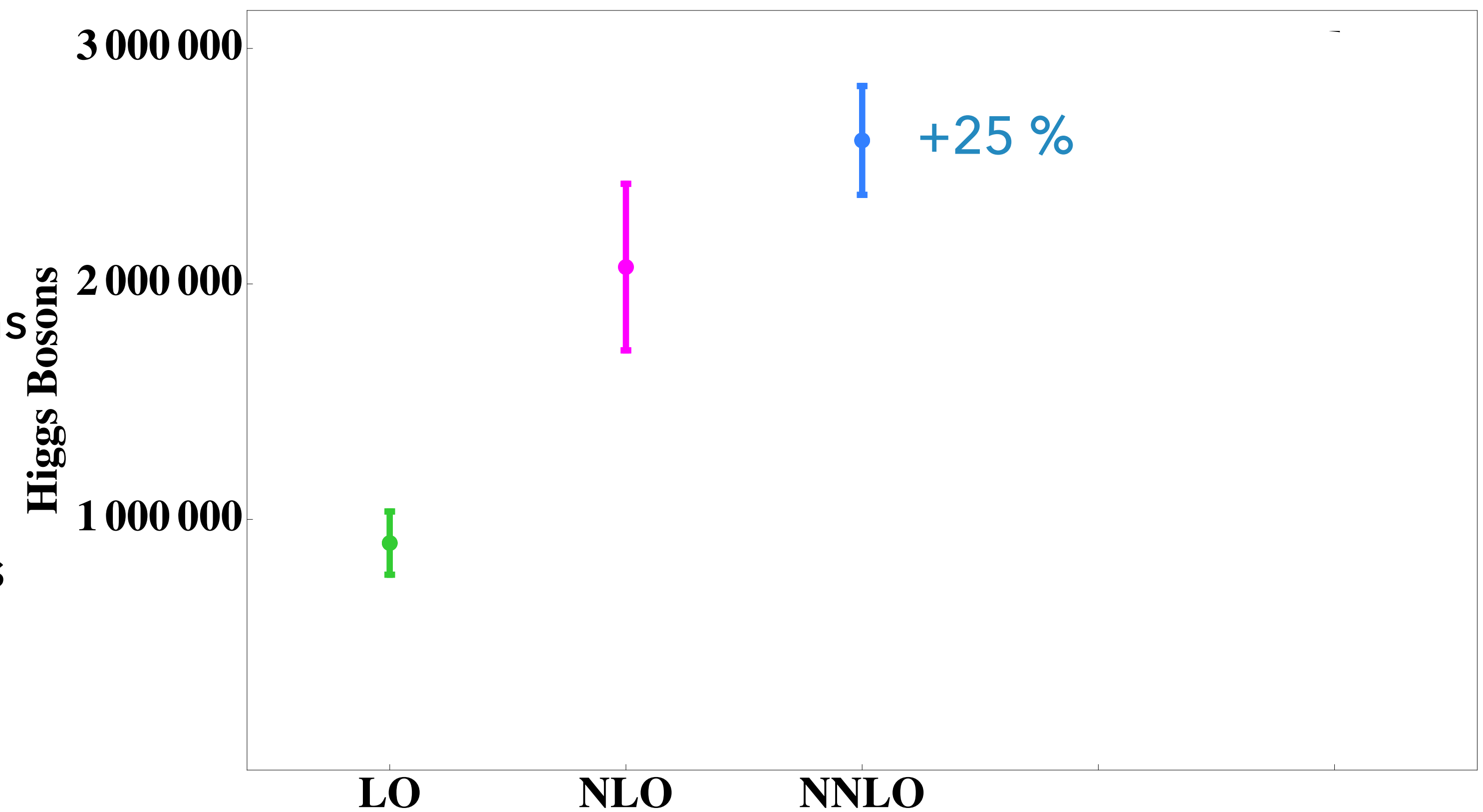
PRODUCING HIGGS BOSONS – THEORY

@ 13 TeV in 2018

$$\sigma = \sigma^{(0)}$$ 900 000 Higgs Bosons

$$+\alpha_S^1 \sigma^{(1)}$$ +1 100 000 Higgs Bosons

$$+\alpha_S^2 \sigma^{(2)}$$ +500 000 Higgs Bosons



PRODUCING HIGGS BOSONS – THEORY

@ 13 TeV in 2018

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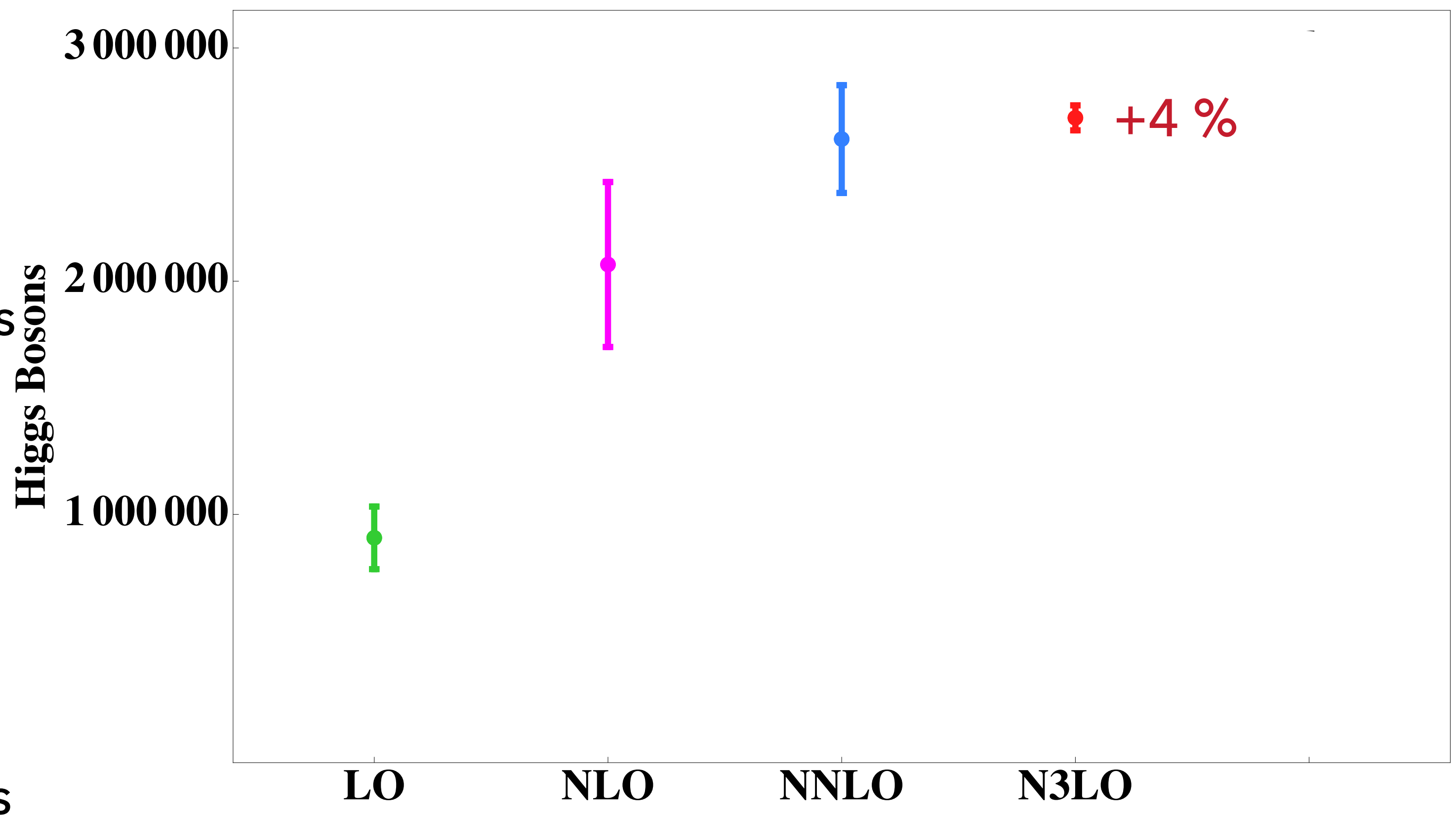
$$+\alpha_S^3 \sigma^{(3)}$$ +100 000 Higgs Bosons



PRODUCING HIGGS BOSONS – THEORY

@ 13 TeV in 2018

$\sigma = \sigma^{(0)}$ 900 000 Higgs Bosons

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$+\alpha_S^2 \sigma^{(2)}$ +500 000 Higgs Bosons

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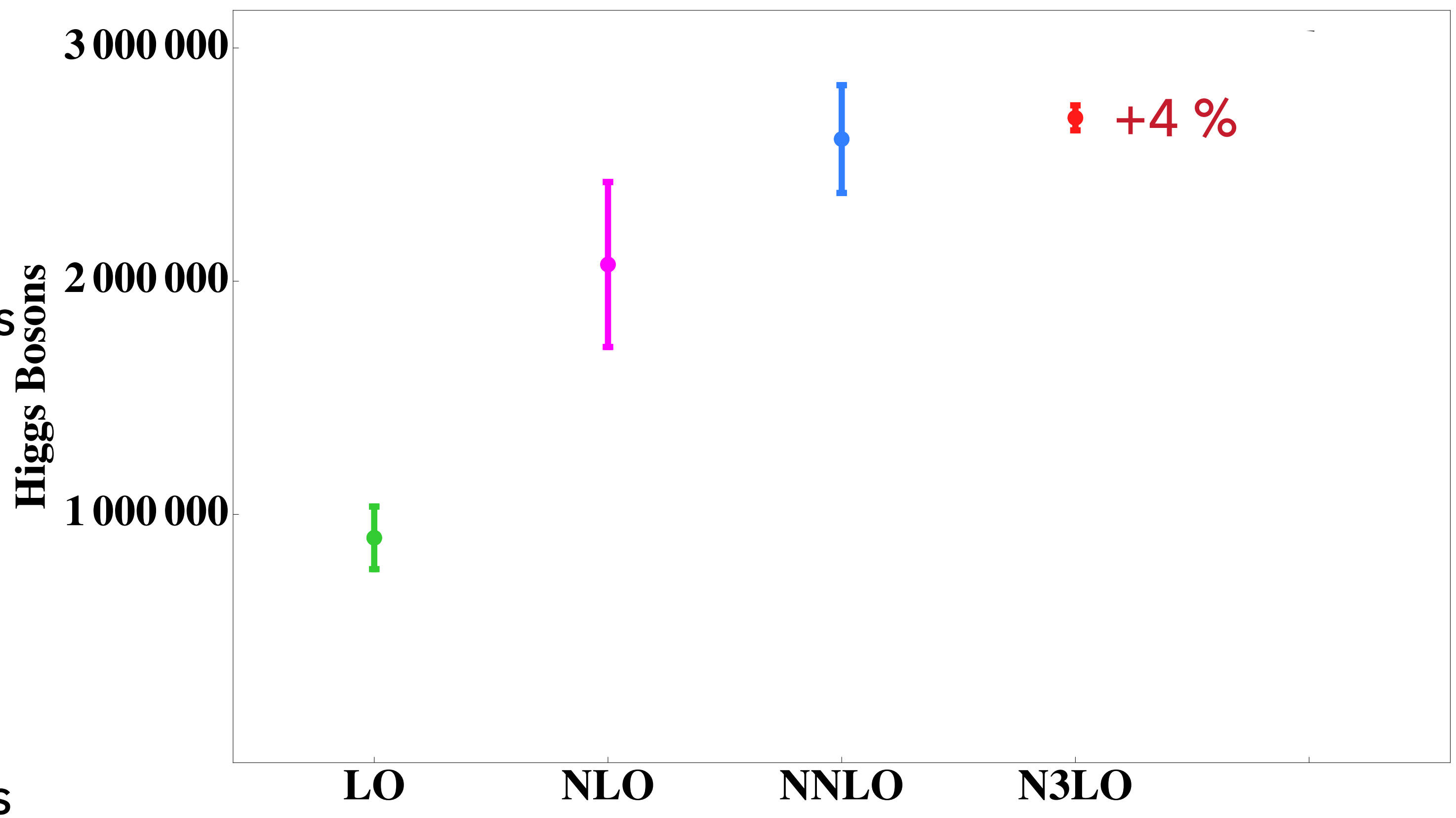


N3LO Corrections:

- Reduce perturbative uncertainty from 9 % -> **2.5** %.
- Stabilize the perturbative expansion.

PRODUCING HIGGS BOSONS – THEORY

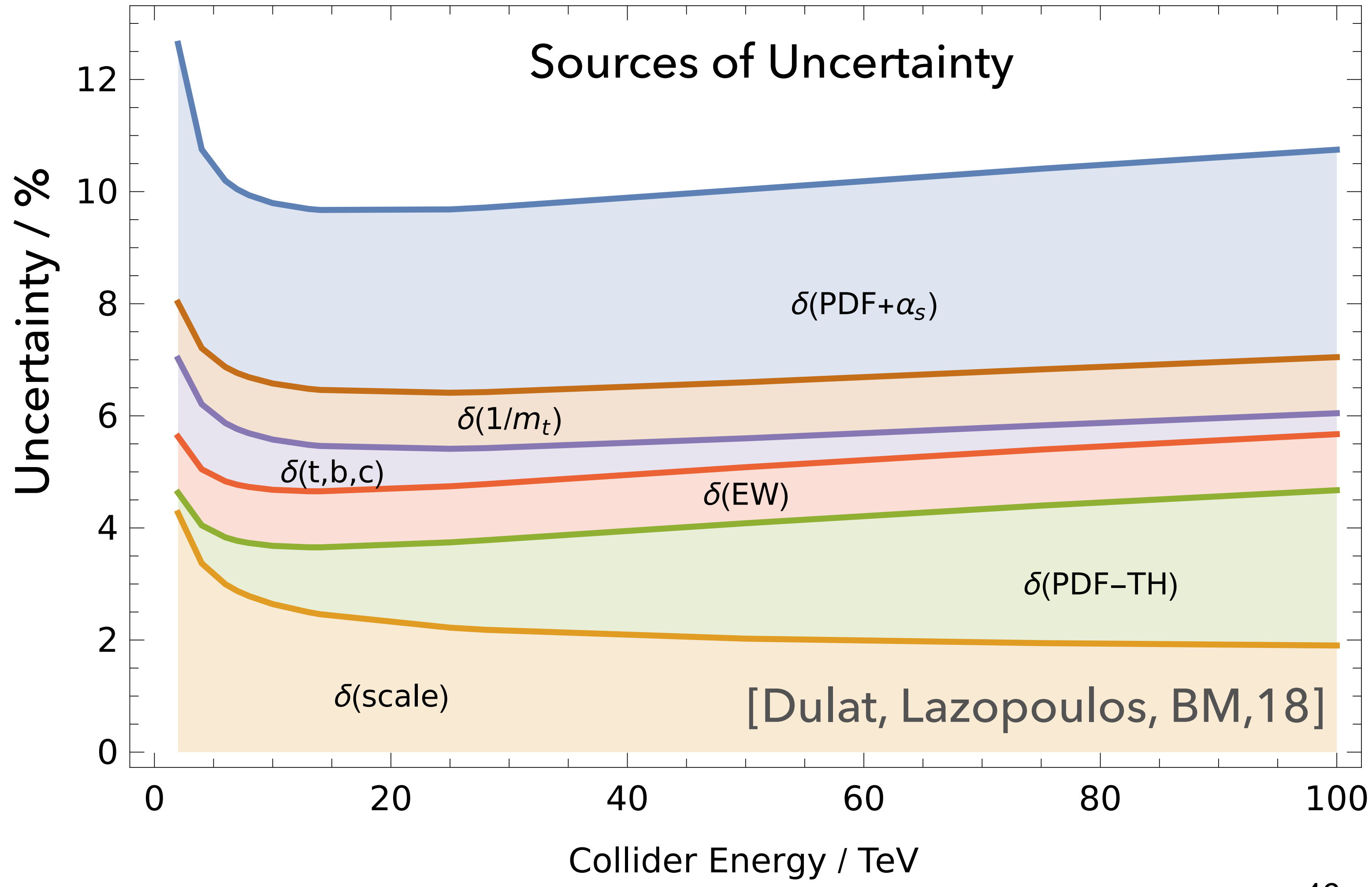
Much more than QCD corrections

- Electro-weak corrections.
- Neglected quark mass effects.
- Coupling to bottom, charm quarks.
- Estimate uncertainties.

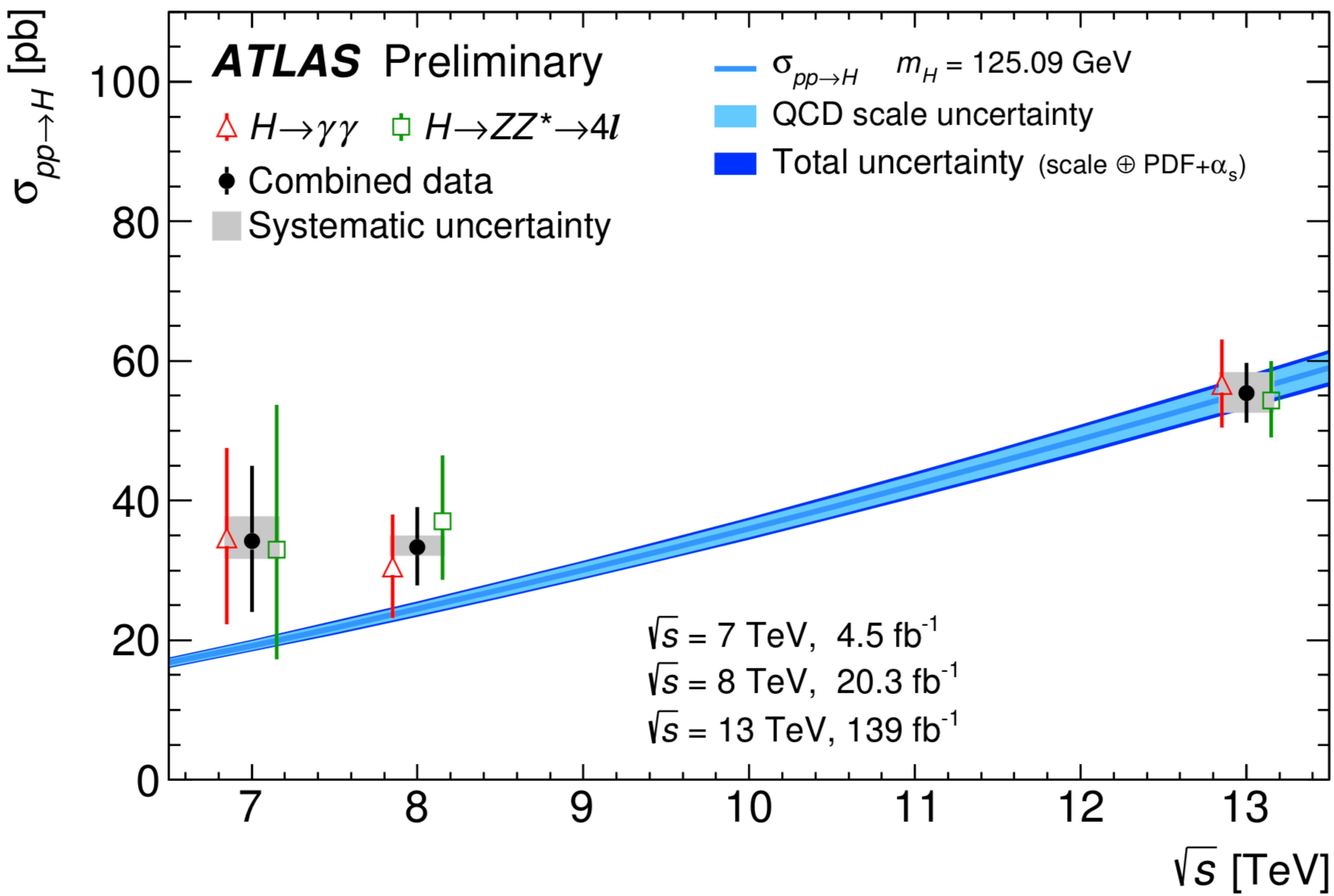
  Truncation of perturbative series

  PDF, α_S

- …

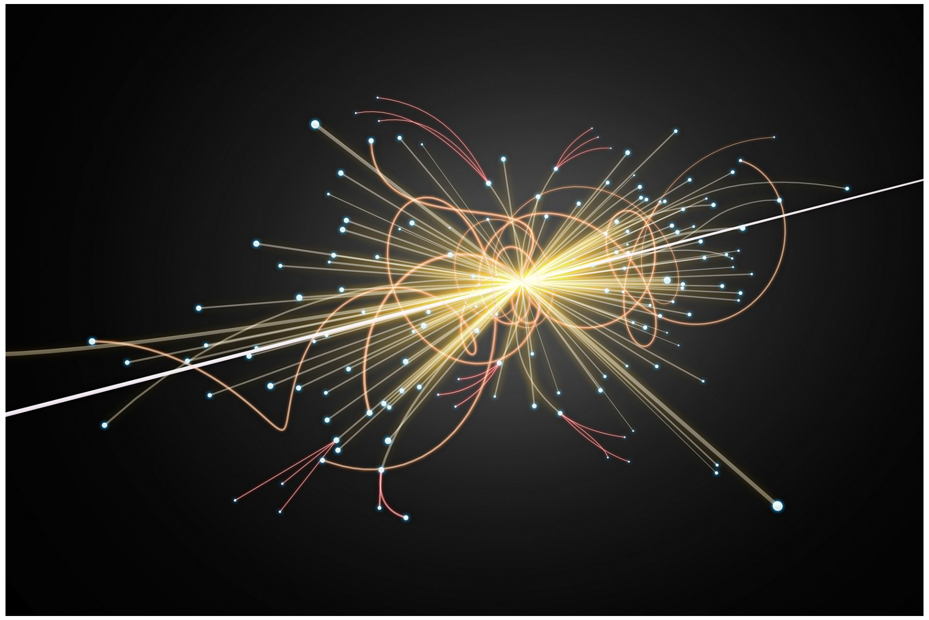


PRODUCING HIGGS BOSONS – THEORY



FULLY DIFFERENTIAL PREDICTIONS TO PRODUCE A HIGGS BOSON

- **Predict distributions as measurable by the LHC**
- **Experimental statistics requires precision.**
- **High degree of flexibility to compute many different observables is required.**



Example:
Cross section predictions for the fiducial di-photon cross section via gluon fusion.

$$P\,P \to H + X \to \gamma\,\gamma + X$$

Realistic selection cuts of final state products in the detector:

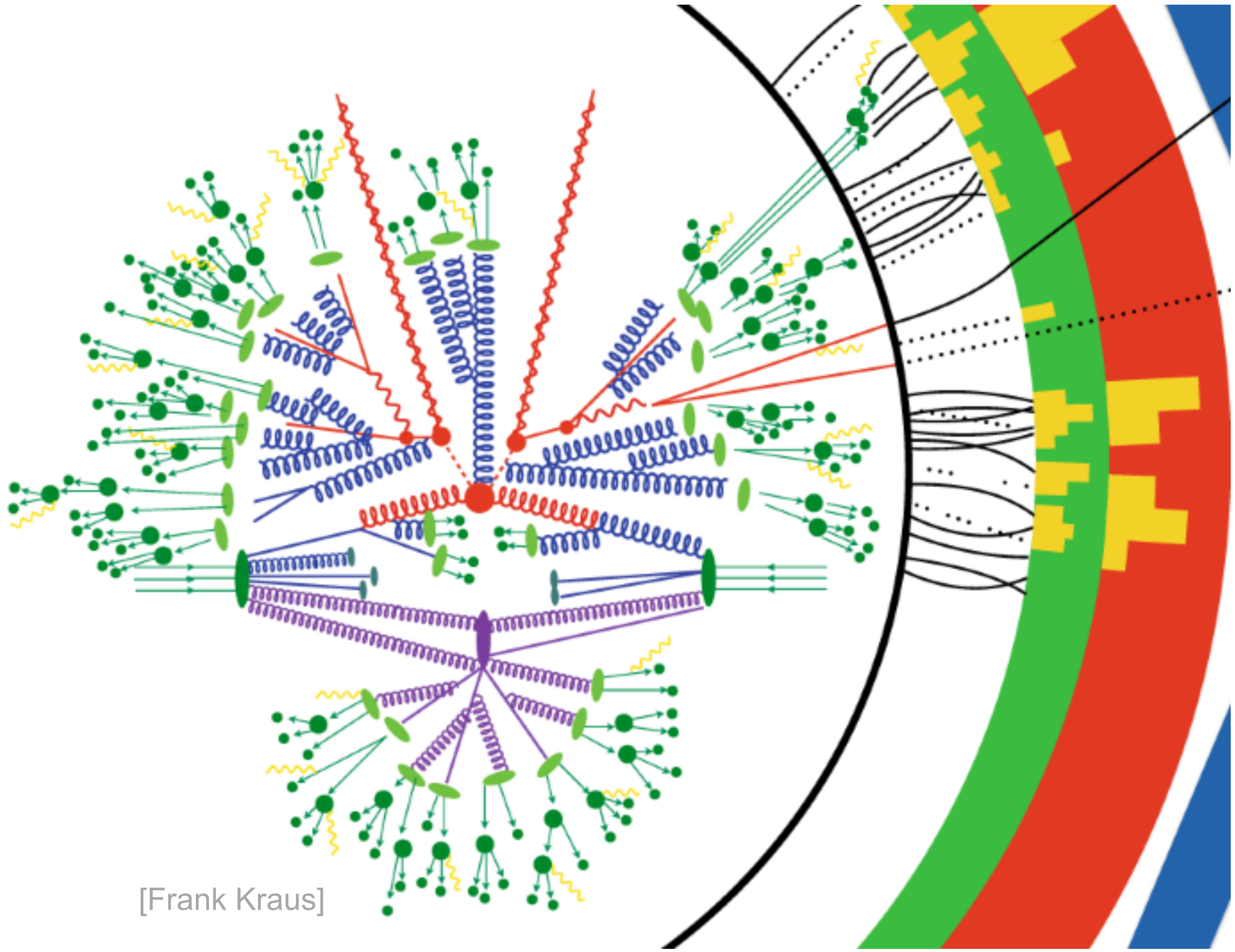
$$p_{\mathrm{T}}^{\gamma_1} > 0.35 \times m_{\gamma\gamma}, \qquad p_{\mathrm{T}}^{\gamma_2} > 0.25 \times m_{\gamma\gamma},$$
$$|\eta^{\gamma}| < 2.37 \;\; \text{excluding} \;\; 1.37 < |\eta^{\gamma}| < 1.52,$$

Ability to compute a large range of observables:

Y_H $\quad p_{\perp}^{\gamma}$ $\quad Y_{\gamma}$ $\quad \cos\theta_{\gamma}$ $\quad \Delta Y_{\gamma}$ $\quad$ Jets $\quad$ ...

NLO

- Scattering Events are complex!
- Calculations involving one loop corrections (QCD or EWK) have been automated to a very large degree: **NLO Revolution**
- Fully automatic generation of NLO events including parton showers hadronization are easily accessible through public tools.
- The challenge: How to move this enormous success to higher precision and higher orders?



[Frank Kraus]

FULLY DIFFERENTIAL PREDICTIONS TO PRODUCE A HIGGS BOSON

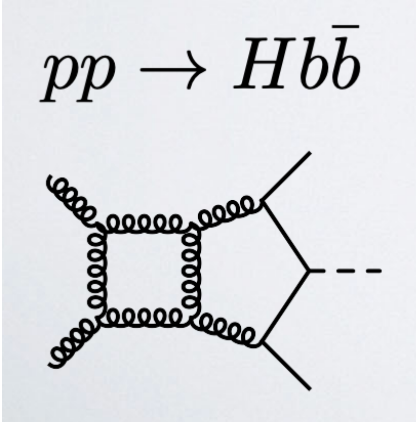
NNLO

Typical strategy:

Loop Integrals: Analytic Computation

- High degree of analytic complexity. (Similar to the case of inclusive cross section)
- Currently used in physical predictions: 2-loop amplitudes for 2 -> 3 scattering massless.
- State of the art in amplitude computations: 5 particle amplitudes at 2 loops.

For example [2107.14733]



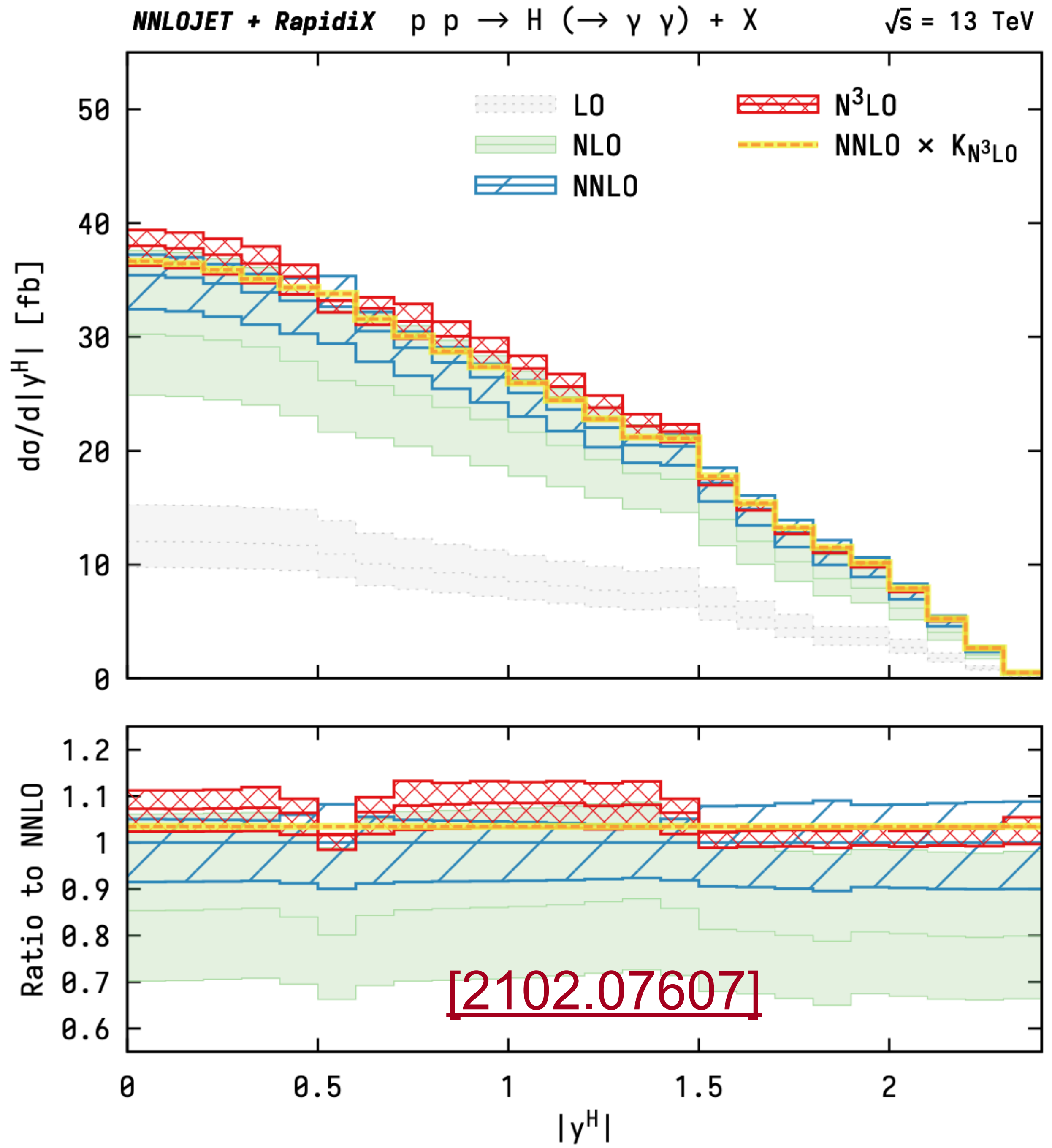
Final state phase space integrals: Numeric Computation

- Numerical integration over final state phase space requires regularization of infrared divergences.
- Fairly intricate and make high precision computations numerically expensive: 10^5 CPU hours / per prediction not uncommon.
- State of the art: Algorithms to operate for any cross section at the LHC at NNLO, color singlet production at N3LO.

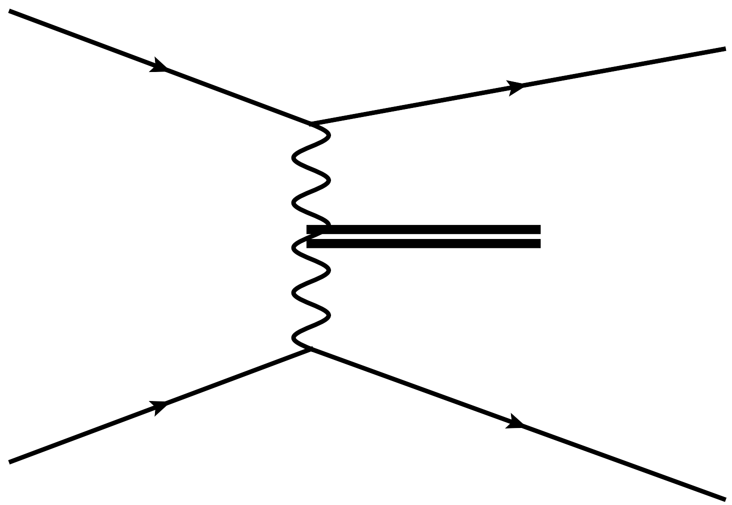
FULLY DIFFERENTIAL PREDICTIONS TO PRODUCE A HIGGS BOSON

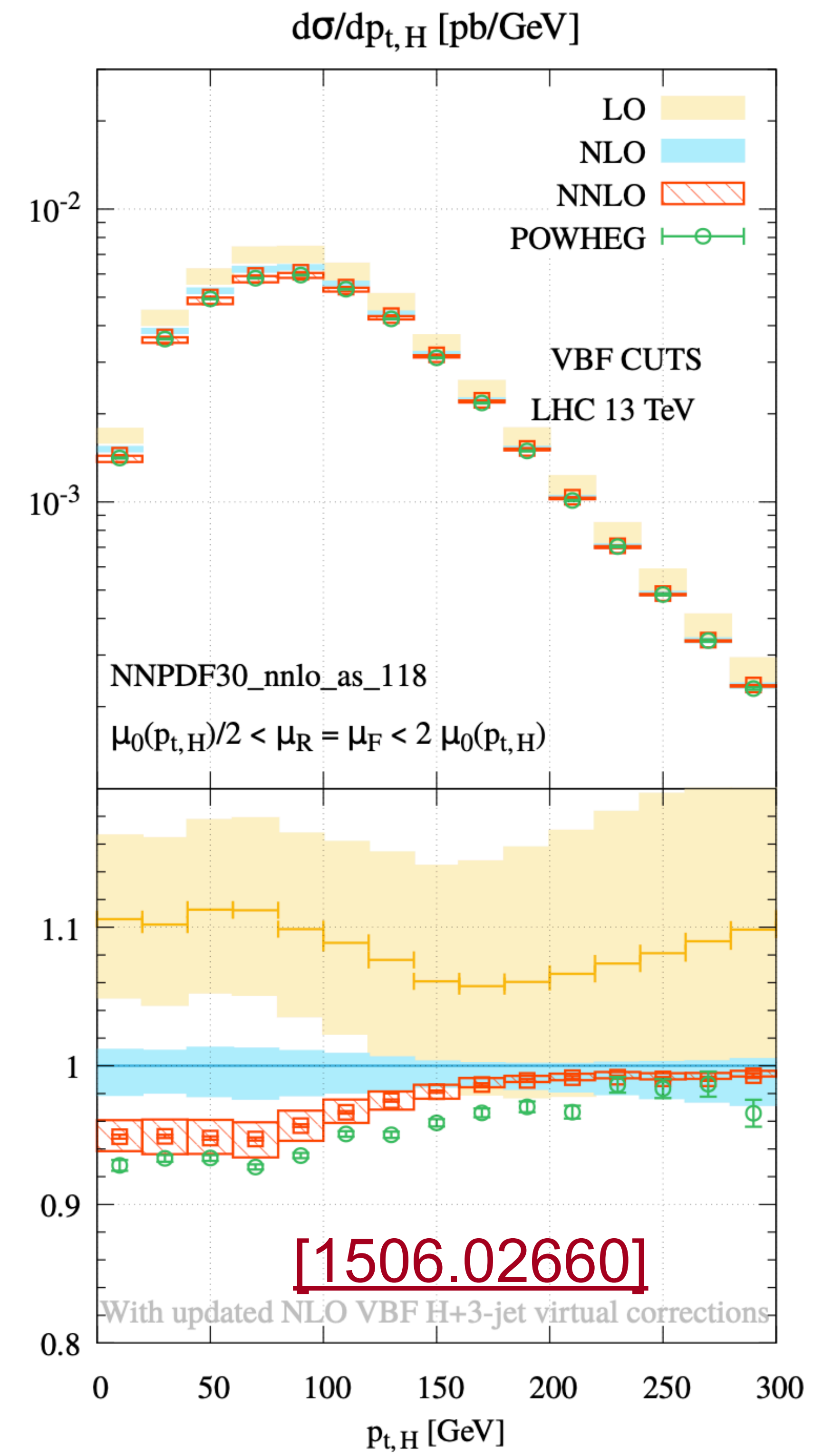
$$P\,P \to H + X \to \gamma\,\gamma + X$$

- Prediction for the rapidity distribution in gluon fusion at N3LO in QCD including cuts on final state photons as imposed by the ATLAS collaboration.
- Enables direct comparison of precise theory and experiment.
- Can study the perturbative progression in fiducial volume.
- Optimize particle cuts.



FULLY DIFFERENTIAL PREDICTIONS TO PRODUCE A HIGGS BOSON





- Vector boson fusion Higgs production at NNLO.
- Realistic selection cuts on jets produced alongside the Higgs.
- Nice example of new ideas being produced for the prediction of differential cross sections: [1506.02660]

FULLY DIFFERENTIAL PREDICTIONS TO PRODUCE A HIGGS BOSON

- Once scattering amplitudes and algorithms for subtraction of IR singularities became available more widespread automatization could begin!
- Public tools for the computation for fully differential NNLO cross sections:

Matrix

- $pp \to Z/\gamma^* (\to l^+l^-)$ ✅
- $pp \to W(\to l\nu)$ ✅
- $pp \to H$ ✅
- $pp \to \gamma\gamma$ ✅
- $pp \to W\gamma \to l\nu\gamma$ ✅
- $pp \to Z\gamma \to l^+l^-\gamma$ ✅
- $pp \to ZZ/WW \to ll\nu\nu$ ✅
- $pp \to WZ \to l\nu ll$ ✅

https://matrix.hepforge.org

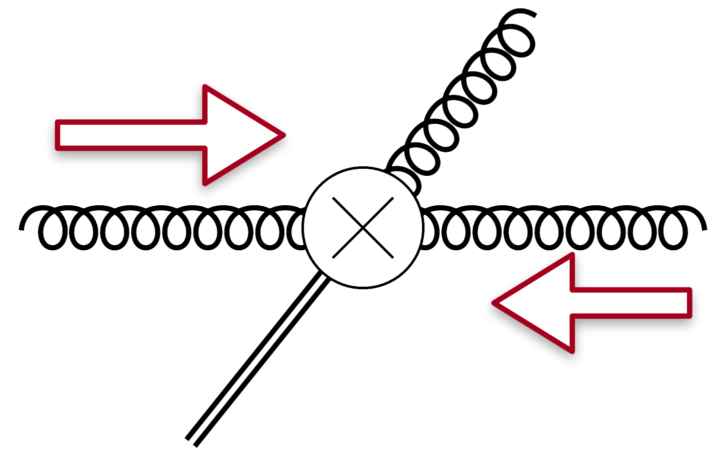
MCFM

Process	nproc	$\sigma_{\rm NLO} \pm \delta\sigma^{\rm MC}_{\rm NLO}$	$\sigma_{\rm NNLO} \pm \delta\sigma^{\rm MC}_{\rm NNLO} \pm \delta\sigma^{\rm pc}_{\rm NNLO}$
W^+	1	4.220 ± 0.002 nb	$4.19 \pm 0.02 \pm 0.043$ nb
W^-	6	3.315 ± 0.001 nb	$3.23 \pm 0.01 \pm 0.033$ nb
Z	31	885.2 ± 0.3 pb	$878 \pm 3 \pm 9$ pb
H	112	1.395 ± 0.001 pb	$1.865 \pm 0.004 \pm 0.019$ pb
$\gamma\gamma$	285	27.94 ± 0.01 pb	$43.60 \pm 0.06 \pm 0.44$ pb
W^+H	91	2.208 ± 0.002 fb	$2.268 \pm 0.007 \pm 0.023$ fb
W^-H	96	1.494 ± 0.001 fb	$1.519 \pm 0.004 \pm 0.015$ fb
ZH	110	0.7535 ± 0.0004 fb	$0.846 \pm 0.001 \pm 0.0085$ fb
$Z\gamma$	300	959 ± 8 fb	1268 ± 22 fb

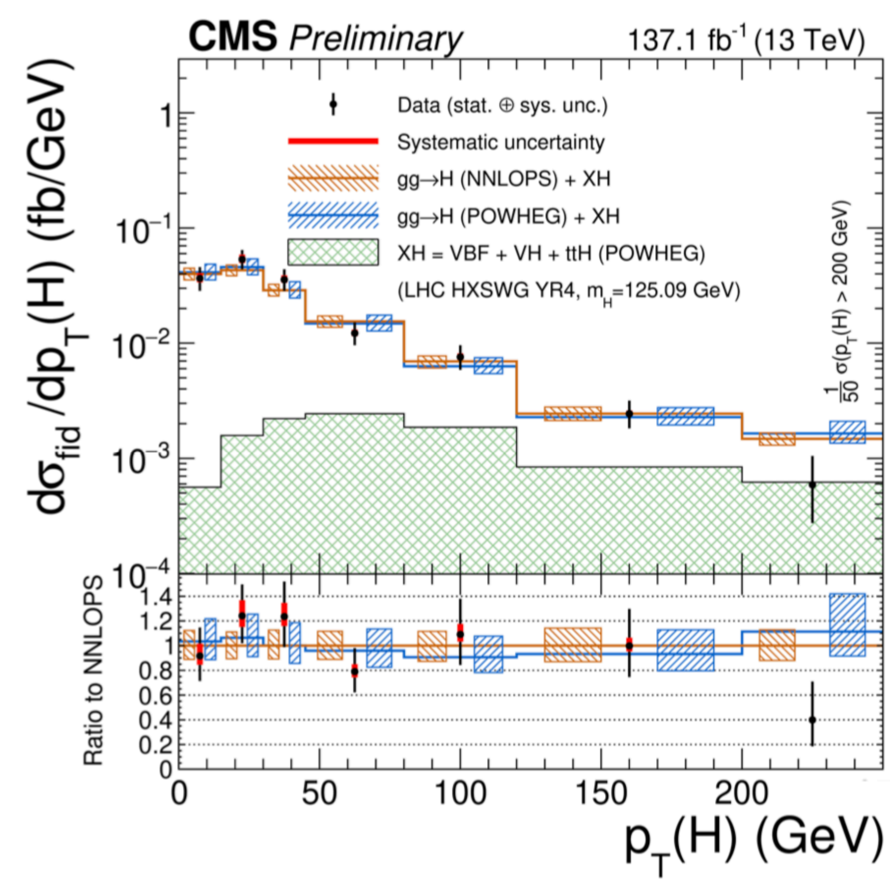
https://mcfm.fnal.gov

TRANSVERSE MOMENTUM DISTRIBUTION

$$\frac{d\sigma}{dp_\perp}$$

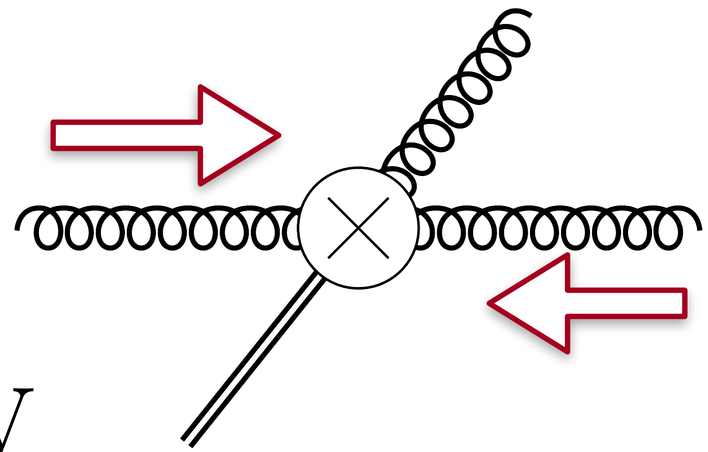


- One of the precision observables in Higgs Boson phenomenology.
- Higgs recoils agains hadronic jets.
- How to predict depends on the kinematic regime!



TRANSVERSE MOMENTUM DISTRIBUTION

$$\frac{d\sigma}{dp_\perp}$$

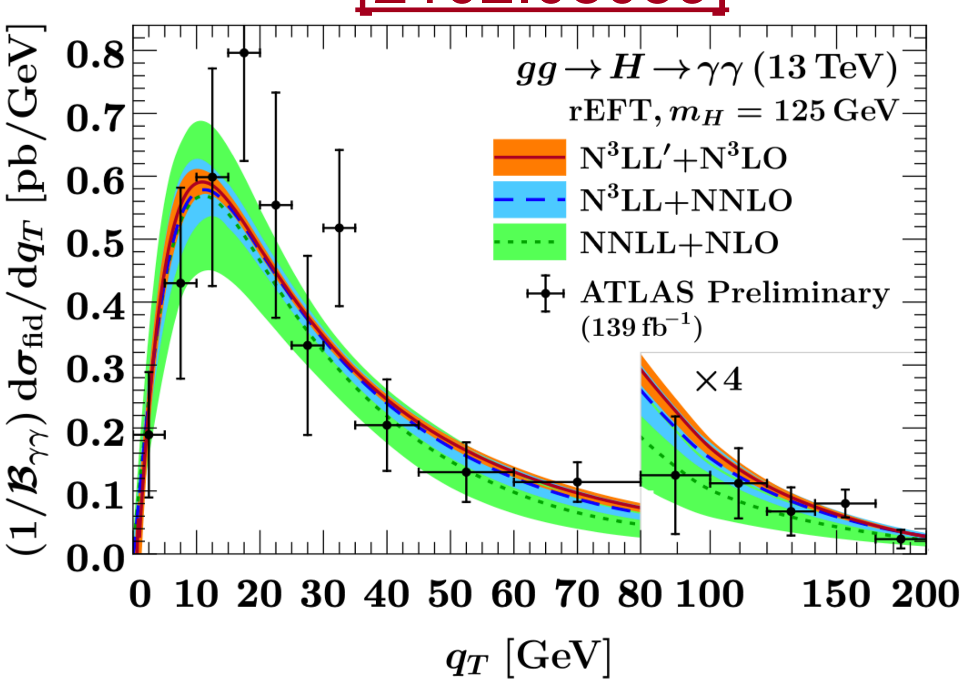


$$p_\perp <\sim 30\text{GeV}$$

- At low transverse momenta, fixed order perturbation theory breaks down: **Resummation!**

$$\sigma^{(n)} \sim \alpha_S^n \log^{(2n-1)}(p_\perp^2)\sigma^{\text{LL}} + \ldots$$

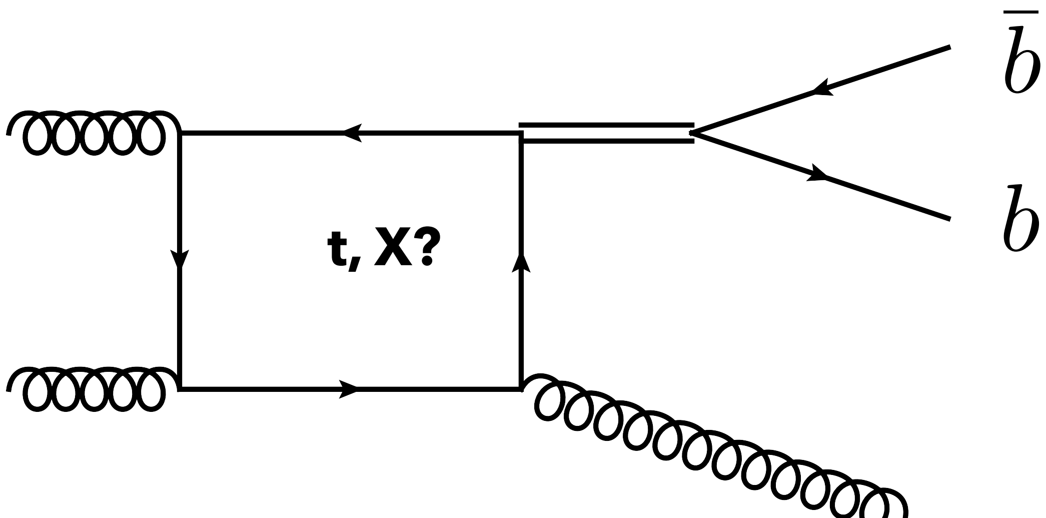
[2102.08039]



- Low pT contains largest bit of cross section.
- Resumed cross sections at very high precision: N3LO +N3LL`

TRANSVERSE MOMENTUM DISTRIBUTION

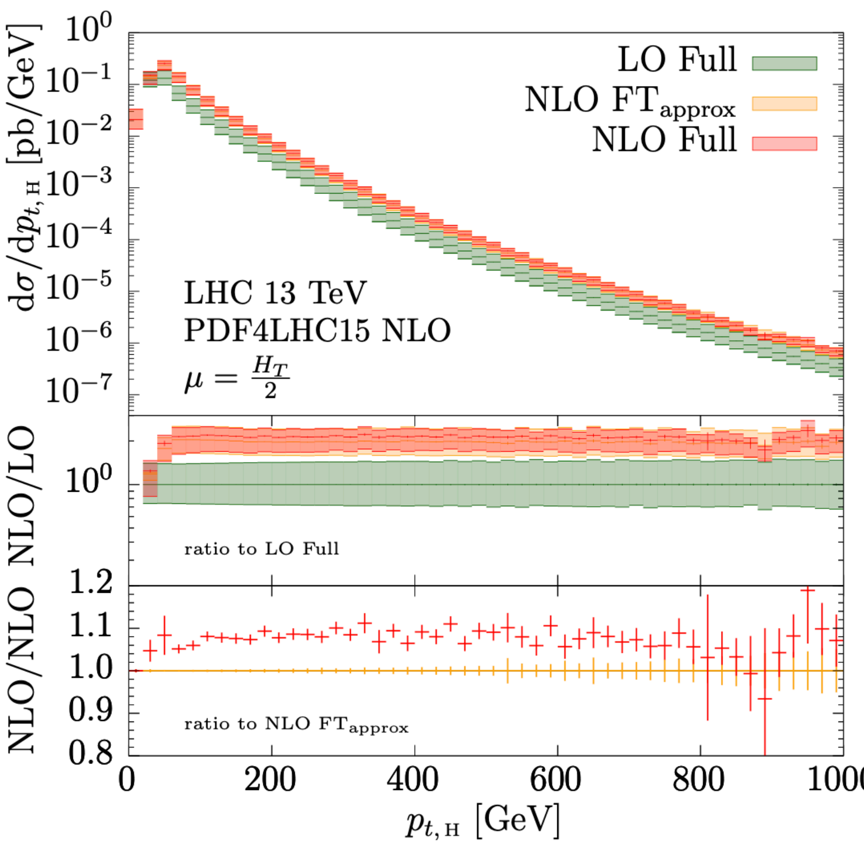
$$\frac{d\sigma}{dp_\perp}$$



$$p_\perp > 200\text{GeV}$$

- Heavy top quark effective theory is not adequate if pT becomes large!
- Very interesting observable:
  Resolve inner workings of the top quark loop.
- Currently:
  NLO corrections are available for large pT
- Very complicated computation using fully numerical techniques for two loop amplitudes!

[1802.00349]



- The Higgs Boson is very exciting and we are able to explore it!
- Thanks to the organizers of the Fermilab-CERN school!

THANK YOU!

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