Introduction to Superposition in Quantum Mechanics

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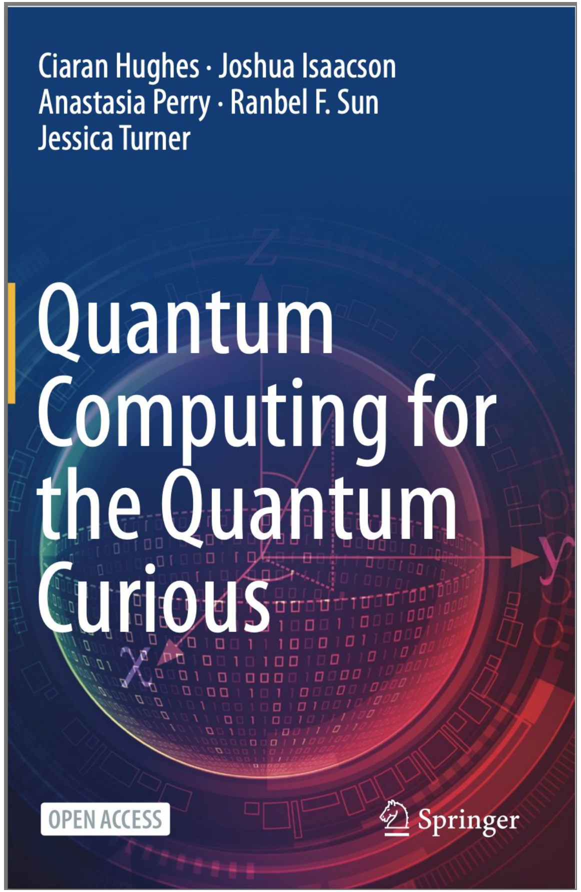
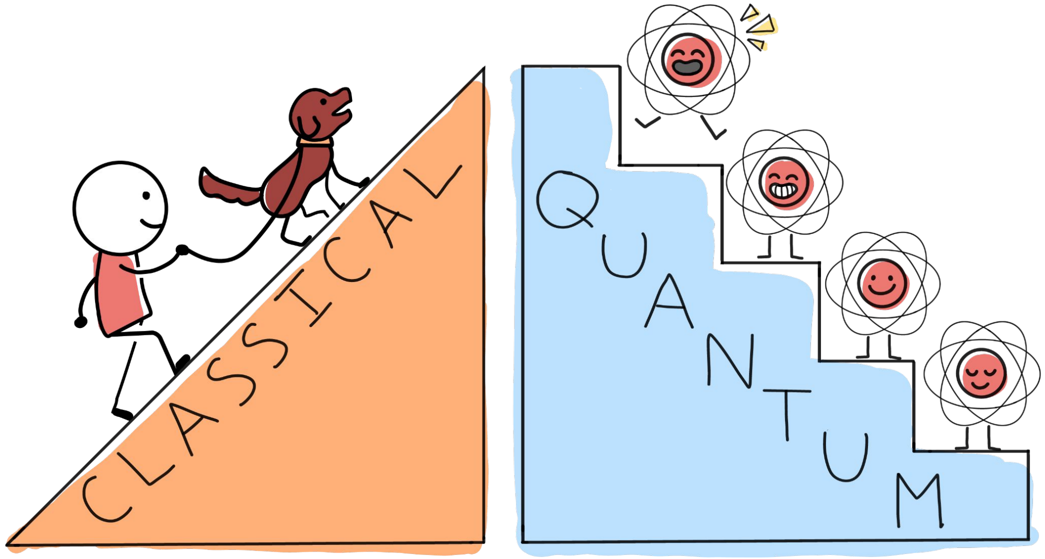
06.23.22

Fermilab



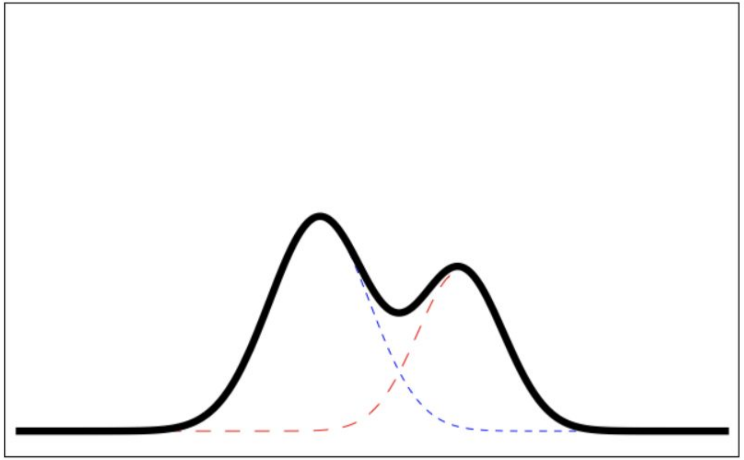
Today we are covering chapter 1 of the course textbook!



Quantum physics, although unintuitive, describes the behaviour of nature at the smallest distance scales

One of the most fundamental pillars of quantum mechanics is the principle of superposition

CLASSICAL SUPERPOSITION



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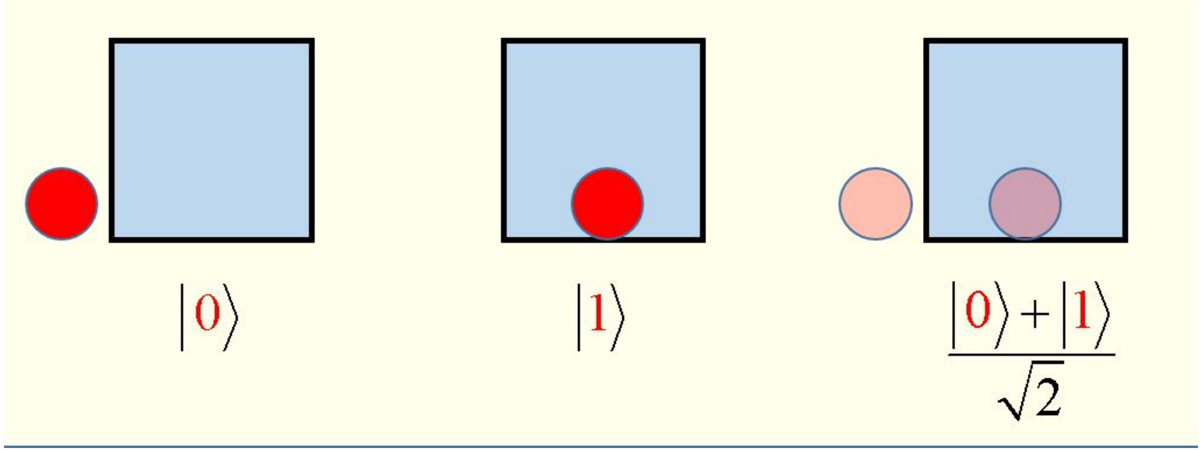
QUANTUM SUPERPOSITION



Topics of today's lecture

1. **Recap: classical superposition**
2. **Quantum vs classical systems**
3. **Quantum superposition**
4. **Measuring quantum systems**

Superposition in classical physics is the stuff we're used to e.g. water waves, waves on a string

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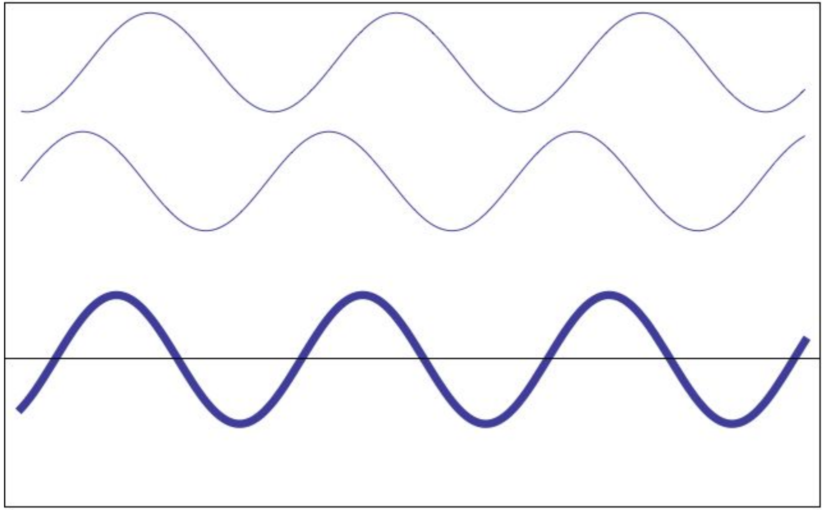

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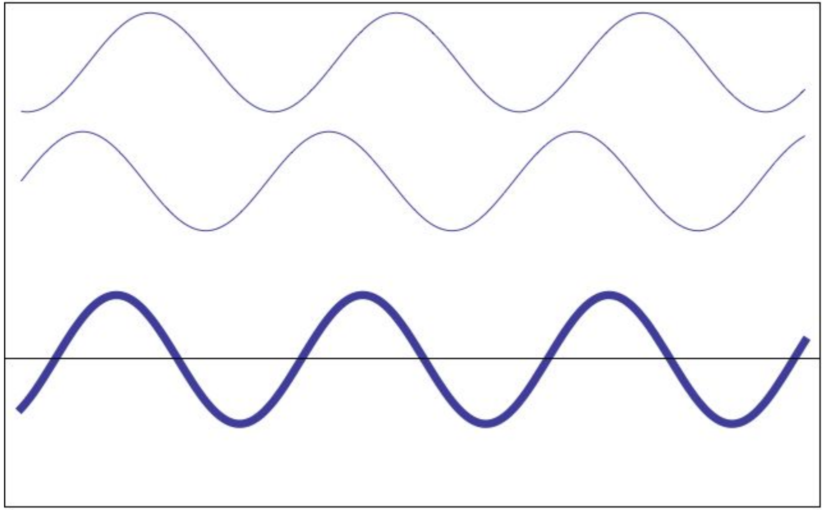
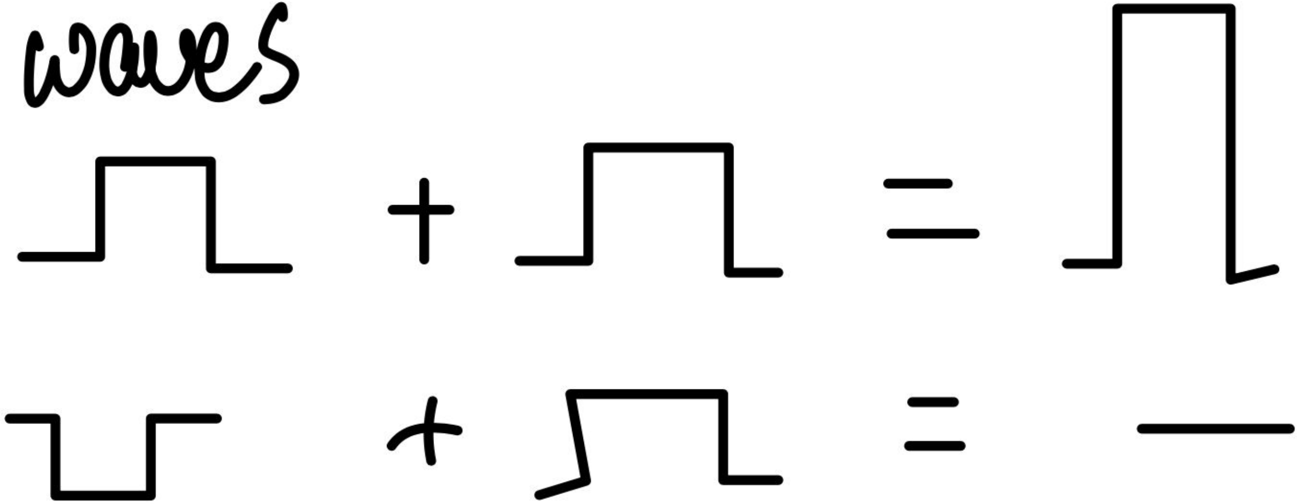


When the waves are exactly out of phase, they cancel!

1. Superposition in Classical Physics

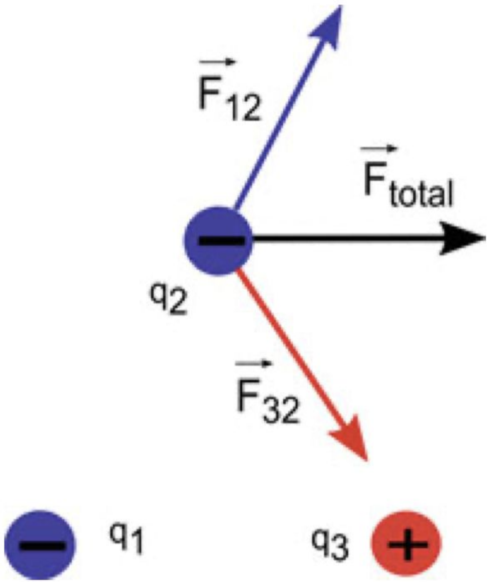
Example

- waves



We simply sum the amplitudes of the waves

The addition of vector quantities is another simple example of classical superposition

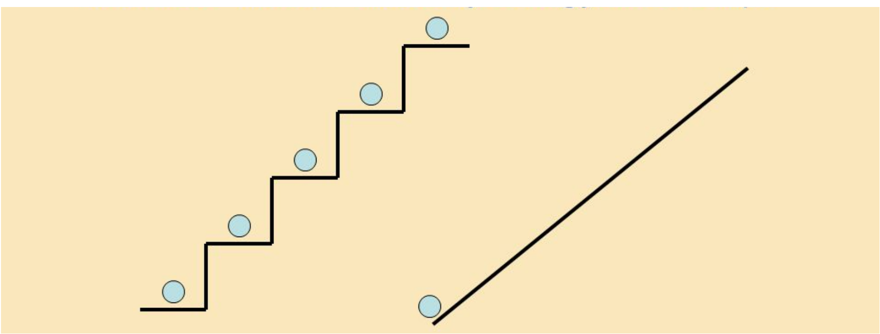


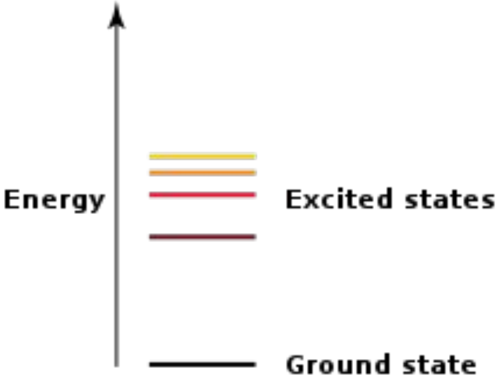
To compute the total force on charge q2 from charges q1 and q3, we sum the forces due to the electric fields

How is a quantum system different to a classical system?

1. **Objects at the very small "particulate" scales exhibit peculiar properties.**

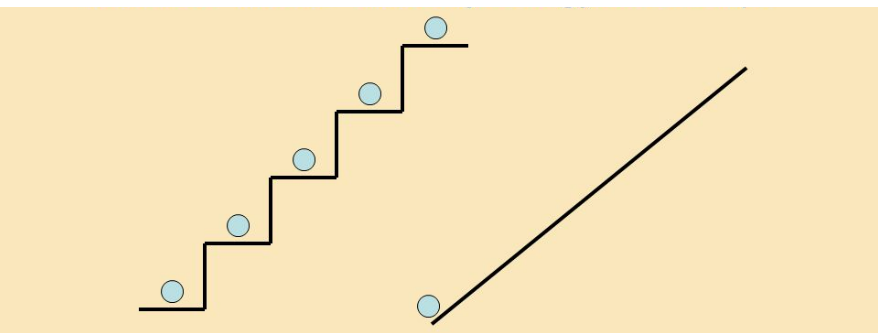
For example, their measurable quantities (energy, momenta, …) are discretely quantised

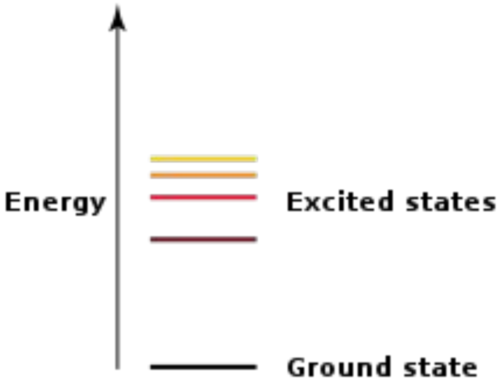




How is a quantum system different to a classical system?

2. A quantum system can be characterised by any of its measurable quantities. For example, we might describe a particle in terms of the energies it can have





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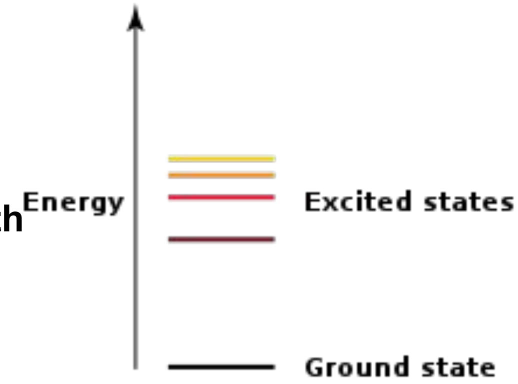
i.e. $E = 0, 1, 2, 3 \ldots hf \rightarrow$ each energy taken by the particle would be described as a **definite state**

The particle may take any of its possible states with some probability!



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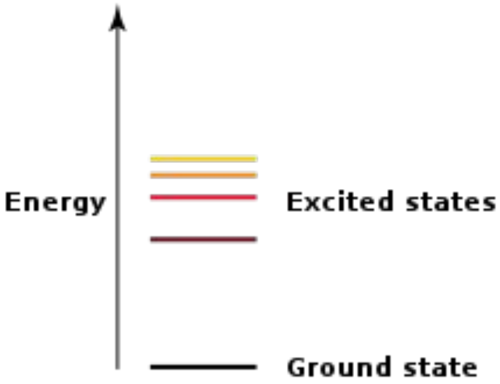
In bra-ket notation, we may write this is as the sum

$$|E\rangle = A_1|E_1\rangle + A_2|E_2\rangle$$

The definite states in this case are E_1 and E_2



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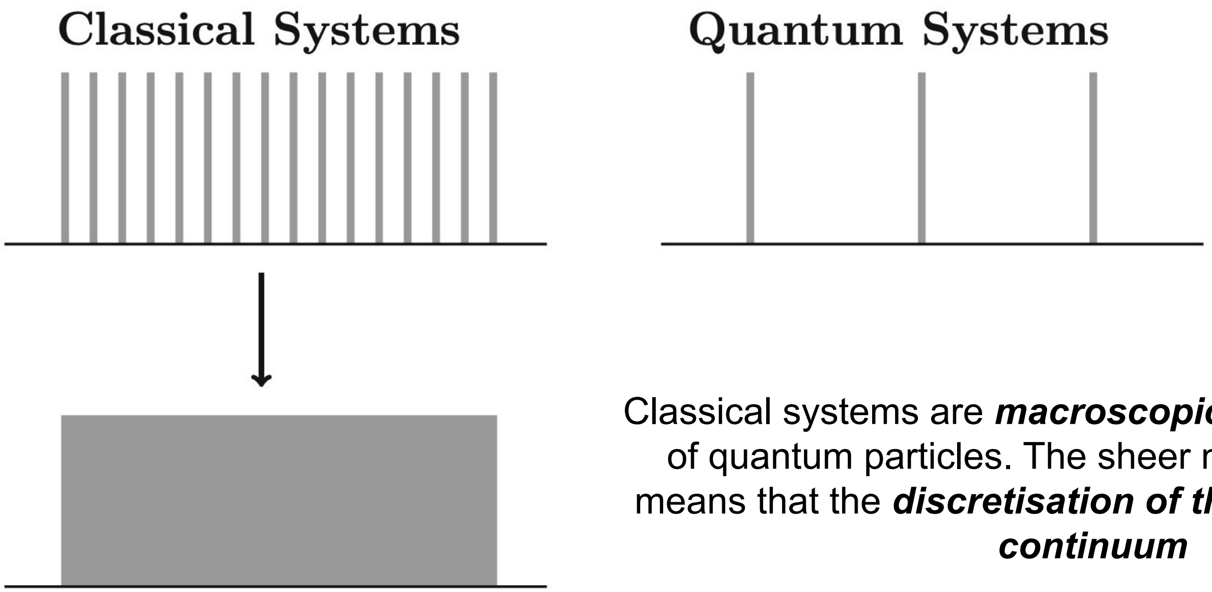

Classical systems are ***macroscopic*** and made up of lots of quantum particles. The sheer number of particles means that the ***discretisation of the states tends to a continuum***

3. Superposition in Quantum Physics

Returning to the two-energy system

$$|E\rangle = A_1|E_1\rangle + A_2|E_2\rangle$$

Until the particle's energy is measured, it is in a **superposition** of the two energy states

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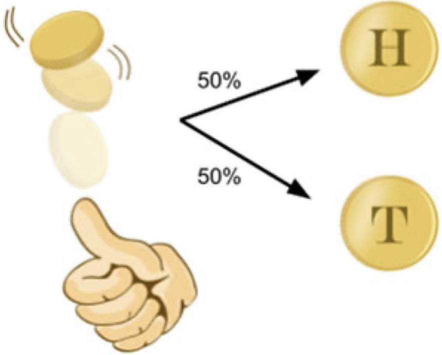
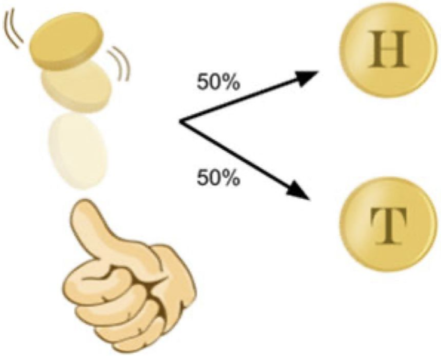
To make this clearer, let's consider a more familiar example



The coin can be written as a superposition of Heads and Tails

$$|\text{coin}\rangle = \frac{1}{\sqrt{2}}|\text{H}\rangle + \frac{1}{\sqrt{2}}|\text{T}\rangle$$

This factor of 1/√2 is a normalization to ensure the total probability of getting either H or T is 1



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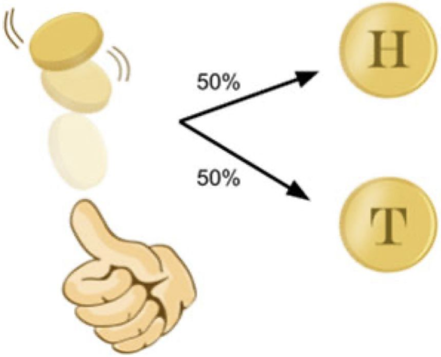


In this example, we know each outcome has a 50% chance, but how do we compute this probability?

3. Superposition in Quantum Physics

Quantum mechanics

let $|\psi\rangle$ be a normalized state (vector in Hilbert space)

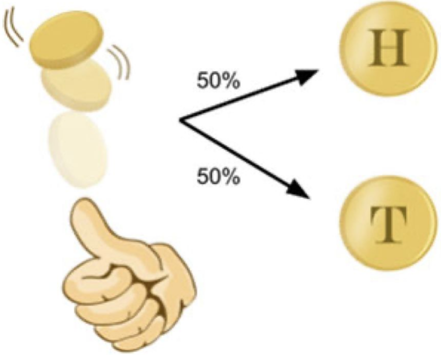
then $|\langle\psi_f|\psi_i\rangle|^2$ = probability (P) that $\psi_i \Rightarrow \psi_f$

state prepared in $|\psi_i\rangle$ will be measured to be in $|\psi_f\rangle$

note $0 \leq P \leq 1$

$\vec{u}\cdot\vec{v} = |u||v|\cos\theta$
$\Rightarrow \frac{\vec{u}\cdot\vec{v}}{|u||v|} \leq 1$

Recall from Mike's lecture how to compute the probability of measuring a state Ψ_f from a state prepared as Ψ_i



In the coin example, $|\Psi_f\rangle = |\mathrm{H}\rangle$ and $|\Psi_i\rangle = |\mathrm{coin}\rangle$

$$|\langle\Psi_f|\Psi_i\rangle|^2 = |\langle\mathrm{H}|\mathrm{coin}\rangle|^2 = \left(\frac{1}{\sqrt{2}}\right)^2 = 50\%$$

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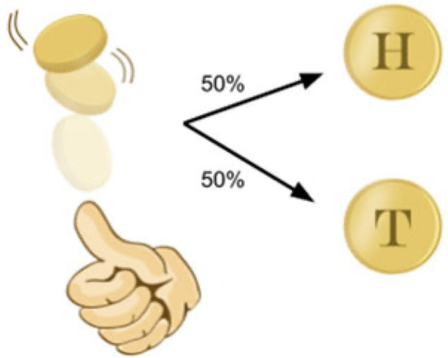
Before we make the measurement of outcome (i.e. while flipping the coin)

do we know what state the coin is in?



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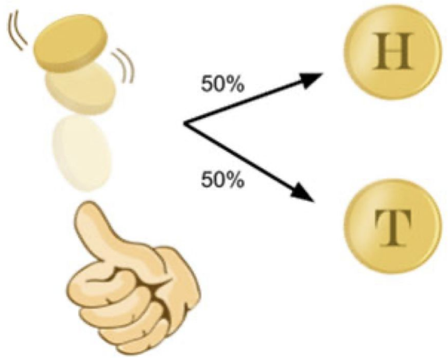
No! The coin is simultaneously in both states…

a quantum superposition



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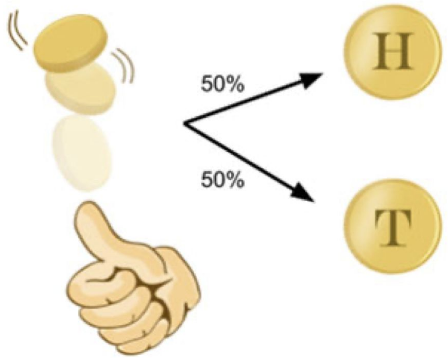

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Measuring the system causes the superposition to break down, i.e. wavefunction "collapse". The measured state exists in a definite state

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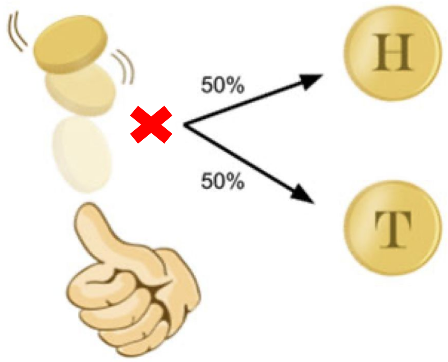
4. Quantum Measurement

We might want to consider a more general example - an unfair coin

The change to the calculation is minimal



$$|\text{coin}\rangle = A|\text{H}\rangle + B|\text{T}\rangle, \; A \neq B$$

since $|\langle\text{coin}|\text{coin}\rangle|^2 = 1 \quad \Rightarrow \quad A^2 + B^2 = 1$

and we have $|\langle\text{T}|\text{coin}\rangle|^2 = B^2$ and $|\langle\text{H}|\text{coin}\rangle|^2 = A^2$

To return to the fair coin example, we can assume that $\text{if } A = B, \; 2A^2 = 1 \Rightarrow A = B = \frac{1}{\sqrt{2}}$

Group problem (breakout room 5 -7 mins +)

1.7 An electron can be in one of two potential wells that are so close that it can "tunnel" from one to the other (see §5.2 for a description of quantum-mechanical tunnelling). Its state vector can be written

$$|\psi\rangle = a|A\rangle + b|B\rangle, \tag{1.45}$$

where $|A\rangle$ is the state of being in the first well and $|B\rangle$ is the state of being in the second well and all kets are correctly normalised. What is the probability of finding the particle in the first well given that: (a) $a = \mathrm{i}/2$; (b) $b = \mathrm{e}^{\mathrm{i}\pi}$; (c) $b = \frac{1}{3} + \mathrm{i}/\sqrt{2}$?

1.8 An electron can "tunnel" between potential wells that form a chain, so its state vector can be written

$$|\psi\rangle = \sum_{-\infty}^{\infty} a_n|n\rangle, \tag{1.46a}$$

where $|n\rangle$ is the state of being in the n^{th} well, where n increases from left to right. Let

$$a_n = \frac{1}{\sqrt{2}}\left(\frac{-\mathrm{i}}{3}\right)^{|n|/2} \mathrm{e}^{\mathrm{i}n\pi}. \tag{1.46b}$$

a. What is the probability of finding the electron in the n^{th} well?
b. What is the probability of finding the electron in well 0 or anywhere to the right of it?

Problems from chapter 1 of

https://www-thphys.physics.ox.ac.uk/people/JamesBinney/qb.pdf

Very good read!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Main Takeaways

1. Quantum physics is unintuitive – we have no macroscopic experience of quantum behaviour!

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2. Particles may exist in quantum superpositions of different states at the same time.

3. Each possible state has a given probability of being observed, but measurement destroys the superposition because only one definite state is seen.