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# MAGIS simulation & testing update

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Oxford Physics Microstructure Detector  
Laboratory

- Theory of OTF & pupil function (reminder)
- Progress on measuring lens OTF & pupil function
- Simulating optics via 3D OTF
- Non-parametric phase estimation via CWT

**Note 1:** apologies for the incompleteness of the slides, I didn't have long to prepare!

**Note 2:** no mention today of detector simulation, more progress has been made there too, for another update

**Note 3:** a lot of details have moved on since the images I show in these slides, we haven't made new plots yet. Please ask if interested in anything and hopefully soon I can give a fuller update!

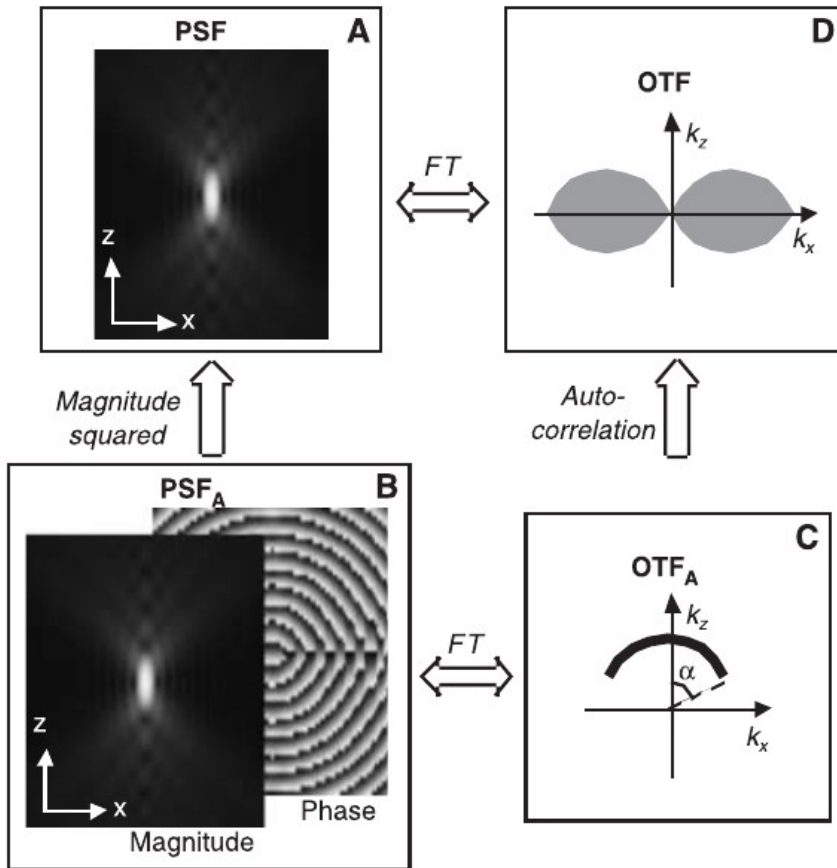
# OTF, PSF & pupil function



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For simulating (and also deconvolving) images, we want to know the incoherent OTF /PSF to reasonable accuracy

To **also** correct for defocus and tilt, it's nice to know the pupil function (from which the above can be derived)



Plan at OPMD is to measure both – incoherent OTF using conventional test (knife edge + laser speckle),

**Also** measure coherent OTF using **shearing interferometry**

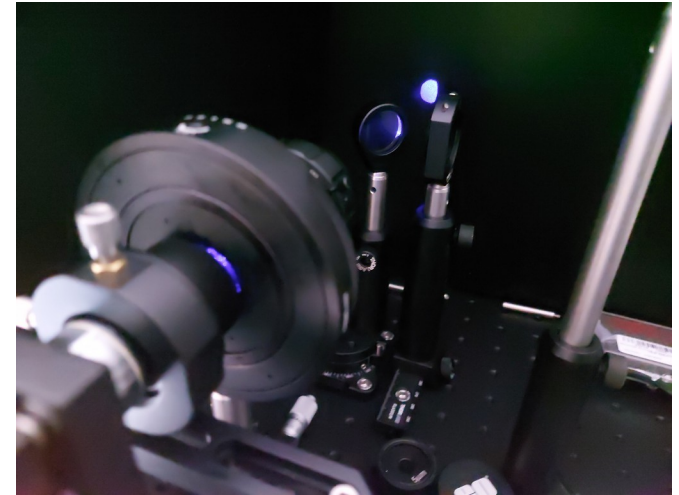
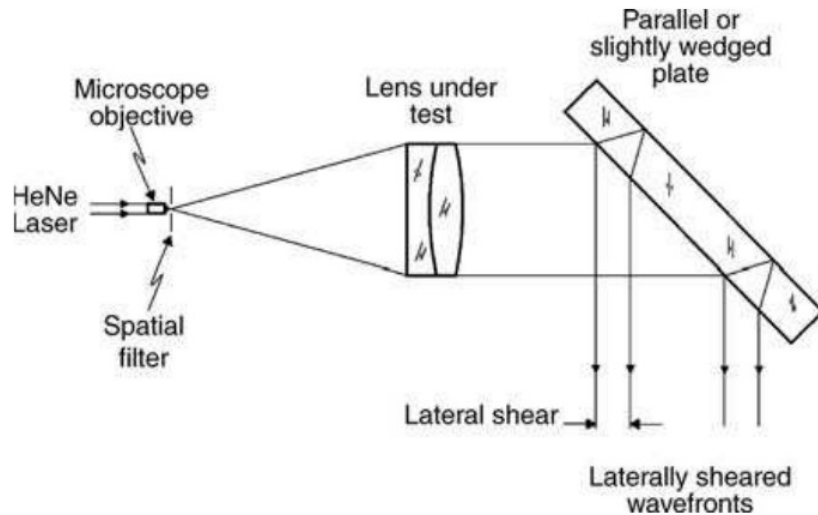
(image from Hanser et al, 2004)

Pupil Function!

# Principle of Murty Interferometer



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**FIGURE 4.32.** Schematic diagram of a lateral shearing interferometer incorporating a laser source and a plane parallel plate of glass.

Use the lens under test to collimate light coming from a pinhole.  
Use a tilted optical flat to laterally shear the wavefronts. They interfere with each other in a manner dictated by the pupil function (& hence aberrations) of the lens under test.

Normally done for larger surfaces – **it's a bit of a pain with the c-mount flange distance being ~15mm**

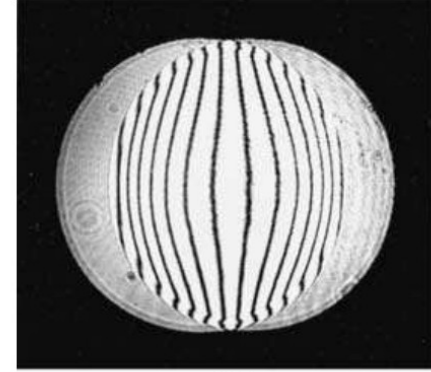
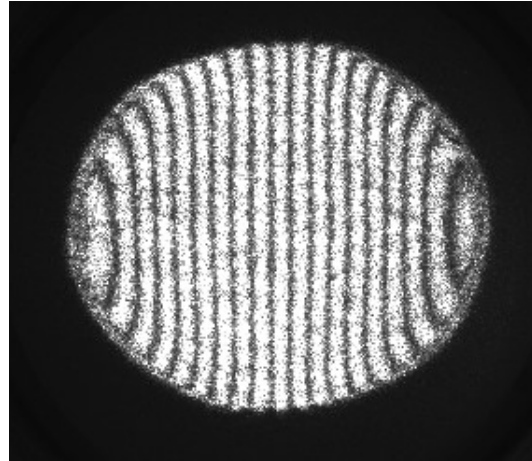
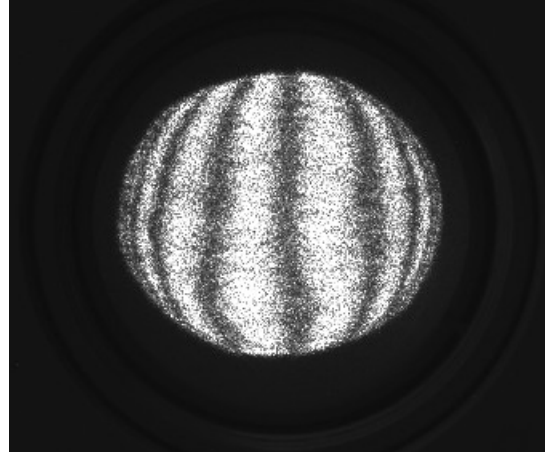
# First Results on Pupil Measurement



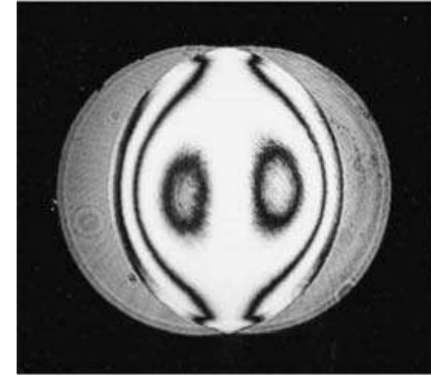
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Example shear interferograms from our setup (MAGIS main camera candidate lens)



(a)



(c)

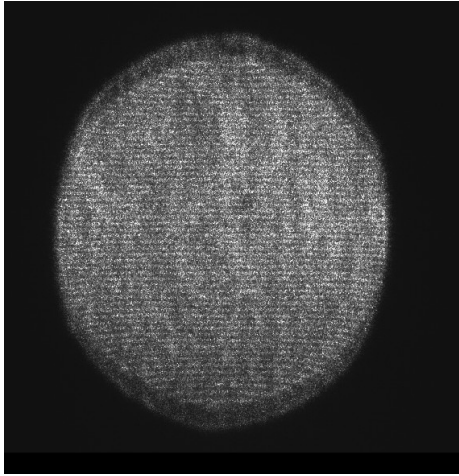
Textbook “spherical aberrated” shear wavefronts (Malacara “Optical Shop Testing”)



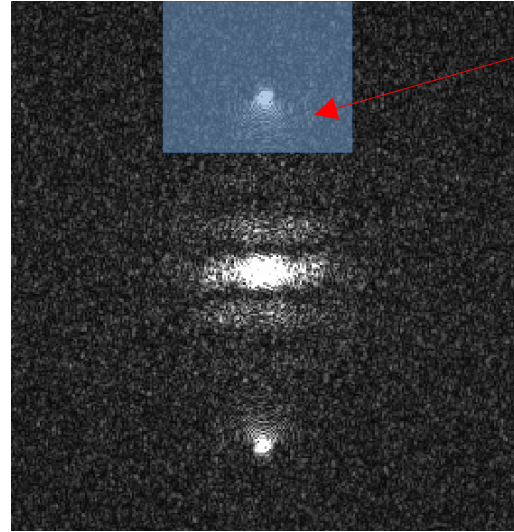
# Data Processing (VERY initial!!)



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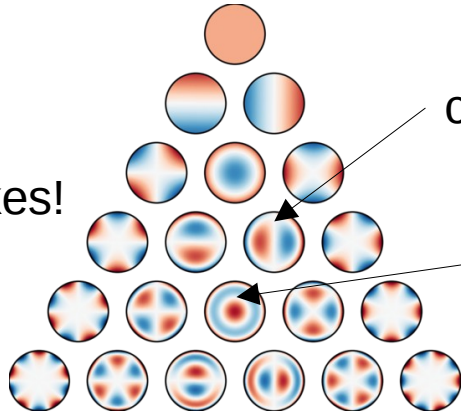


Highly defocused image



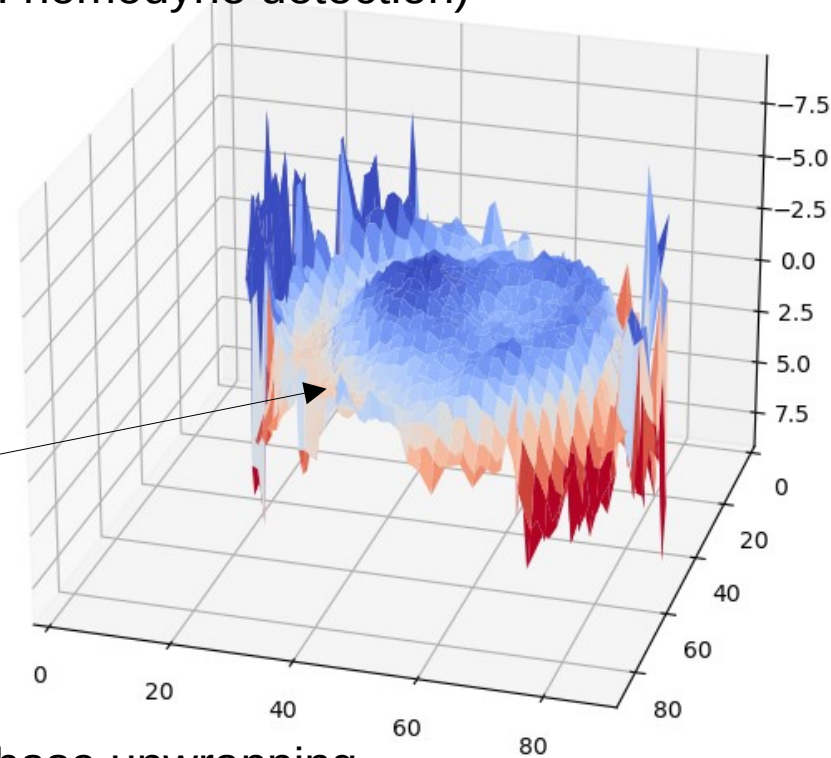
Keep just this bit  
(i.e. homodyne detection)

**TBD:**  
Fit Zernikes!



coma

Primary spherical



2D phase unwrapping  
(courtesy of scikit-image)

# Fast Simulation for MTF & Photon Budget



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Our original simulations (previously described) worked by propagating photons from individual atoms. Good for verifying camera performance, but for parameter space exploration and quick phase extraction tests something (**much**) faster is needed.

## magis\_event\_generator etc

### Atom cloud simulation

- rejection sampled positions (atom shot noise)
- photon recoil (“diffusion”)

~10<sup>4</sup> x speedup



## New fast method

Fringe pattern imprinted **after** atom shot noise incurred. Thus, can approximate shot noise as **White noise** and add in Fourier domain. Recoil/ Diffusion can be well approximated with a Gauss-type kernel (..we think, not tried yet!)

### Optics

- ray trace to exit pupil plane
- construct histogram slices in depth
- apply defocus MTF to each slice
- project to image plane

~10<sup>3</sup> x speedup

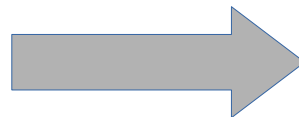


Derive luminosity distribution (can be in 1D/2D/3D) from atom number density distribution. Apply Fresnel propagator to this distribution to get to exit pupil. Apply (3D now!!!) OTF as before

### Detector

ESA Pyxel simulator (QE, noise, Bright pixels, non-linearity, full-well, Etc etc etc)

~0.9x slowdown



**Same as before!**  
(**BUT**) now have to “artificially” add photon shot Noise at this stage. Before it arose naturally Through ray tracing.

# Simulation of images via pupil function

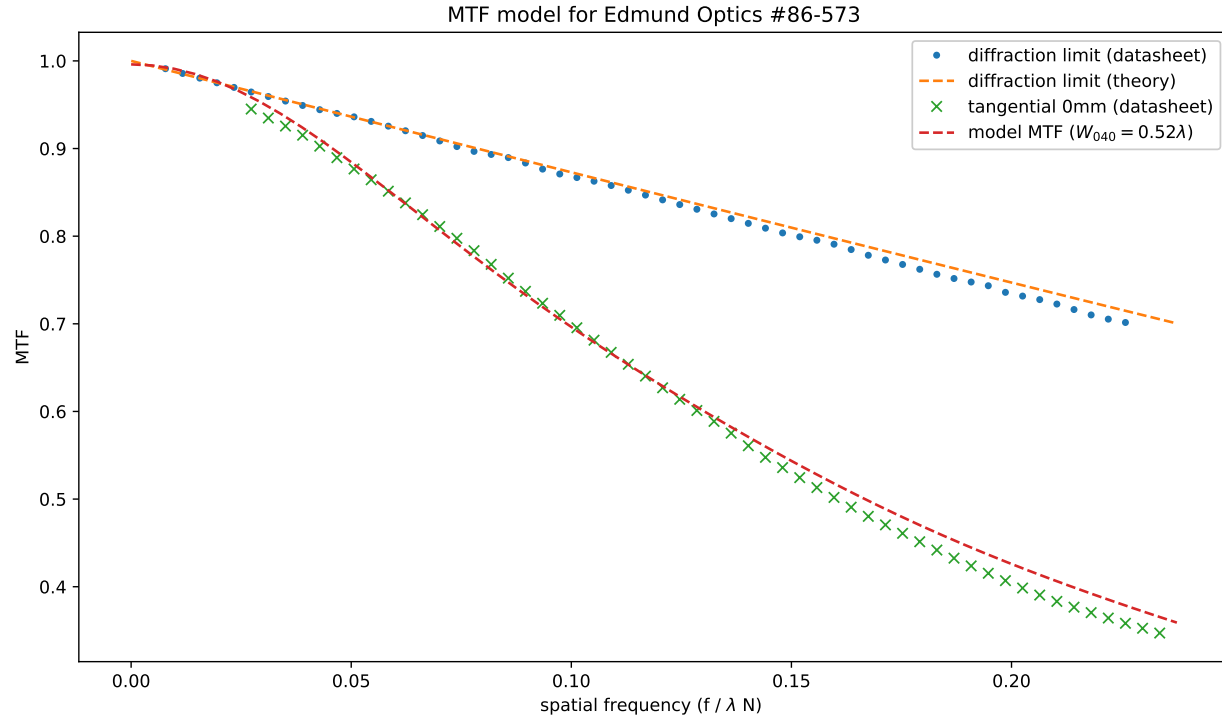


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Realistic pupil function & defocus throughout radiant volume directly applied to pupil function (can also do the same procedure for off-axis tilt)

Derive 3D OTF for image, Use this as a Fourier filter on incident Atom distribution in image space, to get realistic incident irradiance distribution on detector. Then apply ESA pyxel process as normal to get realistic detector output.

Sorry I don't have many pictures to show today – however signs are good **right** is a match of our model's output to the data sheet values for a MAGIS main imager candidate lens





# Non-parametric phase estimation



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For various reasons, we would like some kind of **parameter-free** way to estimate phases from interferometry images:

- Allows relative testing of imaging parameters without having to quibble over what fitting function we are using
- If fast enough, can be used to make (very good) parameter estimates for initialising a non-linear fit so fewer iterations are needed (near real-time performance wanted for MAGIS).

One obvious solution is to use the Hilbert Transform to obtain the analytic signal, unwrap the phase and job done. (Full details for discussion offline).

Unfortunately, generalising this to 2D is difficult. Nobody agrees exactly how best to do it. Best candidate (IMO) is the “monogenic signal”, which we are looking at. Phase unwrapping in 2D is also quite difficult in itself.

Concepts can be illustrated by simple Fourier analysis of the expected cloud shape.

# Non Parametric Phase Extraction (FFT)



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$$h(x, z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x^2 + z^2)}{2\sigma^2}\right) (1 + \cos(kx + \phi))$$

Start from function we are looking at

$$f(x) = 1 + \cos(kx + \phi)$$

$$g(x, z) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(x^2 + z^2)}{2\sigma^2}\right)$$

$$\tilde{H}(u, v) = \tilde{F}(u) * \tilde{G}(u, v)$$

Split into gaussian and cos parts, and apply convolution theorem

$$\tilde{F}(u) = \delta(u) + \frac{1}{2}e^{i\phi}\delta(u - k) + \frac{1}{2}e^{-i\phi}\delta(u + k)$$

$$\tilde{H}(u, v) = \tilde{G}(u, v) + \frac{1}{2}e^{i\phi}\tilde{G}(u + k, v) + \frac{1}{2}e^{-i\phi}\tilde{G}(u - k, v)$$

Can then write down analytical 2D FT quite easily – three shifted Gaussians and a phase factor

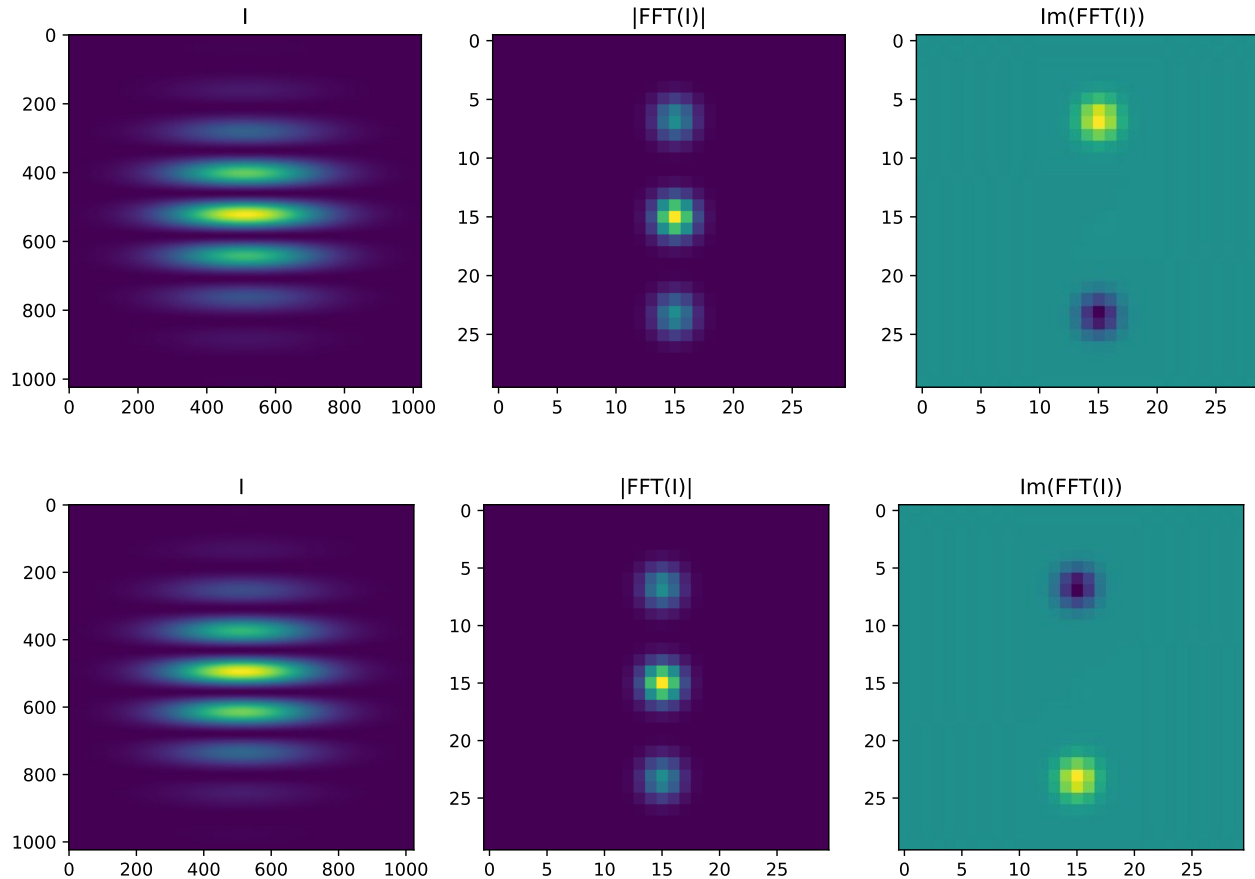
$$\Im\left(\tilde{H}(u, v)\right) = \frac{1}{2}\sin(\phi)\tilde{G}(u + k, v) - \frac{1}{2}\sin(\phi)\tilde{G}(u - k, v)$$

Bottom line – can recover phase information by looking at imaginary part of this

# 2D FFT example



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Toy Examples:

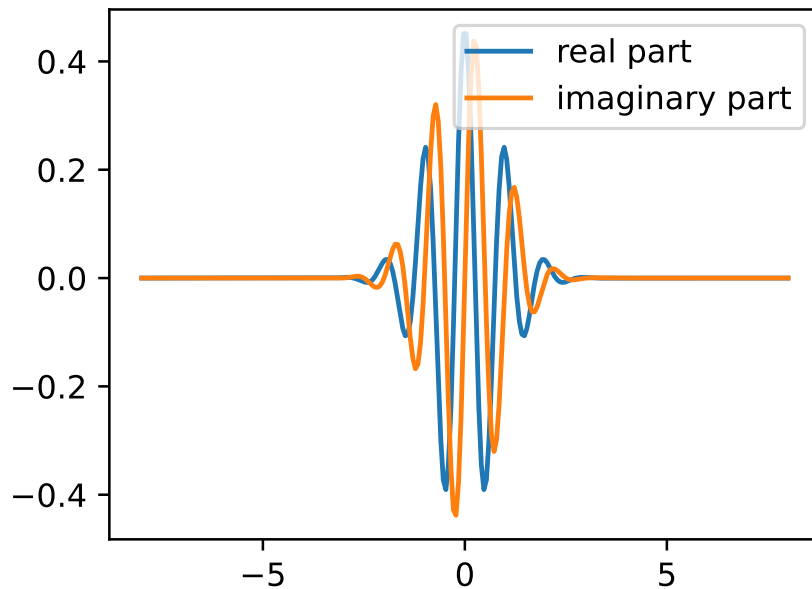
-30 degree phase shift (top)

+50 degree phase shift  
(bottom)

If one were to change frequency or width of gaussian cloud, blobs in fourier space would shift relative to each other (not shown here)

**Problem:** what if we have two (or more) clouds in the image? Would have to either manually segment the images (which is no longer technically “non-parametric”)!

**Wavelets** are functions which have a finite extent in both time and frequency domains. Some wavelets can be scaled arbitrarily, the Morlet shown below is one such.



In a continuous wavelet transform, we produce a highly redundant representation of the original signal in both the time and frequency domains. With a complex wavelet, we also get access to local envelope and phase information.

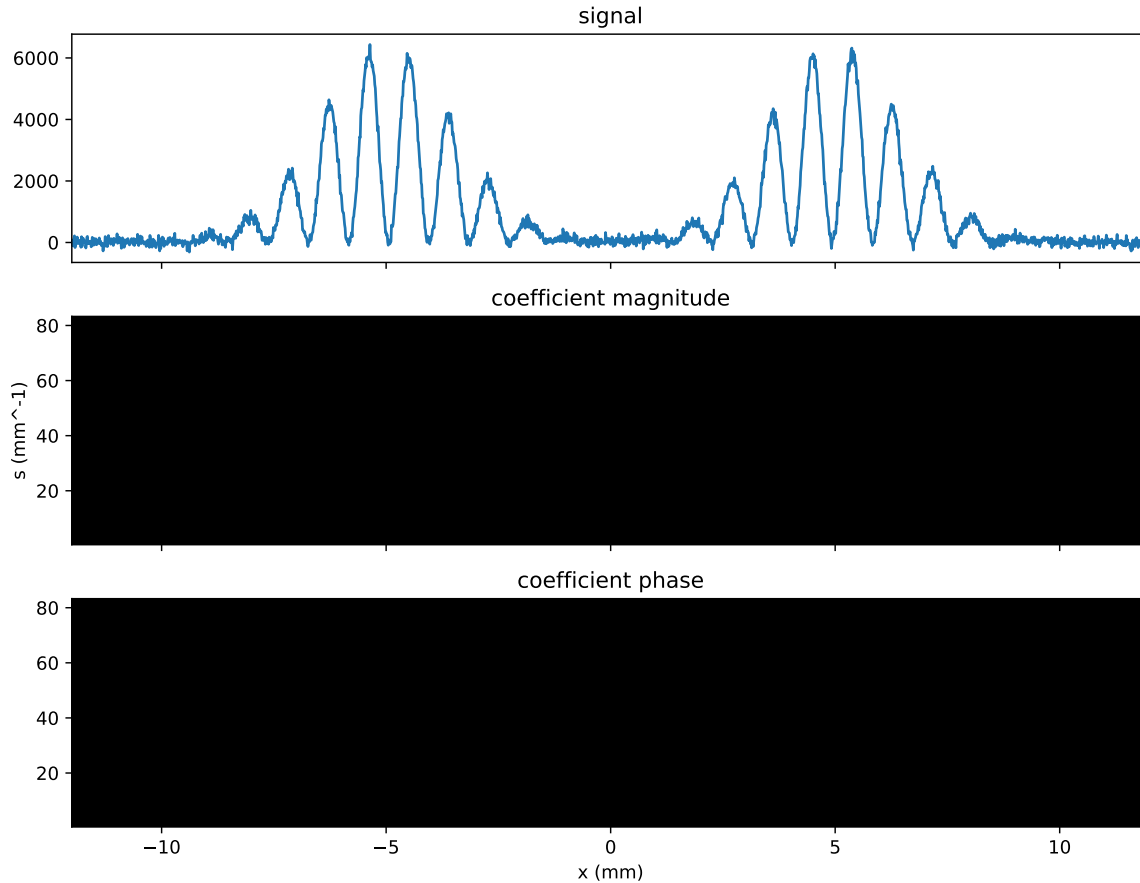
Fast methods exist to calculate the CWT (using FFT based convolution)

$$X_w(a, b) = \frac{1}{\sqrt{a}} \int_{-\infty}^{\infty} x(t) \bar{\psi} \left( \frac{t - b}{a} \right)$$

# CWT applied to atom clouds



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- Since the wavelet can be scaled arbitrarily it is relatively easy to match the resolution of the CWT to the resolution of the system
- A good guess for the values of spatial frequency, amplitude and phase can be extracted by simple peak finding
- If we have a reasonable idea of what spatial frequencies are involved, can calculate a small part of the CWT
- From the bottom image **left**, can apply 2D phase unwrapping algorithm to obtain phase across cloud

# TBD: 2D CWT analysis



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**Unlike Hilbert Transform an obvious, unambiguous and complete extension to 2D exists for the CWT.**

**However, No suitable off the shelf 2D CWT code currently exists for MAGIS image processing. If we want to use this technique we will have to write one. There are plenty of good starting points in the literature**

Two-dimensional Morlet wavelet transform and its application to wave recognition methodology of automatically extracting two-dimensional wave packets from lidar observations in Antarctica

Cao Chen\*, Xinzhao Chu\*

*Cooperative Institute for Research in Environmental Sciences & Department of Aerospace Engineering Sciences, University of Colorado Boulder, 216 UCB, CIRES, Boulder, CO 80309, USA*

## **The fast continuous wavelet transformation (fCWT) for real-time, high-quality, noise-resistant time-frequency analysis**

Lukas P. A. Arts   and Egon. L. van den Broek  



# Thanks!



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# Backup – Gsense laser speckle MTF



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