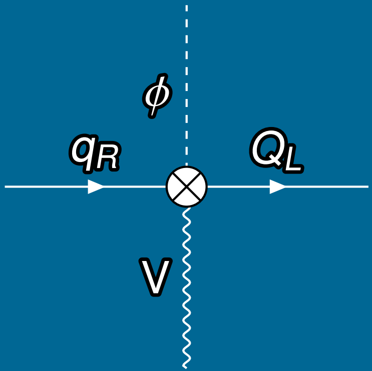

ALP-SMEFT Interference

Theoretical Physics Seminar
Fermilab

Anne Galda

in collaboration with
Matthias Neubert, Sophie Renner

JHEP 06, 135 (2021)

PRISMA+

JOHANNES GUTENBERG UNIVERSITÄT MAINZ

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Outline

- RG Evolution, ALP Lagrangian
- ALP-SMEFT Interference
- Effects on the UV Scale Evolution of the Dipole Coefficients
- Summary and Outlook

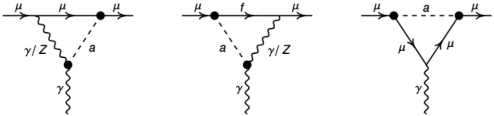
Axions and Axion-Like Particles (ALPs)

Well-motivated candidates for

[1]

- The solution of the **strong CP-problem**
  [Peccei, Quinn (1977); Weinberg (1978); Wilczek (1978)]

- A contribution to $a_\mu^{\mathrm{exp}} - a_\mu^{\mathrm{SM}} = 4.2\sigma$
  [B. Abi et al. (2021)]



- Dark Matter
  [Preskill, Wise, Wilczek (1983)]

[1] https://indico.cern.ch/event/484258/attachments/1213724/1771273/HTJCX.pdf (20/11/2021)

Effective Lagrangian for the Axion-Like Particle

Assume an ALP a that is

- classically shift symmetric ($a \to a + c$)
- a gauge singlet
- a pseudoscalar
- massive with mass m_a

most general Lagrangian:

[H. Georgi, D. B. Kaplan, L. Randall: Phys.Lett.B 169 (1986) 73-78]

$$\mathcal{L}_{\mathrm{ALP}}^{D\leq 5} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 + \frac{\partial^\mu a}{f} \sum_F \bar{\Psi}_F \boldsymbol{c}_F \gamma_\mu \Psi_F$$

$$+ \; c_{GG} \frac{\alpha_s}{4\pi} \frac{a}{f} G_{\mu\nu}^a \tilde{G}^{\mu\nu,a} + c_{WW} \frac{\alpha_2}{4\pi} \frac{a}{f} W_{\mu\nu}^A \tilde{W}^{\mu\nu,A} + c_{BB} \frac{\alpha_1}{4\pi} \frac{a}{f} B_{\mu\nu} \tilde{B}^{\mu\nu}$$

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kinetic and mass term

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coupling to chiral fermion multiplets F

$\boldsymbol{c}_F$: hermitian matrices in generation space

$\hookrightarrow$ allow for flavor off-diagonal couplings

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coupling to gauge fields $G_{\mu\nu,a}$, $W_{\mu\nu}$, $B_{\mu\nu}$

$\tilde{G}^{\mu\nu,a} = \frac{1}{2}\epsilon^{\mu\nu\alpha\beta}G^a_{\alpha\beta}$: dual field strength tensor

Alternative Form of the Effective Lagrangian

Assume an ALP a that is

- classically shift symmetric ($a \to a + c$)
- a gauge singlet
- a pseudoscalar
- massive with mass m_a

alternative form of the Lagrangian:

[M. Bauer, et al.: arXiv:2012.12272]

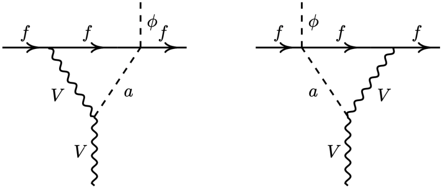
$$\mathcal{L}_{\rm ALP}^{D\leq 5} = \frac{1}{2}(\partial_\mu a)(\partial^\mu a) - \frac{m_{a,0}^2}{2} a^2 - \frac{a}{f}\left(\bar{Q}_L\,\phi\,\hat{\boldsymbol{Y}}_{\boldsymbol{d}}\,d_R + \bar{Q}_L\tilde{\phi}\,\hat{\boldsymbol{Y}}_{\boldsymbol{u}}\,u_R + \bar{L}\phi\,\hat{\boldsymbol{Y}}_{\boldsymbol{e}}\,e_R + \text{h.c.}\right) + C_{GG}\frac{a}{f}\,G_{\mu\nu}^a\tilde{G}^{\mu\nu,a} + C_{BB}\frac{a}{f}\,B_{\mu\nu}\tilde{B}^{\mu\nu} + C_{WW}\frac{a}{f}\,W_{\mu\nu}^A\tilde{W}^{\mu\nu,A}$$

$\Rightarrow$ effective Higgs-Fermion-Fermion-ALP vertex!

$\hat{\boldsymbol{Y}}_{\boldsymbol{d}} = i(\boldsymbol{Y}_{\boldsymbol{d}}\,\boldsymbol{c}_d - \boldsymbol{c}_Q\,\boldsymbol{Y}_{\boldsymbol{d}})$, $\quad C_{GG} = \frac{\alpha_s}{4\pi}\left[c_{GG} + \frac{1}{2}\mathrm{Tr}(\boldsymbol{c}_d + \boldsymbol{c}_u - 2\boldsymbol{c}_Q)\right]$ etc.

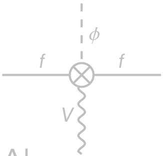
ALP-SMEFT Interference

virtual **ALP** exchange induces **UV-divergent** one-loop graphs, first studied in the case of $(g-2)_\mu$ [Marciano, Masiero, Paradisi, Passera (2016); Bauer, Neubert, Thamm (2017)]



$\sim 1/\epsilon$

→ requires local **dimension-6 operators** as **counterterms**!



↪ generated **independently of the ALP-mass** at Λ!

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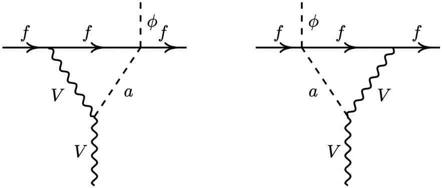

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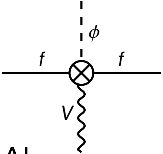

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Standard Model Effective Field Theory

basic idea: SM is the IR limit of the full theory. [Buchmüller, Wyler (1986)]
↪ describe the UV theory in terms of higher dimensional SM operators

$$\mathcal{L}_{\mathrm{SMEFT}} = \mathcal{L}_{\mathrm{SM}} + \frac{1}{\Lambda}\mathcal{L}_5 + \frac{1}{\Lambda^2}\mathcal{L}_6 + \cdots = \sum_d \frac{1}{\Lambda^{d-4}} \sum_{i=1}^{n_d} C_i^{(d)}(\mu)\, Q_i^{(d)}(\mu)$$

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all possible Operators of dimension d

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Wilson coefficients

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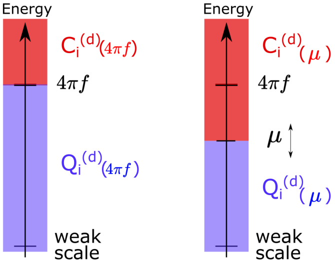


[arXiv:1901.06573 [hep-ph]]

Running of Wilson Coefficients

Factorization Scale μ is arbitrary!
$\hookrightarrow$ non-observable quantity

$$\frac{\mathrm{d}\,\mathcal{L}_{\mathrm{EFT}}}{\mathrm{d}\log\mu} = \frac{\mathrm{d}}{\mathrm{d}\log\mu}\sum_{d}\frac{1}{\Lambda^{d-4}}\sum_{i=1}^{n_d} C_i^{(d)}(\mu)Q_i^{(d)}(\mu) \overset{!}{=} 0$$

Running of Wilson Coefficients

Factorization Scale μ is arbitrary!
$\hookrightarrow$ non-observable quantity

$$\frac{d\,C_i(\mu)}{d\log\mu}Q_i(\mu) + C_i(\mu)\frac{d\,Q_i(\mu)}{d\log\mu} = 0$$

define:

$$\frac{d\,Q_i(\mu)}{d\log\mu} = -\gamma_{ij}(\mu)Q_j(\mu)$$

anomalous dimension matrix

RG Evolution Equation:

$$\frac{d\,C_i(\mu)}{d\log\mu} = \gamma_{ji}(\mu)C_j(\mu)$$

SMEFT Warsaw Basis

minimal dimension-6 basis: 59 operators

[Grzadkowski, Iskrzynski, Misiak, Rosiek (2010)]

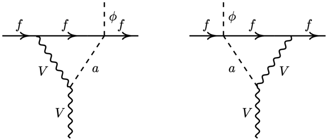
X^3		φ^6 and $\varphi^4 D^2$		$\psi^2\varphi^3$	
Q_G	$f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	Q_φ	$(\varphi^\dagger\varphi)^3$	$Q_{e\varphi}$	$(\varphi^\dagger\varphi)(\bar l_p e_r \varphi)$
$Q_{\widetilde G}$	$f^{ABC} \widetilde G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$	$Q_{\varphi\Box}$	$(\varphi^\dagger\varphi)\Box(\varphi^\dagger\varphi)$	$Q_{u\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p u_r \widetilde\varphi)$
Q_W	$\varepsilon^{IJK} W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$	$Q_{\varphi D}$	$\left(\varphi^\dagger D^\mu\varphi\right)^\star \left(\varphi^\dagger D_\mu\varphi\right)$	$Q_{d\varphi}$	$(\varphi^\dagger\varphi)(\bar q_p d_r \varphi)$
$Q_{\widetilde W}$	$\varepsilon^{IJK} \widetilde W_\mu^{I\nu} W_\nu^{J\rho} W_\rho^{K\mu}$				

$X^2\varphi^2$		$\psi^2 X$	
$Q_{\varphi G}$	$\varphi^\dagger\varphi\, G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eW}	$(\bar l_p \sigma^{\mu\nu}$
$Q_{\varphi\widetilde G}$	$\varphi^\dagger\varphi\, \widetilde G^A_{\mu\nu} G^{A\mu\nu}$	Q_{eB}	$(\bar l_p \sigma$
$Q_{\varphi W}$	$\varphi^\dagger\varphi\, W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uG}	$(\bar q_p \sigma^{\mu\nu}$
$Q_{\varphi\widetilde W}$	$\varphi^\dagger\varphi\, \widetilde W^I_{\mu\nu} W^{I\mu\nu}$	Q_{uW}	$(\bar q_p \sigma^{\mu\nu}$
$Q_{\varphi B}$	$\varphi^\dagger\varphi\, B_{\mu\nu} B^{\mu\nu}$	Q_{uB}	$(\bar q_p \sigma$
$Q_{\varphi\widetilde B}$	$\varphi^\dagger\varphi\, \widetilde B_{\mu\nu} B^{\mu\nu}$	Q_{dG}	$(\bar q_p \sigma^{\mu\nu}$
$Q_{\varphi WB}$	$\varphi^\dagger\tau^I\varphi\, W^I_{\mu\nu} B^{\mu\nu}$	Q_{dW}	$(\bar q_p \sigma^{\mu\nu}$
$Q_{\varphi\widetilde WB}$	$\varphi^\dagger\tau^I\varphi\, \widetilde W^I_{\mu\nu} B^{\mu\nu}$	Q_{dB}	$(\bar q_p \sigma$

$(\bar LL)(\bar LL)$		$(\bar RR)(\bar RR)$		$(\bar LL)(\bar RR)$	
Q_{ll}	$(\bar l_p\gamma_\mu l_r)(\bar l_s\gamma^\mu l_t)$	Q_{ee}	$(\bar e_p\gamma_\mu e_r)(\bar e_s\gamma^\mu e_t)$	Q_{le}	$(\bar l_p\gamma_\mu l_r)(\bar e_s\gamma^\mu e_t)$
$Q_{qq}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar q_s\gamma^\mu q_t)$	Q_{uu}	$(\bar u_p\gamma_\mu u_r)(\bar u_s\gamma^\mu u_t)$	Q_{lu}	$(\bar l_p\gamma_\mu l_r)(\bar u_s\gamma^\mu u_t)$
$Q_{qq}^{(3)}$	$(\bar q_p\gamma_\mu \tau^I q_r)(\bar q_s\gamma^\mu \tau^I q_t)$	Q_{dd}	$(\bar d_p\gamma_\mu d_r)(\bar d_s\gamma^\mu d_t)$	Q_{ld}	$(\bar l_p\gamma_\mu l_r)(\bar d_s\gamma^\mu d_t)$
$Q_{lq}^{(1)}$	$(\bar l_p\gamma_\mu l_r)(\bar q_s\gamma^\mu q_t)$	Q_{eu}	$(\bar e_p\gamma_\mu e_r)(\bar u_s\gamma^\mu u_t)$	Q_{qe}	$(\bar q_p\gamma_\mu q_r)(\bar e_s\gamma^\mu e_t)$
$Q_{lq}^{(3)}$	$(\bar l_p\gamma_\mu \tau^I l_r)(\bar q_s\gamma^\mu \tau^I q_t)$	Q_{ed}	$(\bar e_p\gamma_\mu e_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar u_s\gamma^\mu u_t)$
		$Q_{ud}^{(1)}$	$(\bar u_p\gamma_\mu u_r)(\bar d_s\gamma^\mu d_t)$	$Q_{qu}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar u_s\gamma^\mu T^A u_t)$
		$Q_{ud}^{(8)}$	$(\bar u_p\gamma_\mu T^A u_r)(\bar d_s\gamma^\mu T^A d_t)$	$Q_{qd}^{(1)}$	$(\bar q_p\gamma_\mu q_r)(\bar d_s\gamma^\mu d_t)$
				$Q_{qd}^{(8)}$	$(\bar q_p\gamma_\mu T^A q_r)(\bar d_s\gamma^\mu T^A d_t)$

$(\bar LR)(\bar RL)$ and $(\bar LR)(\bar LR)$		B-violating	
Q_{ledq}	$(\bar l_p^j e_r)(\bar d_s q_t^j)$	Q_{duq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(q_s^{\gamma j})^T C l_t^k\right]$
$Q_{quqd}^{(1)}$	$(\bar q_p^j u_r)\varepsilon_{jk}(\bar q_s^k d_t)$	Q_{qqu}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jk}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{quqd}^{(8)}$	$(\bar q_p^j T^A u_r)\varepsilon_{jk}(\bar q_s^k T^A d_t)$	Q_{qqq}	$\varepsilon^{\alpha\beta\gamma}\varepsilon_{jn}\varepsilon_{km}\left[(q_p^{\alpha j})^T C q_r^{\beta k}\right]\left[(q_s^{\gamma m})^T C l_t^n\right]$
$Q_{lequ}^{(1)}$	$(\bar l_p^j e_r)\varepsilon_{jk}(\bar q_s^k u_t)$	Q_{duu}	$\varepsilon^{\alpha\beta\gamma}\left[(d_p^\alpha)^T C u_r^\beta\right]\left[(u_s^\gamma)^T C e_t\right]$
$Q_{lequ}^{(3)}$	$(\bar l_p^j \sigma_{\mu\nu} e_r)\varepsilon_{jk}(\bar q_s^k \sigma^{\mu\nu} u_t)$		

consistent treatment of the $1/\epsilon$ poles:
embedding of the ALP model in SMEFT via



$$\underbrace{\mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{SM}} + \mathcal{L}_{\text{ALP}} + \mathcal{L}_{\text{SMEFT}}}$$

ALP contributes **source terms** to the $D = 6$ SMEFT Wilson coefficients

$$\frac{d}{d\ln\mu}\, C_i^{\text{SMEFT}} - \gamma_{ji}^{\text{SMEFT}} C_j^{\text{SMEFT}} = \frac{S_i}{(4\pi f)^2} \quad \text{for } \mu < 4\pi f$$

[AG, Neubert, Renner: 2105.01078]

$\hookrightarrow$ SMEFT Wilson coefficients are generated at the scale $\Lambda = 4\pi f$ **independent of the ALP mass**!

ALP-SMEFT Interference: systematic study

Consider an off-shell basis of $D = 6$ SM-operators.

↓

Compute all amputated, one-loop divergent Green's functions with virtual ALP exchange.

↓

Apply the SM EoM to project onto Warsaw basis.

↓

Renormalize the bare Wilson coefficients.

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Consider an off-shell basis of $D = 6$ SM-operators.

Three different classes of operators:

Purely bosonic
X^3
X^2D^2
X^2H^2
XH^2D^2
H^6
H^4D^2
H^2D^4

Single fermion current
ψ^2XD
ψ^2D^3
ψ^2XH
ψ^2H^3
ψ^2H^2D
ψ^2HD^2

4-fermion operators
$(\bar{L}L)(\bar{L}L)$
$(\bar{R}R)(\bar{R}R)$
$(\bar{L}L)(\bar{R}R)$
$(\bar{L}R)(\bar{R}L)$
$(\bar{L}R)(\bar{L}R)$
B-violating

blue: operator NOT present in the Warsaw basis

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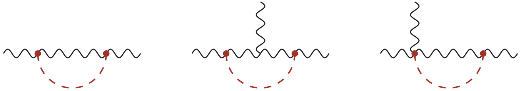
Apply the SM EoM to project onto Warsaw basis.

↓

Renormalize the bare Wilson coefficients.

Some example diagrams ...

Purely bosonic
X^3
$X^2 D^2$
$X^2 H^2$
$X H^2 D^2$
H^6
$H^4 D^2$
$H^2 D^4$



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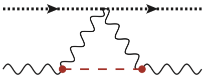
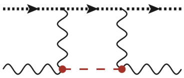
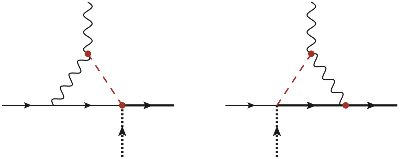




ALP-SMEFT Interference: systematic study

Some example diagrams ...

Single fermion current
$\psi^2 X D$
$\psi^2 D^3$
$\psi^2 X H$
$\psi^2 H^3$
$\psi^2 H^2 D$
$\psi^2 H D^2$



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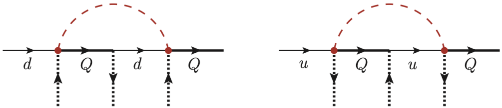

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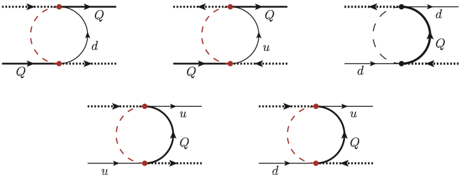
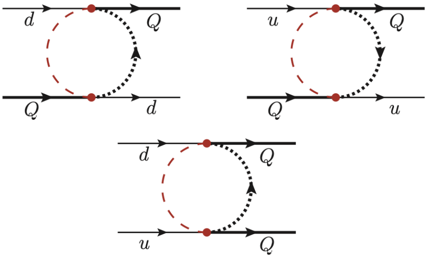


ALP-SMEFT Interference: systematic study

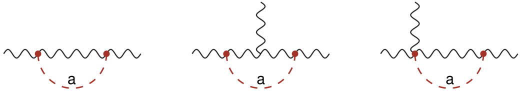
Some example diagrams ...

4-fermion operators
$(\bar{L}L)(\bar{L}L)$
$(\bar{R}R)(\bar{R}R)$
$(\bar{L}L)(\bar{R}R)$
$(\bar{L}R)(\bar{R}L)$
$(\bar{L}R)(\bar{L}R)$
B-violating



Example: Classes X^3 and $X^2 D^2$

Purely bosonic
X^3
$X^2 D^2$
$X^2 H^2$
$X H^2 D^2$
H^6
$H^4 D^2$
$H^2 D^4$



Feynman Diagrams: $\sum_i D_i^{\text{ALP}} \equiv \frac{i\mathcal{A}}{(4\pi f)^2}$

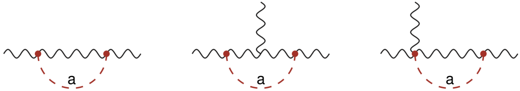
Weinberg-operator: $f^{ABC} G_\mu^{A\nu} G_\nu^{B\rho} G_\rho^{C\mu}$

$$\mathcal{A}(gg(g)) = -\frac{C_{GG}^2}{\epsilon}\left[4g_s \langle Q_G \rangle + \frac{4}{3}\langle \widehat{Q}_{G,2}\rangle - 2m_a^2 \langle G_{\mu\nu}^a G^{\mu\nu,a}\rangle\right] + \text{finite}$$

redundant operator: $\widehat{Q}_{G,2} = (D^\rho G_{\rho\mu})^a \, (D_\omega G^{\omega\mu})^a$

Example: Classes X^3 and X^2D^2

Purely bosonic
X^3
X^2D^2
X^2H^2
XH^2D^2
H^6
H^4D^2
H^2D^4



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redundant operator: $\widehat{Q}_{G,2} = (D^\rho G_{\rho\mu})^a (D_\omega G^{\omega\mu})^a$

ALP-SMEFT Interference: systematic study

What about $\widehat{Q}_{G,1} = (D_\rho G_{\mu\nu})^a \, (D^\rho G^{\mu\nu})^a$?

related via the **Bianchi identity** $D_\alpha\, G_{\beta\gamma} + D_\gamma\, G_{\alpha\beta} + D_\beta\, G_{\gamma\alpha} = 0$ to the Weinberg- and the $\hat{Q}_{G,2}$-operator via

$$2 g_s Q_G + \widehat{Q}_{G,1} - 2\widehat{Q}_{G,2} = 0 .$$

Need to consider only $\widehat{Q}_{G,2}$!

ALP-SMEFT Interference: systematic study

Consider an off-shell basis of $D = 6$ SM-operators.

↓

Compute all amputated, one-loop divergent Green's functions with virtual ALP exchange.

↓

Apply the SM EoM to project onto Warsaw basis.

↓

Renormalize the bare Wilson coefficients.

Transformation to the Warsaw Basis

Example: $\widehat{Q}_{G,2} = (D^\rho G_{\rho\mu})^a \, (D_\omega G^{\omega\mu})^a$

need the **SM equation of motion**

$$D_\rho \, G^{\rho\mu,a} = -\, g_s \, (\bar{Q}_L \, \gamma^\mu \, t^a \, Q_L + \bar{u}_R \, \gamma^\mu \, t^a \, u_R + \bar{d}_R \, \gamma^\mu \, t^a \, d_R)$$

Thus,

$$\begin{aligned} \hat{Q}_{G,2} &= g_s^2 \, (\bar{Q}_L \, \gamma^\mu \, t^a \, Q_L + \bar{u}_R \, \gamma^\mu \, t^a \, u_R + \bar{d}_R \, \gamma^\mu \, t^a \, d_R)^2 \\ &= g_s^2 \Big[\frac{1}{4} \left([Q_{qq}^{(1)}]_{prrp} + [Q_{qq}^{(3)}]_{prrp} \right) - \frac{1}{2N_c} [Q_{qq}^{(1)}]_{pprr} + \frac{1}{2} [Q_{uu}]_{prrp} - \frac{1}{2N_c} [Q_{uu}]_{pprr} \\ &\quad + \frac{1}{2} [Q_{dd}]_{prrp} - \frac{1}{2N_c} [Q_{dd}]_{pprr} + 2[Q_{qu}^{(8)}]_{pprr} + 2[Q_{qd}^{(8)}]_{pprr} + 2[Q_{ud}^{(8)}]_{pprr} \Big] \end{aligned}$$

→ Contribution to purely **fermionic operators**!

Transformation to the Warsaw Basis

Operator class	Warsaw basis	Way of generation	
Purely bosonic			
X^3	yes	direct	—
X^2D^2	no	direct	
X^2H^2	yes	direct	—
XH^2D^2	no	—	
H^6	yes	—	EOM
H^4D^2	yes	—	EOM
H^2D^4	no	—	
Single fermion current			
ψ^2XD	no	—	
ψ^2D^3	no	—	
ψ^2XH	yes	direct	—
ψ^2H^3	yes	direct	EOM
ψ^2H^2D	yes	direct	EOM
ψ^2HD^2	no	—	
4-fermion operators			
$(\bar{L}L)(\bar{L}L)$	yes	—	EOM
$(\bar{R}R)(\bar{R}R)$	yes	—	EOM
$(\bar{L}L)(\bar{R}R)$	yes	direct	EOM
$(\bar{L}R)(\bar{R}L)$	yes	direct	—
$(\bar{L}R)(\bar{L}R)$	yes	direct	—
B-violating	yes	—	—

ALP-SMEFT Interference: systematic study

Consider an off-shell basis of $D = 6$ SM-operators.

Compute all amputated, one-loop divergent Green's functions with virtual ALP exchange.

Apply the SM EoM to project onto Warsaw basis.

Renormalize the bare Wilson coefficients.

From Diagrams to Source Terms

$$\mathcal{A}(gg(g)) = -\frac{1}{\Lambda^2}\,\frac{C_{GG}^2}{\epsilon}\,\Big[4\,g_s\,\langle Q_G\rangle + \tfrac{4}{3}\,\langle \hat{Q}_{G,2}\rangle - 2\,m_a^2\,\langle G^a_{\mu\nu}G^{\mu\nu,a}\rangle\Big] + \text{finite}$$

To cancel the $1/\epsilon$ terms, the *bare* Wilson coefficients must contain

$$C_{G,0} \ni \frac{4g_s}{(4\pi f)^2}\,C_{GG}^2\left(\frac{1}{\epsilon} + \ln\frac{\mu^2}{M^2} + \ldots\right)$$

M: characteristic mass scale of the UV theory

$\ln\mu^2$: generic for one-loop diagrams in dimensional regularization

Thus, after removing the pole: $\frac{d}{d\ln\mu}C_G(\mu) \ni \frac{8g_s}{(4\pi f)^2}\,C_{GG}^2$

$$\frac{d}{d\ln\mu}\,C_i^{\mathrm{SMEFT}} - \gamma_{ji}^{\mathrm{SMEFT}}\,C_j^{\mathrm{SMEFT}} = \frac{S_i}{(4\pi f)^2} \quad \text{for } \mu < 4\pi f$$

$\longrightarrow$ $S_G = 8g_s\,C_{GG}^2$

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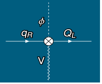


ALP-SMEFT Interference

	Operator Q	Source Term D
Q_G	$g_3\, f^{abc}\, G_\mu^{\ \nu,a} G_\nu^{\ \rho,b} G_\rho^{\ \mu,c}$	$8\left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)^2$
$Q_{\tilde{G}}$	$g_3\, f^{abc}\, \tilde{G}_\mu^{\ \nu,a} G_\nu^{\ \rho,b} G_\rho^{\ \mu,c}$	0
Q_W	$g_2\, \epsilon^{IJK}\, W_\mu^{\ \nu,I} W_\nu^{\ \rho,J} W_\rho^{\ \mu,K}$	$8\left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2$
$Q_{\tilde{W}}$	$g_2\, \epsilon^{IJK}\, \tilde{W}_\mu^{\ \nu,I} W_\nu^{\ \rho,J} W_\rho^{\ \mu,K}$	0
$Q_{\phi G}$	$g_3^2\, \phi^\dagger\phi\, G^a_{\mu\nu} G^{\mu\nu,a}$	0
$Q_{\phi\tilde{G}}$	$g_3^2\, \phi^\dagger\phi\, \tilde{G}^a_{\mu\nu} G^{\mu\nu,a}$	0
$Q_{\phi W}$	$g_2^2\, \phi^\dagger\phi\, W^I_{\mu\nu} W^{\mu\nu,I}$	$-2\left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2$
$Q_{\phi\tilde{W}}$	$g_2^2\, \phi^\dagger\phi\, \tilde{W}^I_{\mu\nu} W^{\mu\nu,I}$	0
$Q_{\phi B}$	$g_1^2\, \phi^\dagger\phi\, B_{\mu\nu} B^{\mu\nu}$	$-2\left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\phi\tilde{B}}$	$g_1^2\, \phi^\dagger\phi\, \tilde{B}_{\mu\nu} B^{\mu\nu}$	0
$Q_{\phi WB}$	$g_1 g_2\, \phi^\dagger\sigma^I\phi\, W^I_{\mu\nu} B^{\mu\nu}$	$-4\left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)\left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)$
$Q_{\phi\tilde{W}B}$	$g_1 g_2\, \phi^\dagger\sigma^I\phi\, \tilde{W}^I_{\mu\nu} B^{\mu\nu}$	0
$Q_{\phi\Box}$	$(\phi^\dagger\phi)\Box(\phi^\dagger\phi)$	$g_1^2\, \frac{8}{3}\, \mathcal{Y}_\phi^2 \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2 + 2\, g_2^2 \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2$
$Q_{\phi D}$	$(\phi^\dagger D_\mu\phi)^*\, (\phi^\dagger D^\mu\phi)$	$g_1^2\, \frac{32}{3}\, \mathcal{Y}_\phi^2 \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$

	Operator Q	Source Term D
$Q_{\underset{ij}{eW}}$	$g_2\, (\bar{L}^i_L\, \sigma^{\mu\nu} e^j_R)\, \tau^I \phi\, W^I_{\mu\nu}$	$-2\, i\, [\hat{\mathbf{Y}}_e]^{ij} \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)$
$Q_{\underset{ij}{eB}}$	$g_1\, (\bar{L}^i_L\, \sigma^{\mu\nu} e^j_R)\, \phi\, B_{\mu\nu}$	$-2\, i\, (\mathcal{Y}_L + \mathcal{Y}_e)[\hat{\mathbf{Y}}_e]^{ij} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)$
$Q_{\underset{ij}{uG}}$	$g_3\, (\bar{Q}^i_L\, \sigma^{\mu\nu}\, t^a u^j_R)\, \tilde{\phi}\, G^a_{\mu\nu}$	$-4\, i\, [\hat{\mathbf{Y}}_u]^{ij} \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)$
$Q_{\underset{ij}{uW}}$	$g_2\, (\bar{Q}^i_L\, \sigma^{\mu\nu} u^j_R)\, \tau^I \tilde{\phi}\, W^I_{\mu\nu}$	$-2\, i\, [\hat{\mathbf{Y}}_u]^{ij} \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)$
$Q_{\underset{ij}{uB}}$	$g_1\, (\bar{Q}^i_L\, \sigma^{\mu\nu} u^j_R)\, \tilde{\phi}\, B_{\mu\nu}$	$-2\, i\, (\mathcal{Y}_Q + \mathcal{Y}_u)\, [\hat{\mathbf{Y}}_u]^{ij} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)$
$Q_{\underset{ij}{dG}}$	$g_3\, (\bar{Q}^i_L\, \sigma^{\mu\nu}\, t^a d^j_R)\, \phi\, G^a_{\mu\nu}$	$-4\, i\, [\hat{\mathbf{Y}}_d]^{ij} \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)$
$Q_{\underset{ij}{dW}}$	$g_2\, (\bar{Q}^i_L\, \sigma^{\mu\nu} d^j_R)\, \tau^I \phi\, W^I_{\mu\nu}$	$-2\, i\, [\hat{\mathbf{Y}}_d]^{ij} \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)$
$Q_{\underset{ij}{dB}}$	$g_1\, (\bar{Q}^i_L\, \sigma^{\mu\nu} d^j_R)\, \phi\, B_{\mu\nu}$	$-2\, i\, (\mathcal{Y}_Q + \mathcal{Y}_d)\, [\hat{\mathbf{Y}}_d]^{ij} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)$
$Q_{\underset{ij}{e\phi}}$	$(\phi^\dagger\phi)\, (\bar{L}^i_L e^j_R\, \phi)$	$-2\, [\hat{\mathbf{Y}}_e\, \mathbf{Y}_e^\dagger\, \hat{\mathbf{Y}}_e]^{ij} - \frac{1}{2}\, [\hat{\mathbf{Y}}_e\, \hat{\mathbf{Y}}_e^\dagger\, \mathbf{Y}_e]^{ij} - \frac{1}{2}\, [\mathbf{Y}_e\, \hat{\mathbf{Y}}_e^\dagger\, \hat{\mathbf{Y}}_e]^{ij} + \frac{4}{3}\, g_2^2 \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2 [\mathbf{Y}_e]^{ij}$
$Q_{\underset{ij}{u\phi}}$	$(\phi^\dagger\phi)\, (\bar{Q}^i_L u^j_R\, \tilde{\phi})$	$-2\, [\hat{\mathbf{Y}}_u\, \mathbf{Y}_u^\dagger\, \hat{\mathbf{Y}}_u]^{ij} - \frac{1}{2}\, [\hat{\mathbf{Y}}_u\, \hat{\mathbf{Y}}_u^\dagger\, \mathbf{Y}_u]^{ij} - \frac{1}{2}\, [\mathbf{Y}_u\, \hat{\mathbf{Y}}_u^\dagger\, \hat{\mathbf{Y}}_u]^{ij} + \frac{4}{3}\, g_2^2 \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2 [\mathbf{Y}_u]^{ij}$
$Q_{\underset{ij}{d\phi}}$	$(\phi^\dagger\phi)\, (\bar{Q}^i_L d^j_R\, \phi)$	$-2\, [\hat{\mathbf{Y}}_d\, \mathbf{Y}_d^\dagger\, \hat{\mathbf{Y}}_d]^{ij} - \frac{1}{2}\, [\hat{\mathbf{Y}}_d\, \hat{\mathbf{Y}}_d^\dagger\, \mathbf{Y}_d]^{ij} - \frac{1}{2}\, [\mathbf{Y}_d\, \hat{\mathbf{Y}}_d^\dagger\, \hat{\mathbf{Y}}_d]^{ij} + \frac{4}{3}\, g_2^2 \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2 [\mathbf{Y}_d]^{ij}$
$Q^{(1)}_{\underset{ij}{\phi L}}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)\, (\bar{L}^i_L \gamma^\mu L^j_L)$	$\frac{1}{4}\, [\hat{\mathbf{Y}}_e\, \hat{\mathbf{Y}}_e^\dagger]^{ij} + \frac{16}{3}\, g_1^2\, \mathcal{Y}_\phi\, \mathcal{Y}_L \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2 \delta_{ij}$
$Q^{(3)}_{\underset{ij}{\phi L}}$	$(\phi^\dagger i \overleftrightarrow{D}^I_\mu \phi)\, (\bar{L}^i_L \sigma^I\, \gamma^\mu L^j_L)$	$\frac{1}{4}\, [\hat{\mathbf{Y}}_e\, \hat{\mathbf{Y}}_e^\dagger]^{ij} + \frac{4}{3}\, g_2^2 \left(\frac{\tilde{c}_{WW}}{4\pi}\right)^2 \delta_{ij}$
$Q_{\underset{ij}{\phi e}}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)\, (\bar{e}^i_R \gamma^\mu e^j_R)$	$-\frac{1}{2}\, [\hat{\mathbf{Y}}_e^\dagger\, \hat{\mathbf{Y}}_e]^{ij} + \frac{16}{3}\, g_1^2\, \mathcal{Y}_e\, \mathcal{Y}_\phi \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2 \delta_{ij}$
$Q^{(1)}_{\underset{ij}{\phi Q}}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)\, (\bar{Q}^i_L \gamma^\mu Q^j_L)$	$\frac{1}{4}\, [\hat{\mathbf{Y}}_d\, \hat{\mathbf{Y}}_d^\dagger]^{ij} - \frac{1}{4}\, [\hat{\mathbf{Y}}_u\, \hat{\mathbf{Y}}_u^\dagger]^{ij} + \frac{16}{3}\, \mathcal{Y}_\phi\, \mathcal{Y}_Q\, g_1^2\, \delta_{ij} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q^{(3)}_{\underset{ij}{\phi Q}}$	$(\phi^\dagger i \overleftrightarrow{D}^I_\mu \phi)\, (\bar{Q}^i_L \sigma^I\, \gamma^\mu Q^j_L)$	$\frac{1}{4}\, [\hat{\mathbf{Y}}_d\, \hat{\mathbf{Y}}_d^\dagger]^{ij} + \frac{1}{4}\, [\hat{\mathbf{Y}}_u\, \hat{\mathbf{Y}}_u^\dagger]^{ij} + \frac{4}{3}\, g_2^2\, \delta_{ij} \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2$
$Q_{\underset{ij}{\phi u}}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)\, (\bar{u}^i_R \gamma^\mu u^j_R)$	$\frac{1}{2}\, [\hat{\mathbf{Y}}_u^\dagger\, \hat{\mathbf{Y}}_u]^{ij} + \frac{16}{3}\, g_1^2\, \mathcal{Y}_\phi\, \mathcal{Y}_u\, \delta_{ij} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ij}{\phi d}}$	$(\phi^\dagger i \overleftrightarrow{D}_\mu \phi)\, (\bar{d}^i_R \gamma^\mu d^j_R)$	$-\frac{1}{2}\, [\hat{\mathbf{Y}}_d^\dagger\, \hat{\mathbf{Y}}_d]^{ij} + \frac{16}{3}\, g_1^2\, \mathcal{Y}_\phi\, \mathcal{Y}_u\, \delta_{ij} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ij}{\phi u}} + \text{h.c.}$	$i\, (\tilde{\phi}^\dagger D_\mu \phi)\, (\bar{u}^i_R \gamma^\mu d^j_R)$	$-[\hat{\mathbf{Y}}_u^\dagger\, \hat{\mathbf{Y}}_d]^{ij}$

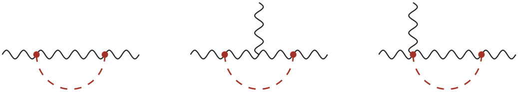
	Operator Q	Source Term D
$Q_{\underset{ijkl}{LL}}$	$(\bar{L}^i_L\, \gamma_\mu L^j_L)\, (\bar{L}^k_L \gamma^\mu L^l_L)$	$\frac{8}{3}\, g_1^2\, \mathcal{Y}_L^2 \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2 \delta_{ij}\delta_{kl} + \frac{2}{3}\, g_2^2 \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2 (2\, \delta_{il}\delta_{jk} - \delta_{ij}\delta_{kl})$
$Q^{(1)}_{\underset{ijkl}{QQ}}$	$(\bar{Q}^i_L\, \gamma_\mu Q^j_L)\, (\bar{Q}^k_L \gamma^\mu Q^l_L)$	$\frac{8}{3}\, g_1^2\, \mathcal{Y}_Q^2 \left(\frac{\tilde{c}_{BB}}{4\pi}\right)^2 \delta_{ij}\delta_{kl} + \frac{2}{3}\, g_3^2 \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)^2 \left(\delta_{il}\delta_{jk} - \frac{2}{N_c}\delta_{ij}\delta_{kl}\right)$
$Q^{(3)}_{\underset{ijkl}{QQ}}$	$(\bar{Q}^i_L\, \gamma_\mu\, \sigma^I\, Q^j_L)\, (\bar{Q}^k_L \gamma^\mu\, \sigma^I\, Q^l_L)$	$\frac{2}{3}\, g_3^2 \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)^2 \delta_{il}\delta_{jk} + \frac{2}{3}\, g_2^2\, \delta_{ij}\delta_{kl} \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2$
$Q^{(1)}_{\underset{ijkl}{LQ}}$	$(\bar{L}^i_L\, \gamma_\mu L^j_L)\, (\bar{Q}^k_L \gamma^\mu Q^l_L)$	$\frac{16}{3}\, g_1^2\, \mathcal{Y}_Q\, \mathcal{Y}_L \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q^{(3)}_{\underset{ijkl}{LQ}}$	$(\bar{L}^i_L\, \gamma_\mu\, \sigma^I\, L^j_L)\, (\bar{Q}^k_L \gamma^\mu\, \sigma^I\, Q^l_L)$	$\frac{4}{3}\, g_2^2\, \delta_{ij}\delta_{kl} \left(\alpha_2 \frac{\tilde{c}_{WW}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{ee}}$	$(\bar{e}^i_R\, \gamma_\mu e^j_R)\, (\bar{e}^k_R \gamma^\mu e^l_R)$	$\frac{8}{3}\, g_1^2\, \mathcal{Y}_e^2\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{uu}}$	$(\bar{u}^i_R\, \gamma_\mu u^j_R)\, (\bar{u}^k_R \gamma^\mu u^l_R)$	$\frac{8}{3}\, g_1^2\, \mathcal{Y}_u^2\, \delta_{ij}\delta_{kl} \left(\frac{\tilde{c}_{BB}}{4\pi}\right)^2 + \frac{4}{3}\, g_3^2 \left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\, \delta_{ij}\delta_{kl}\right) \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{dd}}$	$(\bar{d}^i_R\, \gamma_\mu d^j_R)\, (\bar{d}^k_R \gamma^\mu d^l_R)$	$\frac{8}{3}\, g_1^2\, \mathcal{Y}_d^2\, \delta_{ij}\delta_{kl} \left(\frac{\tilde{c}_{BB}}{4\pi}\right)^2 + \frac{4}{3}\, g_3^2 \left(\delta_{il}\delta_{jk} - \frac{1}{N_c}\, \delta_{ij}\delta_{kl}\right) \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{eu}}$	$(\bar{e}^i_R\, \gamma_\mu e^j_R)\, (\bar{u}^k_R \gamma^\mu u^l_R)$	$\frac{16}{3}\, g_1^2\, \mathcal{Y}_e\, \mathcal{Y}_u\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{ed}}$	$(\bar{e}^i_R\, \gamma_\mu e^j_R)\, (\bar{d}^k_R \gamma^\mu d^l_R)$	$\frac{16}{3}\, g_1^2\, \mathcal{Y}_e\, \mathcal{Y}_d\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q^{(1)}_{\underset{ijkl}{ud}}$	$(\bar{u}^i_R\, \gamma_\mu u^j_R)\, (\bar{d}^k_R \gamma^\mu d^l_R)$	$\frac{16}{3}\, g_1^2\, \mathcal{Y}_u\, \mathcal{Y}_d\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q^{(8)}_{\underset{ijkl}{ud}}$	$(\bar{u}^i_R\, \gamma_\mu\, t^a u^j_R)\, (\bar{d}^k_R \gamma^\mu\, t^a\, d^l_R)$	$\frac{16}{3}\, g_3^2\, \delta_{ij}\delta_{kl} \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{Le}}$	$(\bar{L}^i_L\, \gamma_\mu L^j_L)\, (\bar{e}^k_R \gamma^\mu e^l_R)$	$[\hat{\mathbf{Y}}_e]^{il}\, [\hat{\mathbf{Y}}_e^\dagger]^{kj} + \frac{16}{3}\, g_1^2\, \mathcal{Y}_L\, \mathcal{Y}_e\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{Lu}}$	$(\bar{L}^i_L\, \gamma_\mu L^j_L)\, (\bar{u}^k_R \gamma^\mu u^l_R)$	$\frac{16}{3}\, g_1^2\, \mathcal{Y}_L\, \mathcal{Y}_u\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{Ld}}$	$(\bar{L}^i_L\, \gamma_\mu L^j_L)\, (\bar{d}^k_R \gamma^\mu d^l_R)$	$\frac{16}{3}\, g_1^2\, \mathcal{Y}_L\, \mathcal{Y}_d\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q_{\underset{ijkl}{Qe}}$	$(\bar{Q}^i_L\, \gamma_\mu Q^j_L)\, (\bar{e}^k_R \gamma^\mu e^l_R)$	$\frac{16}{3}\, g_1^2\, \mathcal{Y}_Q\, \mathcal{Y}_e\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q^{(1)}_{\underset{ijkl}{Qu}}$	$(\bar{Q}^i_L\, \gamma_\mu Q^j_L)\, (\bar{u}^k_R \gamma^\mu u^l_R)$	$\frac{1}{N_c}\, [\hat{\mathbf{Y}}_u]^{il}\, [\hat{\mathbf{Y}}_u^\dagger]^{kj} + \frac{16}{3}\, g_1^2\, \mathcal{Y}_u\, \mathcal{Y}_Q\, \delta_{ij}\delta_{kl} \left(\alpha_1 \frac{\tilde{c}_{BB}}{4\pi}\right)^2$
$Q^{(8)}_{\underset{ijkl}{Qu}}$	$(\bar{Q}^i_L\, \gamma_\mu\, t^a\, Q^j_L)\, (\bar{u}^k_R \gamma^\mu\, t^a\, u^l_R)$	$2\, [\hat{\mathbf{Y}}_u]^{il}\, [\hat{\mathbf{Y}}_u^\dagger]^{kj} + \frac{16}{3}\, g_3^2\, \delta_{ij}\delta_{kl} \left(\alpha_s \frac{\tilde{c}_{GG}}{4\pi}\right)^2$
$Q^{(1)}_{\underset{ijkl}{Qd}}$	$(\bar{Q}^i_L\, \gamma_\mu Q^j_L)\, (\bar{d}^k_R \gamma^\mu d^l_R)$	$\frac{1}{N_c}$ [
$Q^{(8)}_{\underset{ijkl}{Qu}}$	$(\bar{Q}^i_L\, \gamma_\mu\, t^a\, Q^j_L)\, (\bar{d}^k_R \gamma^\mu\, t^a\, d^l_R)$	

	Operator Q	Source Te
$Q_{\underset{ijkl}{LedQ}}$	$(\bar{L}^i_L\, e^j_R)\, (\bar{d}^k_R\, Q^l_L)$	$-2\, [\hat{\mathbf{Y}}_e]^{ij}$ [
$Q^{(1)}_{\underset{ijkl}{QuQd}}$	$(\bar{Q}^{i,m}_L\, u^j_R)\, \epsilon_{mn} (\bar{Q}^{k,n}_L\, d^l_R)$	$-2\, [\hat{\mathbf{Y}}_u]^{ij}$ [
$Q^{(8)}_{\underset{ijkl}{QuQd}}$	$(\bar{Q}^{i,m}_L\, t^a\, u^j_R)\, \epsilon_{mn}\, (\bar{Q}^{k,n}_L\, t^a\, d^l_R)$	0
$Q^{(1)}_{\underset{ijkl}{LeQu}}$	$(\bar{L}^{i,m}_L\, e^j_R)\, \epsilon_{mn}\, (\bar{Q}^{k,n}_L\, u^l_R)$	$2\, [\hat{\mathbf{Y}}_e]^{ij}$ [
$Q^{(3)}_{\underset{ijkl}{LeQu}}$	$(\bar{L}^{i,m}_L \sigma_{\mu\nu}\, e^j_R)\, \epsilon_{mn}\, (\bar{Q}^{k,n}_L \sigma^{\mu\nu}\, u^l_R)$	0

Nearly the whole Warsaw basis is sourced by the ALP at one-loop order!

Contributions to the β-Functions

From X^3 and $X^2\,D^2$ diagrams



$$\mathcal{A}\big(gg(g)\big) = -\frac{C_{GG}^2}{\epsilon}\left[4g_s\langle Q_G\rangle + \frac{4}{3}\langle\widehat{Q}_{G,2}\rangle - 2m_a^2\langle G^a_{\mu\nu}G^{\mu\nu,a}\rangle\right] + \text{finite}$$

$\hookrightarrow$ divergent terms contribute to the Z-factors $G^a_{\mu,0} = Z_G^{1/2}\,G^a_\mu$

$$\delta Z_G = \frac{8m_a^2}{(4\pi f)^2}\frac{C_{GG}^2}{\epsilon}$$

enters in

$$\alpha_{s,0} = \mu^{2\epsilon}\,Z_{\alpha_s}\,\alpha_s$$
$$Z_{\alpha_s} = Z_{\bar{q}qg}^2\,Z_q^{-2}\,Z_G^{-1}$$

with $\dfrac{\mathrm{d}\alpha_s}{\mathrm{d}\ln\mu} \equiv -2\alpha_s\beta^{(3)}(\{\alpha^i\})$

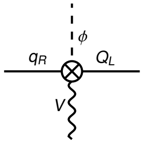
$$\beta^{(3)}(\{\alpha_i\}) = \beta^{(3)}_{\text{SM}}(\{\alpha_i\}) + \frac{8m_a^2}{(4\pi f)^2}\,C_{GG}^2$$

UV Running of the Dipole Coefficients

Dipole Operators above the Weak Scale

$$
\begin{aligned}
\mathcal{L}_{\text{SMEFT}} \supset\, & C_{uB}^{ij}\bar{Q}^i\tilde{\phi}\,\sigma_{\mu\nu}B^{\mu\nu}u_R^j + C_{dB}^{ij}\bar{Q}^i\phi\,\sigma_{\mu\nu}B^{\mu\nu}d_R^j + C_{eB}^{ij}\bar{L}^i\phi\,\sigma_{\mu\nu}B^{\mu\nu}e_R^j \\
& + C_{uW}^{ij}\bar{Q}^i\tau_A\tilde{\phi}\,\sigma_{\mu\nu}W_A^{\mu\nu}u_R^j + C_{dW}^{ij}\bar{Q}^i\tau_A\phi\,\sigma_{\mu\nu}W_A^{\mu\nu}d_R^j \\
& \qquad + C_{eW}^{ij}\bar{L}^i\tau_A\phi\,\sigma_{\mu\nu}W_A^{\mu\nu}e_R^j \\
& + C_{uG}^{ij}\bar{Q}^i\tilde{\phi}\,\sigma_{\mu\nu}G_a^{\mu\nu}t_a u_R^j + C_{dG}^{ij}\bar{Q}^i\phi\,\sigma_{\mu\nu}G_a^{\mu\nu}t_a d_R^j
\end{aligned}
$$

Wilson coefficients C_{fV}^{ij}: 3×3 matrices in generation space

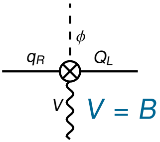


quark-sector dipole operator

Dipole Operators above the Weak Scale

$$\begin{aligned}\mathcal{L}_{\text{SMEFT}} \supset\, & C_{uB}^{ij}\,\bar{Q}^i\tilde{\phi}\,\sigma_{\mu\nu}B^{\mu\nu}u_R^j + C_{dB}^{ij}\,\bar{Q}^i\phi\,\sigma_{\mu\nu}B^{\mu\nu}d_R^j + C_{eB}^{ij}\,\bar{L}^i\phi\,\sigma_{\mu\nu}B^{\mu\nu}e_R^j \\ & + C_{uW}^{ij}\,\bar{Q}^i\tau_A\tilde{\phi}\,\sigma_{\mu\nu}W_A^{\mu\nu}u_R^j + C_{dW}^{ij}\,\bar{Q}^i\tau_A\phi\,\sigma_{\mu\nu}W_A^{\mu\nu}d_R^j \\ & \qquad + C_{eW}^{ij}\,\bar{L}^i\tau_A\phi\,\sigma_{\mu\nu}W_A^{\mu\nu}e_R^j \\ & + C_{uG}^{ij}\,\bar{Q}^i\tilde{\phi}\,\sigma_{\mu\nu}G_a^{\mu\nu}t_a u_R^j + C_{dG}^{ij}\,\bar{Q}^i\phi\,\sigma_{\mu\nu}G_a^{\mu\nu}t_a d_R^j\end{aligned}$$

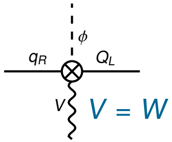
Wilson coefficients C_{fV}^{ij}: 3×3 matrices in generation space



quark-sector dipole operator

$$\mathcal{L}_{\mathrm{SMEFT}} \supset C^{ij}_{uB}\bar{Q}^i\tilde{\phi}\,\sigma_{\mu\nu}B^{\mu\nu}u^j_R + C^{ij}_{dB}\bar{Q}^i\phi\,\sigma_{\mu\nu}B^{\mu\nu}d^j_R + C^{ij}_{eB}\bar{L}^i\phi\,\sigma_{\mu\nu}B^{\mu\nu}e^j_R$$
$$+ C^{ij}_{uW}\bar{Q}^i\tau_A\tilde{\phi}\,\sigma_{\mu\nu}W^{\mu\nu}_A u^j_R + C^{ij}_{dW}\bar{Q}^i\tau_A\phi\,\sigma_{\mu\nu}W^{\mu\nu}_A d^j_R$$
$$+ C^{ij}_{eW}\bar{L}^i\tau_A\phi\,\sigma_{\mu\nu}W^{\mu\nu}_A e^j_R$$
$$+ C^{ij}_{uG}\bar{Q}^i\tilde{\phi}\,\sigma_{\mu\nu}G^{\mu\nu}_a t_a u^j_R + C^{ij}_{dG}\bar{Q}^i\phi\,\sigma_{\mu\nu}G^{\mu\nu}_a t_a d^j_R$$

Wilson coefficients C^{ij}_{fV}: 3×3 matrices in generation space

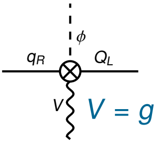


quark-sector dipole operator

Dipole Operators above the Weak Scale

$$\mathcal{L}_{\text{SMEFT}} \supset C^{ij}_{uB} \bar{Q}^i \tilde{\phi}\, \sigma_{\mu\nu} B^{\mu\nu} u^j_R + C^{ij}_{dB} \bar{Q}^i \phi\, \sigma_{\mu\nu} B^{\mu\nu} d^j_R + C^{ij}_{eB} \bar{L}^i \phi\, \sigma_{\mu\nu} B^{\mu\nu} e^j_R$$
$$+ C^{ij}_{uW} \bar{Q}^i \tau_A \tilde{\phi}\, \sigma_{\mu\nu} W^{\mu\nu}_A u^j_R + C^{ij}_{dW} \bar{Q}^i \tau_A \phi\, \sigma_{\mu\nu} W^{\mu\nu}_A d^j_R$$
$$+ C^{ij}_{eW} \bar{L}^i \tau_A \phi\, \sigma_{\mu\nu} W^{\mu\nu}_A e^j_R$$
$$+ C^{ij}_{uG} \bar{Q}^i \tilde{\phi}\, \sigma_{\mu\nu} G^{\mu\nu}_a t_a u^j_R + C^{ij}_{dG} \bar{Q}^i \phi\, \sigma_{\mu\nu} G^{\mu\nu}_a t_a d^j_R$$

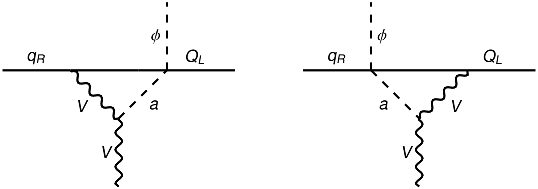
Wilson coefficients C^{ij}_{fV}: 3×3 matrices in generation space



quark-sector dipole operator

UV Evolution in the presence of an ALP

quark-sector:

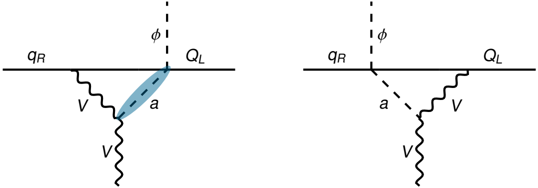


$$
\begin{aligned}
\boldsymbol{S}_{qB} &= 2\, g_1\, C_{BB}\, (\mathcal{Y}_Q + \mathcal{Y}_q)(\boldsymbol{Y}_q \boldsymbol{c}_q - \boldsymbol{c}_Q \boldsymbol{Y}_q) \\
\boldsymbol{S}_{qW} &= g_2\, C_{WW}\, (\boldsymbol{Y}_q \boldsymbol{c}_q - \boldsymbol{c}_Q \boldsymbol{Y}_q) \\
\boldsymbol{S}_{qG} &= 4\, g_s\, C_{GG}\, (\boldsymbol{Y}_q \boldsymbol{c}_q - \boldsymbol{c}_Q \boldsymbol{Y}_q)
\end{aligned}
$$

$q = u,\, d$

UV Evolution in the presence of an ALP

quark-sector:

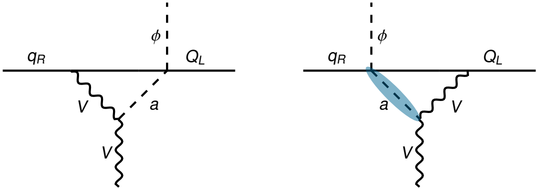


$$\boldsymbol{S}_{qB} = 2\,g_1\,C_{BB}\,(\mathcal{Y}_Q + \mathcal{Y}_q)(\boldsymbol{Y}_q\boldsymbol{c}_q - \boldsymbol{c}_Q\boldsymbol{Y}_q)$$
$$\boldsymbol{S}_{qW} = g_2\,C_{WW}\,(\boldsymbol{Y}_q\boldsymbol{c}_q - \boldsymbol{c}_Q\boldsymbol{Y}_q)$$
$$\boldsymbol{S}_{qG} = 4\,g_s\,C_{GG}\,(\boldsymbol{Y}_q\boldsymbol{c}_q - \boldsymbol{c}_Q\boldsymbol{Y}_q)$$

$q = u,\ d$

UV Evolution in the presence of an ALP

quark-sector:

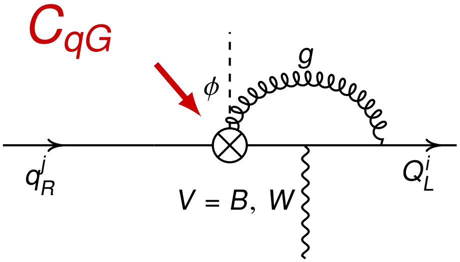


$$\boldsymbol{S}_{qB} = 2\, g_1\, C_{BB}\, (\mathcal{Y}_Q + \mathcal{Y}_q)(\boldsymbol{Y}_q \boldsymbol{c}_q - \boldsymbol{c}_Q \boldsymbol{Y}_q)$$
$$\boldsymbol{S}_{qW} = g_2\, C_{WW}\, (\boldsymbol{Y}_q \boldsymbol{c}_q - \boldsymbol{c}_Q \boldsymbol{Y}_q)$$
$$\boldsymbol{S}_{qG} = 4\, g_s\, C_{GG}\, (\boldsymbol{Y}_q \boldsymbol{c}_q - \boldsymbol{c}_Q \boldsymbol{Y}_q)$$

$q = u,\, d$

Mixing of C_{qB}, C_{qW} and C_{qG}

mixing between the dipole Wilson coefficients:
[E. Jenkins et al: arXiv: 1310.4838, 1312.2014]



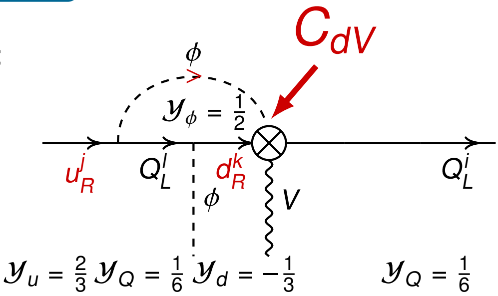
$\hookrightarrow$ QCD-effects mix $\boldsymbol{C_{qG}}$, $\boldsymbol{C_{qW}}$ and $\boldsymbol{C_{qB}}$

Mixing of C_{uV} and C_{dV}

mixing between the dipole Wilson coefficients:
[E. Jenkins et al: arXiv: 1310.4838, 1312.2014]

$$\alpha_t \sim \alpha_s$$

for instance:



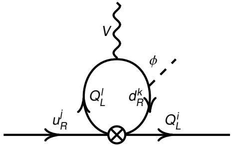
$\hookrightarrow$ the Higgs mixes $\boldsymbol{C_{uV}}$ and $\boldsymbol{C_{dV}}$

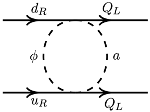
Mixing with other SMEFT coefficients

[E. Jenkins et al: arXiv: 1310.4838, 1312.2014]

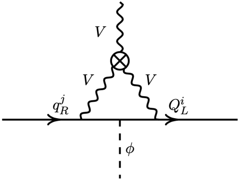
generated **for instance** via:

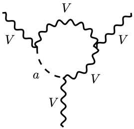
four-fermion operator:



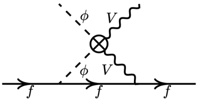


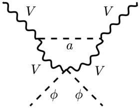
Weinberg operator:





$Q_{HV(V')}$-type operator:





UV Evolution in the presence of an ALP

BUT that's not the end of the story...!

↪ The ALP generates more SMEFT operators that mix into the evolution of the 4-fermion operators! E.g.:

$$\frac{d}{d\ln\mu} C^{(1)}_{QuQd} \propto \begin{cases} + \;\dots \\ \text{since these coefficients themselves} \\ \text{mix with more SMEFT operators!} \end{cases}$$

⟹ Nearly the whole SMEFT operator basis mixes into the evolution of the Wilson coefficients of the dipole operators!

Chromo-magnetic and chromo-electric dipole moments:

$$\mathcal{L}_{\text{eff}} = \hat{\mu}_q \frac{g_3}{2m_q} \bar{q}\,\sigma_{\mu\nu} G_a^{\mu\nu}\, t_a\, q + i\,\hat{d}_q \frac{g_3}{2m_q} \bar{q}\,\sigma_{\mu\nu}\gamma_5 G_a^{\mu\nu}\, t_a q\,,$$

top quark:

$$\hat{\mu}_t = -\frac{y_t v^2}{\Lambda^2} \Re e\, C_{uG}^{33}\,, \qquad \hat{d}_t = -\frac{y_t v^2}{\Lambda^2} \Im m\, C_{uG}^{33}$$

Neglecting contributions $\propto \alpha_1, \alpha_2$ and y_i with $i \neq t$:

$$\frac{d}{d\ln\mu} C_{uG}^{33} = \frac{S_{uG}^{33}}{(4\pi f)^2} + \left(\frac{15\,\alpha_t}{8\pi} - \frac{17\alpha_s}{12\pi}\right) C_{uG}^{33} + \frac{9\alpha_2}{4\pi} y_t (C_G + iC_{\tilde{G}}) + \frac{g_s y_t}{4\pi^2} (C_{HG} + iC_{H\tilde{G}})$$

Relevant source terms: $S_{uG}^{33} = 4 g_s y_t c_{tt} C_{GG}$ and $S_G = 8 g_s C_{GG}^2$
$\hookrightarrow$ both real-valued!

Application: Top Chromo-Magnetic and -Electric Moment

Chromo-magnetic and chromo-electric dipole moments:

$$\mathcal{L}_{\rm eff} = \hat{\mu}_q \, \frac{g_3}{2m_q} \, \bar{q} \, \sigma_{\mu\nu} G_a^{\mu\nu} \, t_a \, q + i \, \hat{d}_q \, \frac{g_3}{2m_q} \bar{q} \, \sigma_{\mu\nu} \gamma_5 G_a^{\mu\nu} \, t_a q \, ,$$

top quark:

$$\hat{\mu}_t = -\frac{y_t v^2}{\Lambda^2} \Re e \, C_{uG}^{33} \, , \qquad \hat{d}_t = -\frac{y_t v^2}{\Lambda^2} \Im m \, C_{uG}^{33}$$

$\frac{d}{d \ln \mu} \, \Im m \, C_{uG}^{33} = 0$
$\rightarrow$ no contribution to $\hat{d}_t$!

Application: Top Chromo-Magnetic and -Electric Moment

Chromo-magnetic and chromo-electric dipole moments:

$$\mathcal{L}_{\text{eff}} = \hat{\mu}_q \frac{g_3}{2m_q} \bar{q}\, \sigma_{\mu\nu} G_a^{\mu\nu}\, t_a\, q + i\, \hat{d}_q \frac{g_3}{2m_q} \bar{q}\, \sigma_{\mu\nu} \gamma_5 G_a^{\mu\nu}\, t_a q\,,$$

top quark:

$$\hat{\mu}_t = -\frac{y_t v^2}{\Lambda^2} \mathfrak{Re}\, C_{uG}^{33}\,, \qquad \hat{d}_t = -\frac{y_t v^2}{\Lambda^2} \mathfrak{Im}\, C_{uG}^{33}$$

$$\frac{d}{d\ln\mu} \mathfrak{Re}\, C_{uG}^{33} = \left(\frac{15\alpha_t}{8\pi} - \frac{17\alpha_s}{12\pi}\right) \mathfrak{Re}\, C_{uG}^{33} + \frac{9\alpha_s}{4\pi}\, y_t\, C_G + \frac{g_s\, y_t}{4\pi^2}\, C_{HG} + \frac{S_{uG}^{33}}{(4\pi f)^2}$$

$$\frac{d}{d\ln\mu} C_G = \frac{15\,\alpha_s}{4\pi}\, C_G + \frac{S_G}{(4\pi f)^2}$$

$$\frac{d}{d\ln\mu} C_{HG} = \left(\frac{3\alpha_t}{2\pi} - \frac{7\alpha_s}{2\pi}\right) C_{HG} + \frac{g_s\, y_t}{4\pi^2} \mathfrak{Re}\, C_{uG}^{33}$$

$\Lambda = 4\pi f$: scale of global symmetry breaking

Application: Top Chromo-Magnetic and -Electric Moment

Chromo-magnetic and chromo-electric dipole moments:

$$\mathcal{L}_{\rm eff} = \hat{\mu}_q \frac{g_3}{2m_q} \bar{q}\, \sigma_{\mu\nu} G_a^{\mu\nu}\, t_a\, q + i\, \hat{d}_q \frac{g_3}{2m_q} \bar{q}\, \sigma_{\mu\nu}\gamma_5 G_a^{\mu\nu}\, t_a q\,,$$

top quark:

$$\hat{\mu}_t = -\frac{y_t v^2}{\Lambda^2} \Re e\, C_{uG}^{33}\,, \qquad \hat{d}_t = -\frac{y_t v^2}{\Lambda^2} \Im m\, C_{uG}^{33}$$

$$\frac{d}{d\ln\mu} \Re e\, C_{uG}^{33} = \left(\frac{15\alpha_t}{8\pi} - \frac{17\alpha_s}{12\pi}\right) \Re e\, C_{uG}^{33} + \frac{9\alpha_s}{4\pi}\, y_t\, C_G + \frac{g_s\, y_t}{4\pi^2}\, C_{HG} + \frac{S_{uG}^{33}}{(4\pi f)^2}$$

$$\frac{d}{d\ln\mu} C_G = \frac{15\,\alpha_s}{4\pi}\, C_G + \frac{S_G}{(4\pi f)^2}$$

$$\frac{d}{d\ln\mu} C_{HG} = \left(\frac{3\alpha_t}{2\pi} - \frac{7\alpha_s}{2\pi}\right) C_{HG} + \frac{g_s\, y_t}{4\pi^2} \Re e\, C_{uG}^{33}$$

not sourced directly

Application: Top Chromo-Magnetic and -Electric Moment

To lowest logarithmic order: [AG, Neubert, Renner: 2105.01078]

$$\hat{\mu}_t \approx -\frac{8\, m_t^2}{(4\pi f)^2} \left[c_{tt}\, C_{GG} \ln \frac{4\pi f}{m_t} - \frac{9\alpha_s}{4\pi}\, C_{GG}^2 \ln^2 \frac{4\pi f}{m_t} \right]$$

$$\approx -(5.87 c_{tt}\, C_{GG} - 1.98 C_{GG}^2) \times \left[\frac{1\,\text{TeV}}{f} \right]^2$$

c_{tt}: ALP-top coupling below EWSB

for $m_t(m_t) = 163.4\,\text{GeV}$, $\alpha_s(m_t) = 0.1084$ and $f = 1\,\text{TeV}$

Combined with experimental bounds from CMS (2019) this gives:

$$-0.68 < (c_{tt}\, C_{GG} - 0.34\, C_{GG}^2) \times \left[\frac{1\,\text{TeV}}{f} \right]^2 < 2.38 \quad (95\%\,\text{CL})$$

Comparable to the strongest bounds following from collider and flavor physics for $m_a > 1\text{GeV}$!

Summary and Outlook

In this talk, we have ...

- ✓ seen the ALP Lagrangian and an alternative form for the coupling to the SM
- ✓ analyzed the effects of an ALP on the $D = 6$ SMEFT operators
- ✓ solved the RG equation of C_{uG}^{33} to lowest logarithmic order
  $\hookrightarrow$ model independent framework for studying virtual ALP contributions to precision measurements

Open Tasks:

- **!** Get an exact solution to the RG evolution equations by solving them numerically.

Take Home Message

The ALP generates SMEFT operators above the weak scale by means of inhomogeneous source terms.

Thank You!