Tianlai Cylinder Pathfinder End-to-End Simulation

Tianlai Collaboration Meeting 2022

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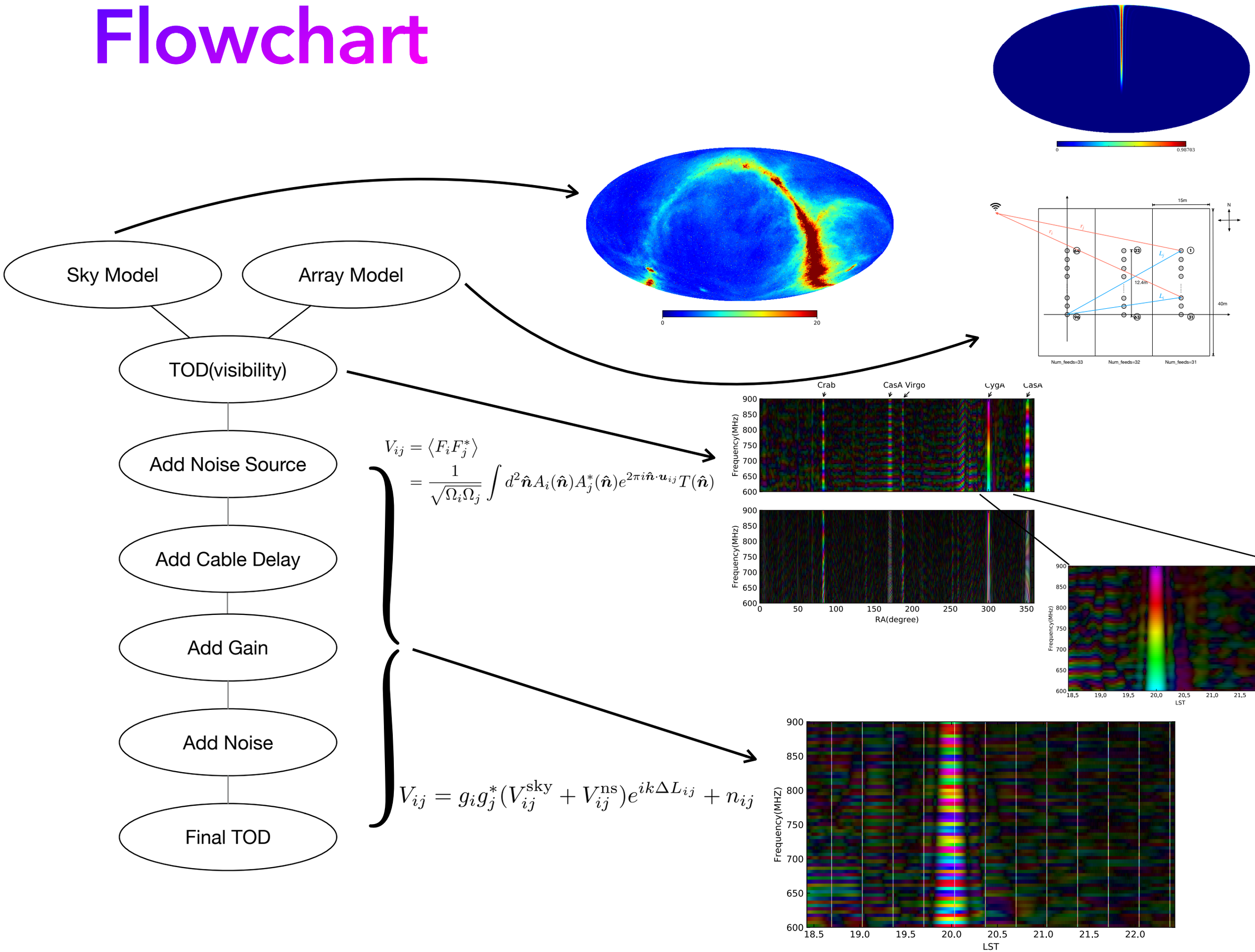
Aug 29, 2022

Flowchart


Sky Model
Array Model
TOD(visibility)
Add Noise Source
Add Cable Delay
Add Gain
Add Noise
Final TOD
$V_{ij} = \langle F_i F_j^* \rangle$
$= \frac{1}{\sqrt{\Omega_i \Omega_j}} \int d^2\hat{\boldsymbol{n}} A_i(\hat{\boldsymbol{n}}) A_j^*(\hat{\boldsymbol{n}}) e^{2\pi i \hat{\boldsymbol{n}} \cdot \boldsymbol{u}_{ij}} T(\hat{\boldsymbol{n}})$
$V_{ij} = g_i g_j^* (V_{ij}^{\rm sky} + V_{ij}^{\rm ns}) e^{ik\Delta L_{ij}} + n_{ij}$
Crab
CasA Virgo
CygA
CasA
Frequency(MHz)
RA(degree)
LST
Num_feeds=33
Num_feeds=32
Num_feeds=31
15m
12.4m
40m

More Realistic?

Beam

beam model

(R. Shaw et al.) Coaxing Cosmic 21cm Fluctuations from the Polarized Sky using m-mode Analysis

First we model the beam amplitude for the unfocused dipole in the E-plane and H-plane as taking the form

$$A_D(\theta; \theta_W) = \exp\left(-\frac{\ln 2}{2}\frac{\tan^2\theta}{\tan^2 \cancel{\theta_W}\ \theta_w/2}\right), \qquad (35)$$

where θ_W is the full width at *half-power* of the beam. For a horizontal dipole mounted a distance $\lambda/4$ over a conducting ground plane (see [46, section 4.7]), we can exactly calculate the widths in the H-plane ($\theta_H = 2\pi/3$) and E-plane ($\theta_E \approx 0.675\theta_H$). We use these value for our fiducial beam model, though we will vary them later in this paper.

In the E-W direction we are solving the Frauhofer diffraction problem of a cylinder feed illuminating an aperture of a finite width. This has the solution

$$A_F(\theta; \theta_W, W) \propto \int_{-\frac{W}{2}}^{\frac{W}{2}} A_D(2\tan^{-1}(\tfrac{2x}{W}); \theta_W) e^{-ikx\sin\theta} dx$$
$$\propto \int_{-1}^{1} e^{-\frac{\ln 2}{\tan^2\theta_W}\frac{u^2}{1-u^2} - i\frac{\pi W}{\lambda} u \sin\theta} du \qquad (36)$$

where we have used the fact that for a cylinder with an f-ratio of 1/4 a ray striking a distance x from the cylinder centre reflects by an angle $\theta = 2\tan^{-1}(2x/W) = 2\tan^{-1} u$ where W is the cylinder width.

N-S direction

E-W direction

Simply adjust two parameters

θ_{ns} and θ_{ew}

to control the beam

Beam

Fitting with Auto-Correlation data

Beam model:

$$A_{\text{amp}}(\hat{\mathbf{n}};\theta_{\text{ew}},\theta_{\text{ns}}) = A_F(\sin^{-1}(\hat{\mathbf{n}}\cdot\hat{\mathbf{x}});\theta_{\text{ew}}) \times A_D(\sin^{-1}(\hat{\mathbf{n}}\cdot\hat{\mathbf{y}});\theta_{\text{ns}})$$

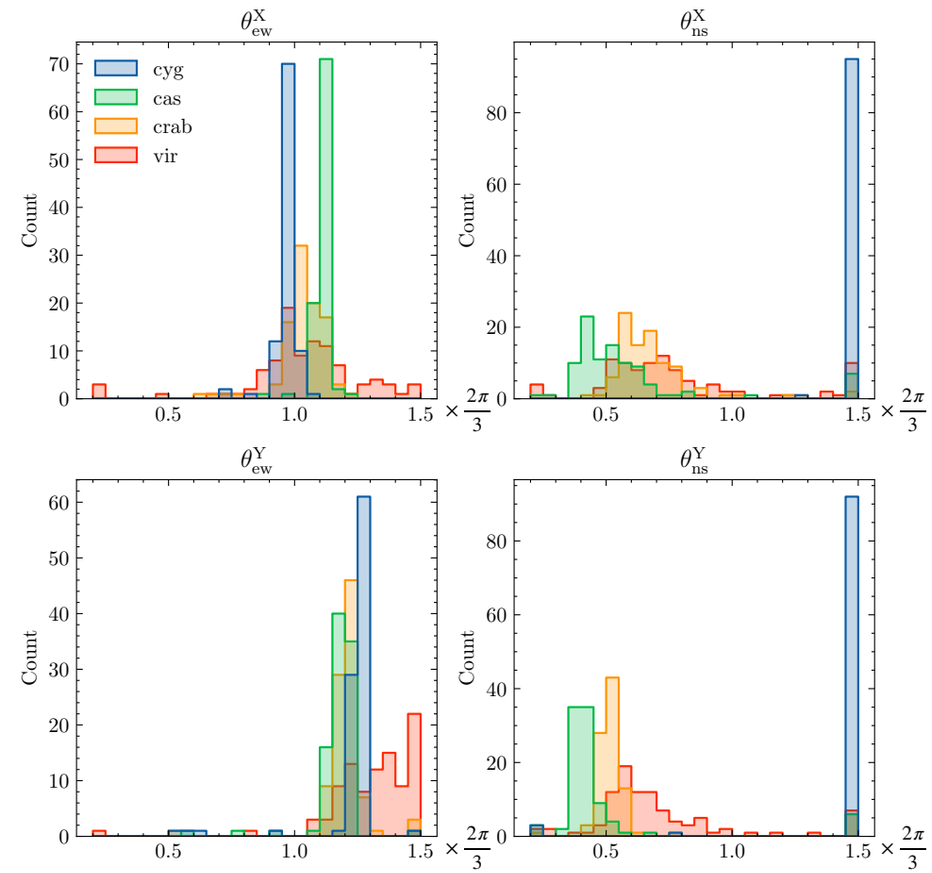
fit the auto-correlation visibility when transitting a calibrator

$$V_i^{\text{auto}} = S_{\text{amp}}[A_F(\theta+\theta_{\text{offset}};\theta_{\text{ew}}) \times A_D(\sin^{-1}\phi_{\text{src}};\theta_{\text{ns}})]^2 + N_{\text{offset}}$$

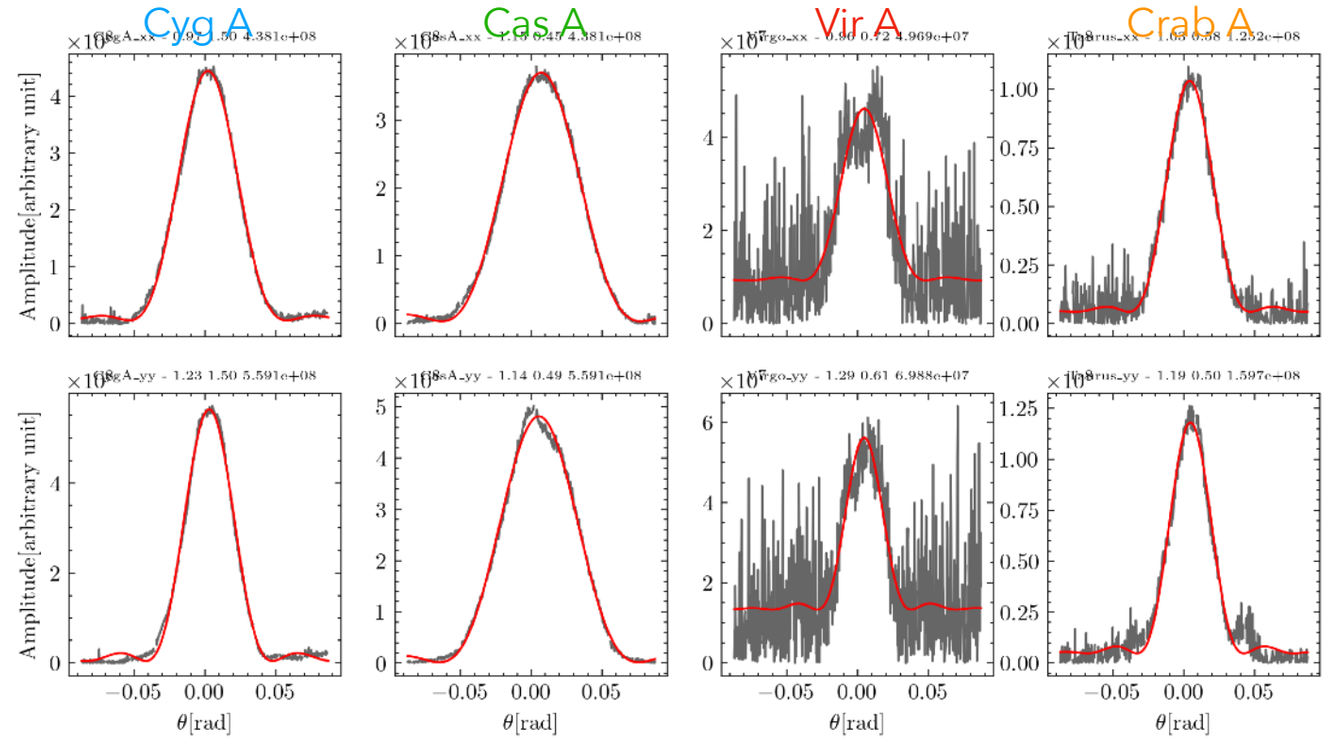
➔

Tianlai: latitude ~ 44.15

1. Cyg A(latitude ~ 40.72). $S_{\text{amp}}^{\text{cyg}}$ as fiducial value
2. Cas A (latitude ~ 58.82) . fitting boundary [$S_{\text{amp}}^{\text{cyg}}$ / 1.2, $S_{\text{amp}}^{\text{cyg}}$ / 1.0]
3. Virgo A (latitude ~ 12.41) [$S_{\text{amp}}^{\text{cyg}}$ / 9.0, $S_{\text{amp}}^{\text{cyg}}$ / 8.0]
4. Crab A (latitude ~ 22.01) [$S_{\text{amp}}^{\text{cyg}}$ / 3.5, $S_{\text{amp}}^{\text{cyg}}$ / 3.0]



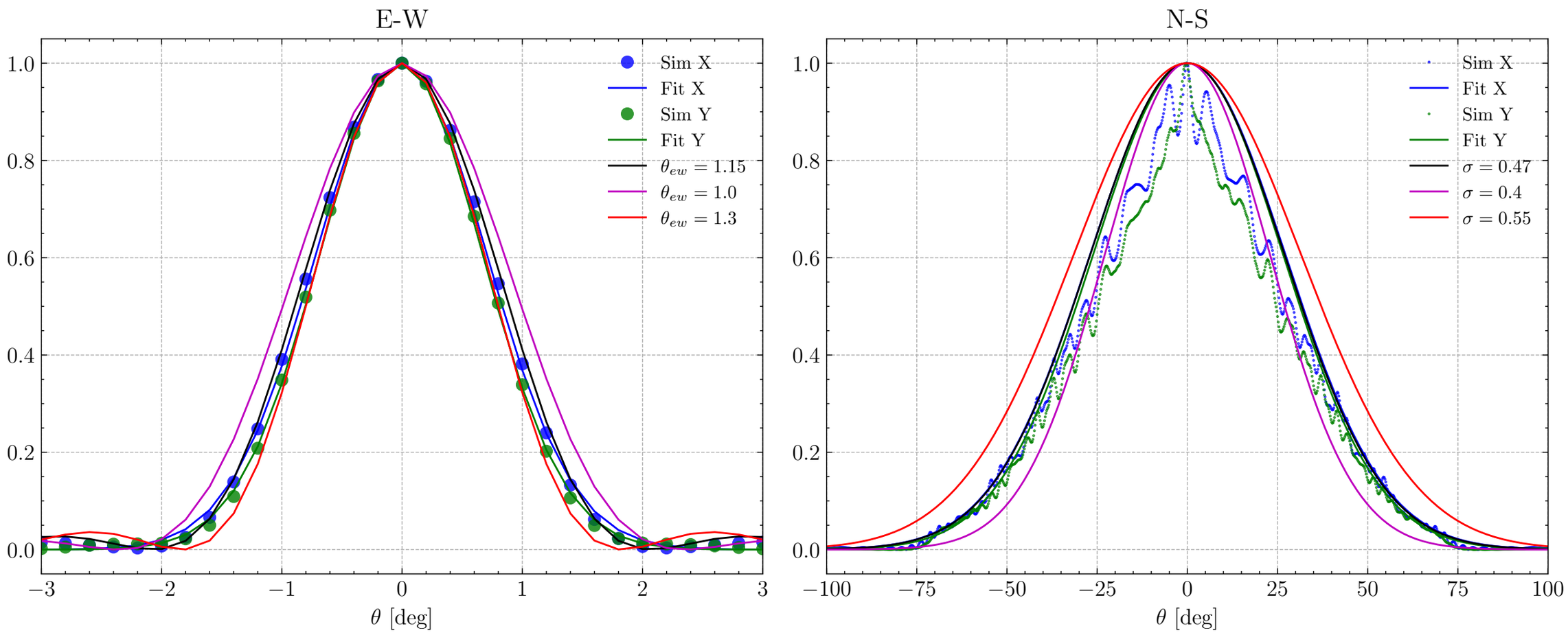
Reference Flux density @ 750MHz:
Cyg A - 2961.24 Jy
Cas A - 2830.90 Jy
Virgo A - 352.57 Jy
Crab A - 950.92 Jy



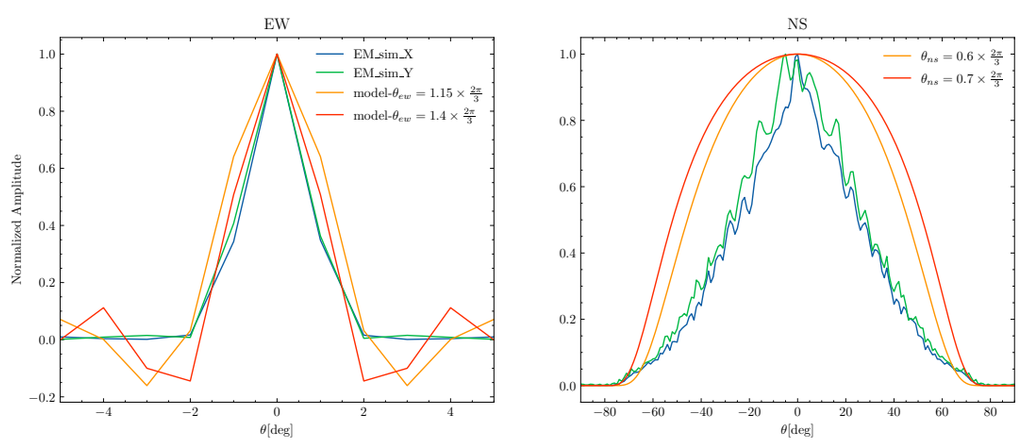
data from cyl_data_[20160927, 20180322]

Beam

EM Simulation Fitting



(Shaw et al.) model



- Change N-S direction ➔ Gaussian beam $e^{-\frac{\theta^2}{2\sigma^2}}$

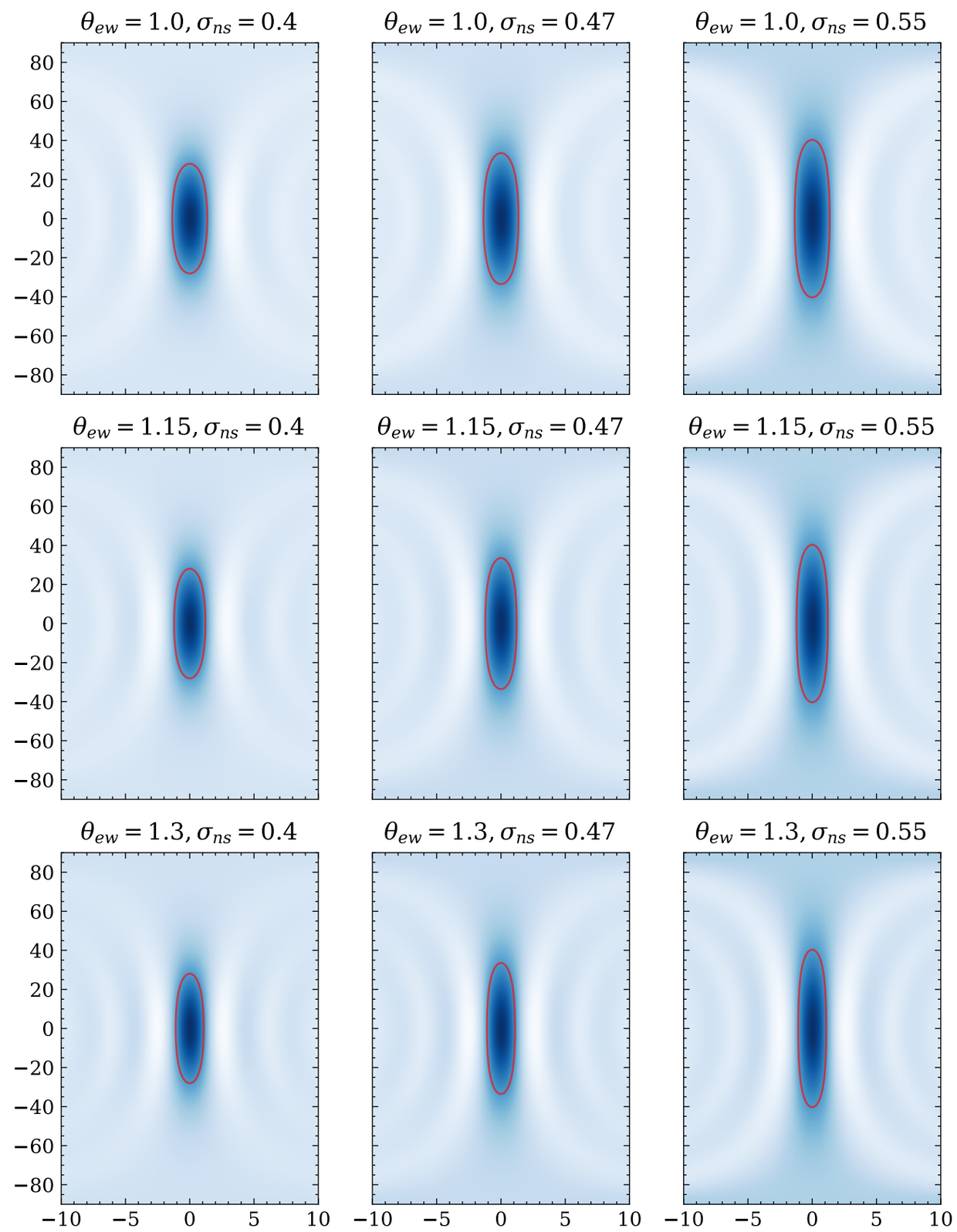
Beam

E-W: θ_{ew} = [1.0, 1.15, 1.3]

N-S: σ = [0.4, 0.47, 0.55]

N-S wider

E-W Narrower

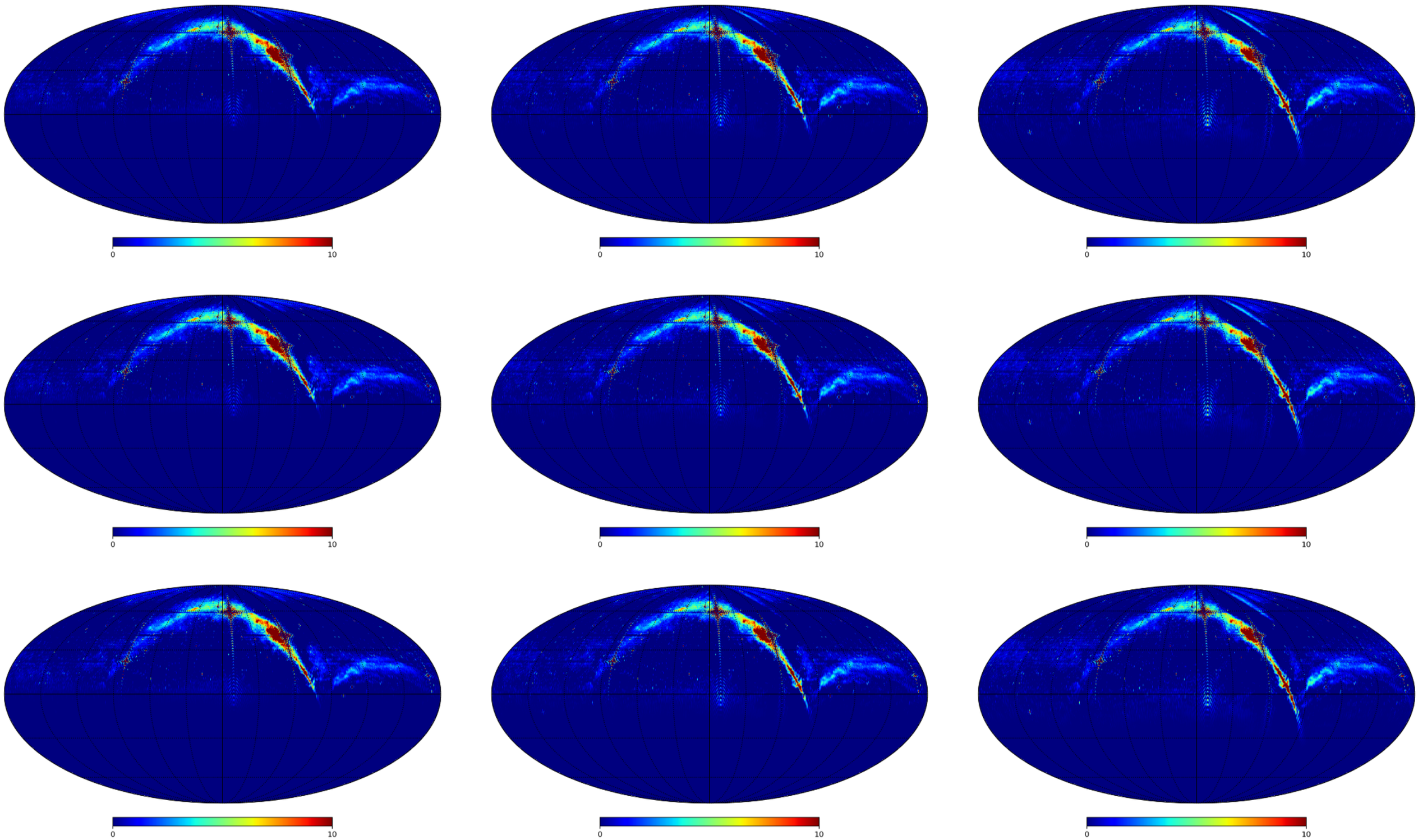


Beam

Consistent Beam

N-S wider

E-W narrower

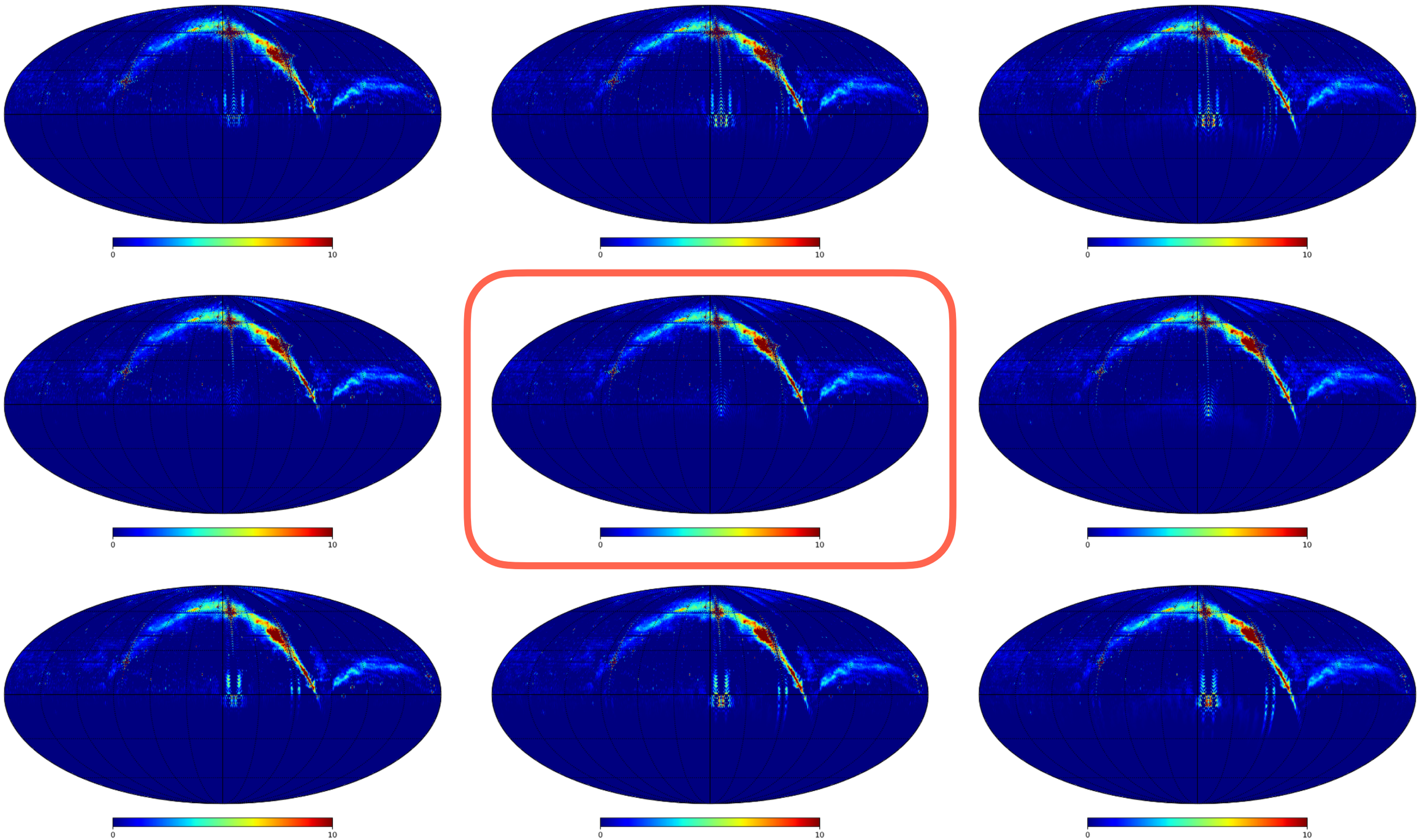


Beam

Inconsistent Beam

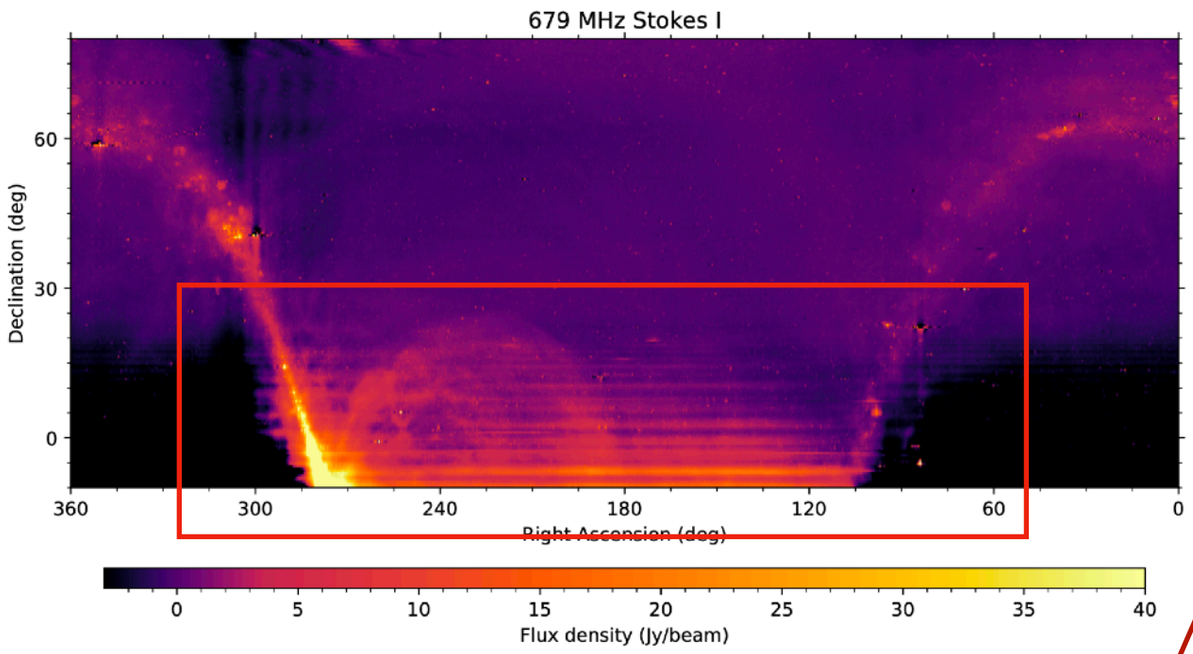
N-S wider

E-W narrower

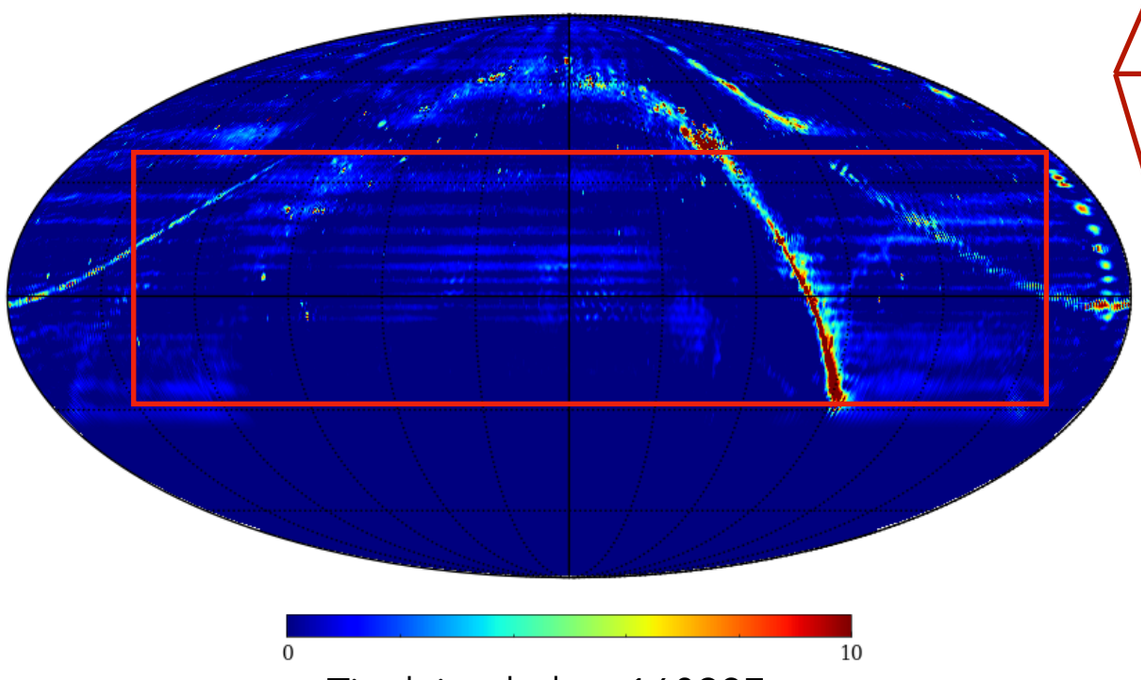


Cross-Coupling

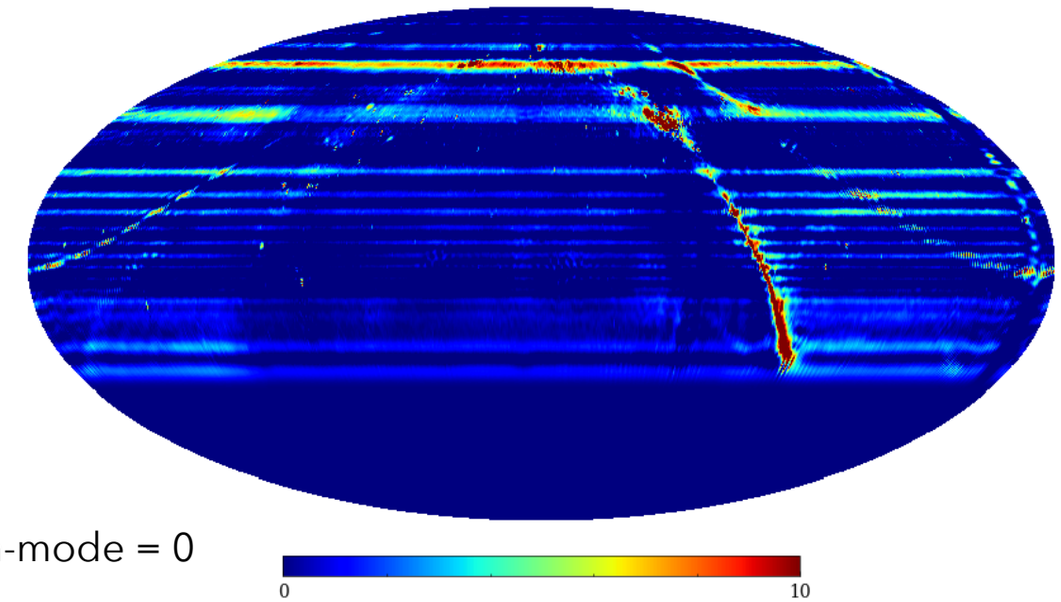
Horizontal stripes in images



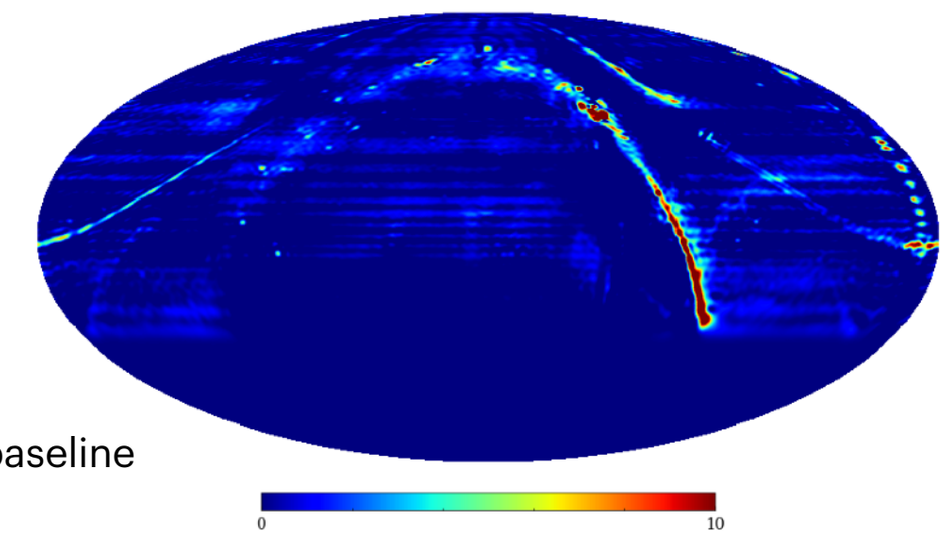
The CHIME Collaboration, 2022a



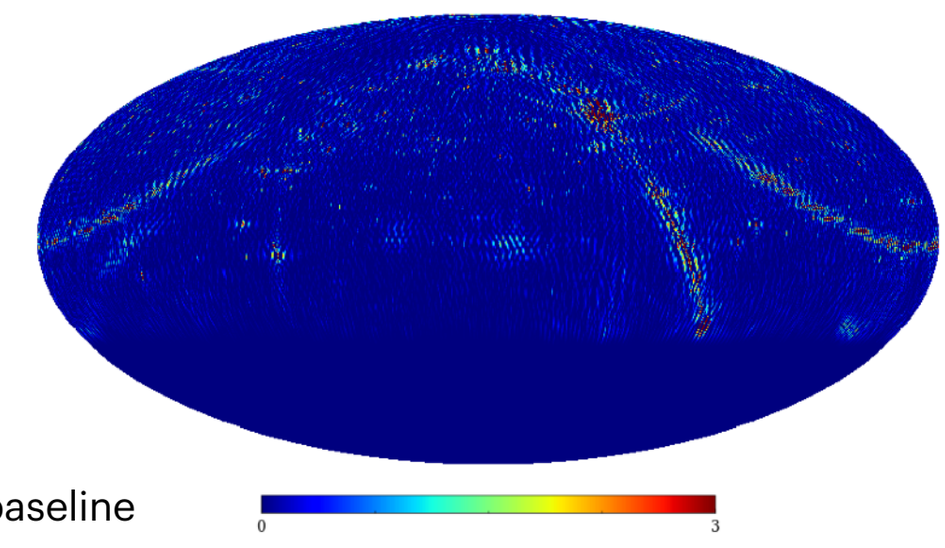
Tianlai, cyl_data 160927



includes m-mode = 0



intra-cylinder baseline



inter-cylinder baseline

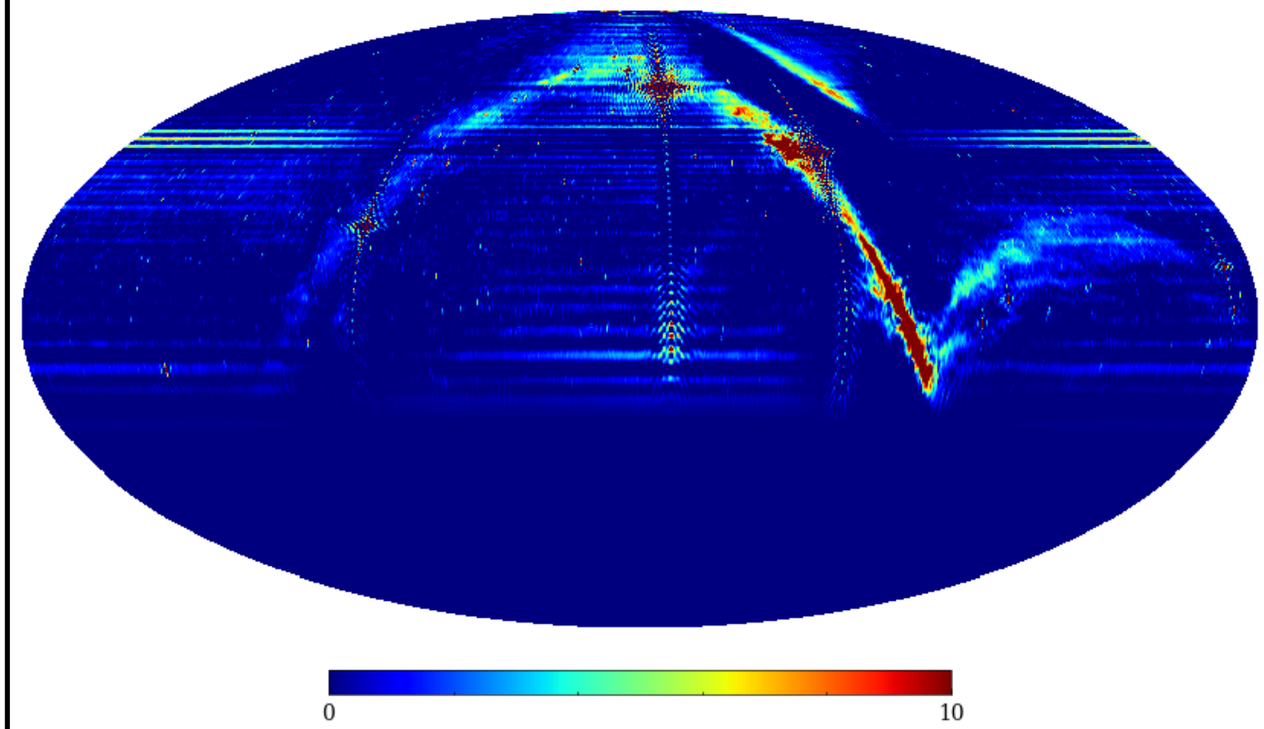
Cross-Coupling

From (Sun et al, 2022), equation (4)

$$
\begin{aligned}
V_{ij} &= \langle \tilde{U}_i^* \tilde{U}_j \rangle \\
&= \langle (U_i + S_{ij} U_j)^* (U_j + S_{ji} U_i) \rangle \\
&= (1 + S_{ij}{}^* S_{ji}) \langle U_i^* U_j \rangle + S_{ij}{}^* \langle |U_j|^2 \rangle + S_{ji} \langle |U_i|^2 \rangle
\end{aligned}
$$

omit $S_{ij}^* S_{ji}$ term, and take $\left\langle \left| U_i \right|^2 \right\rangle = CT^i_{\text{ant}}$

$$V_{ij} = V'_{ij} + S_{ij}^* \times CT_{sys,i} + S_{ji} \times CT_{sys,j}$$



[**JUST**] For illustration, take

transmission coefficients $S_{ij}(D_{ij}) = (e^{2iD_{ij}/\lambda})^{-0.9}$

$C = 0.05$

temperature is modeled as

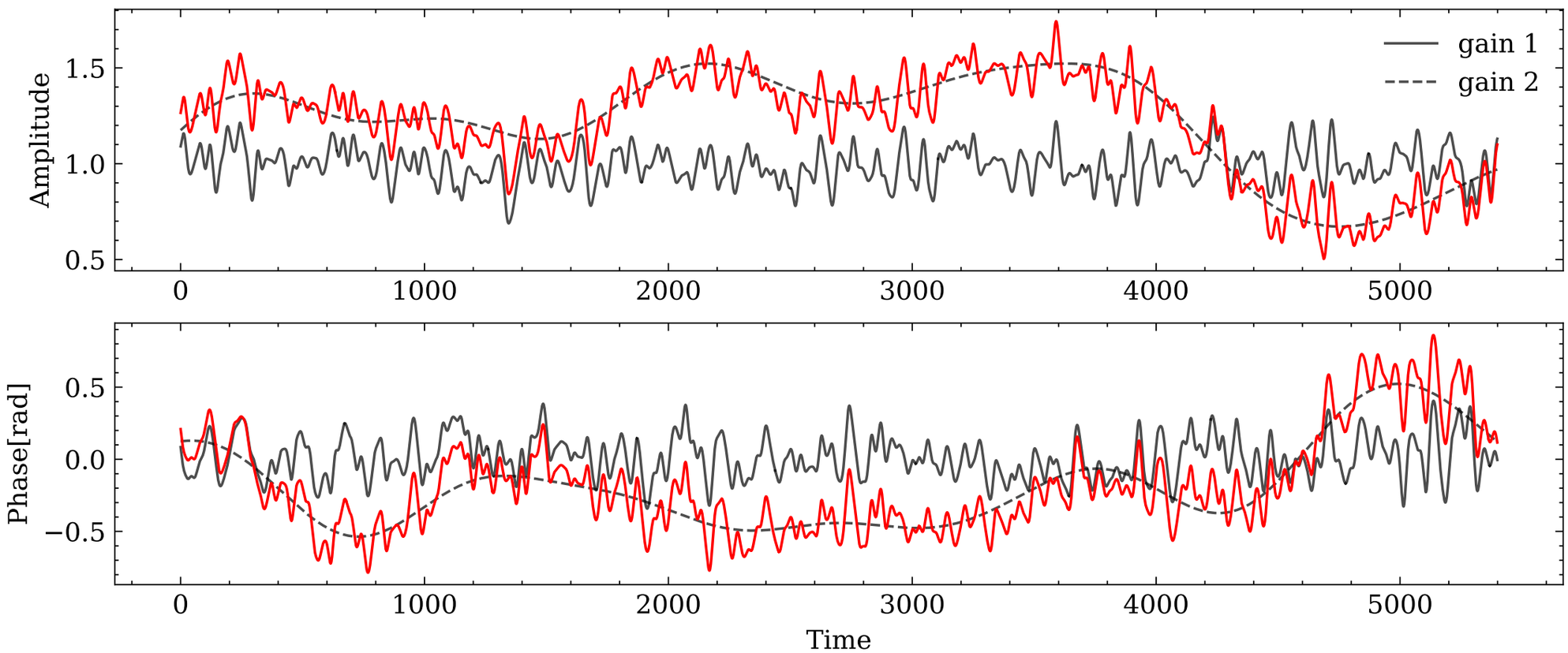
$$[T_i \sim \mathcal{N}(50.0,5.0)] + [5.0 \times \sin(\pi \frac{t}{T})]$$

Gain

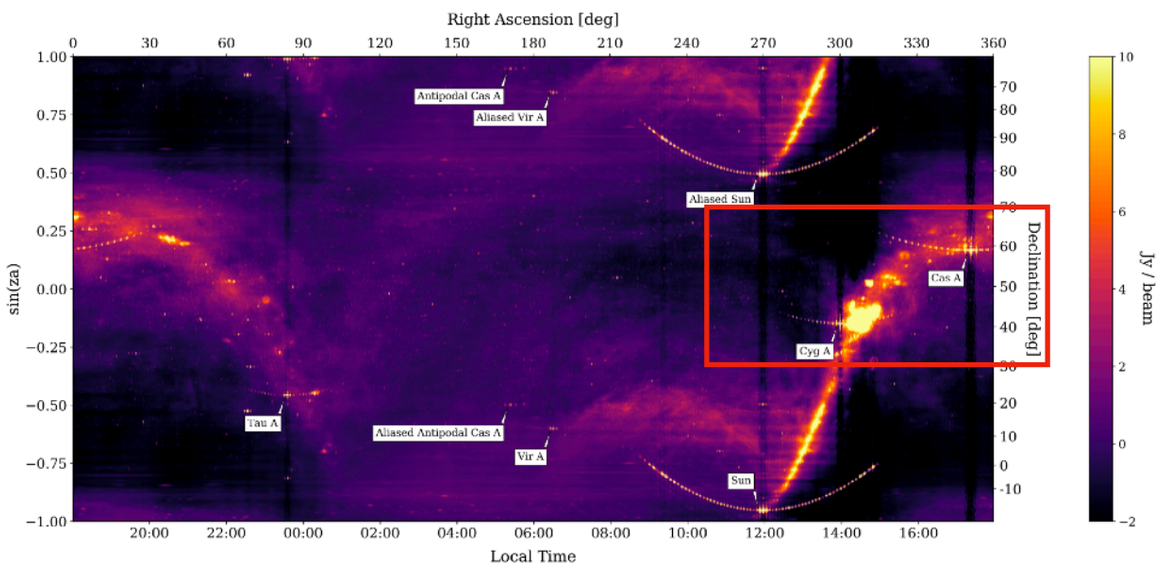
- *Amplitude* and *phase*, is generated by *Gaussian process* with covariance functions

$$C_{ij} = \sigma^2 \exp\left(-\frac{1}{2\xi^2}(t_i - t_j)^2\right)$$

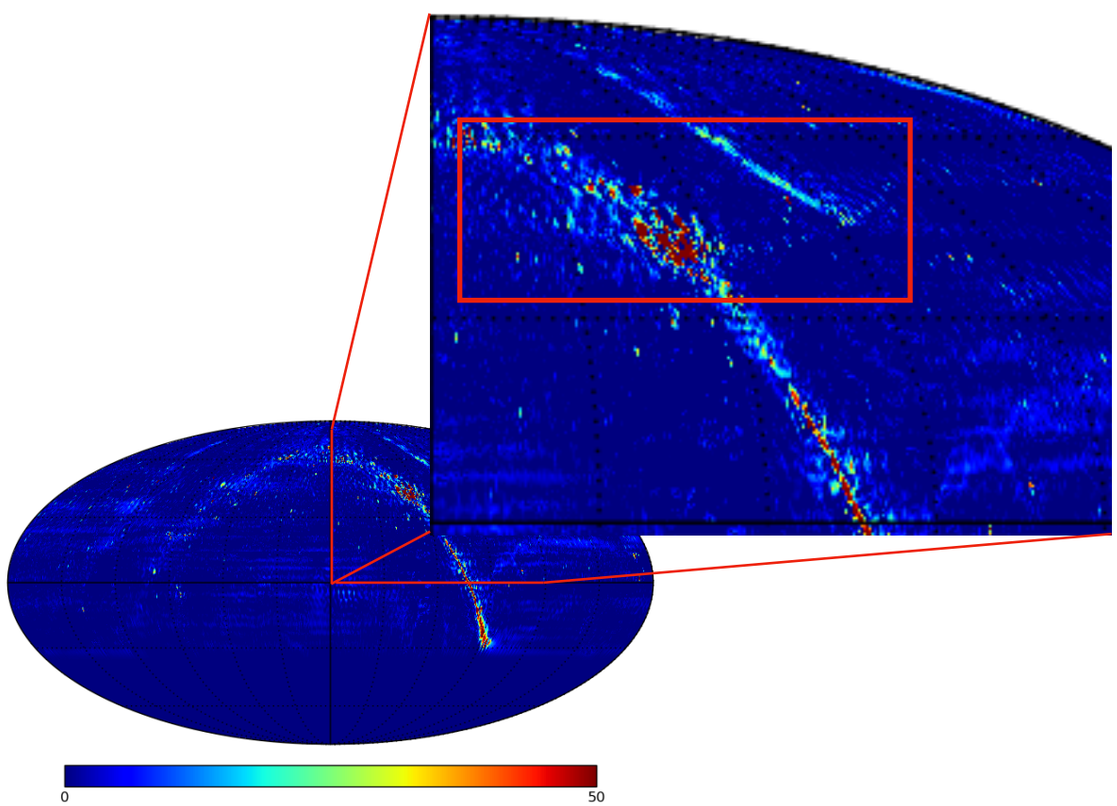
- Improved from **single** correlation length **to multiple**…



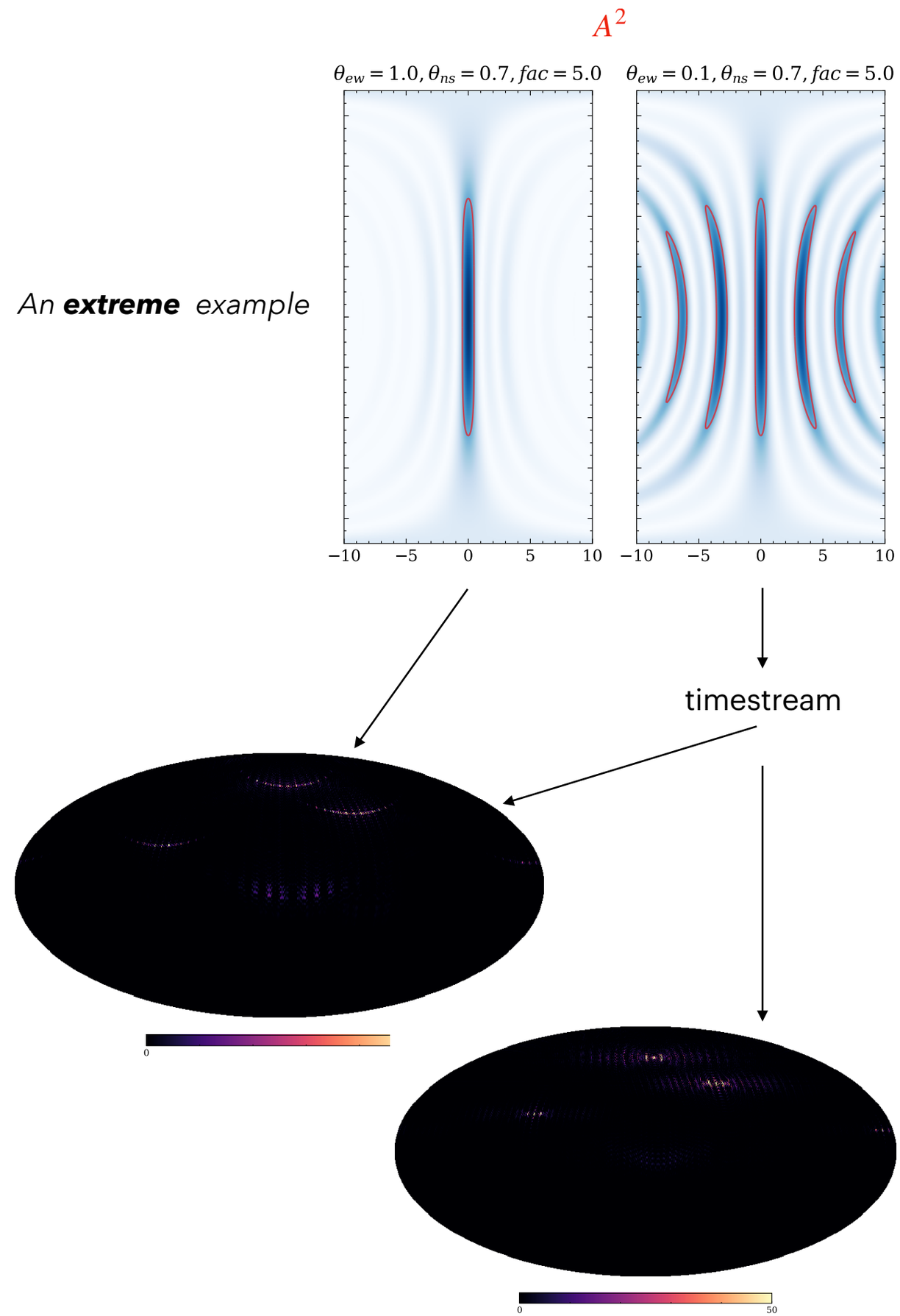
Side lobes



The CHIME Collaboration, 2022a



*An **extreme** example*



tlpipe

Calibration

NOT include Cross-Coupling…

ANS Calibration

$$V_{ij}^{\text{ns_on}} = g_i g_j^* (V_{ij}^{\text{sky}} + V_{ij}^{\text{ns}}) e^{ik\Delta L_{ij}} = |G_{ij}|(V_{ij}^{\text{sky}} + V_{ij}^{\text{ns}}) e^{i(k\Delta L_{ij} + \varphi_i - \varphi_j)} \tag{18}$$

$$V_{ij}^{\text{ns_off}} = |G_{ij}| V_{ij}^{\text{sky}} e^{i(k\Delta L_{ij} + \varphi_i - \varphi_j)} \tag{19}$$

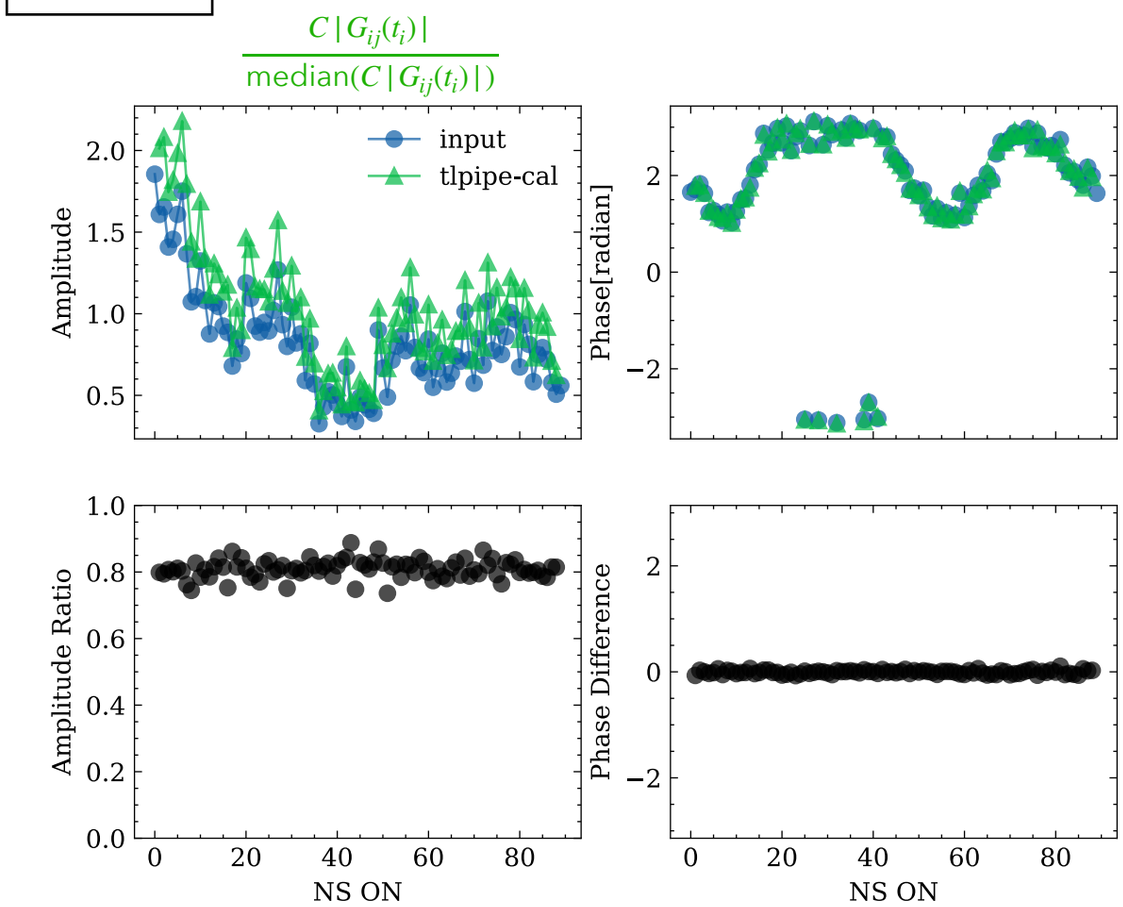
where $g_i = |g_i| e^{i\varphi_i}$ is the complex gain of feed i, $|G_{ij}| = |g_i||g_j|$, and ΔL_{ij} is the equivalent instrument delay difference between the feed i and j due to the variance in cable length we have introduced in section 2.5, then we have

$$V_{ij}^{\text{ns_on}} - V_{ij}^{\text{ns_off}} = |G_{ij}| V_{ij}^{\text{ns}} e^{i(k\Delta L_{ij} + \varphi_i - \varphi_j)} = C|G_{ij}| e^{i\phi_{ij}}, \tag{20}$$

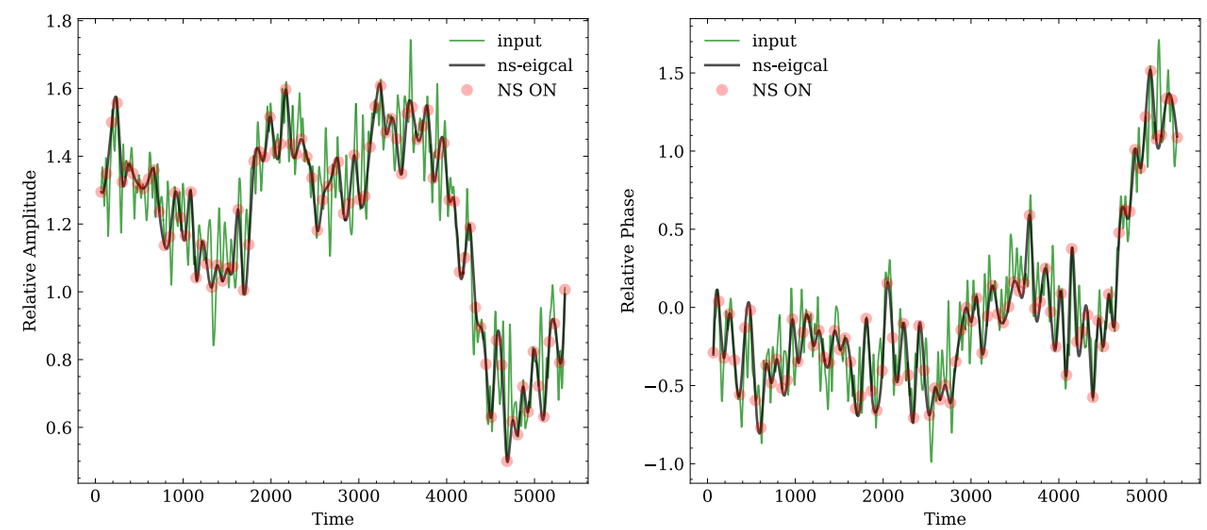
where $\phi_{ij} = k(\Delta L_{ij} + r_i - r_j) + \varphi_i - \varphi_j$ and using the reasonable

If we assume the NS is known exactly…

Feed Pair



single feed relative to feed 0



tlpipe

Calibration

Point Source Calibration

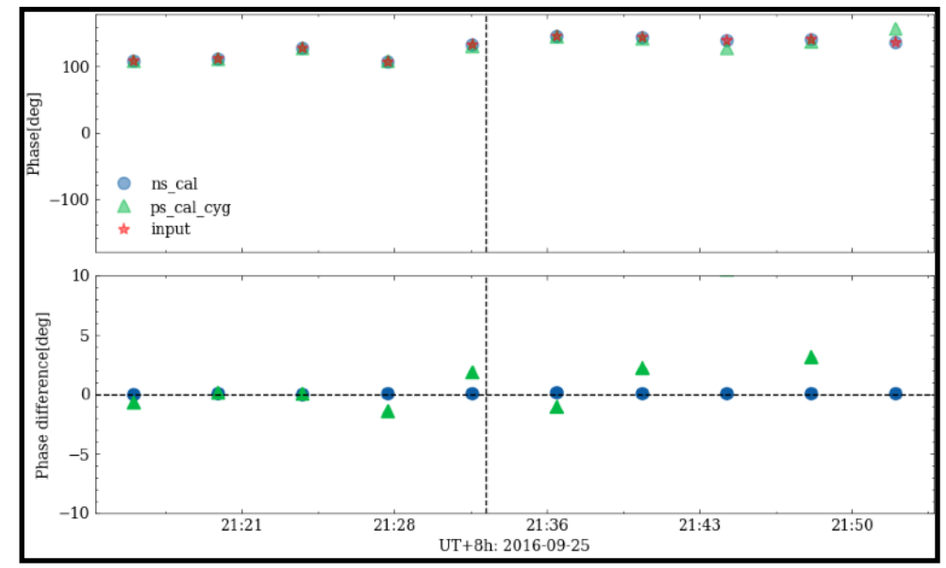
$$V_{ij}^{\mathrm{ps}} = S_c(|g_i|A_i(\hat{\boldsymbol{n}}_0)e^{i(kL_i+\varphi_i)+2\pi i\hat{\boldsymbol{n}}_0\cdot\vec{\boldsymbol{u}}_i})(|g_j|A_j(\hat{\boldsymbol{n}}_0)e^{-ik(kL_j+\varphi_j)+2\pi i\hat{\boldsymbol{n}}_0\cdot\vec{\boldsymbol{u}}_j})^*$$
$$= S_c G_i G_j^*$$

solved $G_i = |g_i|A_i(\hat{\boldsymbol{n}}_0)e^{-ik(L_i+\varphi_i)+2\pi i\hat{\boldsymbol{n}}_0\cdot\vec{\boldsymbol{u}}_i}$

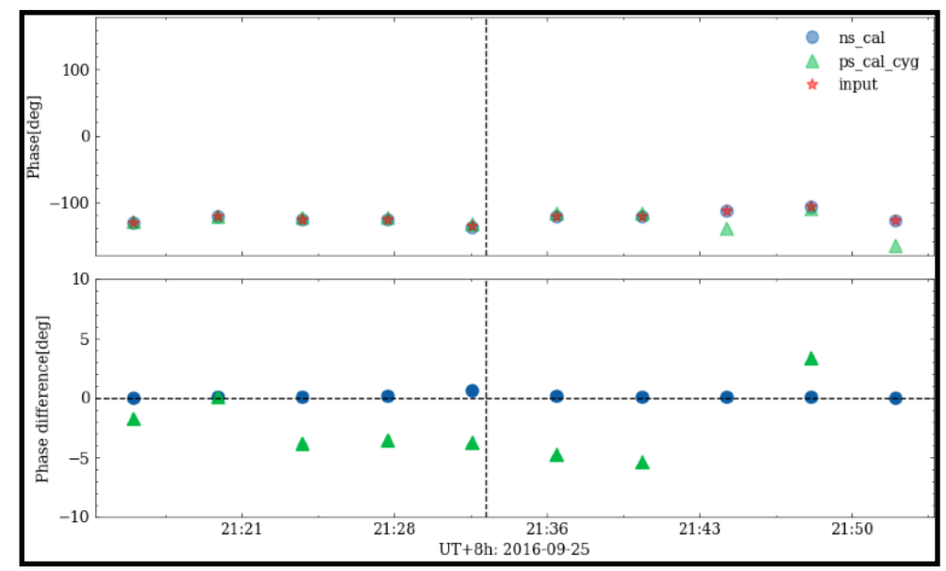
+

model: $A_i(\boldsymbol{n}_0)e^{2\pi i\hat{\boldsymbol{n}}_0\cdot\vec{\boldsymbol{u}}_i}$

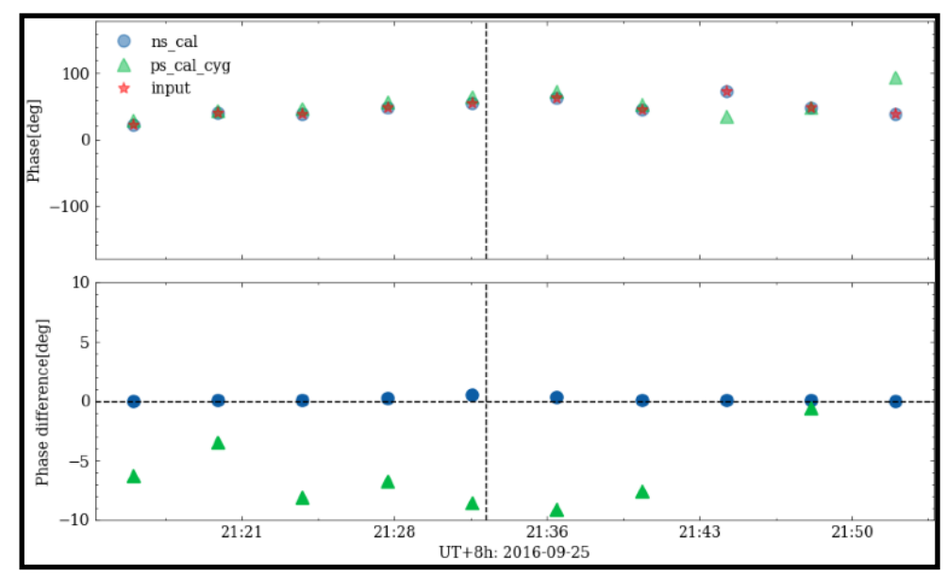
Phase Comparison



A



B



C

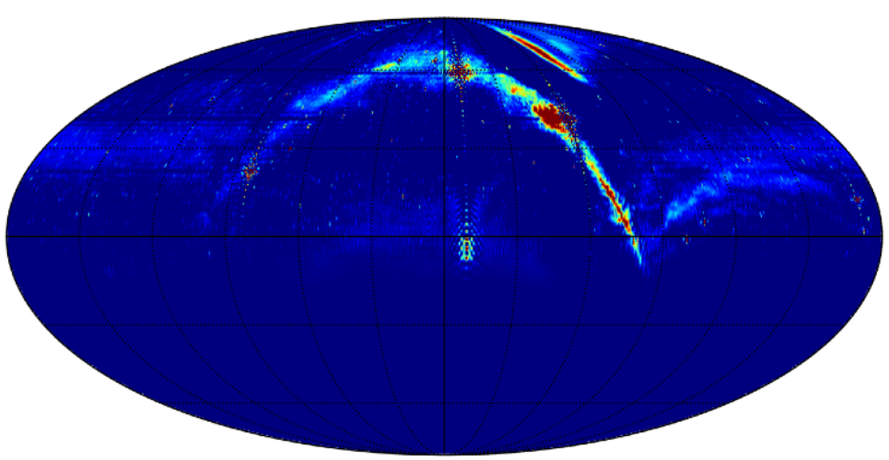
Map-Making

Tikhonov Regularization

$$\hat{\boldsymbol{a}}_{\text{Tikhonov}} = \left(\boldsymbol{B}^{\dagger}\boldsymbol{N}^{-1}\boldsymbol{B} + \varepsilon\boldsymbol{I}\right)^{-1}\boldsymbol{B}^{\dagger}\boldsymbol{N}^{-1}\boldsymbol{v}$$

Auto-Correlation fitting beam

N-S Gaussian beam



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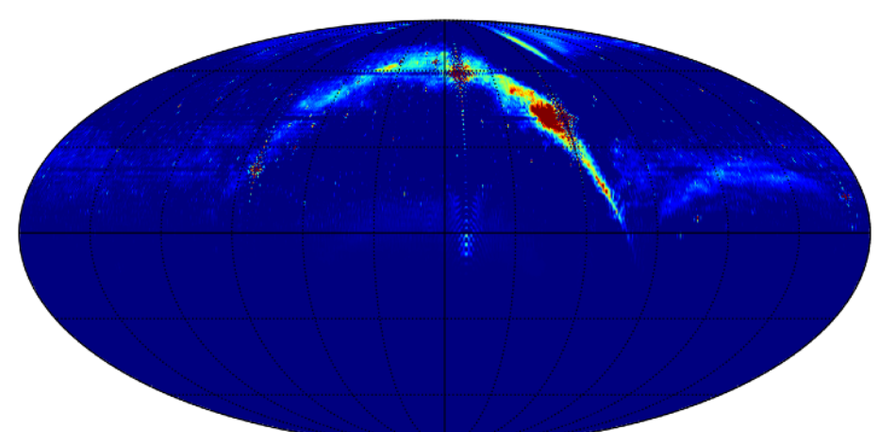

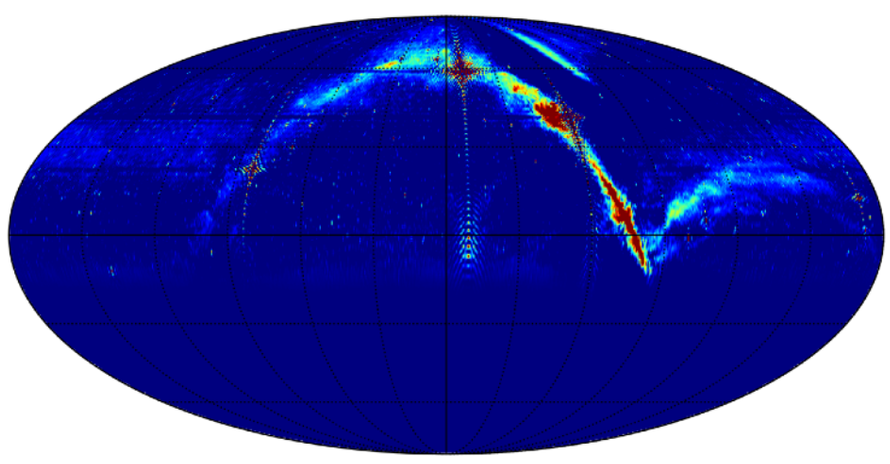

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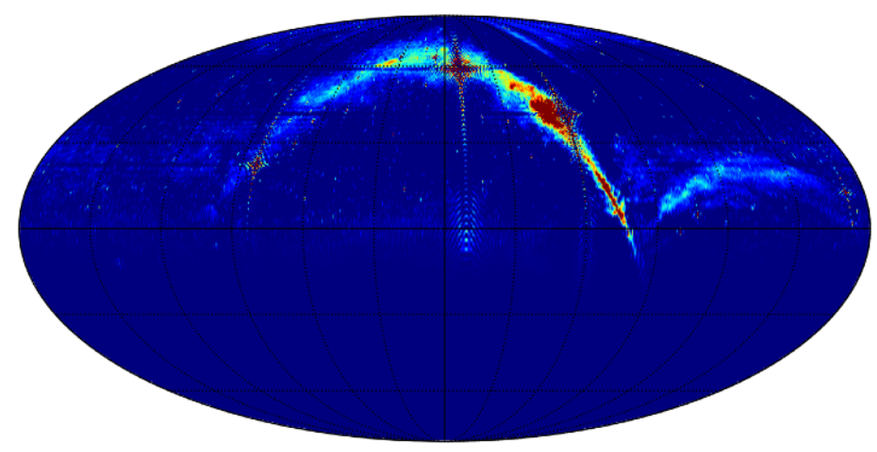

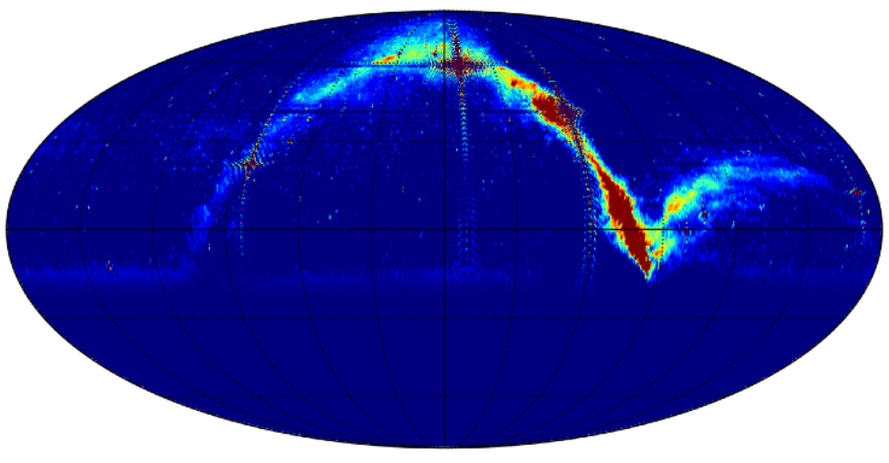

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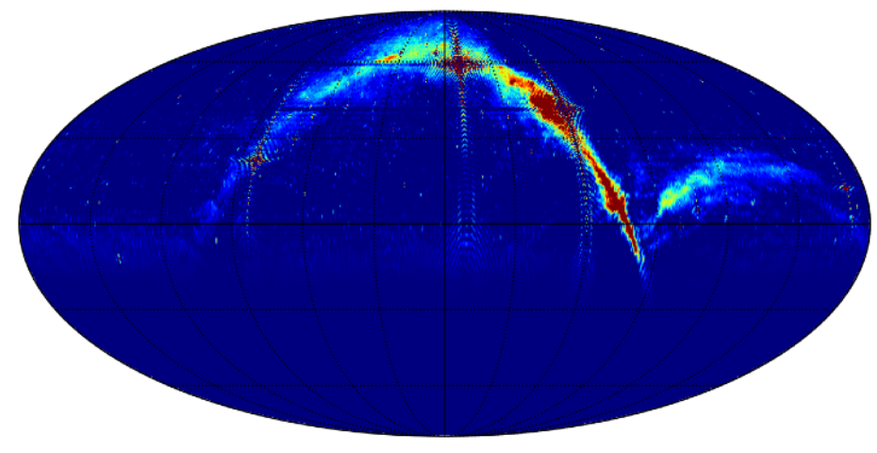

Summary

- A more “realistic” beam model and gain
- Cross-coupling and side-lobes in images
- Precision of Calibration?

- **NEXT?**
  - Multi-Frequencies
  - Cosmological aspects

THANKS