Radiative Correction: Update

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Overview

What kind of calculations or improvements are needed where we can help?

- Neutrino-Nucleon scattering (NC/CC) for DIS regions
- Elastic scattering of electrons and neutrinos off Nucleons
- Real photon emission in neutrino Nucleon scattering events
- Effects of lepton masses when considering low energy cross sections

Collaboration with Fermilab's Joint Theoretical Experimental Group with the goal to improve on the NLO corrections to neutrino nucleon scattering.

- Begin with the simpler process of electron proton scattering using low energy EM proton form factors
- This is a great way to calibrate the method because of access to JLAB data on e-p scattering
- The idea is to perform the full one loop calculation plus real radiation and propose this as an addition to GENIE

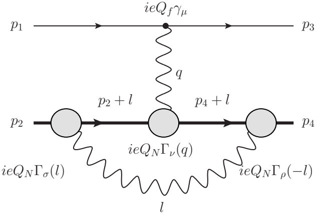
Low Energy Electron Proton Scattering

As far as we know GENIE is based on work by Vanderhaegen, and Maximon and Tjon. These results however rely on certain approximations

- Both works use a “soft photon approximation” in the box diagrams
- In Vanderhaegen’s work the angular peaking approximation is used for the radiative tail
- These approximations are to be avoided and the full virtual and real NLO amplitude will be computed.

Overview

The following serves as an example in where difficulties arise, take for instance the proton vertex correction:



Corresponding to the following integral

$$
\begin{aligned}
&(-\Lambda^2)^4 e^4 Q_f^2 Q_N^2 \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{\mathcal{N}^\mu(l)}{(l^2-\Lambda^2)^4 (l^2-m_0^2)((p_2+l)^2-M^2)((p_4+l)^2-M^2)} \\
= \quad & \frac{(-\Lambda^2)^4}{3!} e^4 Q_f^2 Q_N^2 \frac{\partial^3}{\partial(\Lambda^2)^3} \left(\frac{1}{\Lambda^2-m_0^2} \int \frac{\mathrm{d}^4 l}{(2\pi)^4} \frac{\mathcal{N}(l)}{((p_2+l)^2-M^2)((p_4+l)^2-M^2)} \left[\frac{1}{l^2-\Lambda^2} - \frac{1}{l^2-m_0^2} \right] \right)
\end{aligned}
$$

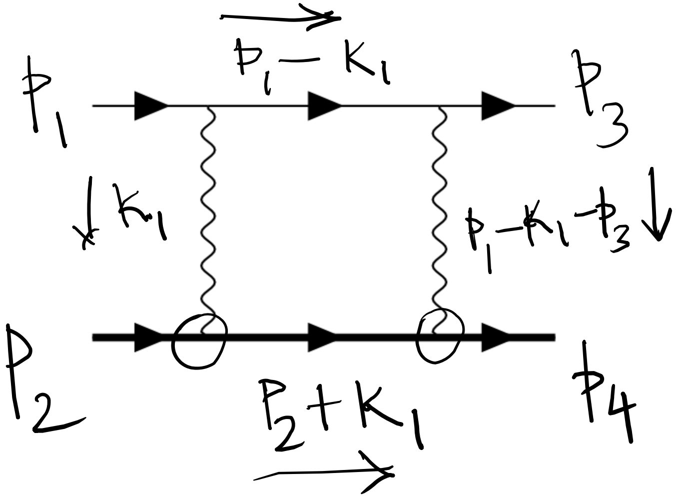
Where

$$
\begin{aligned}
\mathcal{N}^\mu(l) &= \Gamma_{r\nu}(l)(\not{p}_4 + \not{l} + M)\Gamma_r^\mu(q)(\not{p}_2 + \not{l} + M)\Gamma_r^\nu(-l) \\
\Gamma_r^\mu(q) &= \gamma^\mu + \kappa \frac{i\sigma^{\mu\nu} q_\nu}{2M}
\end{aligned}
$$

Because the integrals themselves can be simplified, this leads to being able to use simpler methods to evaluate them.

- FeynCalc: Mathematica package for QFT related algebra (Performs the Passarino-Veltman reduction)
- Package X: Mathematica package for QFT related algebra (Used to analytically evaluate Passarino-Veltman functions)
- FeynHelper: A package to interlink FeynCalc to Package-X (Used to merge the above two points)
- Vegas Algorithm: FORTRAN code MC to numerically evaluate the resulting amplitudes from the above programs
- COLLIER Integral Library to numerically evaluate scalar functions for $n \geq 4$

Strategies for box diagrams



$$p_1^2 = p_3^2 = m^2$$
$$p_2^2 = p_4^2 = M^2$$

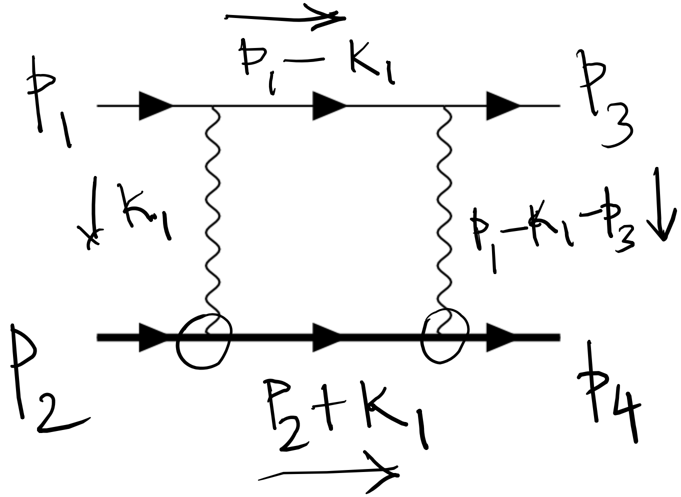
Proton vertex: $\gamma^\mu \rightarrow \Gamma_\mu = F_1(q^2)\gamma_\mu + \kappa F_2(q^2)\dfrac{i\sigma_{\mu\nu}q^\nu}{2M}$

Structure of the proton form factor: $F_1(q^2) = F_2(q^2) = \left(\dfrac{-\Lambda^2}{q^2 - \Lambda^2}\right)^n$

n=1 implies monopole
n=2 implies dipole

Extra propagator terms: $\dfrac{1}{k_1^2 - \Lambda^2}, \quad \dfrac{1}{(k_1 + p_3 - p_1)^2 - \Lambda^2}$

Strategies for box diagrams



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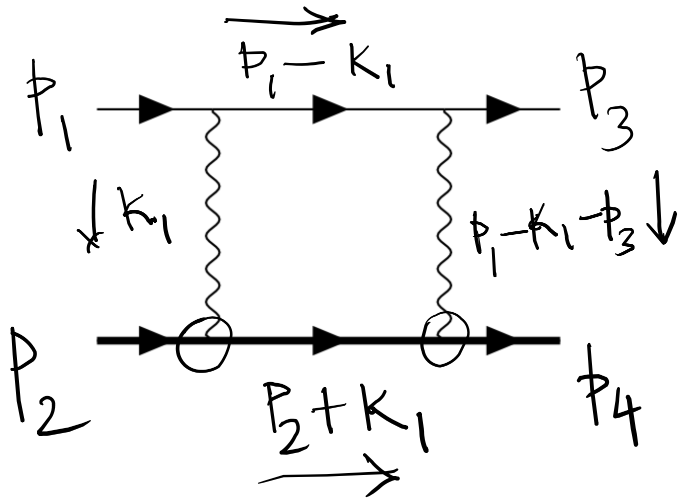
Possible list of denominators:

$$d_1 = k_1^2$$
$$d_2 = (k_1 - p_1)^2$$
$$d_3 = (k_1 + p_2)^2 - M^2$$
$$d_4 = (k_1 + p_3 - p_1)^2$$
$$d_5 = k_1^2 - \Lambda^2$$
$$d_6 = (k_1 + p_3 - p_1)^2 - \Lambda^2$$

Typical term under integration:

$$\frac{N}{d_1^{a\,1}\, d_2^{a\,2}\, d_3^{a\,3}\, d_4^{a\,4}\, d_5^{a\,5}\, d_6^{a\,6}}$$

Strategies for box diagrams



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$$\frac{N}{d_1^{a1}\, d_2^{a2}\, d_3^{a3}\, d_4^{a4}\, d_5^{a5}\, d_6^{a6}}$$

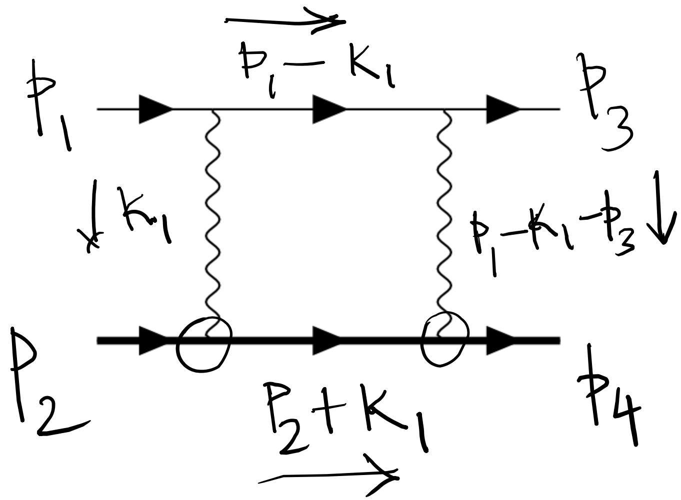
Issue! : 5 and 6 point functions are not easily handled in available automated programs.

We can use partial fraction to decompose

$$\frac{1}{k_1^2(k_1^2-\Lambda^2)} \equiv \frac{1}{d_1 d_5} \quad \text{and} \quad \frac{1}{(k_1+p_3-p_1)^2((k_1+p_3-p_1)^2-\Lambda^2)} \equiv \frac{1}{d_4 d_6}$$

at the expense of getting derivative operators.

Strategies for box diagrams



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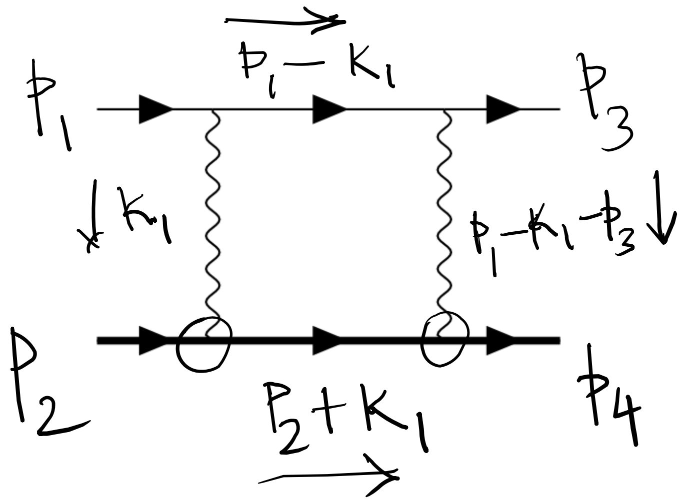
at the expense of getting derivative operators.

$$\frac{1}{k^2-\lambda^2}\left(\frac{-\Lambda^2}{k^2-\Lambda^2}\right)^m = \frac{(-\Lambda^2)^m}{(m-1)!}T^{m-1}\left(\frac{1}{\Lambda^2-\lambda^2}\left[\frac{1}{k^2-\Lambda^2}-\frac{1}{k^2-\lambda^2}\right]\right)$$

where $$T \equiv \frac{\partial}{\partial(\Lambda^2)}$$

$\lambda = 0$ in dimreg and nonzero
in mass regularization for IR singularity

Strategies for box diagrams



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\end{aligned}
$$

- Now everything can be written up to D_0 functions and its derivatives!

- To simplify the power of the denominators, we can use IBP (integration by parts) reduction techniques. To do so, we have to define well defined family. For the above diagram there will be four such families.

x: d_1, d_2, d_3, d_4

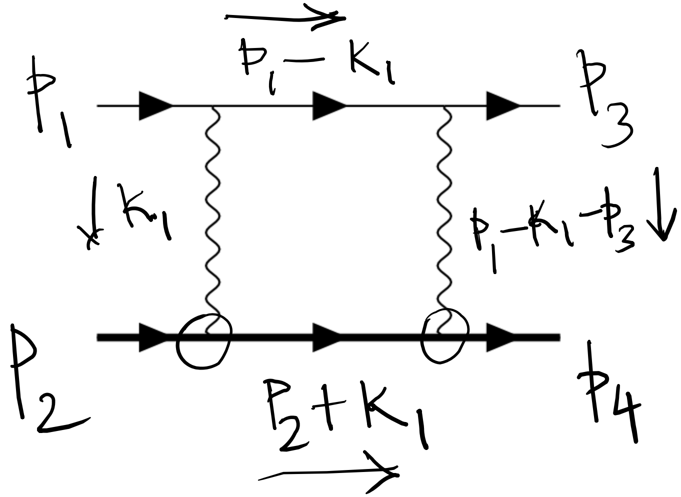
z: d_5, d_2, d_3, d_4

y: d_5, d_2, d_3, d_6

w: d_1, d_2, d_3, d_6

But, there can be further issues with derivatives when too many scales are involved!

Strategies for box diagrams



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x: d_1, d_2, d_3, d_4 z: d_5, d_2, d_3, d_4
y: d_5, d_2, d_3, d_6 w: d_1, d_2, d_3, d_6

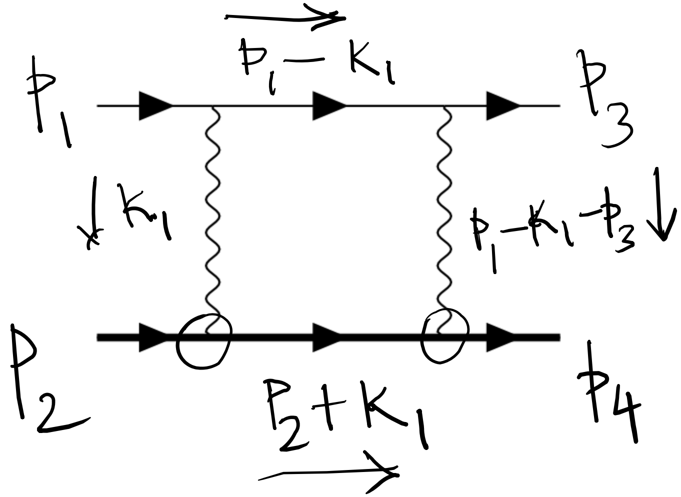
But, there can be further issues with derivatives when too many scales are involved!

For example, the "y" family has scales "m, M, Λ" , which can be an issue to evaluate the derivatives of the D_0 functions.

This can be tackled if we can remove one of the scales. We can write:

$$\frac{1}{k^2-\Lambda^2} = \frac{1}{k^2-(1-\xi)M^2}, \text{ where } \xi \equiv \frac{M^2-\Lambda^2}{M^2}$$

Strategies for box diagrams



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$$\frac{1}{k^2-\Lambda^2} = \frac{1}{k^2-(1-\xi)M^2}, \text{ where } \xi \equiv \frac{M^2-\Lambda^2}{M^2}$$

Which allows us to expand the denominator w.r.t. small parameter ξ and which essentially removes the parameter Λ.

$$\frac{1}{k^2-\Lambda^2} = \frac{1}{k^2-M^2} - \frac{\xi M^2}{(k^2-M^2)^2} + \frac{\xi^2 M^4}{(k^2-M^2)^3} - \cdots$$

It eventually leads to simpler D_0 functions at the expense of higher power in the denominator; which again can be tackled using IBP reductions.

Overview

Preliminary Results

Soft + Virtual Corrections

	$\epsilon_1 = 4.4$ GeV $Q^2 = 6\ (\text{GeV/c})^2$		$\epsilon_1 = 12$ GeV $Q^2 = 16\ (\text{GeV/c})^2$		$\epsilon_1 = 21.5$ GeV $Q^2 = 31.3\ (\text{GeV/c})^2$	
	CW	MTj	CW	MTj	CW	MTj
Z^0	−0.2187	−0.2187	−0.2330	−0.2330	−0.2323	−0.2323
Z^1	−0.0569	−0.0569	−0.0517	−0.0517	−0.0625	−0.0625
$Z^2 + \delta_{el}^{(1)}$	−0.0146(C) −0.0146(W)	−0.0174 −0.0174	−0.0165(C) −0.0185(W)	−0.0243 −0.0243	−0.0173(C) −0.0202(W)	−.0267 −.0267
$\delta_{el}^{(1)}$	+0.0096(C) +0.0096(W)	+0.0068 +0.0068	+0.0194(C) +0.0174(W)	+0.0116 +0.0116	+0.0279(C) +0.0250(W)	+0.0185 +0.0185
δ	−0.2902(C) −0.2902(W)	−0.2930 −0.2930	−0.3012(C) −0.3032(W)	−0.3090 −0.3090	−0.3121(C) −0.3150(W)	−0.3214 −0.3214

Figure: Comparision of Maximon and Tjon (MTj)[1] to our work, Crowe and Wackeroth (CW)

Future Steps

- Comparison of our box calculation to Oleksandr Tomalak's work on Two Photon exchange corrections .
- The next phase will be to add onto this core machinery the QED corrections to neutrino-Nucleon scattering
- Possibility to institute a switch to move from a lower energy regime to higher energy DIS regime using in house EW corrections to neutrino-Nucleon scattering [1]

[1] Kwangwoo P., Baur U., Wackeroth D. , "Electroweak radiative corrections to neutrino–nucleon scattering at NuTeV," arxiv.org/abs/0910.5013

Overview

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