

Simulating Qubits Coupled to a Cavity

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Motivation

Scalable quantum computers must utilize arrays of qubits to hold information. Unlike classical computers, qubits are prone to noise and interference that decrease fidelity. In an array, this noise prevents functionality. The proposed solution to this is coupling multiple qubits to one cavity, while retaining fidelity. We simulate a single qubit coupled to a cavity and two qubits coupled to a cavity, as well as comparing qubit detuning to fidelity.

Multi-Qubit un-tuned Oscillations

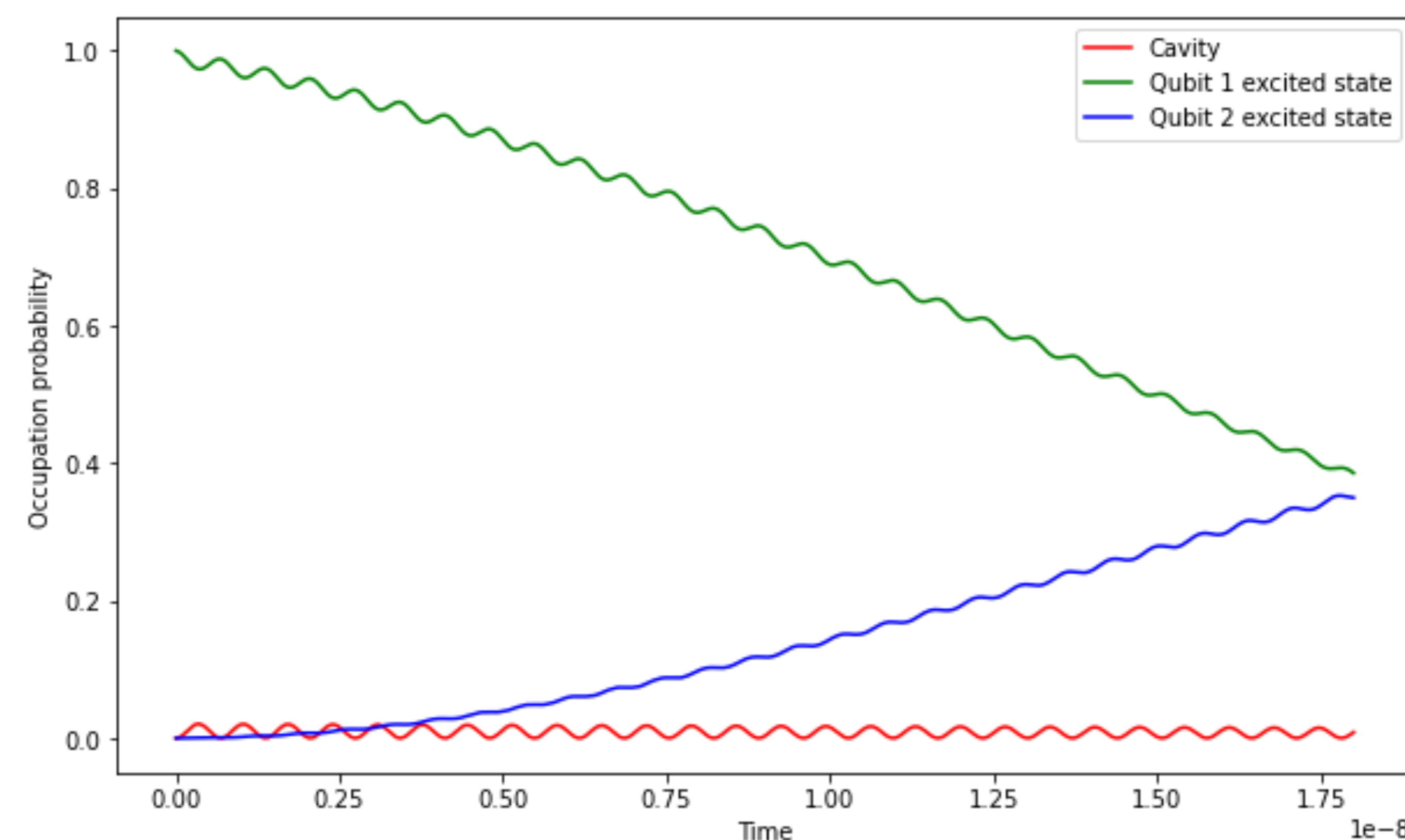


Fig 1. Two qubits coupled to one cavity using experimental values from [2] in the strong coupling, resonance regime. The oscillations represent the Rabi oscillations of the qubit-cavity-qubit system, and the noise present is due to qubit and cavity relaxation.

Theory

The relevant Hamiltonians are the Jaynes and Tavis Cummings models. The Jaynes model is when $N = 1$ qubit:

$$H = \hbar\omega_c \hat{a}^\dagger \hat{a} + \sum_{i=1}^N \hbar\omega_i \hat{\sigma}_i^+ \hat{\sigma}_i^- + g_i (\hat{a}^\dagger \hat{\sigma}_i^- + \hat{a} \hat{\sigma}_i^+)$$

Where ω_c is cavity frequency, ω_i are qubit frequencies. In the two-qubit case, qubit detuning led to drift of $\frac{g^2}{\Delta}$, where $\Delta = \omega_c - \omega_i$. Rabi oscillations for the system:

$$\Omega_i = \sqrt{4Ng_i^2 + \Delta_i^2}, \text{ for the same } g \text{ and } \Delta \text{ for all qubits}$$

Multi-Qubit Tuned Oscillations

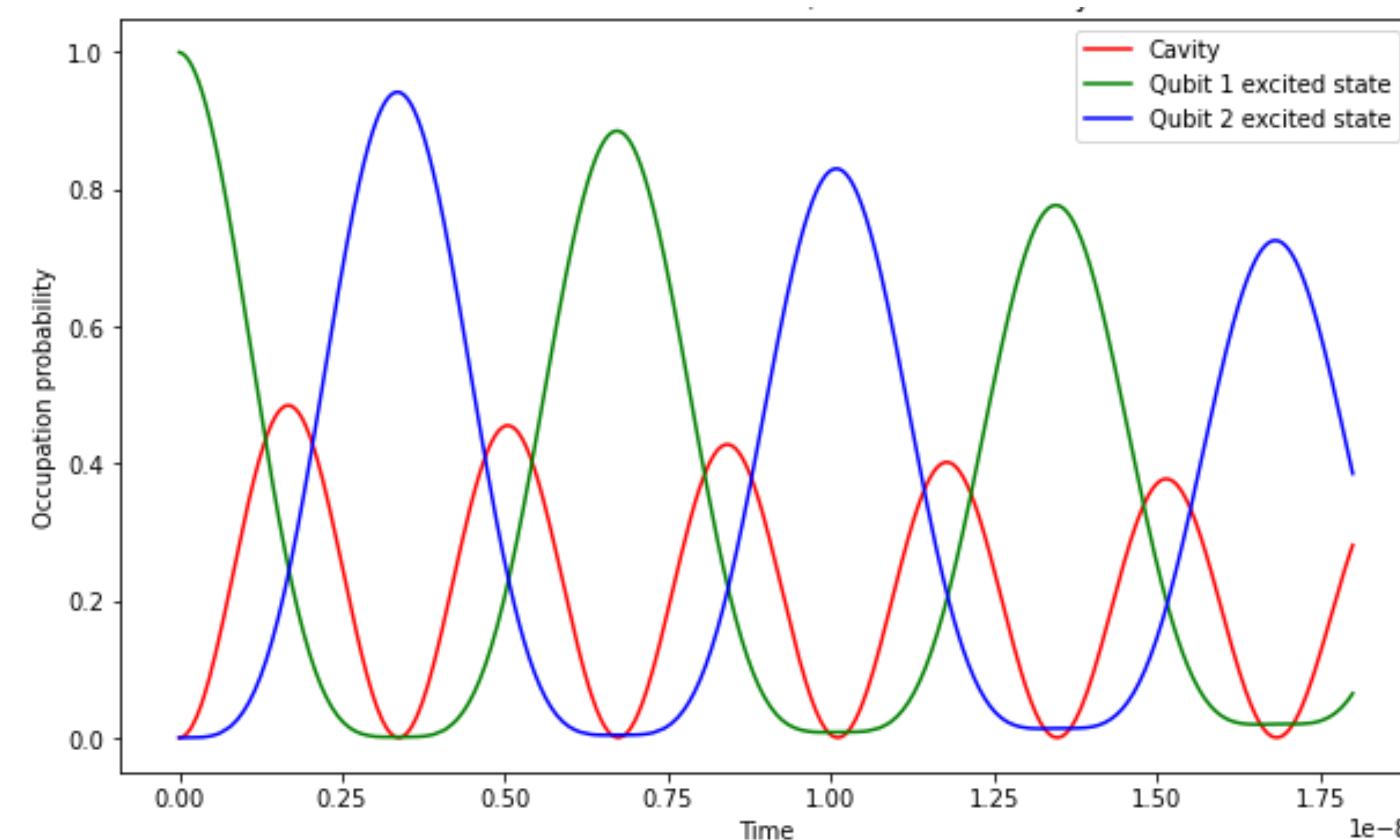


Fig 2. Two qubits coupled to one cavity with identical coupling ($g=0.22\text{GHz}$) and no detuning. The noise added is due to qubit and cavity relaxation. On-resonance qubits result in accurate Rabi calculations.

Simulation Methods

QuTip was used to simulate occupation probability of both qubits and the cavity. The Rabi Oscillation frequency was first calculated with the theory approach and compared to a simulation output. The output frequencies were obtained by fitting a secondary curve. Qubit dephasing and relaxation were added via exponential damping and as evolution operators to the Hamiltonian Lindblad master equation. In more complex cases, the Processor method and relevant quantum circuits were used.

Rabi Frequency vs Single Qubit Detuning

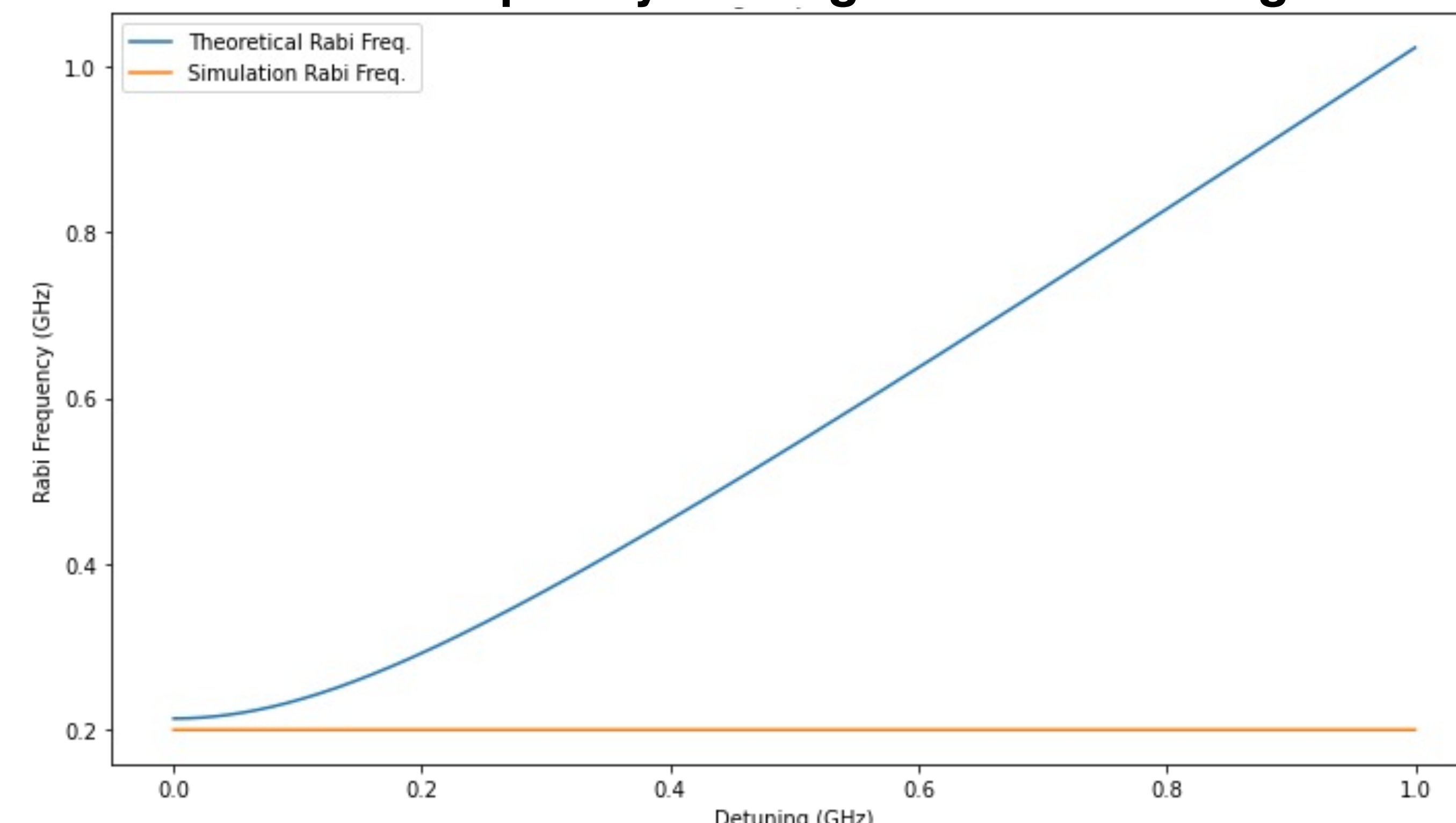


Fig 3. For every value of detuning in the dispersive limit (ie g is much less than detuning), the Rabi frequency is calculated. As the detuning increases, the Rabi frequency theory and simulation do not agree.

Results

The multi-qubit simulations are accurate and tunable for the desired cavity and qubit conditions. In the single qubit case, this is also true. Both results are reflected in fig (3) and (4), in the case of single qubit detuning. However, in the both cases, the Rabi analysis loses accuracy when qubits are detuned different amounts from the resonator cavity. It is suspected that the Rabi calculations are not approximated by the Tavis-Cummings model when detuning is not equal for the qubits.

Cavity Wigner Function in Excited State

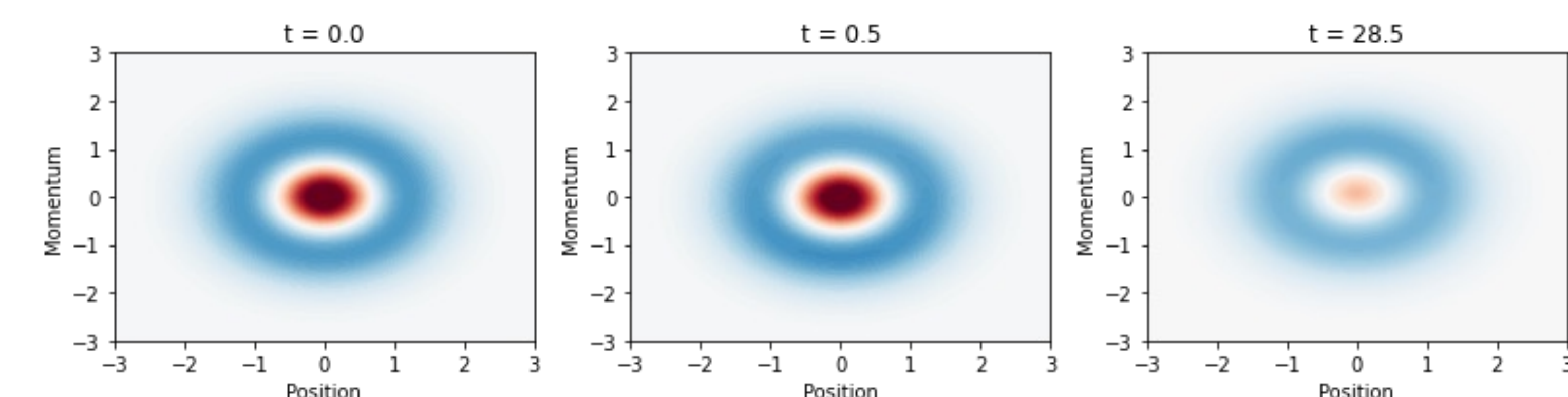


Fig 4. Wigner function 2-dimensional visualization for the cavity in the single qubit regime. The Wigner function represents the quasi-probability density function of occupying the excited state in the cavity. The function is graphed at every time the cavity excited state is occupied, so the color is the probability the quanta occupies that position and momentum. The decay of the function is visible, with decay parameters for qubit and cavity relaxation.

Acknowledgements

This manuscript has been authored by Fermi Research Alliance, LLC under Contract No. DE-AC02-07CH11359 with the U.S. Department of Energy, Office of Science, Office of High Energy Physics.

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