# Recap

wave functions $|\psi(x,t)\rangle$

SE : $\hat{H}|\psi(x,t)\rangle = i\hbar \frac{\partial}{\partial t}|\psi(x,t)\rangle$

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Time-indepedant SE : $V(x,t) = V(x)$

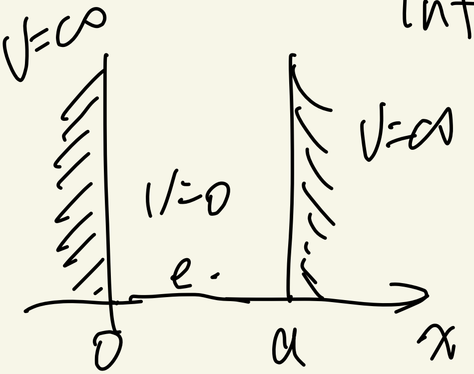
$$\psi(x,t) = \psi(x)\,\varphi(t)$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\,\underline{\psi(x)} = E\cdot\psi(x)$$

$$\boxed{\psi(x,t) = \psi(x)\,e^{-iEt/\hbar}}$$

$\downarrow$ stationary states

# Infinite square well




[particle in a box]

$\psi(x)$ $x<0$ or $x>a$.

RHS: $E\cdot\psi(x)$ finite.

LHS: $V(x)\cdot\psi(x)=\infty$

$KE=\frac{1}{2}mv^2$

$\Longrightarrow$ $\psi=0$

$0<x<a$. $-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2}=E\psi$

$$\underline{\psi_n(x)}=\sqrt{\frac{2}{a}}\sin\left(\frac{n\pi}{a}x\right)$$

$n=1,2,3,\ldots$

stationary states!

1). $\psi_n(x,t)=\psi_n(x)\cdot e^{-iEt/\hbar}$

prob: $|\psi_n(x,t)|^2=\underline{|\psi_n(x)|}^2$ $\boxed{E_1=\frac{\pi^2\hbar^2}{2ma^2}}$

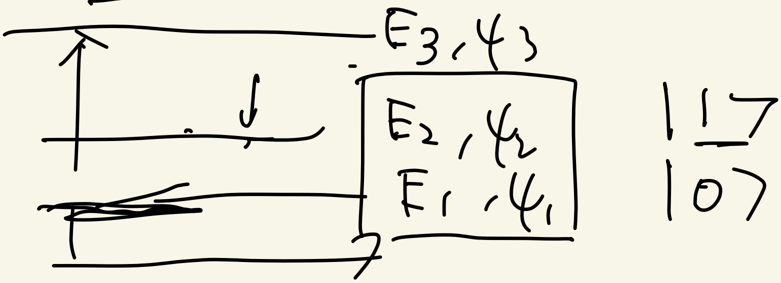
2) $\hat{H}\psi_n(x,t)=E_n\cdot\psi_n$

$$E_n=\frac{n^2\pi^2\hbar^2}{\underline{2ma^2}}$$ $n=1,2,3,\ldots$

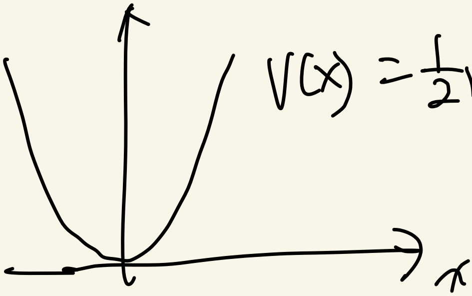
3) $$\psi(x,t) = \sum_{n=1,2,\dots} c_n \psi_n(x,t)$$

$\psi_1, \psi_2, \psi_3 \dots$

$$\psi = \frac{1}{\sqrt{2}}\psi_1 e^{-iE_1 t/\hbar} + \frac{1}{\sqrt{2}}\psi_2 e^{-iE_2 t/\hbar}$$



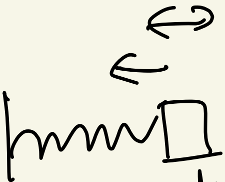
# Simple harmonic oscillator.




$V(x) = \frac{1}{2} m \omega^2 x^2$

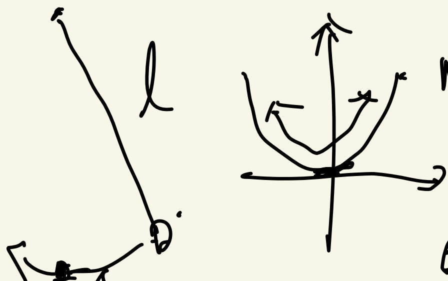
$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + \frac{1}{2} m \omega^2 x^2 \psi(x) = E \cdot \psi(x)$$

$\left\{ \begin{array}{l} \text{ODE.} \\ \text{raising / lowering operators / ladder operators.} \end{array} \right.$

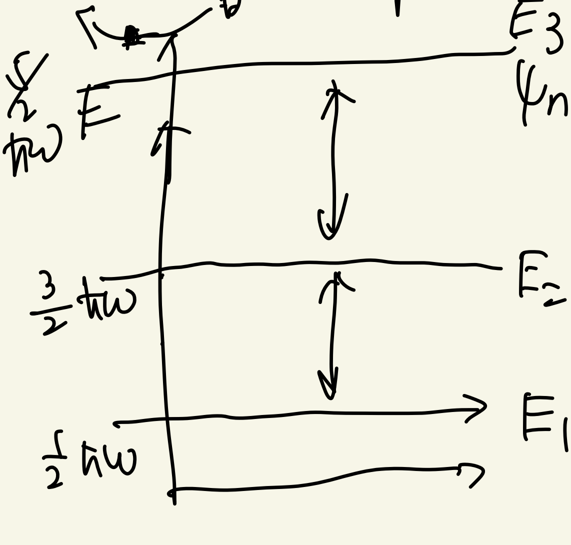


$F = -kx$

$F = -\frac{dV}{dx} \Rightarrow V(x) = \frac{1}{2} k x^2$



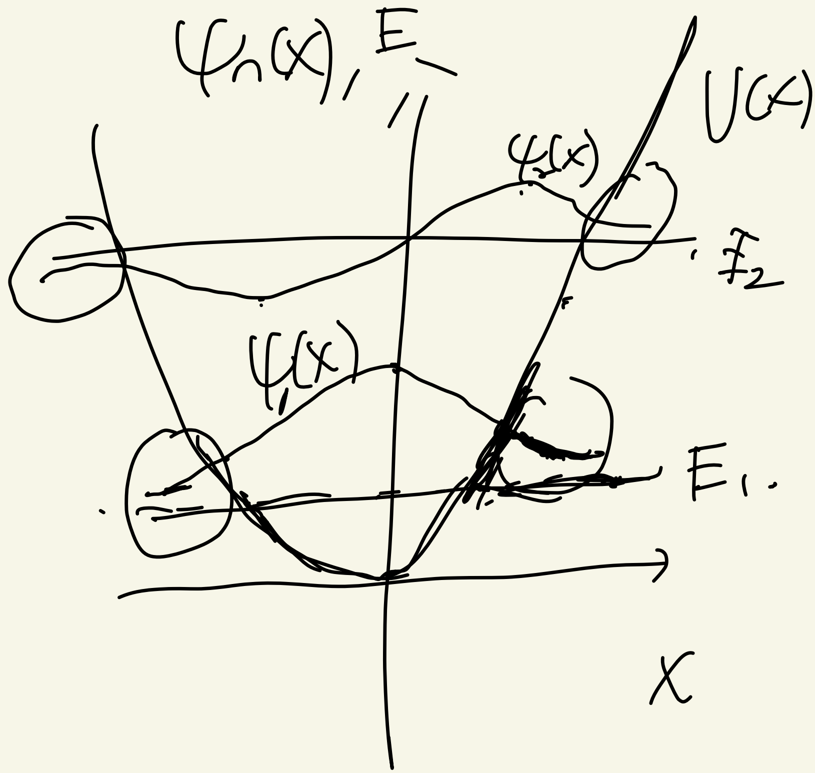

mgh.



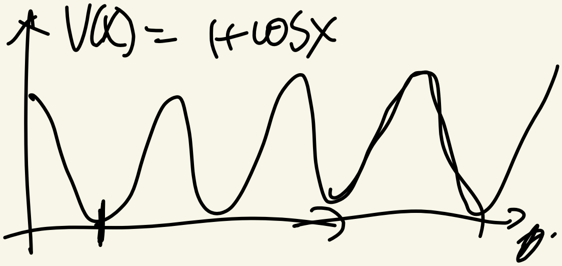

$\psi_n(x)$ : stationary states.

$E_n = (n + \frac{1}{2})\hbar\omega$

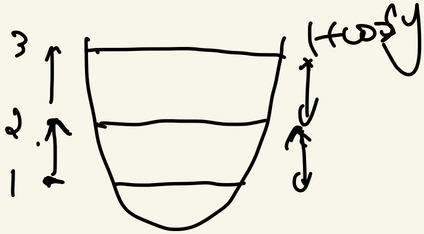
$n: 0, 1, 2 \ldots$




$$V(x) = x^2.$$




$x \simeq \pi$




$$V(x) = 1+\cos x$$

$$y : \pi - x$$

$$V(x) = 1+\cos(\pi - y)$$

$$= 1-\cos y$$

$$= 1-\left(1-\frac{y^2}{2}+O(y^4)\right)$$

$$V(y) = \underline{\frac{y^2}{2}} + O(\underline{y^4})$$

# Hydrogen atom

3D instead of 1D

$p^{+}$ $e^{-}$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x)$$

$$\hookrightarrow \frac{d^2\psi}{dx^2} + \frac{d^2\psi}{dy^2} + \frac{d^2\psi}{dz^2}$$

$$V(r) = -\frac{e^2}{4\pi\varepsilon_0}\cdot\frac{1}{r}$$

$$\frac{p_x^2}{2m} + \frac{p_y^2}{2m} + \frac{p_z^2}{2m}$$

$$\psi(x, y, z) \longrightarrow \psi(r, \theta, \phi) = R(r) \times Y(\theta, \phi)$$

$$\hookrightarrow \vec{L} = \vec{r} \times \vec{p}$$

$Y_l$, $Y_0, Y_1, Y_2 \ldots$

$$\boxed{-\frac{\hbar^2}{2m}\frac{d^2}{dr^2}(r\cdot R) + \left[V(r) + \frac{\hbar^2}{2m}\frac{l(l+1)}{r^2}\right](r\cdot R) = E\,r\,R}$$

$R_{nl}(r)$ $n = 1, 2, 3 \ldots$

$$E_n = -\left[\frac{m}{2\hbar^2}\left(\frac{e^2}{4\pi\varepsilon_0}\right)^2\right]\frac{1}{n^2} = -\frac{13.6\,eV}{n^2}$$

1. $R_{20}(r)\,Y_0(\theta, \phi)$ $E_2 = -3.4\,eV$
2. $R_{21}(r)\,Y_1(\theta, \phi)$ $E_2 = -3.4\,eV$