

**A third TDR-era paper:
deep dive into θ_{13} , θ_{23} , δ_{CP} and
parameter degeneracies**

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$P(\nu_\mu \rightarrow \nu_e)$ depends on four parameters in a complicated way

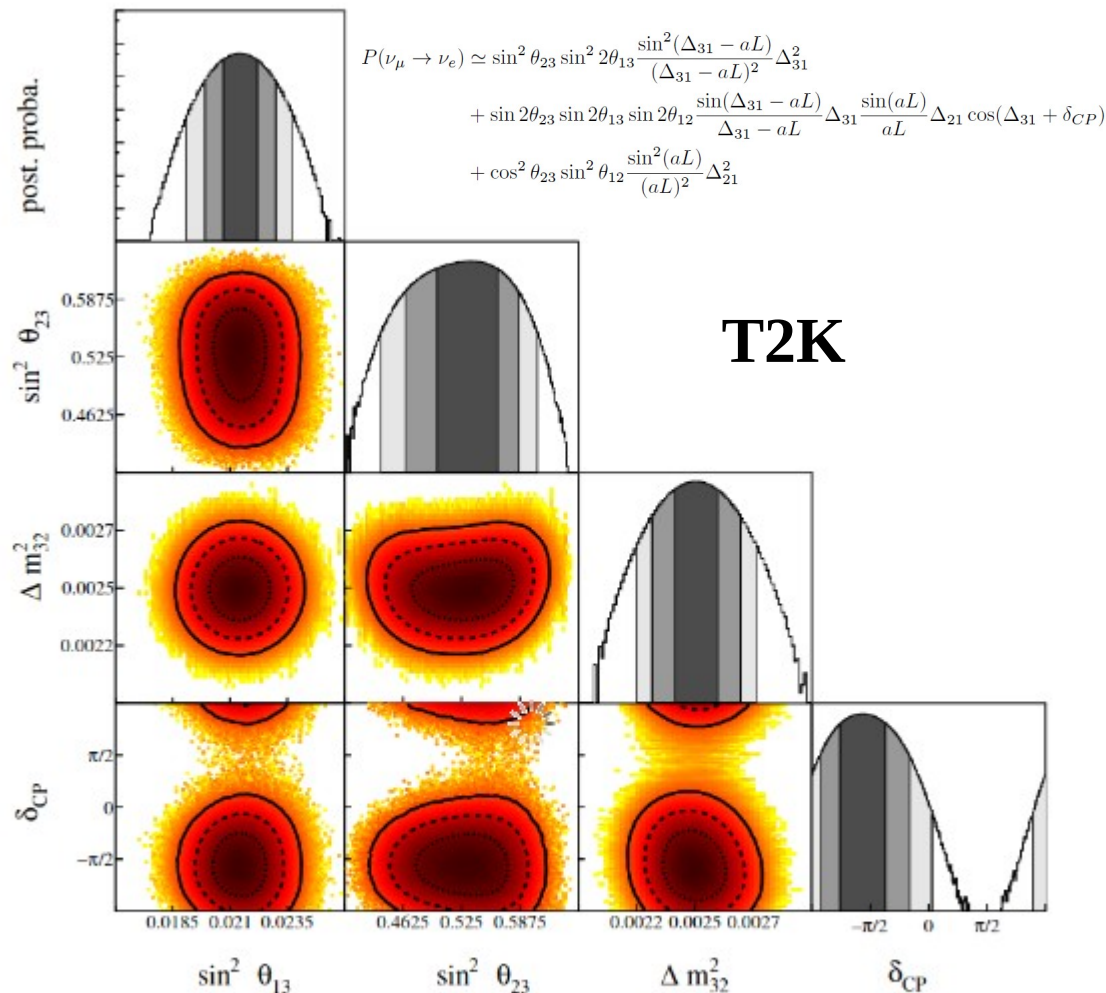
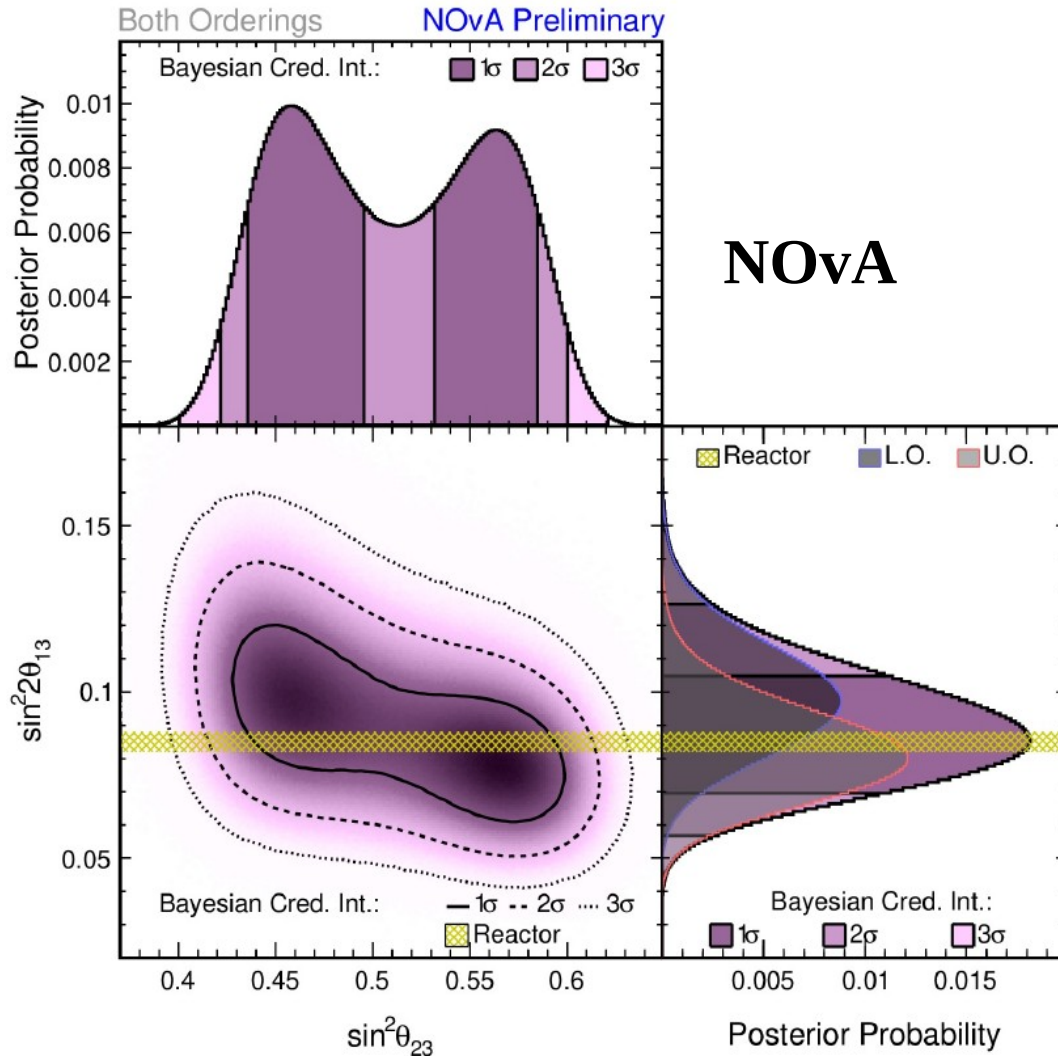


FIG. 64: Posteriors probabilities together with 1, 2 and 3 σ credible intervals for all the oscillation parameters of interest and their combinations in the normal mass ordering. A logarithmic scale is used for the axis corresponding to the posterior probability density, and darker colors correspond to larger probabilities.

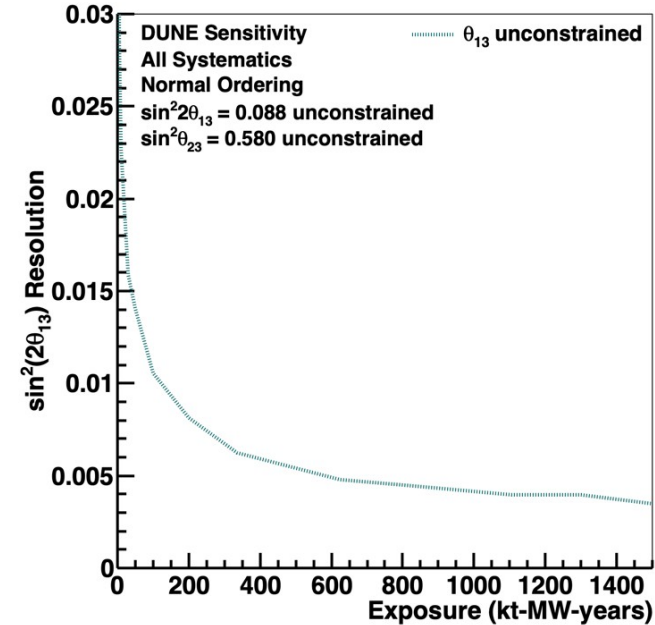
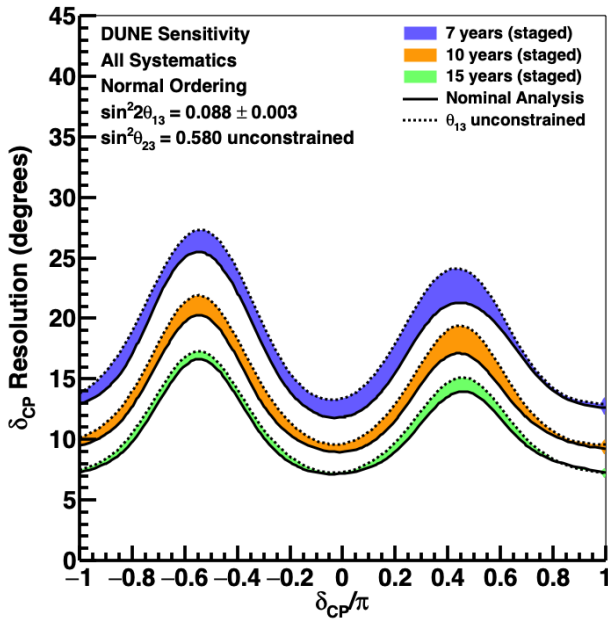
- Three major open questions in neutrino oscillations can be addressed by measuring $\nu_\mu \rightarrow \nu_e$ transitions
 - CP violation (δ_{CP})
 - Mass ordering
 - θ_{23} octant
- These can be measured with accelerator (NOvA, T2K, T2HK, DUNE) and atmospheric (IceCUBE, SK, HK, km3net) neutrinos
- All of these experiments measure something that is sensitive to four parameters (θ_{13} , θ_{23} , δ_{CP} , Δm^2_{atm}) in a complicated way
- All measurements are actually allowed regions in this 4D parameter space, which can then be projected down to 1 or 2 parameters

There are several places where allowed regions can be disjoint



- To leading order, experiments are sensitive to the product $\sin^2 2\theta_{13} \sin^2 \theta_{23}$, so it is common for these parameters to be highly correlated in fits
- ν_μ disappearance constrains $\sin^2 2\theta_{23}$; when it is non-maximal, postfit probability density distributions become bimodal
- This makes it especially challenging for appearance experiments to measure θ_{13}

What is “resolution” when there are disjoint allowed regions?



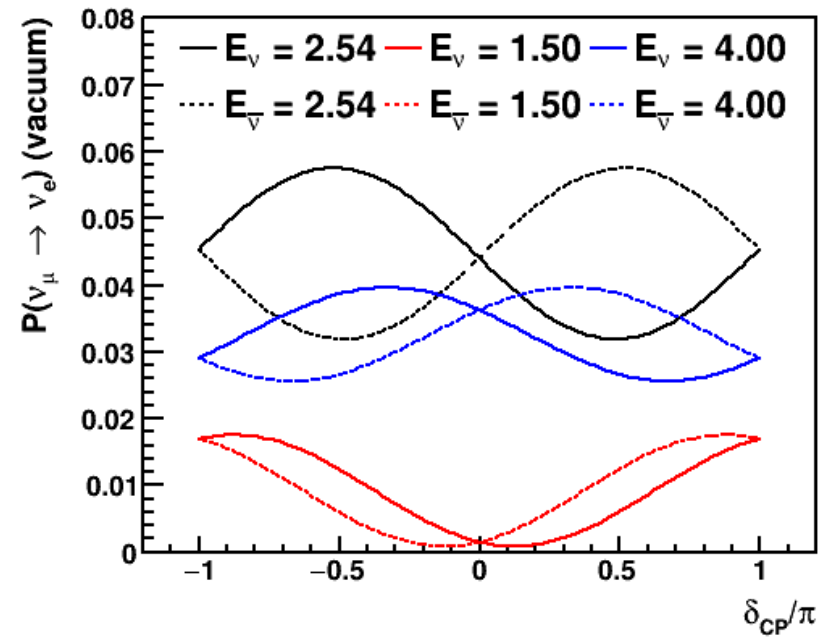
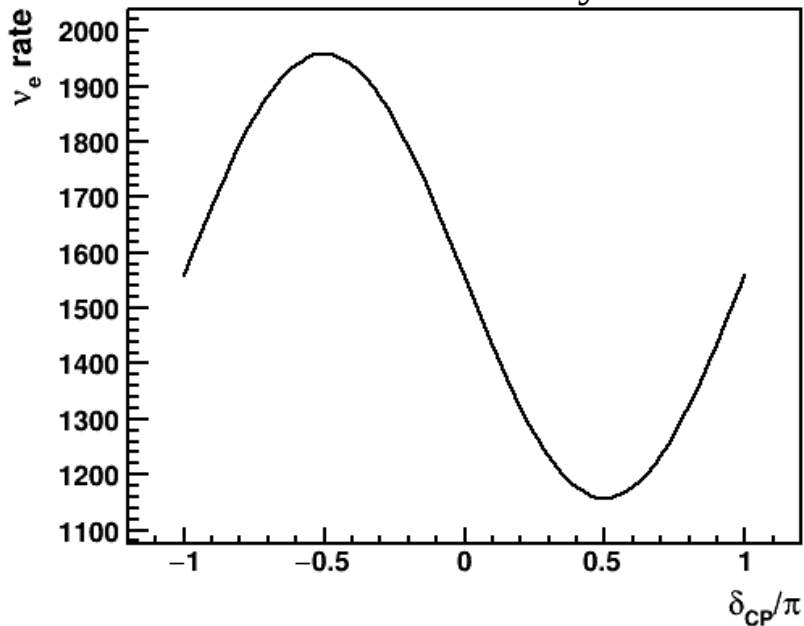
- One can define a parameter resolution as the width of the 68% allowed region, or the width of the $\Delta\chi^2 < 1$ region, but it's not obvious what to do when there are two disconnected regions
- This happened not to be a problem for us at 68%, but it is a problem at 90%

Example: the “sin δ degeneracy”

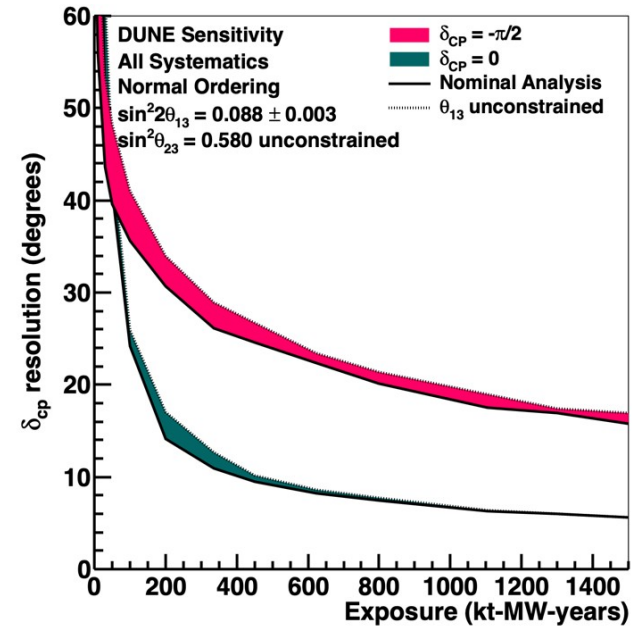
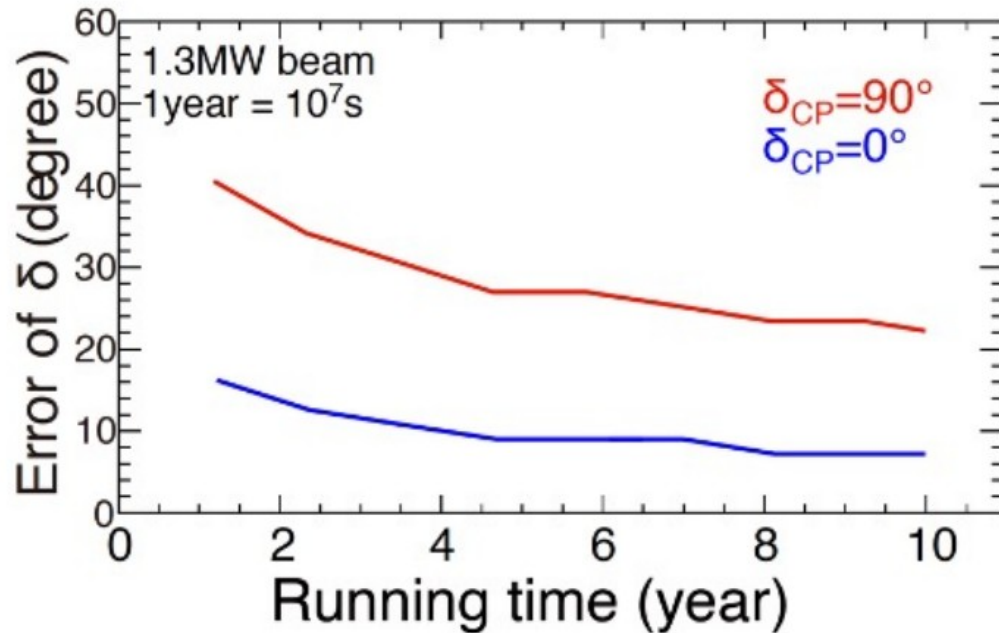
$$\begin{aligned}
 P(\nu_\mu \rightarrow \nu_e) \simeq & \sin^2 \theta_{23} \sin^2 2\theta_{13} \frac{\sin^2(\Delta_{31} - aL)}{(\Delta_{31} - aL)^2} \Delta_{31}^2 \\
 & + \sin 2\theta_{23} \sin 2\theta_{13} \sin 2\theta_{12} \frac{\sin(\Delta_{31} - aL)}{\Delta_{31} - aL} \Delta_{31} \frac{\sin(aL)}{aL} \Delta_{21} \cos(\Delta_{31} + \delta_{CP}) \\
 & + \cos^2 \theta_{23} \sin^2 \theta_{12} \frac{\sin^2(aL)}{(aL)^2} \Delta_{21}^2
 \end{aligned}$$

$\pi/2$ at flux peak
↓

DUNE FHC 3.5yrs



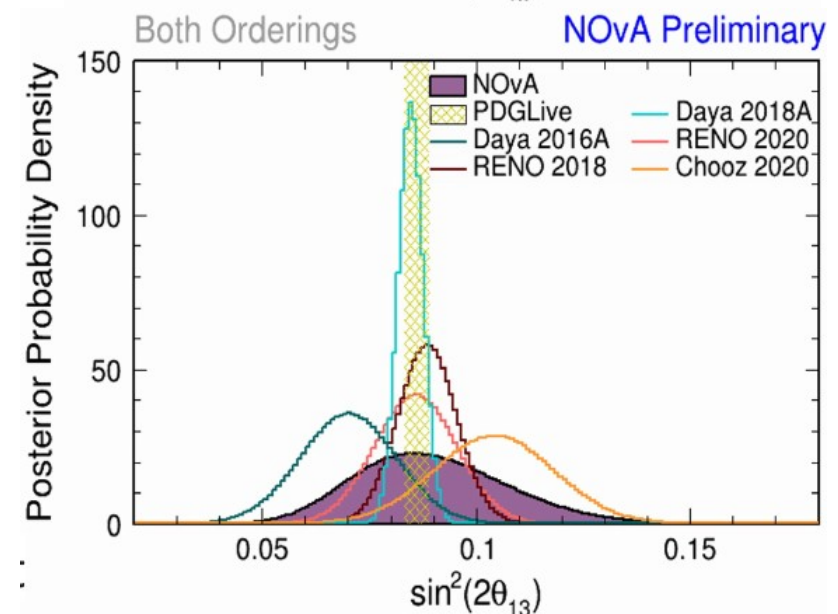
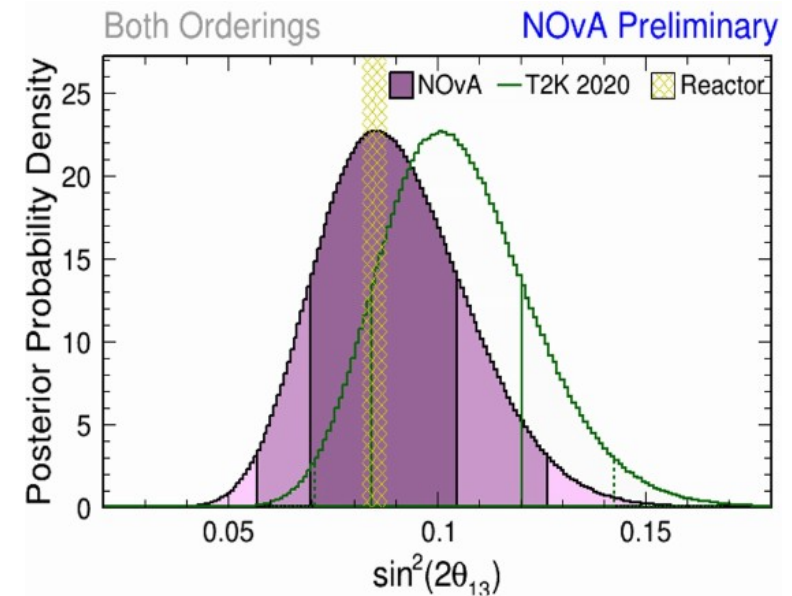
By focusing on 0° and 90° , we are sidestepping this issue



- DUNE has slightly better δ resolution than T2HK at 0 and 90 , which are the points where shape matters the least
- I strongly suspect that T2HK has almost no ability to distinguish 45° from 135° , whereas DUNE can
- T2HK has never said what “error of δ ” means, or shown any plots of resolutions at other true values of δ (this issue doesn’t affect CPV significance)
- It is likely that δ is not a multiple of $\pi/2$, and it is bad for DUNE/T2HK comparisons that the conversation is so focused on maximal CPV

θ_{13} : cross check and non-unitarity

- DUNE has ~ 0.004 resolution to $\sin^2 2\theta_{13}$, which is roughly twice as good as T2HK reports
- Daya Bay resolution is 0.003, and has no meaningful cross-check currently (other measurements are $\sim 10x$ worse)
- A deviation between θ_{13} from appearance and disappearance would be indirect evidence of PMNS non-unitarity \rightarrow DUNE is the best way to do this



Proposal

- Choose a few especially challenging true oscillation points (NO, lower octant, $\delta \sim -\pi/4$?) to run a large number of throws, and look at the 3D or 4D distribution of best fit values
- Look at the 1σ -90%- 3σ allowed regions for δ , θ_{13} , θ_{23} ; at least the 90% and 3σ will have tricky correlations
- Look at DUNE-Daya Bay non-unitarity test for different values of θ_{23} , for what region(s) of parameter space are we sensitive to non-unitarity
- I think this could make a third paper where we purposely seek out the hard regions and show how DUNE's spectral information ultimately resolves them

A little about me

- Undergraduate student at Carleton College, MN
- REU student at the University of Rochester
- Previously worked with the NSD neutrino group at Berkeley Lab



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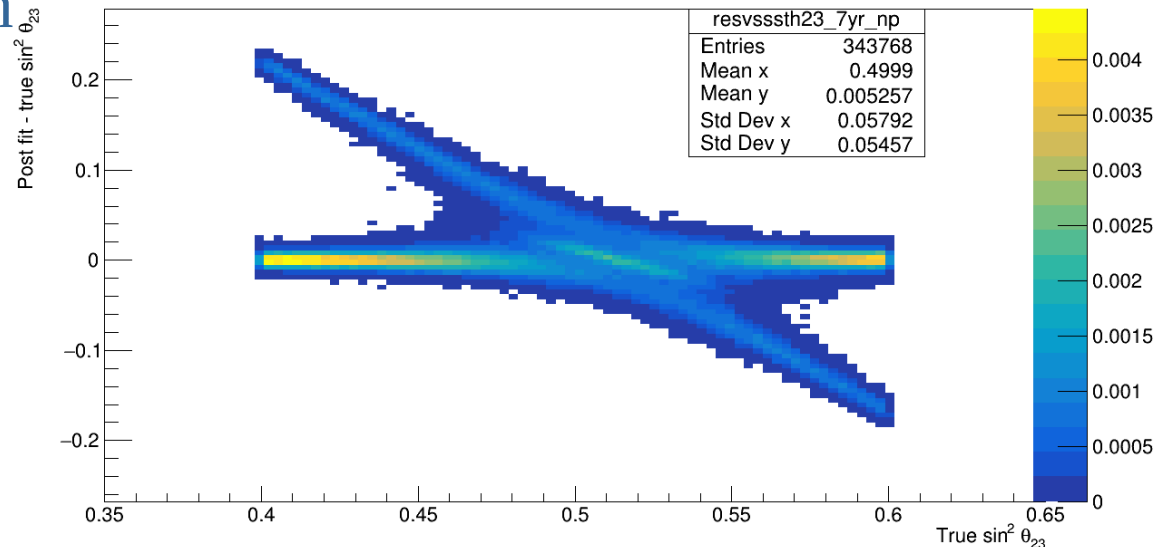
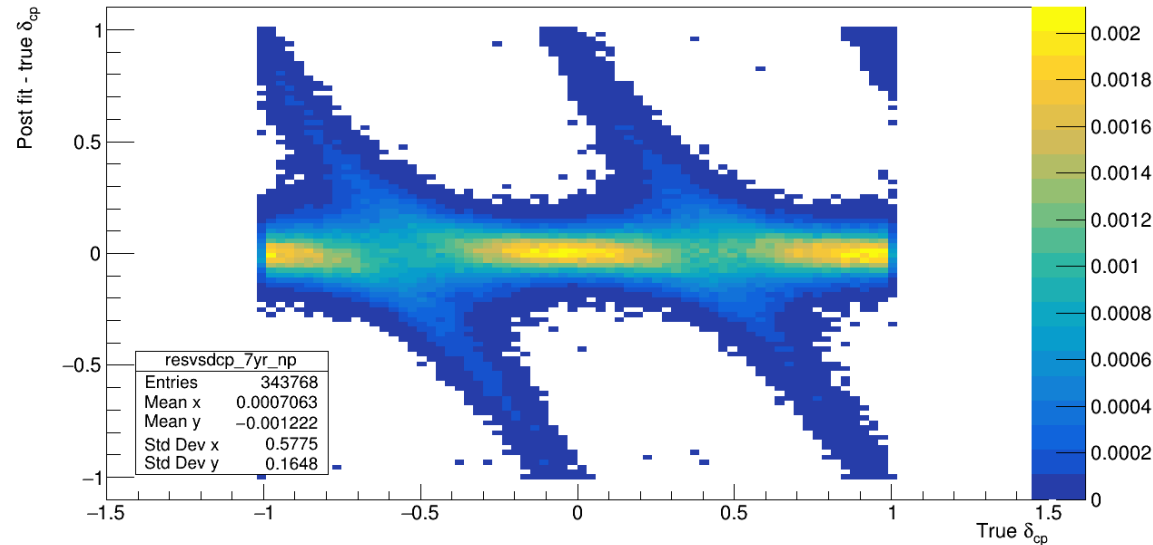


Why this study?

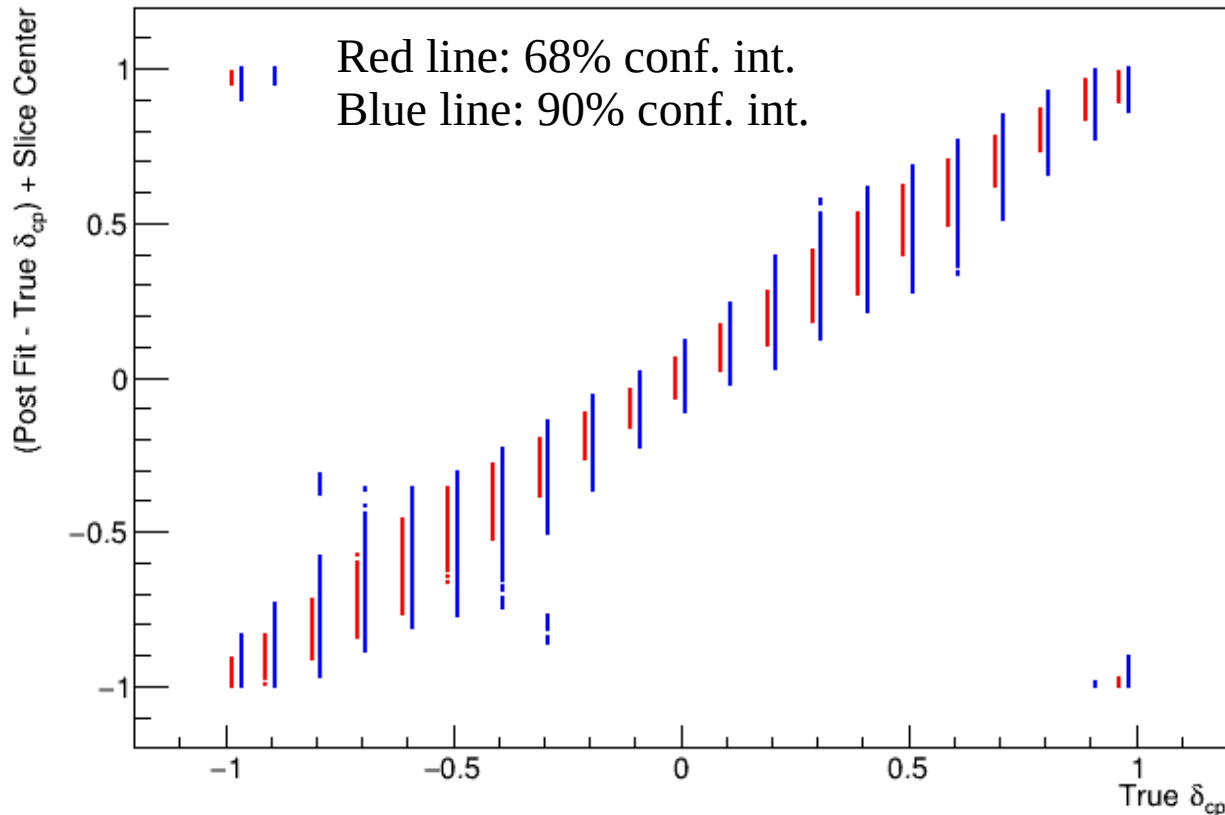
- Previous measurements of oscillation parameters have been treated independently, omitting possible correlations.
- As we develop the next generation of precision neutrino experiments, including DUNE, these correlations become significant
- Understanding how DUNE fits of oscillation parameters are affected by these correlations enables more accurate evaluation of DUNE's sensitivity to new physics.

Initial TDR Analysis

- Resolution plots using LBL TDR analysis data.
- TOP: δ_{cp} post fit (pf) – true vs true
 - Sine dependence at flux peak---high precision at minimal cp violation, with $0-\pi$ octant degeneracy
- BOTTOM: $\sin^2\theta_{23}$ pf – true vs true
 - Octant flip at non-maximal mixing

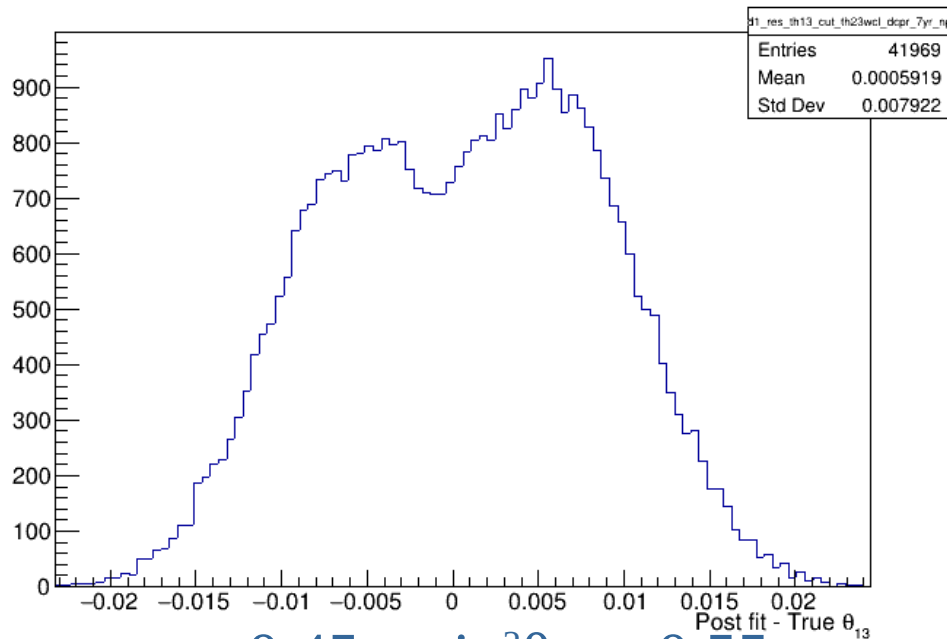


δ_{cp} octant degeneracy

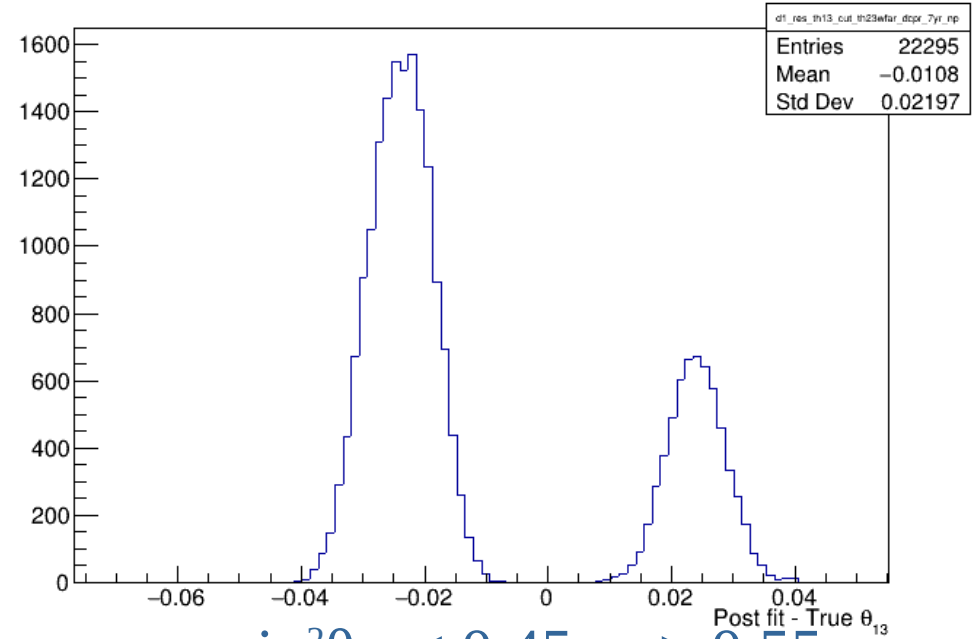


- $\nu_\mu \rightarrow \nu_e$ oscillation eqn has sine dependence on δ_{cp} (at flux peak)
 - Higher precision at minimal cp violation ($-\pi, 0, \pi$)
 - Octant degeneracy
- 90% confidence interval captures degeneracy
 - Interval asymmetry about δ_{cp} slice value
- Octant asymmetry about $\delta_{cp} = 0$

θ_{23} octant flip effect on θ_{13}



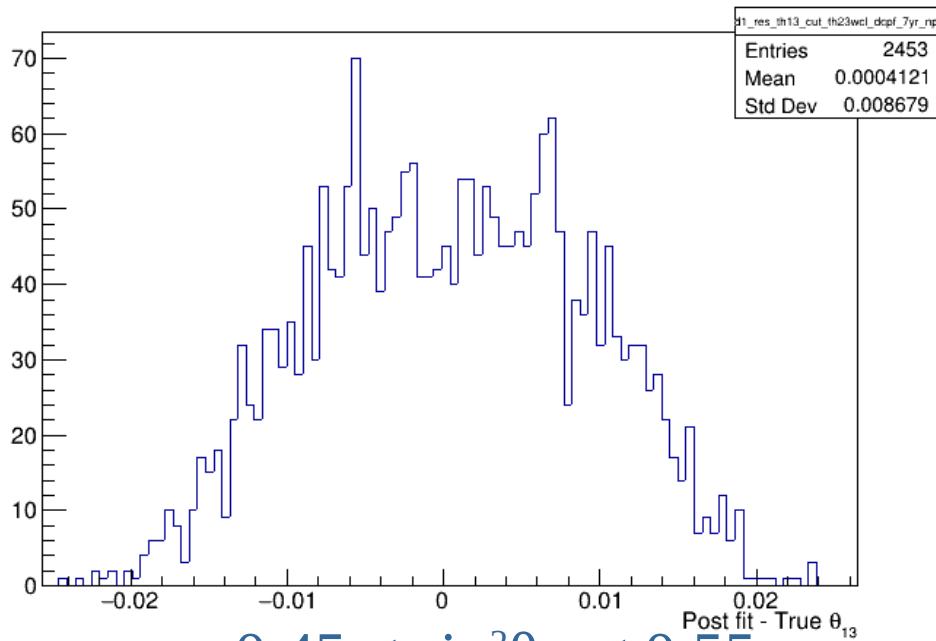
$0.45 < \sin^2\theta_{23} < 0.55$



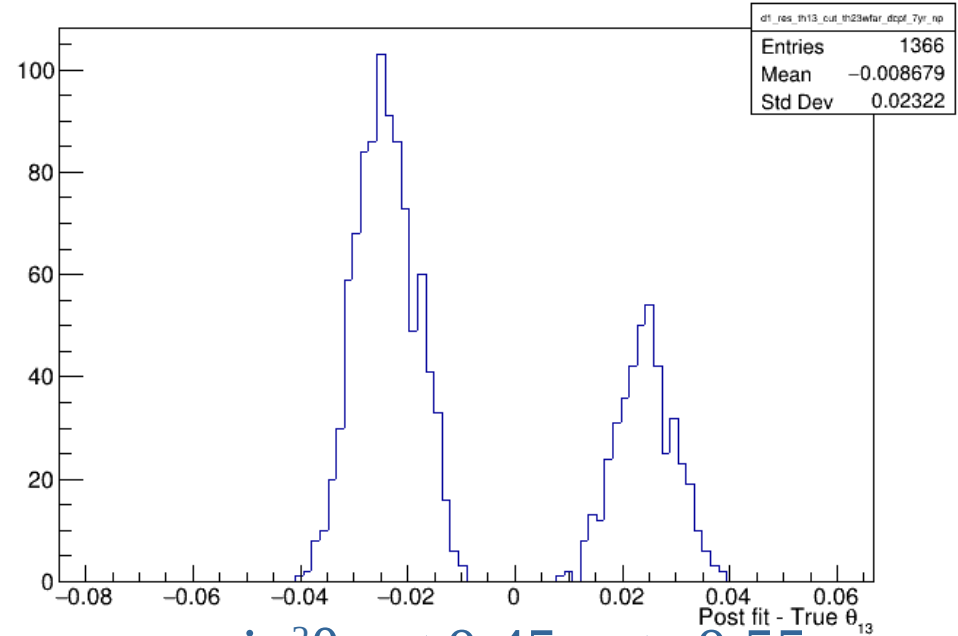
$\sin^2\theta_{23} < 0.45$ or > 0.55

- Above: θ_{13} Post fit - true distributions, θ_{23} measured in wrong octant
- θ_{23} octant error leads to bimodality in θ_{13} measurement
- Less maximal θ_{23} = greater bimodality
- Asymmetry between modes on right plot: what favors under- vs over-estimation?

δ_{cp} effect?



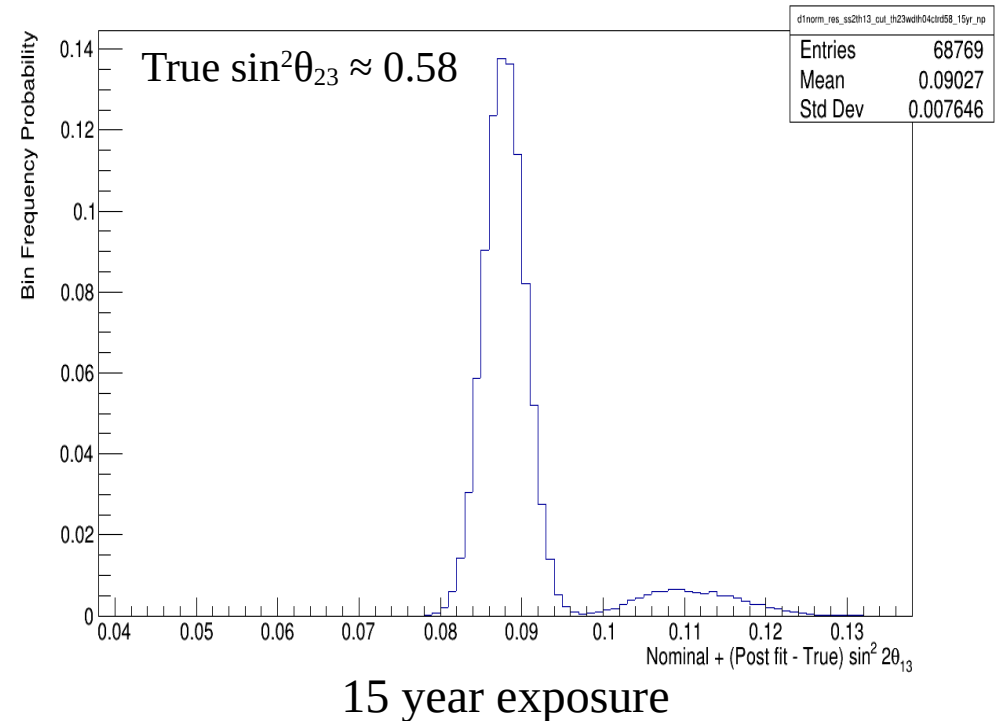
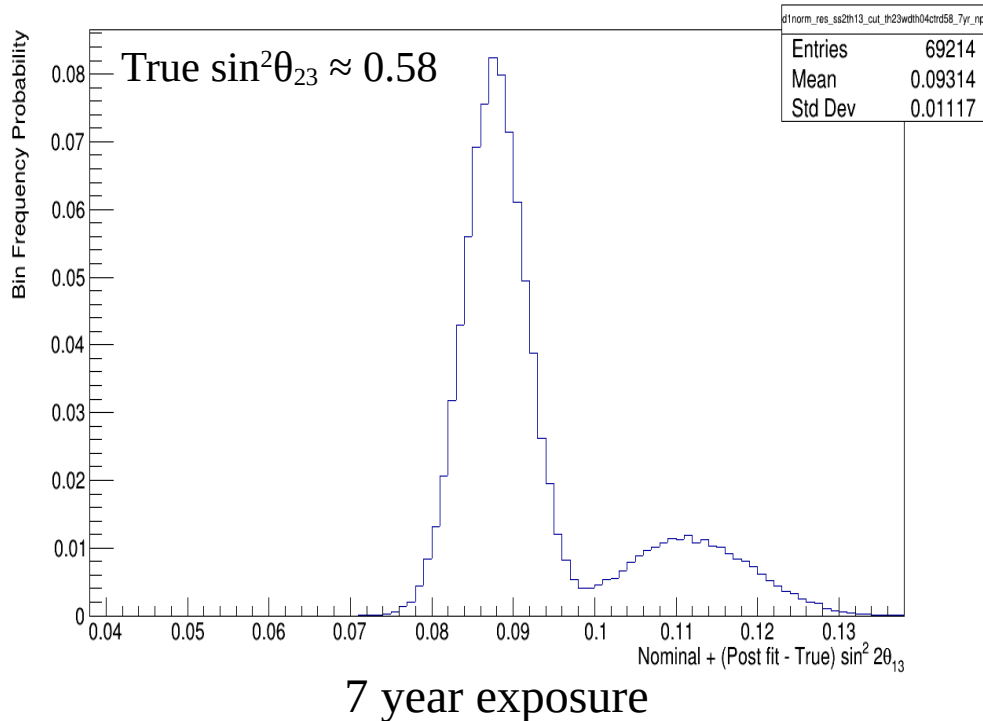
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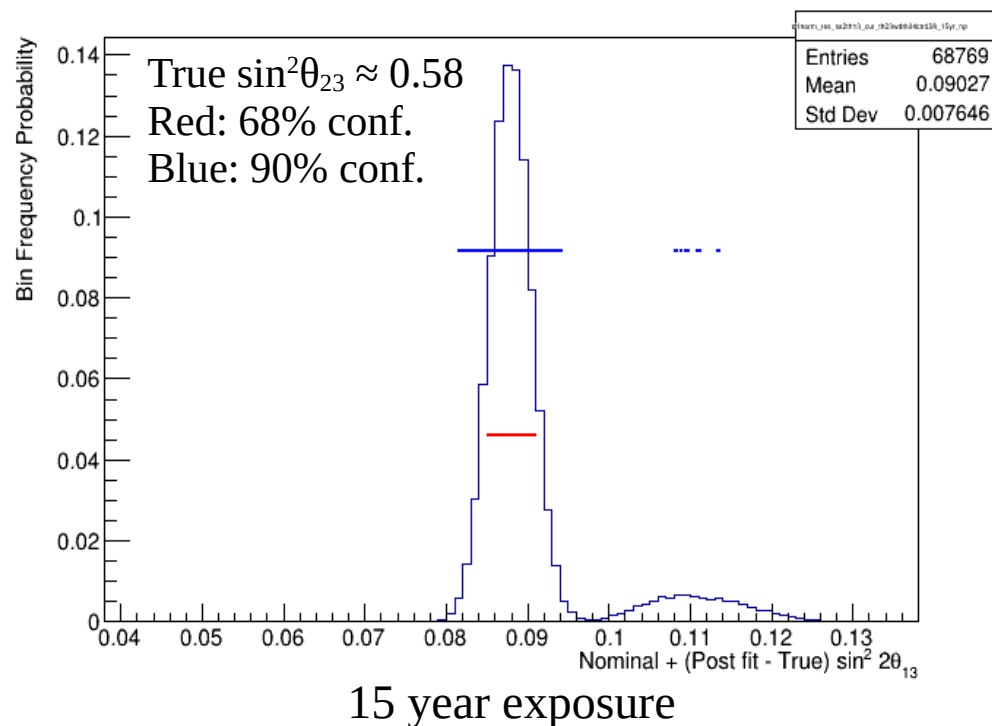
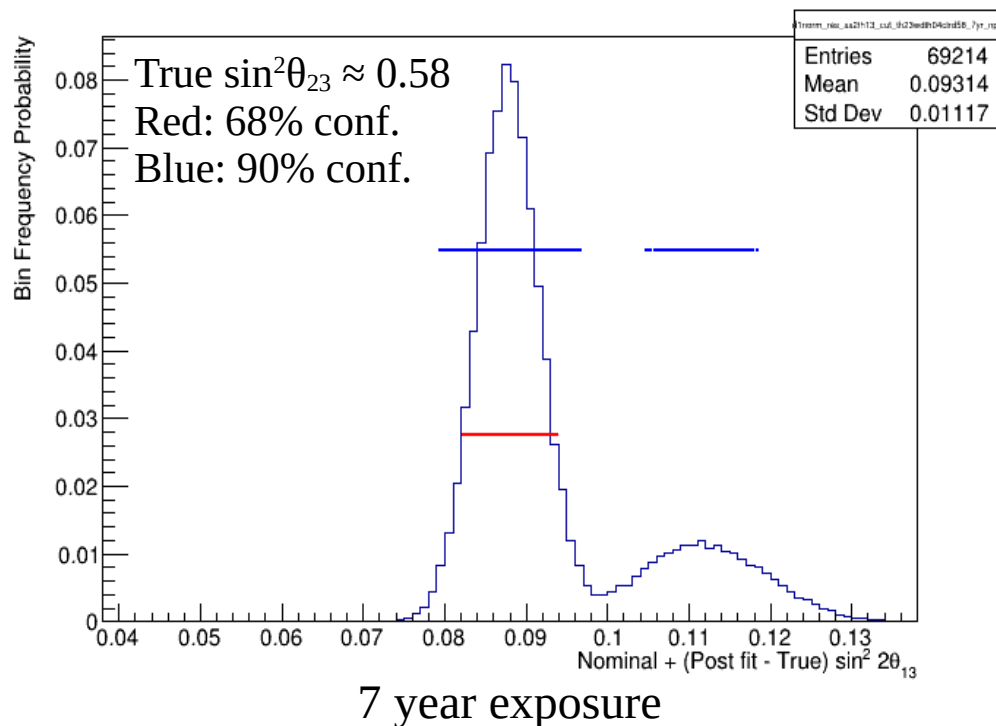
- Flipping δ_{cp} appears to be uncorrelated with θ_{13} measurement
- δ_{cp} degeneracy appears to be independent of θ_{13} - θ_{23} correlation

θ_{23} octant flip effect on θ_{13}



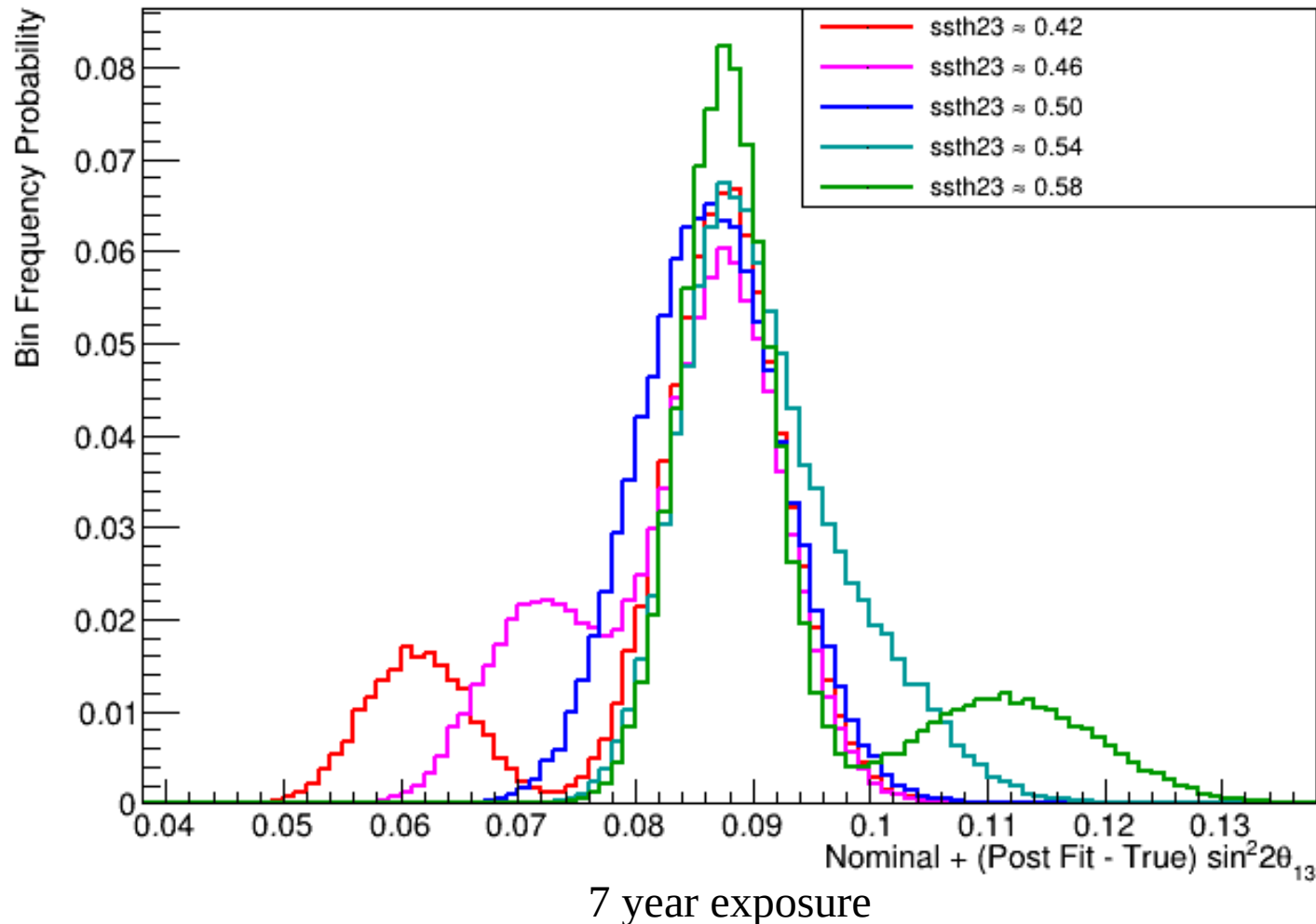
- $\sin^2 2\theta_{13}$ postfit distribution shown at fixed $\sin^2\theta_{23} \approx 0.58$.
- Underestimated $\sin^2\theta_{23}$ corresponds to overestimated $\sin^2 2\theta_{13}$, gap between modes due to disfavored maximal θ_{23}
- Increasing exposure decreases octant error significance

θ_{23} octant flip effect on θ_{13}



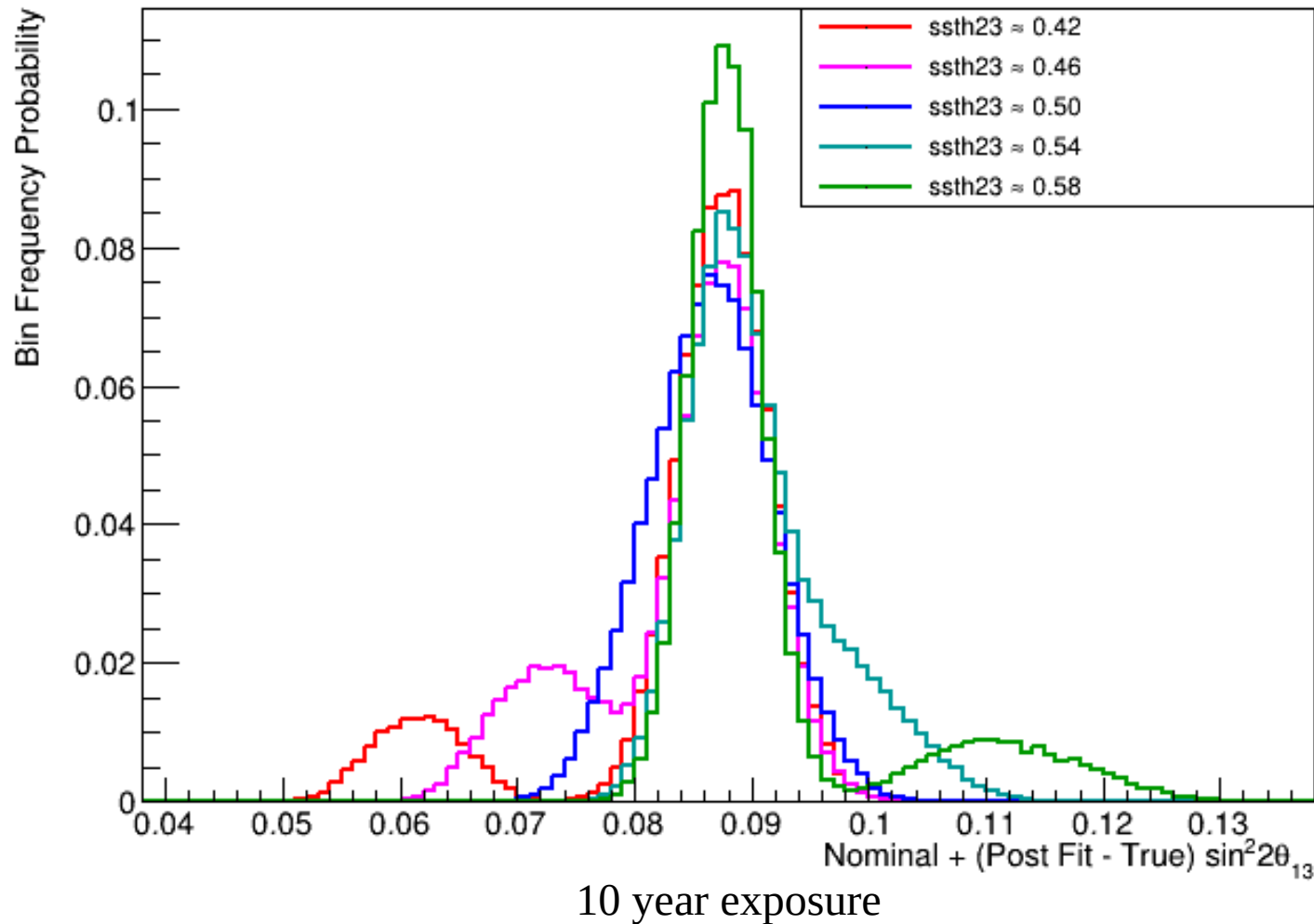
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PF θ_{13} distribution depends on θ_{23}



- Narrower true mode peak, greater true-error mode separation at non-maximal θ_{23}
- Broader true mode peak, no bimodality at maximal θ_{23}

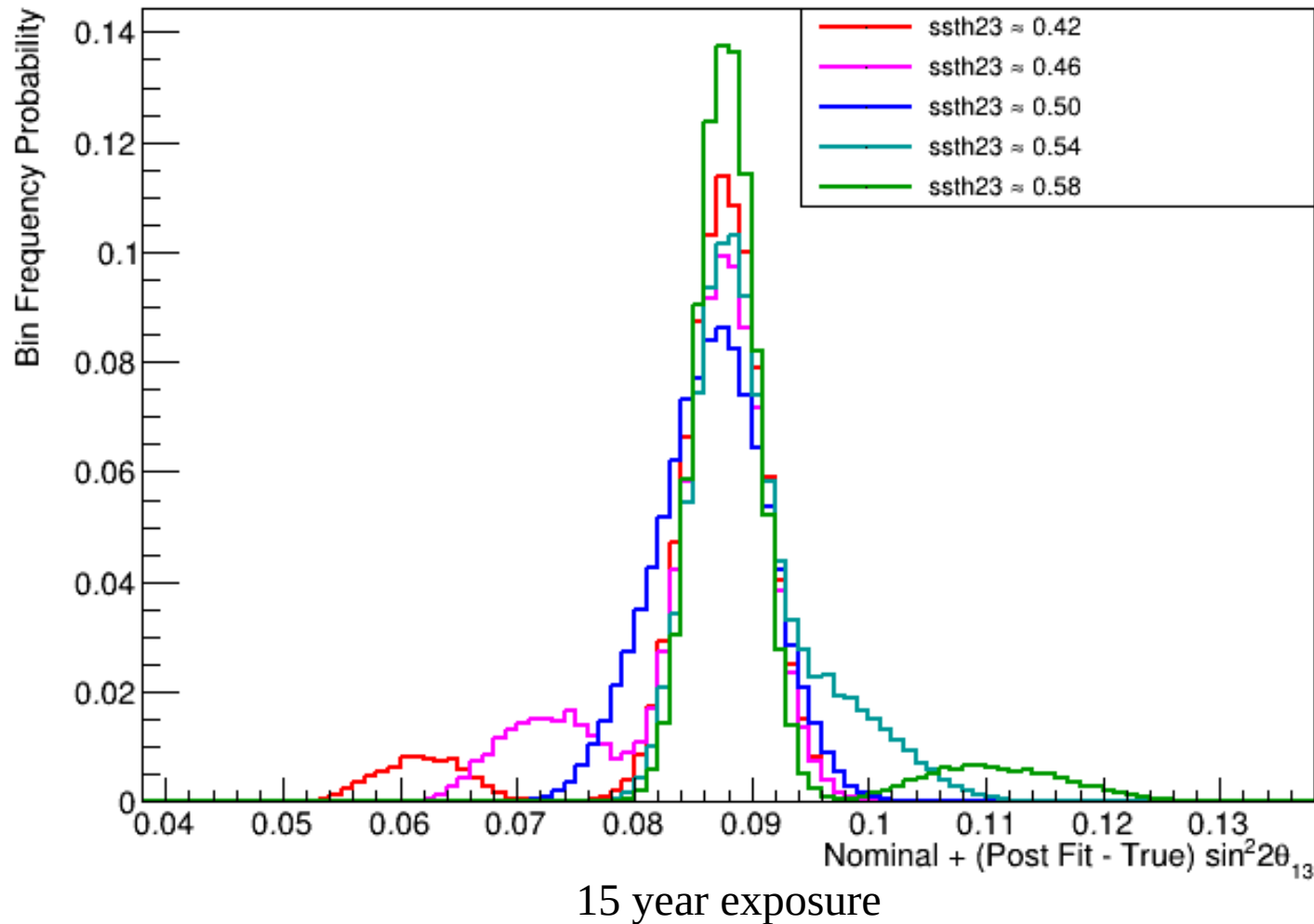
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Relative size of error mode decreases with exposure

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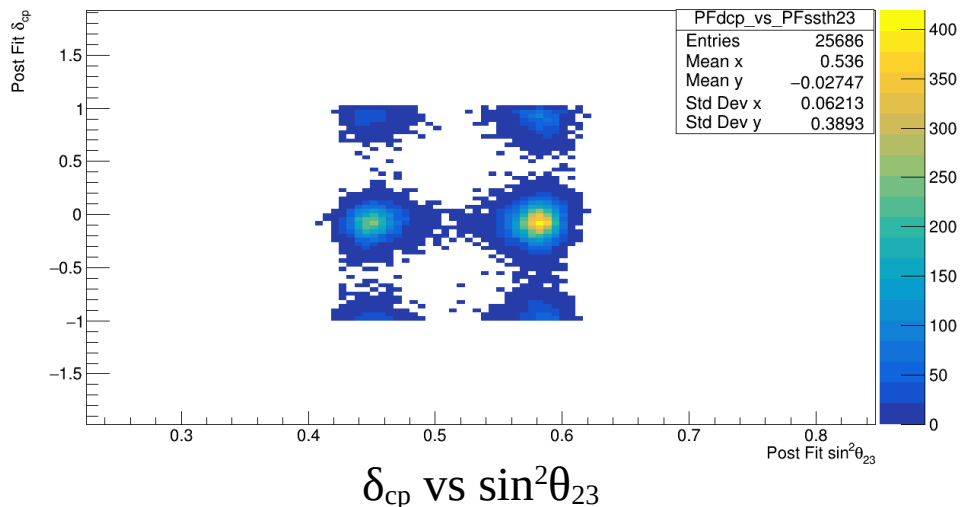
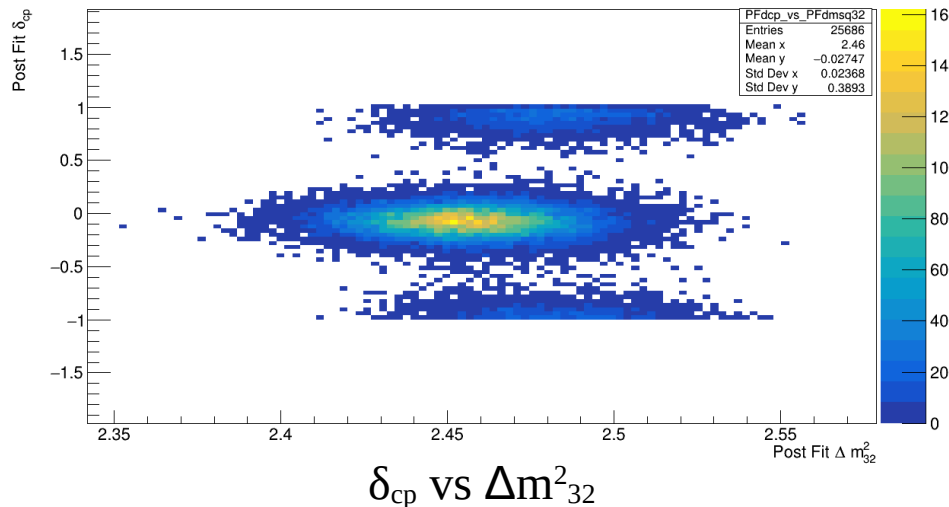
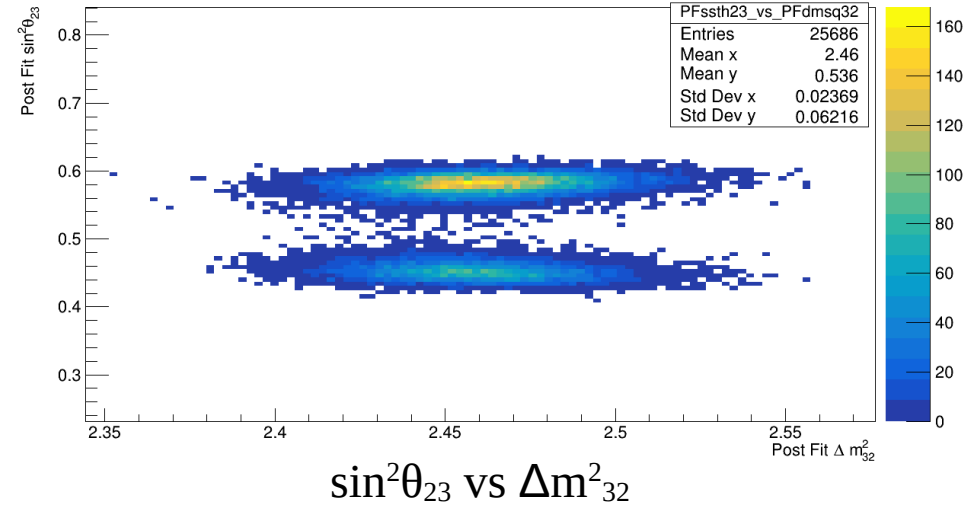
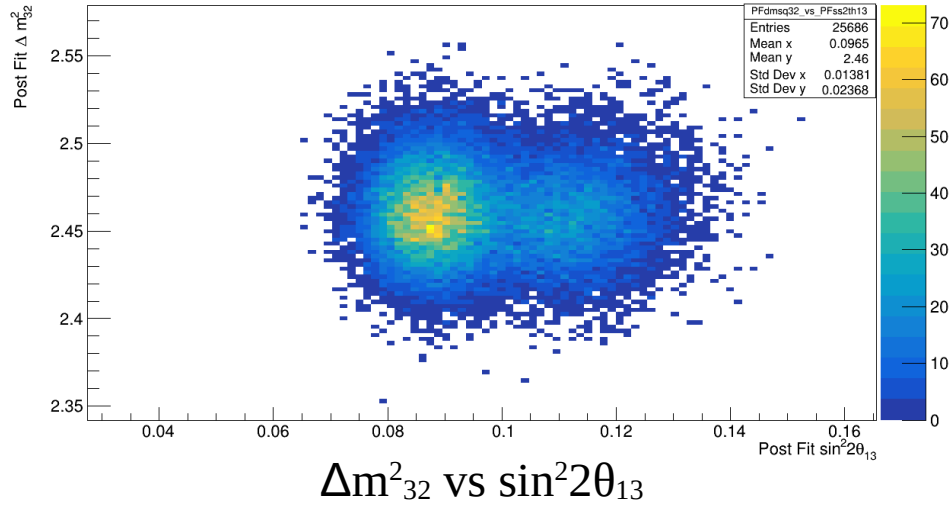
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First single point analysis

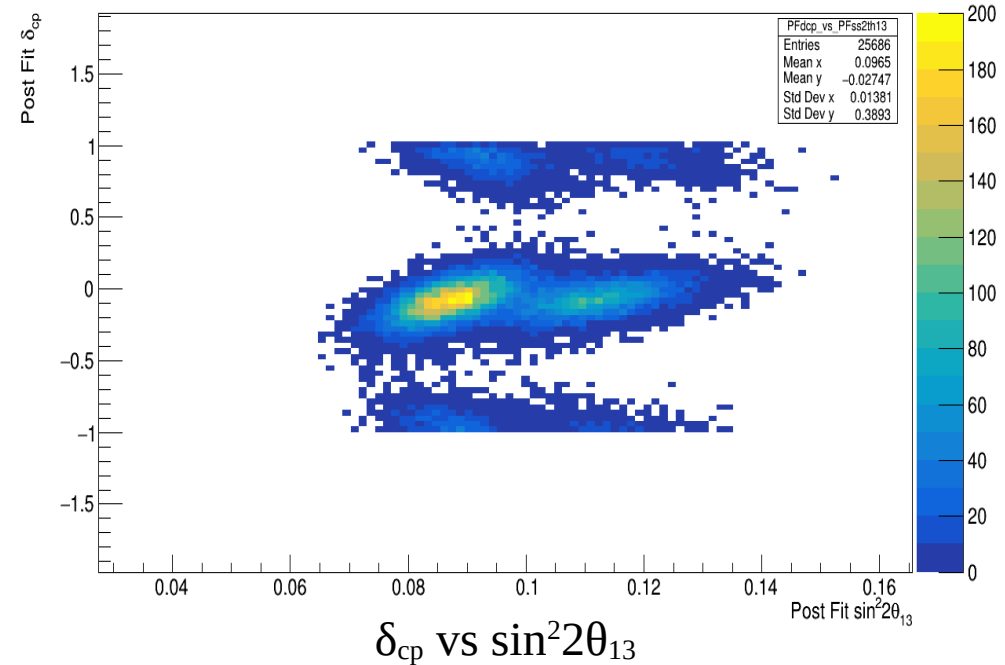
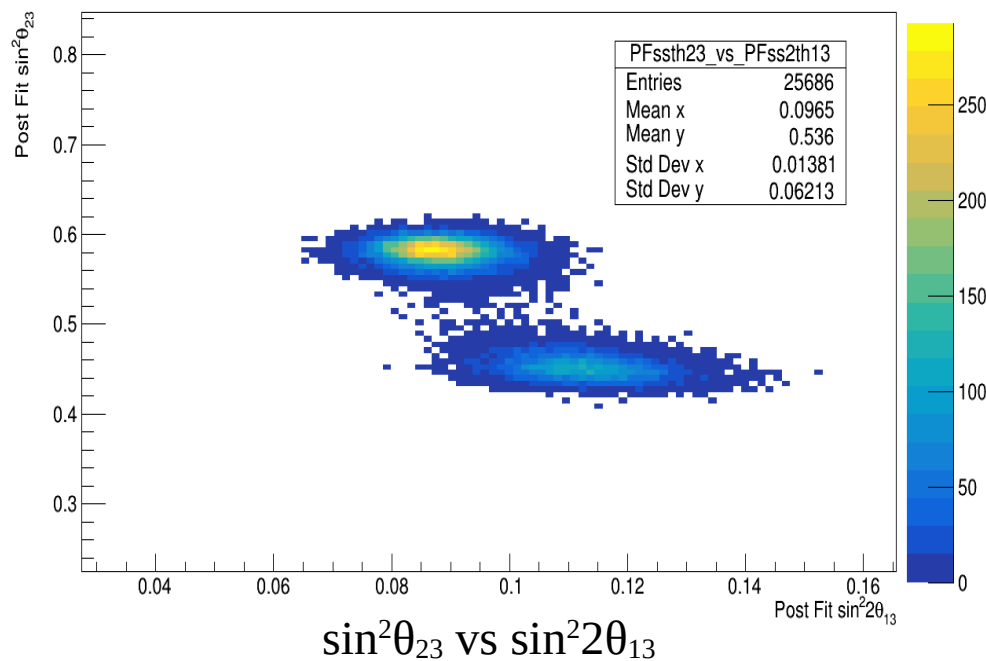
- Toy analysis fixed at single point in 4d true parameter space

Parameter	True Value
$\sin^2 2\theta_{13}$	0.088
Δm^2_{32}	$2.45 \times 10^{-3} \text{ eV}$
$\sin^2 \theta_{23}$	0.58
δ_{cp}	-0.08π

Parameters that appear weakly correlated or uncorrelated

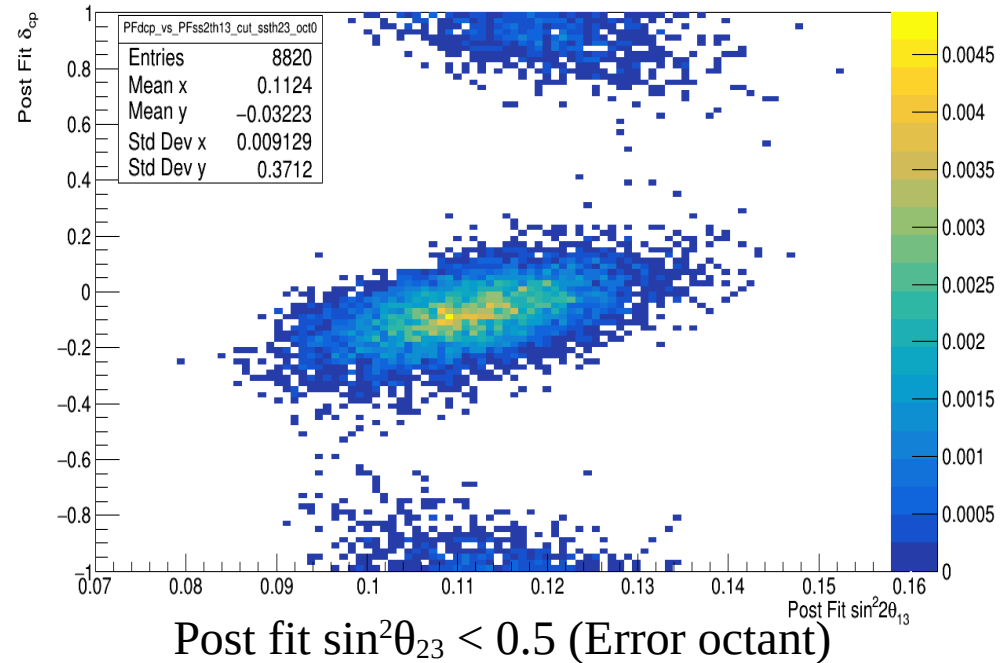
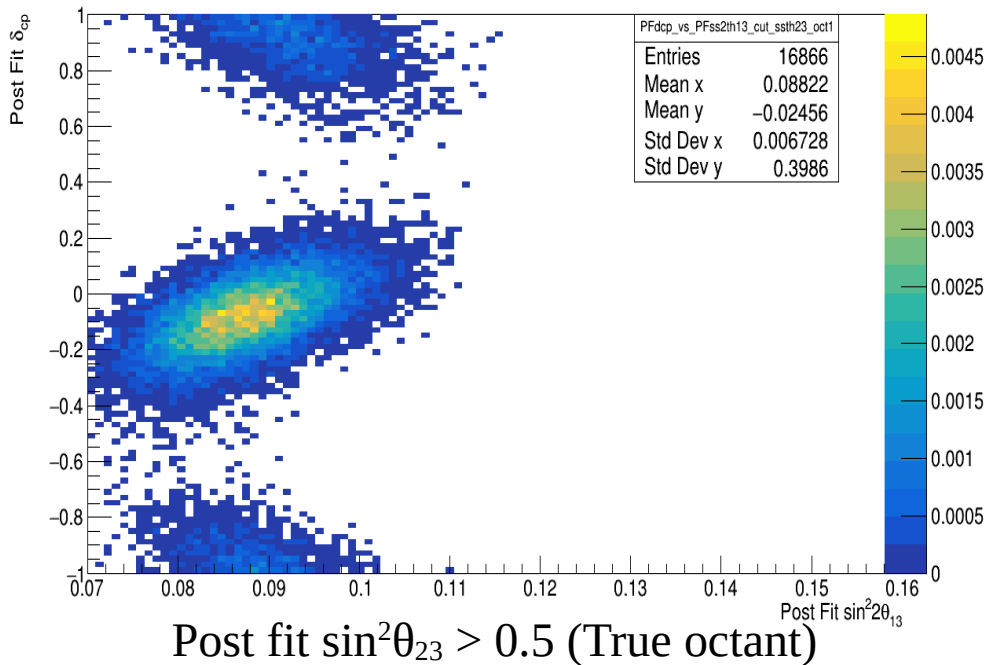


Candidate correlated parameters



- Left: θ_{13} - θ_{23} octant error
- Right: Octant error visible in θ_{13} , correlation due to dominant ν_e appearance inverse proportionality to δ_{cp}

Fixed point δ_{cp} vs $\sin^2 2\theta_{13}$



- Correlation strength weakly dependent on θ_{23} octant
 - Stronger in true octant
 - Allowed populations closer together in true octant

Next steps

- Investigate DUNE θ_{13} sensitivity to indirect test of PMNS non-unitarity when combined with Daya Bay θ_{13} constraint
- Pick interesting allowed points in true parameter space, run throws, explore correlations in 3d/4d post fit parameter space, determine how correlations affect 1σ , 90%, and 3σ confidence intervals
- Explore how increasing energy spectral information may resolve parameter degeneracies. Compare DUNE and HK resolution ability.