ORIGIN OF NONTOPOLOGICAL SOLITON DARK MATTER

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Fermilab Theory Seminar
September 22, 2022

Carleton University

MOTIVATION

Dark matter could be complex, even in simple theories!

With attractive interactions, DM could form large/heavy structures:

- Nontopological solitons (of bosons or fermions) [this talk]
- Dark nuclei [Wise Zhang 1407.4121, 1411.1772; Gresham Lou Zurek 1707.02313, 1707.02316, 1805.04512]
- Dark quark nuggets [Bai Long 1804.10249; Bai Long Lu 1810.04360; Liang Zhitnitsky 1606.00435]
- Dark monopoles [Bai Korwar **NO** 2005.00503]
- Dark/mirror stars [Curtin Setford 1909.04071, 1909.04072; +many follow-ups]
- Boson or axion stars
- Microhalos, dark disks, dissipative DM…
- …many of which could form black holes
  (Even "light" extremal black holes could be stable [Bai **NO** 1906.04858; Bai Berger Korwar **NO** 2007.03703])

MOTIVATION

Nontopological solitons (NTSs):

1. Representative model for several types of dark compact objects.
2. Require minimal new fields.
   Scalar NTSs are generic with *just one* new complex scalar field!

Long history of study:

Early work: [Rosen 1968; Friedberg Lee Sirlin 1976; Coleman 1986].
In SUSY: [Kusenko hep-ph/9704273; Kusenko Shaposhnikov hep-ph/9709492].
With gauge [Lee Stein-Schabes Watkins Widrow 1989; Gulamov Nugaev Panin Smolyakov 1506.05786; Brihaye Cisterna Hartmann Luchini 1511.02757; Heek Rajaraman Riley Verhaaren 2103.06905, 2107.10280]
or topological [Bai Lu **NO** 2111.10360] charge.
Fermion solitons: [Lee Pang 1987; Macpherson Campbell hep-ph/9408387; Hong Jong Xie 2008.04430]
Other works: [Kusenko hep-ph/9704073; Dvali Kusenko Shaposhnikov hep-ph/9707423; Kusenko Shaposhnikov Tinyakov hep-th/9801041; Berkooz Chung Volansky hep-ph/0507218; Bishara Johnson Lennon March-Russell 1708.04620; Heek Rajaraman Riley Verhaaren 2009.08462]
Reviews: [Lee Pang 1992; Nugaev Shkerin 1905.05146]

MOTIVATION & OUTLINE

Questions about NTSs:

- How do they form?
- How big can they get?
- Can they dominate the dark sector?

Outline:

1. Models and properties of scalar NTSs (aka Q-balls)
2. Solitosynthesis
3. Phase transitions
4. Interplay with magnetic monopoles

NTS MODELS AND PROPERTIES

Q-BALLS

Q-balls appear when the potential for a complex scalar field satisfies
[Coleman 1986]

$$\frac{\mathrm{d}V}{\mathrm{d}|\phi|} = 0\,, \qquad m_\phi^2 \equiv \left.\frac{\mathrm{d}^2 V}{\mathrm{d}\phi\,\mathrm{d}\phi^*}\right|_{\phi=0} > 0\,,$$

$$V(|\phi|)/|\phi|^2 \text{ has a minimum at } |\phi| = \phi_0/\sqrt{2} > 0$$

$$0 \leq \frac{\sqrt{2V(\phi_0/\sqrt{2})}}{\phi_0} < m_\phi\,.$$

For a single-field model, a nonrenormalizable potential is required to satisfy this. A simple model is:

$$V(\phi) = m_\phi^2|\phi|^2 - \beta|\phi|^4 + \frac{\xi}{m_\phi^2}|\phi|^6$$

Let's construct a renormalizable multi-field model and see how the solutions work.

Q-BALLS

Two-complex-scalar renormalizable potential:

$$V(S,\phi) = \frac{1}{4}\lambda_\phi(|\phi|^2 - v^2)^2 + \frac{1}{4}\lambda_{\phi S}|S|^2|\phi|^2 + \lambda_S|S|^4 + m_{S,0}^2|S|^2$$

S: $U(1)_S$ global symmetry—Q-ball constituents

ϕ: Any global or gauge symmetry. Here a global $U(1)_\phi$ for simplicity.

- Could even be the Higgs doublet [Pontón Bai Jain 1906.10739]

EQUATIONS OF MOTION

Use dimensionless rescaling of fields, with time-dependent phase for S:

$$\phi = v\, f(r)\,, \quad S = e^{-i\omega t}\frac{v}{\sqrt{2}}\, s(r)$$

Classical EOM:

$$f'' + \frac{2}{\bar{r}}\, f' - \lambda_\phi\, f\,(f^2 - 1) - \frac{1}{4}\,\lambda_{\phi S}\, s^2 f = 0\,,$$
$$s'' + \frac{2}{\bar{r}}\, s' + \Omega^2\, s - \frac{1}{4}\,\lambda_{\phi S}\, f^2\, s - \lambda_S\, s^3 - \mu_0^2\, s = 0\,,$$

$$\boxed{U_{\rm eff} = \Omega^2 s^2/2 - V(s, f)}$$

Boundary conditions:

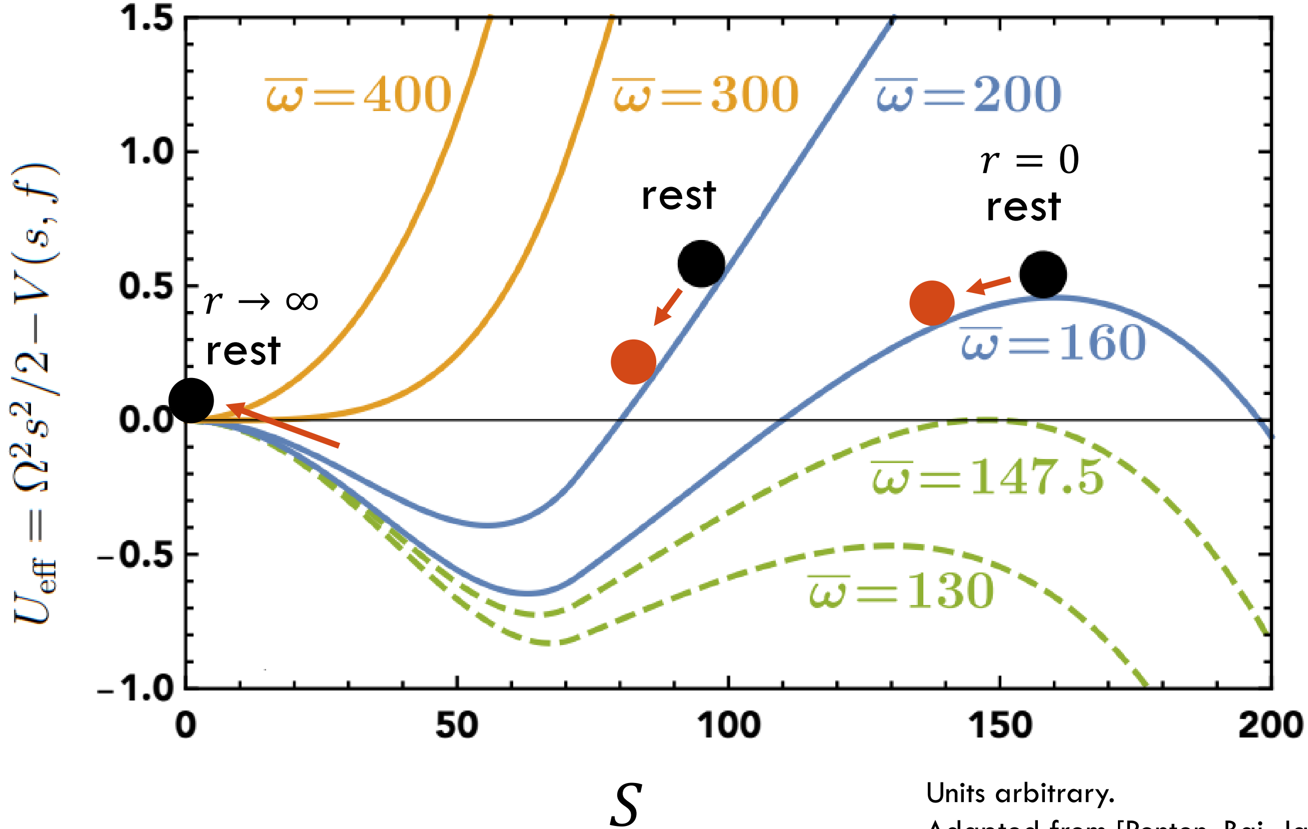
$$f'(0) = 0\,,\ f(\infty) = 1\,,\ s'(0) = 0\,,\ s(\infty) = 0$$

Rescalings:

$$\bar{r} = v\, r \qquad \Omega = \omega/v \qquad \mu_0 = m_{S,0}/v$$

EFFECTIVE POTENTIAL

$$V(S,\phi) = \frac{1}{4}\lambda_\phi(|\phi|^2 - v^2)^2 + \frac{1}{4}\lambda_{\phi S}|S|^2|\phi|^2 + \lambda_S|S|^4 + m_{S,0}^2|S|^2$$



Units arbitrary.
Adapted from [Ponton, Bai, Jain 1906.10739]

Conditions for field to roll and come to rest at $S = 0$:

$$\boxed{\overline{\Omega}^2 \equiv \Omega^2 - \mu_0^2 < \frac{\lambda_{\phi S}}{4}}$$

$$\boxed{\overline{\Omega}^4 > \overline{\Omega}_c^4 \equiv \lambda_S\,\lambda_\phi}$$

MASS & CHARGE

Once a solution is determined, its mass and charge are calculated by:

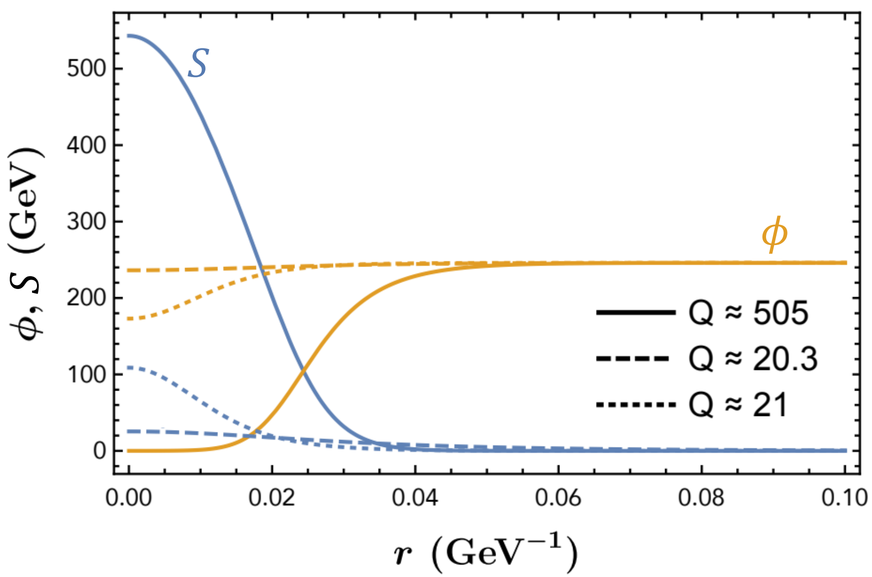
$$Q = i \int d^3x \, \left(S^\dagger \, \partial_t \, S - S \, \partial_t \, S^\dagger\right) = 4\pi \, \Omega \int_0^\infty d\bar{r} \, \bar{r}^2 s^2$$

$$m_Q/v = 4\pi \int d\bar{r}\bar{r}^2 \left(\frac{1}{2}s'^2 + f'^2 + \frac{1}{2}\Omega^2 s^2 + \frac{1}{8}\,\lambda_{\phi S}\, s^2\, f^2 + \frac{1}{4}\lambda_\phi(1 - f^2)^2 + \frac{1}{4}\lambda_S s^4 + \frac{1}{2}m_{S,0}^2 s^2\right)$$

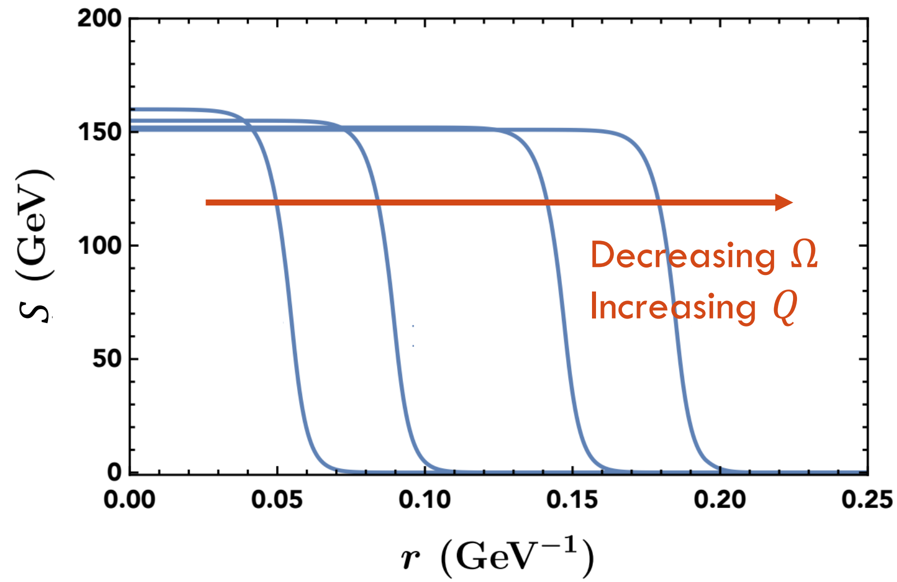
FIELD PROFILES

Valid approx at small Q.
Unphysical due to radiative corrections.

$\lambda_S = 0$



$\lambda_S > 0$



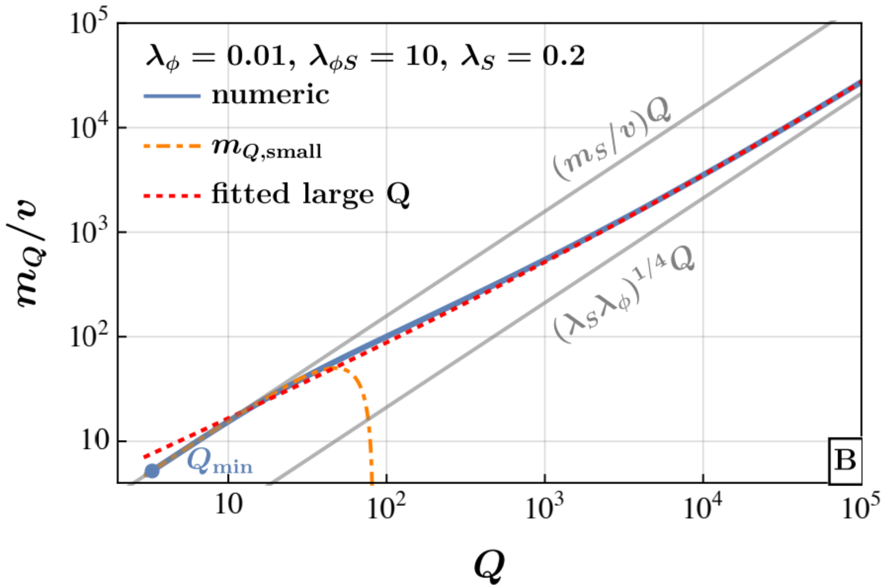
Adapted from [Ponton, Bai, Jain 1906.10739]

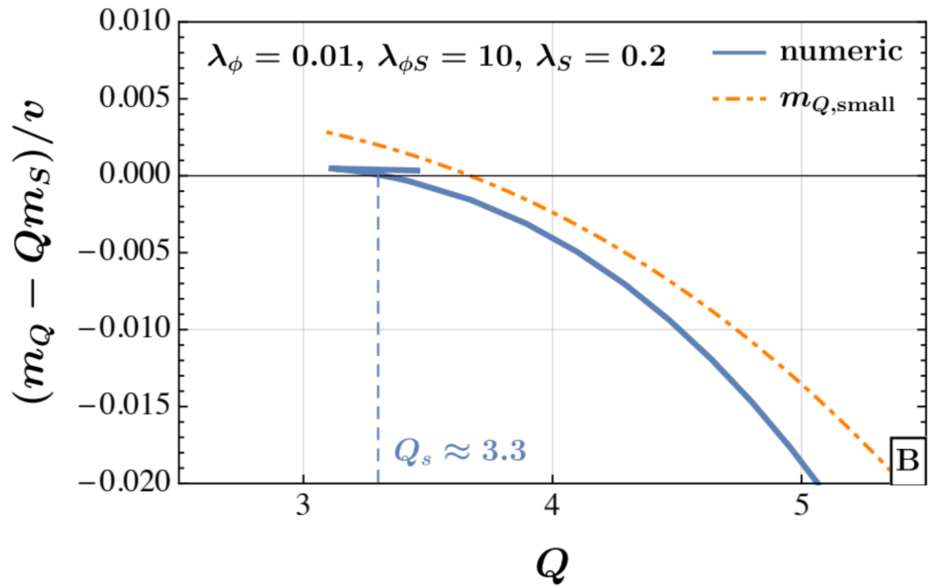
"Building up"

"Building out"

Trade-off of increased vacuum energy for ϕ and decreased mass for S inside Q-ball.

MASS AND STABILITY





$$m_{Q,\mathrm{large}} \approx v\,Q\left[(\lambda_S\lambda_\phi)^{1/4} + c_2 Q^{-1/3}\right] ,$$

$$R_{Q,\mathrm{large}} \approx \frac{3^{1/3}\,\lambda_S^{1/12}}{(4\pi)^{1/3}\lambda_\phi^{1/4}\,v}\,Q^{1/3}.$$

$$m_{Q,\mathrm{small}} \approx \left(\frac{1}{2}\,\lambda_{\phi S}^{1/2} + \lambda_\phi\,\lambda_{\phi S}^{-1/2}\right) v\,Q$$
$$- \frac{(\pi^2-9)}{2048\,\pi^2(\pi^2-6)^3}\,\lambda_{\phi S}^{5/2}\,v\,Q^3 .$$

$$R_{Q,\mathrm{small}} \approx 64\pi(\pi^2-6)/(\lambda_{\phi S}^{3/2}\,v\,Q).$$

OTHER MODELS

Call prior model with $\lambda_S > 0$ "Model B."

Will compare to "Model A" used in previous literature—very similar but no self-quartic term for S:

$$V(S,\sigma) = \frac{1}{8}\lambda\,(\sigma^2 - \sigma_0^2)^2 + \frac{1}{3}\lambda_2\,\sigma_0(\sigma-\sigma_0)^3 + \frac{m_S^2}{(\sigma_- - \sigma_0)^2}|S|^2(\sigma-\sigma_0)^2 + \Lambda\,,$$

$$\lambda_2 = 0.15\lambda$$

$$m_Q = 5.15\sigma_0\lambda^{1/4}Q^{3/4}\,,$$

$$R_Q = 0.8\lambda^{-1/4}\sigma_0^{-1}Q^{1/4}$$

$$m_S = 5.15\sigma_0\lambda^{1/4}Q_{\min}^{-1/4}$$

Q-BALL INTERACTIONS

$$S + S^\dagger \leftrightarrow \phi + \phi^\dagger \,, \qquad \text{S annihilation}$$

$$\left.\begin{aligned} (Q) + S &\leftrightarrow (Q+1) + X \,, \\ (Q) + S^\dagger &\leftrightarrow (Q-1) + X \,, \end{aligned}\right\} \text{Q-ball charge/discharge}$$

$$\left.(Q_{\rm min}) + S^\dagger \leftrightarrow \underbrace{S + S + \cdots + S}_{Q_{\rm min}-1} + X \,.\right\} \text{S fusion/Q-ball destruction}$$

$$\left.\begin{aligned} (Q_1) + (Q_2) &\leftrightarrow (Q_1 + Q_2) + X \,, \\ (Q_1) + (-Q_2) &\leftrightarrow \begin{cases} (Q_1 - Q_2) + X & \text{for } Q_1 - Q_2 \geq Q_{\rm min} \,, \\ \underbrace{S + S + \cdots + S}_{Q_1 - Q_2} + X & \text{for } Q_{\rm min} > Q_1 - Q_2 \geq 0 \,. \end{cases} \end{aligned}\right\} \text{Q-ball fusion}$$

Assumption: take Q-ball radiative capture cross sections $\sigma_Q \sim \pi R_Q^2$

Q-BALL INTERACTIONS

$$S + S^\dagger \leftrightarrow \phi + \phi^\dagger \ ,$$

S annihilation

$$\begin{aligned}(Q) + S &\leftrightarrow (Q+1) + X \ , \\ (Q) + S^\dagger &\leftrightarrow (Q-1) + X \ ,\end{aligned}$$

Q-ball charge/discharge

$$(Q_{\rm min}) + S^\dagger \leftrightarrow \underbrace{S + S + \cdots + S}_{Q_{\rm min}-1} + X \ .$$

S fusion/Q-ball destruction

$$(Q_1) + (Q_2) \leftrightarrow (Q_1 + Q_2) + X \ ,$$

$$(Q_1) + (-Q_2) \leftrightarrow \begin{cases} (Q_1 - Q_2) + X & \text{for } Q_1 - Q_2 \geq Q_{\rm min} \ , \\ \underbrace{S + S + \cdots + S}_{Q_1 - Q_2} + X & \text{for } Q_{\rm min} > Q_1 - Q_2 \geq 0 \ . \end{cases}$$

Q-ball fusion

Not efficient for sufficiently large Q

Assumption: take Q-ball radiative capture cross sections $\sigma_Q \sim \pi R_Q^2$

Q-BALL INTERACTIONS

For Q-ball fusion, compare interaction rate to Hubble:

$$\Gamma = n_Q \sigma v_{\rm rel} \sim n_Q \pi R^2 \sqrt{T/m_Q}$$

Assuming Q-balls make up all dark matter (a generous assumption at intermediate scales during solitosynthesis) and $T \sim v$ or σ_0

$$\begin{aligned} Q \lesssim\ & 2 \times 10^3 (\sigma_0/10^3\ \text{GeV})^{-16/5} && \text{Model A} \\ & 6 \times 10^4 (v/10^3\ \text{GeV})^{-12/5} && \text{Model B} \end{aligned}$$

for fusion to be important.

SOLITOSYNTHESIS

SOLITOSYNTHESIS

The process of building up NTSs from free particle fusion and accumulation.

Prior work:

[Griest Kolb 1989; Frieman Olinto Gleiser Alcock 1989; Kusenko Shaposhnikov hep-ph/9709492; Postma hep-ph/0110199; Gresham Lou Zurek 1707.02316]

MAXIMUM CHARGE IN EQUILIBRIUM

Upper limit on Q-ball charge:

$$\tau_{Q_{\min}\to Q_{\max}} = \sum_{Q=Q_{\min}}^{Q_{\max}} \frac{1}{n_S\,(\sigma v_{\mathrm{rel}})_Q} \lesssim H^{-1}$$

With an initial asymmetry η, charge-up exceeds charge-down at sufficiently small $T < m_S$.

IN EQUILIBRIUM

Equilibrium number densities:

$$n_i^{\rm eq} = \frac{1}{2\pi^2} T\, m_i^2 \exp(\mu_i/T) K_2(m_i/T)\,,$$

Chemical potential relationship:

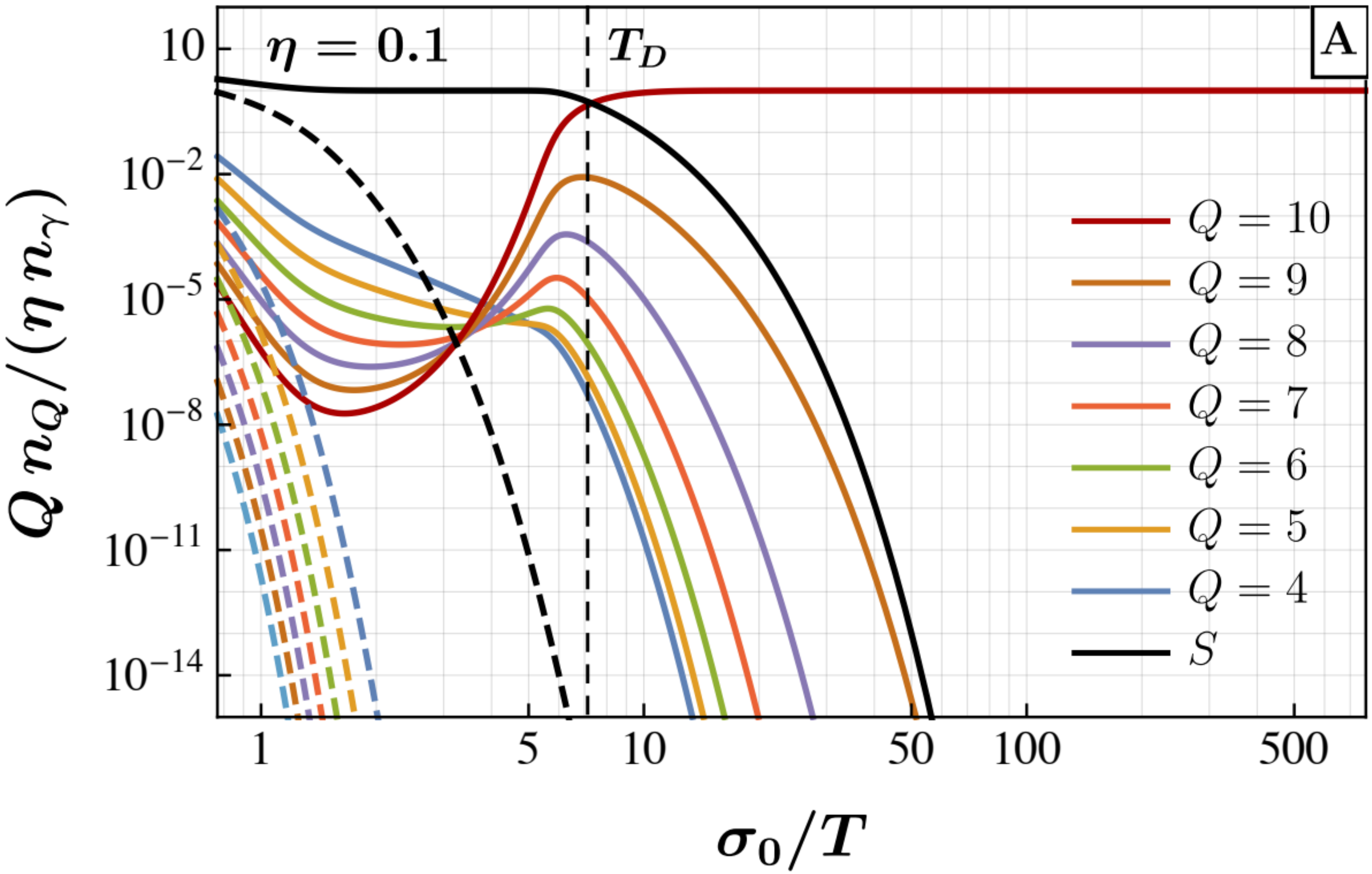
$$\begin{aligned}\mu_{S^\dagger} &= -\mu_S\,,\\ \mu_Q &= Q\,\mu_S\,,\\ \mu_{-Q} &= -\mu_Q\,.\end{aligned}$$

Defines a single chemical potential $\mu \equiv \mu_S$, which is determined by

$$Z_S - Z_{S^\dagger} + \sum_{Q=Q_{\rm min}}^{Q_{\rm max}} Q\,(Z_Q - Z_{-Q}) = 1\,.$$

$$Z_i \equiv n_i/(\eta\, n_\gamma)$$

IN EQUILIBRIUM



IN EQUILIBRIUM

What charge dominates at low temperature?

$$m_Q = m_1 Q^p$$

$$r = \frac{(Q+1)n_{Q+1}}{Q\,n_Q} = \left(\frac{Q+1}{Q}\right)^{\frac{3p}{2}+1} \exp\left(\frac{m_Q - m_{Q+1} + \mu}{T}\right)$$

For $p < 1$, Q_{max} dominates:

$$r \xrightarrow{Q\to\infty} e^{\mu/T} > 1$$

For $p = 1$, a low to intermediate charge dominates:

$$\frac{dr}{dQ} = \frac{2p\,m_1\left(Q^p(Q+1) - (Q+1)^p Q\right) - (2+3p)T}{2\,Q^2\,T}\left(\frac{Q+1}{Q}\right)^{3p/2} \exp\left(\frac{m_Q - m_{Q+1} + \mu}{T}\right)$$

$$< 0$$

IN EQUILIBRIUM

Analytic estimate of *charge*-domination temperature T_D:

$$Z_S \approx Q_{\max} Z_{Q_{\max}} \approx 1/2$$

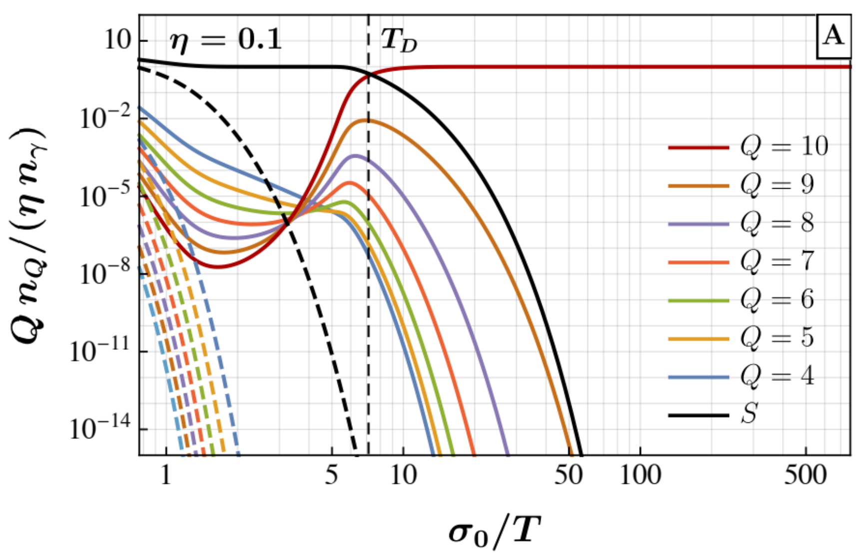
$$\boxed{B_{Q_{\max}} = Q_{\max}\, m_S - m_Q}$$

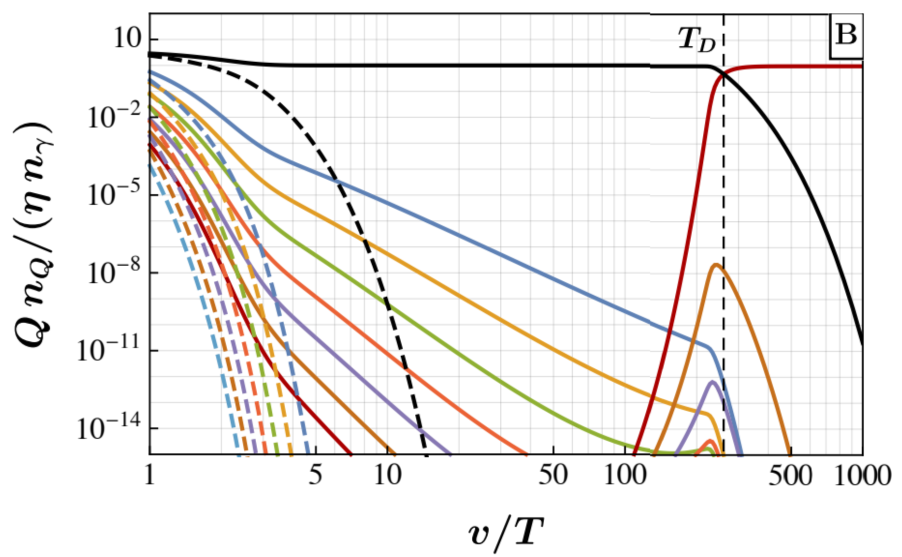
$$T_D = \frac{B_{Q_{\max}}}{\log\left\{ \frac{1}{Q_{\max}} \left[\frac{2}{\eta c_\gamma} \left(\frac{m_S}{2\pi T_D} \right)^{\frac{3}{2}} \right]^{Q_{\max}-1} \left(\frac{m_S}{m_{Q_{\max}}} \right)^{\frac{3}{2}} \right\}}$$

$$\xrightarrow{Q_{\max}\to\infty} \frac{m_S - m_{Q_{\max}}/Q_{\max}}{\log\left\{ \frac{2}{\eta c_\gamma} \left(\frac{m_S}{2\pi T_D} \right)^{\frac{3}{2}} \right\}}$$

No Q_{max} dependence for $p \leq 1$

IN EQUILIBRIUM





IN EQUILIBRIUM

Analytic estimate of *energy-density*-domination temperature T_ρ:

$$n_{Q_{\rm max}} \simeq \eta\, n_\gamma / Q_{\rm max}$$

$$\mu \simeq \frac{1}{Q_{\rm max}} \left(m_{Q_{\rm max}} + T \log \left[\frac{\eta\, c_\gamma}{Q_{\rm max}} \left(\frac{2\pi T}{m_{Q_{\rm max}}} \right)^{3/2} \right] \right) , \quad T < T_D$$

Solving for $\rho_S = \rho_{Q_{max}}$

$$T_\rho \xrightarrow{Q_{\rm max} \to \infty} \frac{m_S - m_{Q_{\rm max}}/Q_{\rm max}}{\log \left[\frac{Q_{\rm max} m_S}{m_{Q_{\rm max}}} \frac{1}{\eta\, c_\gamma} \left(\frac{m_S}{2\pi T_\rho} \right)^{3/2} \right]}$$

FREEZE OUT

Boltzmann equations:

$$
\begin{aligned}
\dot{n}_Q + 3Hn_Q = &- \delta_{Q,Q_{\min}} (\sigma v_{\rm rel})_{Q_{\min}} \left(n_{Q_{\min}} n_{S^\dagger} - n_{Q_{\min}}^{\rm eq} n_{S^\dagger}^{\rm eq} \left(\frac{n_S}{n_S^{\rm eq}} \right)^{Q_{\min}-1} \right) \\
&- (1 - \delta_{Q,Q_{\max}}) (\sigma v_{\rm rel})_Q \left(n_Q n_S - n_Q^{\rm eq} n_S^{\rm eq} \left(\frac{n_{Q+1}}{n_{Q+1}^{\rm eq}} \right) \right) \\
&+ (1 - \delta_{Q,Q_{\min}}) (\sigma v_{\rm rel})_{Q-1} \left(n_{Q-1} n_S - n_{Q-1}^{\rm eq} n_S^{\rm eq} \left(\frac{n_Q}{n_Q^{\rm eq}} \right) \right) \\
&- (1 - \delta_{Q,Q_{\min}}) (\sigma v_{\rm rel})_Q \left(n_Q n_{S^\dagger} - n_Q^{\rm eq} n_{S^\dagger}^{\rm eq} \left(\frac{n_{Q-1}}{n_{Q-1}^{\rm eq}} \right) \right) \\
&+ (1 - \delta_{Q,Q_{\max}}) (\sigma v_{\rm rel})_{Q+1} \left(n_{Q+1} n_{S^\dagger} - n_{Q+1}^{\rm eq} n_{S^\dagger}^{\rm eq} \left(\frac{n_Q}{n_Q^{\rm eq}} \right) \right)
\end{aligned}
$$

FREEZE OUT

Summed Boltzmann equations:

$$\dot{n}_{\mathrm{NTS}} + 3\,H\,n_{\mathrm{NTS}} = -(\sigma v_{\mathrm{rel}})_{Q_{\min}} \left(n_{Q_{\min}} n_{S^\dagger} - n^{\mathrm{eq}}_{Q_{\min}} n^{\mathrm{eq}}_{S^\dagger} \left(\frac{n_S}{n_S^{\mathrm{eq}}} \right)^{Q_{\min}-1} \right)$$

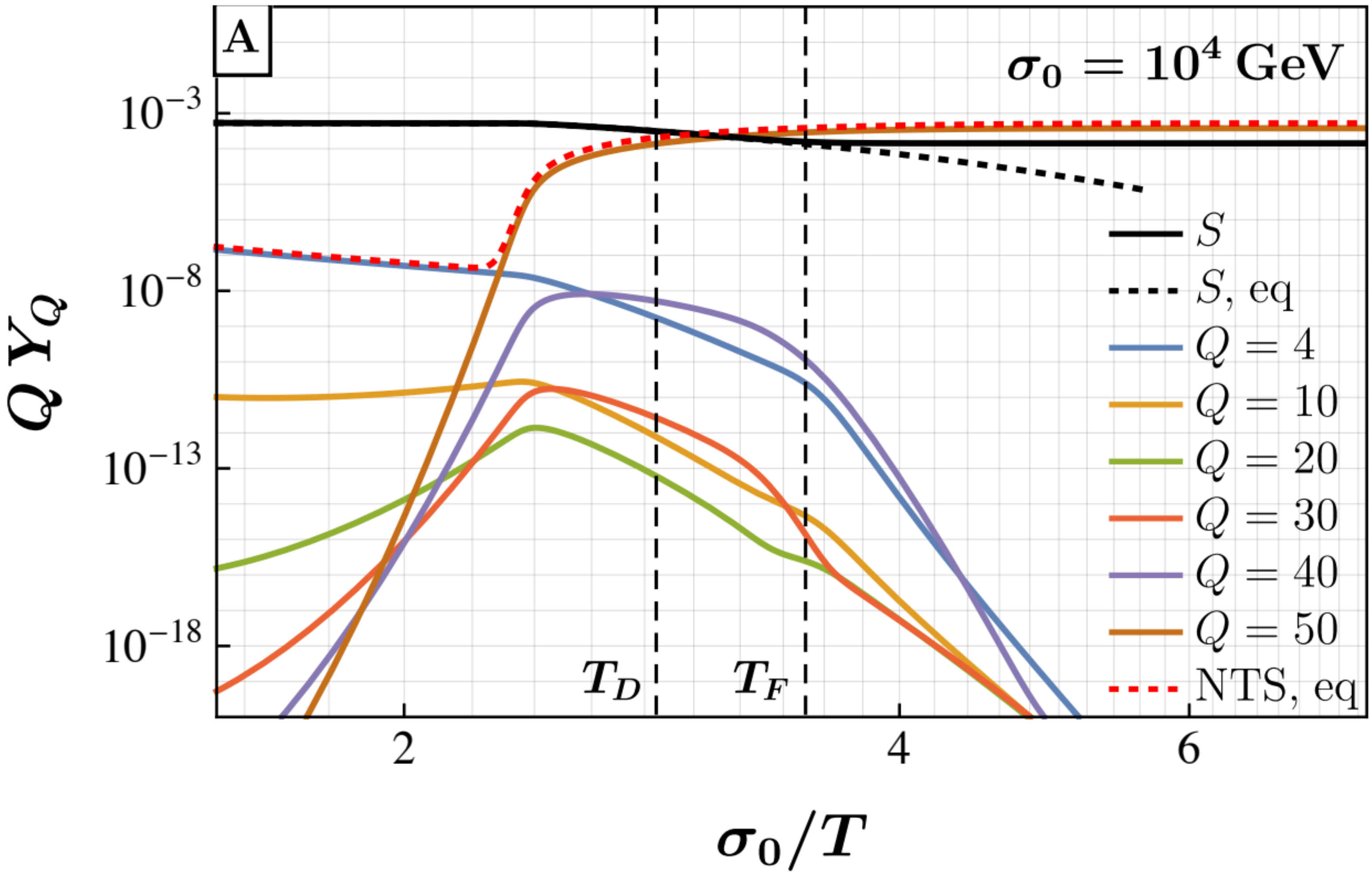
$$n_{\mathrm{NTS}} \equiv \sum_{Q=Q_{\min}}^{Q_{\max}} n_Q$$

Freeze-out temperature:

$$H\, n^{\mathrm{eq}}_{\cancel{\mathrm{NTS}}\,Q_{\max}} \sim (\sigma v_{\mathrm{rel}})_{Q_{\min}}\, n^{\mathrm{eq}}_{Q_{\min}}\, n^{\mathrm{eq}}_{S^\dagger}\, \big|_{T=T_F}$$

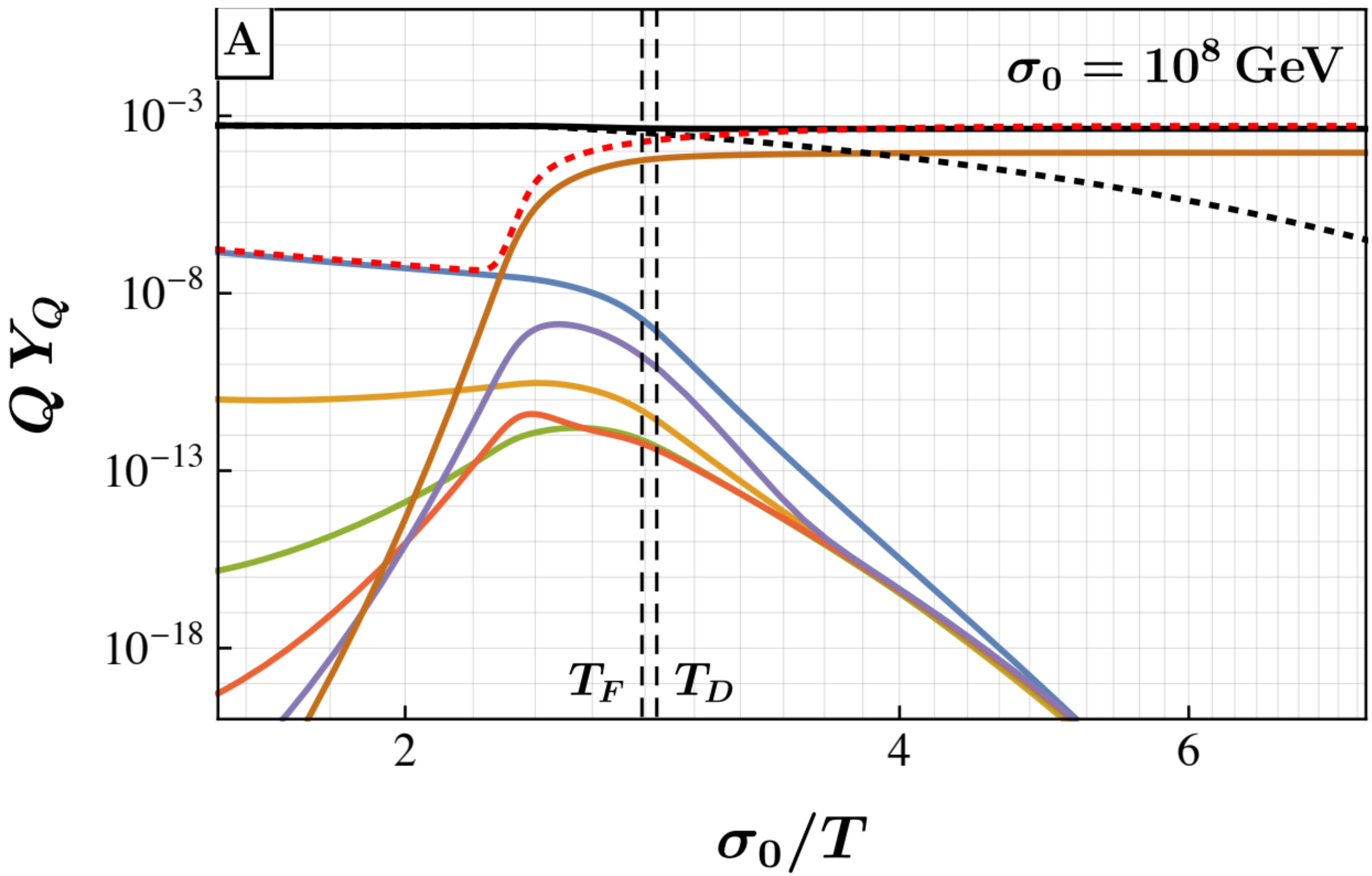
Get an analytic estimate for $T < T_D$ using $n^{\mathrm{eq}}_{\mathrm{NTS}} \approx n^{\mathrm{eq}}_{Q_{\max}}$.

FREEZE OUT



A
$\sigma_0 = 10^4$ GeV
10^{-3}
10^{-8}
10^{-13}
10^{-18}
$Q\,Y_Q$
T_D
T_F
2
4
6
σ_0/T
S
S, eq
$Q = 4$
$Q = 10$
$Q = 20$
$Q = 30$
$Q = 40$
$Q = 50$
NTS, eq

FREEZE OUT

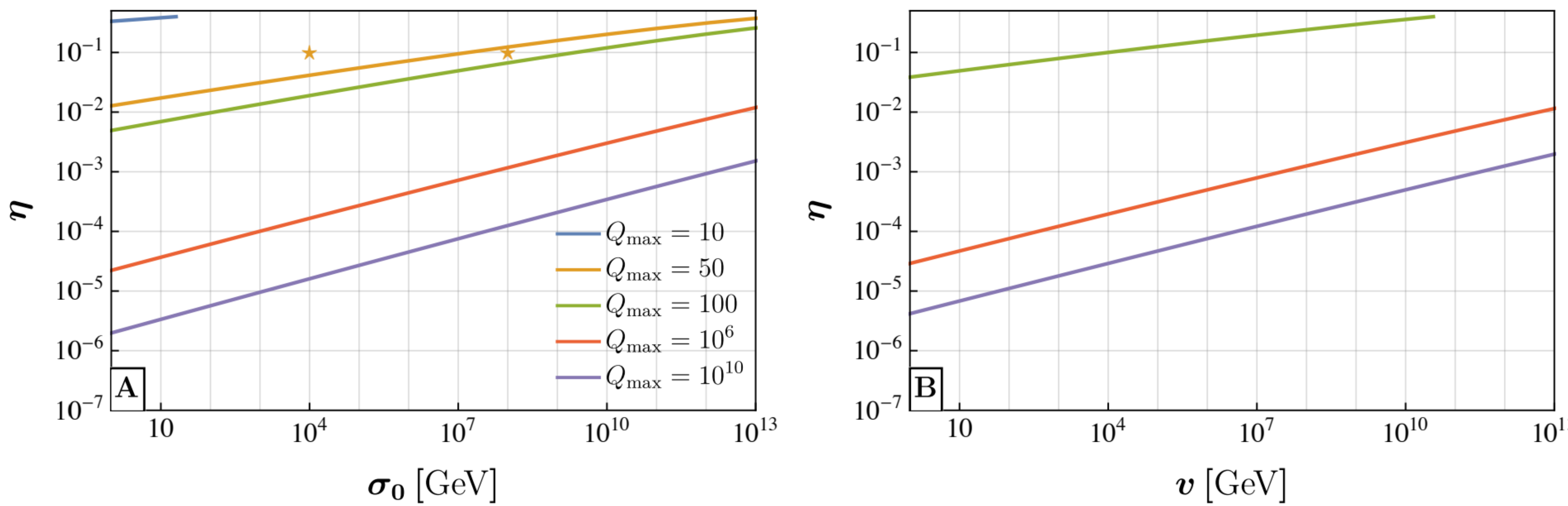


Q-BALL DOMINATION AFTER FREEZE OUT

When is $T_F < T_D$?

Equate our analytic estimates for each:

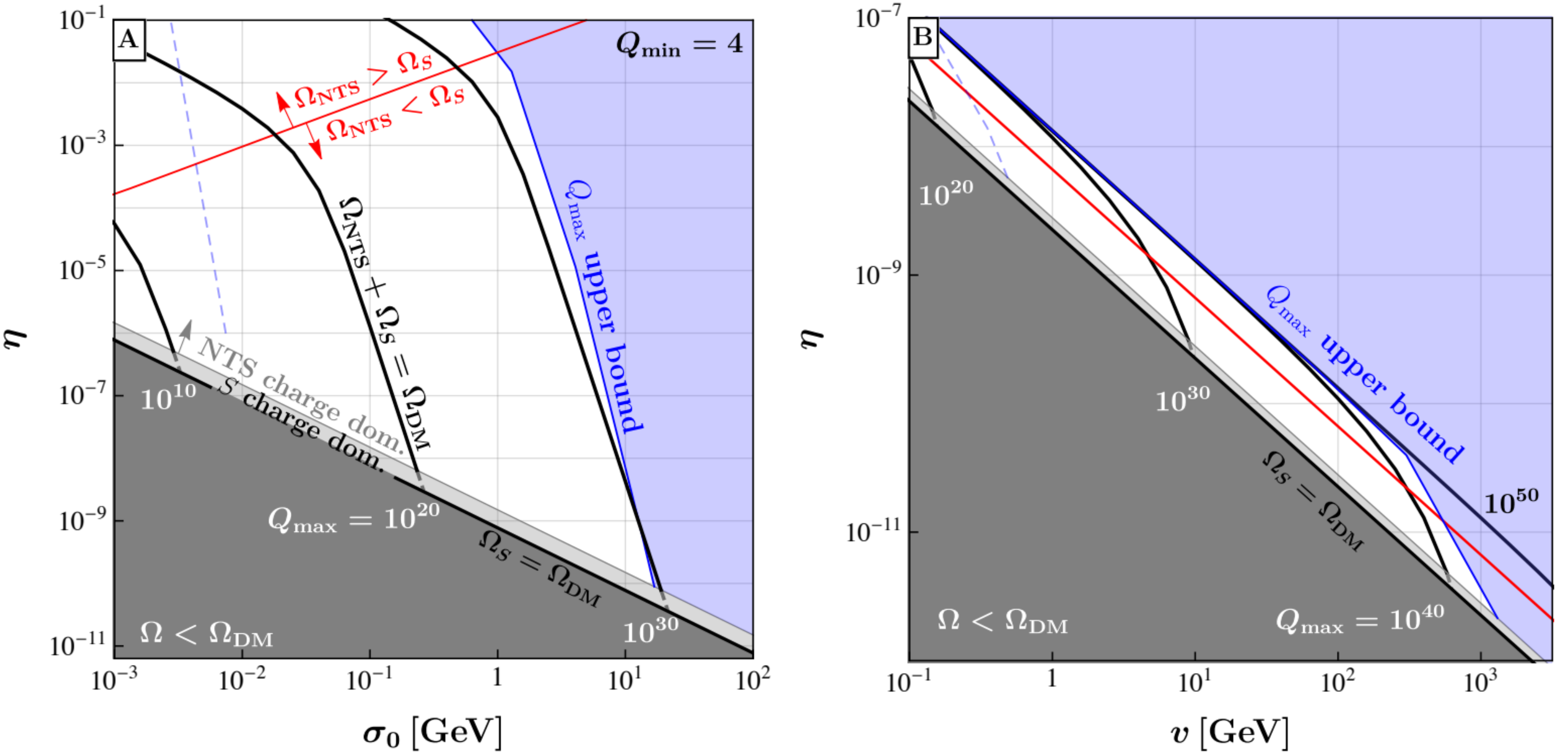
Plots: $Q_{min} = 4$



Derived scaling relationship at large Q_{max}:

$$\eta \propto \left[\frac{v}{Q_{\max} M_{\rm pl}}\right]^{\frac{m_S}{m_{Q_{\min}}} \approx 1/Q_{\min}}$$

DARK MATTER ABUNDANCE

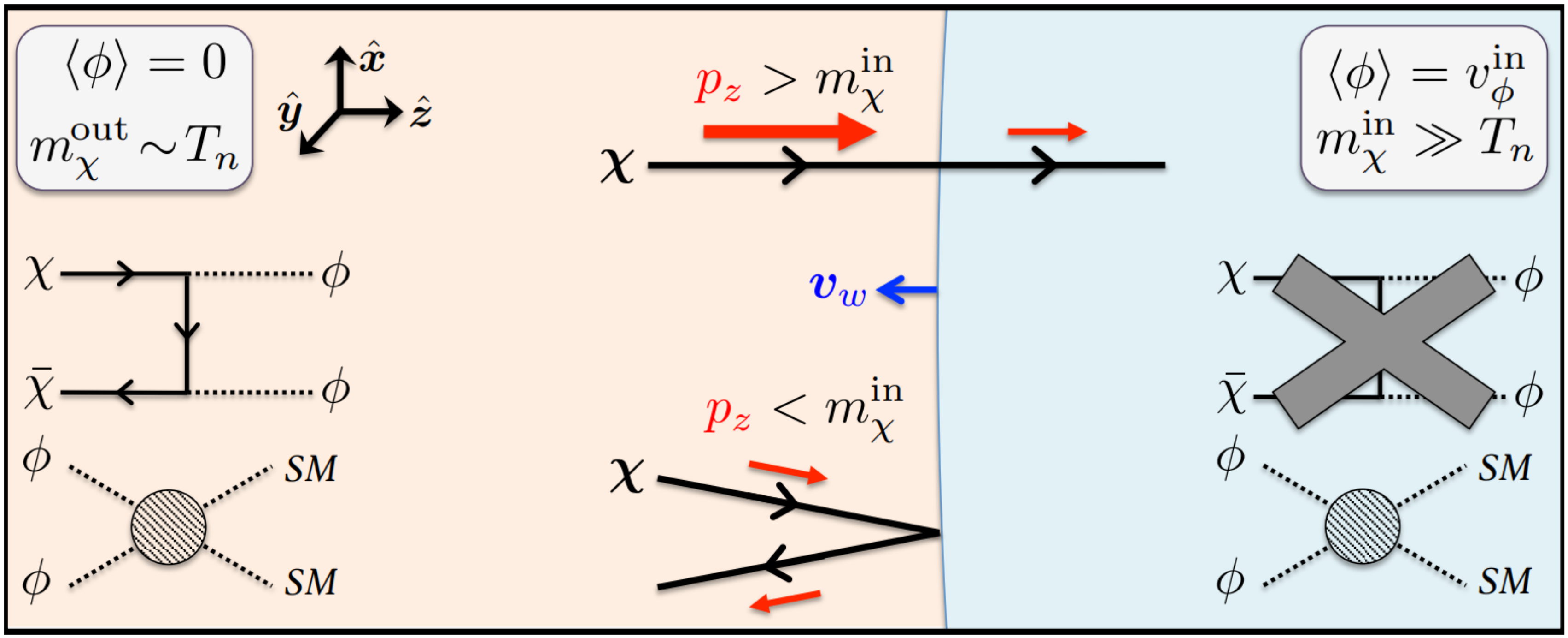


ϕ PHASE TRANSITION

1ST ORDER PHASE TRANSITION FORMATION

Phase transition in ϕ

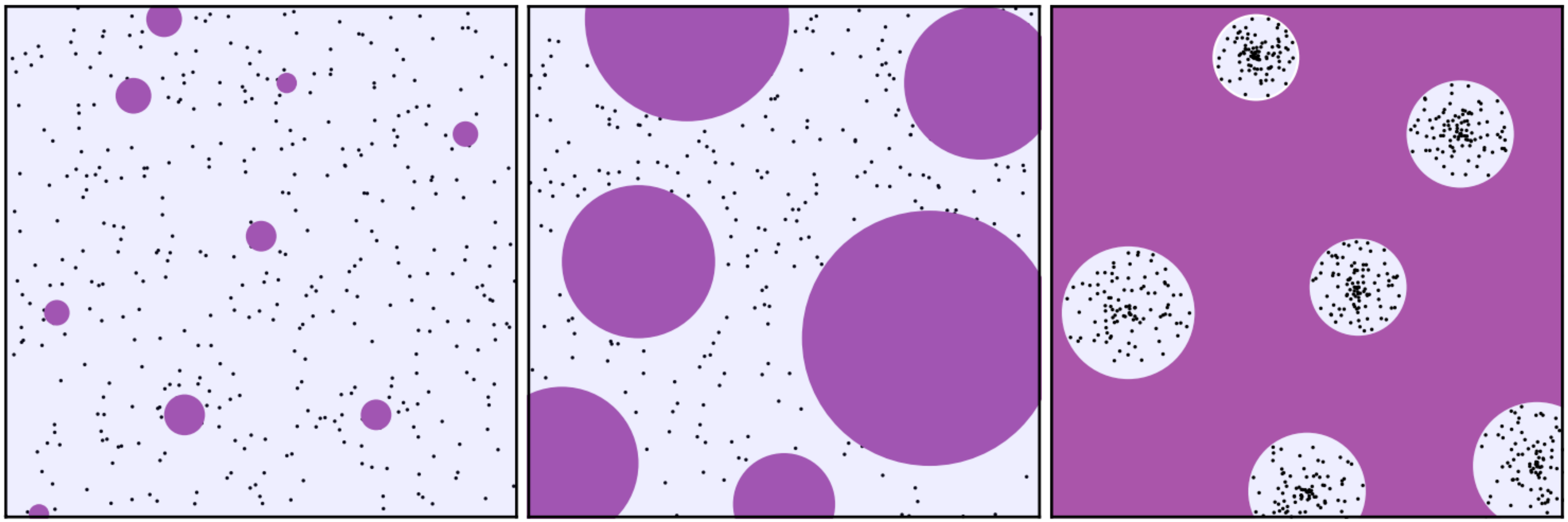
$$V(S,\phi) = \frac{1}{4}\lambda_\phi(|\phi|^2 - v^2)^2 + \frac{1}{4}\lambda_{\phi S}|S|^2|\phi|^2 + \lambda_S|S|^4 + m_{S,0}^2|S|^2$$



[Baker Kopp Long 1912.02830]

$\chi = S$

1ST ORDER PHASE TRANSITION FORMATION



Adapted from [Asadi, *et. al.* 2103.09827]

See [Frieman Gelmini Gleiser Kolb 1988; Griest Kolb Massarotti 1989; Frieman Olinto Gleiser Alcock 1989; Macpherson Campbell hep-ph/9408387; Bai Long Lu 1810.04360; Pónton Bai Jain 1906.10739; Hong Jung Xie 2008.04430; Bai Lu **NO** 2111.10360;…]

1^{ST} ORDER PHASE TRANSITION FORMATION

Bubble nucleation rate per unit volume:

$$\gamma \approx T^4 \left(\frac{S_3}{2\pi T}\right)^{3/2} e^{-S_3/T}$$

Related to the 3D bounce action:

$$S_3 = 4\pi \int r^2 dr \left[\frac{1}{2}\left(\frac{\partial\phi}{\partial r}\right)^2 + V(\phi, T)\right]$$

Thin-wall approximation gives a leading term and series expansion:

$$\frac{S_3}{T} = \frac{a}{\epsilon_c^2} + \frac{b}{\epsilon_c} + \ldots \qquad \epsilon_c(T) \equiv (T_c - T)/T_c$$

Fraction of space in the false vacuum:

$$h(t) = \exp\left[-\frac{4\pi}{3}\int_{t_c}^{t} dt'\, v_{\rm sh}^3\,(t-t')^3\,\gamma(t')\right]$$

1^{ST} ORDER PHASE TRANSITION FORMATION

Define the bubble nucleation time:

$$h(t_n) = 1/e$$

The number density of bubble nucleation sites at this time:

$$n_{\rm nuc} = \int_{t_c}^{t_n} dt'\, \gamma(t')\, h(t') \;\approx\; \left(8\pi v_{\rm sh}^3\, \beta^{-3}\right)^{-1}$$

$$\beta/H \approx T\ d(S_3/T)/dT = (1-\epsilon_c)(2a/\epsilon_c^3 + b/\epsilon_c^2)$$

Taking only the leading (a) term:

$$n_{\text{Q-ball}}(T_n) \sim n_{\rm nuc} \approx (4\pi v_{\rm sh}^3 a^{1/2})^{-1} H_n^3 \left(\log\left[\frac{v_{\rm sh}^3\, \epsilon_n^9\, T_n^4}{8\sqrt{2\pi}\, a^{5/2}\, H_n^4}\right]\right)^{3/2}$$

2^{ND} ORDER PHASE TRANSITION FORMATION

Probability to be in false or true vacuum at Ginzburg temperature:

$$p_{\text{false}}/p_{\text{true}} \sim \exp[-\Delta V(T_G)\,(2\xi)^3/T_G]$$
$$\xi \simeq (\lambda_\phi T_G)^{-1}$$

False vacuum regions can become Q-balls:

$$n_{\text{Q-ball}}(T_G) \sim \frac{1}{1+p_{\text{true}}/p_{\text{false}}}\xi^{-3} \sim 10^{-1}\lambda_\phi^{3/2}v^3\,, \quad \text{(SOPT)}$$

FOPT makes <u>fewer</u> Q-balls than SOPT:

$$n_{\text{Q-ball}}(T_n) \sim n_{\text{nuc}} \approx (4\pi v_{\text{sh}}^3 a^{1/2})^{-1} H_n^3 \left(\log\left[\frac{v_{\text{sh}}^3\,\epsilon_n^9\,T_n^4}{8\sqrt{2\pi}\,a^{5/2}\,H_n^4}\right]\right)^{3/2}, \quad \text{(FOPT)}$$
$$v \gg H_n \sim v^2/M_{\text{pl}}$$

PHASE TRANSITION FORMATION

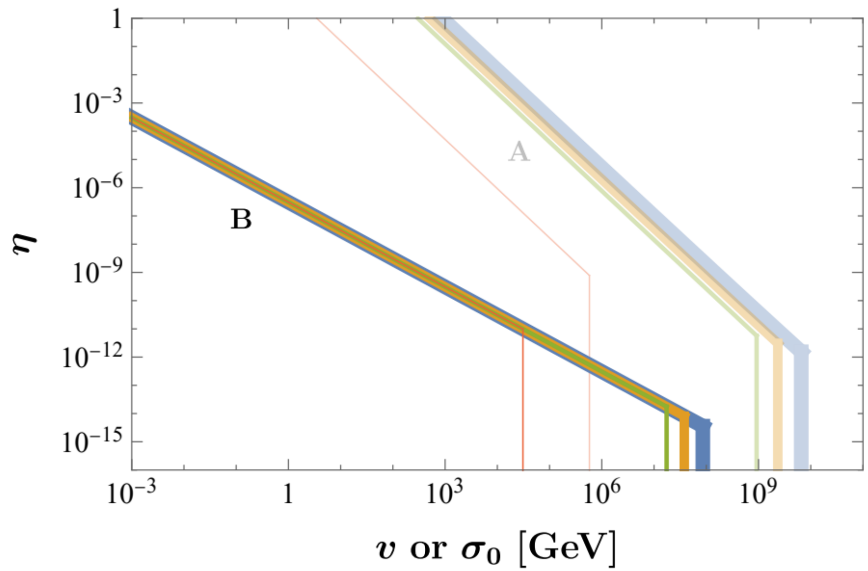
Q-ball properties determined by number of S particles in each false vacuum region:

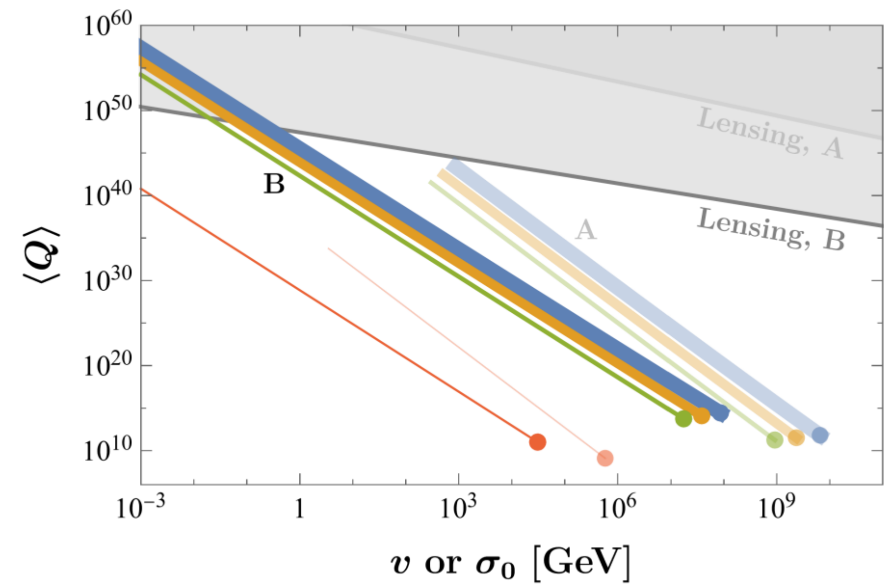
$$N_S^{\text{Q-ball}} \sim p_{\text{in}}\, n_S / n_{\text{Q-ball}}$$

$$\langle Q \rangle \sim \max\left[\underbrace{\eta N_S^{\text{Q-ball}}}_{\text{Asymmetric component}},\ \underbrace{(N_S^{\text{Q-ball}})^{1/2}}_{\text{Symmetric component}} \right]$$

1ST ORDER PHASE TRANSITION FORMATION

Q-ball abundance making up all dark matter:

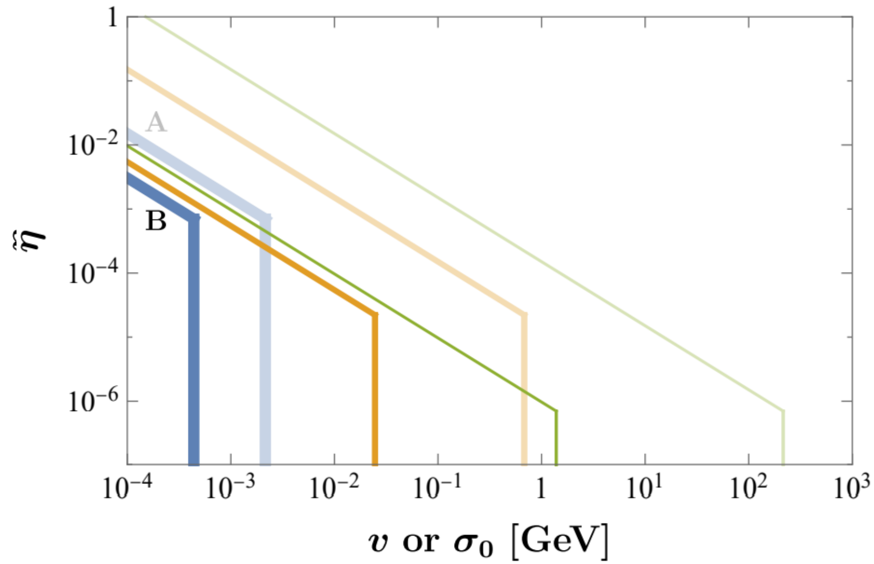


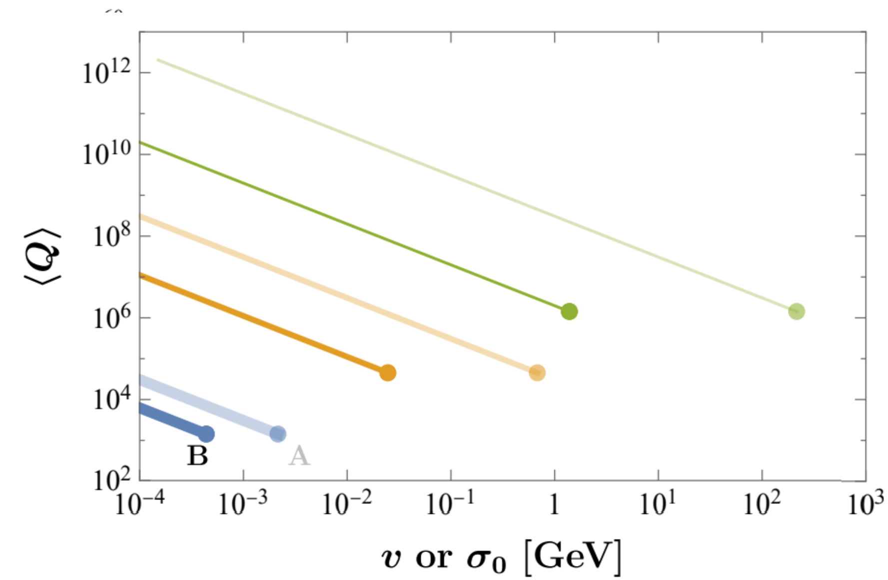


Colors: varying a

2^{ND} ORDER PHASE TRANSITION FORMATION

Q-ball abundance making up all dark matter:





Colors: varying λ_ϕ and taking $T_G = \lambda_\phi^{-1/2} v$

IS SOLITOSYNTHESIS "EFFICIENT"?

Are phase transition initial conditions important?
Or does solitosynthesis erase all initial conditions?

Initial conditions are modified by:

If both occur and $\langle Q \rangle < Q_{\mathrm{max}}$, PT unimportant, solitosynthesis dominates.

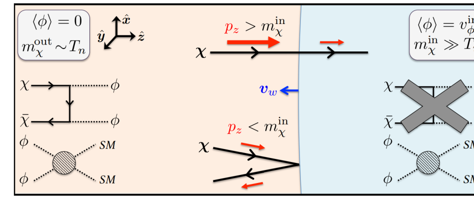
1. Building up of Q-balls starting from free-particle fusion.

   Depends on the phase transition and solitosynthesis freeze-out temperatures, T_f and T_F.

2. Evolution of Q-balls seeded during phase transition.

   Suppressed if free particles in true vacuum regions are suppressed.
   Depends on the free particle mass m_S compared to the phase transition temperature T_f.

IS SOLITOSYNTHESIS "EFFICIENT"?



2. For no evolution of Q-balls seeded during phase transition:

   a) Free particles remain inside the false vacuum bubbles during the phase transition.

      Boltzmann suppressed $\sim e^{-m_S/T}$. Therefore, $m_S/T_f \gtrsim \mathcal{O}(10)$.

   b) Free particles not thermally produced. $S + S^\dagger \quad \leftrightarrow \quad \phi + \phi^\dagger$

      $(\sigma v_{\rm rel}) n_S^{\rm eq} \lesssim H$ $\qquad m_S/T_f \gtrsim 31 + (3/2)\log[m_S/(31 T_f)] + \log(T_f \cdot {\rm TeV}/v^2)$

   c) Particles cannot be dislodged from Q-balls by the thermal bath.

$$(Q) + S \quad \leftrightarrow \quad (Q+1) + X$$

$$m_S/T_f \gtrsim 50 + (3/2)\log[m_S/(50\,T_f)] + \log(T_f \cdot {\rm TeV}/v^2) + (1/2)\log(\langle Q\rangle/10^{10}) \qquad \text{Model A}$$
$$m_S/T_f \gtrsim 53 + (3/2)\log[m_S/(53\,T_f)] + \log(T_f \cdot {\rm TeV}/v^2) + (2/3)\log(\langle Q\rangle/10^{10}) \qquad \text{Model B}$$

BENCHMARKS

Points where free particles + Q-balls make up all of dark matter:

Mechanism	Model	η	m_Q (g)	R_Q (m)	$\langle Q \rangle$	σ_0 or v (GeV)
Solitosynthesis	A	10^{-10}	3	3×10^{-10}	6×10^{29}	10
	B	10^{-10}	5×10^{22}	2×10^{-3}	1×10^{45}	1×10^{2}
	B	10^{-6}	6×10^{30}	3×10^{5}	1×10^{57}	1×10^{-2}
FOPT	A	0	9×10^{-6}	5×10^{-23}	3×10^{11}	2×10^{9}
	B	0	2×10^{-3}	4×10^{-19}	1×10^{14}	4×10^{7}
	B	10^{-4}	8×10^{26}	1×10^{5}	7×10^{53}	3×10^{-3}
SOPT	A	0	2×10^{-20}	3×10^{-15}	5×10^{4}	7×10^{-1}
	B	0	1×10^{-20}	5×10^{-13}	5×10^{4}	2×10^{-2}

Q-balls dominate the $U(1)_S$ global charge density in all cases.

Q-balls dominate the energy density in all cases except first row.

MAGNETIC Q-BALLS

MOTIVATION

Magnetic monopoles could explain the observed quantization of electric charge, but we have not detected them.

One possibility: we are looking for the wrong type of monopole.

Larger bound states of monopoles?

Plan:

- Start with unit magnetic charge $\rightarrow$ spherically symmetric solutions
- Generalize to Q-monopole-balls with larger magnetic charge
- Formation

THEORY

$$\begin{aligned}\mathcal{L} &= |\partial_\mu S|^2 + \frac{1}{2}(D_\mu \phi^a)^2 - \frac{1}{4}F^a_{\mu\nu}F^{a\mu\nu} - V(S,\phi)\,,\\ V(S,\phi) &= \frac{1}{8}\lambda_\phi(\phi^a\phi^a - v^2)^2 + \frac{1}{2}\lambda_{\phi S}|S|^2(\phi^a\phi^a) + \lambda_S|S|^4 + m^2_{S,0}|S|^2\,,\end{aligned}$$

ϕ^a: SU(2) gauge triplet scalar
S: U(1) global complex scalar

Spontaneous SU(2) breaking leads to monopoles:

$$\pi_2[SU(2)/U(1)] = \mathbb{Z}$$

(All parameters are taken positive)

EQUATIONS OF MOTION

Use dimensionless rescaling of fields, with time-dependent phase for S and "hedgehog gauge" for ϕ^a:

$$\phi^a = \hat{r}^a\, v\, f(r)\,, \quad S = e^{-i\omega t}\frac{v}{\sqrt{2}}\, s(r)\,, \quad A_0 = 0\,, \quad A_i^a = \epsilon^{aij}\frac{\hat{r}^j}{e\, r}\, a(r)$$

Classical EOM:

$$\begin{aligned}
&a'' - \frac{1}{\bar{r}^2}\, a(1-a)(2-a) + e^2\,(1-a)\, f^2 = 0\,,\\
&f'' + \frac{2}{\bar{r}}\, f' - \frac{2}{\bar{r}^2}\,(1-a)^2\, f - \frac{1}{2}\,\lambda_\phi\, f\,(f^2-1) - \frac{1}{2}\,\lambda_{\phi S}\, s^2 f = 0\,,\\
&s'' + \frac{2}{\bar{r}}\, s' + \Omega^2\, s - \frac{1}{2}\,\lambda_{\phi S}\, f^2\, s - \lambda_S\, s^3 - \mu_0^2\, s = 0\,,
\end{aligned}$$

$$\boxed{U_{\text{eff}} = \Omega^2 s^2/2 - V(s,f)}$$

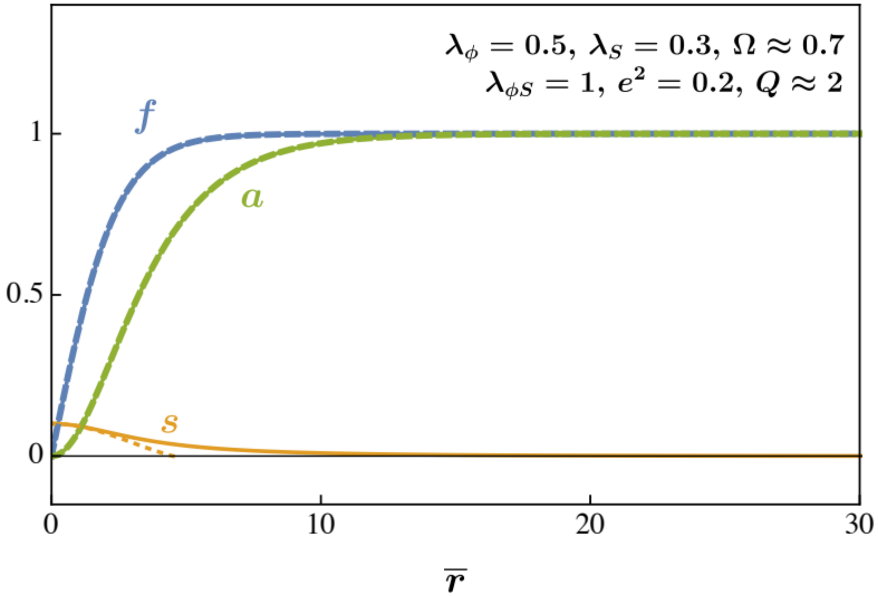
Boundary conditions:

$$f(0) = 0\,,\ f(\infty) = 1\,,\ s'(0) = 0\,,\ s(\infty) = 0\,,\ a(0) = 0\,,\ a(\infty) = 1$$

Rescalings: $\bar{r} = v\, r$ $\qquad \Omega = \omega/v$ $\qquad \mu_0 = m_{S,0}/v$

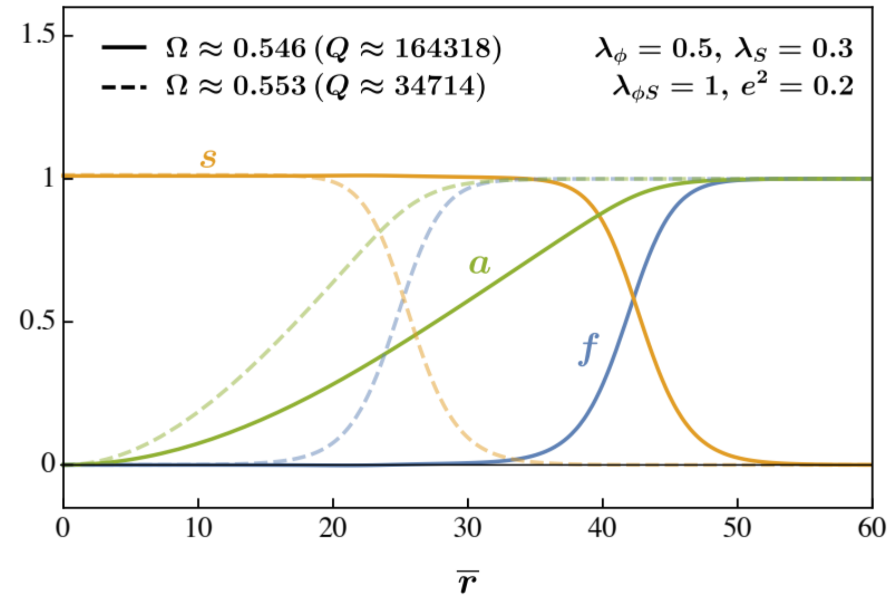
FIELD PROFILES

Small Q



"S particles bound in a monopole"

Large Q



"Monopole bound in a Q-ball"

LARGE Q

Ansatz:

$$f(\overline{r}) \approx \begin{cases} 0\,, & \overline{r} < \overline{r}_b \\ 1\,, & \overline{r} > \overline{r}_b \end{cases}\,, \quad s(\overline{r}) \approx \begin{cases} s_0\,, & \overline{r} < \overline{r}_b \\ 0\,, & \overline{r} > \overline{r}_b \end{cases}\,, \quad a(\overline{r}) \approx \begin{cases} \overline{r}^2/\overline{r}_b^2\,, & \overline{r} < \overline{r}_b \\ 1\,, & \overline{r} > \overline{r}_b \end{cases}\,.$$

Mass ($\mu_0 = 0$):

$$M_{(2,Q)} \approx \underbrace{\frac{304\,\pi\,v}{35\,e^2\,\overline{r}_b}}_{\int d^3x B^2/2} + \underbrace{\frac{4\pi}{3}\overline{r}_b^3 v\left(\frac{1}{4}\lambda_S s_0^4 + \frac{1}{8}\lambda_\phi + \frac{1}{2}\Omega^2 s_0^2\right)}_{\text{Vacuum energy}}$$

Global charge Q:

$$Q \approx \frac{4\pi}{3}\,\overline{r}_b^3\,\Omega\,s_0^2$$

LARGE Q

Ansatz:

$$f(\overline{r}) \approx \begin{cases} 0\,, & \overline{r} < \overline{r}_b \\ 1\,, & \overline{r} > \overline{r}_b \end{cases}\,, \quad s(\overline{r}) \approx \begin{cases} s_0\,, & \overline{r} < \overline{r}_b \\ 0\,, & \overline{r} > \overline{r}_b \end{cases}\,, \quad a(\overline{r}) \approx \begin{cases} \overline{r}^2/\overline{r}_b^2\,, & \overline{r} < \overline{r}_b \\ 1\,, & \overline{r} > \overline{r}_b \end{cases}\,.$$

Mass ($\mu_0 = 0$):

$$M_{(2,Q)} \approx \frac{304\,\pi\,v}{35\,e^2\,\overline{r}_b} + \frac{4\pi}{3}\overline{r}_b^3 v\left(\frac{1}{4}\lambda_S s_0^4 + \frac{1}{8}\lambda_\phi + \frac{1}{2}\Omega^2 s_0^2\right)$$

Eliminate Ω for Q, minimize w.r.t. $\bar{r}_b, s_0$:

$$\begin{aligned} \overline{r}_b &\approx \frac{(3/\pi)^{1/3}}{2^{5/12}}\,\frac{\lambda_S^{1/12}}{\lambda_\phi^{1/4}}\,Q^{1/3}\,, \\ M_{(2,Q)} \approx M_{(0,Q)} &\approx Q\,\Omega_c\,v\,, \\ s_0 &\approx \left(\frac{\lambda_\phi}{2\,\lambda_S}\right)^{1/4}\,, \\ \Omega &\approx \Omega_c\,. \end{aligned}$$

$$\boxed{\overline{\Omega}^4 > \overline{\Omega}_c^4 \equiv \frac{1}{2}\,\lambda_S\,\lambda_\phi}$$

LARGE Q STABILITY

"Monopole bound in a Q-ball":

$$\Delta M = M_{(2,0)} + M_{(0,Q)} - M_{(2,Q)} \approx \frac{4\pi v}{e} Y - \frac{304 \times 2^{5/12}\,\pi^{4/3}}{35 \times 3^{1/3}} \frac{\lambda_\phi^{1/4}}{e^2\,\lambda_S^{1/12}} Q^{-1/3}\, v$$

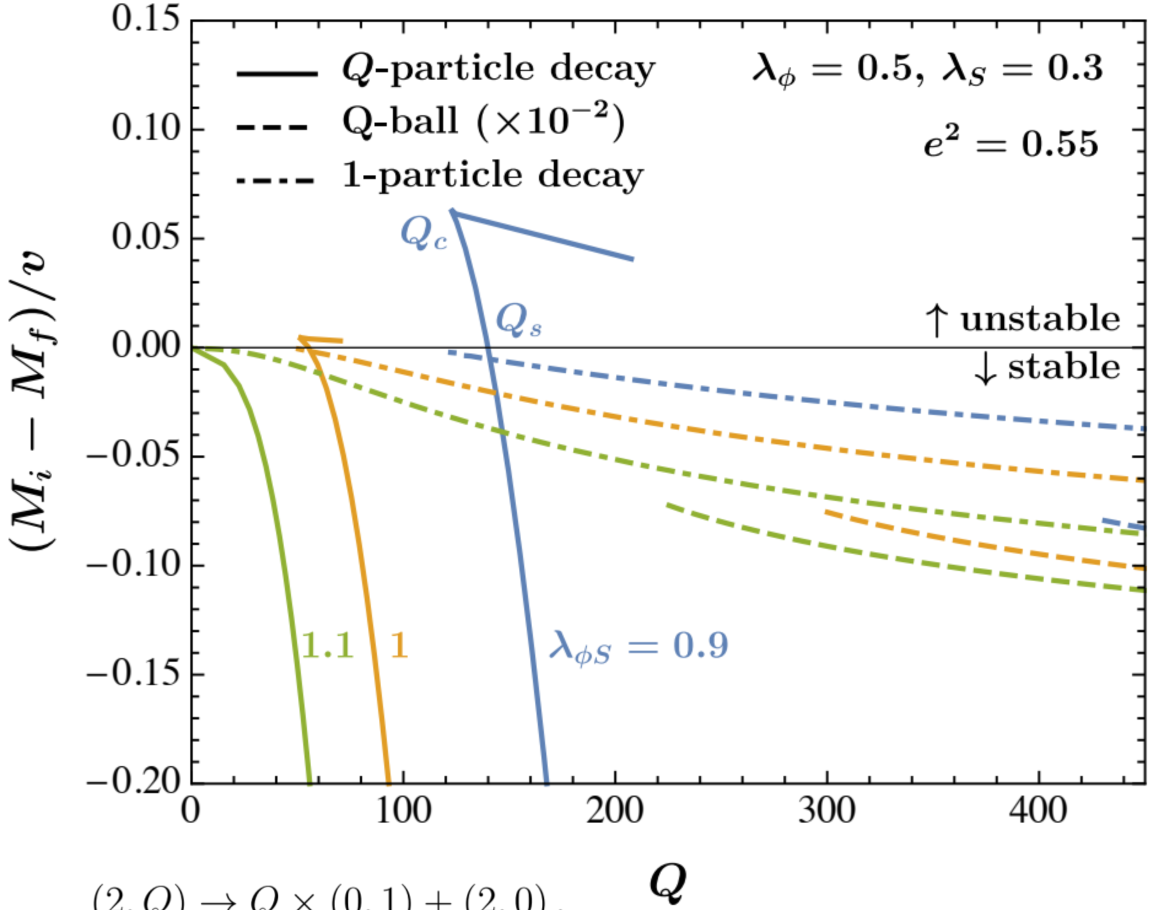
Evaporation into free S particles stable if:

$$(M_{(2,Q)} - M_{(2,0)})/Q \; < \; m_S \; = \; v\sqrt{\lambda_{\phi S}/2},$$

$$\left[\begin{array}{c} M_{(2,Q)} \approx M_{(0,Q)} \;\; \approx \;\; Q\,\Omega_c\,v\,, \\ \overline{\Omega}^2 \equiv \Omega^2 - \mu_0^2 < \dfrac{\lambda_{\phi S}}{2} \end{array}\right]$$

<u>Always stable at sufficiently large Q.</u>

MASS AND STABILITY

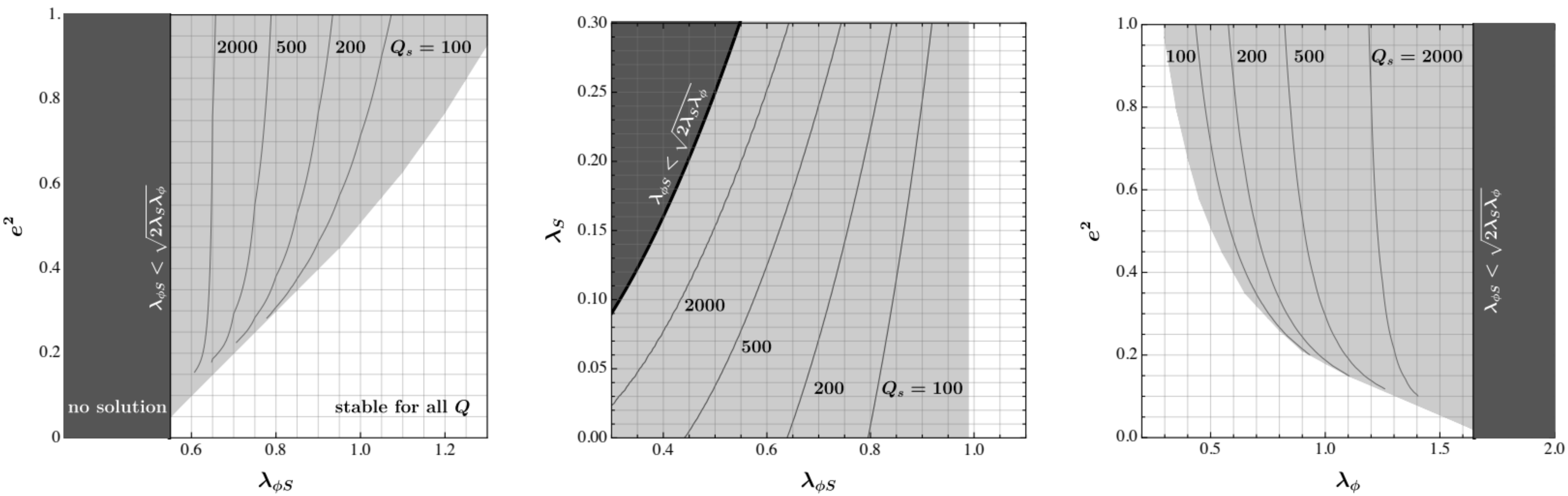


Q-particle decay : $(2, Q) \to Q \times (0, 1) + (2, 0)\,,$

Q-ball decay : $(2, Q) \to (0, Q) + (2, 0)\,,$

1-particle decay : $(2, Q) \to (2, Q - 1) + (0, 1)\,.$

STABILITY



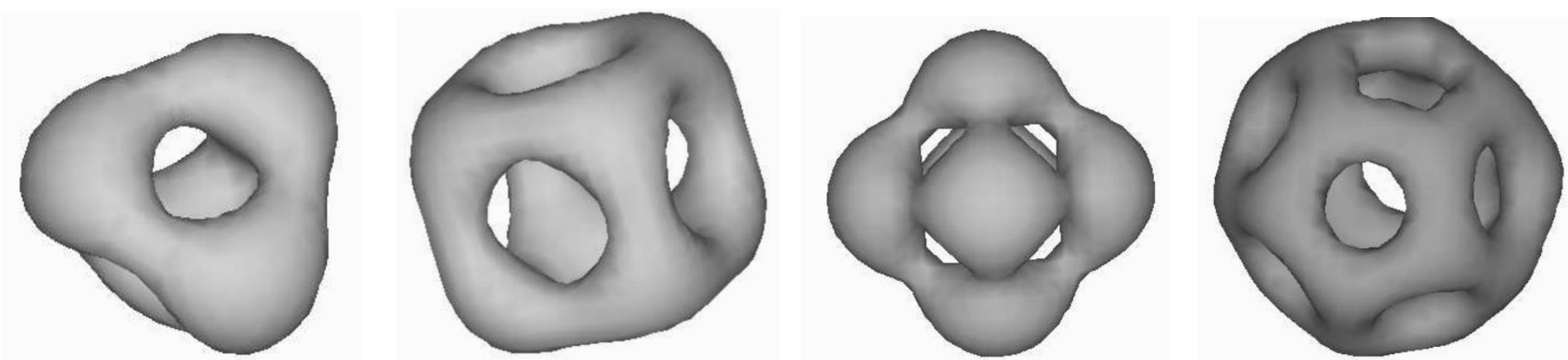
When not being varied:

$$\lambda_\phi = 0.5,\ e^2 = 0.5,\ \lambda_S = 0.3,\ \lambda_{\phi S} = 1, \text{ and } m_{S,0} = 0.$$

LARGER CHARGE

Like-charged monopoles repel and <u>only form bound states in the "BPS limit"</u> (attractive Yukawa interaction cancels the repulsive magnetic force)

Larger-charged monopoles cannot be spherically symmetric [Weinberg Guth 76]



[Houghton Sutcliffe hep-th/9601146, hep-th/9601147]

Or larger "magnetic bags" [Bolognesi hep-th/0512133; Lee Weinberg 0810.4962]

LARGE MONOPOLE CHARGE $q > 2$

$$M_{(2,Q)} \approx \frac{304\,\pi\,v}{35\,e^2\,\overline{r}_b} + \frac{4\pi}{3}\overline{r}_b^3 v\left(\frac{1}{4}\lambda_S s_0^4 + \frac{1}{8}\lambda_\phi + \frac{1}{2}\Omega^2 s_0^2\right)$$

$$\Downarrow$$

$$M_{(q,Q)} \sim \frac{304\pi v(q/2)^2}{35e^2\overline{r}_b} + \frac{4\pi}{3}\overline{r}_b^3 v\left(\frac{1}{4}\lambda_S s_0^4 + \frac{1}{8}\lambda_\phi + \frac{1}{2}\Omega^2 s_0^2\right)$$

Stability against $(q, Q) \rightarrow (q-2, Q) + (2,0)$:

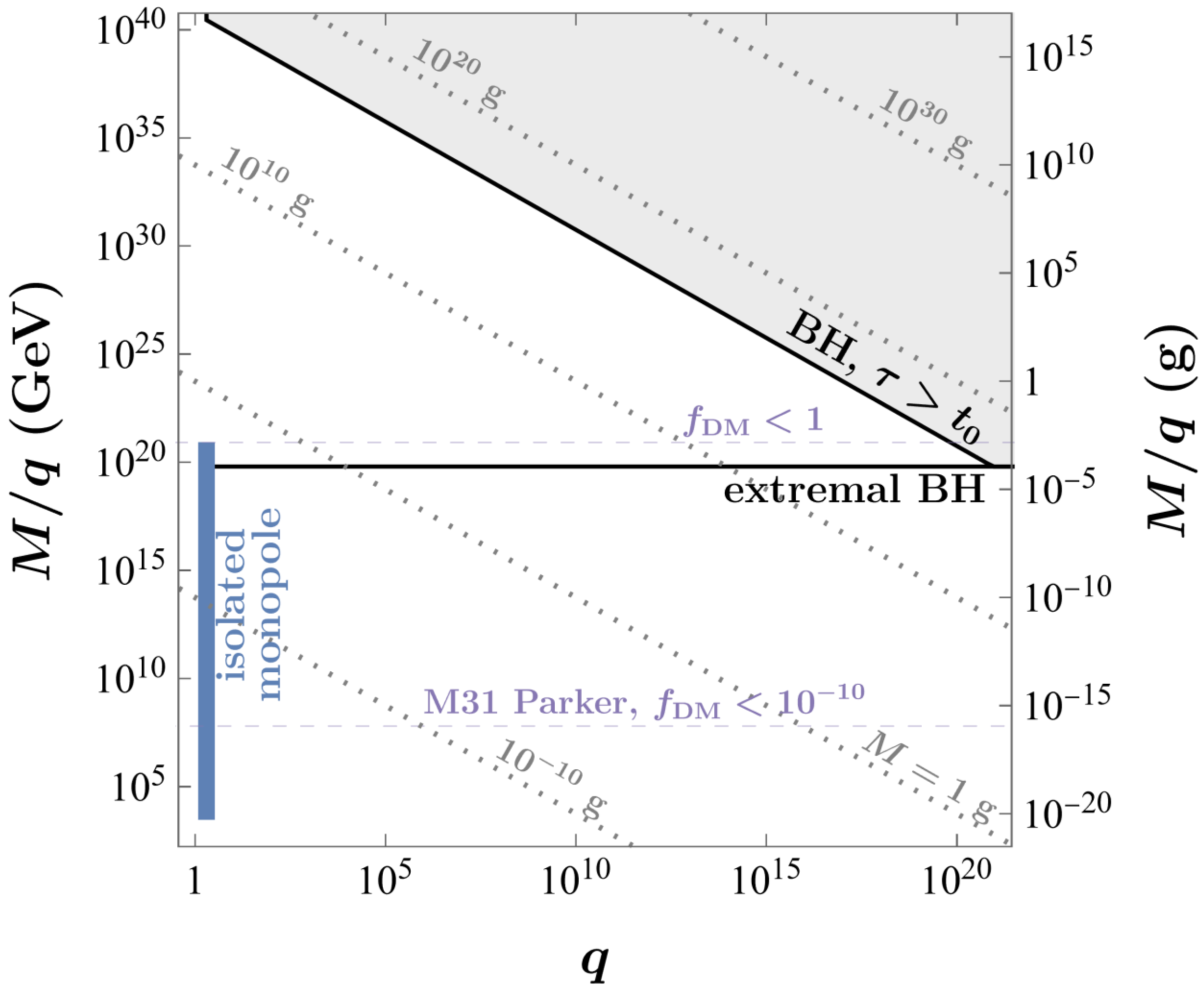
$$q \lesssim 1 + \frac{35}{76}\,e\,\overline{r}_b\,Y \approx \frac{35(3/\pi)^{1/3}}{76\times 2^{5/12}}\,e\,Y\,\frac{\lambda_S^{1/12}}{\lambda_\phi^{1/4}}Q^{1/3}$$

Minimum stable mass for stability:

$$M_{(q,Q)} \gtrsim 21\,\frac{\lambda_\phi}{e^3Y^3}q^3 v$$

ALLOWED MASS AND CHARGE

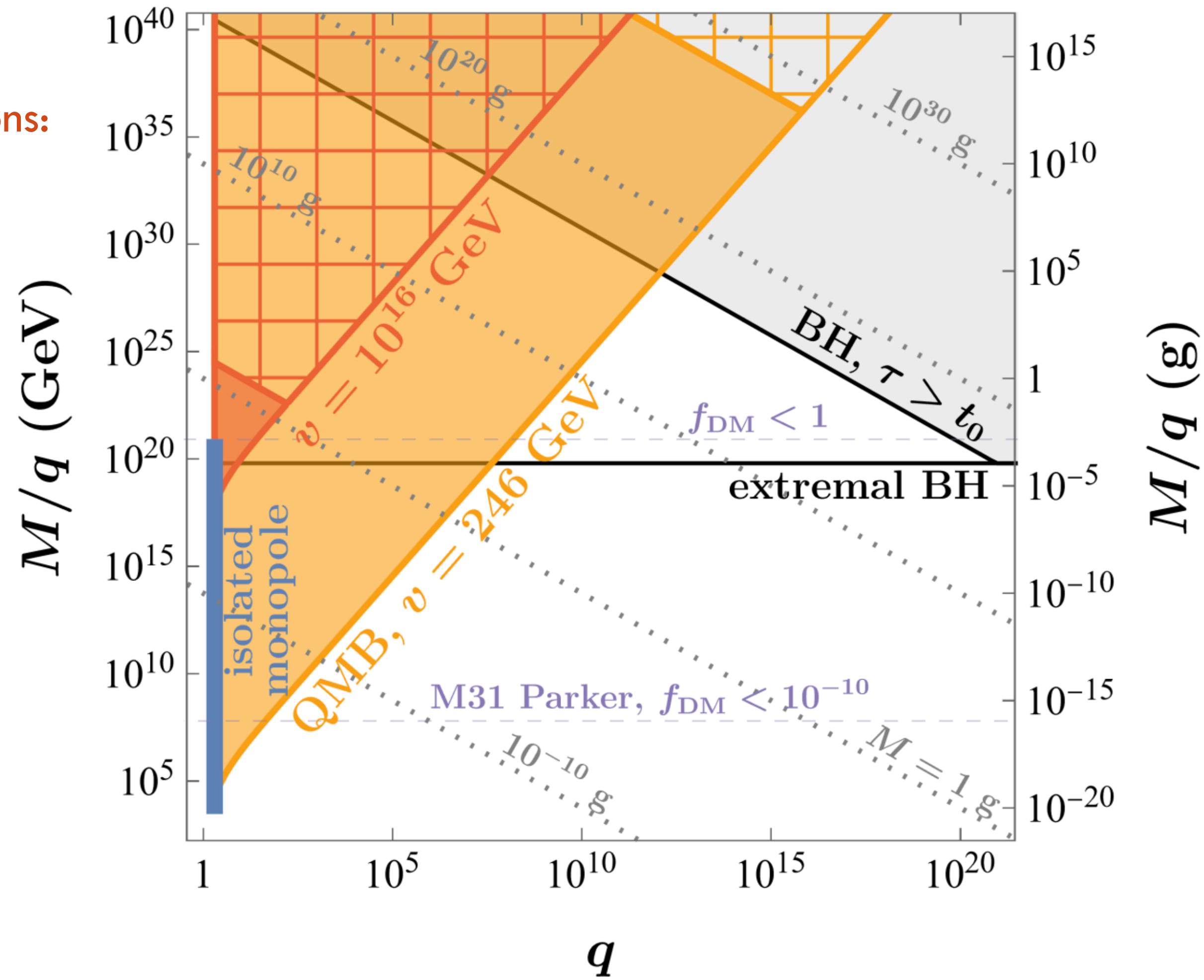
For magnetic BHs, see:
[Lee Weinberg hep-th/9406021;
Maldacena 2004.06084;
Bai Berger Korwar **NO** 2007.03703;
Diamond Kaplan 2103.01850]



ALLOWED MASS AND CHARGE

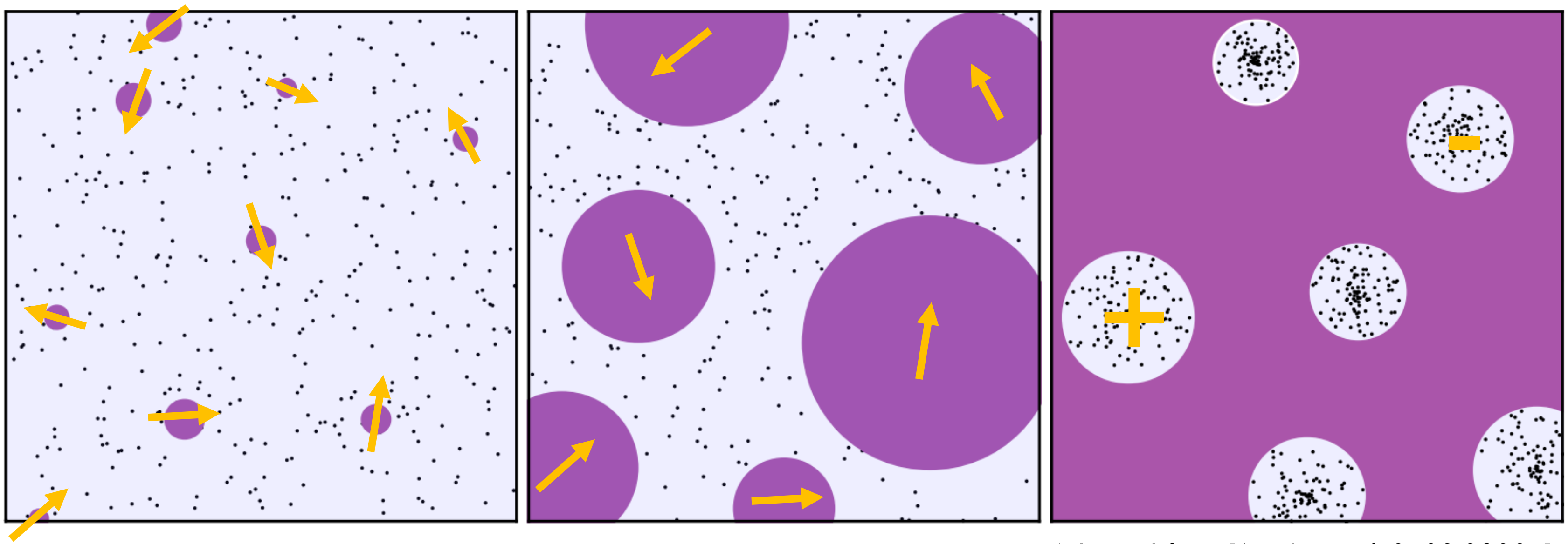
For magnetic BHs, see:
[Lee Weinberg hep-th/9406021;
Maldacena 2004.06084;
Bai Berger Korwar **NO** 2007.03703;
Diamond Kaplan 2103.01850]

Hatched regions:
$R > r_s$



FORMATION

In a first order phase transition, monopoles and Q-balls tend to form in the same places.



Adapted from [Asadi, *et. al.* 2103.09827]

Q-monopole-balls have smaller Q_S than Q-balls
$\Rightarrow S$ fusion is easier on monopoles during solitosynthesis.

But, expect small monopole charges.

SUMMARY

SUMMARY

- Q-balls are generic in theories with multiple scalar fields.
  - Q-monopole balls are generally expected when monopoles exist with a “Higgs-portal type” coupling to a scalar.
- Both solitosynthesis and phase transitions can make macroscopically large Q-balls.
  - Refined analytic estimates for Q-ball properties from solitosynthesis.
  - For solitosynthesis, particle-antiparticle asymmetry could match SM.
    For phase transitions, no particle-antiparticle asymmetry required.
  - Magnetic monopoles could be hiding inside Q-balls.
- These Q-balls can explain dark matter.