Transition Radiation from a Metal Foil. OTR and COTR

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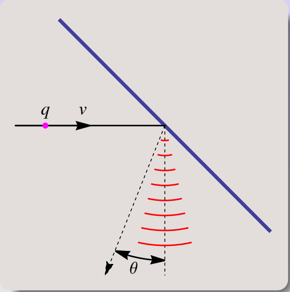
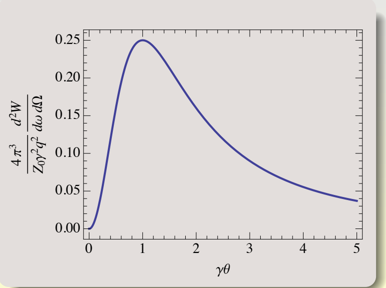
Transition radiation - angular distribution



Energy radiated per unit frequency ω per unit solid angle, $\beta = v/c$

$$\frac{d^2 W}{d\omega d\Omega} = \frac{Z_0 q^2}{4\pi^3} \frac{\beta^2 \sin^2\theta}{(1-\beta^2\cos^2\theta)^2}$$
$$\approx \frac{Z_0 q^2}{4\pi^3} \frac{\theta^2}{(\gamma^{-2}+\theta^2)^2}$$



TR is localized at small angles $\theta \sim 1/\gamma$ relative to the forward direction. For 45 MeV beam, $\gamma = 88$. The spectrum does not depend on frequency ω.

Radiation energy on the detector

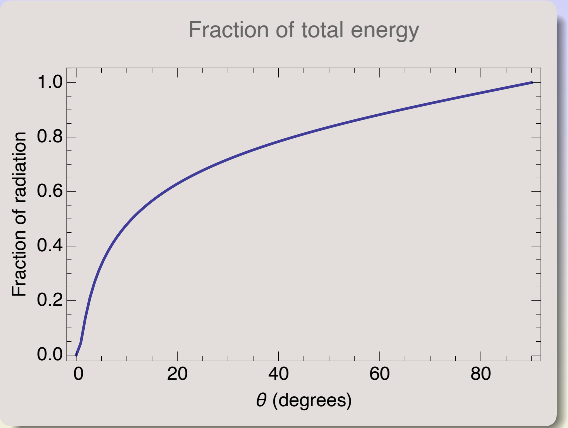
Integrating $d^2W/d\omega d\Omega$ over the solid angle gives the spectrum of radiation

$$\frac{d\mathcal{W}}{d\omega} = 2\pi \int_0^{\pi/2} \sin\theta d\theta \frac{d^2\mathcal{W}}{d\omega d\Omega} = \frac{Z_0 q^2}{4\pi^2}\left[\left(\frac{1}{\beta}+\beta\right)\operatorname{arctanh}(\beta) - 1\right]$$

We assume incoherent radiation, then the energy radiated by the beam $\propto N_e$ in the bunch.
Assume $Q = 1$ nC and the collection angle 90°.

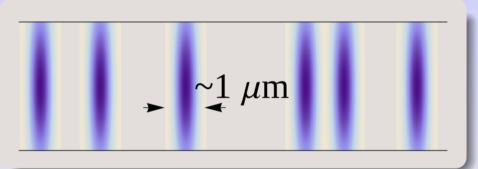
	0.5 − 1 μm	1 − 5 μm
Energy [nJ]	0.027	0.021

Energy on the detector depends on collection angle



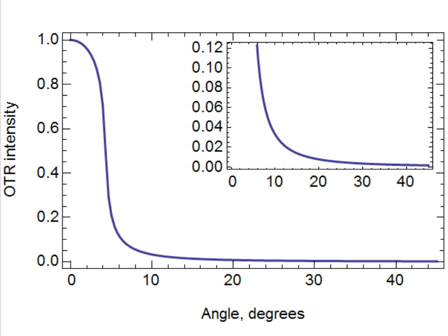
If spectral sensitivity of the detector is available, it can be taken into account.

COTR versus OTR

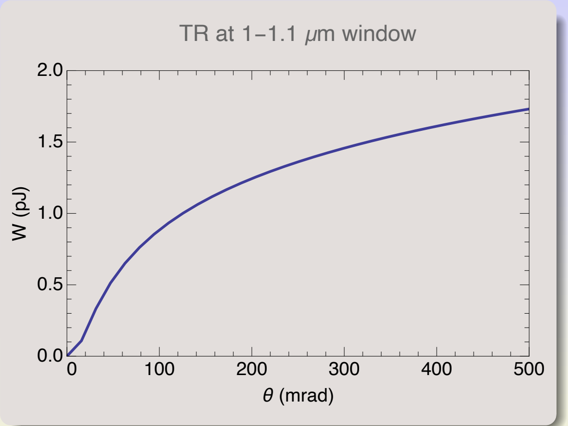


Incoherent radiation (no microbunching) is proportional to N_e, coherent radiation is proportional to N_e^2.

COTR will have a different angular dependence which can be measured by rotating the foil. (Calculations were done for $\gamma = 500$, 75 mrad collection angle, $\lambda \approx 0.6$ μm).



Energy on the detector depends on collection angle



Radiated TR energy for 1nC beam as a function of the collection angle θ in 100 nm window.