Superconducting Qubit Primer

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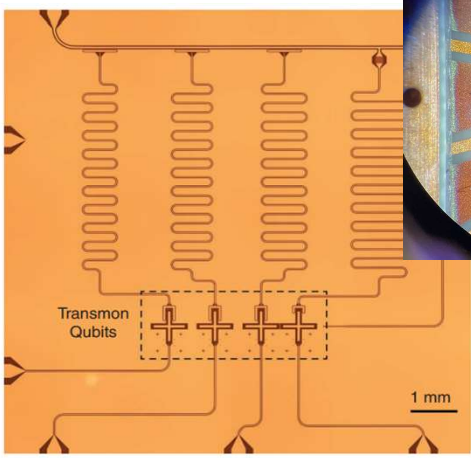
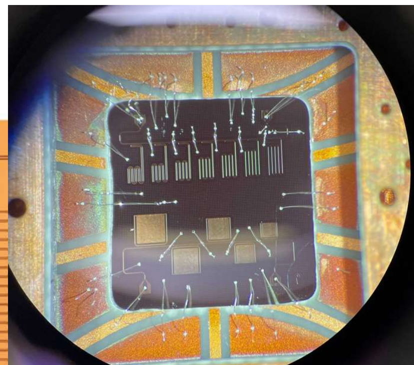
QSC training, 09/15/2022

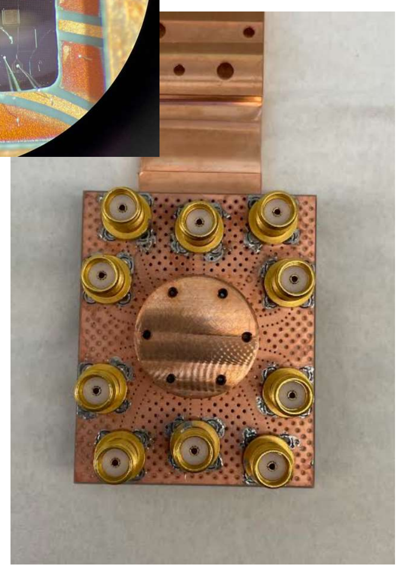
From device to measurement setup

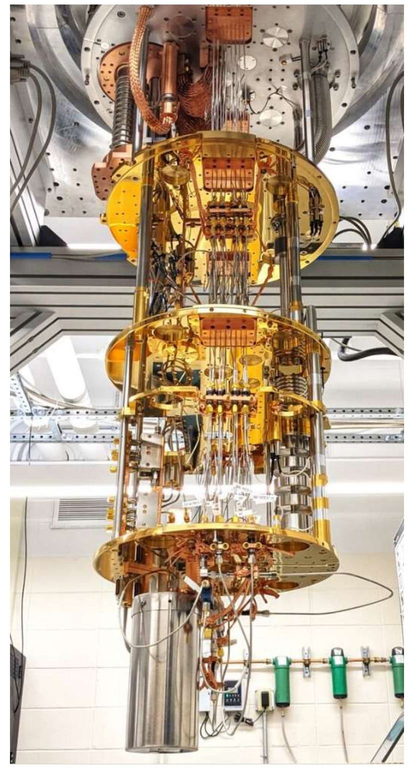
Operating frequency: 1-10GHz

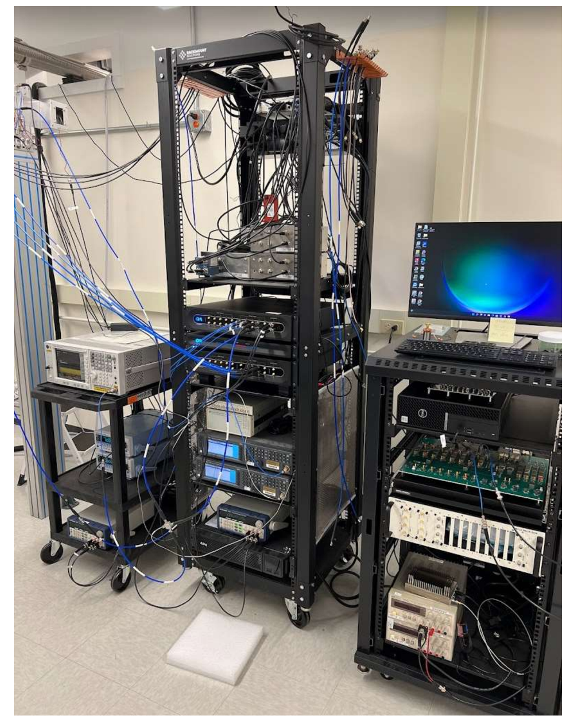
5GHz ~ 200mK ~ 20 ueV











Overview

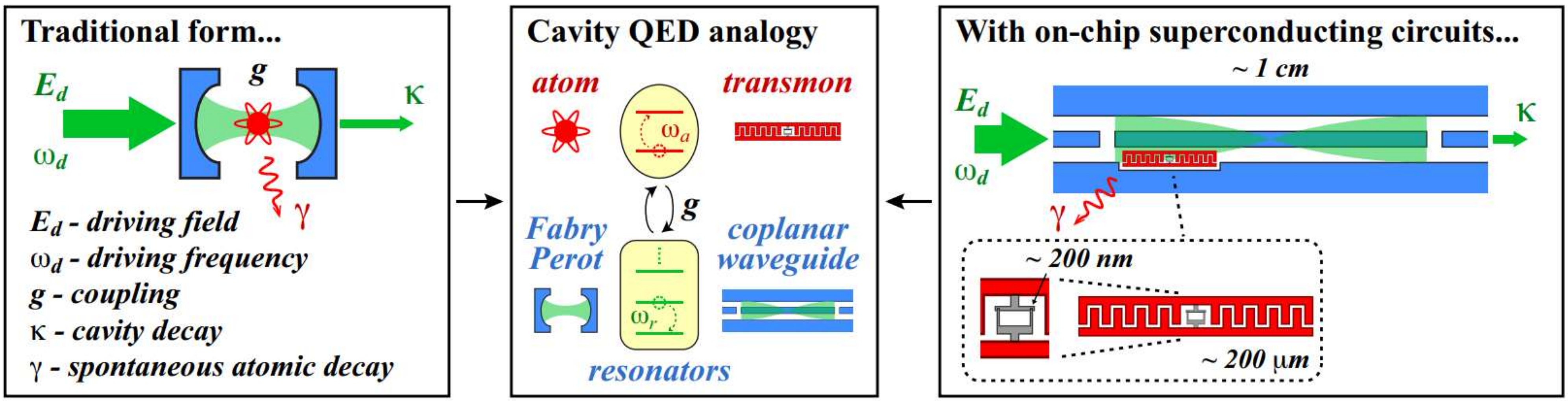
- What is circuit QED?
- What is a transmon qubit?
- A minimal cQED system for qubit control/readout

- Qubit characterization
  - Where do I start – basic experiments
  - Other things to do / think about

- Qubit as sensors –
  - Qubit as photon counter
  - Qubit as charge detector
  - Qubit as flux-noise detector

Circuit QED

- Superconducting qubits as “artificial atoms”

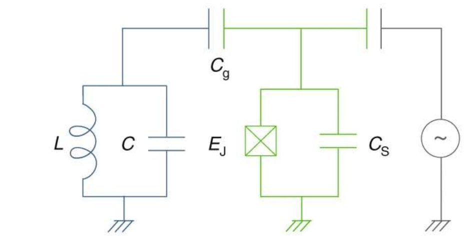


From Nathan K. Langford

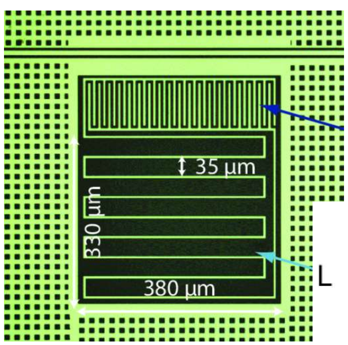
Circuit components

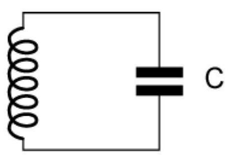
- Capacitors (C, E_c)
- Linear inductors (L, E_L)
- Josephson junctions (non-linear inductor, E_J)
- Resistors (intentional and unintentional)

Everything you need in circuit QED: linear resonators, non-linear resonators, and maybe some transmission lines..

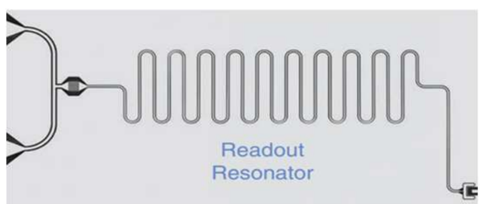


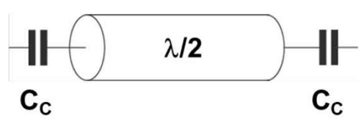
Lumped vs distributed



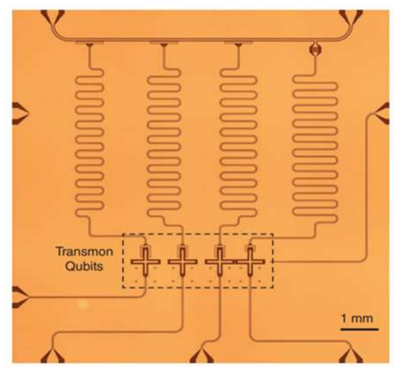


$\omega = [LC]^{-1/2}$

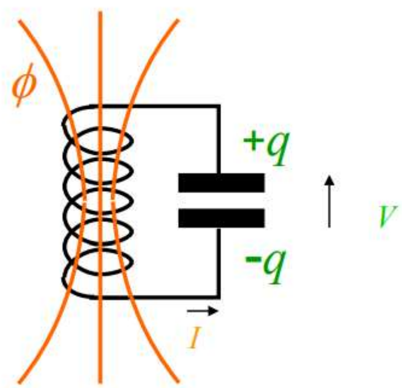




2D vs 3D



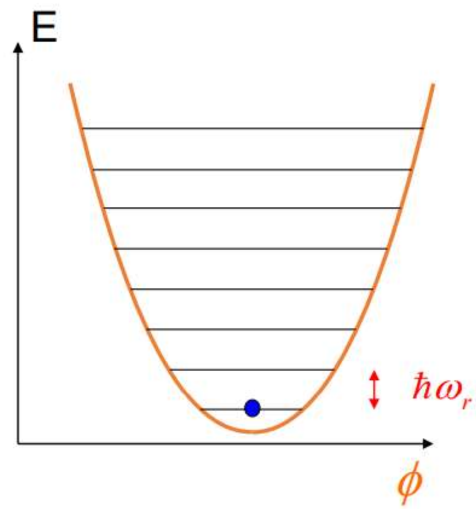
LC circuit as a quantum harmonic oscillator



$$[\phi, q] = i\hbar$$

$$\phi = LI$$

$$q = CV$$



$$\hbar\omega_r > k_B T$$

5GHz (~ 230 mK)

10mK

$E = E_c + E_L$

- $E_c = q^2/2C$ ("kinetic energy")
- $E_L = L\,I^2/2 = \phi^2/2L$ ("potential energy")
- $\omega_r = 1/\mathrm{Sqrt}(LC)$

Transmon qubit as nonlinear LC resonator

Common materials

Josephson junction:

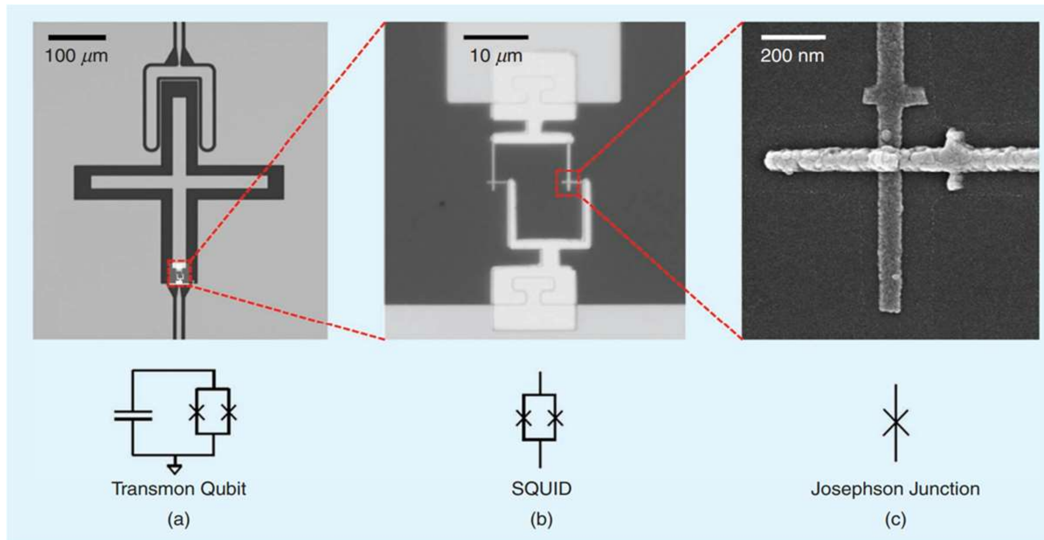
Al/AlOx/Al

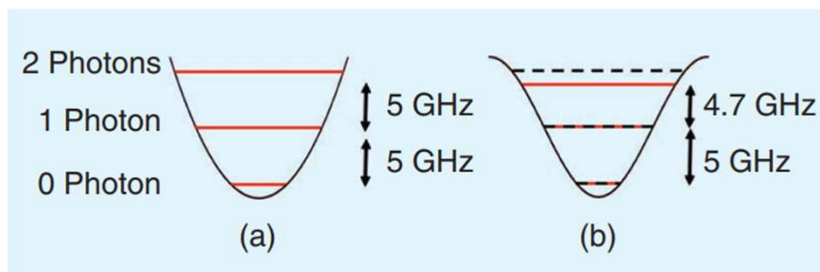
Rest of the circuit:

Nb, Al, Ta..

Substrate:

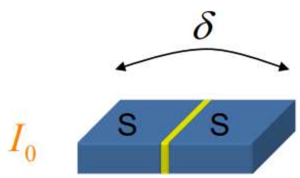
Sapphire or Si



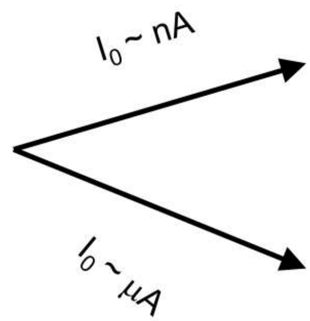


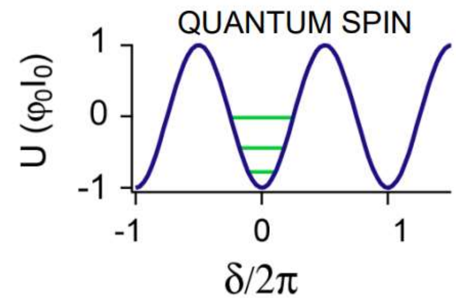
Linear → Nonlinear (“qubit/qudit”)

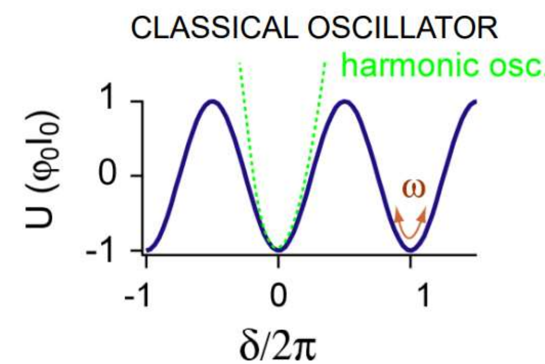
Josephson junction as nonlinear inductor

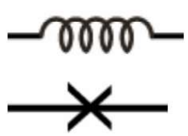


$I = I_0 \sin \delta$









- $I(t) = \frac{1}{L}\Phi(t)$
- $I(t) = I_0 \sin[\Phi(t)/\varphi_0]$

$$L_J \equiv \frac{\varphi_0}{I_0} = \frac{\hbar R_T}{\pi \Delta_{BCS}} (BCS)$$

$$\delta \equiv \frac{\Phi}{\varphi_0}$$

Reminder: 5GHz ~ 200mK ~ 20 ueV

Aluminum: T_c 1.2K, SC gap ~ 170 ueV

Transmon energy spectrum

E = E_c + E_J

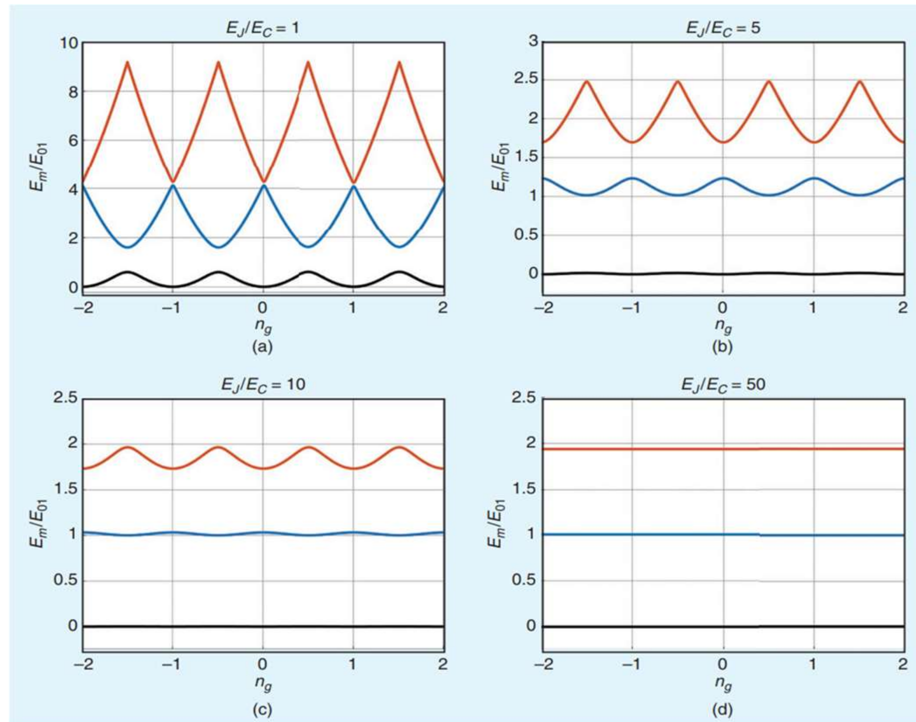
In the transmon regime ($E_J >> E_c$)

- Charge dispersion is negligible
- Qubit freq: ω_{01} = Sqrt($8E_c E_J$)
- Anharmonicity:
  $\alpha = \omega_{12} - \omega_{01} \sim - E_c$

(Example next page)

Energy levels vs charge "q"

"tight binding lattice"



A typical transmon

- Typical numbers
- Look up link below for definitions and formula

Capacitance

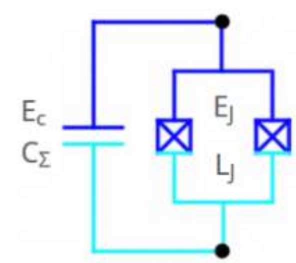
$E_c/2\pi =$	0.250000	GHz
$C_\Sigma =$	77.4817	fF

Al Josephson Junction/SQUID

$E_J/2\pi =$	10.00000	GHz
$L_J =$	16.3332	nH
$I_c =$	20.1415	nA
$R =$	13725.9	Ω

Qubit frequencies

$f_{ge} \approx$	4.22214	GHz
$f_{ef} \approx$	3.97214	GHz
$f_{gf}/2 \approx$	4.09714	GHz
$\alpha \approx$	0.250000	GHz



Schematic of a transmon qubit.

Purcell decay and χ-shift

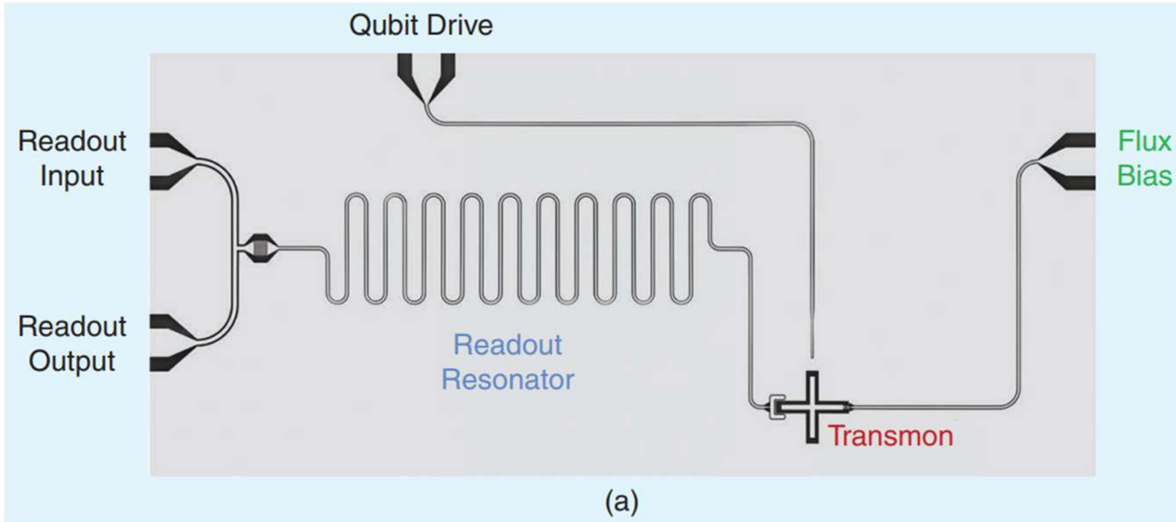
$f_{res} =$	6.00	GHz
$Q_{res} =$	10.000	k
$\kappa_{res} =$	0.6000	MHz
$g/2\pi =$	100	MHz

$\Delta = f_{ge} - f_{res} =$	-2.250	GHz
$T_{Purcell} \approx$	134.3	μs
$\kappa_{Purcell} \approx$	1.185	kHz
χ-shift $\approx$	-0.4444	MHz
$n_{crit} \approx$	126.6	

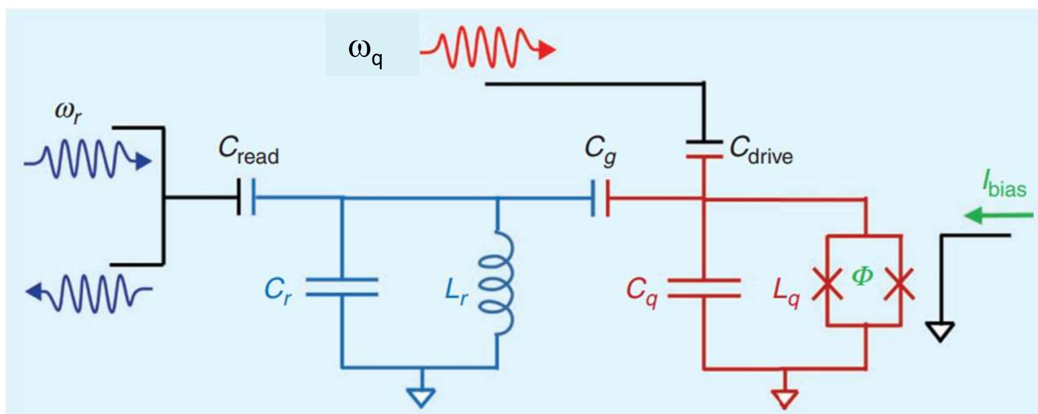
Transmon calculator: http://antonpotocnik.com/?p=560257

A minimal circuit QED setup

Layout:



Effective circuit diagram:



Jaynes-Cummings model:

"linear resonator coupled to non-linear resonator (qubit)"

***Dispersive regime/limit*:**

- large detuning (Δ) >> qubit-resonator coupling (g), **no resonant energy exchange**
- Eigenstates are dressed states
  - qubit lifetime can be limited by resonator lifetime,
  - resonator can inherit small non-linearity from qubit,
  - the coupling leads to "dispersive shifts" ~ g^2/Δ
- State-dependent dispersive shift
  - Resonator frequency depends on qubit state,
  - Qubit frequency depends on resonator occupancy,
  - This is called the "χ-shift"

$$H = \hbar\omega_r\, a^\dagger a + \tfrac{1}{2}\hbar\omega_q\,\sigma_z + \hbar\chi\,\sigma_z\, a^\dagger a \qquad g \ll \Delta$$

A typical transmon

Formulas

$E_c = \frac{e_0^2}{2C_\Sigma}$

$E_J = \left(\frac{\Phi_0}{2\pi}\right)^2 \frac{1}{L_J}$

$E_J = \frac{\Phi_0}{2\pi} I_c$

$I_c = \frac{\pi \Delta}{2R}$

$L_J = \frac{\Phi_0}{2\pi} \frac{1}{I_c}$

$h f_{ge} \approx \sqrt{8 E_c E_J} - E_c$

$h f_{ef} \approx \sqrt{8 E_c E_J} - 2E_c$

$\kappa_{\mathrm{Purcell}} \approx \kappa (g/\Delta)^2$

$\chi\mathrm{-shift} \approx g^2/\Delta - g^2/(\Delta - E_c)$

$\Phi_0 = 2.067834 \cdot 10^{-15}\,\mathrm{Wb}$

$\Delta_0 = 176 \cdot 10^{-6}\,\mathrm{V}$

$e_0 = 1.60218 \cdot 10^{-19}\,\mathrm{As}$

$h = 2\pi\hbar = 6.62607 \cdot 10^{-34}\,\mathrm{Js}$

$df_{\mathrm{fwhm}} = 1/(\pi T_2)$

$Q = f/df_{\mathrm{fwhm}} = \pi f T_2 = 2\pi f T_1$

$1/T_2 = 1/(2T_1) + 1/T_\varphi$

$T_{\mathrm{Purcell}} = 1/(2\pi \kappa_{\mathrm{Purcell}})$

$n_{\mathrm{crit}} \approx \Delta^2/(2g)^2$

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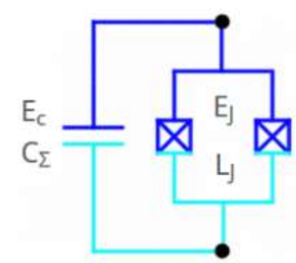

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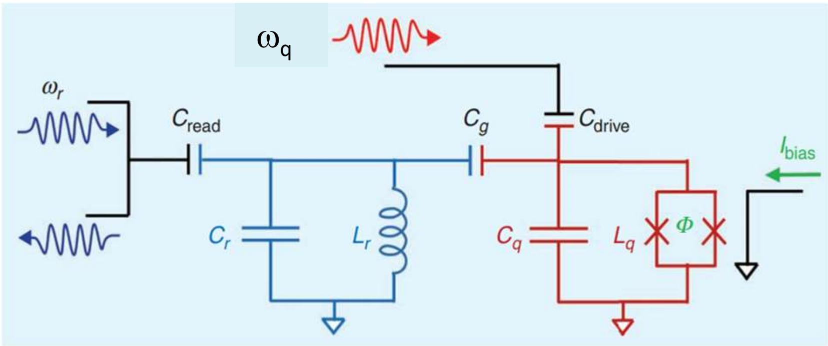
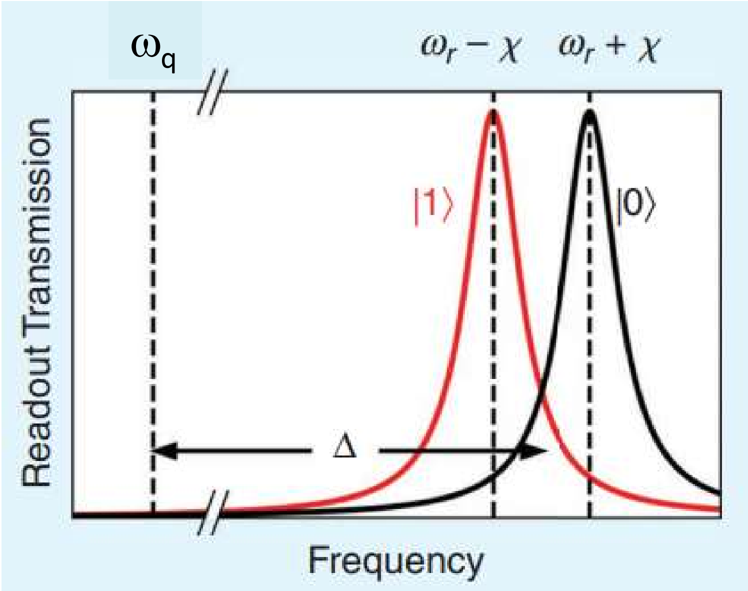
Qubit control and readout – one slider

- Qubit readout:

Use qubit-state-dependent resonator frequency, to read out occupancy of qubit. (We focus here on strong projective measurement)

- Qubit control: charge drive: “xy”; flux drive “z”





Now the fridge is cold… What do I do… Is my qubit alive?

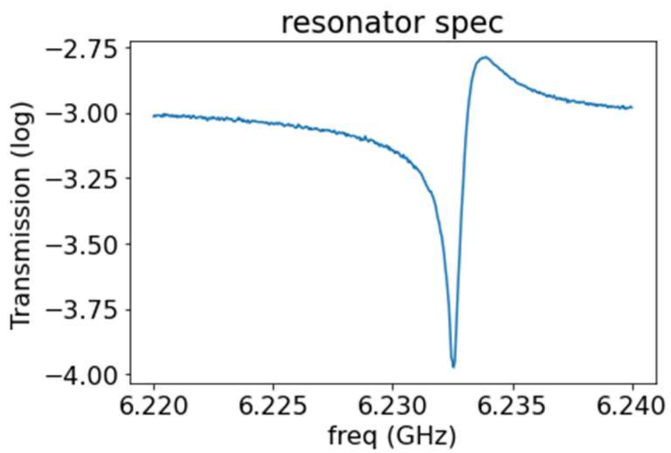
- Strong suggestion – always use a two-tone network analyzer (e.g. Keysight PNAX) for initial qubit calibration, before moving to a pulsed measurement setup (e.g. RFSoC).

Resonator spectroscopy – what information can I extract?

- Reflection vs transmission vs hanger
- What shape do you expect, what does the width and height of the peak/dip tell you?
- Why do typical resonators have asymmetric shoulders in the spectra? (“Feno resonance”)
- How to fit high Q resonators properly to extract quality factors?

Interactive notebook from Alex’s lab:

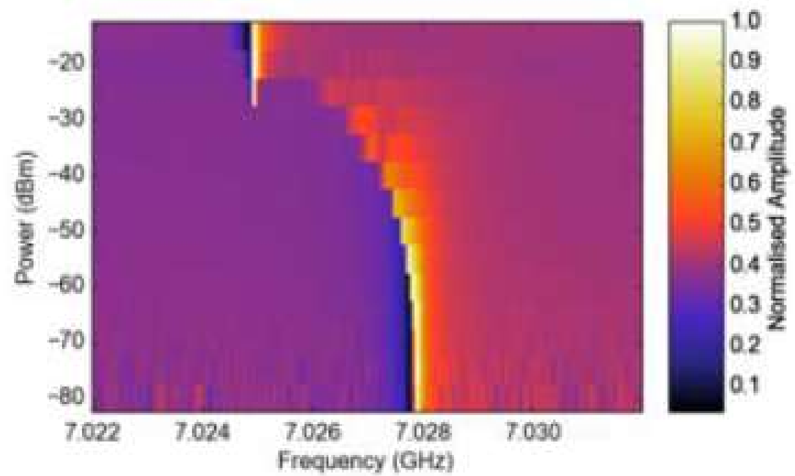
https://colab.research.google.com/drive/1sLLlLYMNsGkx8GDkBFPCNZmclJiPkWvr?usp=sharing



I see the readout resonator... Is my qubit alive?

- "Power-dependence of the resonator"
  - low power: linear regime (observe: dressed state freq)
  - high power: nonlinear regime
  - very high power: saturation regime (observe: bare resonator freq)

  - for most purposes, we operate in the linear regime, where the system is in the dispersive limit.

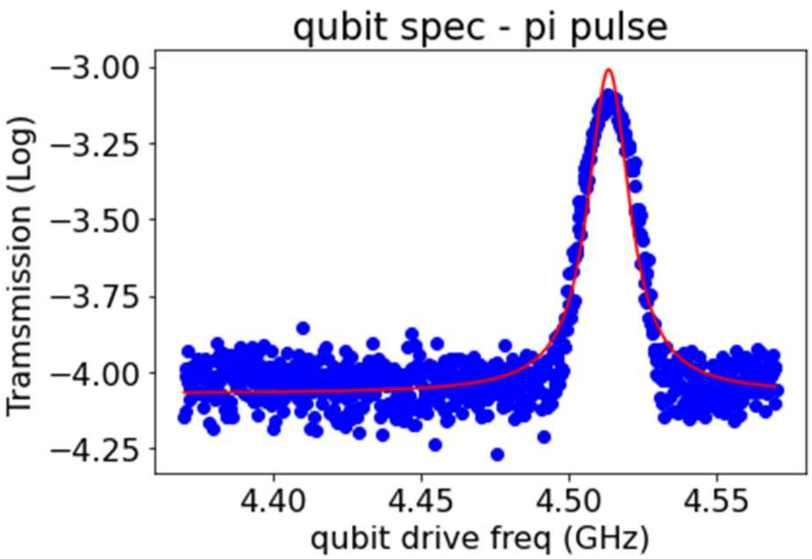


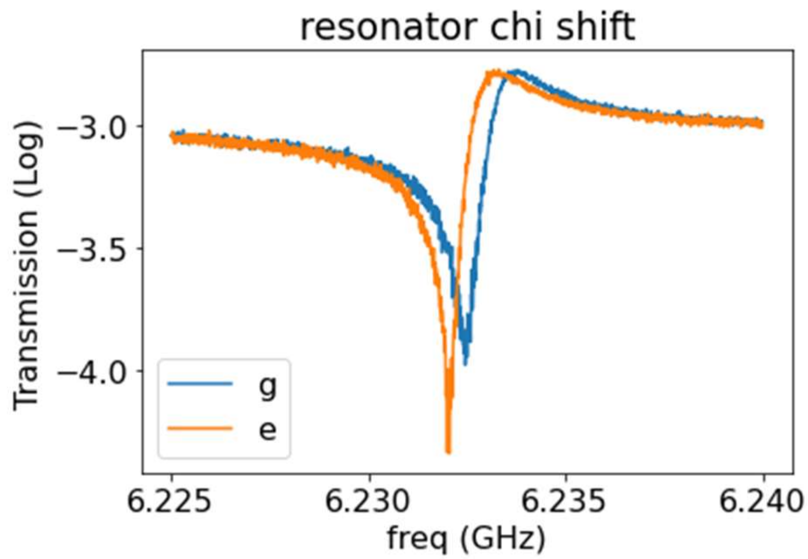
Ref: (no clear intuition why it should snap to bare freq.)
Exp: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.105.173601
Theory: https://journals.aps.org/prl/abstract/10.1103/PhysRevLett.105.100505

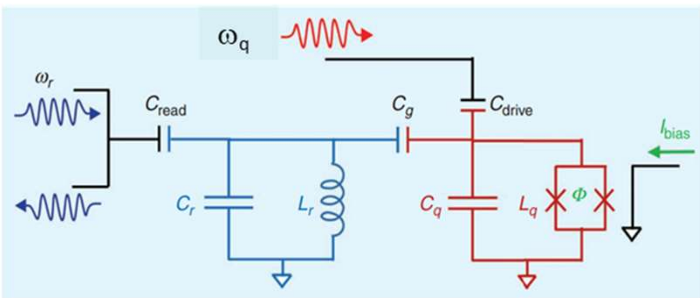
Remark:
These powers are still in the "quantum" regime. At even higher powers, kinetic inductance of the cooper pairs becomes non-negligible, and the resonator can show classical Kerr non-linearity.

Qubit spectroscopy – “two-tone” spectroscopy:

- Consider continuous excitation (CW)

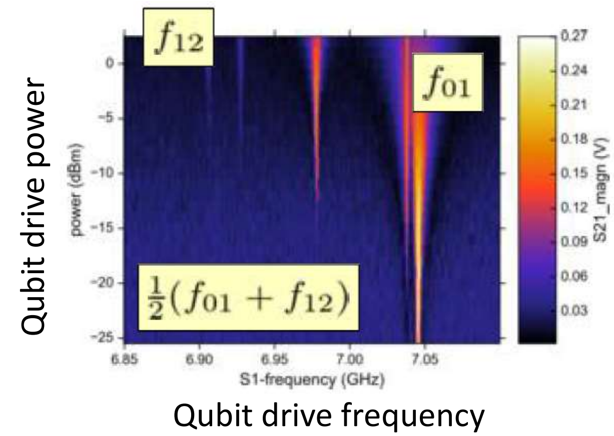






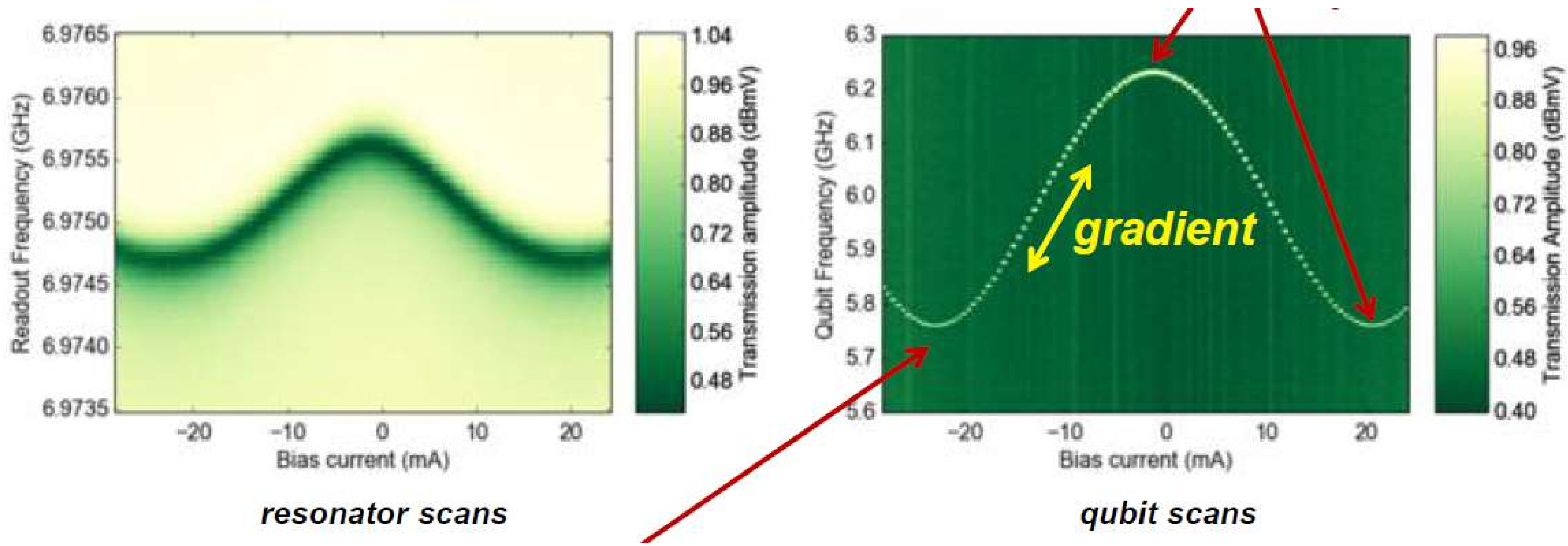
Qubit spectroscopy – What information can I extract

- Qubit frequency (ω_{01})
- Anharmonicity via two photon transition (ω_{12} +ω_{01})/2



- Sanity check for qubit temperature
- Sanity check for resonator temperature ("Number splitting peaks")
- Estimate of T1, T2

Resonator/qubit spectra for frequency tunable transmon

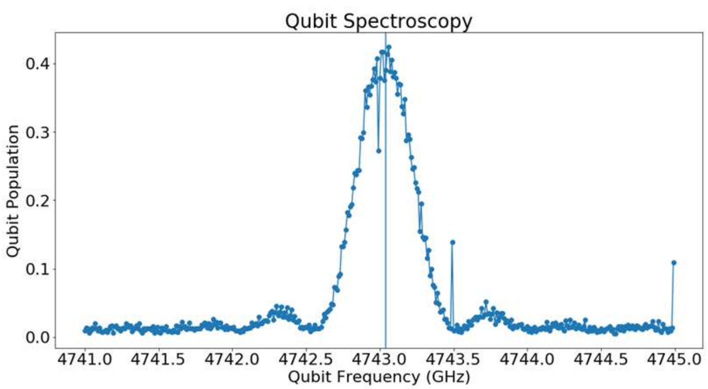


Nathan K. Langford

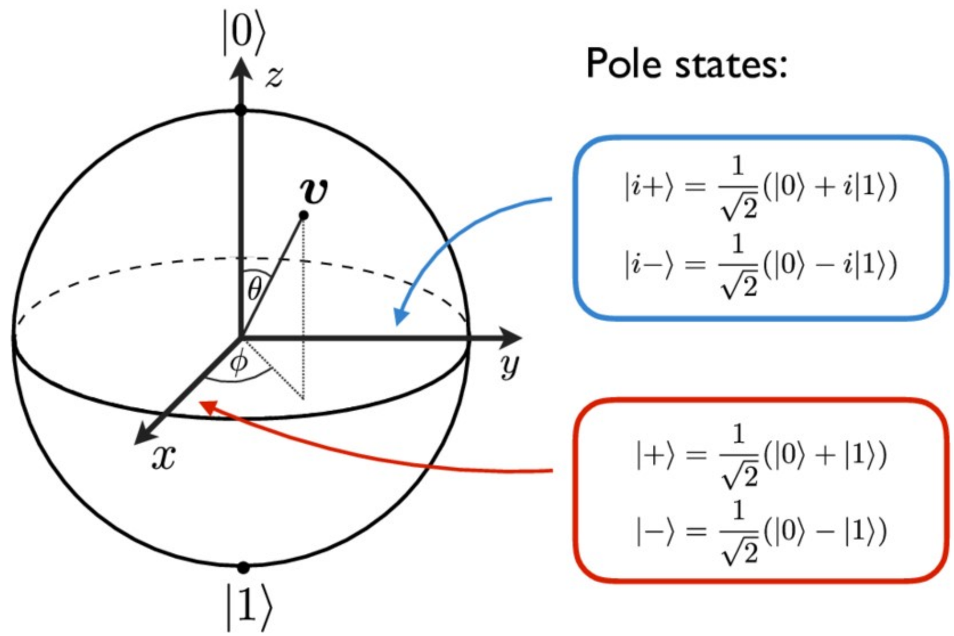
Controlling the qubit

CW spectroscopy (so-far) vs pulsed

- Pulsed measurements – pump, then probe.
- Need AWG for generating pulses, and precise timing.



← Sinc from square pulse

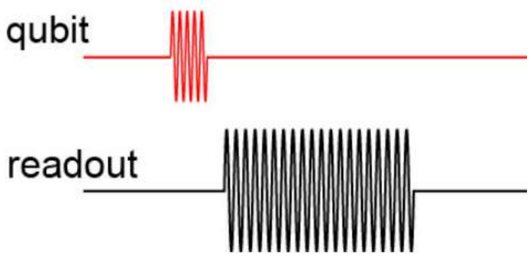


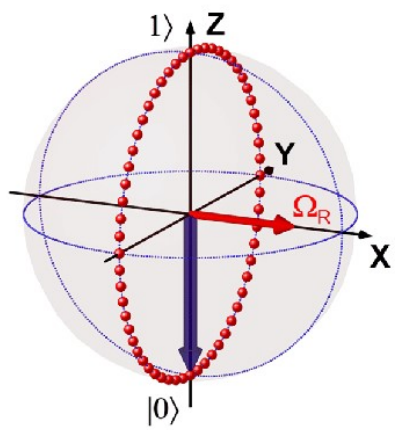
Rabi oscillation

- Drive qubit on resonance (ω_{01}), then read (projection along z)
- Angle of rotation depends on Rabi rate (amplitude), and duration.
- What happens when the drive is (slightly) off-resonant? "Chevron pattern"

Questions:

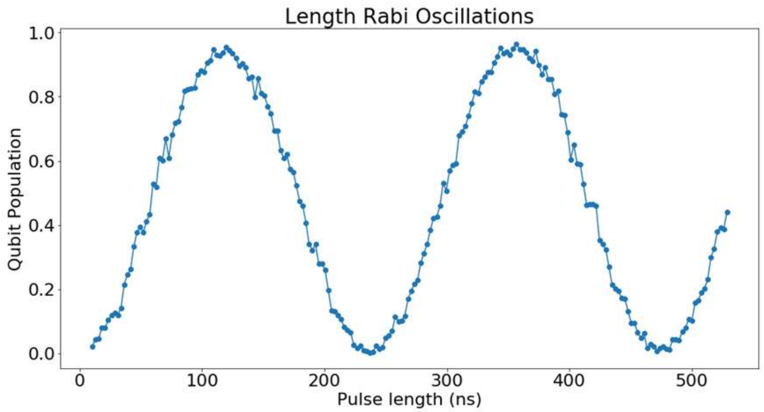
- Effect of decoherence on Rabi?
- How to perform arbitrary rotations on the Bloch sphere? There is only one drive..
- How fast can you perform single rotations? DRAG pulses.
- How to quantify the gate fidelity? Randomized benchmarking etc.

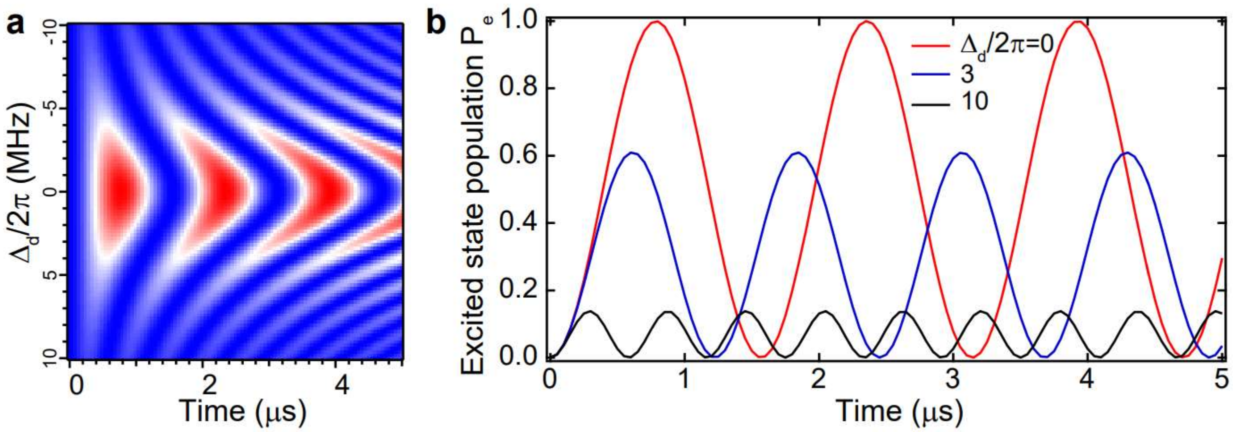




$\Delta = 0$: $\Omega_R^{eff} = \Omega_R$

Rabi

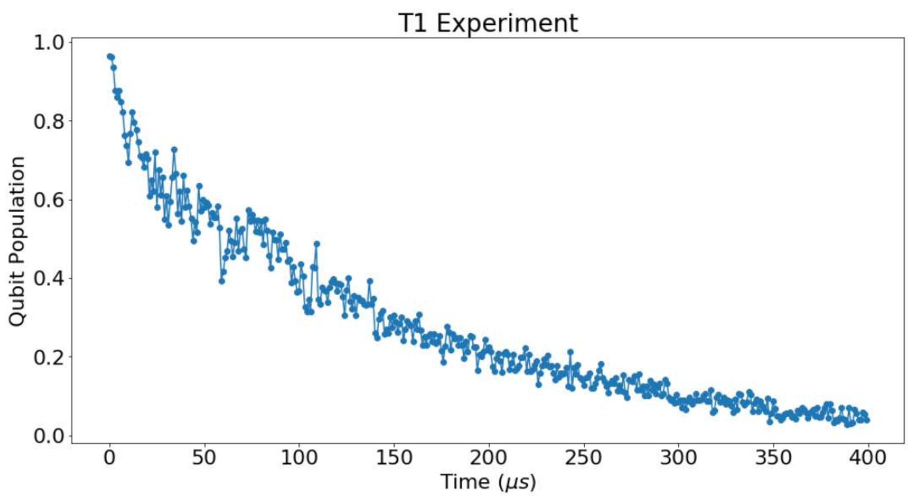




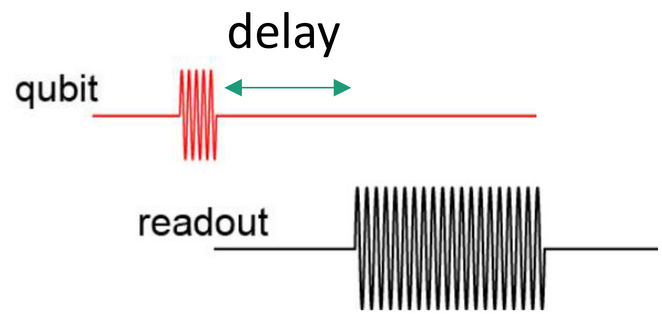
M. Naghiloo thesis

T1 measurement

Qubit relaxation



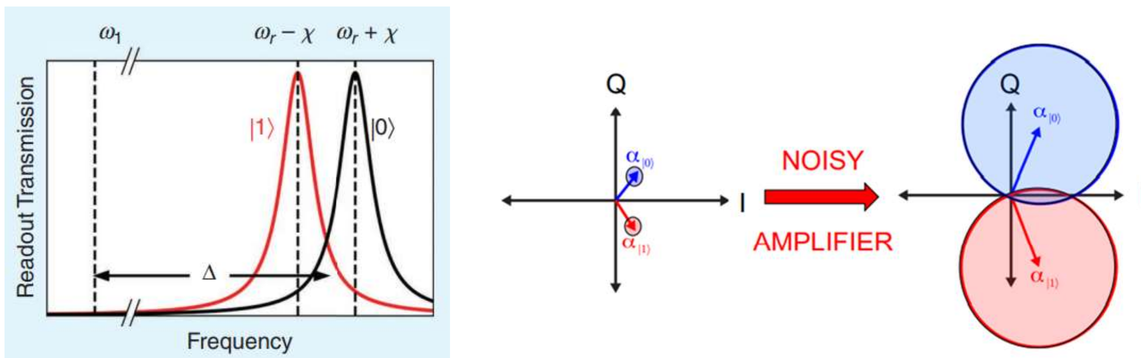
delay



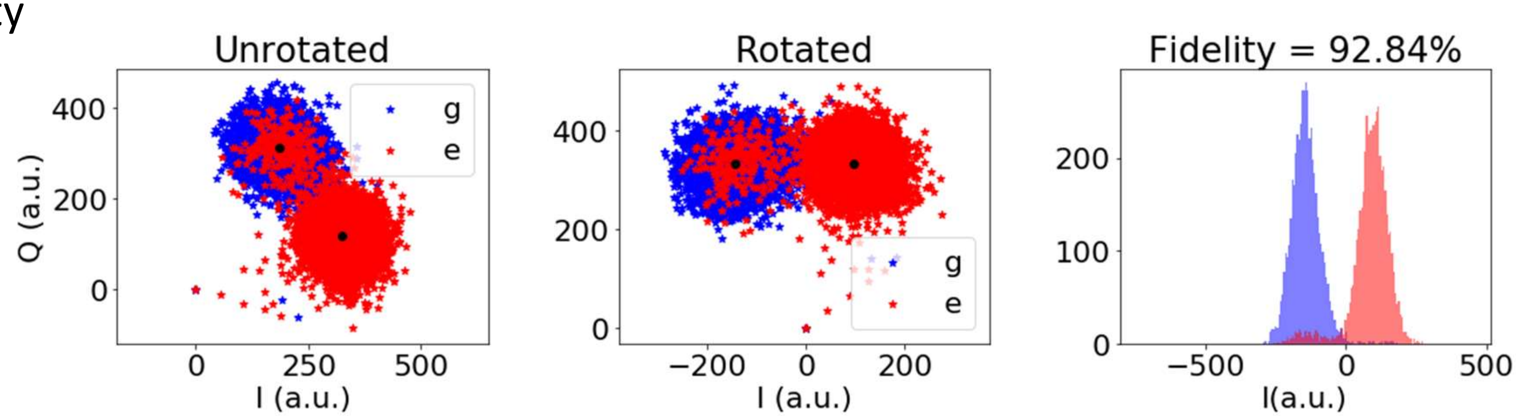
Question:

- What limits T1?
- Intrinsic (fab…), external (Purcell limit, Purcell filter)

Readout – calibrated

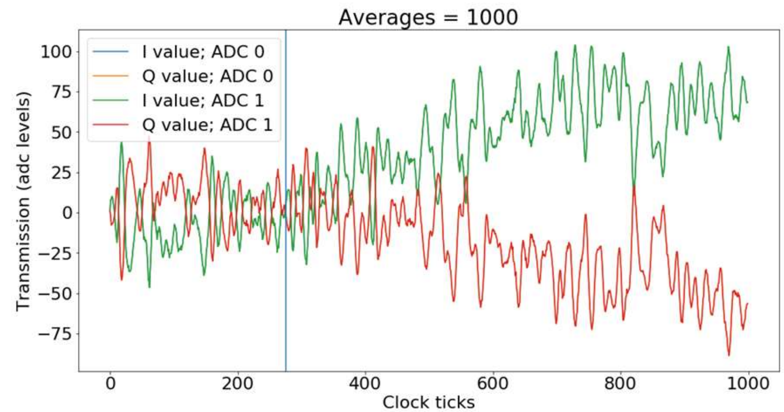


View in the IQ plane,
Single shot fidelity



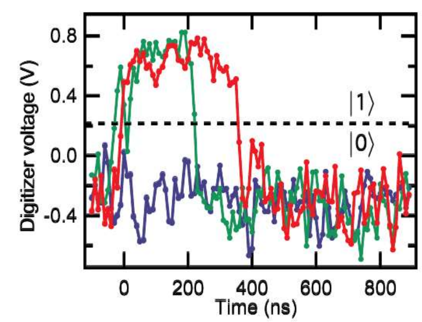
Readout – calibrated cont.

View as readout trajectories – how to weigh your trajectory signal for best SNR? What other information is contained in the trajectories?



Others

- How to readout much faster, without hurting qubit T1? (Purcell filter)
- How to populate and clear resonator faster? (CLEAR pulses)
- Weak measurements, and other things…



PRL 106, 110502 (2011) PHYSICAL REVIEW LETTERS week ending 18 MARCH 2011

Observation of Quantum Jumps in a Superconducting Artificial Atom

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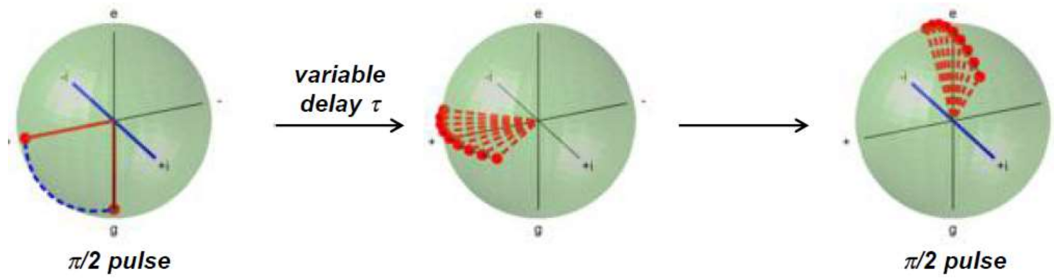
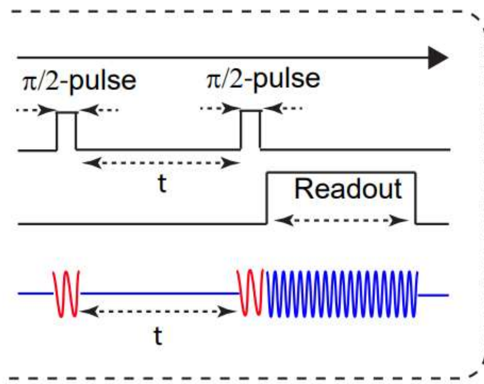
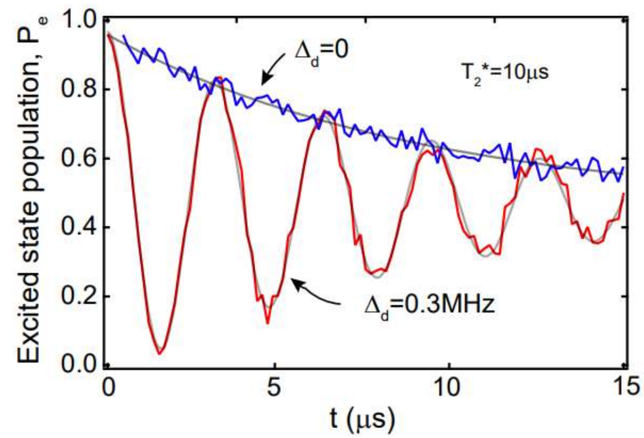
T2* (dephasing time) - Ramsey sequence



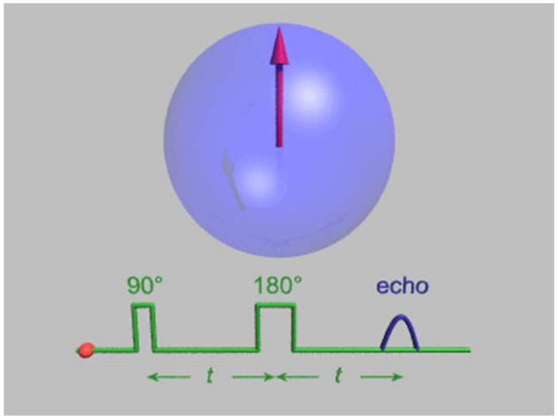


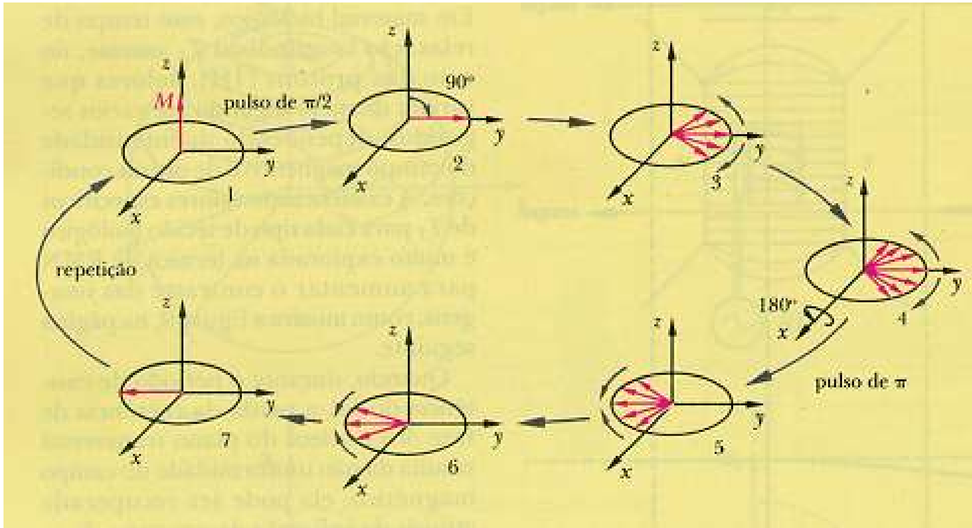


M. Naghiloo thesis

- Where does the fringe come from?
  - By detuning the qubit drive – bad approach
  - By adding a controlled phase to the qubit second pulse

T2 – Spin echo sequence





Insensitive to DC/low frequency phase noise (i.e. fluctuations in qubit frequency)

Question:

- CPMG sequence
- Spin echo with multiple pi pulses – what's different? See flux noise spectroscopy
- Dynamical decoupling sequences

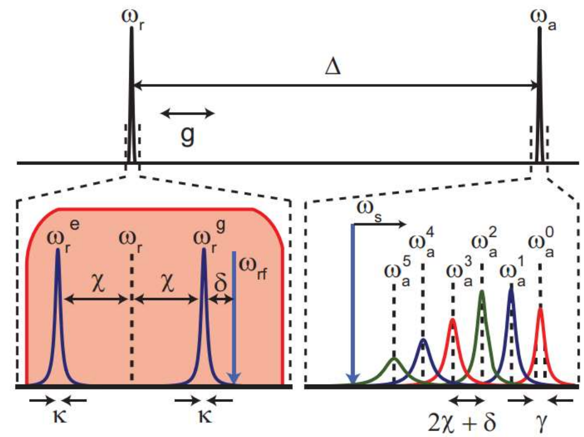
Qubit as photon-counter

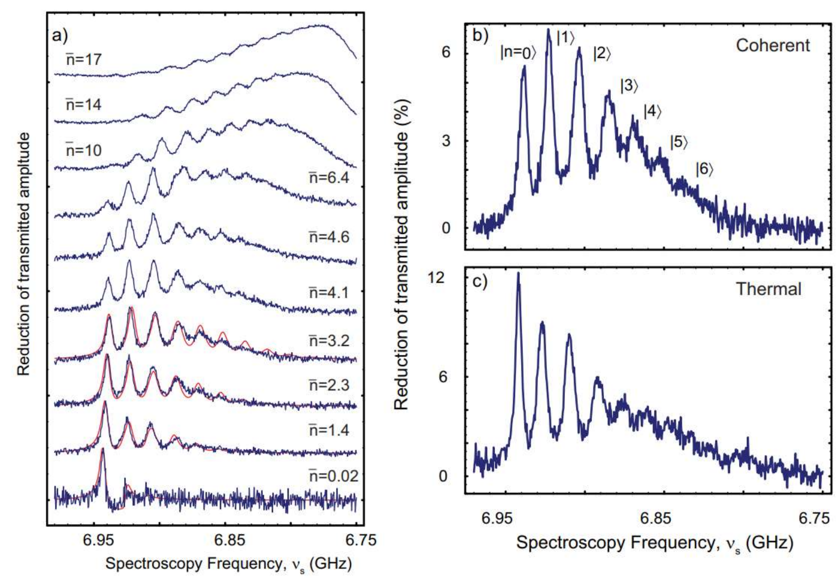
How many photons are in my resonator?

- Photon-number splitting of the qubit frequency in the "strongly dispersive limit" (PRA, 74:042318, 2006)

- Qubit frequency shift/broadening in the "weakly dispersive limit"(PRL, 94:123602, 2005)

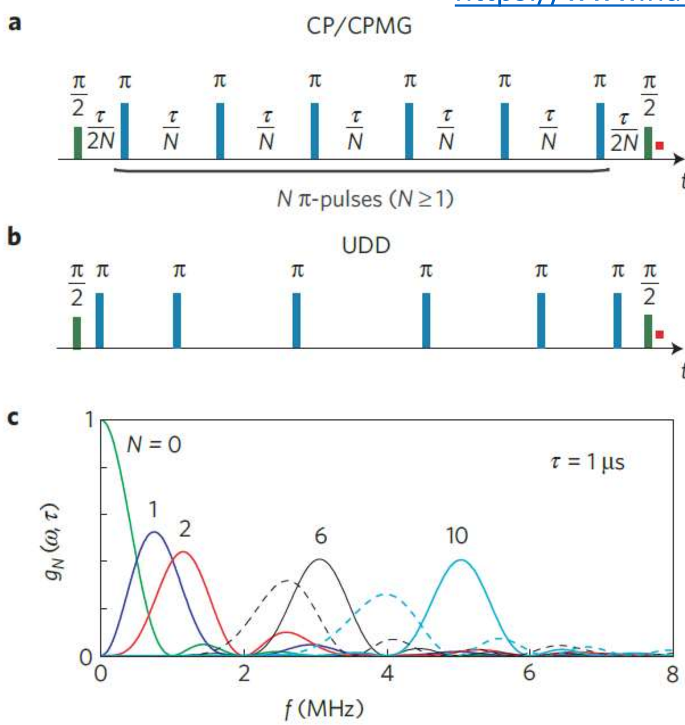
Question:
- Measure resonator temperature?
- What about measuring qubit temperature?

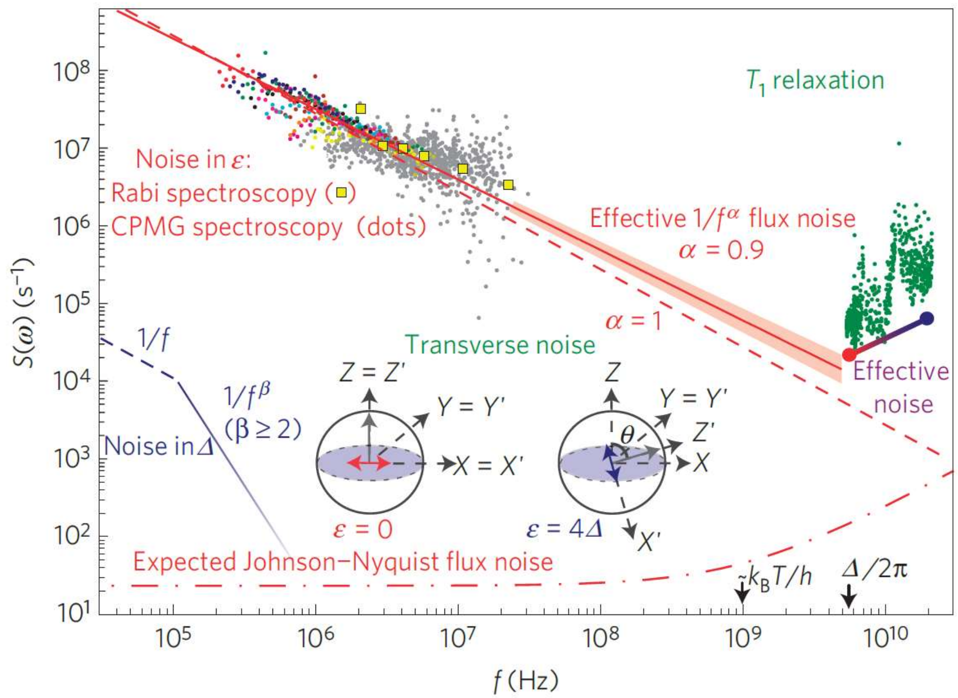




Transmon as flux (phase) noise spectrum analyzer

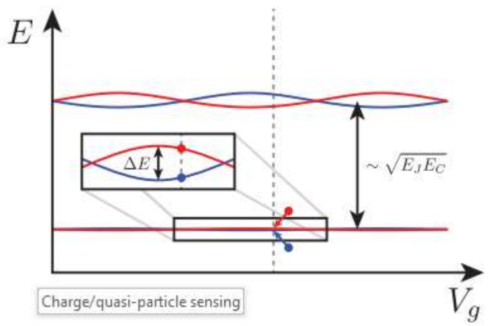
https://www.nature.com/articles/nphys1994

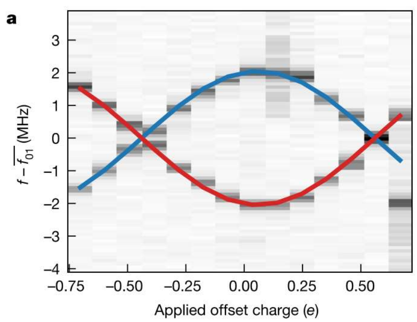




https://www.nature.com/articles/ncomms3337.pdf
https://web.physics.ucsb.edu/~martinisgroup/theses/OMalley2016.pdf
https://www.nature.com/articles/s41467-021-21098-3 + others from Will Oliver's group

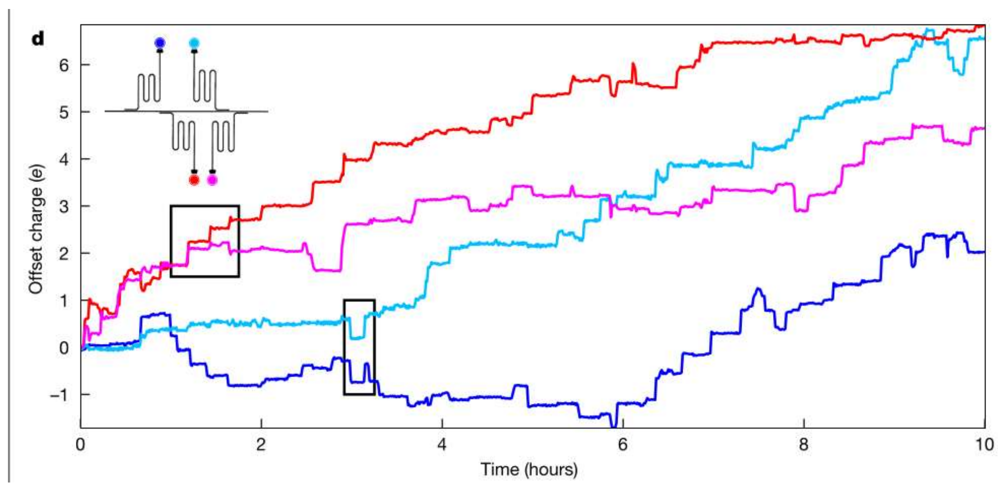
Transmon as charge detector / charge noise spectra analyzer





Ej/Ec at a moderate ~ 20, charge dispersion of qubit freq ~ few-10 MHz
Still long enough coherence times

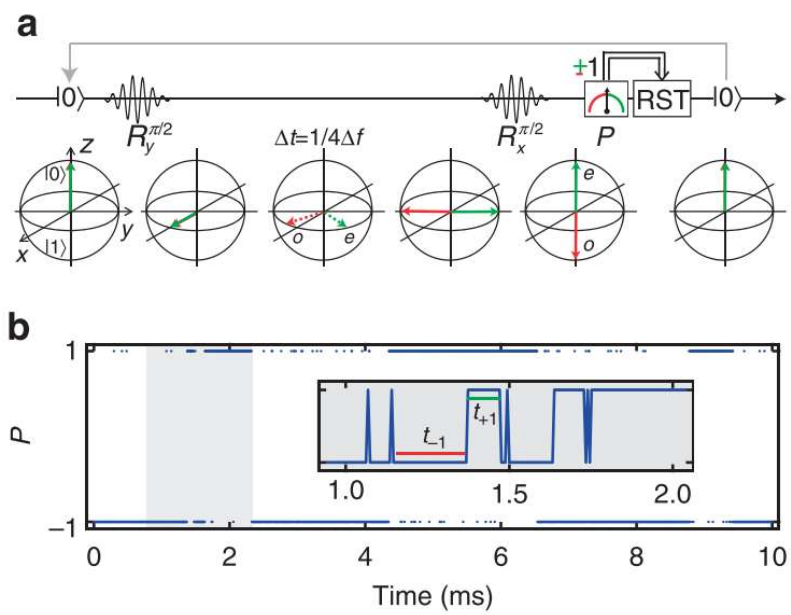
Ramsey type experiment to distinguish charge (parity) states.
Track charge fluctuation over long times.

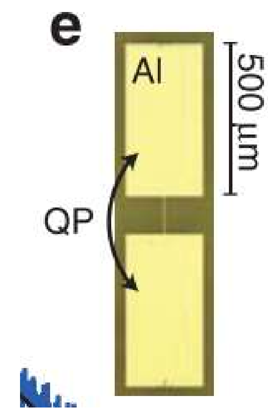


Wilen, C. D. *et al.* Correlated charge noise and relaxation errors in superconducting qubits. *Nature* **594**, 369–373 (2021).

Transmon as charge detector – cont.

Real time detection of quasiparticle tunneling





Floating islands in 3D cavity; less fluctuations in induced offset charge

Ristè, D. *et al.* Millisecond charge-parity fluctuations and induced decoherence in a superconducting transmon qubit. *Nat. Commun.* **4**, 1913 (2013).

Sorted Reading list

Getting started:

- M. Naghiloo, “Introduction to Experimental Quantum Measurement with Superconducting Qubits”. PhD Dissertation (2019). (Chapters 1-3, and 4 if interested)
- Practical Guide for Building Superconducting Quantum Devices (PRX 2021)

Circuit QED bibles:

- David Schuster’s PhD thesis.
- Steve Girvin Les Houches Notes

Textbooks:

- Exploring the Quantum: Atoms, Cavities, and Photons (Serge Haroche, Jean-Michel Raimond)
- Quantum information and optics with Superconducting Circuits (Juan Jose Garcia Ripoll)

To look up references and terminology…

- A quantum engineer's guide to superconducting qubits

Numerical packages to play with

- Qutip https://qutip.org/docs/latest/guide/guide.html
  Qutip lecture notebooks
- https://scqubits.readthedocs.io/
- https://sequencing.readthedocs.io/en/latest/index.html
- Qiskit

Credit:

Some materials in this presentation are directly from these sources:

- M. Naghiloo, *"Introduction to Experimental Quantum Measurement with Superconducting Qubits"*. PhD Dissertation (2019).

- Irfan Siddiqi's lecture slides at Okinawa (2019): PowerPoint Presentation (oist.jp) (intro to circuit QED), PowerPoint Presentation (oist.jp) (intro to quantum measurement).

- Presentation by Nathan K. Langford (2016): "Engineering the quantum: probing atoms with light & light with atoms in a transmon circuit QED system"