Raytheon BBN

Superconducting parametric amplifiers

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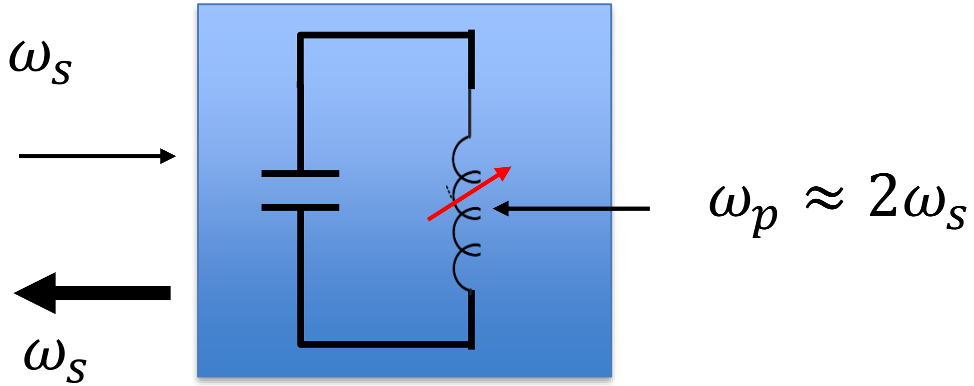
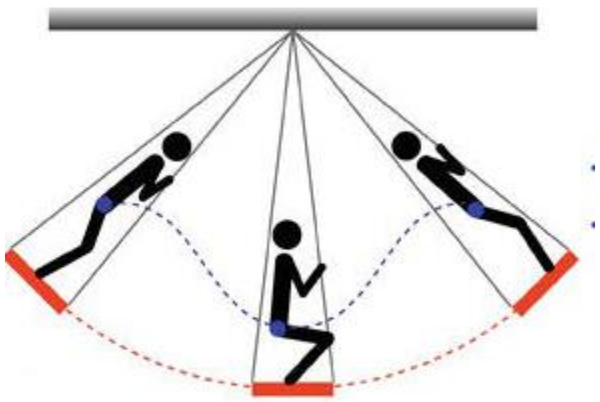
Parametric Amplifiers

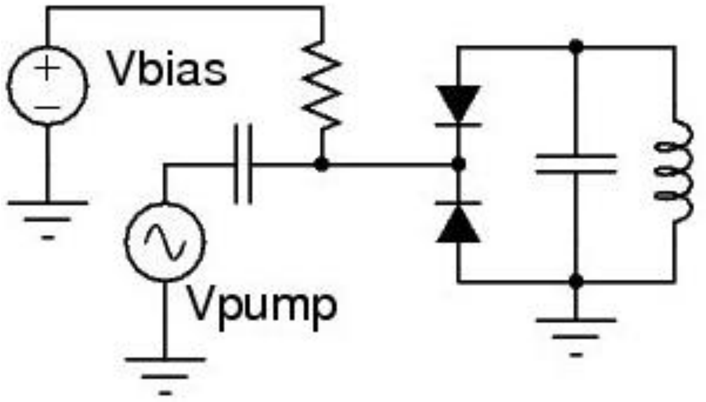
- Gain obtained by modulating a nonlinear resonant element around twice its resonant frequency.
- Noise performance typically close to the standard quantum limit.
- Lumped-element vs traveling wave.
- They have enabled high readout fidelity for superconducting qubits.



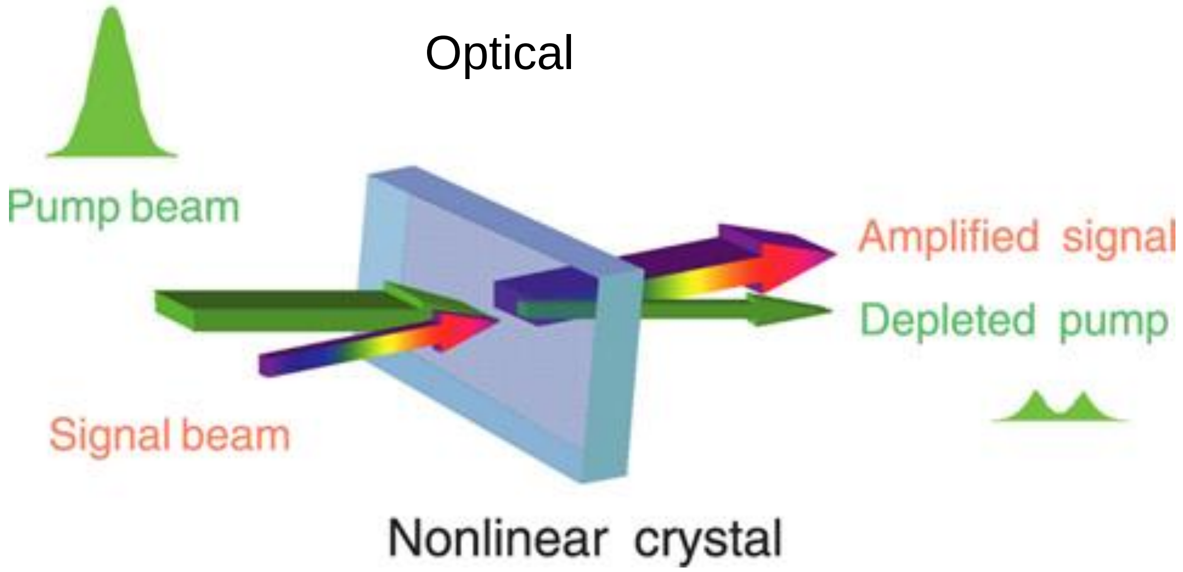


Parametric amplifiers

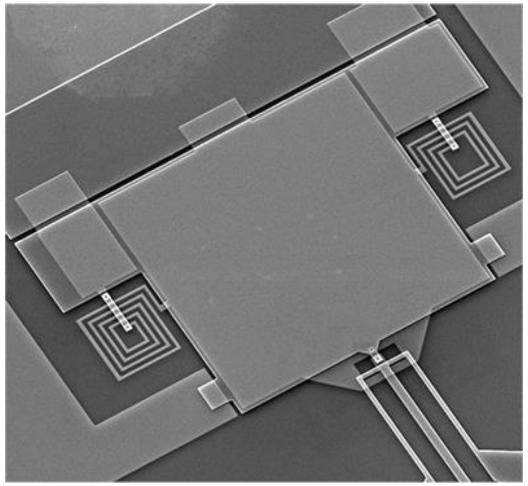
Varactor-based



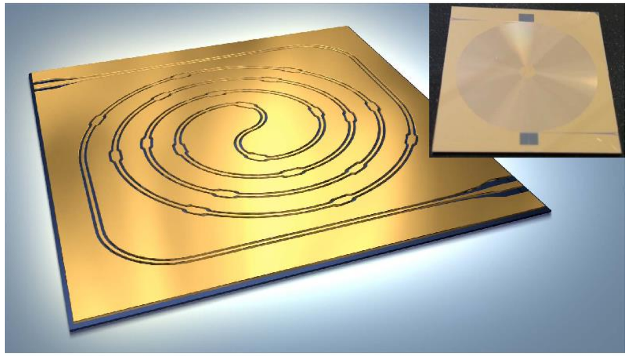
Optical



Superconducting lumped-element



Superconducting traveling-wave



Josephson parametric amplifiers

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$$H = \frac{Q^2}{2C} - E_J \cos\left(2\pi \frac{\hat{\phi}}{\phi_0}\right)$$

$$\cos(\phi) = 1 - \frac{\phi^2}{2!} + \frac{\phi^4}{4!} + \cdots$$

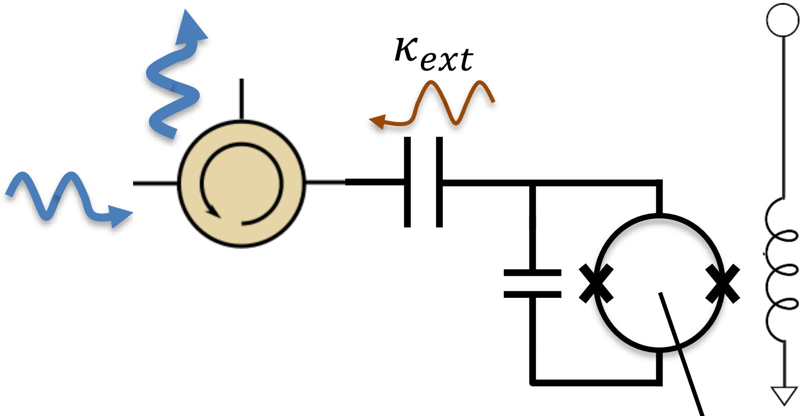
$$\hat{\phi} = \sqrt{\frac{\hbar}{2C\omega_0}}(\hat{a}^\dagger + \hat{a})$$

$$\hat{Q} = \sqrt{\frac{\hbar C\omega_0}{2}}(\hat{a}^\dagger - \hat{a})$$

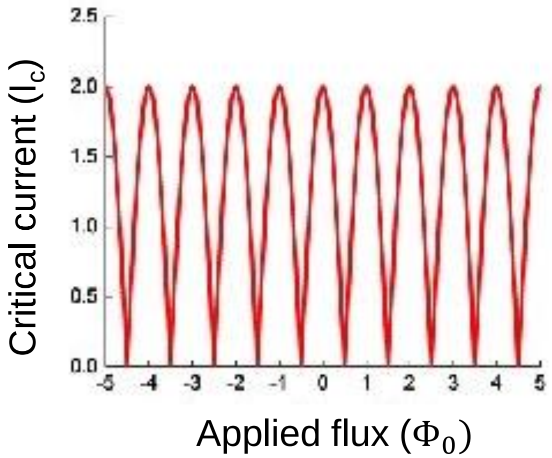
$$\longrightarrow \quad H \approx \omega_0 \hat{a}^\dagger \hat{a} + \frac{\Lambda}{6}(\hat{a}^\dagger + \hat{a})^4 \qquad \Lambda = -\frac{e^2}{4C}$$

↓

Kerr Hamiltonian



$$\Phi_p(t) = \delta\Phi \sin(\omega_p t)$$



$$L_J = \frac{\Phi_0}{2\pi I_{c0}\cos(\pi\Phi/\Phi_0)}$$

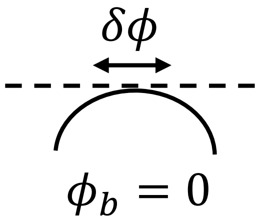
Josephson parametric amplifiers

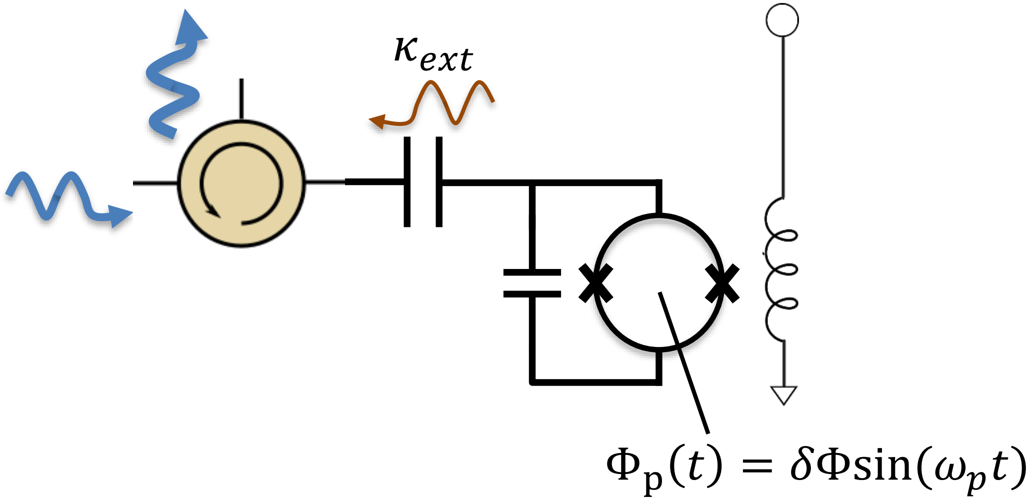
$$H \approx \omega_0 \hat{a}^\dagger \hat{a} + \frac{\Lambda}{6}\left(\hat{a}^\dagger + \hat{a}\right)^4$$

$$\hat{a} = \alpha_p \sin(\omega_p t) + \hat{d}$$

$$\omega_p = \omega_s - \Delta$$

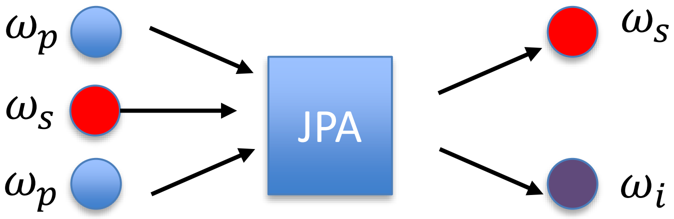
Rotating Wave Approximation (RWA)

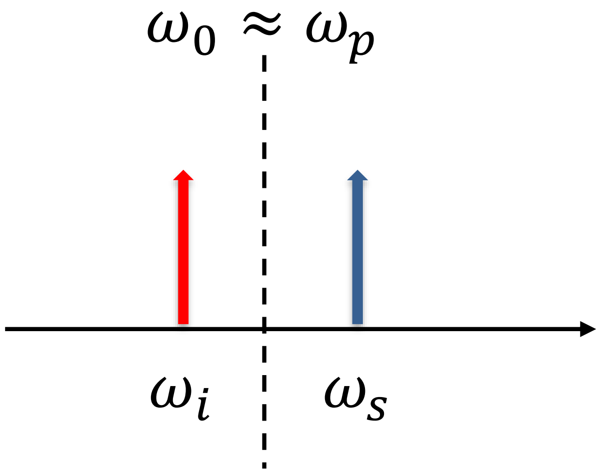




$$H' \approx \Delta \hat{d}^\dagger \hat{d} + \frac{\Lambda}{2}\left(\alpha_p^2 \hat{d}^{\dagger 2} + \alpha_p^{*2} \hat{d}^2\right)$$

4-wave mixing





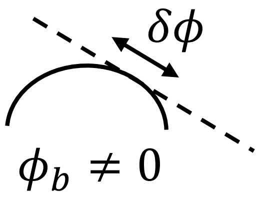
3-wave mixing

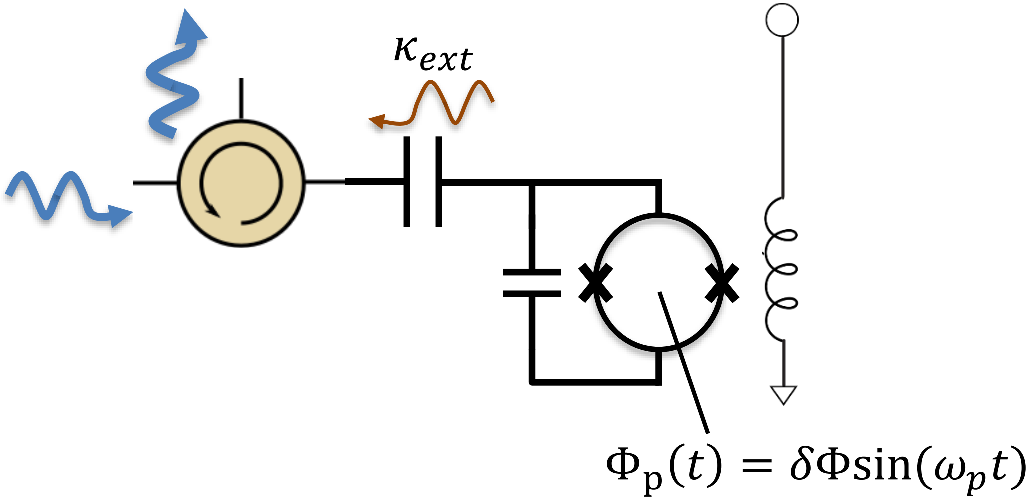
$$H \approx \omega_0 \hat{a}^\dagger \hat{a} + \frac{\Lambda}{6}\left(\hat{a}^\dagger + \hat{a}\right)^4$$

$$\hat{a} = \alpha_b + \alpha_p \sin(\omega_p t) + \hat{d}$$

$$\omega_p = 2\omega_0 - \Delta$$

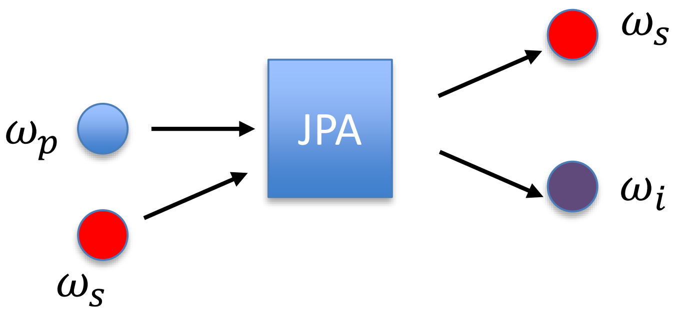
Rotating Wave Approximation (RWA)

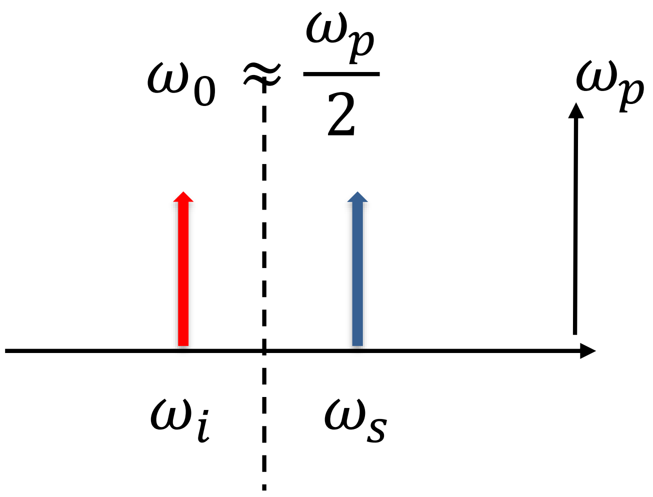




$$H' \approx \Delta \hat{d}^\dagger \hat{d} + \Lambda \alpha_b \left(\alpha_p \hat{d}^{\dagger 2} + \alpha_p^* \hat{d}^2\right)$$

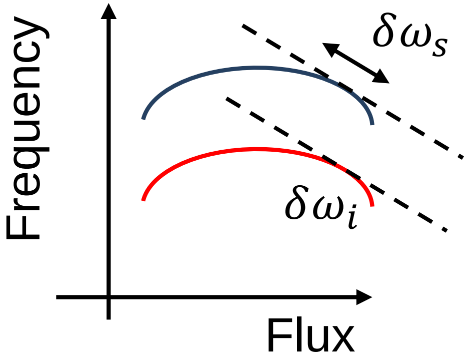
3-wave mixing



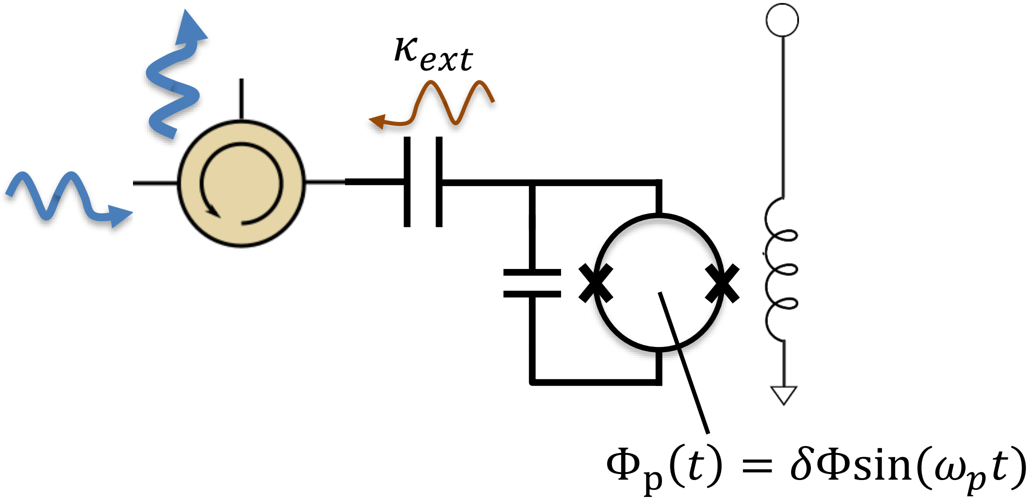


Coupling rate

$$H = \omega_0 \hat{d}^\dagger \hat{d} + g_p(\hat{d}^2 + \hat{d}^{\dagger 2})$$

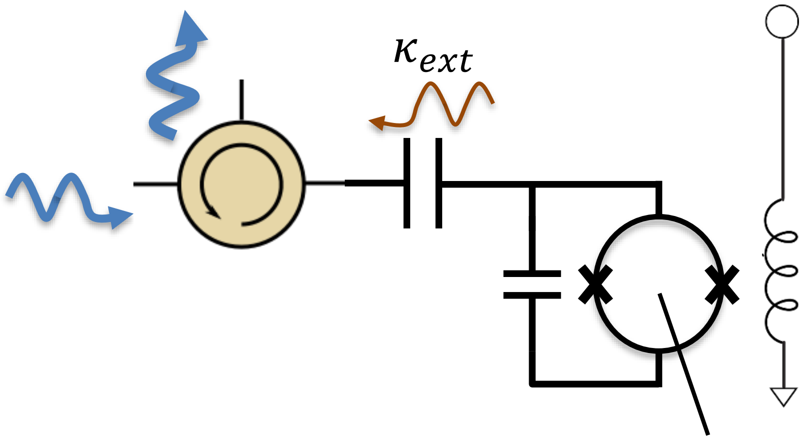


$$g_p = \Lambda \alpha_b \alpha_p = \frac{1}{2}\sqrt{\frac{\partial \omega_s}{\partial \Phi}\frac{\partial \omega_i}{\partial \Phi}}\,\delta\Phi$$



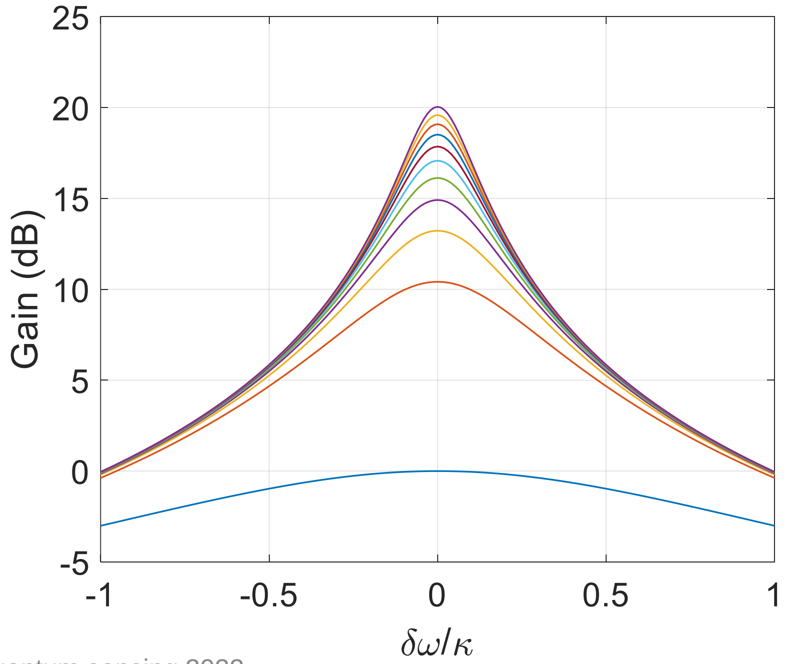
The coupling rate is the rate at which energy is transferred from the pump to the signal and it is related to the rate of change of the resonance frequency with flux.

Parametric amplifier and gain-bandwidth product



$$\Phi(t) = \delta\Phi \sin(\omega_p t)$$

- Typically embed SQUID into resonant circuit, capacitively coupled to the input line.
- Gain profile is Lorentzian.
- Constant gain-bandwidth product $G_0 B = \kappa$

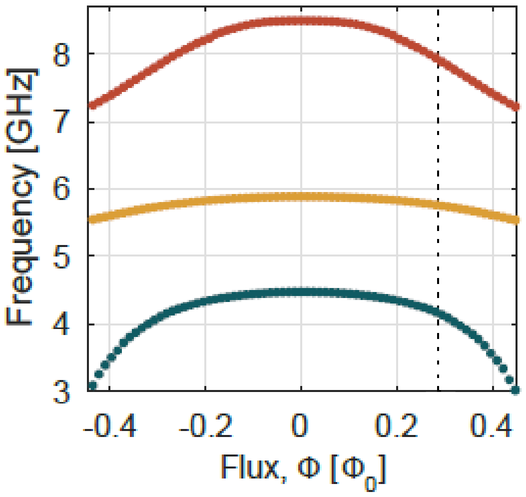


$$\frac{da_s}{dt} = \left(j\omega_s - \frac{\kappa}{2}\right) a_s + j g_p a_s^* + \sqrt{\kappa_{ext}}\, a_{s,in}$$

$$a_{s,out} = -a_{s,in} + \sqrt{\kappa_{ext}}\, a_s$$

$$G = \frac{j\left(\frac{\kappa_{ext}}{\kappa}\right)\left(\frac{\delta\omega}{\kappa} + \frac{j}{2}\right)}{\frac{\delta\omega^2}{\kappa^2} + \frac{j\delta\omega}{\kappa} + \frac{g_p^2}{4\kappa^2} - \frac{1}{4}} \longrightarrow \text{Pole near } g_p \to \kappa$$

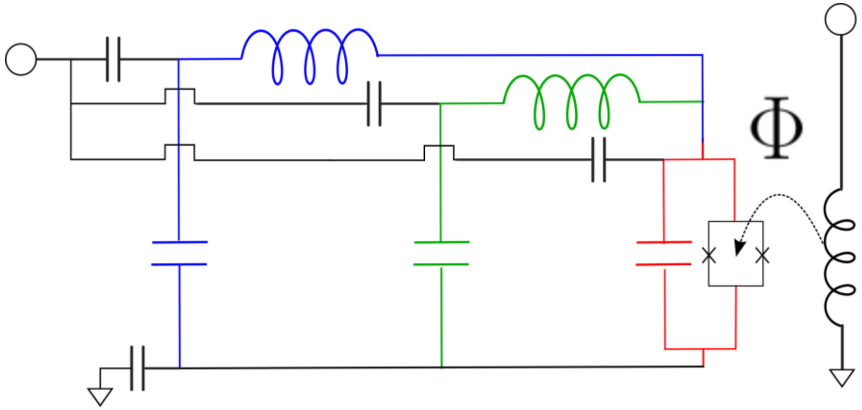
Signal and idler can live in different resonators too

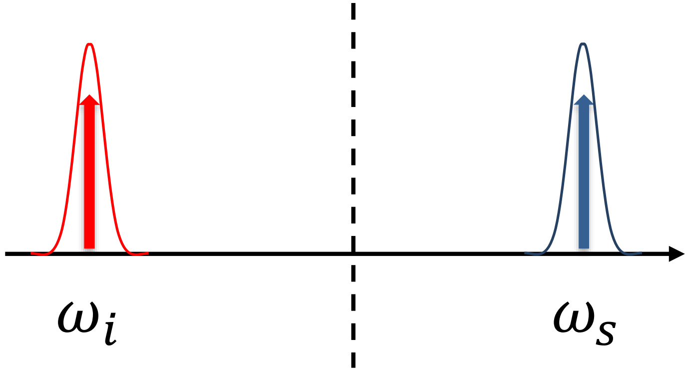


$$H = \omega_0 \hat{a}^\dagger \hat{a} + g_p(\hat{a}^2 + \hat{a}^{\dagger 2})$$

$$H = \omega_s \hat{a}_s{}^\dagger \hat{a}_s + \omega_i\, \hat{a}_i{}^\dagger \hat{a}_i + g_p(\hat{a}_s \hat{a}_i + \hat{a}_s{}^\dagger \hat{a}_i{}^\dagger)$$

$$g_p = \frac{1}{2}\sqrt{\delta\omega_s \delta\omega_i} = \frac{\delta L}{4\sqrt{L_s L_i}}\sqrt{\omega_s \omega_i}$$

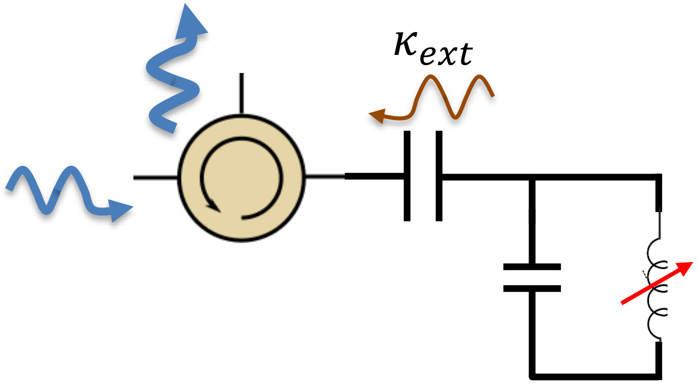




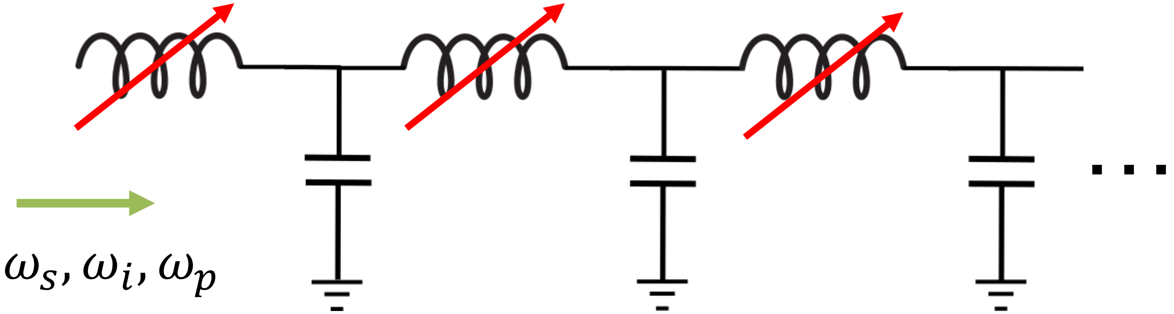
$$\omega_s + \omega_i \approx \omega_p$$

Lumped-element vs traveling-wave

Lumped-element



Traveling-wave



- Small size.
- Typically 1-500MHz bandwidth.
- Gain-bandwidth product (if single-tone modulation).
- Energy conservation: $\omega_p = \omega_s + \omega_i$

- Large area.
- Typically ~2-8GHz bandwidth.
- No gain-bandwidth product.
- Frequency and phase matching (energy and momentum conservation):

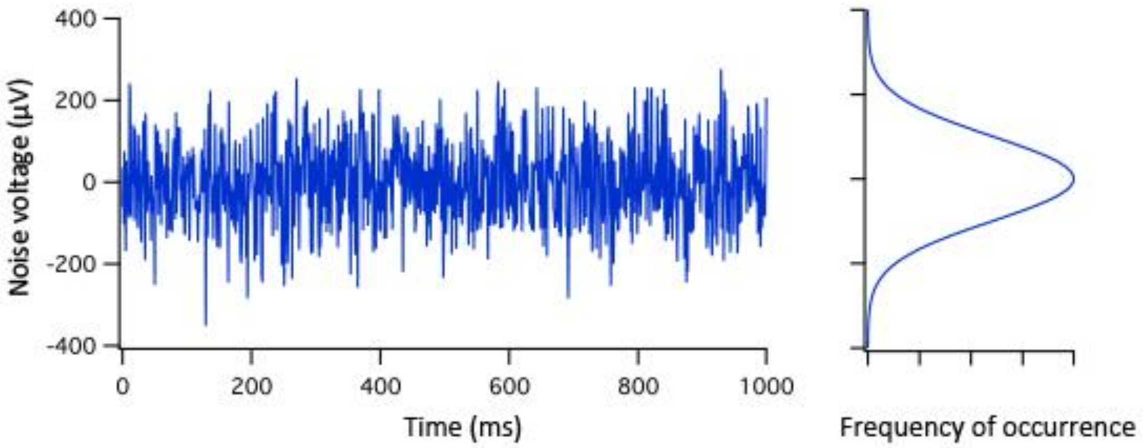
$$\omega_p = \omega_s + \omega_i \, , \, k_p = k_s + k_i$$

Noise

$$\langle x \rangle = 0$$

$$\langle x^2 \rangle \neq 0$$

$$\frac{1}{2} k \langle x^2 \rangle = \frac{1}{2} k_B T$$



Noise

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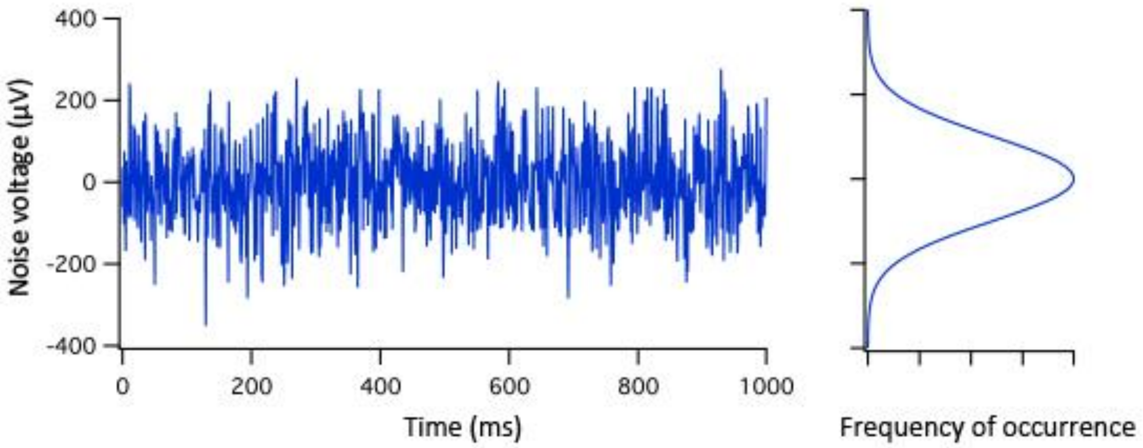


$$R(t) = < x(t)x^*(t-\tau) > = \lim_{T\to\infty} \frac{1}{T}\int_0^T x(t)x^*(t-\tau)\, d\tau$$

Autocorrelation function

$R(t) = \delta(t) \longrightarrow$ White noise.

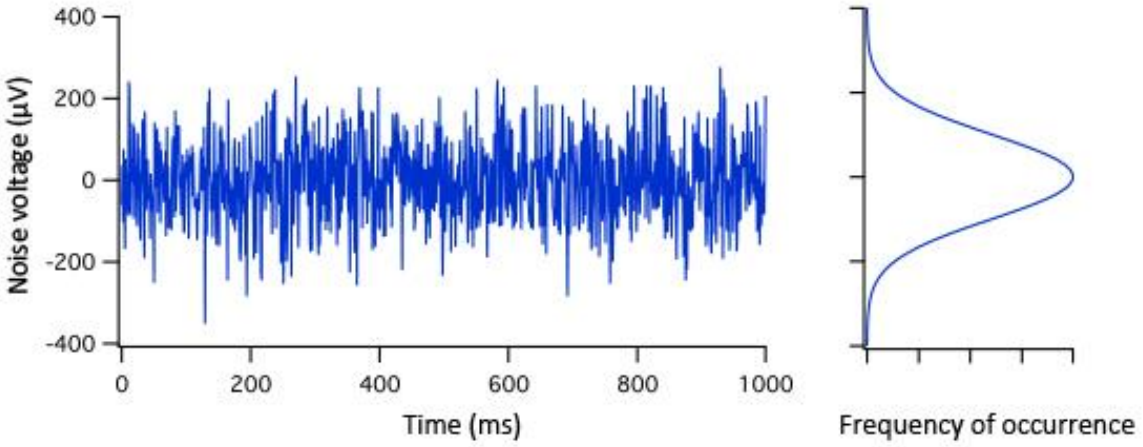
Power spectral density

$x(t) \Leftrightarrow \tilde{x}(\omega)$ Fourier transform of random noise

$S(\omega) =< |\tilde{x}(\omega)|^2 >$ Power spectral density

The power spectral density measures the amount of noise power per unit bandwidth in W/Hz

Wiener-Khinchin theorem



$$
\begin{aligned}
S_\psi(\omega) &\equiv \lim_{T\to\infty} \frac{\tilde\psi^*(\omega)\tilde\psi(\omega)}{T} \\
&= \lim_{T\to\infty} \frac{1}{T}\int_{-\infty}^{+\infty}\int_{-\infty}^{+\infty} dt_1 dt_2\; e^{-i\omega(t_2-t_1)}\psi^*(t_2)\psi(t_1) \\
&= \int_{-\infty}^{+\infty} d\Delta\; e^{-i\omega\Delta} \lim_{T\to\infty}\frac{1}{T}\int_{-\infty}^{+\infty} dt_1 \psi^*(t_1+\Delta)\psi(t_1) \\
S_\psi(\omega) &= \int_{-\infty}^{+\infty} d\Delta\; e^{-i\omega\Delta}\; \langle \psi^*(t_1+\Delta)\psi(t_1)\rangle
\end{aligned}
$$

Time domain

$$\langle \psi^*(t+\Delta)\psi(t)\rangle \sim \delta(\Delta)$$

$$\langle x(t+\tau)x(\tau)\rangle \propto \exp\left(-\omega_c t/2Q\right)$$

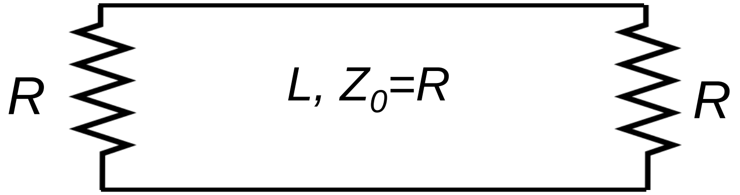
Frequency domain

constant $S_\psi(\omega)$

(white noise)

$$S_x = \frac{S_F/m^2}{(\omega^2-\omega_c^2)^2 + (\omega\omega_c/Q)^2}$$

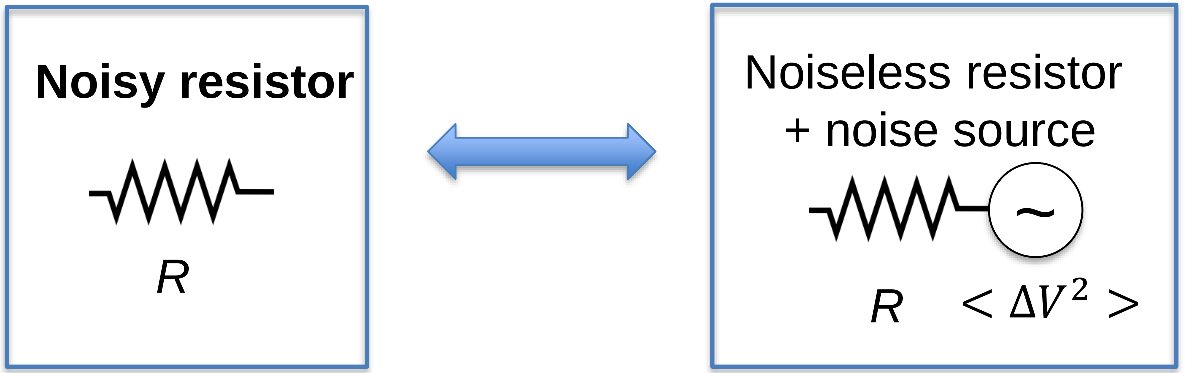
Johnson – Nyquist noise



$$N = \frac{2Ldf}{v}$$ Number of TL modes

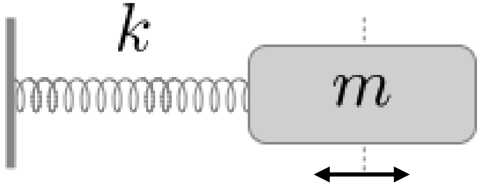
$$E = \frac{hfdf}{e^{hf/k_BT} - 1}$$ Energy per mode

$$< \Delta V^2 > = 4R \frac{hf}{e^{hf/k_BT} - 1} \approx 4k_BTR$$



Nyquist, Harry. "Thermal agitation of electric charge in conductors." *Physical review* 32.1 (1928): 110.

Quantum noise



$$< \Delta x^2 > = \frac{\hbar}{2m\omega}$$

$$< \Delta p^2 > = \frac{m\hbar\omega}{2}$$

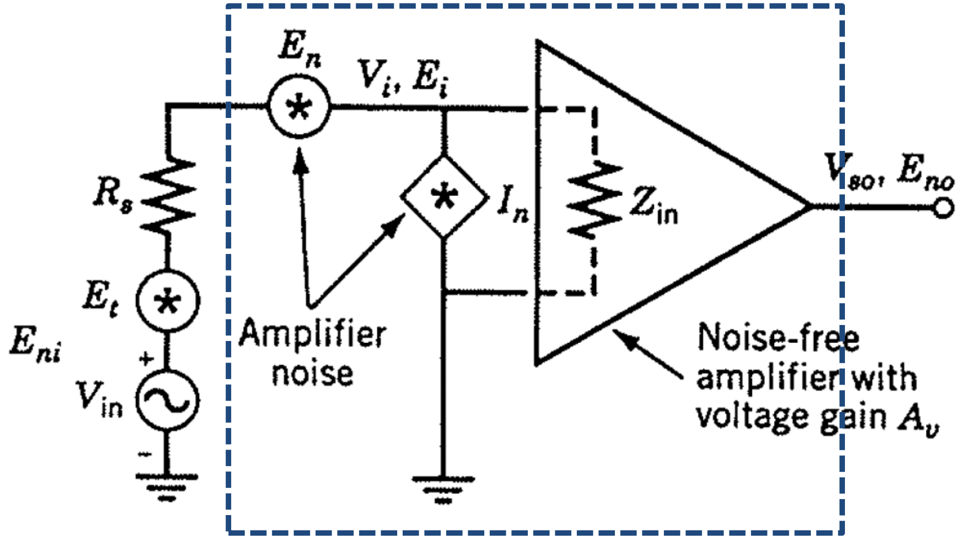
Necessary to satisfy Heisenberg uncertainty principle:

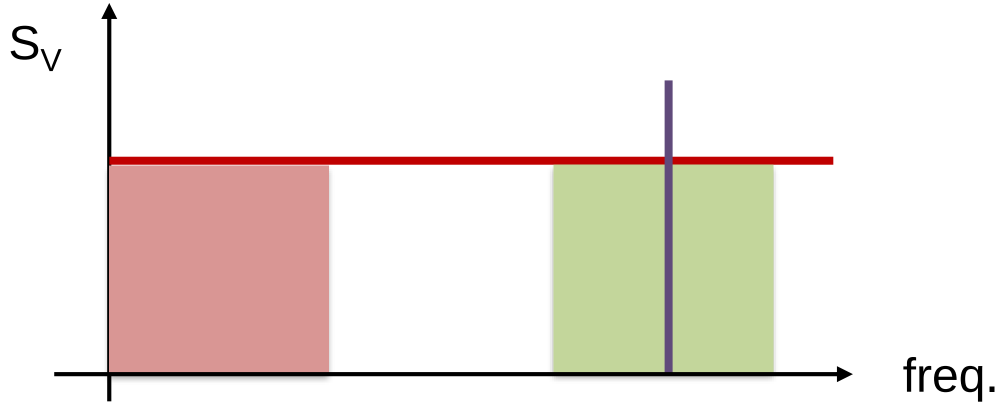
$$\Delta x \Delta p \geq \frac{\hbar}{2}$$

For electromagnetic fields: $< \Delta n^2 > = n_{th} + \frac{1}{2}$ $\qquad n_{th} = \frac{k_B T}{\hbar\omega}$

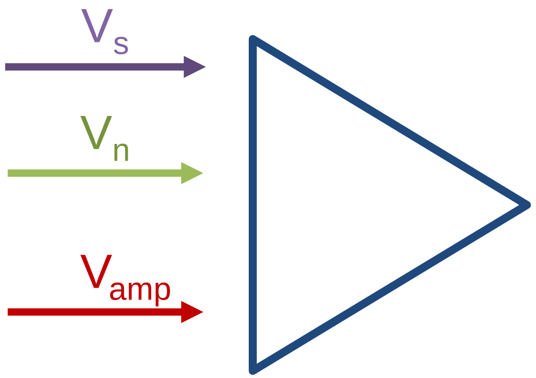
Thermal photons **Vacuum noise**

Amplifier noise figure





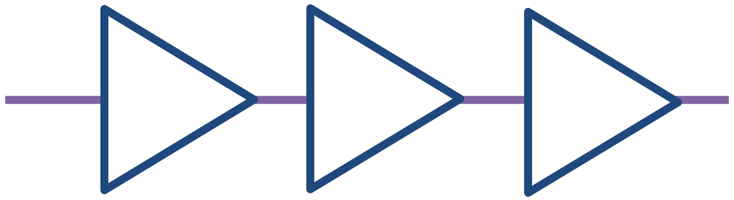
$$\mathrm{SNR} = \frac{\mathrm{V_{signal}}}{\sqrt{\int \mathrm{S_v df}}}$$



Gain * (V_s + V_n + V_{amp})

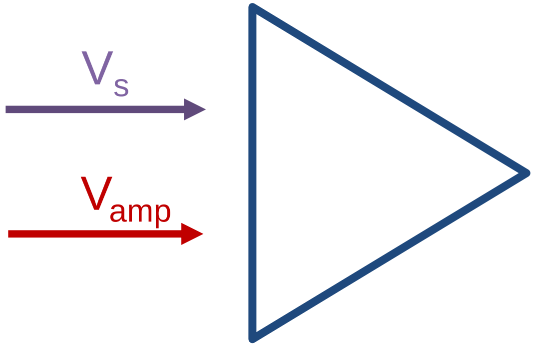
Noise figure

$$\mathcal{F} = \frac{\mathrm{SNR_{in}}}{\mathrm{SNR_{out}}}$$



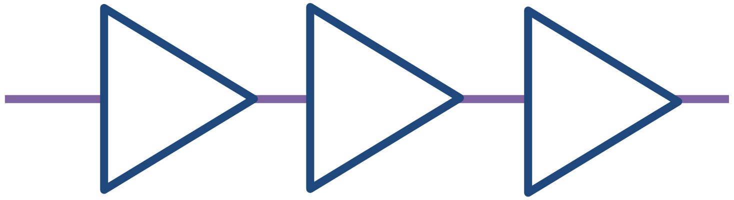
$$\mathcal{F}_{\mathrm{total}} = \mathcal{F}_1 + \frac{(\mathcal{F}_2 - 1)}{G_1} + \frac{(\mathcal{F}_3 - 1)}{G_1 G_2}$$

Noise temperature

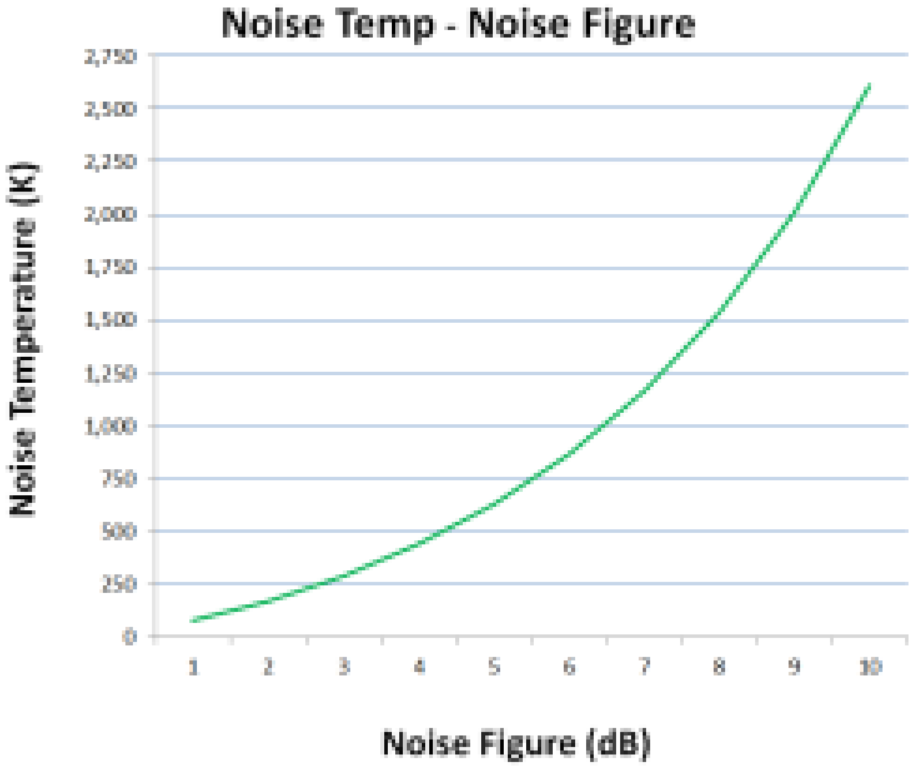


Gain * (V_s + V_{amp})

$$S_V/R = 4k_B(T + T_{\mathrm{amp}})$$



$$T_{\mathrm{total}} = T_{n1} + \frac{T_{n2}}{G_1} + \frac{T_{n3}}{G_1 G_2}$$



$$F = \frac{T_n}{T_{ref}} + 1$$

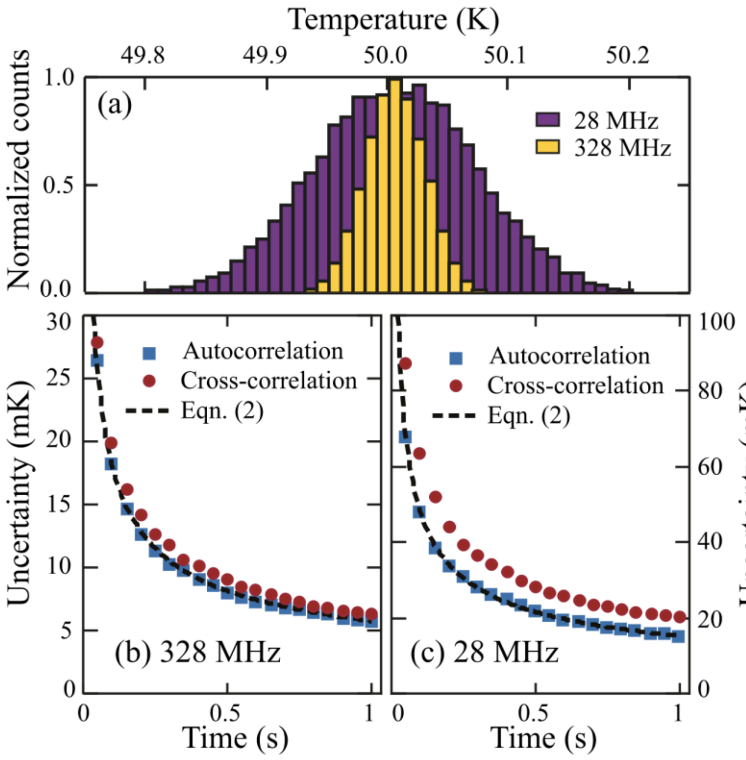
$T_{ref} = 290\ K$

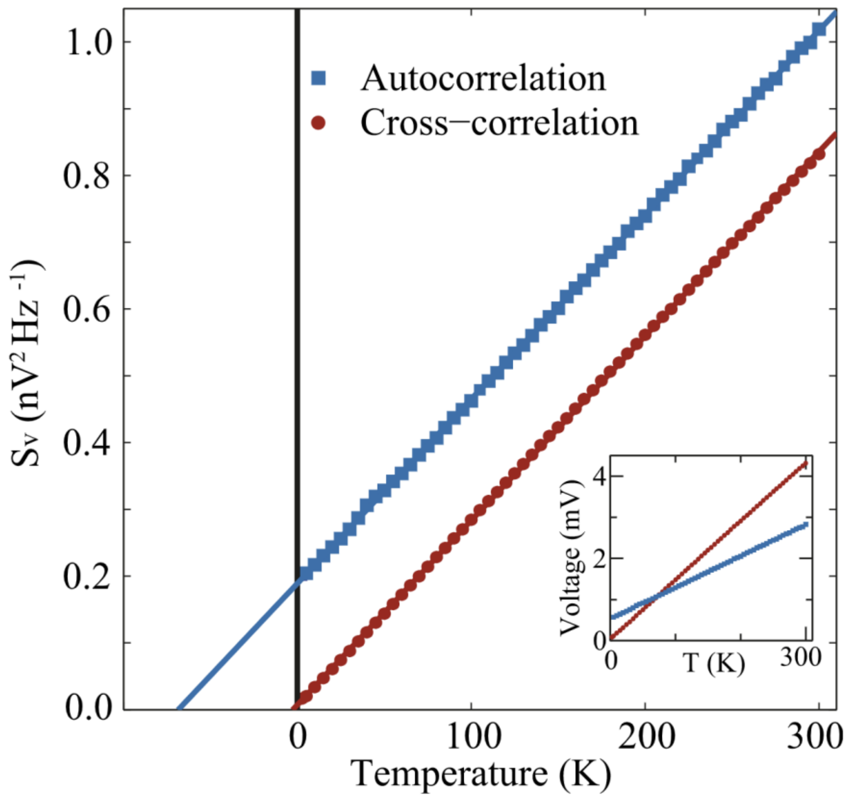
Noise and measurement time

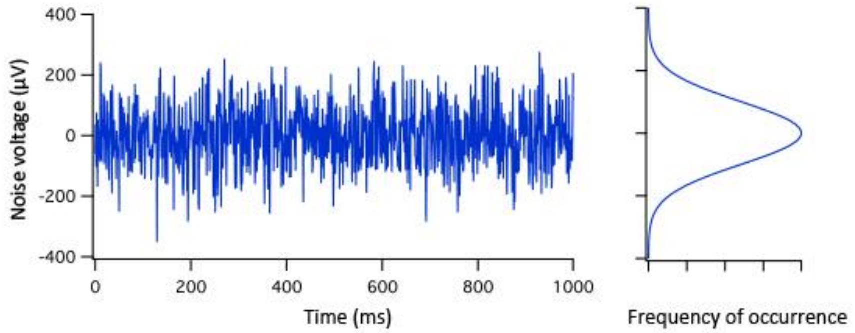
$$S_V/R = 4k_B(T + T_{amp})$$

$$\delta T = \frac{T + T_{amp}}{\sqrt{B\tau_{avg}}}$$

Doubling of the amplifier noise means quadruple the measurement time!!

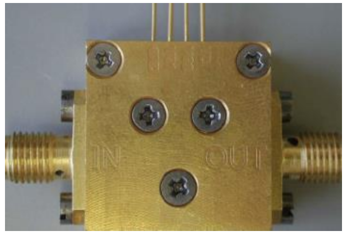




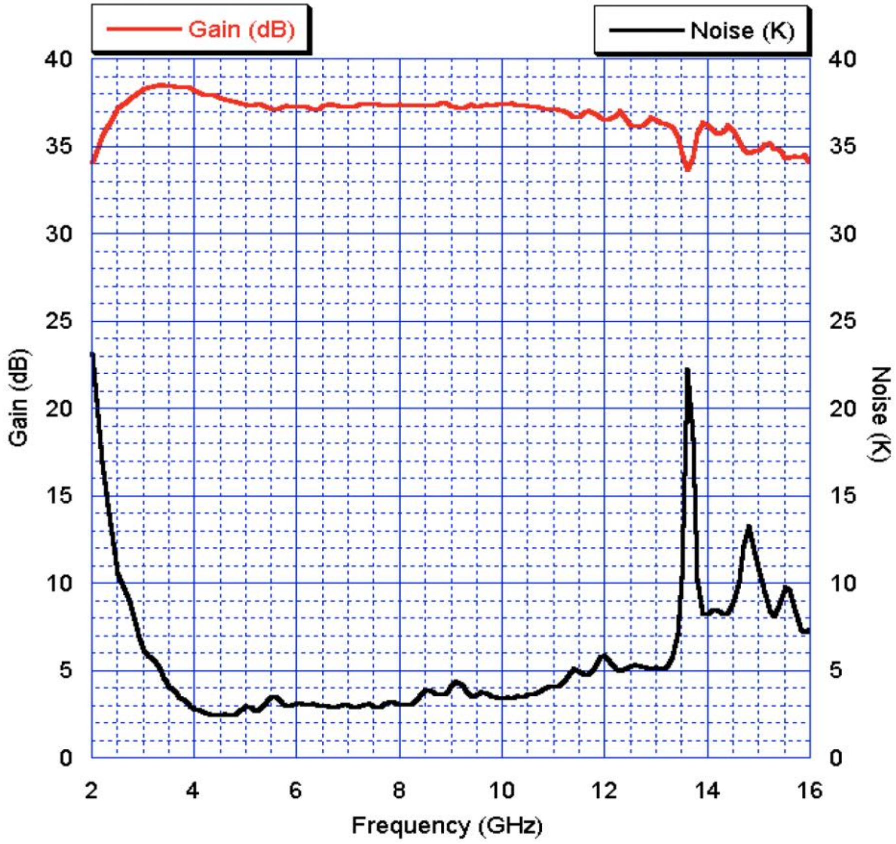


Crossno, APL (2015)

Low noise amplifiers



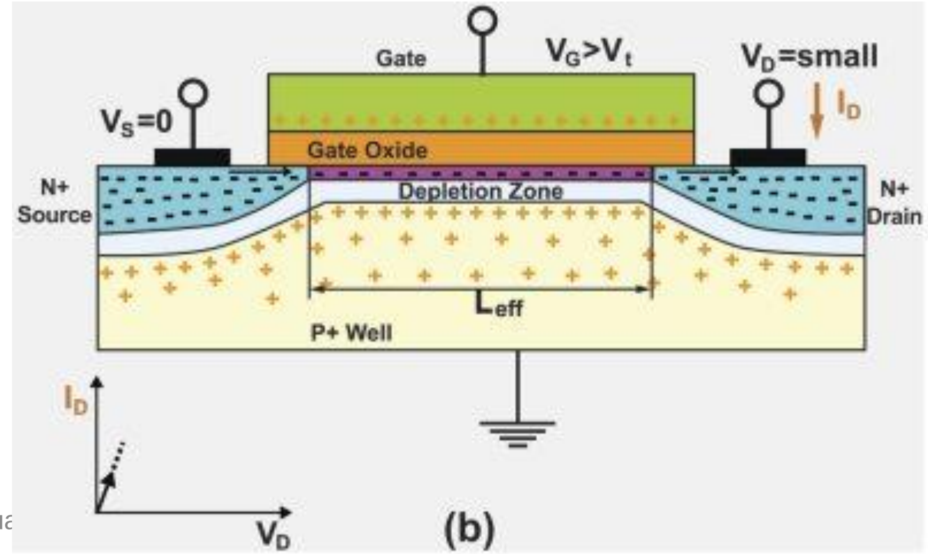
Caltech HEMT



$$S_V/R = 4k_B(T + T_{\rm amp})$$

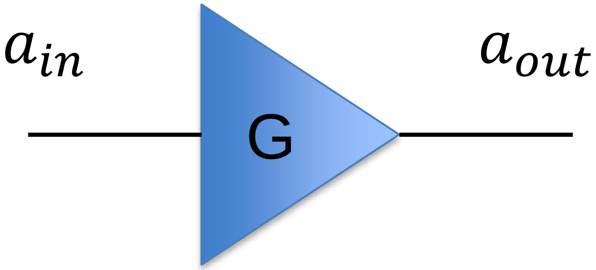
Example: at 6 K, T_{HEMT} = 3 K
Added quanta = 10

Amplification by MOSFET



Quantum limit of phase-insensitive linear amplifiers

Raytheon BBN

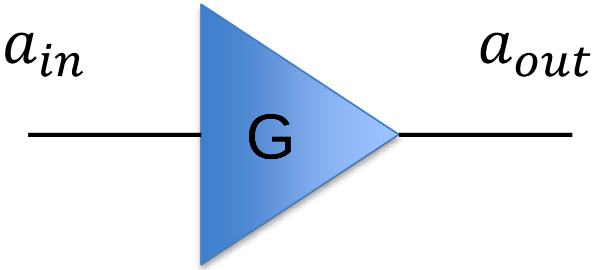


$\hat{a}_{out} = \sqrt{G}\hat{a}_{in}$ $\hat{a}_{out}^{\dagger} = \sqrt{G}\hat{a}_{in}^{\dagger}$ → $[\hat{a}_{out}, \hat{a}_{out}^{\dagger}] = G[\hat{a}_{in}, \hat{a}_{in}^{\dagger}] = G > 1$

Not unitary!

Caves, C. M. (1982). Quantum limits on noise in linear amplifiers. *Physical Review D*, *26*(8), 1817.

Quantum limit of phase-insensitive linear amplifiers



$\hat{a}_{out} = \sqrt{G}\hat{a}_{in}$ $\quad \hat{a}_{out}^{\dagger} = \sqrt{G}\hat{a}_{in}^{\dagger}$ $\Rightarrow$ $[\hat{a}_{out}, \hat{a}_{out}^{\dagger}] = G[\hat{a}_{in}, \hat{a}_{in}^{\dagger}] = G > 1$

Not unitary!

$$\hat{a}_{out} = \sqrt{G}\hat{a}_{in} + \hat{\mathcal{F}}$$

↘ Additional noise

$[\hat{a}_{out}, \hat{a}_{out}^{\dagger}] = 1$ $\Rightarrow$ $[\hat{\mathcal{F}}, \hat{\mathcal{F}}^{\dagger}] = 1 - G$

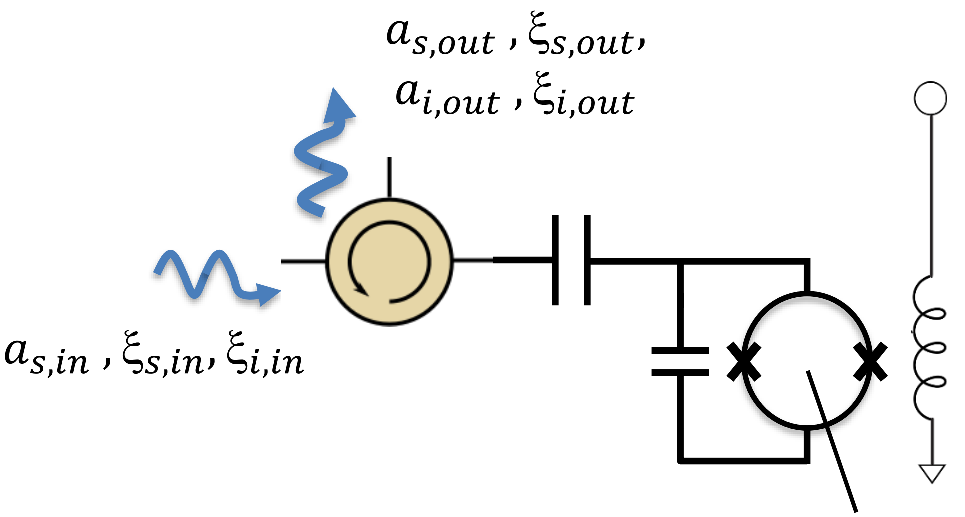
$$\Rightarrow \Delta a_{out}^2 = G\Delta a_{in}^2 + \frac{1}{2}\langle\{\hat{\mathcal{F}}, \hat{\mathcal{F}}^{\dagger}\}\rangle \geq G\Delta a_{in}^2 + \frac{1}{2}\langle[\hat{\mathcal{F}}, \hat{\mathcal{F}}^{\dagger}]\rangle = G\Delta a_{in}^2 + \frac{G-1}{2}$$

$$\boldsymbol{n_{add} \geq \frac{1}{2}\left(1 - \frac{1}{G}\right)}$$

$$T_N \geq \frac{\hbar\omega}{2k_B}$$

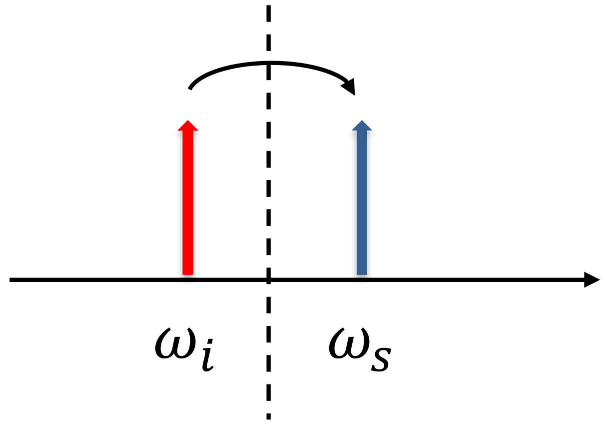
Caves, C. M. (1982). Quantum limits on noise in linear amplifiers. *Physical Review D*, *26*(8), 1817.

Noise in parametric amplifiers



$$\Phi(t) = \Phi_p \sin(\omega_p t)$$

$$\xi_{s,out} = \sqrt{G}\xi_{s,in} + \sqrt{G-1}\xi_{i,in}$$



$$\langle \xi^\dagger_{j,in}[\omega']\xi_{j,in}[\omega]\rangle = n_{th}\delta(\omega-\omega')$$

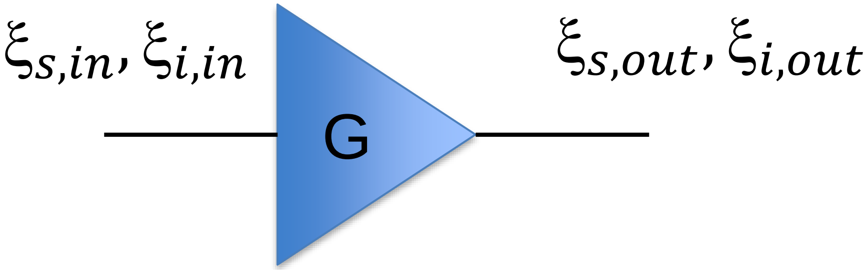
$$j = i, s$$

$$\langle \xi_{j,in}[\omega]\xi^\dagger_{j,in}[\omega']\rangle = (n_{th}+1)\delta(\omega-\omega')$$

- Thermal and vacuum noise sources at signal and idler.
- Use signal scattering parameters to compute output noise.
- Note: ½ photon added noise at the signal frequency comes from the idler!

$$\langle \mathbf{A}^\dagger_{\text{out}}[\omega']\mathbf{A}^T_{\text{out}}[\omega]\rangle = \mathbf{S}^*[\omega']\langle \mathbf{A}^\dagger_{\text{in}}[\omega']\mathbf{A}^T_{\text{in}}[\omega]\rangle\mathbf{S}^T[\omega]$$

Example



Parametric amplifier scattering matrix:

$$\begin{bmatrix} a_{s,out} \\ a_{i,out}^{\dagger} \end{bmatrix} = S \begin{bmatrix} a_{s,in} \\ a_{i,in}^{\dagger} \end{bmatrix} \qquad S = \begin{bmatrix} \sqrt{G} & \sqrt{G-\eta} \\ \sqrt{G-\eta} & \sqrt{G} \end{bmatrix}$$

Output noise operators:

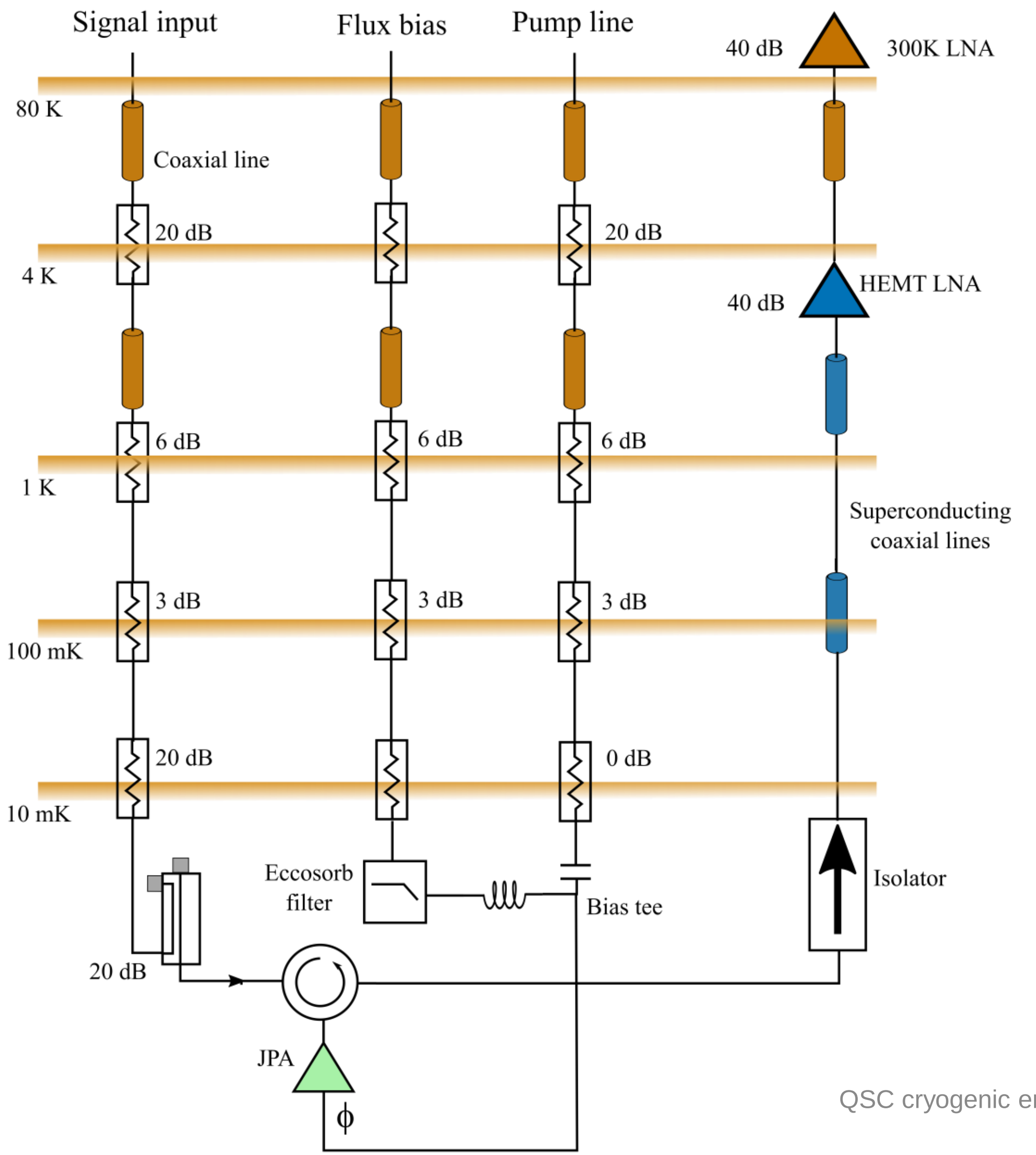
$$\xi_{s,out} = \sqrt{G}\xi_{s,in} + \sqrt{G-\eta}\xi_{i,in} \qquad \xi_{i,out} = \sqrt{G-\eta}\xi_{s,in} + \sqrt{G}\xi_{i,in}$$

$$\left\langle \xi_{s\backslash i,in}^{\dagger}[\omega']\xi_{s\backslash i,in}[\omega] \right\rangle = n_{th}\delta(\omega-\omega'), \qquad \left\langle \xi_{s\backslash i,in}[\omega]\xi_{s\backslash i,in}^{\dagger}[\omega'] \right\rangle = (n_{th}+1)\delta(\omega-\omega')$$

$$n_{add} = \frac{\Delta\xi_{s,out}^2}{G} - \Delta\xi_{s,in}^2 = \frac{\mathbf{1}}{\mathbf{2}}\left(\mathbf{1} - \frac{\boldsymbol{\eta}}{\boldsymbol{G}}\right)$$

→ Quantum limited amplifier for $\eta = 1$

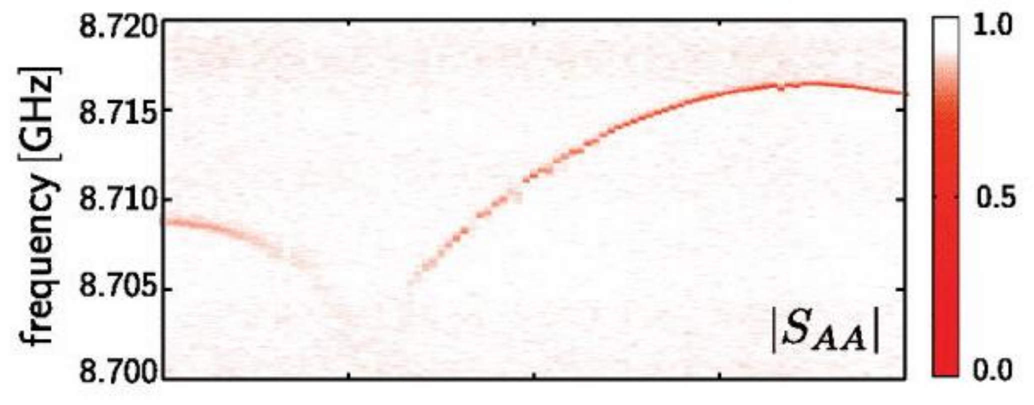
Amplifier characterization



- ~70-80 dB insertion loss at the input.
- Attenuation distributed to suppress thermal noise from the previous stage.
- Less attenuation on pump line (~20-40dB).
- Lowpass filters at the MC on the bias line.
- Cryogenically compatible bias tee (we use Marki BT-0018 and Anritsu K250\V250).

Parametric amplifier tuneup

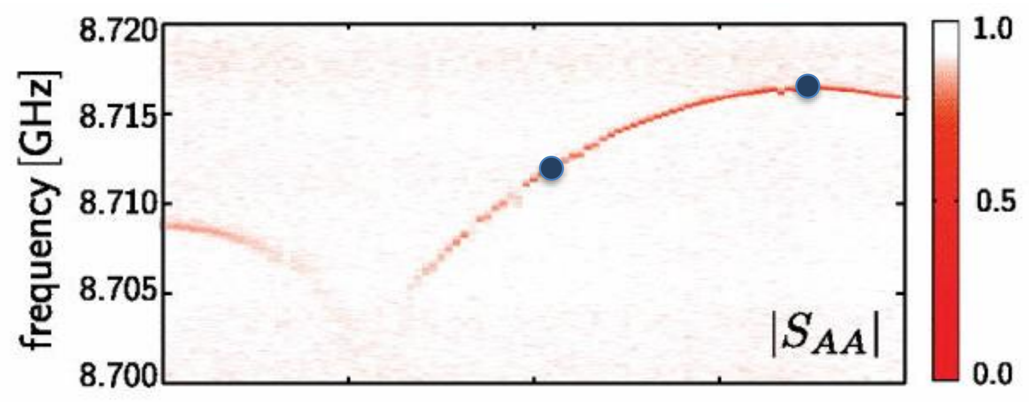
Step 1: Measure input reflection coefficient in a VNA vs flux bias.
Determine resonant frequency vs flux.



Parametric amplifier tuneup

Step 1: Measure input reflection coefficient in a VNA vs flux bias. Determine resonant frequency vs flux.

Step 2: Choose a flux bias and start pumping at resonance (4-wave mixing) or at twice the resonant frequency (3-wave mixing).



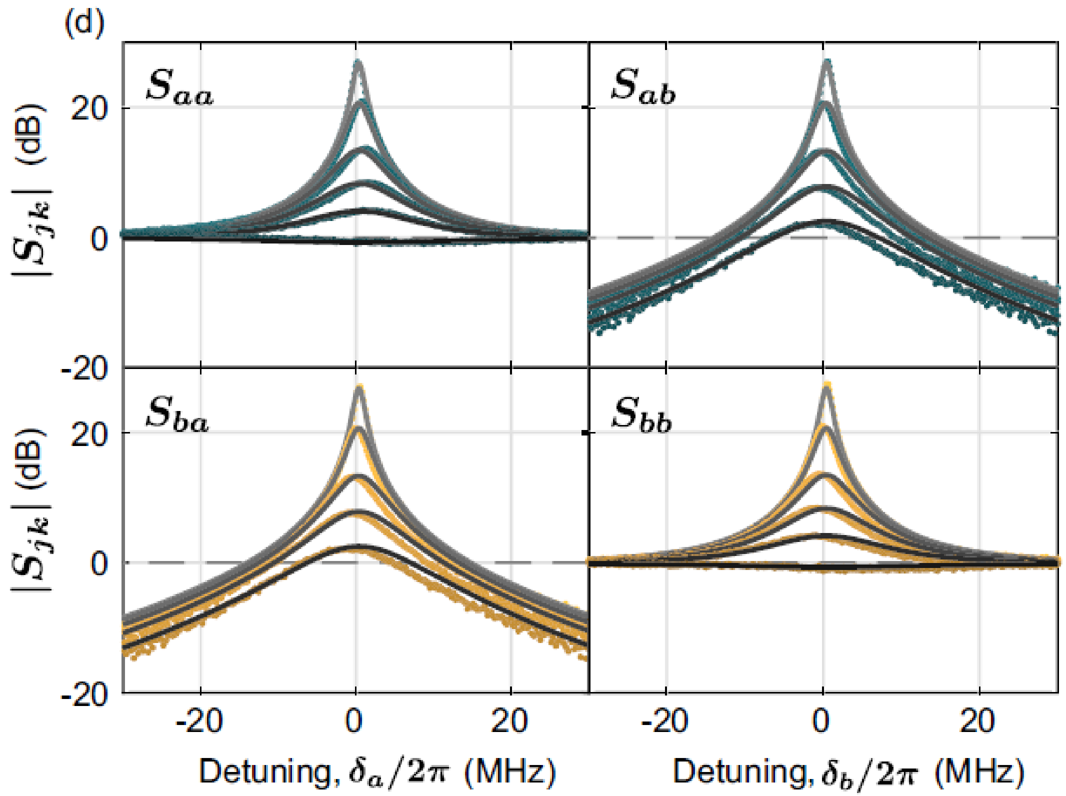
For 4-wave mixing bias at the top of the flux curve (or bottom for asymmetric SQUIDs), for 3-wave mixing bias on the slope.

Parametric amplifier tuneup

Step 1: Measure input reflection coefficient in a VNA vs flux bias. Determine resonant frequency vs flux.

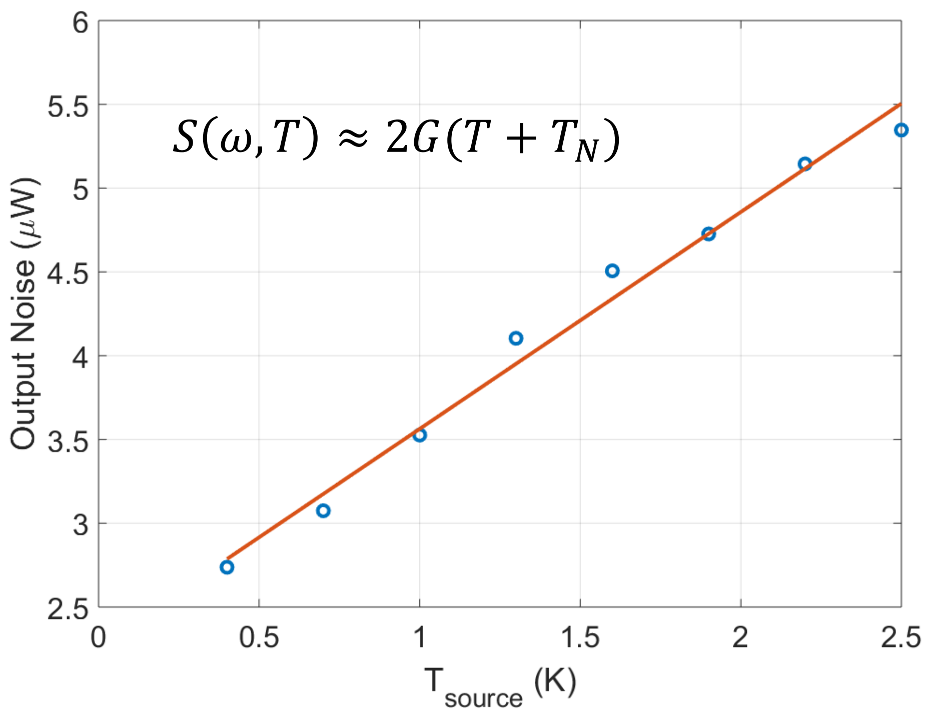
Step 2: Choose a flux bias and start pumping at resonance (4-wave mixing) or at twice the resonant frequency (3-wave mixing).

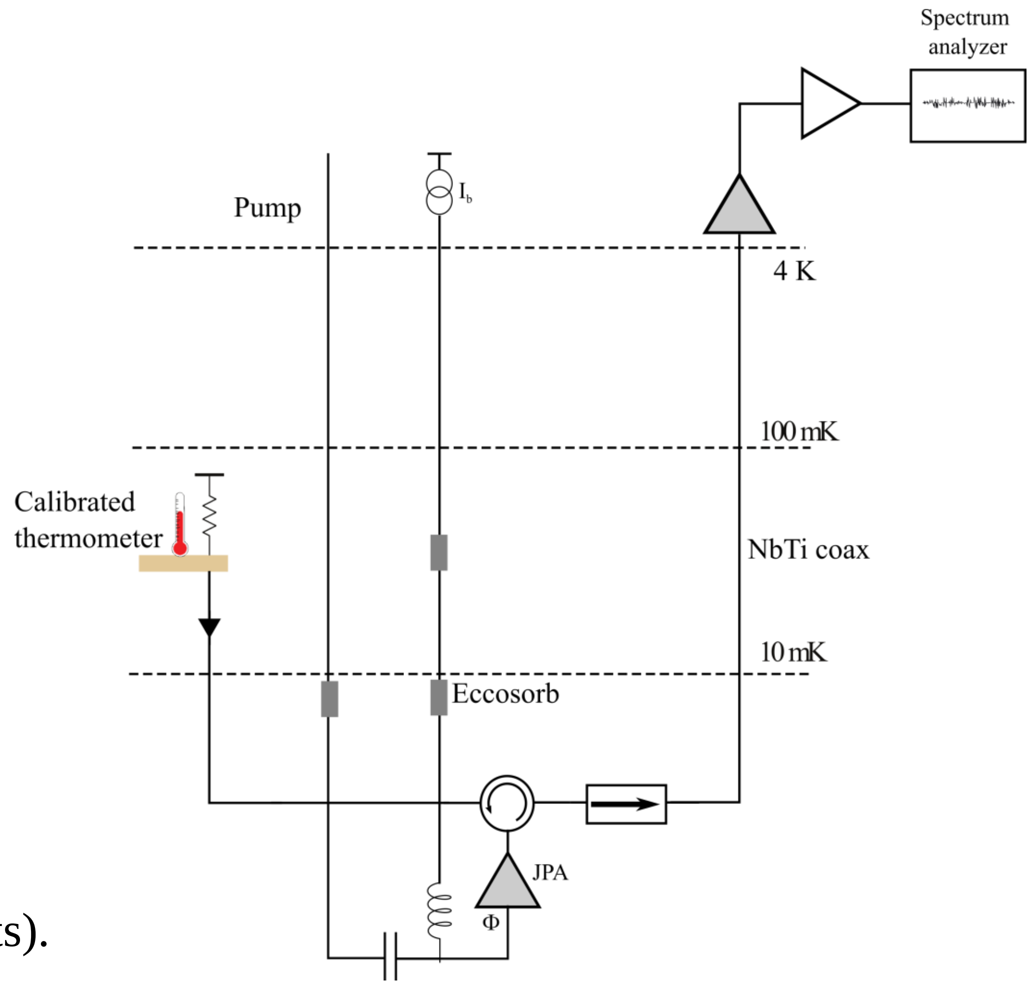
Step 3: Increase pump amplitude up to desired gain. Finely adjust pump frequency and amplitude to achieve Lorentzian gain profile.



Note: transmission gain requires frequency offset functionality in the VNA or external downconversion mixer.

Noise measurement





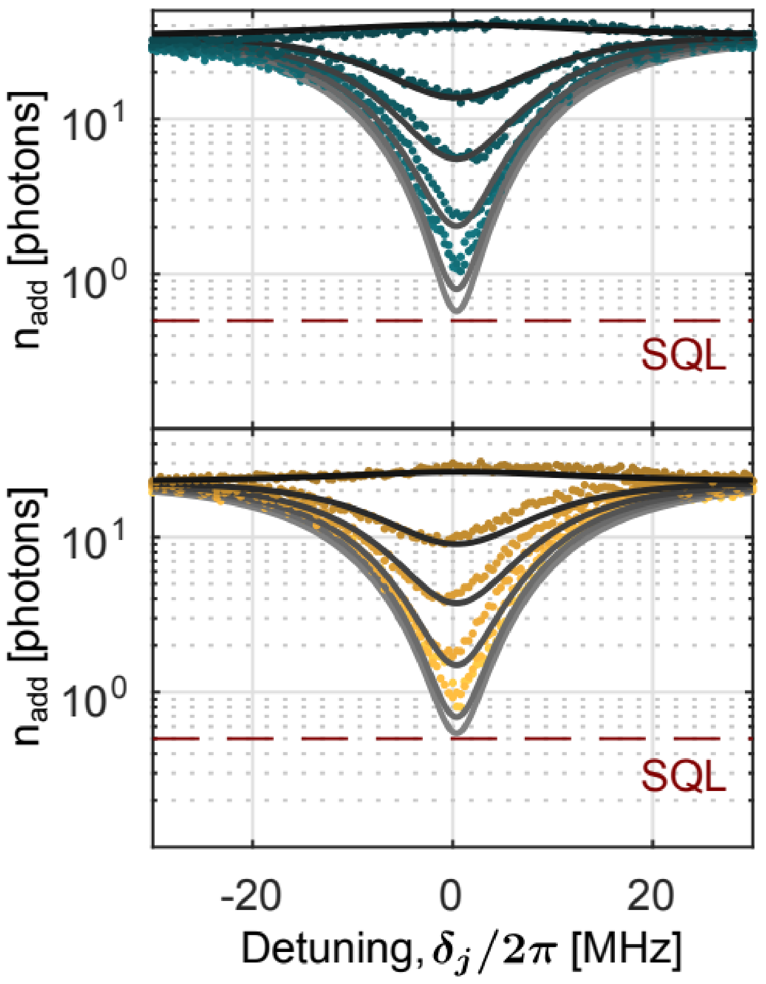
- Noise characterization with tunable temperature load.
- More accurate than hot-cold load measurement (more sample points).
- Wideband measurement (2-12GHz).

Noise in JPA

- Acquire noise spectrum vs frequency at different temperatures for the noise source $T_1 \ldots . T_N$.
- Fit to $S(\omega, T) \approx 2G(T + T_N)$ at each frequency point and compute the effective temperature T_N.
- We can also express the noise as number of added noise photons $n_{add} = k_B T_N / \hbar\omega$.

$$T_N \geq \frac{\hbar\omega}{2k_B}$$

$$n_{add} \geq \frac{1}{2}$$



Degenerate parametric amplifier

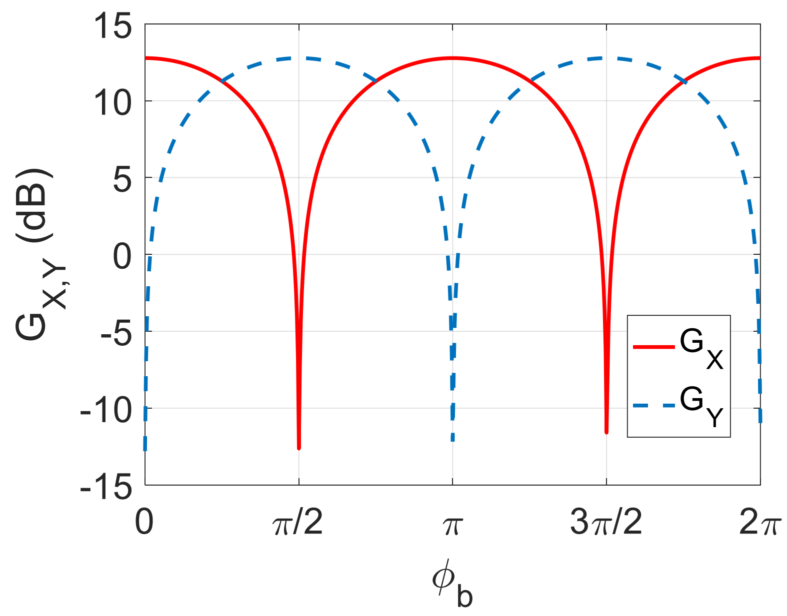
- “Degenerate” Parametric amplifier when $\omega_s = \omega_i$
- Gain is phase sensitive $G = G(\phi)$.

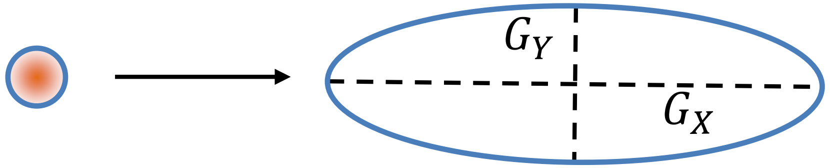
$$a_{out} = \sqrt{G}a_{in} + \sqrt{G-1}a_{in}^{\dagger}$$

$$G_X \neq G_Y$$

$$X = \frac{a + a^{\dagger}}{\sqrt{2}}$$

$$Y = \frac{a - a^{\dagger}}{\sqrt{2}i}$$





Phase sensitive amplification and squeezing

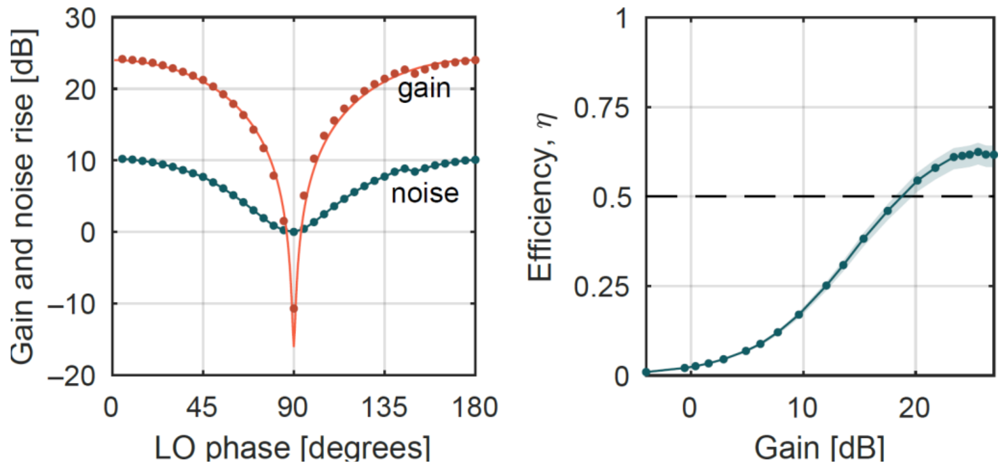
Raytheon BBN

If gain is phase sensitive ($G_X \neq G_Y$) we have a different formula for the added noise:

$$A_X A_Y \geq \frac{1}{4}\left(1 - \frac{1}{\sqrt{G_X G_Y}}\right)$$

If $G_X G_Y = 1$ we can have **noiseless** amplification.

$$< \Delta X_\theta \Delta Y_\theta > \leq \frac{1}{4}$$



Lecocq, F., et al. , *Physical Review Applied* 13.4 (2020): 044005.

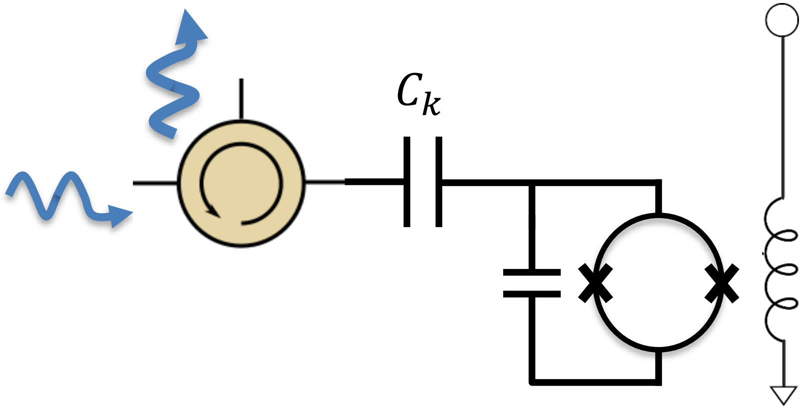
Wideband Parametric Amplifiers

Narrowband amplifiers:

Capacitive coupling to input line:

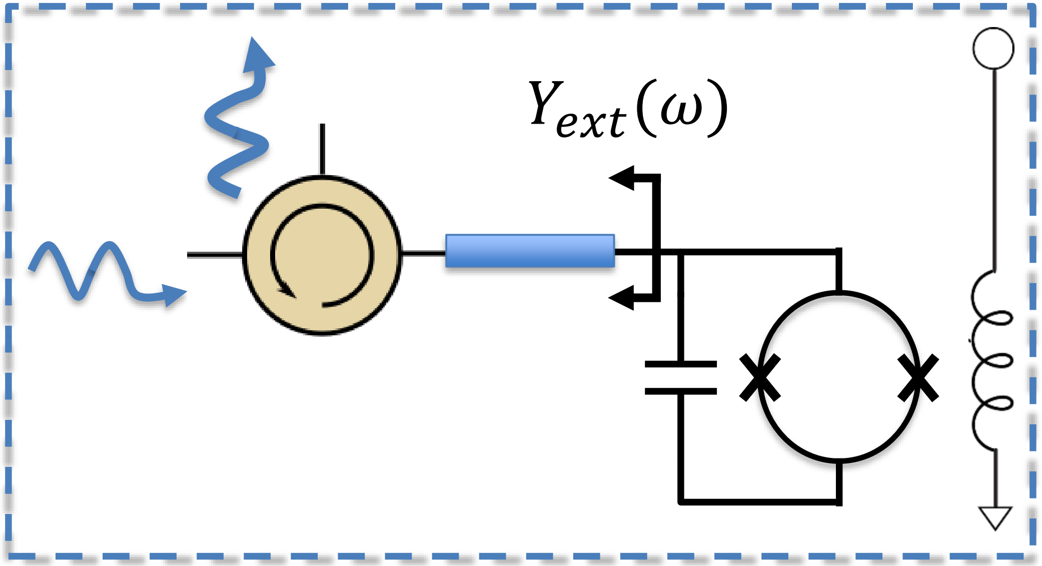
$$Y_{ext} \approx j\omega_0 C_k + \omega_0^2 C_k^2 Z_0$$

detuning damping



Wideband amplifiers

- Direct (strong) coupling to input\output lines increases device bandwidth.
- Amplifier interacts with **wideband** environment.
- The equivalent admittance seen by the amplifier can be a generic function of frequency $Y_{ext}(\omega)$.



Modeling wideband JPAs

We use the pumpistor model*.

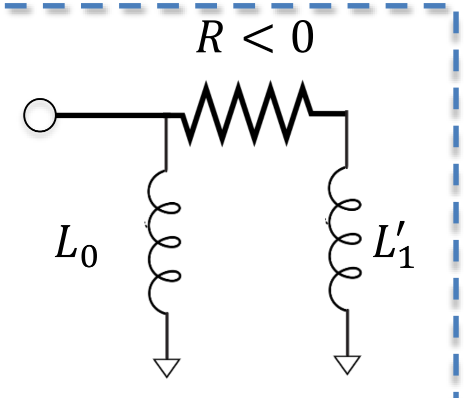
$$X = -\left(\frac{\Phi_0}{2\pi I_c}\right)^2 \frac{4\omega_s\omega_i Y_{ext}^*(\omega_i)}{\pi^2 \sin^2\left(\frac{\pi\Phi_{dc}}{\Phi_0}\right)} \frac{\Phi_0^2}{\Phi_{ac}^2}$$

$$L_0 = \frac{\Phi_0}{2\pi I_c \cos(\frac{\pi\Phi_{dc}}{\Phi_0})}$$

$$L_1 = -\frac{\Phi_0}{2\pi I_c} \frac{4\cos\left(\frac{\pi\Phi_{dc}}{\Phi_0}\right)}{\pi^2 \sin^2\left(\frac{\pi\Phi_{dc}}{\Phi_0}\right)} \frac{\Phi_0^2}{\Phi_{ac}^2}$$

$$Y_{JPA}(\omega_s) = \frac{1}{j\omega_s L_0} + \frac{1}{j\omega_s L_1 + X}$$

$Y_{JPA} \equiv$



Equivalent negative resistance depends on external impedance at **idler** frequency.

*Sundqvist, Kyle M., and Per Delsing., *EPJ Quantum Technology* 1.1 (2014): 1-21.

Mutus, Josh Y., et al. , *Applied Physics Letters* 104.26 (2014): 263513.

JPA impedance matching

Raytheon BBN

Transducer power gain in reflection can be computed directly from the JPA equivalent circuit and the source impedance:

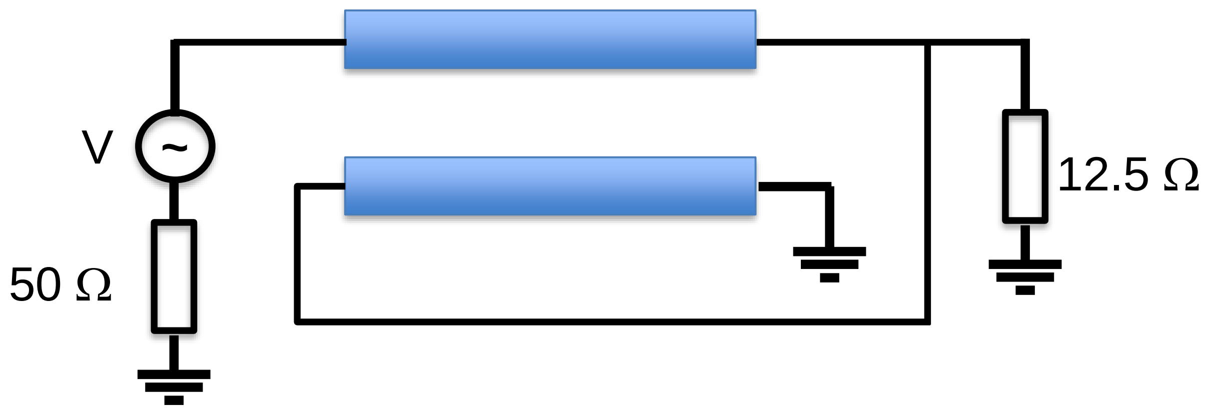
$$G \approx \frac{4[Re(Y_{ext})]^2}{\left(Y_{ext} + Y_{JPA}\right)^2}$$

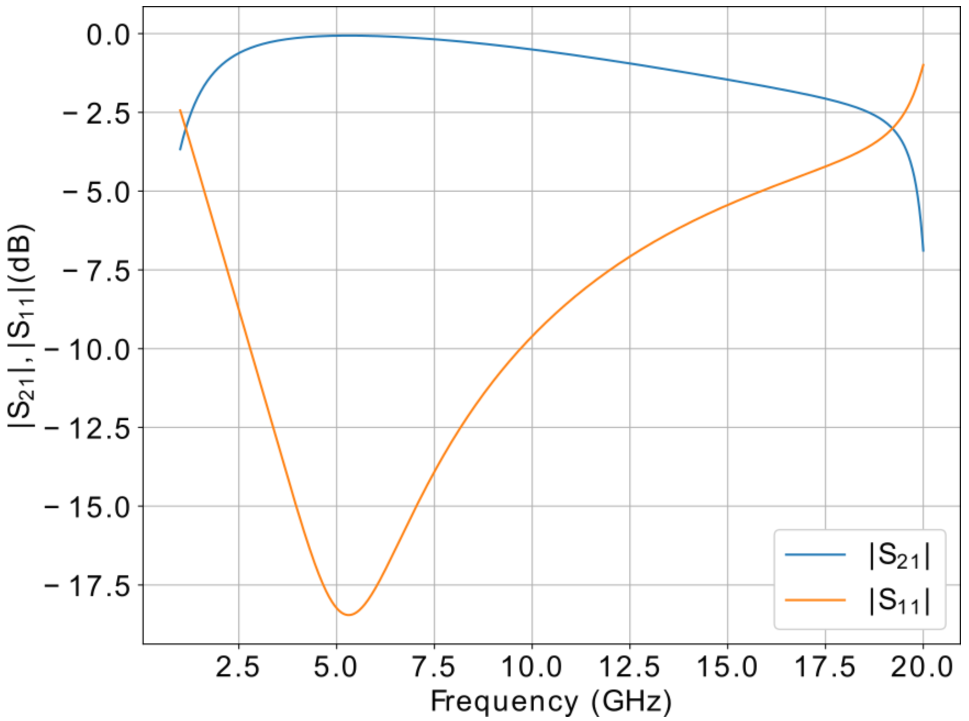
Maximum gain is achieved when $Y_{ext}(\omega_s) \approx -Y_{JPA}(\omega_s)$

$$Re(Y_{ext}) \approx -Re\left(Y_{JPA}\right) > 0$$

$$Im(Y_{ext}) = -Im\left(Y_{JPA}\right)$$

Transmission Line Transformer



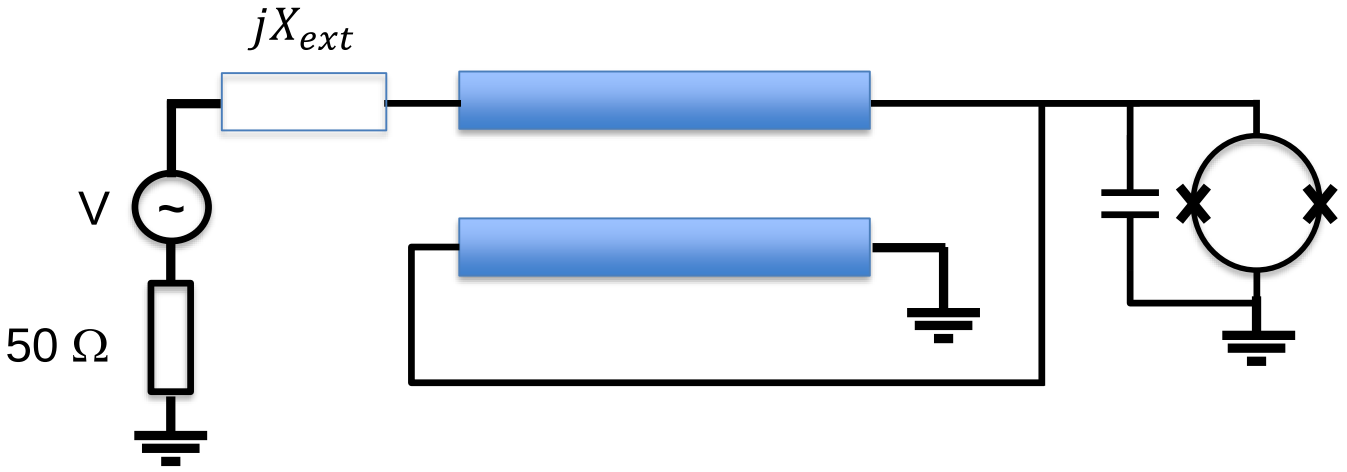


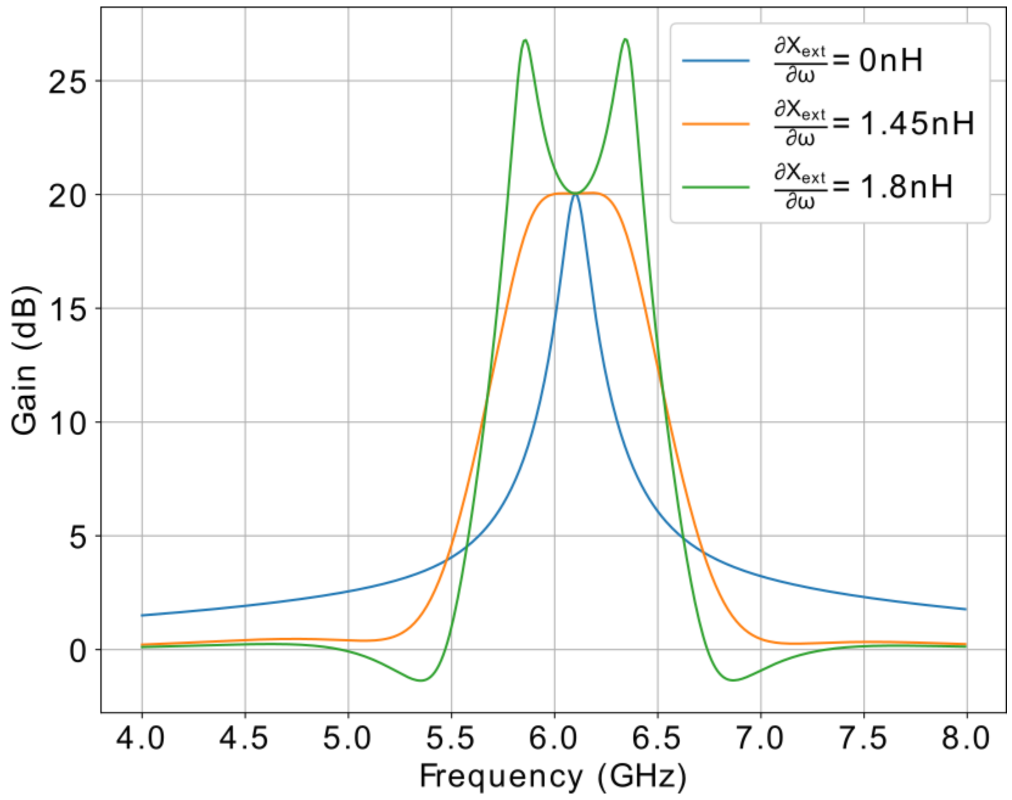
- 1-20GHz 3-dB bandwidth.
- <1dB insertion loss 2-12GHz, $Re(Z) = 6 - 12.5\ \Omega$

Flat gain with impedance matching

Transformer provides real part.

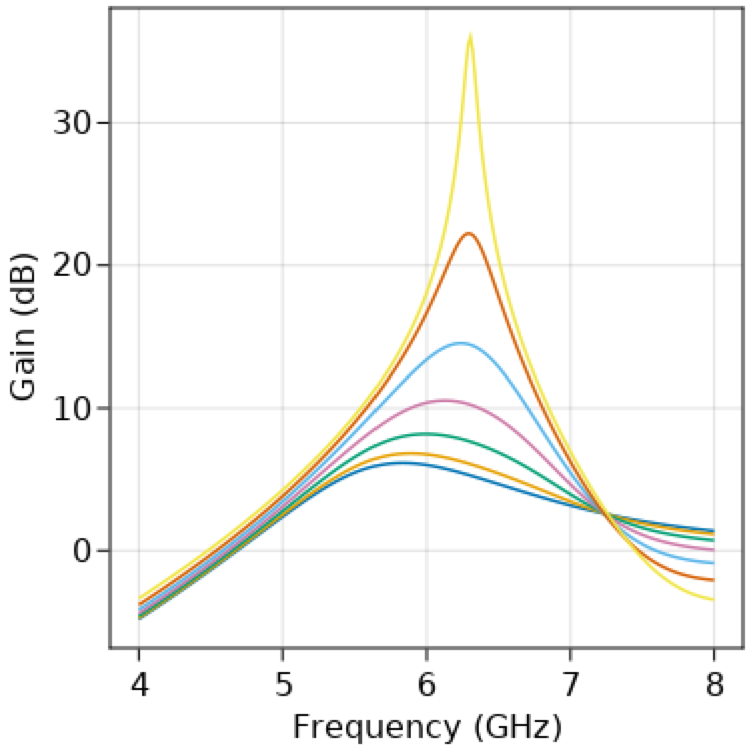
Additional imaginary part provided by input cable.



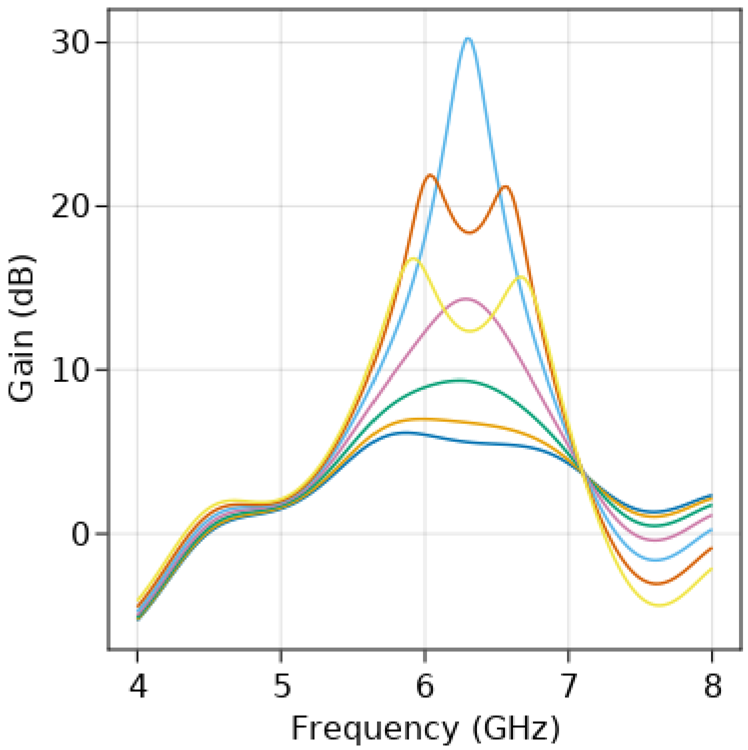


Effect of cables on wideband JPA

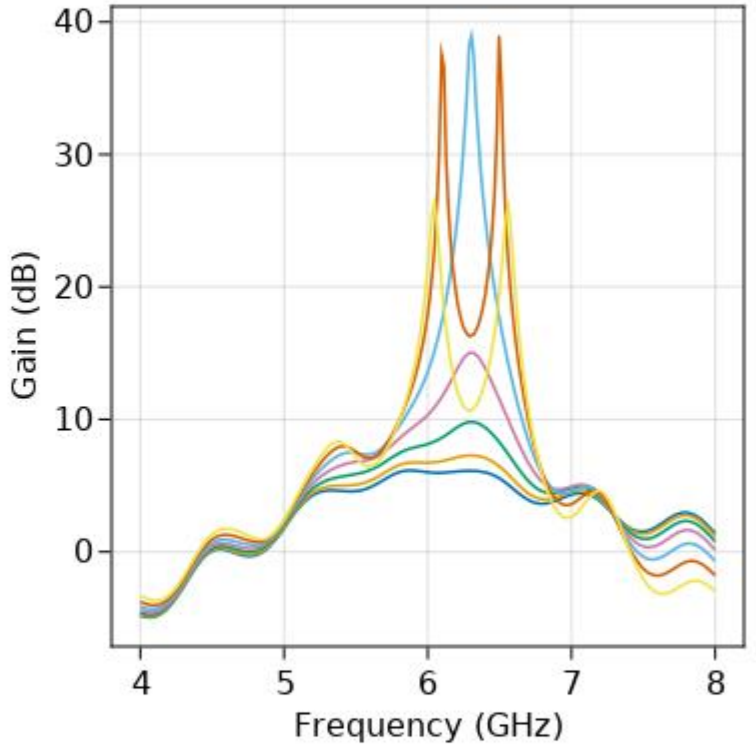
L = 0 cm



L = 9 cm

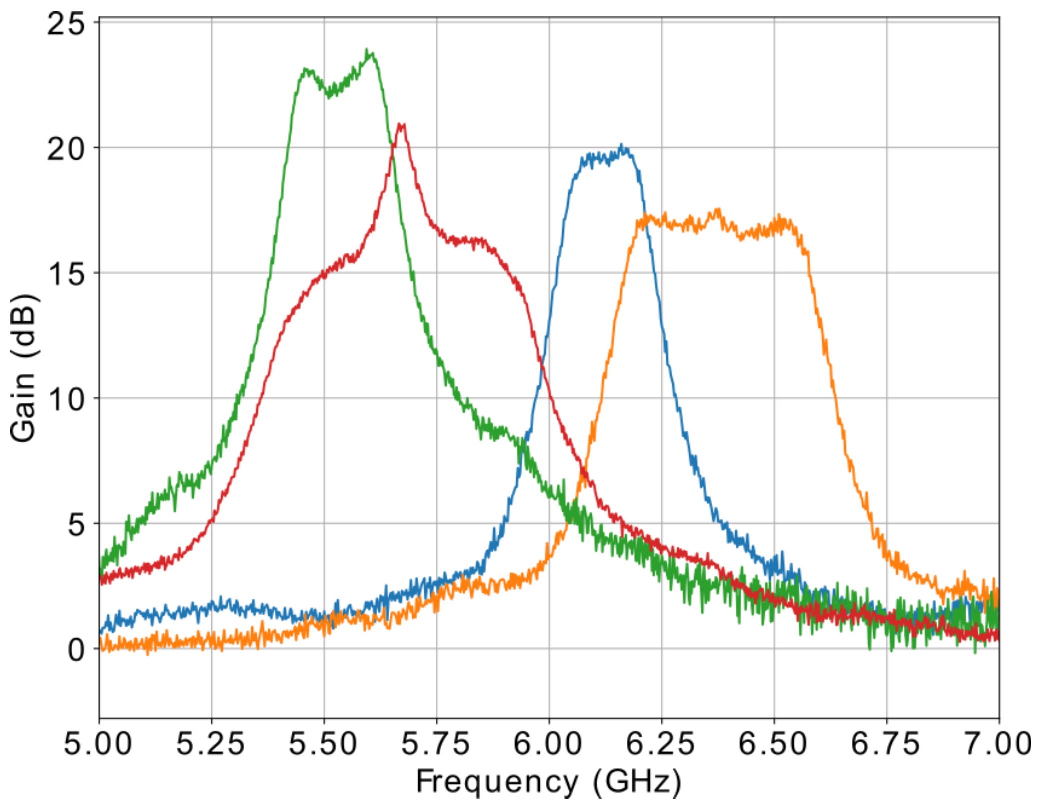


L = 15 cm



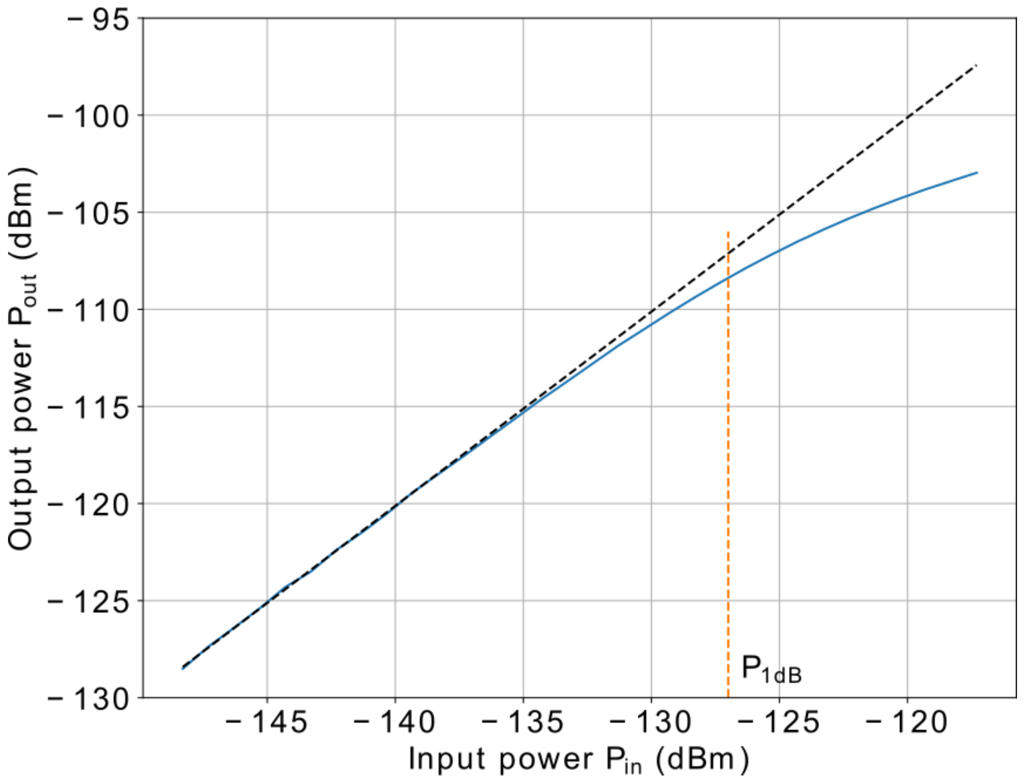
Cable can provide additional reactance

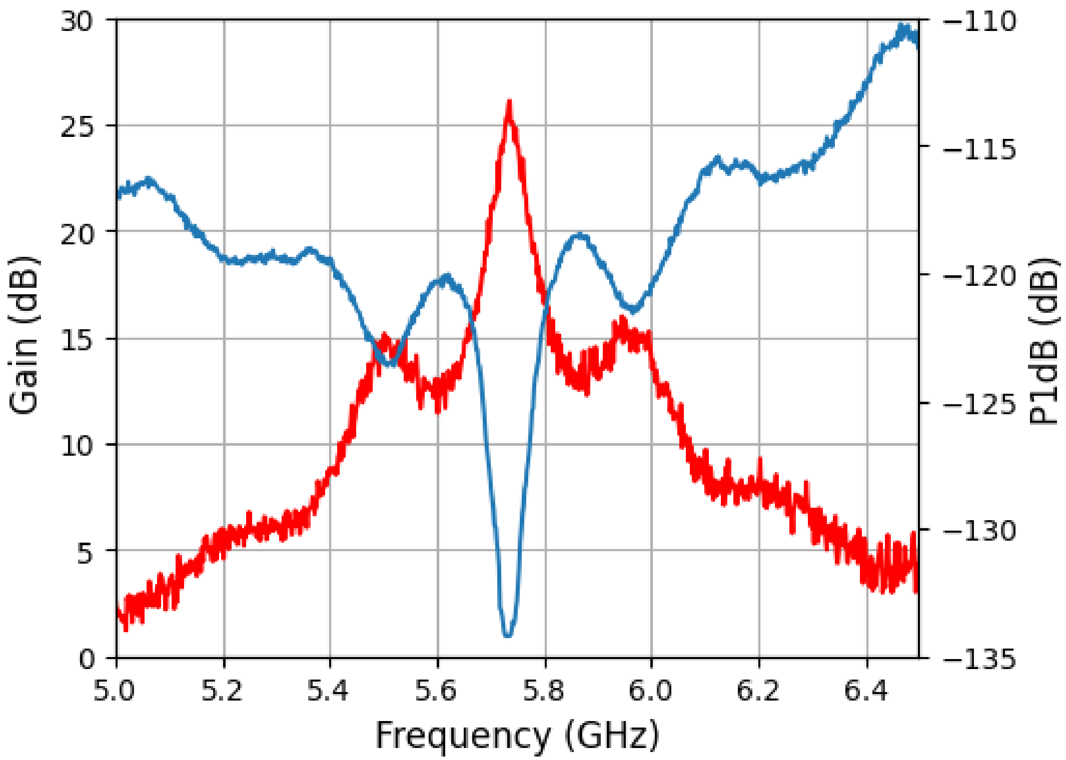
Amplifier Gain Measurements



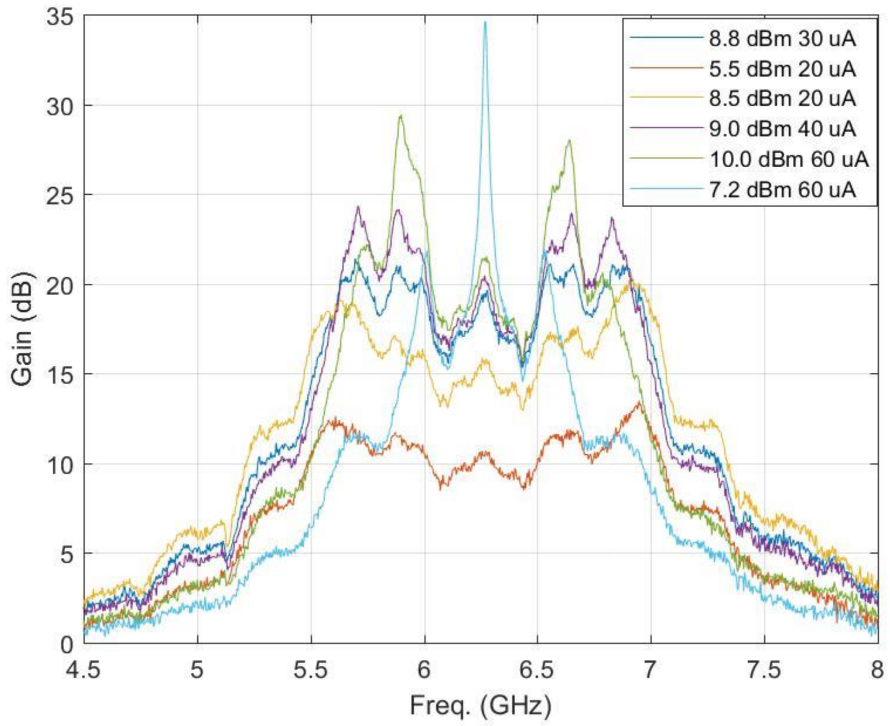
- 2-3Ghz gain-bandwidth product.
- 200MHz bandwidth @20 dB gain, 450MHz bandwidth @17 dB gain.
- Tunable in the 5-7Ghz range.
- Bandwidth about a factor of 2 smaller than simulations, probably due to unaccounted impedance mismatch.

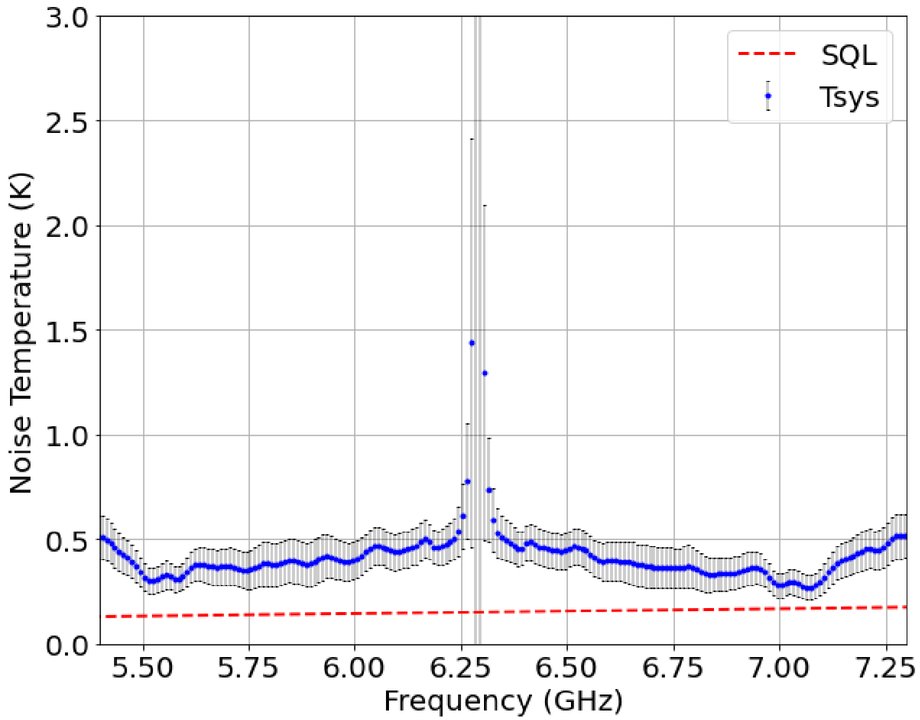
Saturation Power





Wideband JPA Noise





System noise temperature 0.5-1.1 added noise photons from 5.4GHz to 7.23GHz

Acknowledgements

G. Ribeill

B. Hassick

M. Gustafsson

KC Fong

G. Rowlands