operators - Θ

Θ is linear transformation

from $\mathcal{H}$ to $\mathcal{H}'$

1) for any $|v\rangle \in \mathcal{H}$,

$\Theta|v\rangle \in \mathcal{H}'$

2) $\Theta|v+u\rangle = \Theta|v\rangle + \Theta|u\rangle$

orthonormal basis

$|e_1\rangle, \ldots, |e_n\rangle$

$\begin{pmatrix} 1 \\ 0 \\ \vdots \\ 0 \end{pmatrix}, \ldots, \begin{pmatrix} 0 \\ \vdots \\ 0 \\ 1 \end{pmatrix}$

$$|v\rangle = \sum_i v^i |e_i\rangle$$

$$\Rightarrow \langle e_j|v\rangle = \sum_i v^i \langle e_j|e_i\rangle$$

$$= \sum_i v^i \delta_{ij}$$

$$= v^j$$

$$|w\rangle = \Theta|v\rangle$$

$$w^i = \langle e_i|w\rangle = \langle e_i|\Theta|v\rangle$$

$$= \langle e_i|\Theta\left(\sum_j v^j |e_j\rangle\right)$$

$$= \sum_j v^j \underbrace{\langle e_i|\Theta|e_j\rangle}_{\Theta_{ij}}$$

matrix elements of Θ

$$= \sum_j v^j \Theta_{ij}$$

$$\langle e_i | \Theta | e_j \rangle = \Theta_{ij}$$

$$|w\rangle = \Theta |v\rangle$$

$$w^i = \sum_j \Theta_{ij} v^j$$

$$\begin{pmatrix} w^1 \\ \vdots \\ w^N \end{pmatrix} = \begin{pmatrix} \Theta_{11} & \cdots & \Theta_{1N} \\ \Theta_{21} & \cdots & \Theta_{2N} \\ & \vdots & \\ \Theta_{N1} & \cdots & \Theta_{NN} \end{pmatrix} \begin{pmatrix} v^1 \\ v^2 \\ \vdots \\ v^N \end{pmatrix}$$

$$w^1 = \Theta_{11} v^1 + \Theta_{12} v^2 + \cdots + \Theta_{1N} v^N$$

$$H|\psi\rangle = E|\psi\rangle$$

operator number

$$\sum_j H_{ij}\psi^j = E\psi^i$$

$$\begin{pmatrix} H_{11} & H_{12} \\ H_{21} & H_{22} \end{pmatrix}\begin{pmatrix} \psi^1 \\ \psi^2 \end{pmatrix} = E\begin{pmatrix} \psi^1 \\ \psi^2 \end{pmatrix}$$

eigenvalue equation

if $|\psi\rangle$ satisfies 〃, then $|\psi\rangle$ is an eigenvector

$$H|\psi\rangle = E|\psi\rangle$$

Time independent Schrodinger eqn

$$H = -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)$$

$$\{|x\rangle\} \qquad x \in \mathbb{R}$$

$$\langle x|\psi\rangle = \psi(x)$$

$$\langle x|H|\psi\rangle = E\langle x|\psi\rangle$$

$$\langle x| -\frac{\hbar^2}{2m}\frac{d^2}{dx^2} + V(x)|\psi\rangle = E\,\psi(x)$$

$$-\frac{\hbar^2}{2m}\frac{d^2\psi}{dx^2} + V(x)\psi(x) = E\psi(x)$$