# Polarisation and calibration

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Tianlai collaboration meeting, Madison, WI, August 2022

# A word of caution

- I am not an expert on polarisation and radio interferometer polarisation calibration
- Peter asked me ton introduce this subject for the discussion
- And it would be indeed nice if we start thinking about Tianlai calibration including the polarisation and possibly make polarisation radio maps
- Fortunately, I found summer school lecture slides by Rick Perley (NRAO) which I will use extensively in the following slides
- I will still mostly raise questions that I hope we will be able to answer in the coming months

**Rick Perley** 2014 interferometer polarisation calibration course:
https://www.atnf.csiro.au/research/radio-school/2014/talks/ATCAPolarization2014.pdf

# Polarisation

- Electric field for strictly monochromatic radiation would draw an ellipse
- Polarisation state can be described either as the superposition of two orthogonal linear polarisation , or two circular (left, right) polarisation states
- We do not deal with pure monochromatic EM radiation in real life, and there is usually some finite bandwidth for detectors.
- One has usually antenna which are sensitive to linear polarisation , or sometimes circular polarisation
- Polarisation state is often described as the four Stokes parameters (I,Q,U,V) or (S0,S1,S2,S3)

Stokes parameter expression using linear polarisation complex (amplitude and phase) electric field components. **Four real Stokes parameters**

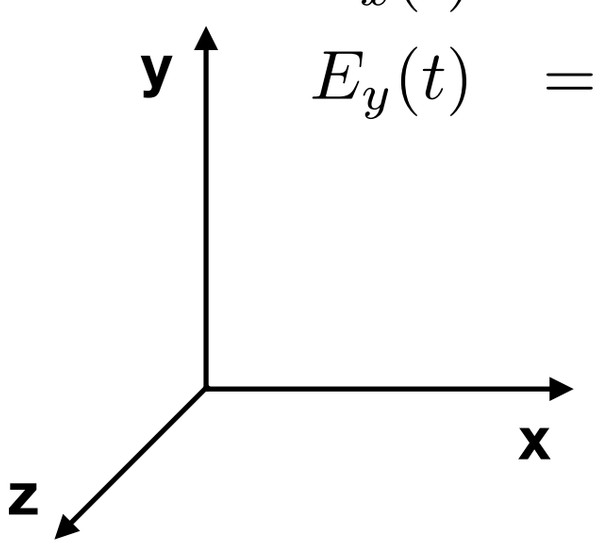
$$
\begin{aligned}
I &= \langle E_x E_x^* + E_y E_y^* \rangle \\
Q &= \langle E_x E_x^* - E_y E_y^* \rangle \\
U &= 2\langle \mathrm{Re}(E_x E_y^*) \rangle = \langle E_x E_y^* + E_x^* E_y \rangle \\
V &= -2\langle \mathrm{Im}(E_x E_y^*) \rangle = +i\langle E_x E_y^* - E_x^* E_y \rangle
\end{aligned}
$$

Monochromatic coherent radiation: $I^2 = Q^2 + U^2 + V^2$

General case : $Q^2 + U^2 + V^2 = I_p^2 < I^2$

$$
\begin{aligned}
E_x(t) &= E_x^0 \cos(\omega t + \phi) \\
E_y(t) &= E_y^0 \sin(\omega t + \psi)
\end{aligned}
$$

$\frac{I_p}{I}$ : polarised frac




**Unpolarised :**

$I > 0 \quad ; \quad Q = U = V = 0$

# Stokes Parameters – Monochromatic Case



- Perfectly monochromatic EM waves have an E-vector which traces a perfect ellipse in a fixed plane.
- We utilize in radio astronomy the parameters defined by George Stokes (1852), and introduced to astronomy by Chandrasekhar (1946):

$$
\begin{aligned}
I &= A_X^2 + A_Y^2 &&= A_R^2 + A_L^2 \\
Q &= A_X^2 - A_Y^2 &&= 2A_R A_L \cos\delta_{RL} \\
U &= 2A_X A_Y \cos\delta_{XY} &&= -2A_R A_L \sin\delta_{RL} \\
V &= -2A_X A_Y \sin\delta_{XY} &&= A_R^2 - A_L^2
\end{aligned}
$$

Units of power: Jy, or Jy/beam

where $A_X$ and $A_Y$ are the cartesian amplitude components of the E-field, and $\delta_{XY}$ is the phase lag between them, and

$A_R$ and $A_L$ are the opposite circular amplitude components of the E-field, and $\delta_{RL}$ the phase lag between them.

- By (IAU) convention, the 'X' axis points to the NCP, the 'Y' axis to the east.
- Also by IAU convention, LCP has the E-vector rotating clockwise for approaching radiation.
- Monochromatic radiation is 100% polarized: $I^2 = Q^2 + U^2 + V^2$



**From R. Perley (2014)**

# Stokes Visibilities



- In terms of the (complex) fields at antennas1 and 2, the Stokes Visibilities can be written as

$$\mathcal{I} = \left(\mathcal{E}_{X1}\mathcal{E}^*_{X2} + \mathcal{E}_{Y1}\mathcal{E}^*_{Y2}\right) \quad = \left(\mathcal{E}_{R1}\mathcal{E}^*_{R2} + \mathcal{E}_{L1}\mathcal{E}^*_{L2}\right)$$
$$\mathcal{Q} = \left(\mathcal{E}_{X1}\mathcal{E}^*_{X2} - \mathcal{E}_{Y1}\mathcal{E}^*_{Y2}\right) \quad = \left(\mathcal{E}_{R1}\mathcal{E}^*_{L2} + \mathcal{E}_{L1}\mathcal{E}^*_{R2}\right)$$
$$\mathcal{U} = \left(\mathcal{E}_{X1}\mathcal{E}^*_{Y2} + \mathcal{E}_{Y1}\mathcal{E}^*_{X2}\right) \quad = -i\left(\mathcal{E}_{R1}\mathcal{E}^*_{L2} - \mathcal{E}_{L1}\mathcal{E}^*_{R2}\right)$$
$$\mathcal{V} = -i\left(\mathcal{E}_{X1}\mathcal{E}^*_{Y2} - \mathcal{E}_{Y1}\mathcal{E}^*_{X2}\right) = \left(\mathcal{E}_{R1}\mathcal{E}^*_{R2} - \mathcal{E}_{L1}\mathcal{E}^*_{L2}\right)$$

where I have dropped the angle brackets <> to reduce clutter.

- The script symbols $\mathcal{I}, \mathcal{Q}, \mathcal{U}, \mathcal{V}$ remind us that these Stokes Visibilities are complex numbers, related to the (real) source brightness through Fourier transform.
- $\mathcal{I} \Longleftrightarrow$ I, $\mathcal{Q} \Longleftrightarrow$ Q, $\mathcal{U} \Longleftrightarrow$ U, $\mathcal{V} \Longleftrightarrow$ V



From R. Perley (2014)

The four stokes parameters in visibility (Fourier) space

$$R_{X1X2} = \langle V_{X1} V_{X2}^* \rangle = (\mathcal{I} + \mathcal{Q})/2$$

$$R_{Y1Y2} = \langle V_{Y1} V_{Y2}^* \rangle = (\mathcal{I} - \mathcal{Q})/2$$

$$R_{X1Y2} = \langle V_{X1} V_{Y2}^* \rangle = (\mathcal{U} + i\mathcal{V})/2$$

$$R_{Y1X2} = \langle V_{Y1} V_{X2}^* \rangle = (\mathcal{U} - i\mathcal{V})/2$$

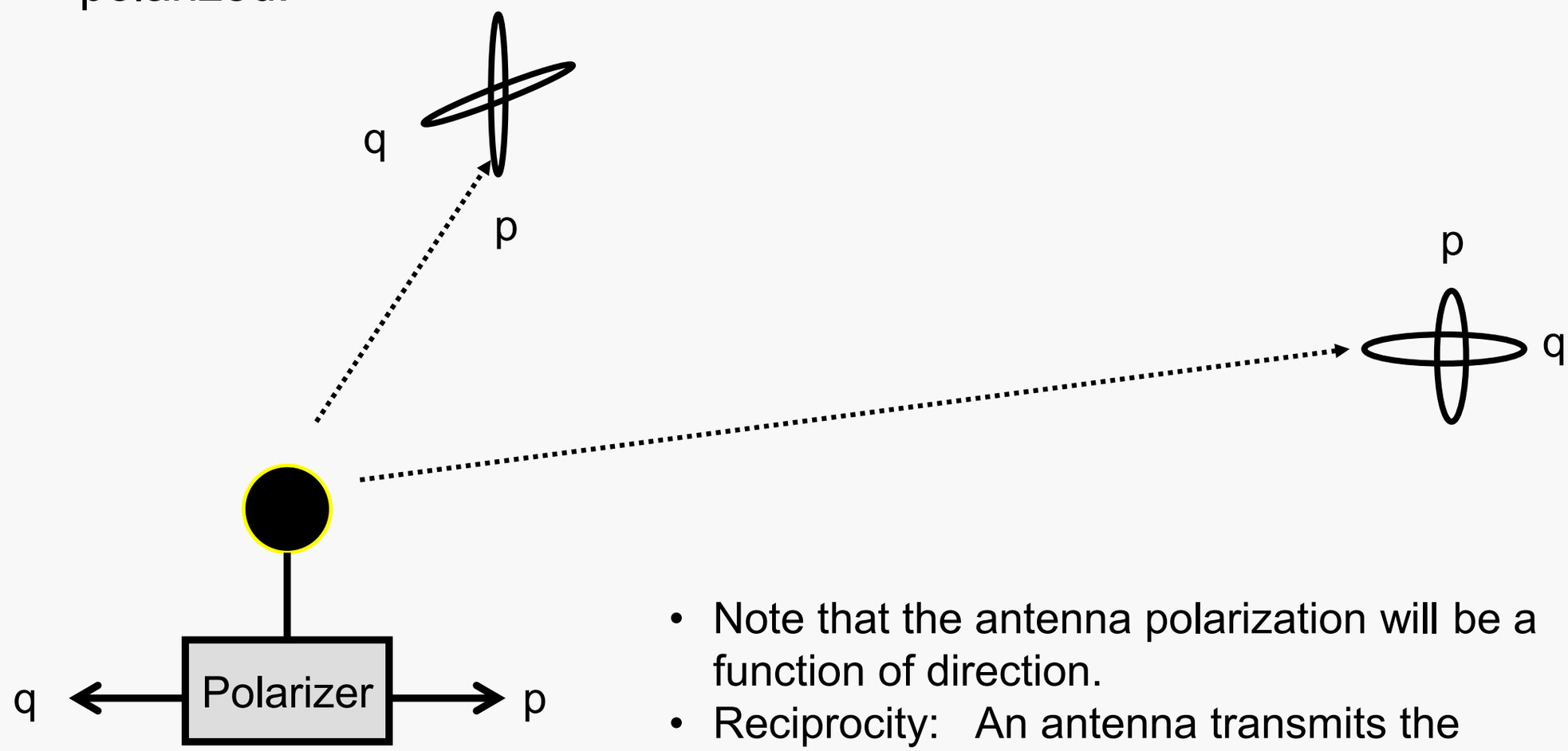
The four visibilities computed from the two linear polarisations (X,Y) of antenna 1,2

- Determine the instrument response assuming known sky or some point source polarisation
- We have four complex parameters (8 real values), and a number of unknowns -
- Gains of each four signals X1, Y1, X2, Y2,
- but also orientation of the antenna, cross-polarisation terms…
- The mathematical formalism uses Jones matrices (for a given antenna) and Mueller matrices for the four parallel and perpendicular visibility terms fo a given pair of antenna
- No bright polarised source in sky, except Cygnus A & Crab
- Galactic synchrotron is however significantly polarised at high galactic latitudes
- Many highly polarised much fainter (~10 mJy) radio sources, mostly pulsars

# Polarization of Real Antennas



- Unfortunately, antennas never provide perfectly orthogonal outputs.
- In general, the two outputs from an antenna are elliptically polarized.




- Note that the antenna polarization will be a function of direction.
- Reciprocity: An antenna transmits the same polarization that it receives.

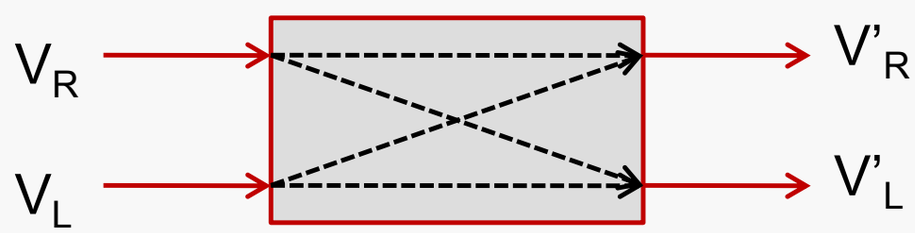


**From R. Perley (2014)**

# Jones Matrix Algebra



- The analysis of how a real interferometer, comprising real antennas and real electronics, is greatly facilitated through use of Jones matrices.
- In this, we break up our general system into a series of 4-port components, each of which is presumed to be linear.
- We consider each component to have two inputs and two outputs:




- And write:

$$\begin{pmatrix} V'_{R'} \\ V'_L \end{pmatrix} = \begin{pmatrix} G_{RR} & G_{LR} \\ G_{RL} & G_{LL} \end{pmatrix} \begin{pmatrix} V_R \\ V_L \end{pmatrix}$$

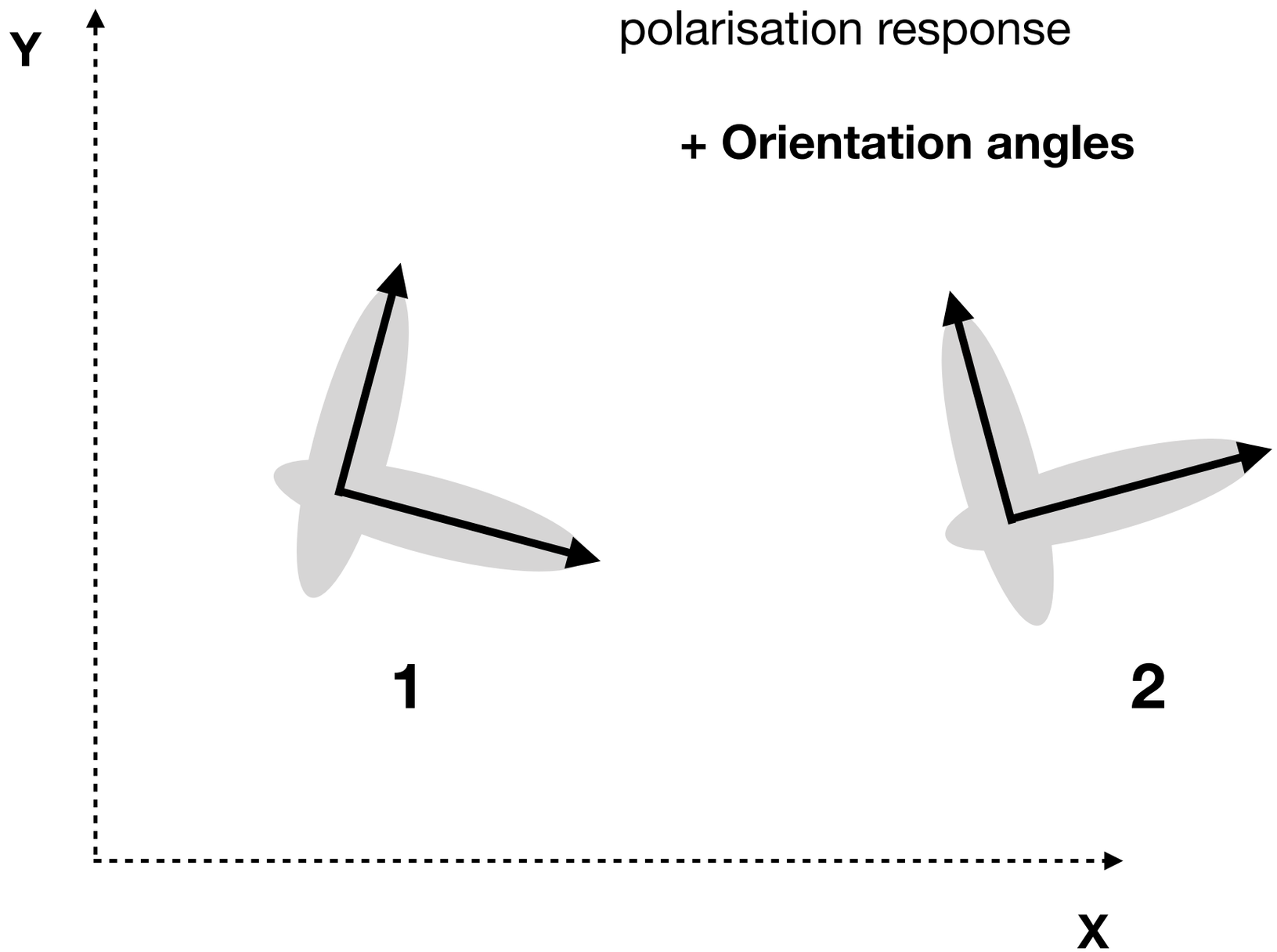
- Or, in shorthand **V' = JV**
- The four G components of the Jones matrix describe the linkages within the 'grey box'.



**From R. Perley (2014)**

Jone's Matrix characterise each antenna polarisation response

**+ Orientation angles**



# The Mueller Matrix



- There are four cross products, and four complex Stokes visibilitities.
- The results (details at end of this talk) can be compactly written as a matrix equation:

$$\mathbf{R} = \mathbf{G} \cdot \mathbf{P} \cdot \mathbf{\Psi} \cdot \mathbf{S}$$

Where

- **R** = the response vector – the correlator output.
- **G** = the gain matrix – effect of post-polarizer amplifier
- **P** = the polarization mixing matrix (Mueller matrix)
- $\Psi$ = the antenna rotation matrix (can include propagation)
- **S** = the Stokes vector – what we want.



**From R. Perley (2014)**

- Planck Galactic synchrotron polarisation maps (2015, 2018)

https://www.aanda.org/articles/aa/full_html/2016/10/aa25967-15/aa25967-15.html

https://www.aanda.org/articles/aa/full_html/2020/09/aa33881-18/aa33881-18.html

- Cygnus A polarisation measurement

https://arxiv.org/pdf/2009.06554.pdf

- High polarisation radio sources (pulsars …)

https://aas.aanda.org/articles/aas/pdf/1999/09/ds1597.pdf

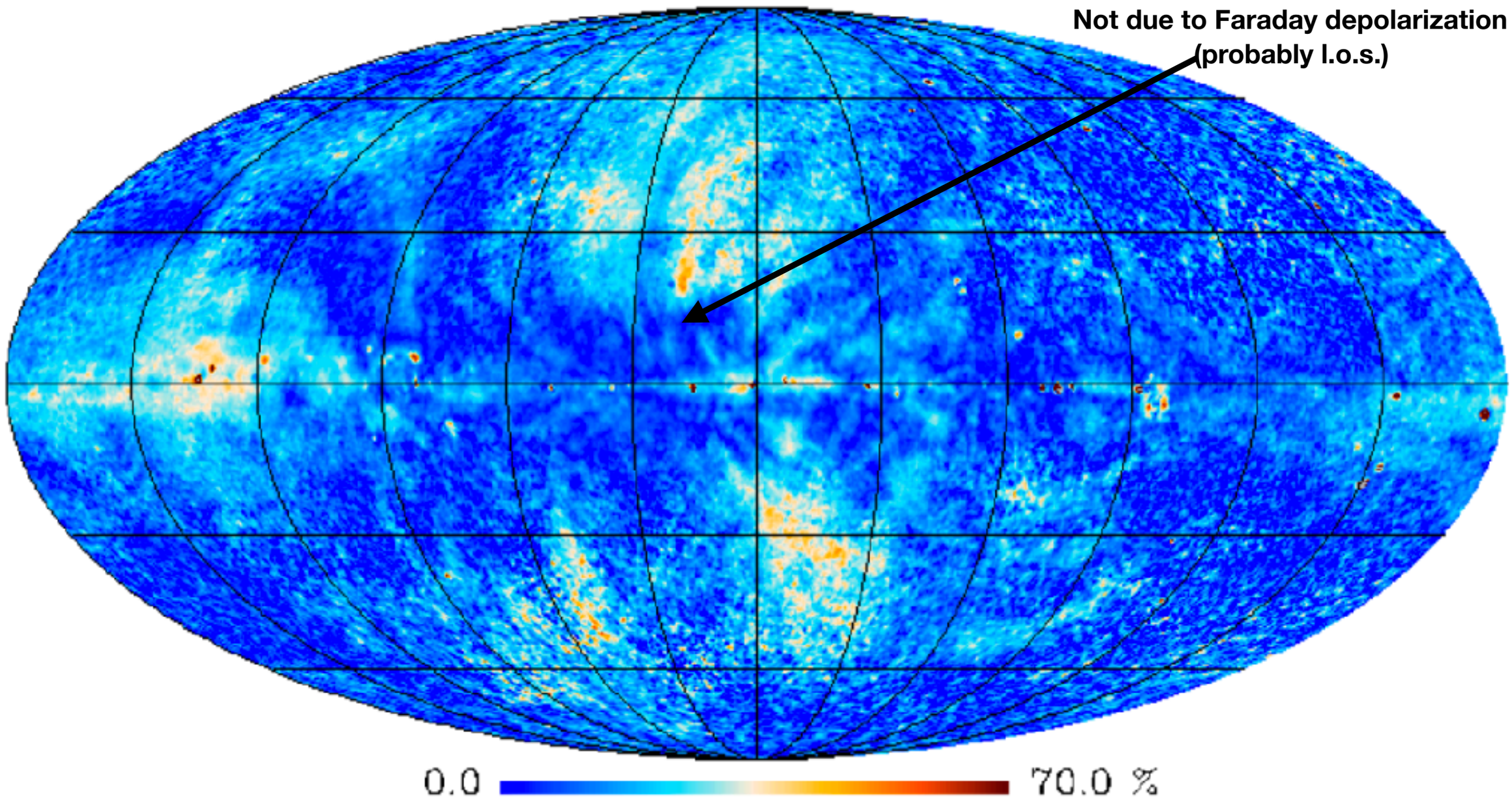
https://arxiv.org/abs/1007.2367

- Polarised point source catalog by LOFAR

https://www.aanda.org/articles/aa/full_html/2018/05/aa32228-17/aa32228-17.html#R45

# Synchrotron polarization fraction

**Polarization fractions up to ~40-50% suggest very coherent magnetic field (theoretical max~75% therefore very little depolarization)**




Planck Collaboration et al. XXV (2015)

**Slide by Clive Dickinson (2017)** / Data Planck 2015 , Galactic synchrotron polarisation maps

- Use unpolarised sky sources to determine as many parameters as possible
- Can not determine orientation angle, maybe orientation difference between antenna ?
- We need to reduce the full problem into a simplified case that we could solve using  several non polarised bright sources and one or two polarised sources (Crab and Cygnus A )
- Could we use then the diffuse synchrotron polarisation map to further refine the polarisation calibration and reconstruction ?
- What if we just reconstruct intensity (no Q,U,V) , what are the science cases we will miss ?
- Any suggestion / comment / question welcome …