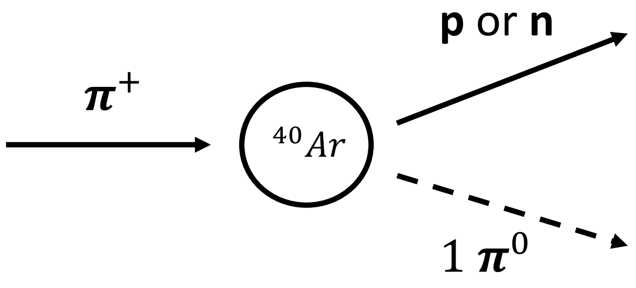
Charge Exchange Inclusive Differential Cross Section Measurement

Hadron Analysis Meeting
Kang Yang, University of Oxford
01 Sep. 2022

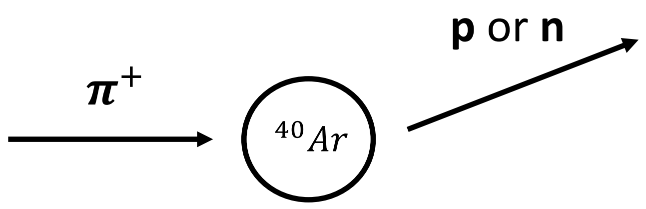
Signal & Background Definition

- 1GeV π^+ beam events in ProtoDUNE-SP.

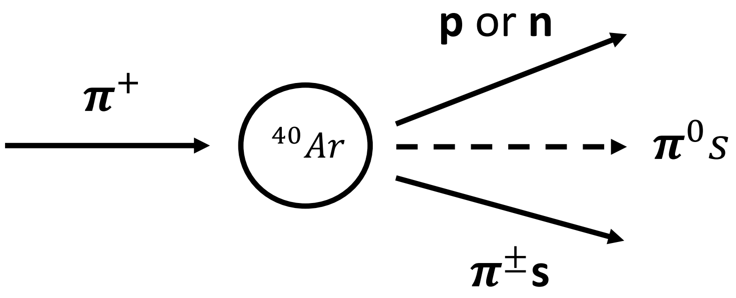
Pion Charge Exchange (signal)

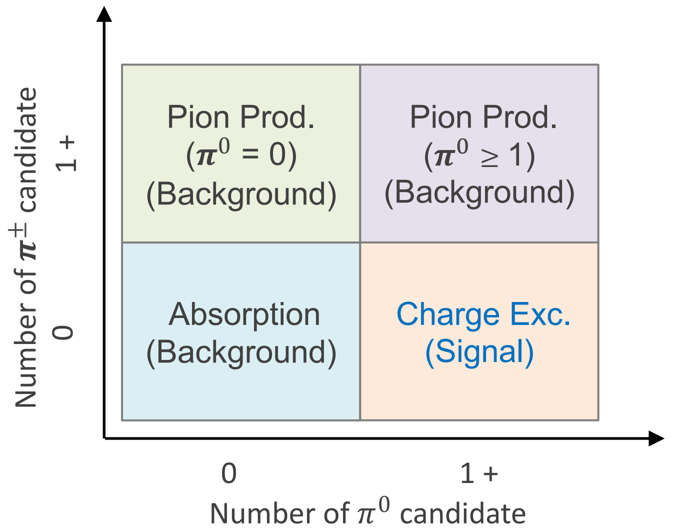


Pion Absorption (background)



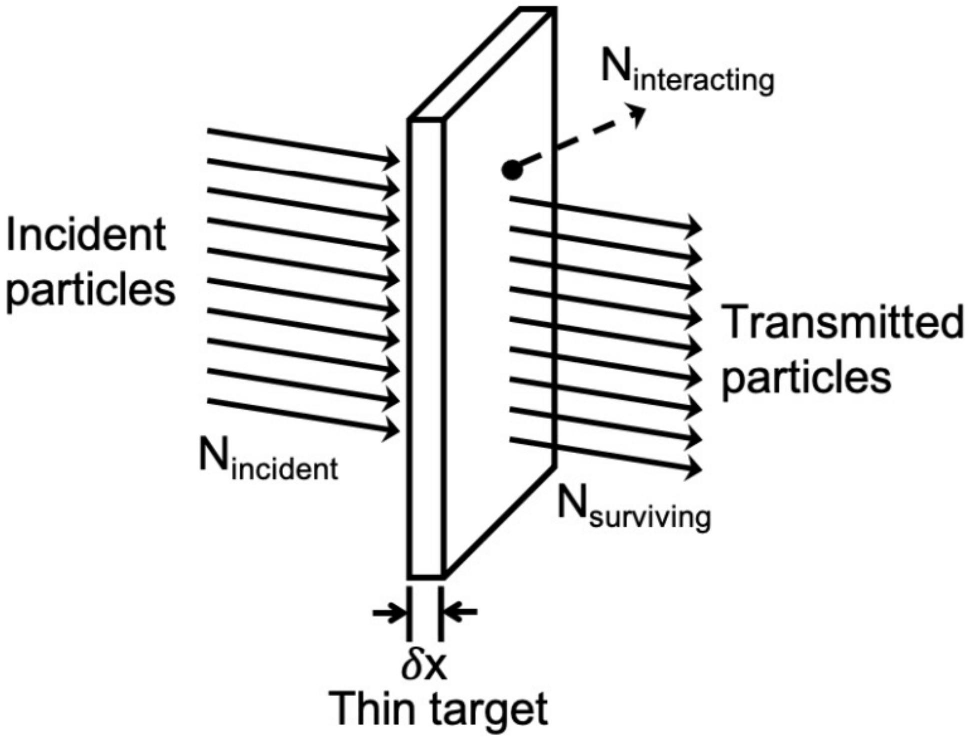
Pion Production (background)



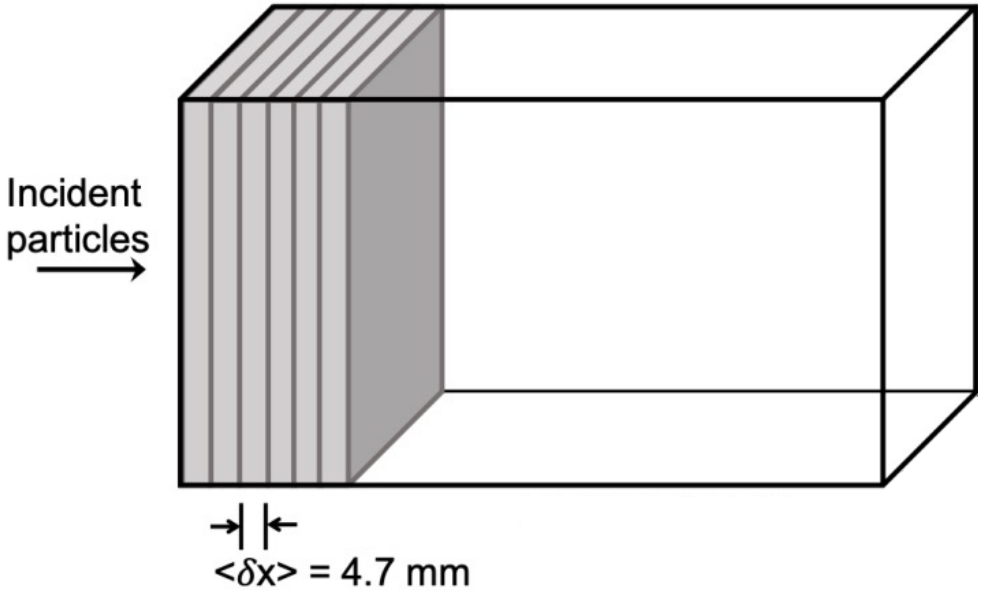


Topology	$\pi^\pm$	π^0
Pion Charge Exchange (signal)	No	Yes (only 1 π^0)
Pion Absorption (background)	No	No
Pion Production (π^0 = 0) (background)	Yes	No
Pion Production (π^0 = 1) (background)	Yes	Yes (only 1 π^0)
Pion Production (π^0 > 1) (background)	Yes	Yes (> 1 π^0)

Thin-Slice Method

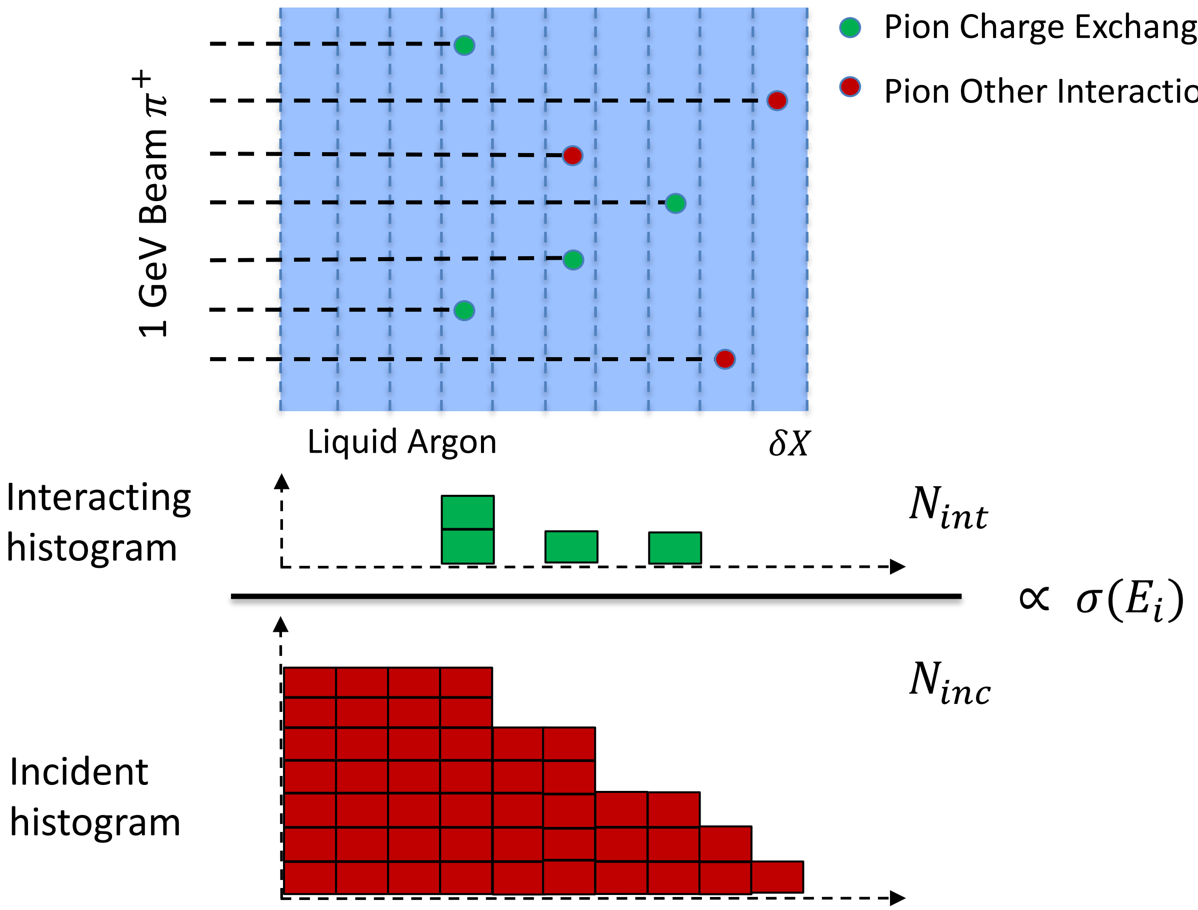


arXiv:2108.00040v1



- Interaction probability $\frac{N_{int}}{N_{inc}} = P_{Int} = 1 - e^{-\sigma_{Tot}\, n\, \delta X}$, $n = \frac{\rho N_A}{m_{Ar}}$ is the density of the target.
- The interaction length of pions in liquid argon is of the order of $\sim$ 50 cm.
- Treat the argon volume as a sequence of many adjacent thin targets.

Thin-Slice Method



$$\sigma = \frac{m_{Ar}}{\rho \delta X N_A} \ln\left(\frac{N_{inc}}{N_{inc} - N_{int}}\right)$$

- m_{Ar} is the mass of argon atom
- N_A is the Avogadro constant
- ρ is the density of liquid argon
- δX is the thickness of the slice
- N_{inc} is the number of incident beam pions in a slice
- N_{int} is the number of beam pions which have interactions in a slice

Francesca's thesis

Method Validation (Truth Level)

Method Validation in Truth Level

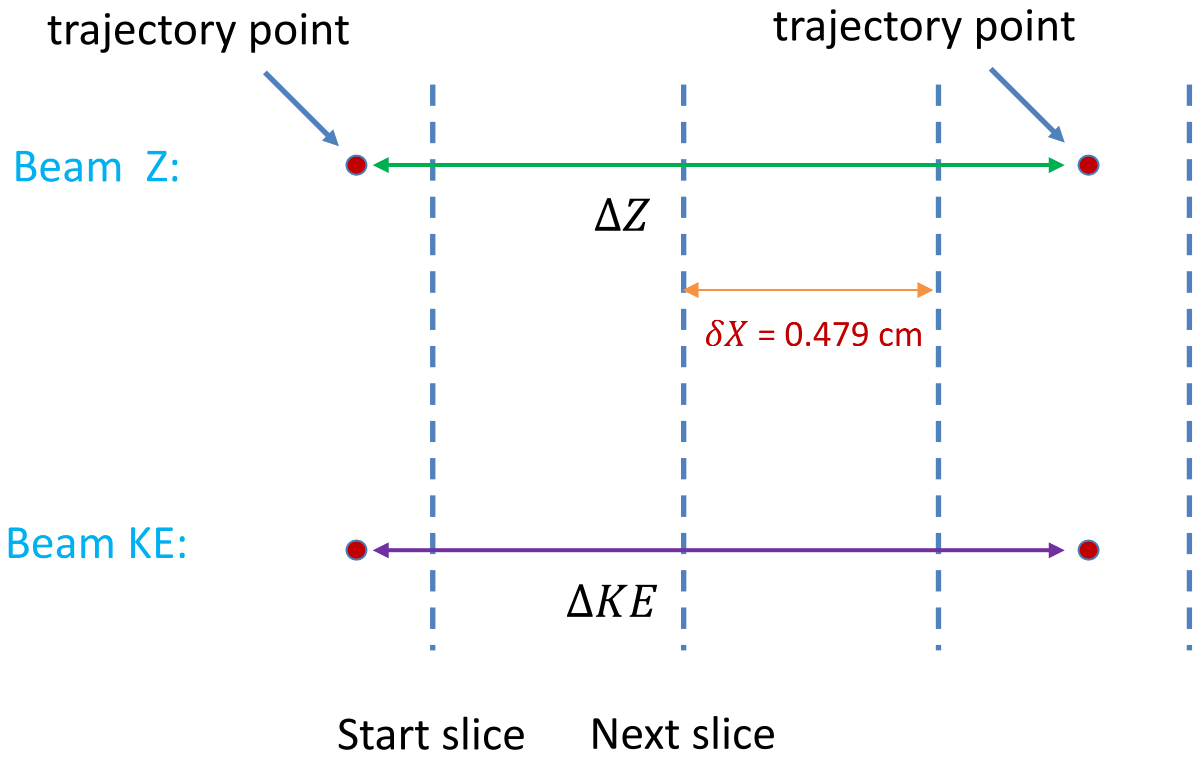
- N_{inc} and N_{int} are two 1D histogram in terms of beam pion kinetic energy (KE or T).

- Two PDSP Analyzer variables are used to fill the histograms:
  - true_beam_traj_Z (true beam trajectory points - z coordinates)
  - true_beam_traj_KE (true beam trajectory points – kinetic energy)

- Spatial slicing δX = 0.479 cm - wire spacing/pitch

- Each histogram has a range of 0 – 1000 MeV with a bin width of 50 MeV

Method Validation in Truth Level

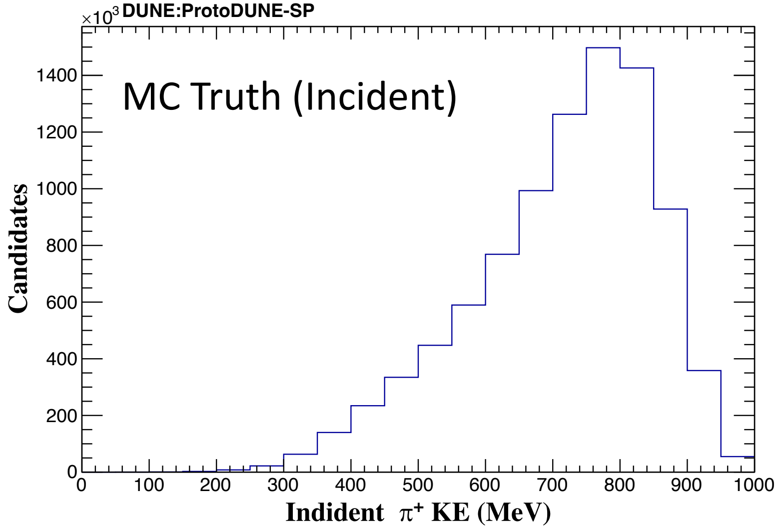


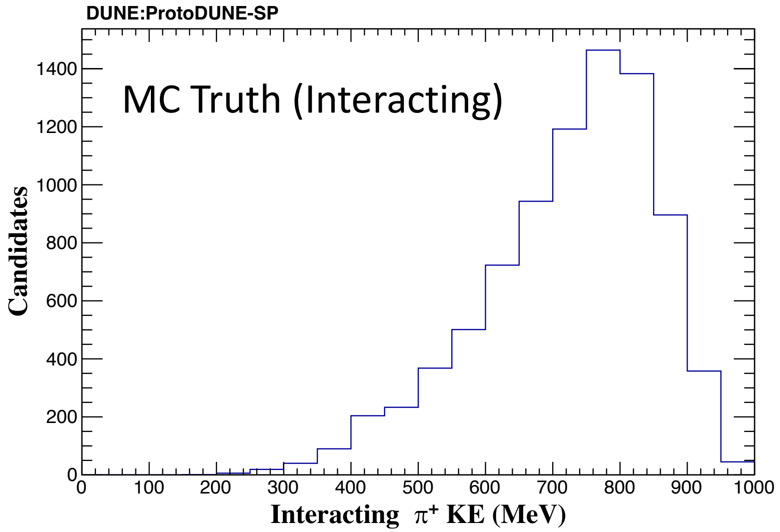
- Only include trajectory points starting in the active volume
- Only include trajectory points less than slice 464 ≈ 222 cm (the end of APA3)
- Loop over all trajectory points
- Calculate ΔZ and ΔKE between adjacent points
- Use $\frac{\delta X}{\Delta Z} = \frac{\delta E}{\Delta KE}$ to get energy slices

- Obtain a new beam KE vector with wire pitch (0.479 cm) slicing
- If the last trajectory point ends within the APA3, the beam interacting energy is assigned using the last element of the new beam KE vector.

Total Truth Cross Section σ_{CEX}

- If beam PDG is 211, then loop over the new KE vector and fill N_{inc} histogram.
- If beam end process is "pi+Inelastic" and with charge exchange topology, then fill N_{int} histogram with the last element of the new KE vector .
- For each bin i, the cross section is calculated as:
- $\sigma_i = \frac{m_{Ar}}{\rho \delta X N_A} \ln\left(\frac{N_{inc}}{N_{inc}-N_{int}}\right)_i$
- m_{Ar}= 39.95 g/mol, ρ = 1.39 g/cm^3
- N_A = 6.02 *10^23 1/mol, δX = 0.479 cm

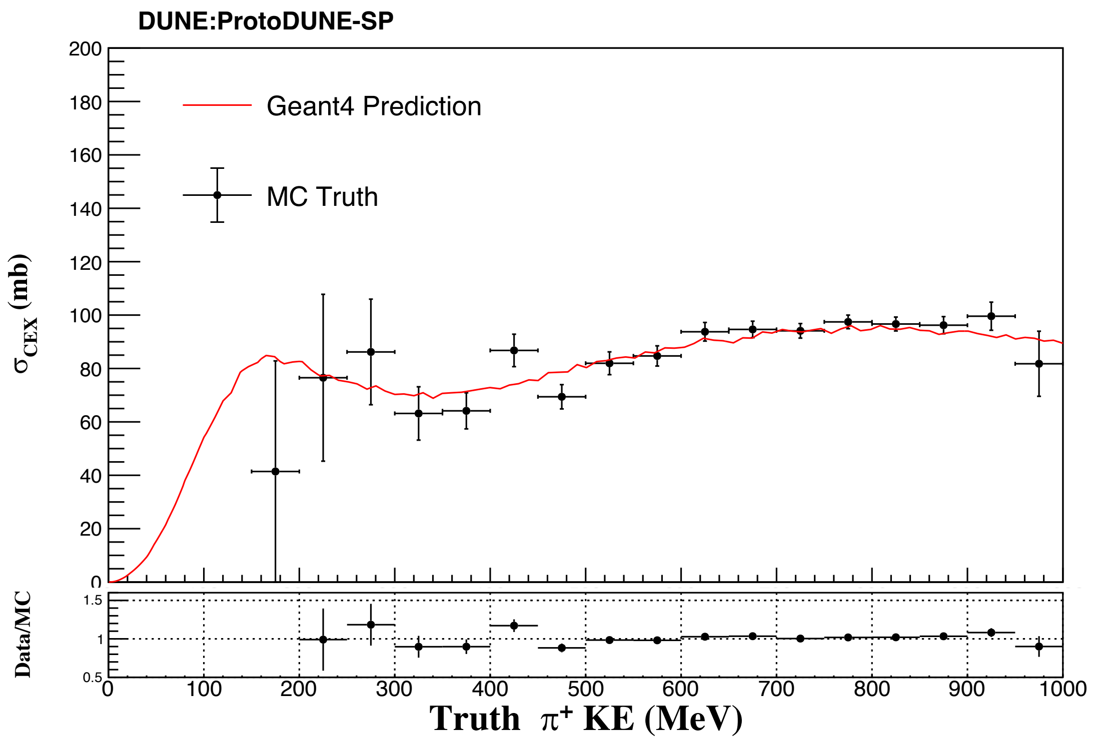




WARWICK THE UNIVERSITY OF WARWICK UNIVERSITY OF OXFORD DUNE

Total Truth Cross Section σ_{CEX}

- If beam PDG is 211, then loop over the new KE vector and fill N_{inc} histogram.
- If beam end process is "pi+Inelastic" and with charge exchange topology, then fill N_{int} histogram with the last element of the new KE vector .
- For each bin i, the cross section is calculated as:
- $\sigma_i = \frac{m_{Ar}}{\rho \delta X N_A} \ln\left(\frac{N_{inc}}{N_{inc} - N_{int}}\right)_i$
- m_{Ar}= 39.95 g/mol, ρ = 1.39 g/cm^3
- N_A = 6.02 *10^23 1/mol, δX = 0.479 cm



Differential Cross Section Formula

$$\sigma_i \approx \frac{m_{Ar}}{\rho \delta X N_A} \frac{N_{int}^{i}}{N_{inc}^{i}}$$

- Calculate the differential cross section as,

$$\left(\frac{d\sigma}{dT_{\pi^0}}\right)_{ij} = \frac{m_{Ar}}{\rho \delta X N_A} \frac{1}{(\Delta T_{\pi^0})} \frac{N_{int}^{ij}}{N_{inc}^{i}} \quad \text{(Eq. 1)}$$

ΔT_{π^0} is the bin width of π^0 KE

Index i : beam T_{π^+} bin,

Index j : daughter T_{π^0} bin

- Thin slice total CEX cross section is,

$$\sigma_i = \frac{m_{Ar}}{\rho \delta X N_A} \frac{N_{int}^{i}}{N_{inc}^{i}} \quad \text{(Eq. 2)}$$

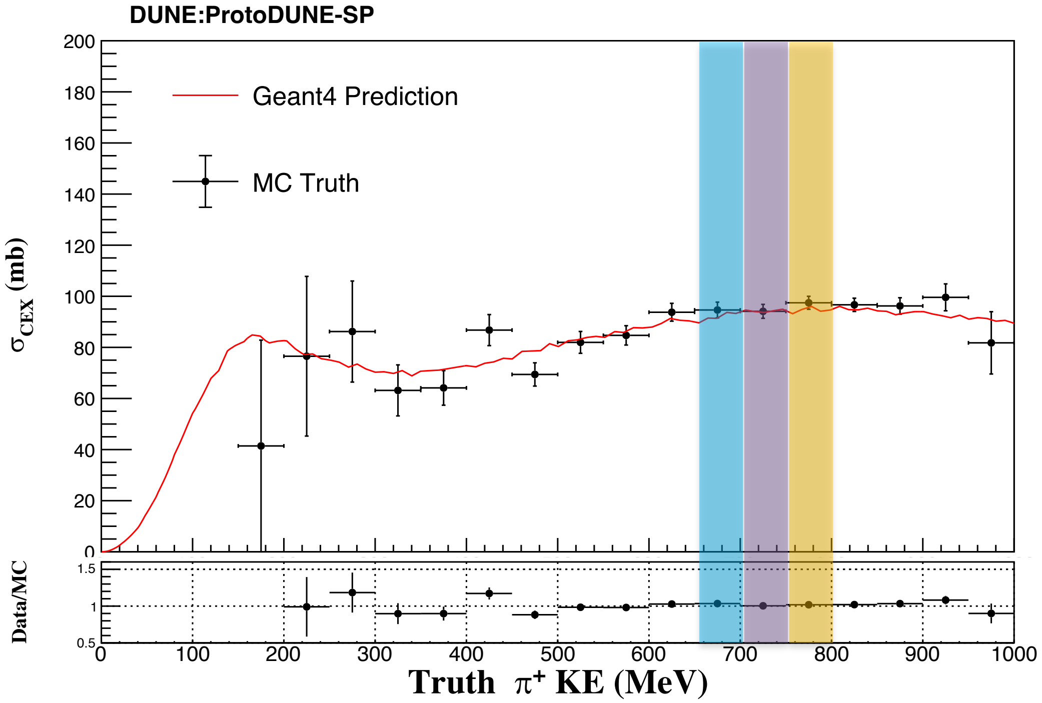
- Then the differential cross section formula is (Sub. Eq. 2 into Eq. 1),

$$\left(\frac{d\sigma}{dT_{\pi^0}}\right)_{ij} = \frac{1}{(\Delta T_{\pi^0})} \frac{N_{int}^{ij}}{N_{int}^{i}} \sigma_i$$

Thank you Jonathon Sensenig for useful discussions!

GEANT4 and Truth $d\sigma$ Calculation

- The Geant4Rewight package is used to extract the differential cross section, $\frac{d\sigma}{dT}$
- Measure differential XS at each bin i (for a given pion KE).



- Pion KE = 675 MeV → Evaluate $\sigma_{KE=675\ MeV}$
- Pion KE = 775 MeV → Evaluate $\sigma_{KE=775\ MeV}$
- Pion KE = 875 MeV → Evaluate $\sigma_{KE=875\ MeV}$

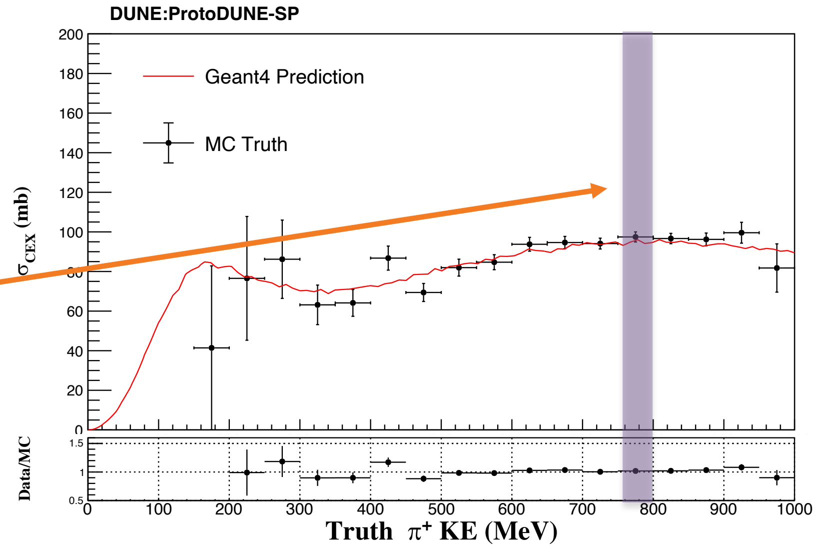
- Differential XS for bin j (daughter T_{π^0} bin) is,

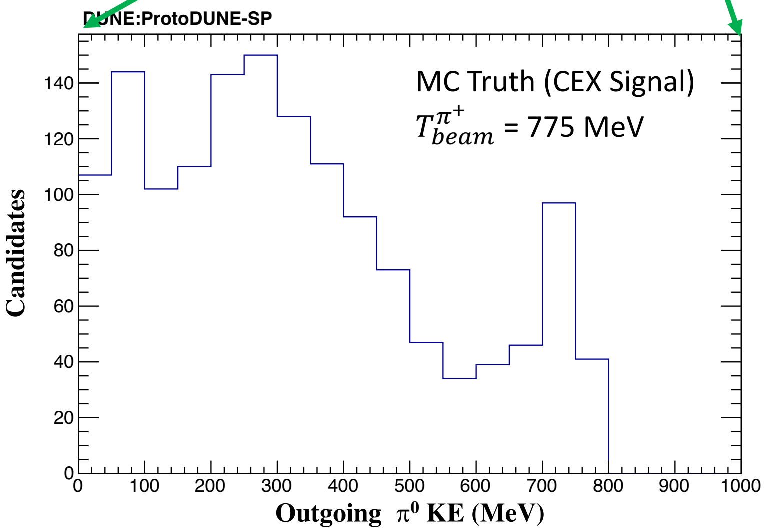
$$\left(\frac{d\sigma}{dT_{\pi^0}}\right)_j = \frac{1}{(\Delta T_{\pi^0})} \frac{N_{int}^{j}}{N_{int}} \sigma_{KE=775\ MeV}$$

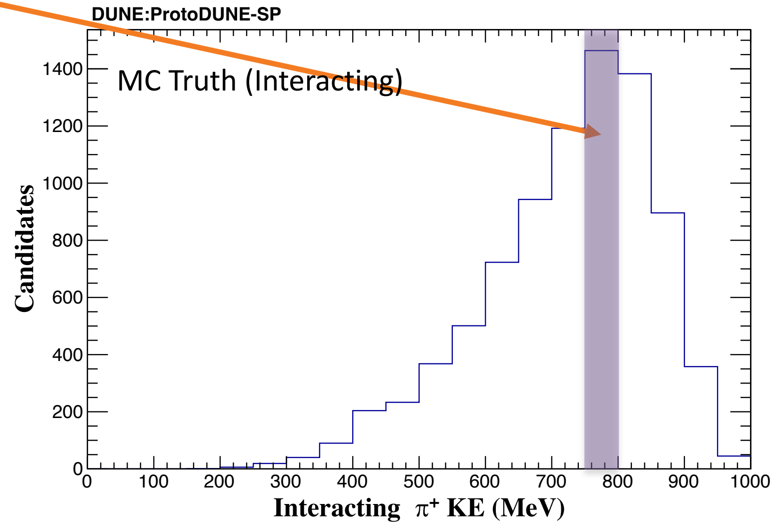
dσ Validation in Truth Level

- Differential XS for bin j (daughter T_{π^0} bin) with $T^{\pi^+}_{beam}$ = 775 MeV is,

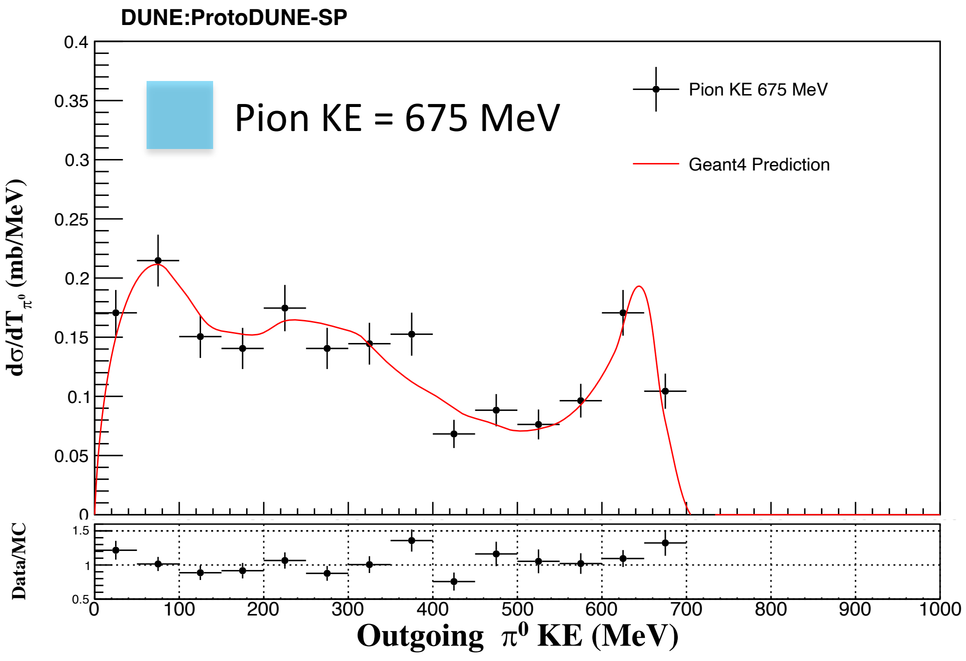
$$\left(\frac{d\sigma}{dT_{\pi^0}}\right)_j = \frac{1}{(\Delta T_{\pi^0})} \frac{N^j_{int}}{N_{int}} \sigma_{KE=775\ MeV}$$

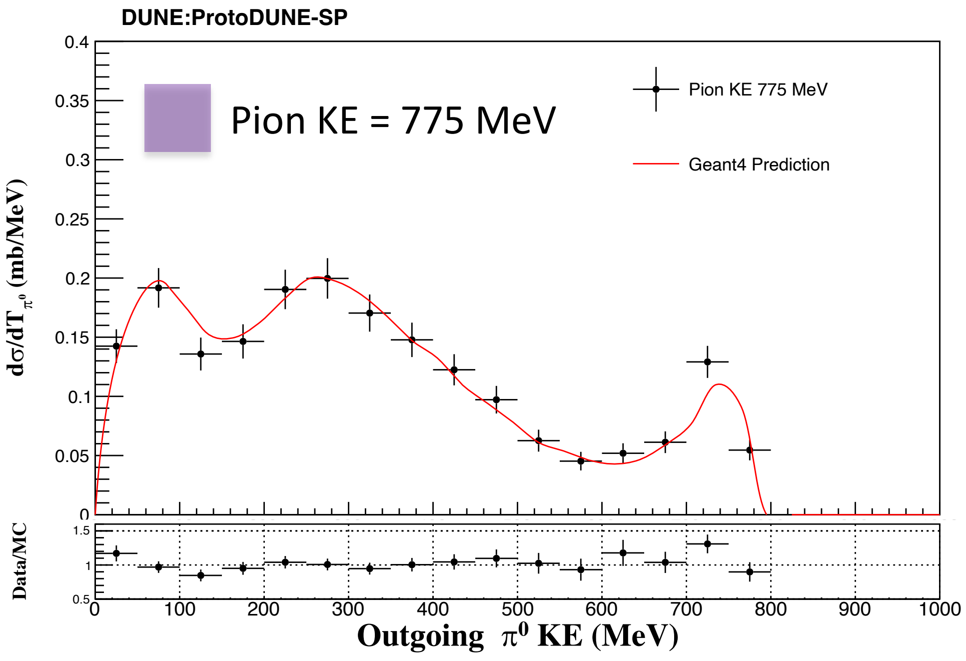






dσ Validation in Truth Level

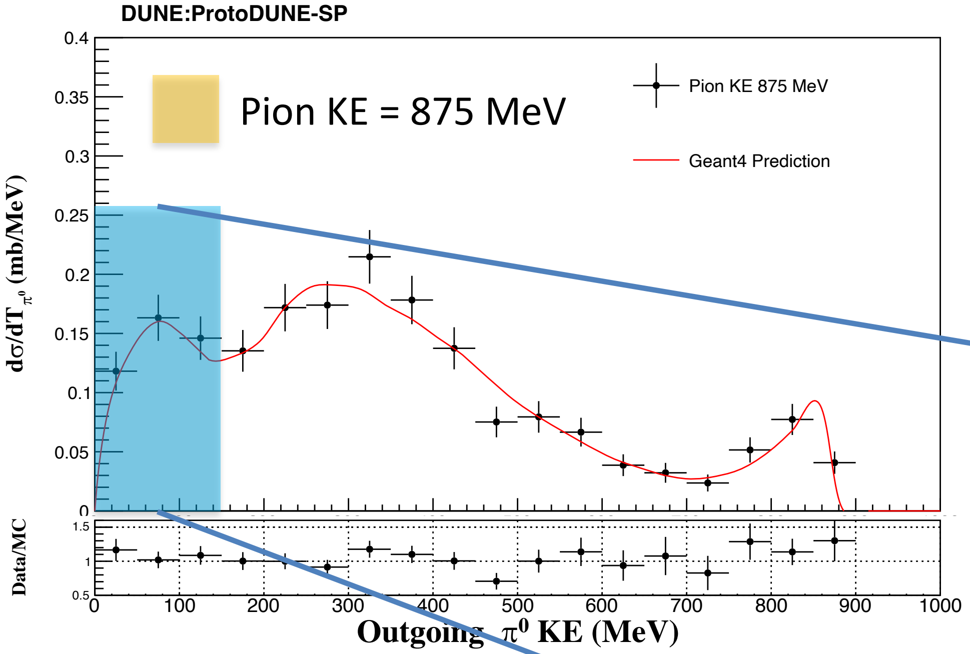


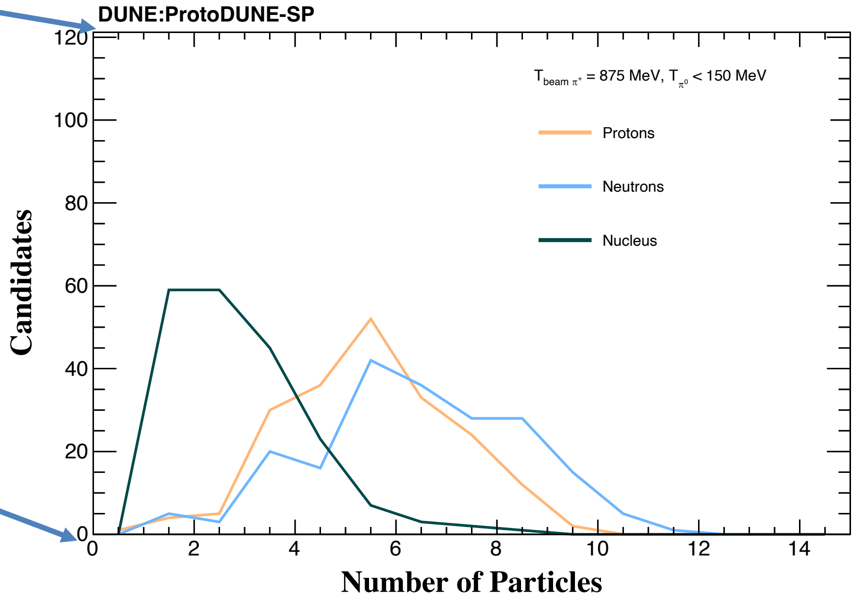


- The differential cross section formula works well in truth level.
- There are three peaks in the differential cross section distribution.

Low T_{π^0} Region

- Nucleus is defined as PDG > 2212, such as a deuteron, He^3, He^4 and nucleus.
- Many daughter nucleons and 1 π^0.
- The direction of π^0 is arbitrary.



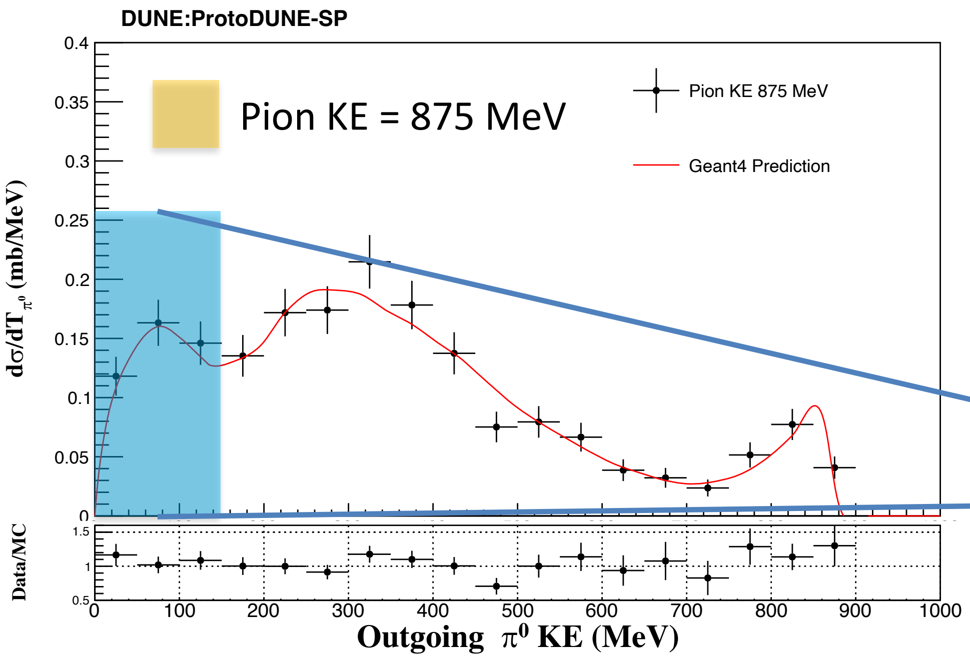


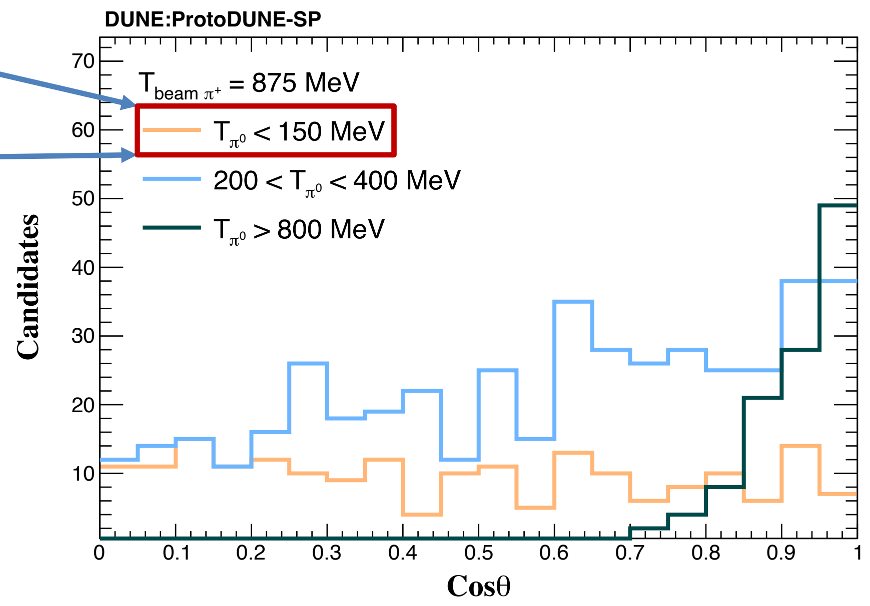
3 nucleus

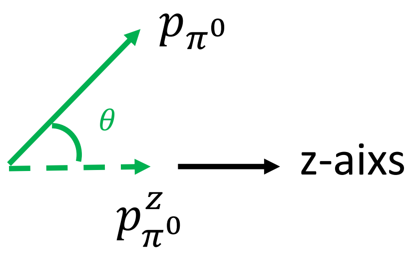
$$\pi^+ + {}^{40}_{18}Ar \rightarrow {}^{24}_{12}Mg + {}^{2}_{1}D + {}^{2}_{1}D + 5p + 7n + \pi^0$$

Low T_{π^0} Region

- Nucleus is defined as PDG > 2212, such as a deuteron, He^3, He^4 and nucleus.
- Many daughter nucleons and 1 π^0.
- The direction of π^0 is arbitrary.

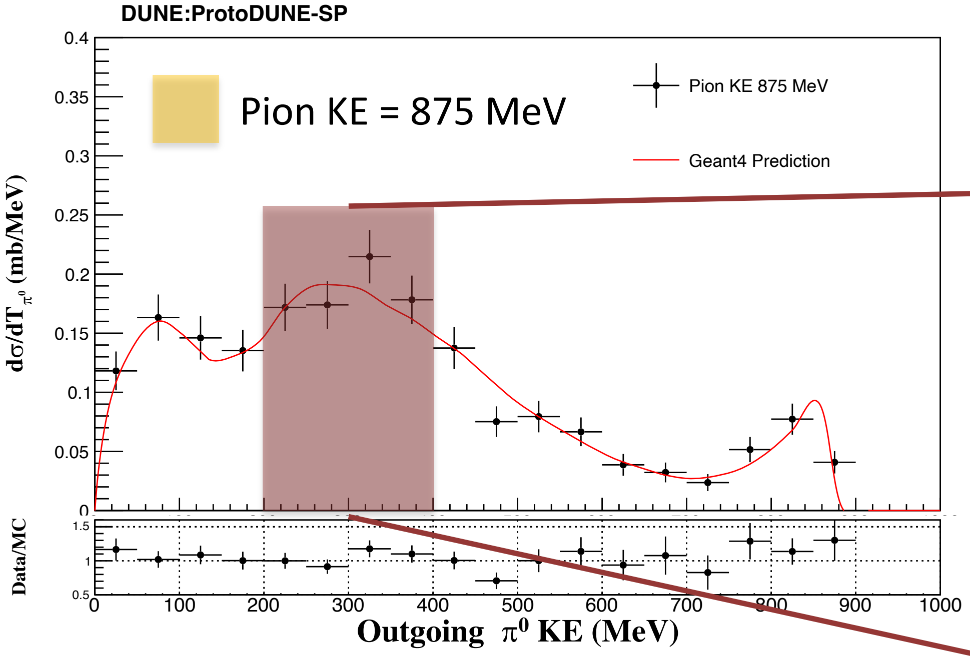


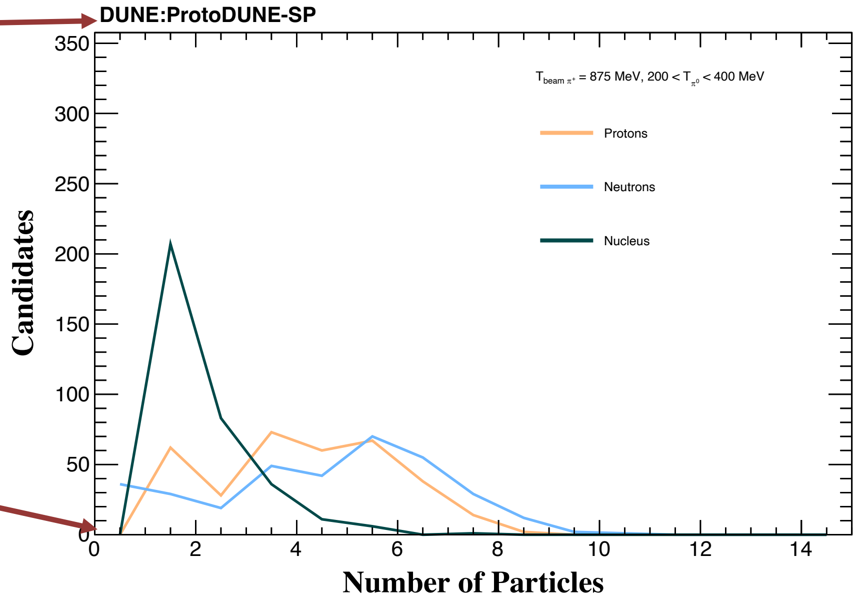




Middle T_{π^0} Region

- Less nucleons are produced.



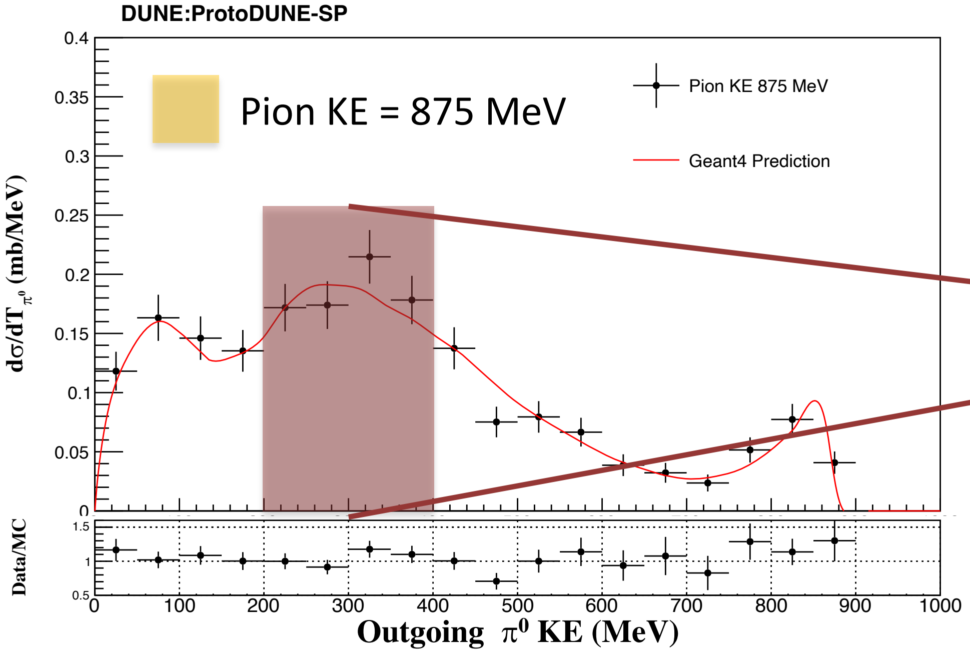


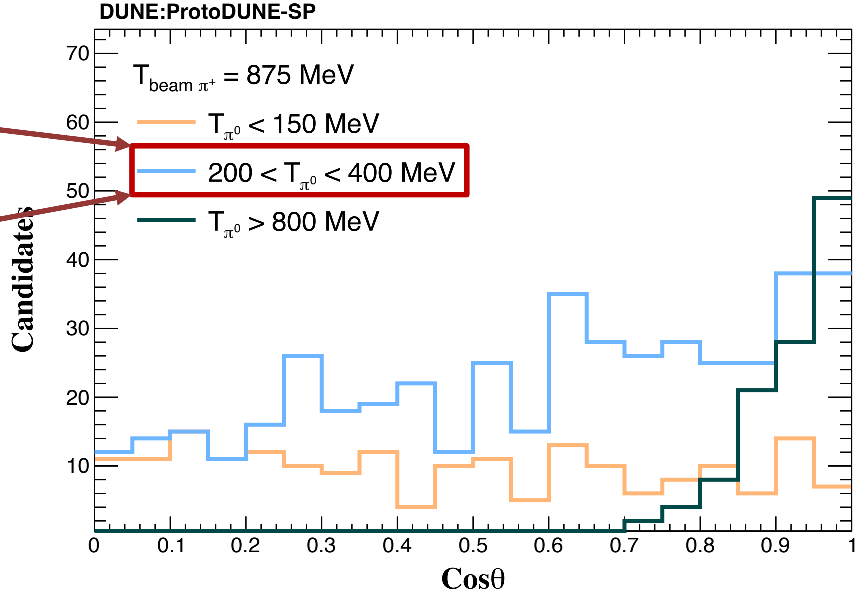
2 nucleus

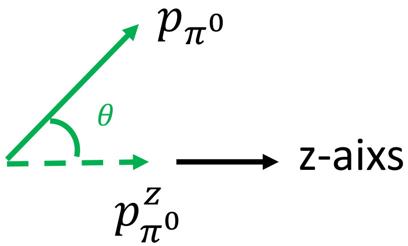
$$\pi^+ + {}^{40}_{18}Ar \rightarrow {}^{28}_{14}Si + {}^{2}_{1}D + 4p + 6n + \pi^0$$

Middle T_{π^0} Region

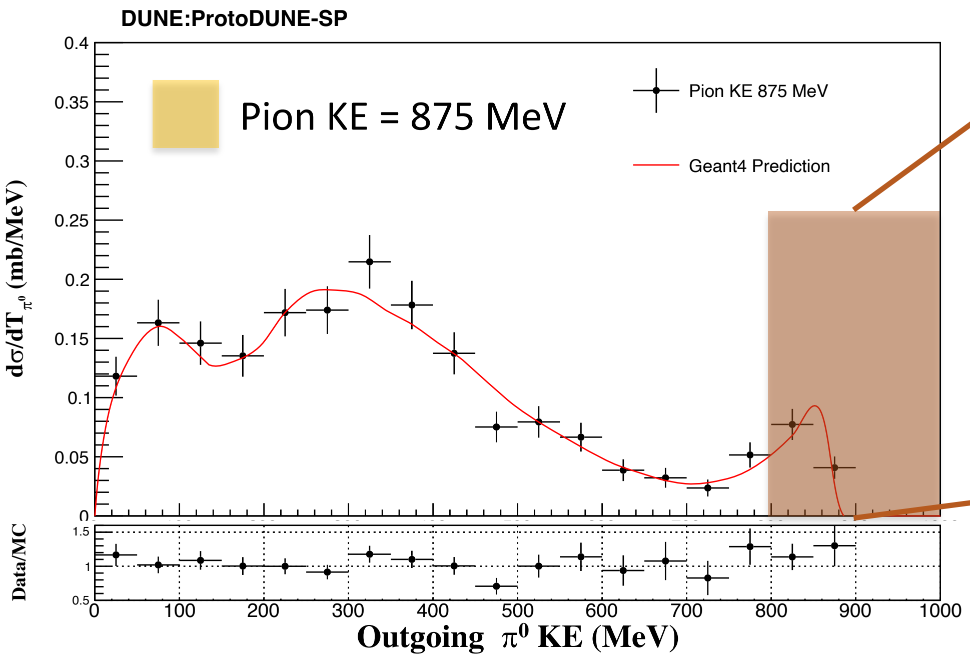
- Less nucleons are produced.

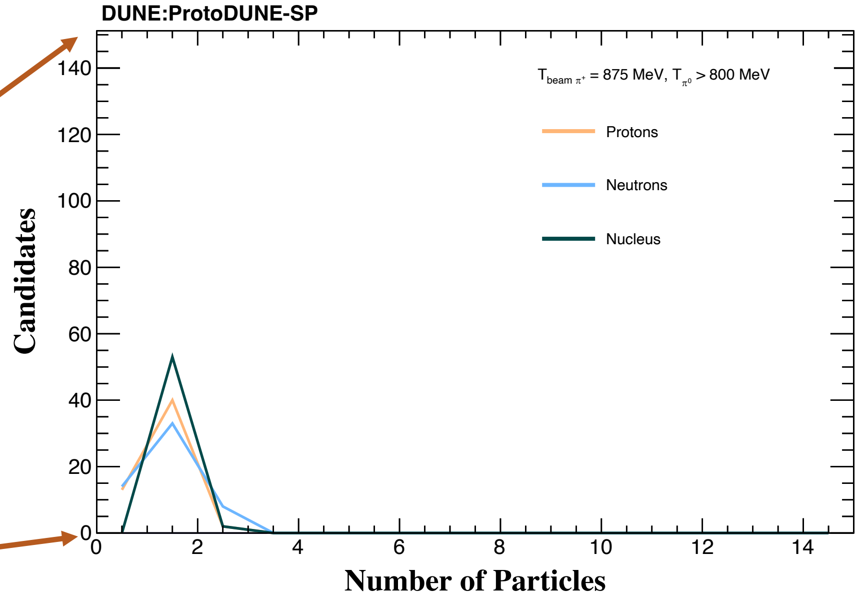






High T_{π^0} Region

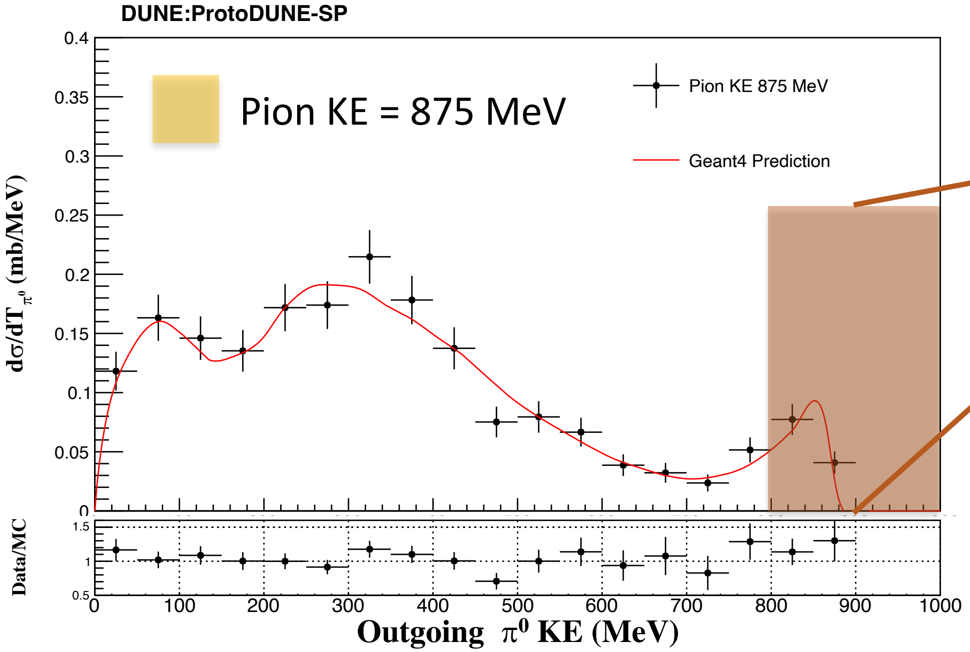


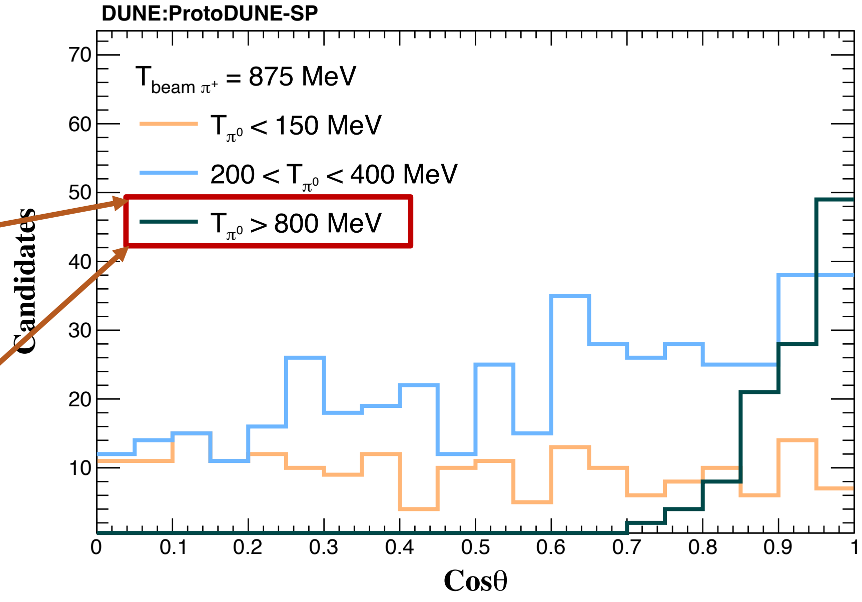


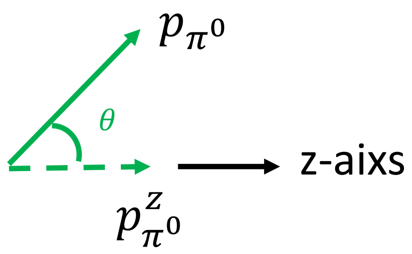
$$\pi^+ + {}^{40}_{18}Ar \rightarrow {}^{38}_{18}Ar + \text{p} + \text{n} + \pi^0$$

- Only one Argon isotope (such as Ar^{39}, Ar^{38}) is produced.
- One or two nucleons and 1 π^0.
- The direction of π^0 is forward.

High T_{π^0} Region







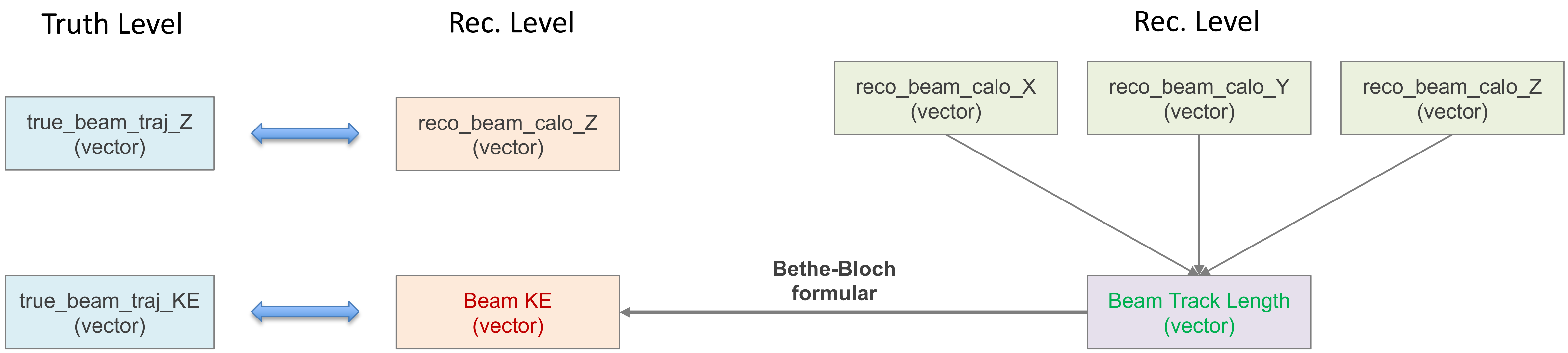
- Only one Argon isotope (such as Ar^{39}, Ar^{38}) is produced.
- One or two nucleons and 1 π^0.
- The direction of π^0 is forward.

Fake Data Test (Rec. Level)

Fake Data

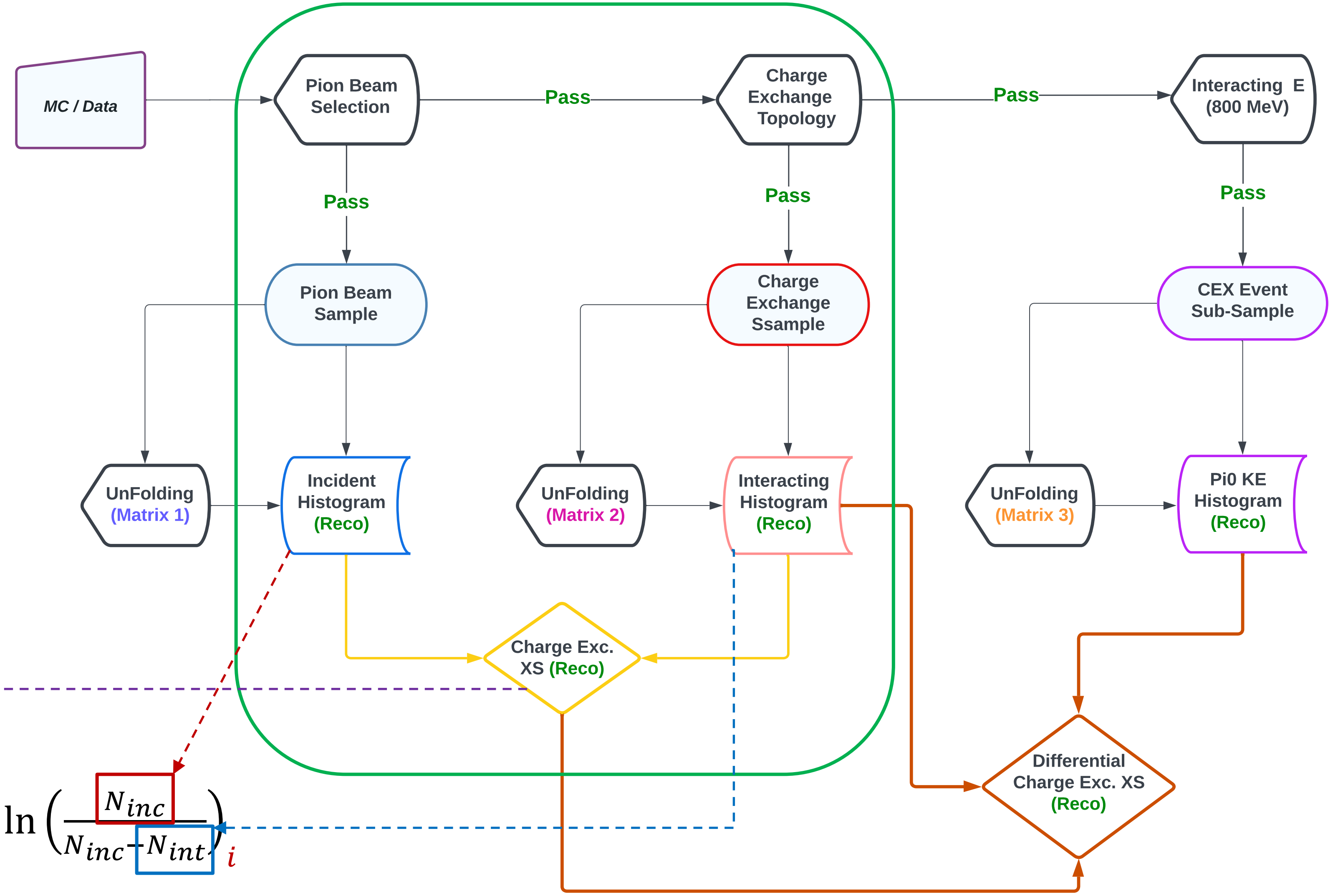
- Full 1GeV Pion MC sample is divided into 2 sub-samples.
  1. Fake Data
  2. MC
- MC sample is used to train the mapping response matrix for unfolding.
- Fake Data is used to validate the cross section extraction procedures.
- The unfolding is done using RooUnfold package (iterative Bayesian method, 4 iteration)

Rec. Beam Kinetic Energy



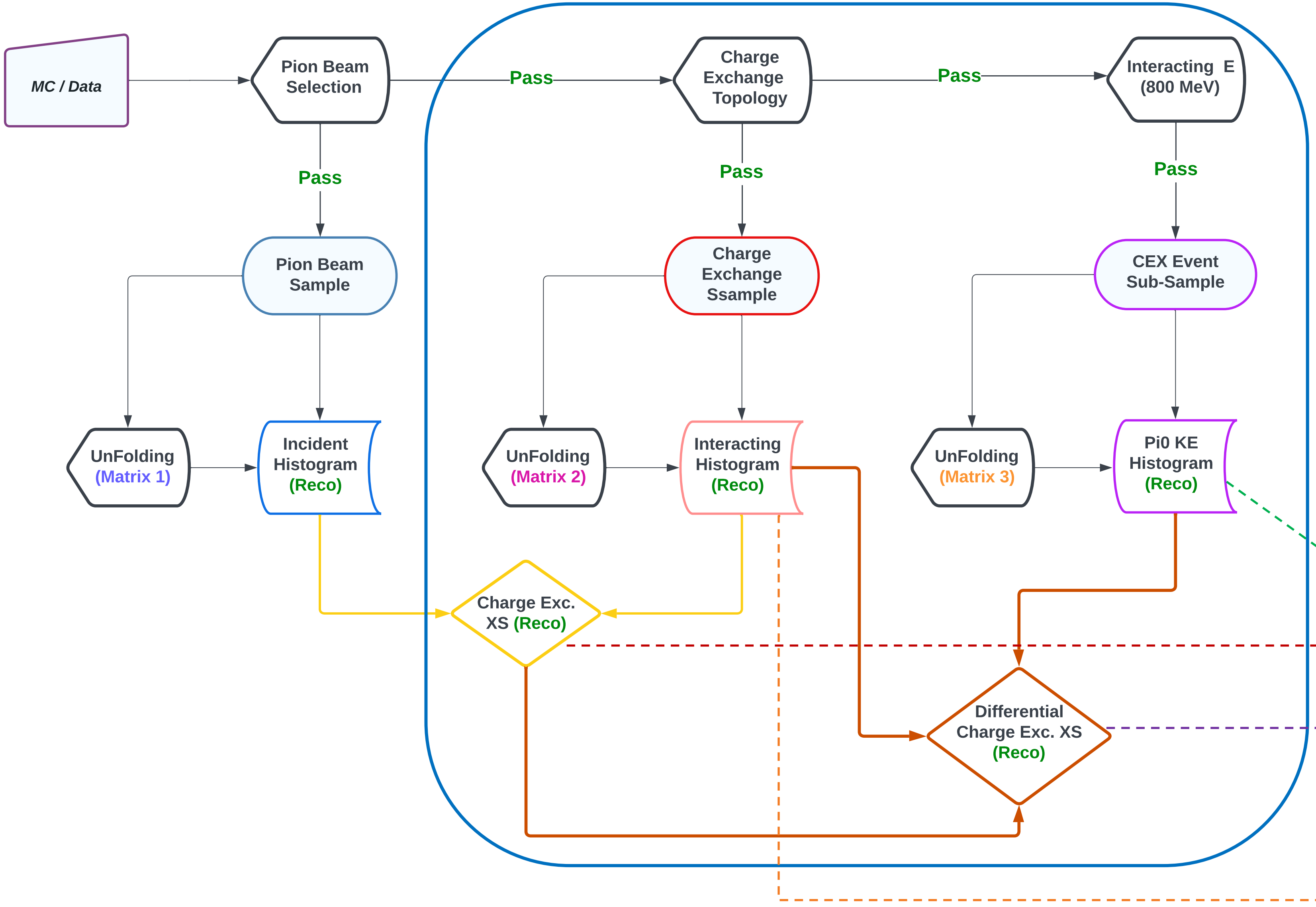
- There is no PDSP Analyzer variable to describe the kinetic energy of the beam pion.
- The beam track length is calculated using XYZ positions at each space point.
- Convert the track length to kinetic energy using pion assumption in the Bethe-Bloch formular.

Analysis Flowchart



$$\sigma_i = \frac{m_{Ar}}{\rho \delta X N_A} \ln\left(\frac{N_{inc}}{N_{inc} - N_{int}}\right)_i$$

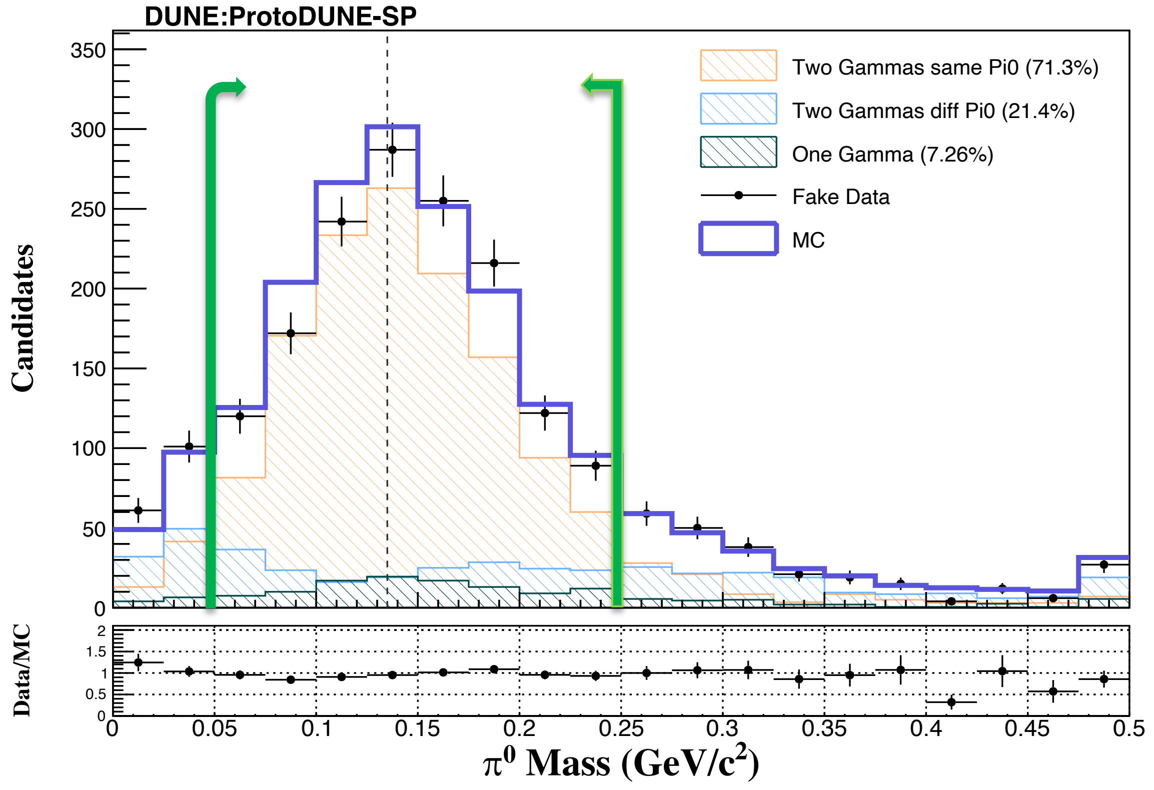
Analysis Flowchart



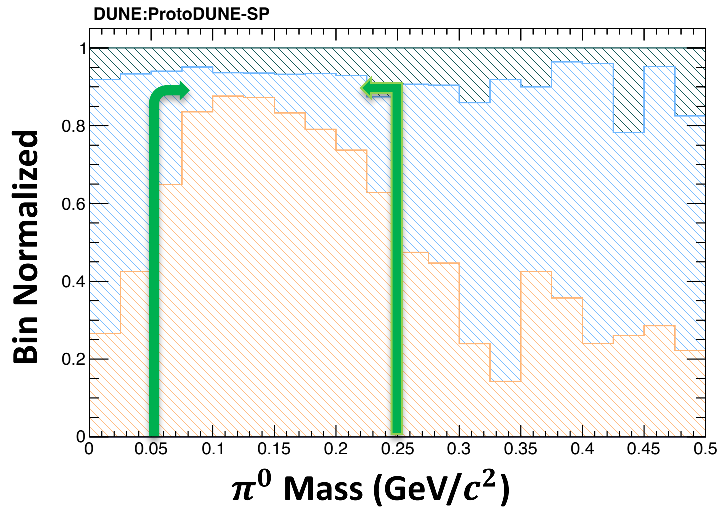
$$\left(\frac{d\sigma}{dT_{\pi^0}}\right)_j = \frac{1}{(\Delta T_{\pi^0})} \frac{N^j_{int}}{N_{int}} \sigma_{KE=775\ MeV}$$

- The final state π^0 need to be reconstructed.
- The reconstructed π^0 mass is used to reduce background.
- The kinematic fitting is used to improve the reconstructed π^0 energy.

Reconstructed π^0 Mass



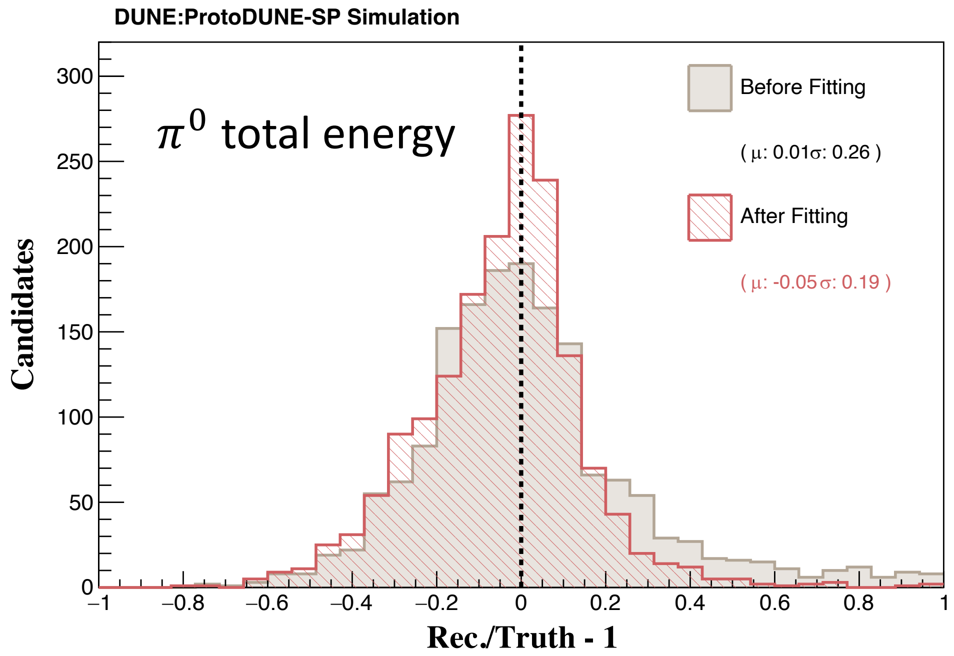
- Truth π^0 Mass is indicated by the dash line.
- The invariant mass peak from the signal (two gammas from same π^0) looks good!
- There is a continuous background from two gammas coming from different π^0 .
- One Gamma background means there is at least one non-photon candidate.

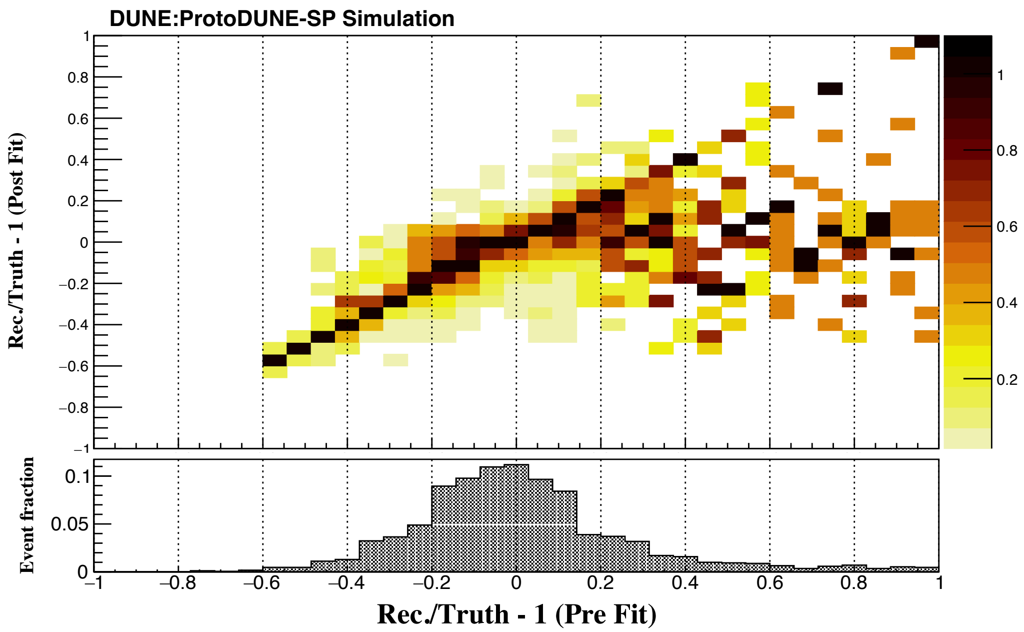


- π^0 Mass distribution **after** shower energy and angle correction.

π^0 Kinematic Fitting

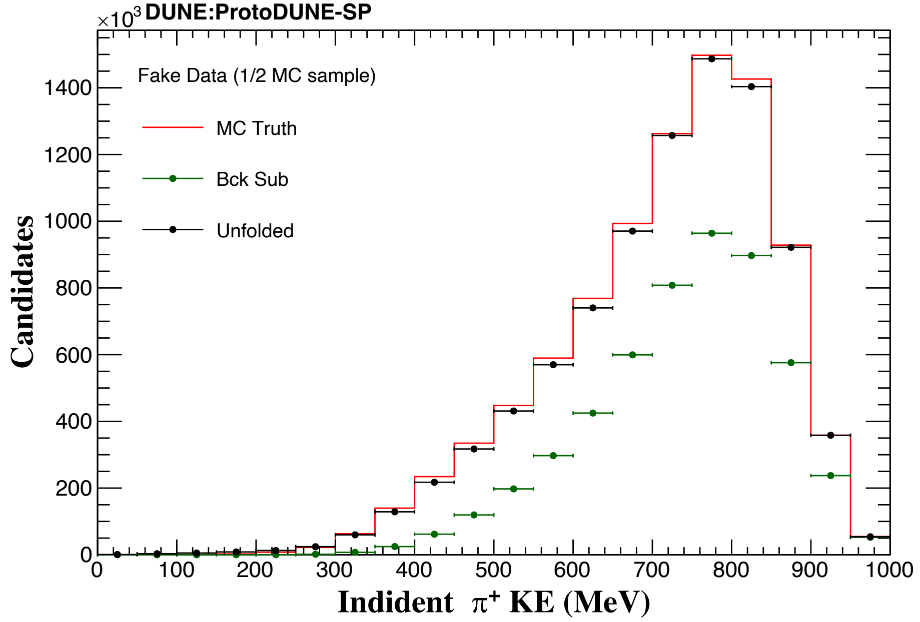
$$E_\pi = E_1 + \frac{m_{\pi^0}^2}{2E_1(1-cos\theta)}$$

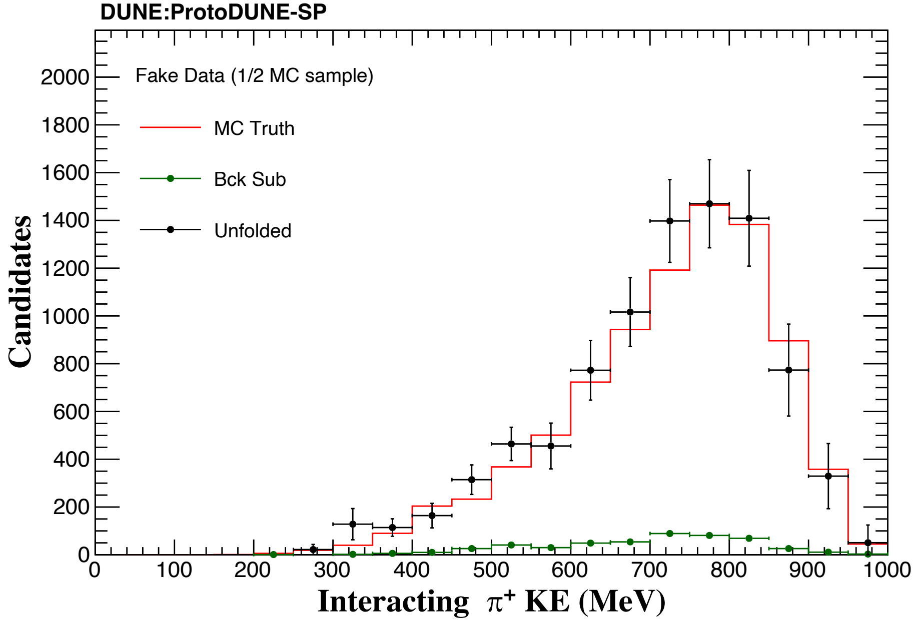




- The reconstructed π^0 energy bias looks pretty good after fitting.
- The spread of the distribution is reduced from 0.26 to 0.19 (see in legend).
- KF has a larger impact on events where the π^0 energy is overestimated.

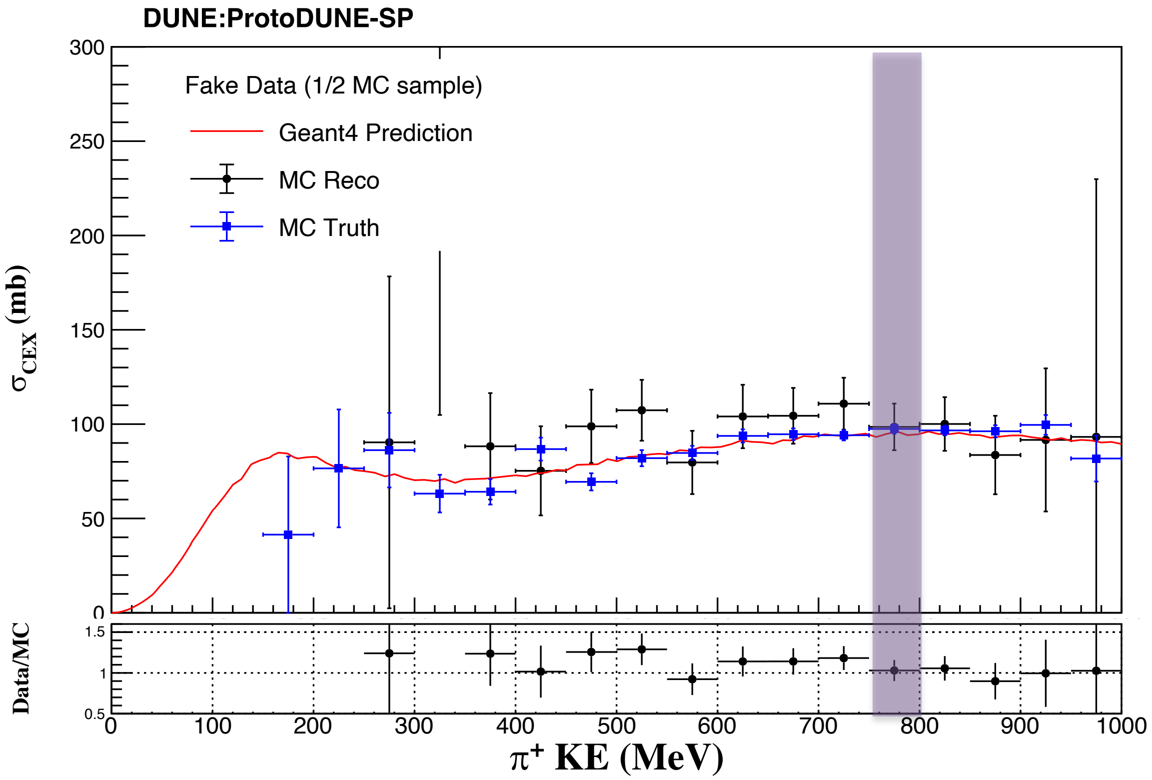
Background Subtraction & Unfolding





- Mapping response matrix is trained using half of the total MC.
- The other half of the MC is used to test the unfolding procedures.
- The red histogram is computed using the truth information of the fake data sample.

Total Cross Section σ_{CEX} Measurement



- MC Reco is the fake data sample after background subtraction and unfolding.
- MC Truth is derived using the truth information of the fake data sample.
- Above 400 MeV, both Rec. and Truth agree with the Gean4 predication well.

Differential Cross Section Formula

$$\sigma_i \approx \frac{m_{Ar}}{\rho \delta X N_A} \frac{N^i_{int}}{N^i_{inc}}$$

- Calculate the differential cross section as,

$$\left(\frac{d\sigma}{dT_{\pi^0}}\right)_{ij} = \frac{m_{Ar}}{\rho \delta X N_A} \frac{1}{(\Delta T_{\pi^0})} \frac{N^{ij}_{int}}{N^i_{inc}} \quad \text{(Eq. 1)}$$

ΔT_{π^0} is the bin width of π^0 KE

Index i : beam T_{π^+} bin,
Index j : daughter T_{π^0} bin

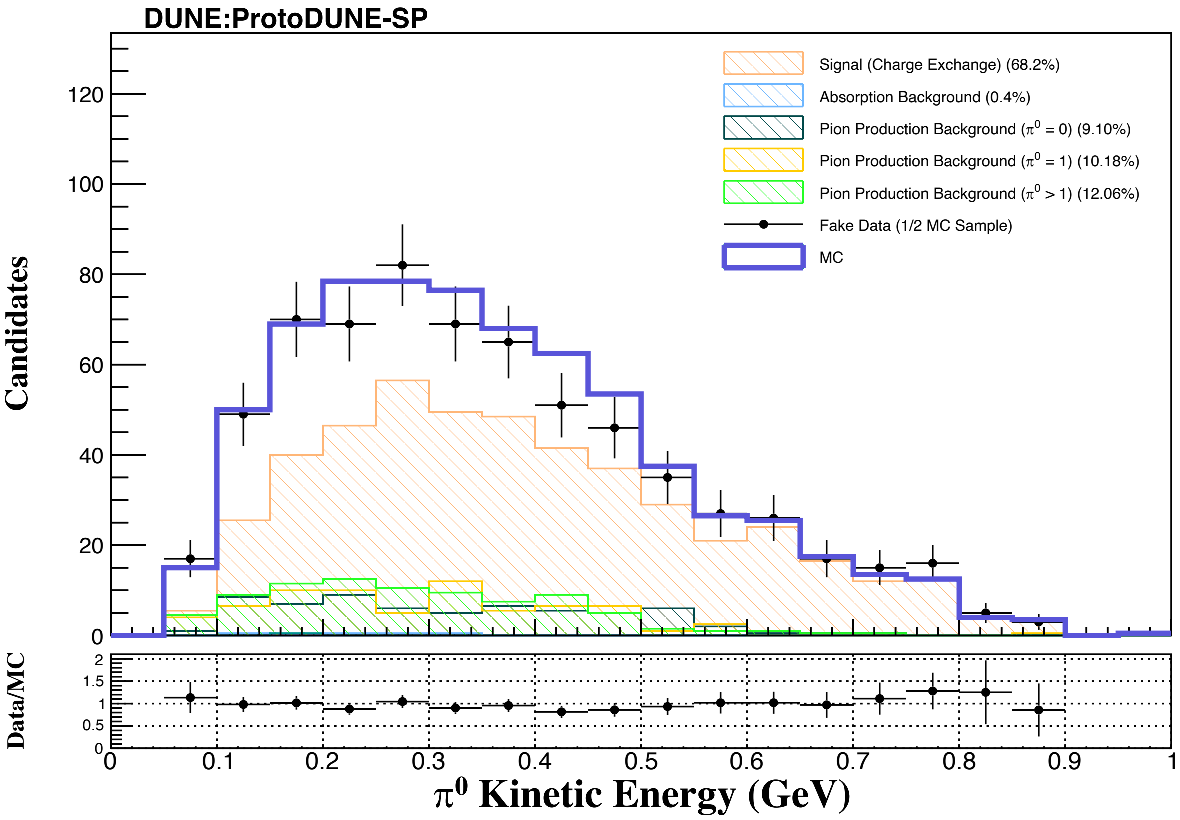
- Thin slice total CEX cross section is,

$$\sigma_i = \frac{m_{Ar}}{\rho \delta X N_A} \frac{N^i_{int}}{N^i_{inc}} \quad \text{(Eq. 2)}$$

- Then the differential cross section formula is (Sub. Eq. 2 into Eq. 1),

$$\left(\frac{d\sigma}{dT_{\pi^0}}\right)_{ij} = \frac{1}{(\Delta T_{\pi^0})} \frac{N^{ij}_{int}}{N^i_{int}} \sigma_i$$

π^0 Kinetic Energy Reconstruction



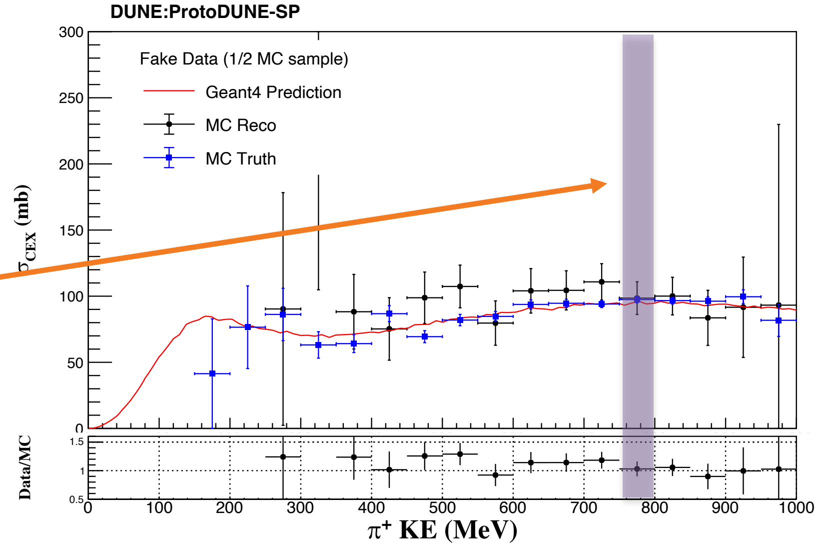
! All Rec. Pion Beam Sample !

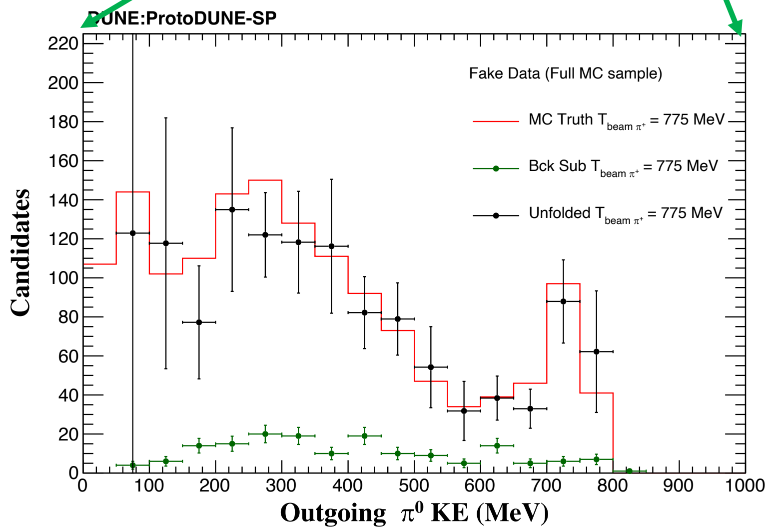
- Reconstructed π^0 kinetic energy distribution from after event topology cut.
- Event topology definitions can be found in page 2.
- Signal (charge exchange) is around 68%.
- The largest background is pion production with $\pi^0 > 1$ (12%)
- No phase space cut on daughter $\pi^{+/-}$ (cannot detect pion with mom. < 150 MeV)

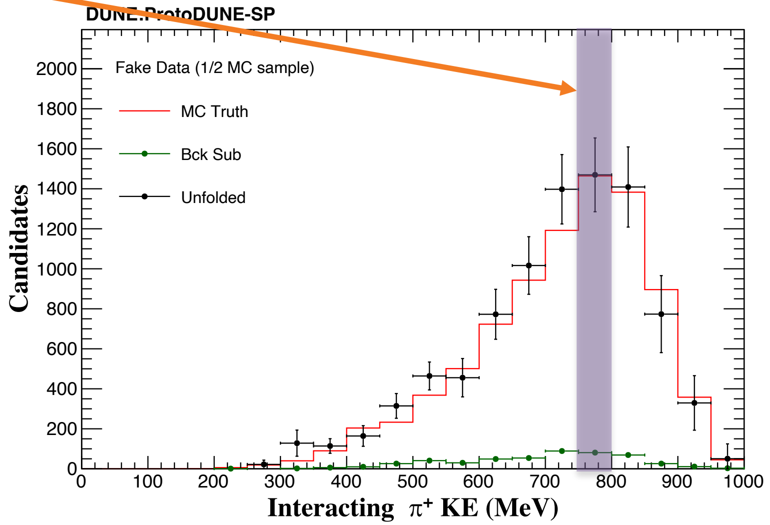
Towards dσ Measurement

- Differential XS for bin j (daughter T_{π^0} bin) is,

$$\left(\frac{d\sigma}{dT_{\pi^0}}\right)_j = \frac{1}{(\Delta T_{\pi^0})} \frac{N^j_{int}}{N_{int}} \sigma_{KE=775\ MeV}$$

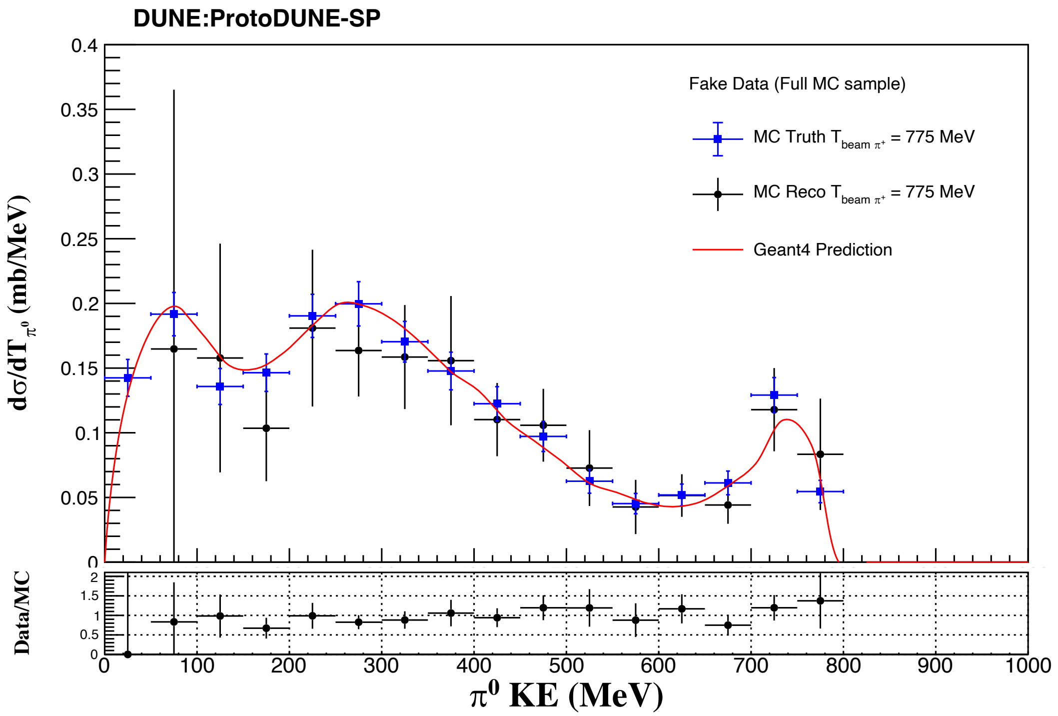






Diff. Cross Section $d\sigma/dT_{\pi^0}$

- Beam pion interacts between 750 - 800 MeV and produces one π^0(Charge Ex.).

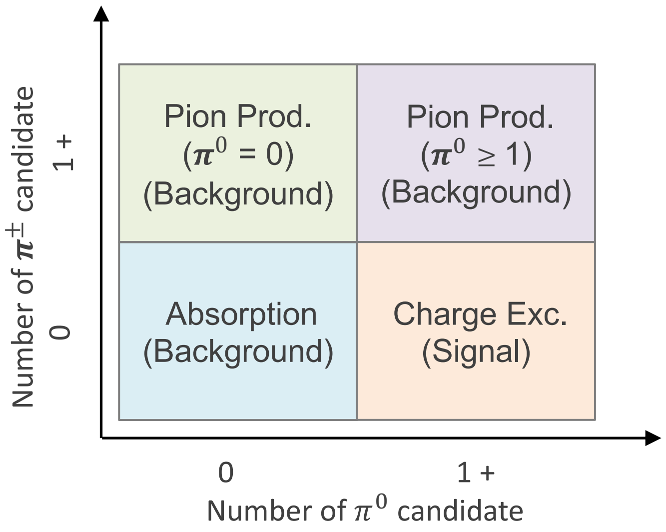


- The full MC sample was used in the rec. calculation to increase the statistics.
- The statistical error is quite large for the low T_{π^0} region.
- Both Rec. and Truth agree well with the Gean4 predication.

Towards Real Data dσ Measurement

- The differential cross section measurements were validated using fake data.

- Background constraints need to be done using sideband channels (data-driven method):
  - ❖ Fit three scaling factors in each background channel.
  - ❖ Find scaling factors as a function of T_{π^0}
  - ❖ Use scaling factors to tune the background component in signal channel.

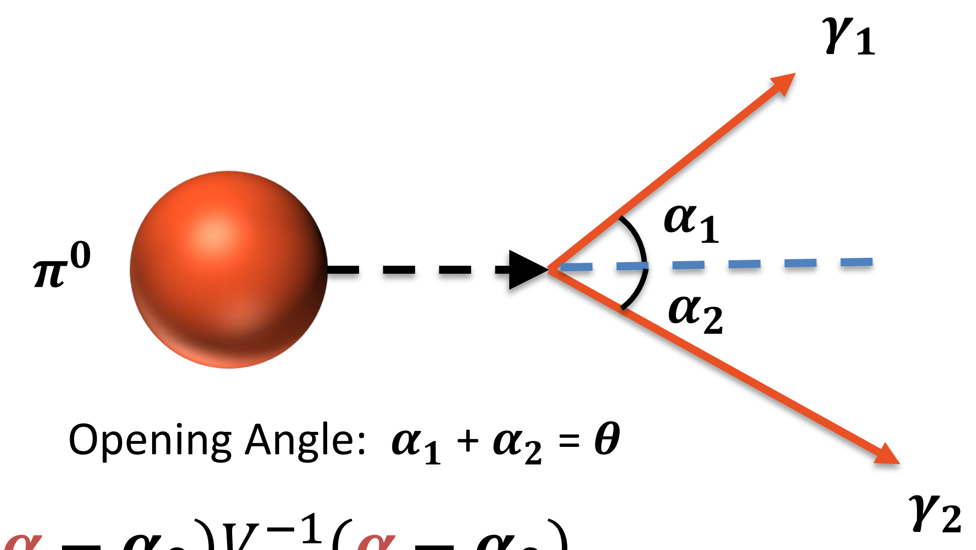


- One-dimensional unfolding may not be enough, need more studies and tests.

Summary

- The formula for computing the differential cross section is discussed.
- Truth-level validation is performed and shows good agreement with Geant4 input.
- Fake data sample (1/2 of MC sample) is used to perform the charge exchange cross section.
- After unfolding, both MC Reco and Truth of the fake data sample agrees well with the Geant4 input.
- Need to study more on background constraints and unfolding when move on to the real data sample.

π^0 Decay Kinematics



- Invariant mass of two photons:
  - $m_{\gamma\gamma}^2 = 2E_1E_2(1 - cos(\theta))$
- Minimizing (Full CVM):
  - $\chi^2 = \lambda * \{2 * E_1E_2[1 - cos(\theta)] - m_{\gamma\gamma}^2\} + (\boldsymbol{\alpha} - \boldsymbol{\alpha_0})V_{\alpha_0}^{-1}(\boldsymbol{\alpha} - \boldsymbol{\alpha_0})$
  - where $\boldsymbol{\alpha} = (E_1 \quad E_2 \quad \theta)$
  - $V_{\alpha_0}^{-1}$ is the inverse of the full CVM matrix
- Minimizing (Only diagonal CVM):
  - $\chi^2 = \lambda * \{2 * E_1E_2[1 - cos(\theta)] - m_{\gamma\gamma}^2\} + \Sigma_i(\alpha_i - \alpha_{0_i})^2/\sigma_i^2$
  - we set the off diagonal elements to be 0

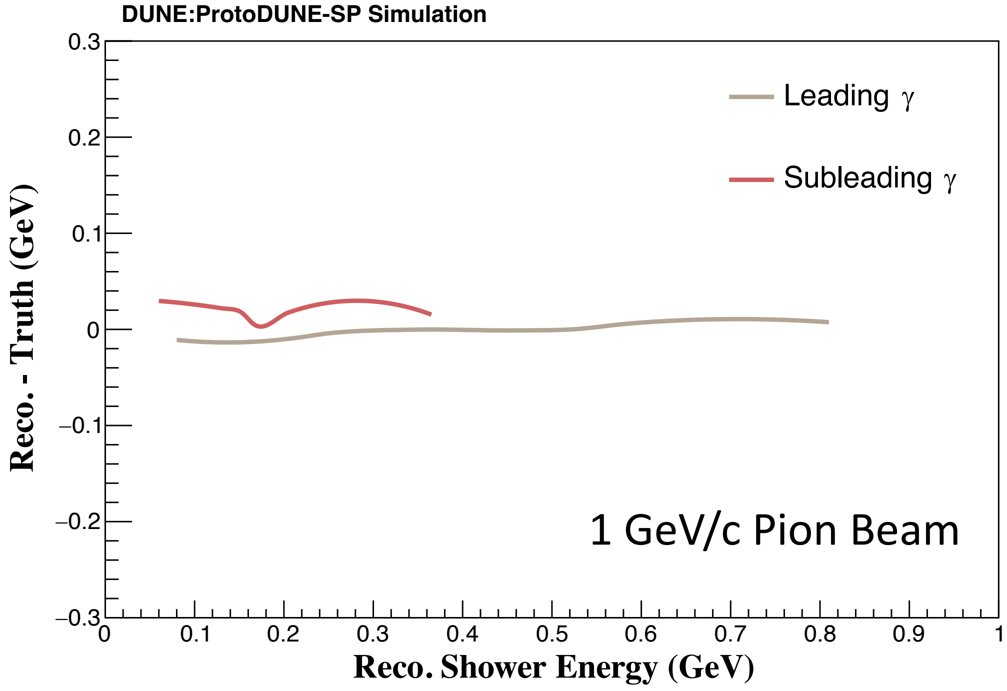
CVM Calculation

Variables	Matched Terms
Truth $\boldsymbol{E_1}$	$E[X_i]$
Truth $\boldsymbol{E_2}$	$E[X_j]$
Measured $\boldsymbol{e_1}$	X_i
Measured $\boldsymbol{e_2}$	X_j

CVM element: $V_{X_i X_j} = E[(X_i - E[X_i])(X_j - E[X_j])^T]$

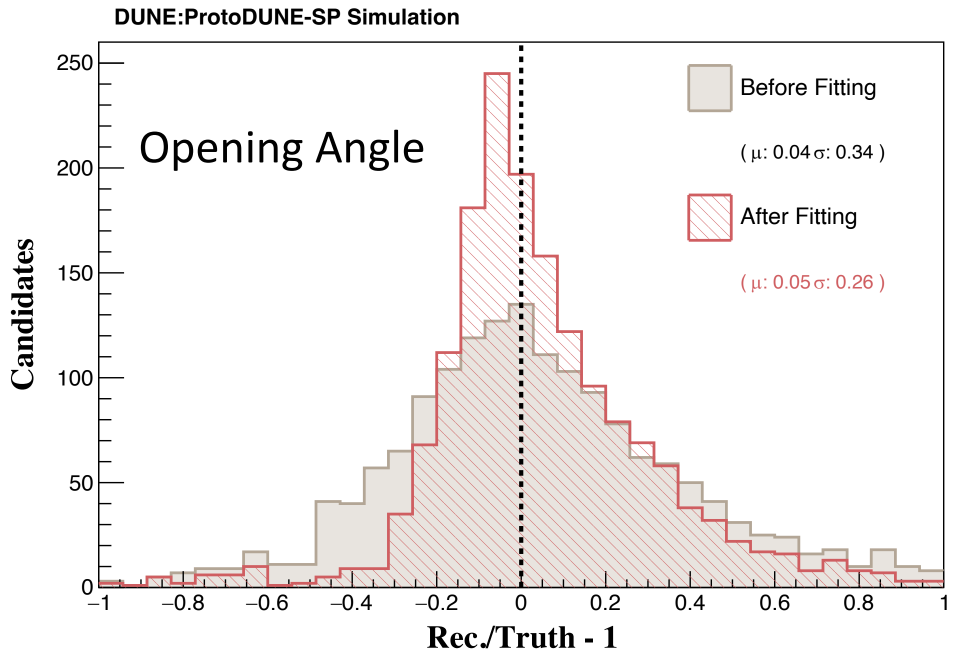
$$V_{e1e2} = \frac{\sum(e_1^i - E_1^i)(e_2^i - E_2^i)}{N}$$

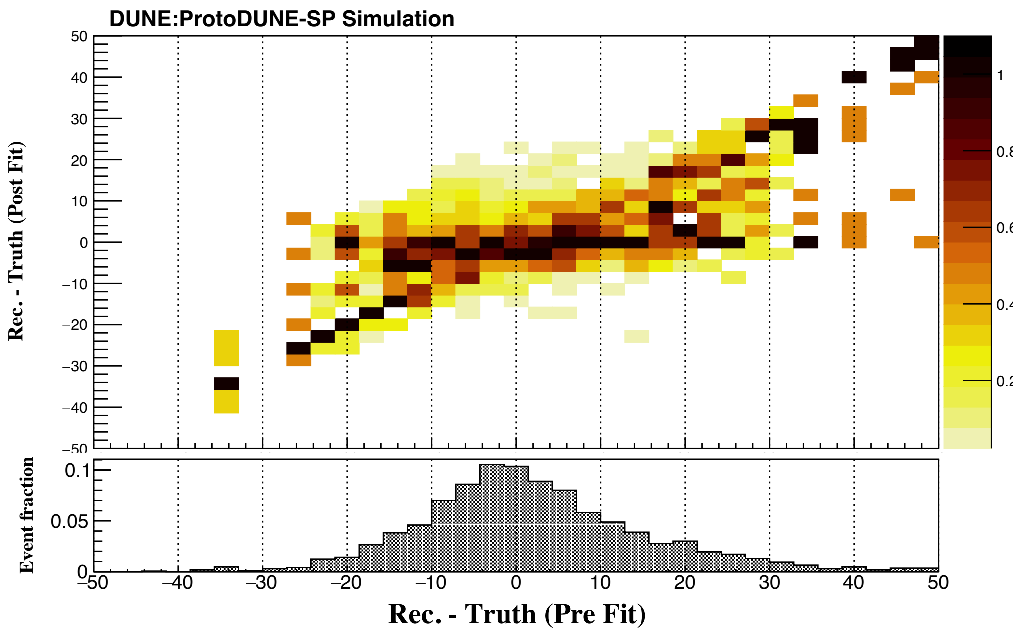
$$i = 0, 1, \ldots, N$$
$$N = \text{sample size}$$



- No obvious energy dependence is found on the absolute difference of reco. and truth shower energy.
- Use the shower-energy independent CVM.

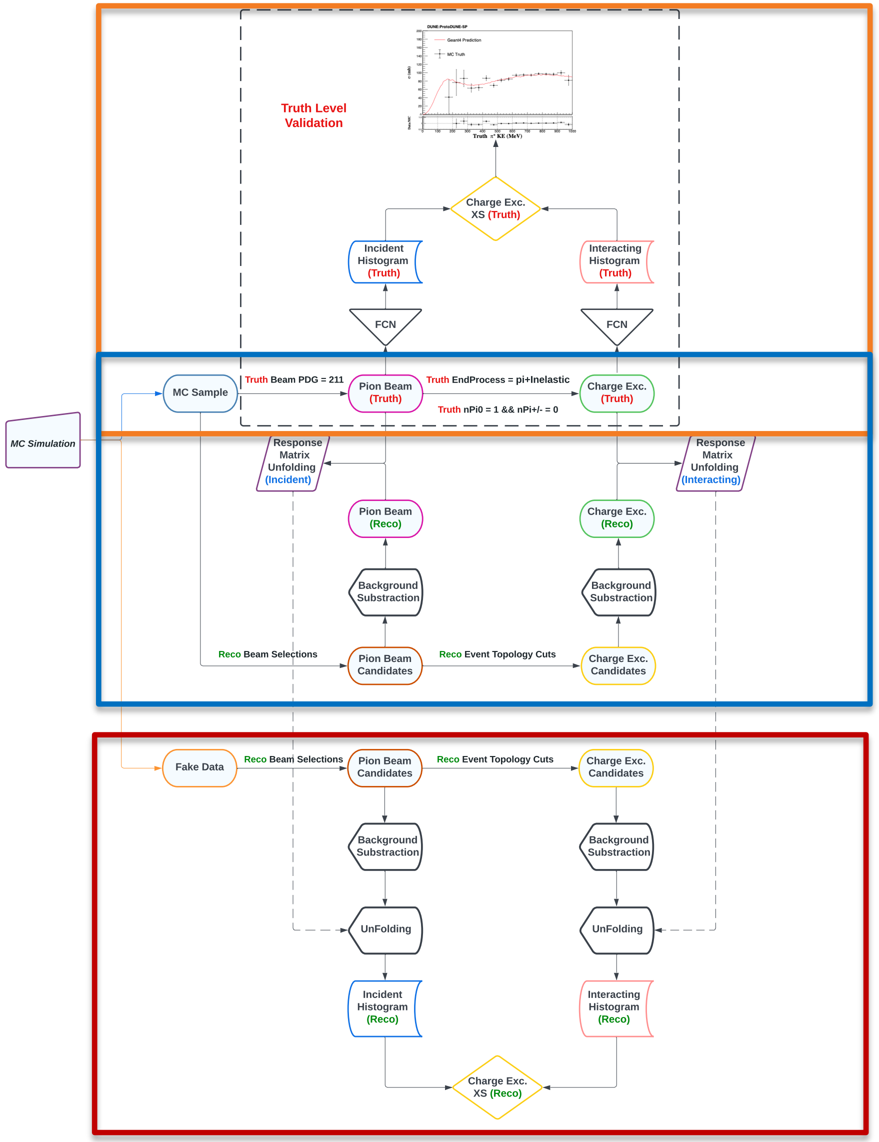
Kinematic Fitting Results





- After the kinematic fitting, the opening angle between two showers is improved a lot.
- Events with an extreme absolute angle difference are not improved via the fitting.
- Can reconstruct π^0 energy using leading shower E and opening angle

Truth Level Validation and Unfolding

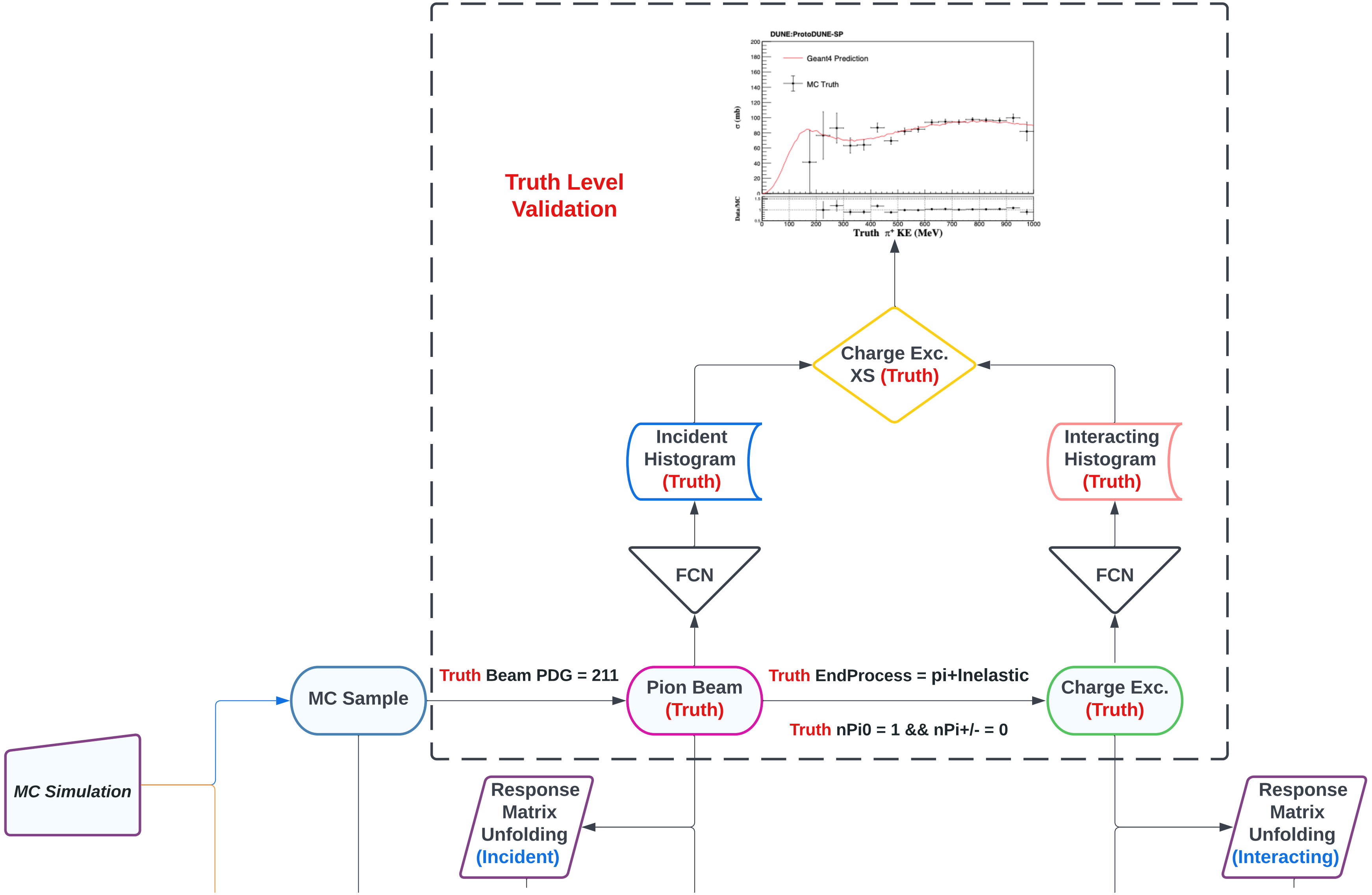


1 1. Truth Level Validation

2 2. Train Response Matrix

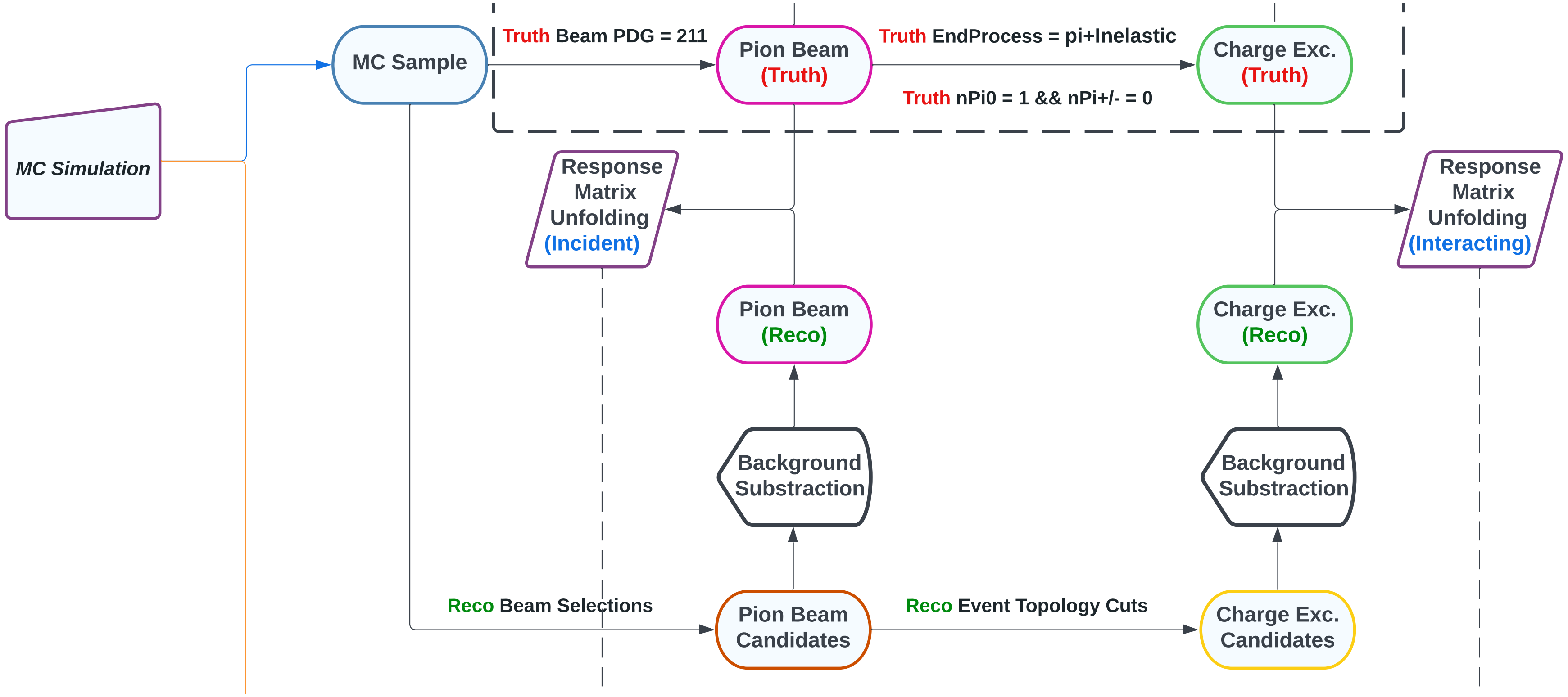
3 3. Unfolding Data

Truth Level Validation and Unfolding

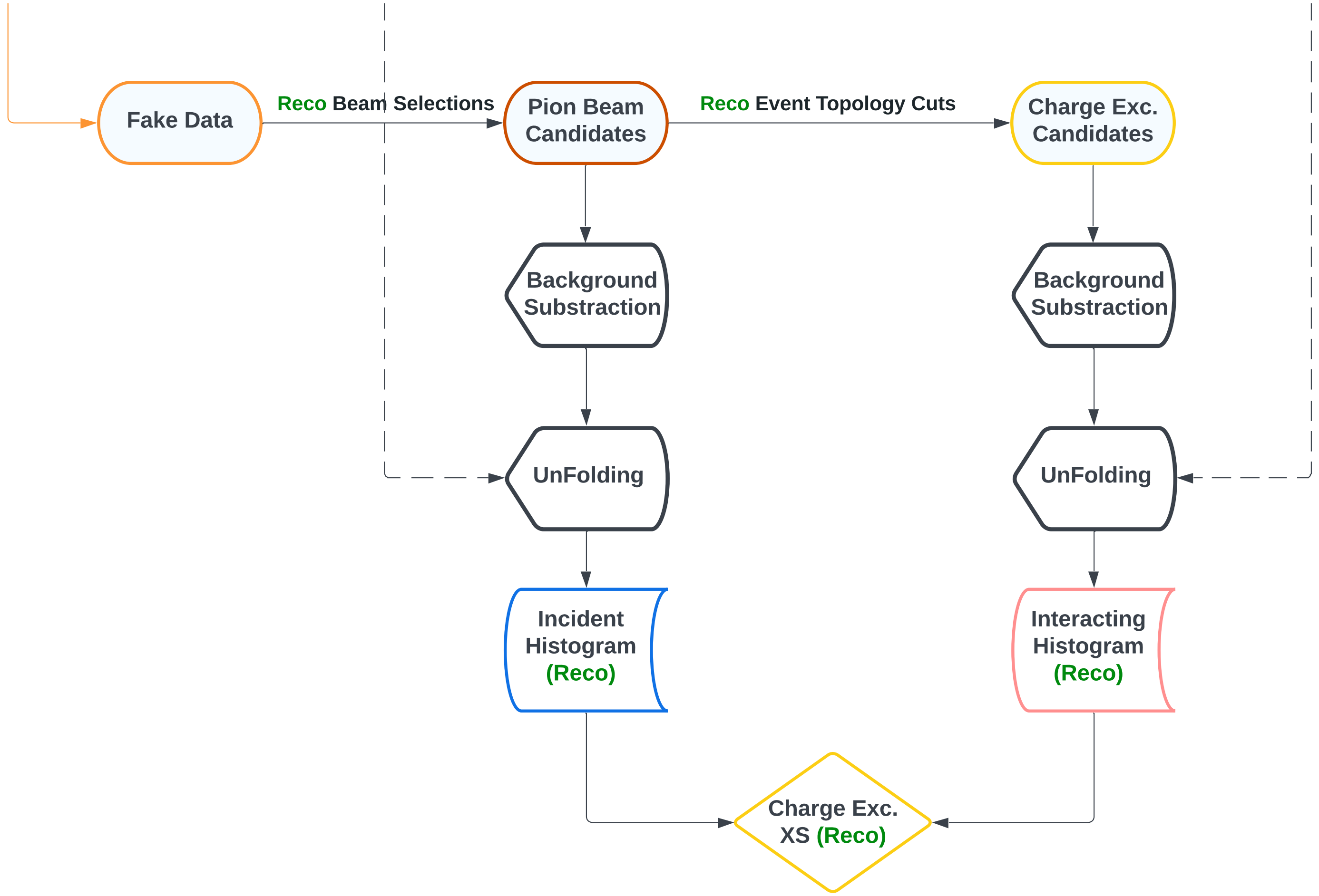


WARWICK THE UNIVERSITY OF WARWICK
UNIVERSITY OF OXFORD
DUNE

Truth Level Validation and Unfolding



Truth Level Validation and Unfolding



Response Matrix