Generators

Lecture 1: Leading order and parton shower basics

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Introduction

- I am a theorist interested in collider phenomenology.
- Main interest: higher order corrections in QCD.
- Author of next-to-leading order Monte Carlo code MCFM.

- Two lectures, today and Monday morning.
- Discussion session on Monday afternoon.
- If anything else comes up:
  - catch me in my office: 3rd floor, East (cafeteria) side
  - or email: johnmc@fnal.gov

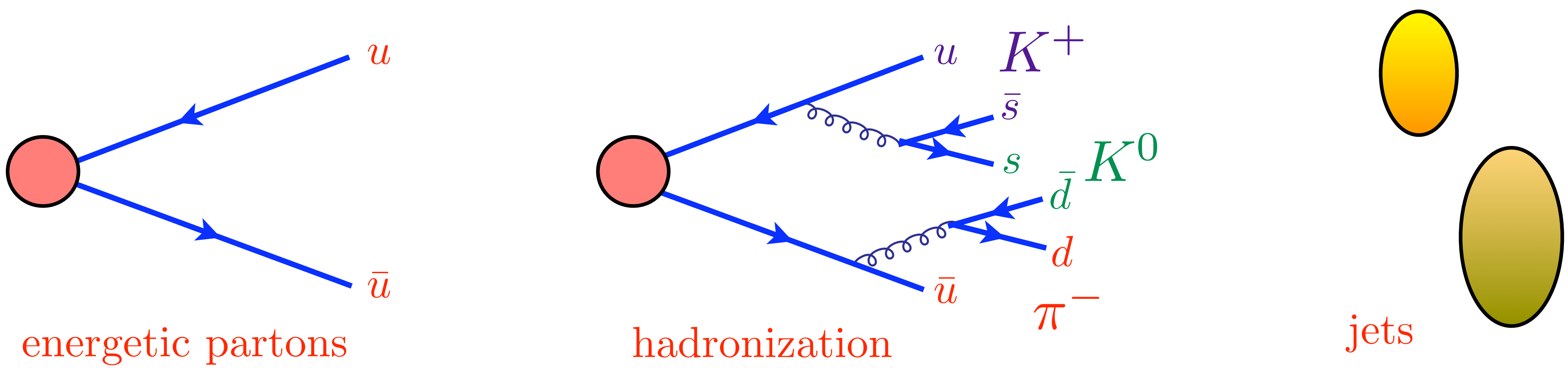
Outline of lectures

- Review of leading order predictions.
- Investigation of soft and collinear kinematic limits.
- Theoretical underpinnings of parton showers.
- Modern parton showers.
- Higher order tools.

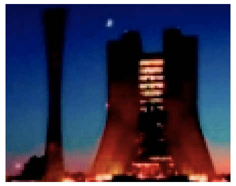
Reference: **“General purpose event generators for LHC physics”**
A. Buckley et al, arXiv: 1101.2599

Setting the scene

- All particles observed in experiments should be color neutral
  $\rightarrow$ no quarks or gluons.
- How then can we mesh experimental observations with the QCD Lagrangian, which necessarily involves the fundamental quark and gluon fields?
- A scattering can be described in terms of energetic quarks and gluons (partons) that subsequently hadronize, combining into color-neutral mesons and baryons, without too much loss of energy.
- This concept is often referred to as local parton-hadron duality.



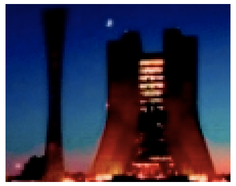
- This naturally accommodates the replacement of jets of particles in the final state by an equivalent number of quarks or gluons $\rightarrow$ LO picture.



Leading order tools

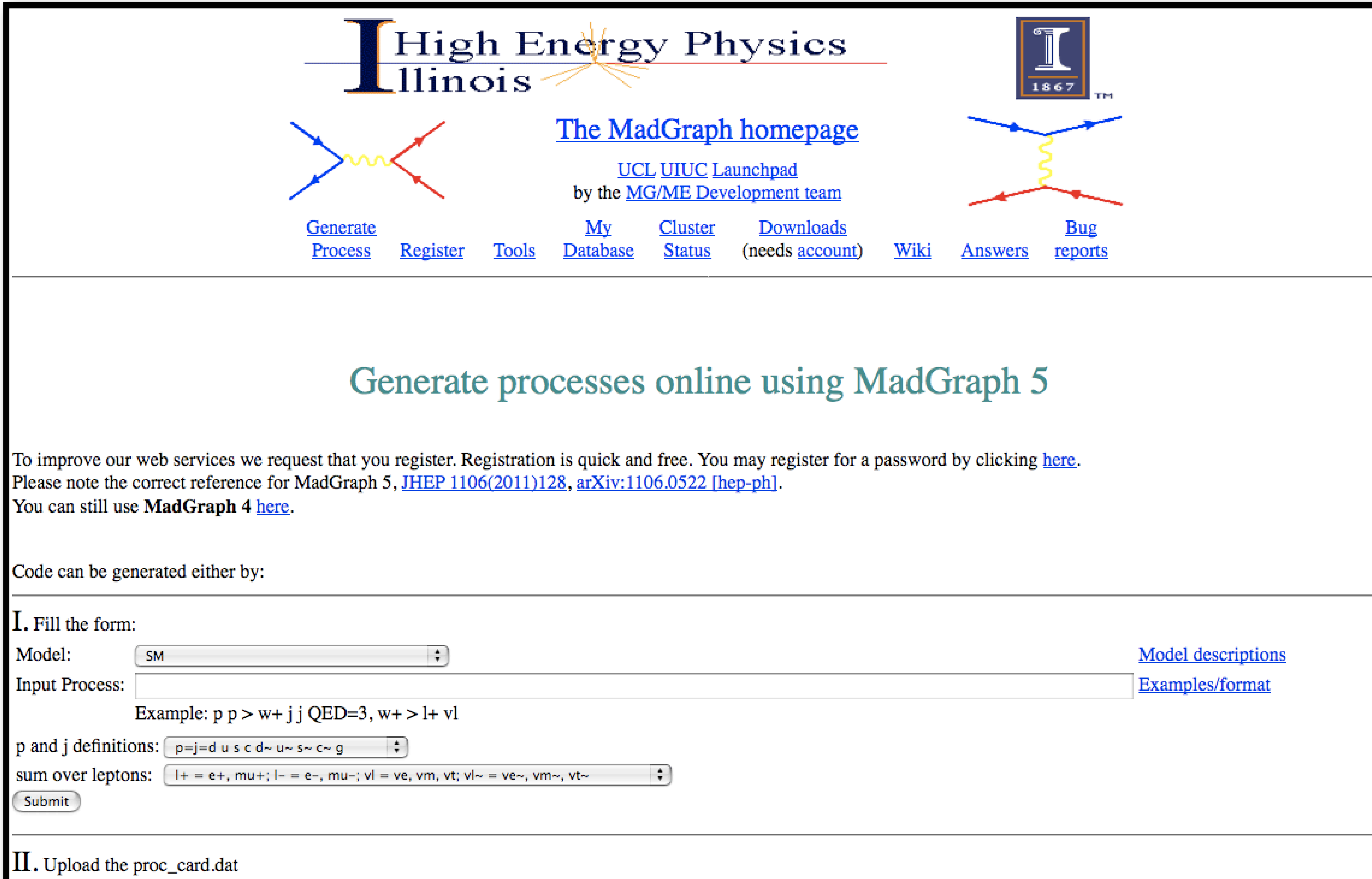
- The leading order estimate of the cross section is obtained by computing all relevant tree-level Feynman diagrams (i.e. no internal loops).
- Nowadays this is practically a solved problem - many suitable tools available.

ALPGEN	M. L. Mangano et al. http://alpgen.web.cern.ch/alpgen/
AMEGIC++	F. Krauss et al. http://projects.hepforge.org/sherpa/dokuwiki/doku.php
CompHEP	E. Boos et al. http://comphep.sinp.msu.ru/
HELAC	C. Papadopoulos, M. Worek http://helac-phegas.web.cern.ch/helac-phegas/helac-phegas.html
MadGraph	F. Maltoni, T. Stelzer http://madgraph.hep.uiuc.it/



MadGraph

The version 5 of the MadGraph Matrix Element Generator for the simulation of parton-level events for decay and collision processes at high energy colliders. Allows matrix element generation and event generation for any model that can be written as a Lagrangian, using the output of the FeynRules Feynman rule calculator. Provides output in multiple formats and languages, including Fortran MadEvent, Fortran Standalone matrix elements, C++ matrix elements, and Pythia 8 process libraries.



High Energy Physics Illinois

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The MadGraph homepage

UCL UIUC Launchpad
by the MG/ME Development team

Generate Process | Register | Tools | My Database | Cluster Status | Downloads (needs account) | Wiki | Answers | Bug reports

Generate processes online using MadGraph 5

To improve our web services we request that you register. Registration is quick and free. You may register for a password by clicking here.
Please note the correct reference for MadGraph 5, JHEP 1106(2011)128, arXiv:1106.0522 [hep-ph].
You can still use **MadGraph 4** here.

Code can be generated either by:

I. Fill the form:

Model: SM — Model descriptions

Input Process: — Examples/format

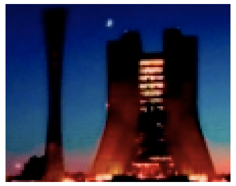
Example: p p > w+ j j QED=3, w+ > l+ vl

p and j definitions: p=j=d u s c d~ u~ s~ c~ g

sum over leptons: l+ = e+, mu+; l- = e-, mu-; vl = ve, vm, vt; vl~ = ve~, vm~, vt~

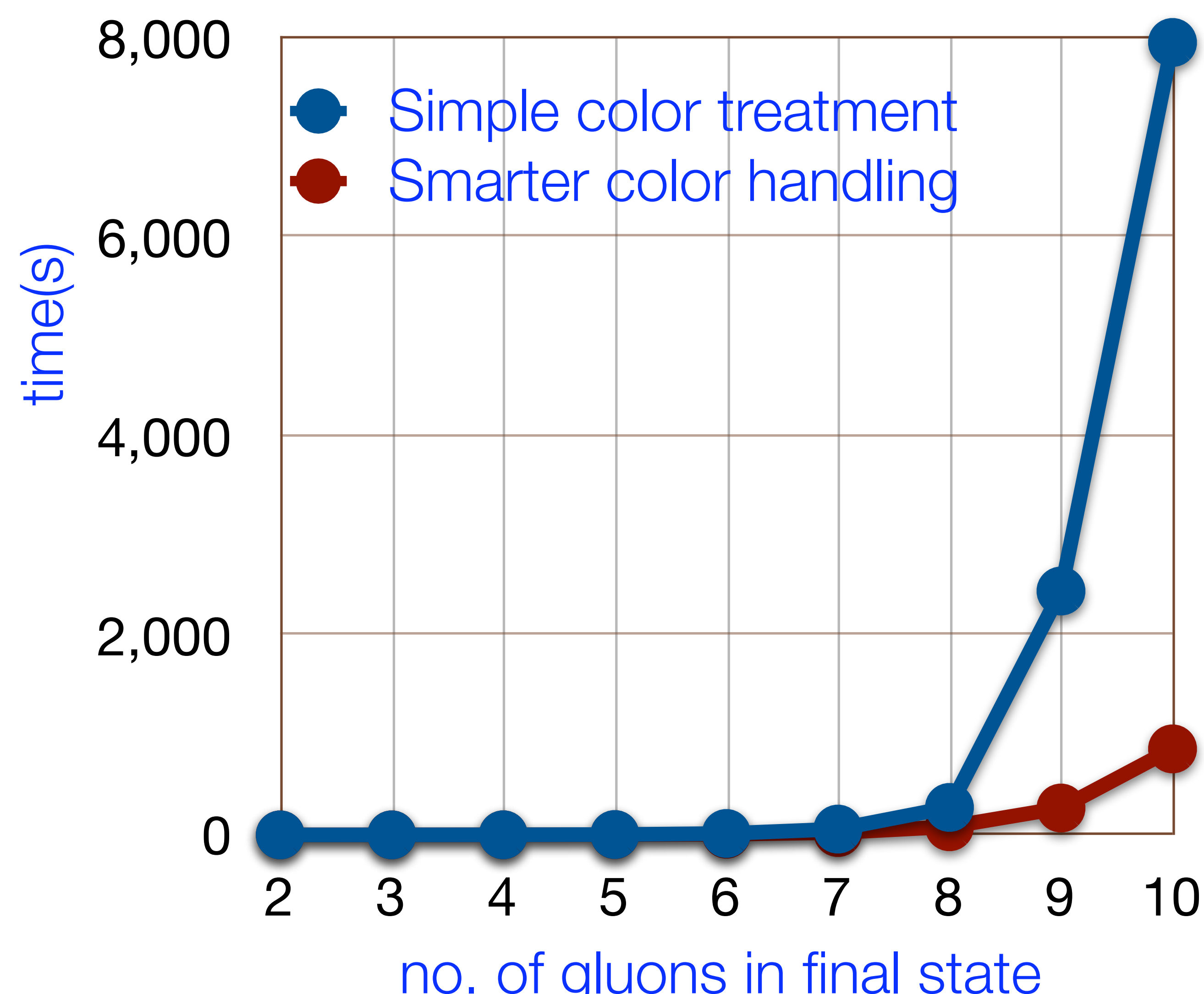
Submit

II. Upload the proc_card.dat



Limiting factors

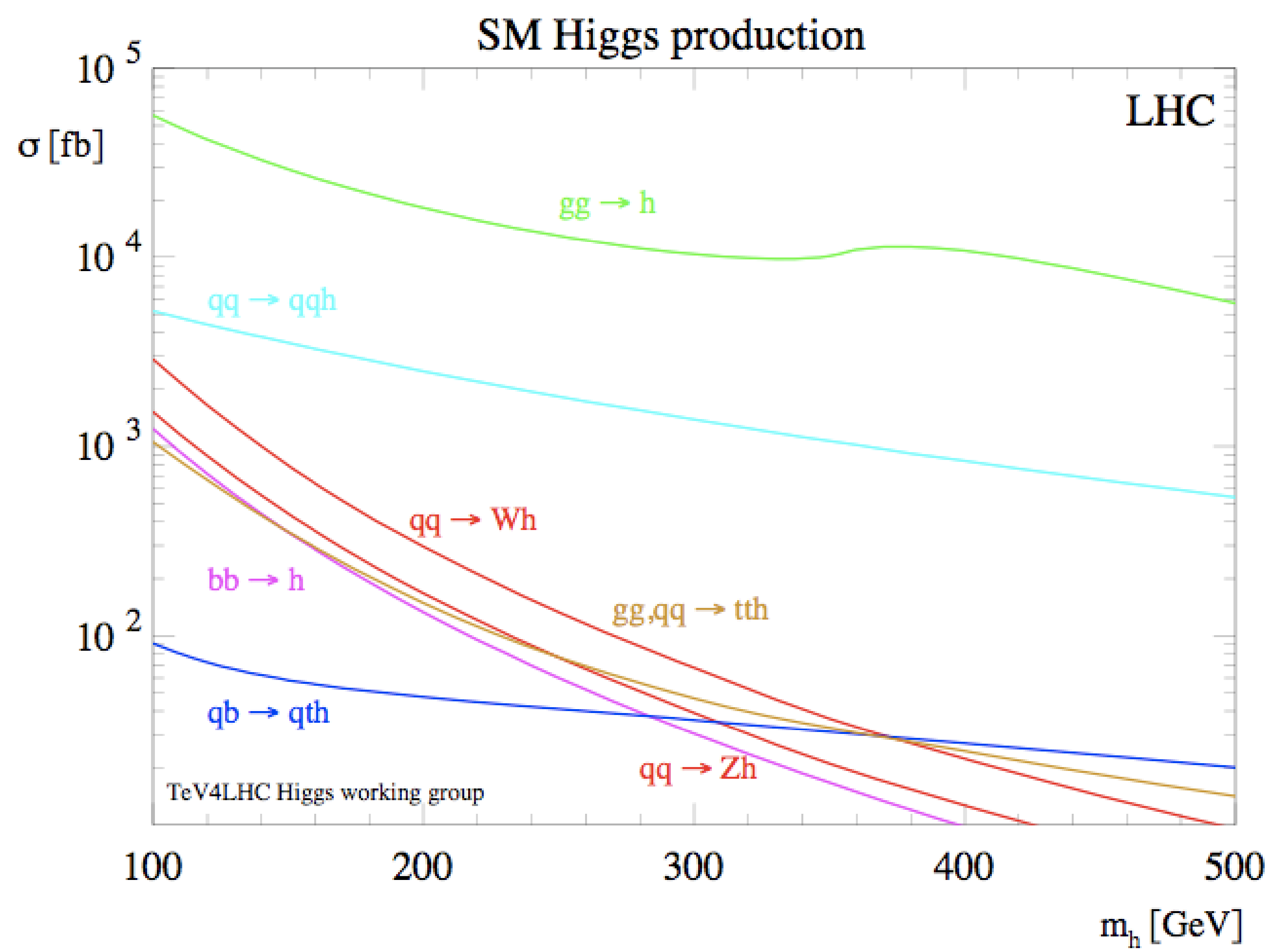
- Solved problem in principle, but computing power is still an issue.
- This is mostly because the number of Feynman diagrams entering the amplitude calculation grows factorially with the number of external particles.
  - hence smart (recursive) methods to generate matrix elements.
- Demonstrated by the time taken to generate 10,000 events involving 2 gluons in the initial state and up to 10 in the final state.
- The lower curve shows a smarter treatment of color factors, which become a limiting factor too.
  - active research area.



(adapted from C. Duhr et al., 2006)

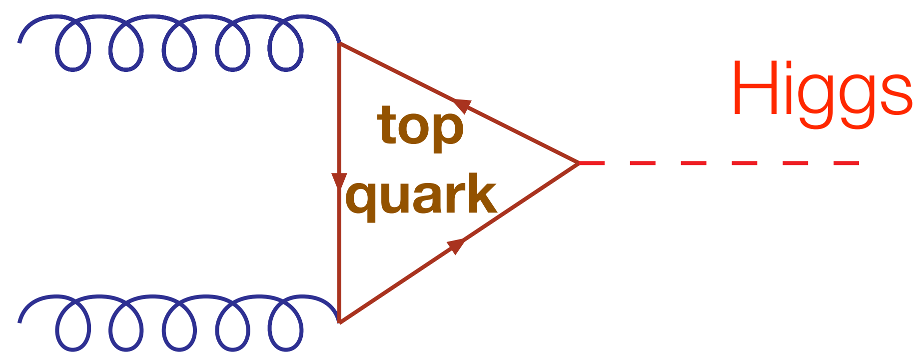
Beyond fixed order

- Ten gluons doesn't come close to the typical multiplicity in a collider event.
- Moreover, we want a tool that says something about hadrons, not partons.
- How can we hope to build something like this from scratch, using QCD?
  - use universal behaviour of QCD cross sections to build parton shower.
  - combine perturbative calculations with non-perturbative modelling.
- Begin with an unlikely topic: theory of Higgs production.
- Shown here are cross sections for different Higgs production modes at the (14 TeV) LHC.
- Here we are interested in the mode with the largest cross section: gluon fusion.



Higgs coupling to gluons

- How does this coupling take place? Certainly not directly!
- The answer is through a loop, with the Higgs coupling preferentially to the heaviest quark available: the top quark.



- In general, loop-induced processes are suppressed compared to tree-level contributions - but at the LHC, gluons are plentiful (especially compared to antiquarks).
- We're not going to perform this computation here, but note that in the limit that the top mass is infinite the result is formally equivalent to the coupling obtained by adding a term to the Lagrangian:

$$\mathcal{L}_{ggH} = \frac{C}{2} H \, F^A_{\mu\nu} F^{\mu\nu}_A$$

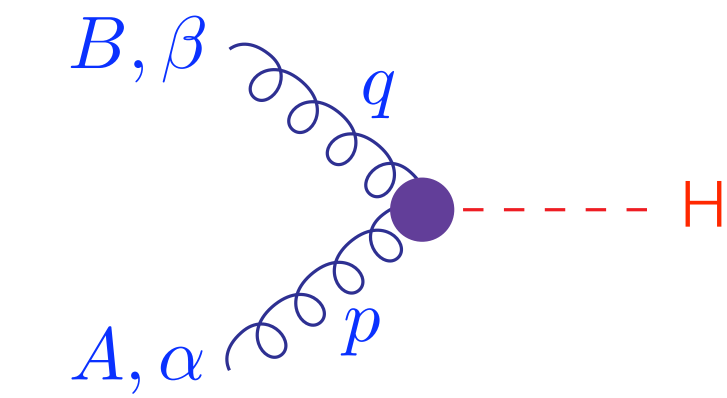
$C = \frac{\alpha_s}{6\pi v}$ — Higgs field — gluon field strength

"Effective Theory" gives rise to ggH coupling and new Feynman rules.

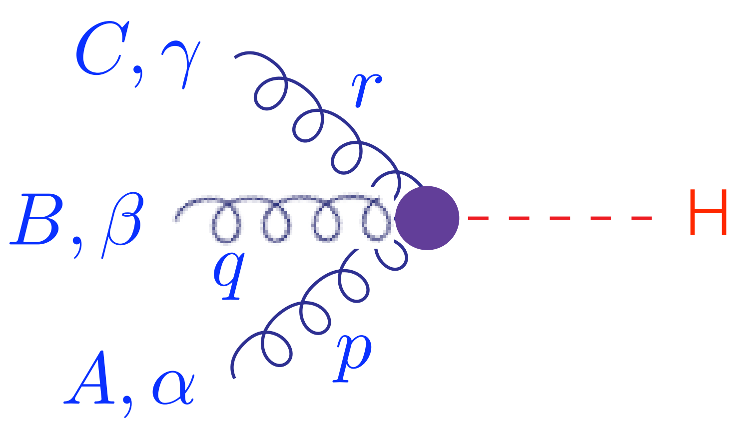
→ **Frank Petriello lectures**

Feynman rules: effective theory

- Also get 3- and 4-point vertices that mimic the structure of the pure QCD case.

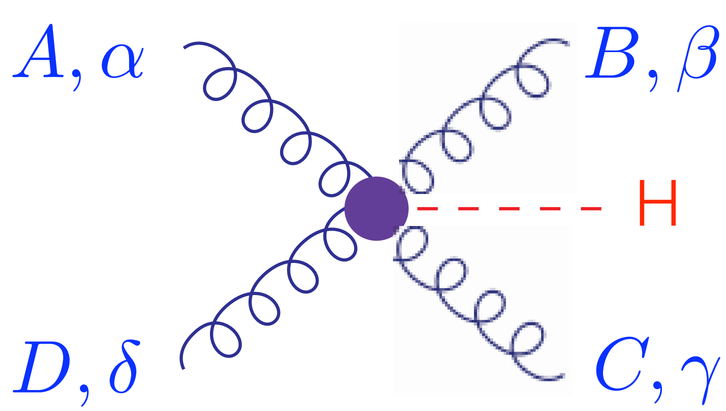


$$2Ci\delta^{AB}\left(p\cdot q\, g^{\alpha\beta} - p^\beta q^\alpha\right)$$



$$-2Cg_s f^{ABC}\Big[g^{\alpha\beta}(p^\gamma - q^\gamma) + g^{\beta\gamma}(q^\alpha - r^\alpha) + g^{\gamma\alpha}(r^\beta - p^\beta)\Big]$$

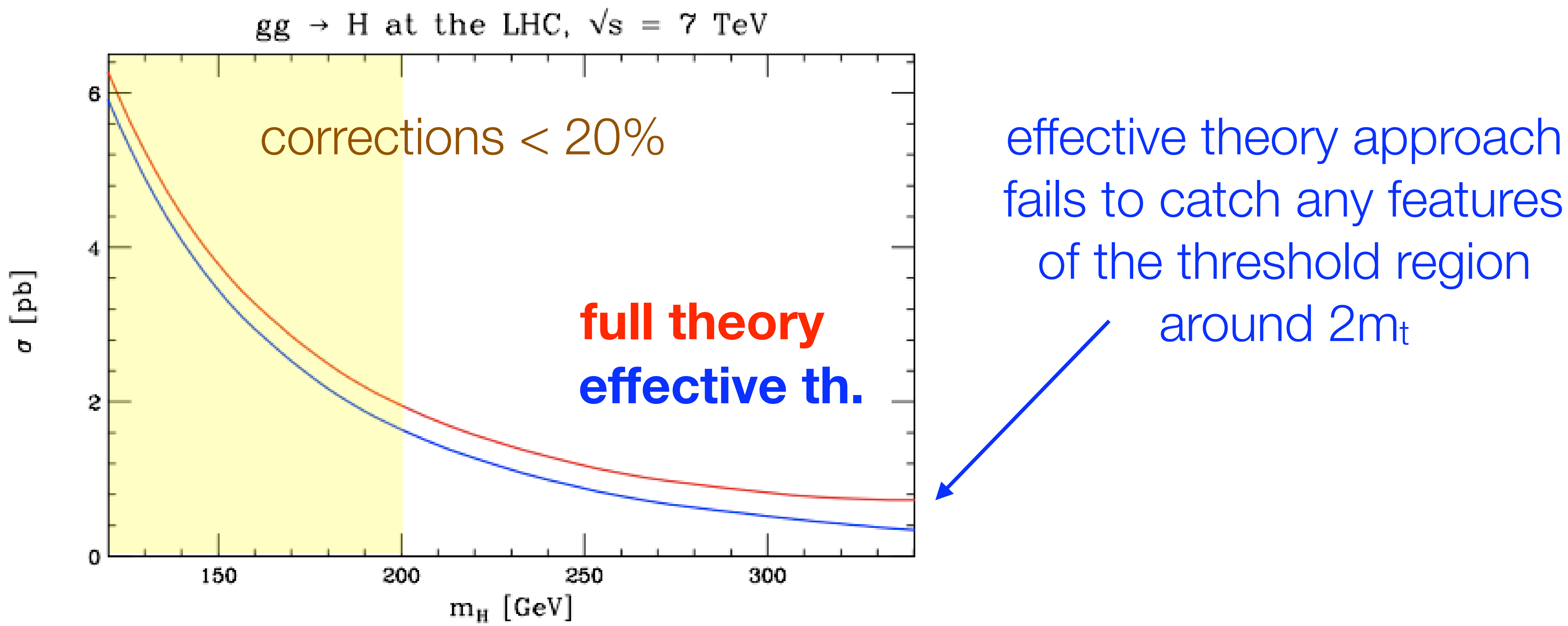
(all momenta incoming)



$$-2Cig_s^2 f^{ABX} f^{XCD}\left[g^{\alpha\gamma}g^{\beta\delta} - g^{\alpha\delta}g^{\gamma\beta}\right]$$
$$-2Cig_s^2 f^{BCX} f^{XAD}\left[g^{\beta\alpha}g^{\gamma\delta} - g^{\beta\delta}g^{\alpha\gamma}\right]$$
$$-2Cig_s^2 f^{BCX} f^{XAD}\left[g^{\gamma\beta}g^{\alpha\delta} - g^{\gamma\delta}g^{\beta\alpha}\right]$$

Effective theory

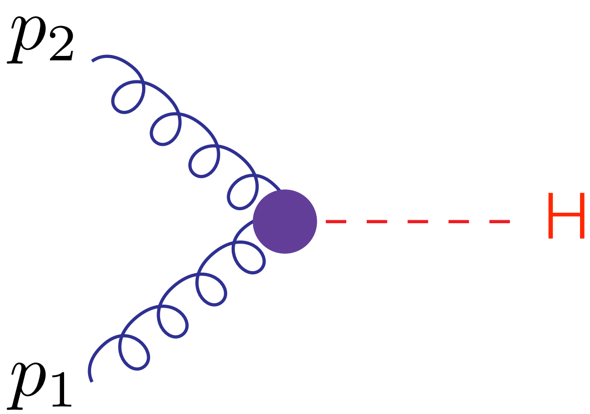
- This effective theory is a good approximation.



- Moreover it is very useful for more complicated calculations
  - chain new vertices together in order to compute cross sections that would be intractable in the full (finite top mass) theory.
  - e.g. producing additional quarks or gluons (i.e. jets).

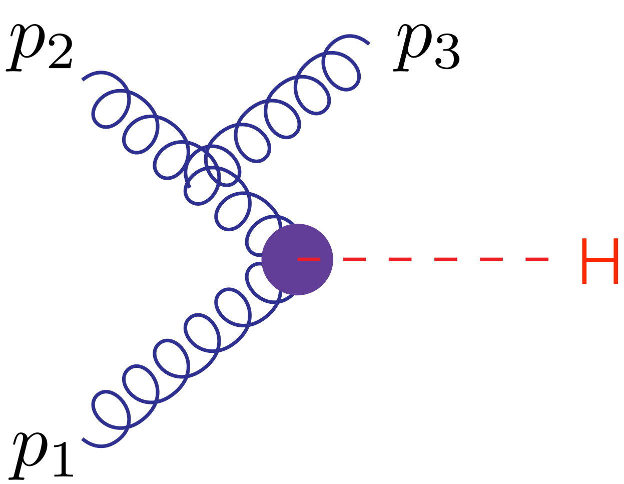
Matrix elements

- First look at the squared matrix elements for this process (exercise: check!).



$$|\mathcal{M}_{Hgg}|^2 = 2(N_c^2 - 1)C^2 m_H^4$$

- Now consider adding a gluon (total of 4 diagrams - remember triple-gluon+H).



$$|\mathcal{M}_{Hggg}|^2 = 4N_c(N_c^2 - 1)C^2 g_s^2 \times \left(\frac{m_H^8 + (2p_1.p_2)^4 + (2p_1.p_3)^4 + (2p_2.p_3)^4}{8p_1.p_2\, p_1.p_3\, p_2.p_3} \right)$$

- Inspect this in the limit that gluons 2 and 3 are collinear:

$$p_2 = zP \;, \quad p_3 = (1 - z)P$$

Collinear limit: gluons

- Under this transformation we can make the replacements:

$$2p_1.p_2 \to z m_H^2 \ , \quad 2p_1.p_3 \to (1-z) m_H^2 \ , \quad 2p_2.p_3 \to 0 \ ,$$

and simply read off the answer:

$$|\mathcal{M}_{Hggg}|^2 \xrightarrow{\text{coll.}} 4N_c(N_c^2-1)C^2 g_s^2 m_H^4 \left(\frac{1+z^4+(1-z)^4}{2z(1-z)p_2.p_3} \right)$$

- This clearly shares some features with the *Hgg* matrix element squared we just calculated, which we can exploit to write it in a new way:

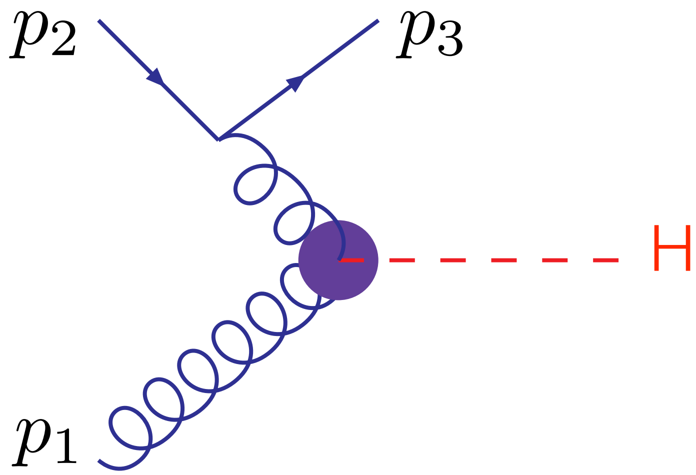
$$|\mathcal{M}_{Hggg}|^2 \xrightarrow{\text{coll.}} \frac{2g_s^2}{2p_2.p_3} |\mathcal{M}_{Hgg}|^2 P_{gg}(z)$$

Here the collinear splitting function, which only depends on the relative weight in the splitting (*z*), is defined by:

$$P_{gg}(z) = 2N_c \left(\frac{z^2+(1-z)^2+z^2(1-z)^2}{z(1-z)} \right)$$

Collinear limit: quarks

- Same trick with the two collinear gluons replaced by quark-antiquark pair.



$$|\mathcal{M}_{Hg\bar{q}q}|^2 = 4T_R(N_c^2-1)C^2g_s^2 \times \left(\frac{(2p_1.p_2)^2+(2p_1.p_3)^2}{2p_2.p_3}\right)$$

- We find a similar result. In the collinear limit, the matrix element squared is again proportional to the matrix element with one less parton:

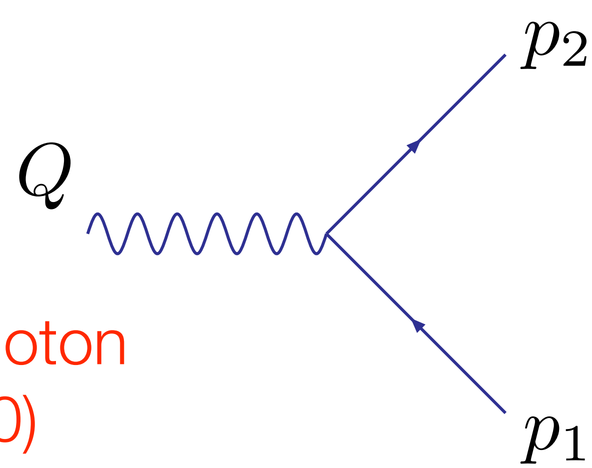
$$|\mathcal{M}_{Hg\bar{q}q}|^2 \xrightarrow{\text{coll.}} \frac{2g_s^2}{2p_2.p_3}|\mathcal{M}_{Hgg}|^2 P_{qg}(z)$$

The splitting function this time is given by:

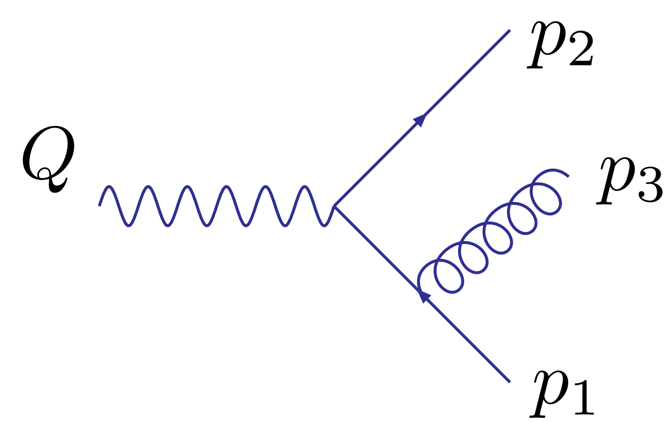
$$P_{qg}(z) = T_R\left(z^2+(1-z)^2\right)$$

Collinear limit: quark-gluon

- To investigate this last case, we don't need the Higgs interaction.



virtual photon
(Q^2>0)



(2 diagrams)

$$|\mathcal{M}_{\gamma^* \bar{q} q}|^2 = 4 N_c e_q^2 Q^2$$

$$|\mathcal{M}_{\gamma^* \bar{q} q g}|^2 = 8 N_c C_F e_q^2 g_s^2 \times \left(\frac{(2p_1.p_3)^2 + (2p_2.p_3)^2 + 2Q^2(2p_1.p_2)}{4\, p_1.p_3\, p_2.p_3} \right)$$

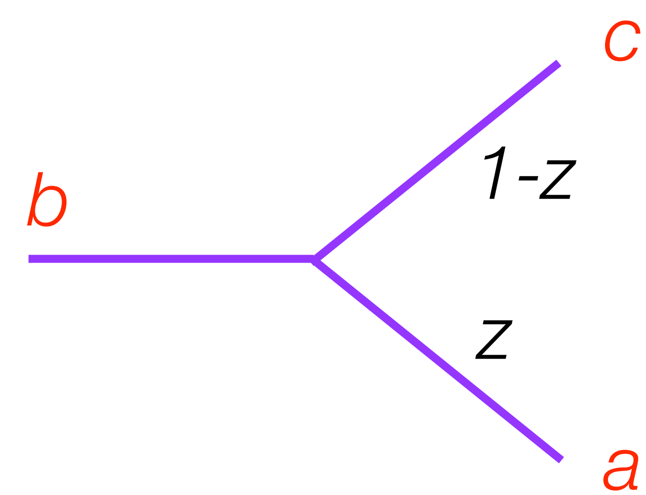
- A similar analysis, with the gluon carrying momentum fraction *(1-z)*, leads to the result:

$$P_{qq}(z) = C_F \left(\frac{1+z^2}{1-z} \right)$$

Universal factorization

- The important feature of these results is that they are universal, i.e. they apply to the appropriate collinear limits in all processes involving QCD radiation.
- They are a feature of the QCD interactions themselves.

$$|\mathcal{M}_{ac\ldots}|^2 \xrightarrow{a,\,c\text{ coll.}} \frac{2g_s^2}{2p_a.p_c} |\mathcal{M}_{b\ldots}|^2 P_{ab}(z)$$



collinear singularity

$$P_{qq}(z) = C_F \left(\frac{1+z^2}{1-z} \right)$$

additional soft singularity as $z \to 1$

$$P_{gg}(z) = 2N_c \left(\frac{z^2 + (1-z)^2 + z^2(1-z)^2}{z(1-z)} \right)$$

soft for $z \to 0$, $z \to 1$

$$P_{qg}(z) = T_R \left(z^2 + (1-z)^2 \right)$$

Infrared singularities

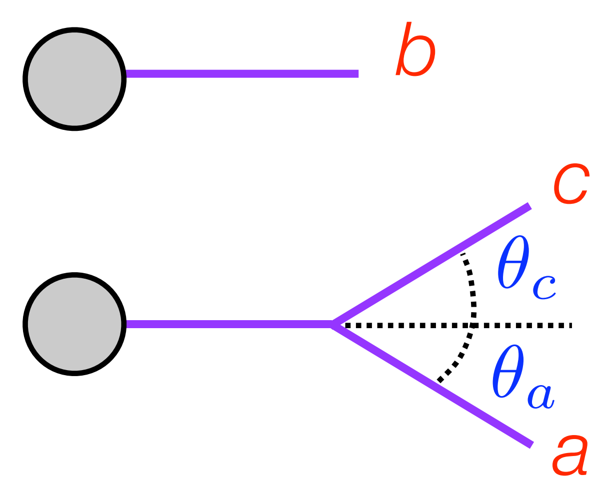
- These are called infrared singularities, which occur when relevant momenta become small.
  - they are thus indicative of long-range phenomena which are, by definition, not well described by perturbation theory.
  - at such scales are approached, hadronization takes over and apparent singularities are avoided.
- In perturbative QCD we avoid such issues by restricting our attention to infrared safe quantities that are insensitive to such regions.
  - for example: in our leading order calculations, we try to describe jets with large transverse momenta, not arbitrarily soft particles.
  - useful to regularize such singularities: they appear in intermediate steps of a calculation, but must disappear at the end (for physical observables).
    - this is a statement of the Kinoshita-Lee-Nauenberg (KLN) theorem.

The silver lining

- On the positive side:
  - we have learned that emission of soft and collinear partons is favoured;
  - we know exactly the form of the required matrix elements when that occurs.
- In fact it also applies to the phase space too.
- Start from the standard phase space formula:

$$d\mathrm{PS}_{(\ldots)b} = (\ldots)\,\frac{d^3\vec{p}_b}{(2\pi)^3 2E_b}$$

$$d\mathrm{PS}_{(\ldots)ac} = (\ldots)\,\frac{d^3\vec{p}_a}{(2\pi)^3 2E_a}\,\frac{d^3\vec{p}_c}{(2\pi)^3 2E_c}$$



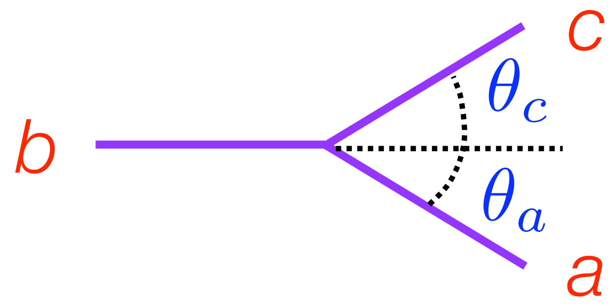
and note that, if we fix the momentum of *a*, we can relate these by:

$$d\mathrm{PS}_{(\ldots)ac} = d\mathrm{PS}_{(\ldots)b}\,\frac{d^3\vec{p}_a}{(2\pi)^3 2E_a}\,\frac{E_b}{E_c} \approx d\mathrm{PS}_{(\ldots)b}\,\frac{1}{(2\pi)^2}\,\frac{E_a E_b}{2E_c}\,dE_a\,\theta_a d\theta_a$$

(for $\theta_a \sim 0$)

Small angle approximation

- "Small angle" kinematics of the collinear limit:



$$p_a = z p_b \,, p_c = (1-z) p_b$$

$$\Longrightarrow \quad E_a = z E_b \,, E_c = (1-z) E_b$$

$$z\theta_a - (1-z)\theta_c = 0 \qquad \Longrightarrow \quad \theta_a = (1-z)(\theta_a + \theta_c)$$

- Now relate *t*, the virtuality of *b*, to the opening angle $\theta = \theta_a + \theta_c$:

$$t = (p_a + p_c)^2 = 2E_a E_c (1 - \cos\theta^2) = E_b^2 z(1-z)\theta^2 = \frac{z E_b^2 \theta_a^2}{1-z}$$

- Hence we can write the factorized form in this limit as,

$$d\mathrm{PS}_{(\ldots)ac} = d\mathrm{PS}_{(\ldots)b} \, \frac{1}{(2\pi)^2} \, \frac{E_a E_b}{2E_c} \, \frac{(1-z)E_b}{2zE_b^2} \, dz \, dt \; = d\mathrm{PS}_{(\ldots)b} \, \frac{dz\, dt}{16\pi^2}$$

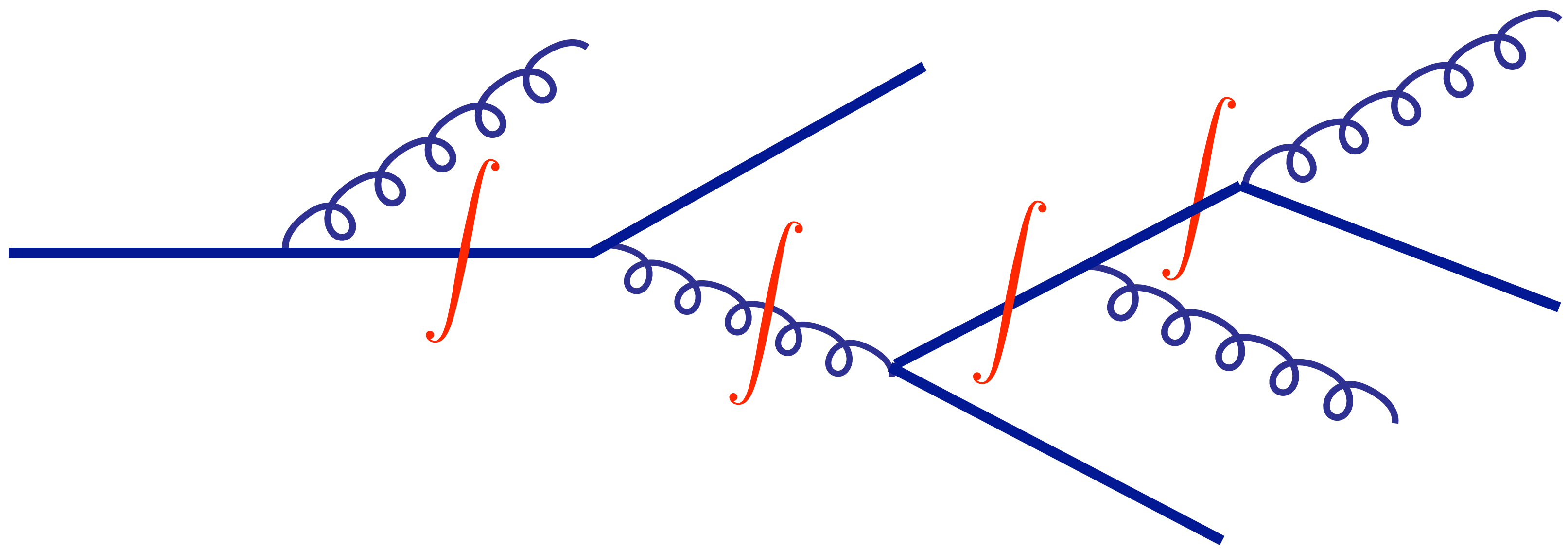
- Combining this with our previous matrix element factorization formula gives:

$$d\sigma_{(\ldots)ac} = |\mathcal{M}_{(\ldots)ac}|^2 d\mathrm{PS}_{(\ldots)ac} = d\sigma_{(\ldots)b} \left(\frac{\alpha_s}{2\pi}\right) \frac{dt}{t} \, P_{ab}(z) \, dz$$

Parton showers

$$d\sigma_{n+1} = d\sigma_n \left(\frac{\alpha_s}{2\pi}\right) \frac{dt}{t} P_{ab}(z)\, dz$$

- This is an important equation: it tells us how we can generate additional soft and collinear radiation ad infinitum.



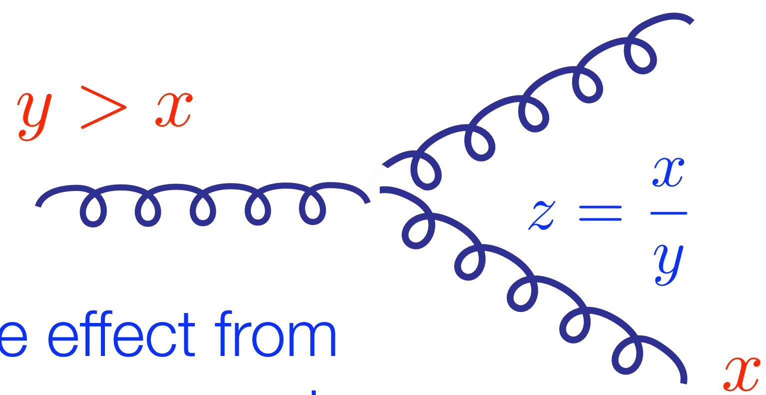
- Technically this is called timelike branching since we have implicitly assumed that all particles are outgoing ($t>0$).
  - extension to the spacelike case (radiation on an incoming line) is similar.
- This is the principle upon which all parton shower simulations are based.

Popular parton shower programs

PYTHIA	T. Sjöstrand et al. http://home.thep.lu.se/~torbjorn/Pythia.html
HERWIG	G. Corcella et al. http://hepwww.rl.ac.uk/theory/seymour/herwig/
HERWIG++	S. Gieseke et al. http://projects.hepforge.org/herwig/
SHERPA	F. Krauss et al. http://projects.hepforge.org/sherpa/dokuwiki/doku.php
ISAJET	H. Baer et al. http://www.nhn.ou.edu/~isajet/

Inside a parton shower

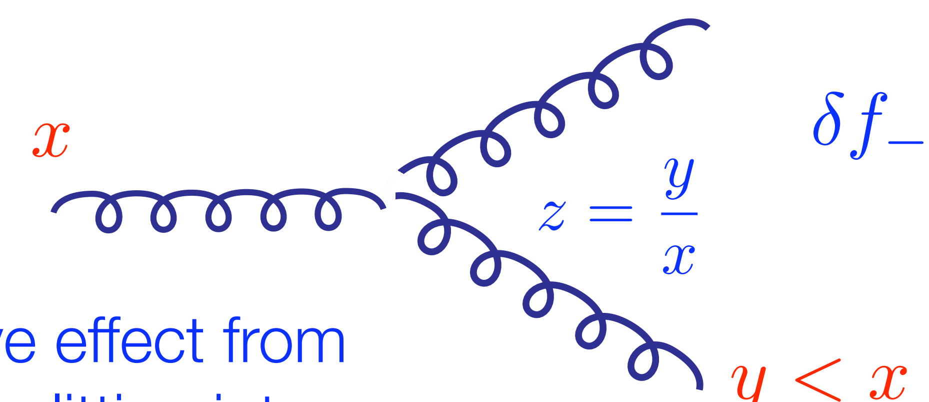
- The defining equation can be interpreted in terms of the probability of having a parton branching with given *(x,t)* at some point in the shower: let's call it *f(x,t)*.
- For simplicity, let's assume that the evolution doesn't change the parton species, e.g. an all-gluon shower (extension is straightforward).
- Now consider a small change from *t* to *t+δt* and its effect on *f(x,t)*.



+ve effect from higher momenta splitting

$$\delta f_+(x,t) = \frac{\delta t}{t} \int_x^1 dy\, dz \left(\frac{\alpha_s}{2\pi}\right) P_{gg}(z) f(y,t) \delta(x - zy)$$

$$= \frac{\delta t}{t} \int_x^1 \frac{dz}{z} \left(\frac{\alpha_s}{2\pi}\right) P_{gg}(z) f(x/z,t)$$



-ve effect from splitting into smaller momenta

$$\delta f_-(x,t) = \frac{\delta t}{t} f(x,t) \int_0^x dy\, dz \left(\frac{\alpha_s}{2\pi}\right) P_{gg}(z) \delta(y - zx)$$

$$= \frac{\delta t}{t} f(x,t) \int_0^1 dz \left(\frac{\alpha_s}{2\pi}\right) P_{gg}(z)$$

The DGLAP equation

- By taking the difference can reinterpret this as a differential equation for *f(x,t)*:

$$t\,\frac{\partial f(x,t)}{\partial t} = \int_0^1 dz \left(\frac{\alpha_s}{2\pi}\right) P_{ab}(z) \left(\frac{1}{z} f(x/z,t) - f(x,t)\right)$$

- This is called the Dokshitzer-Gribov-Lipatov-Altarelli-Parisi (DGLAP) equation.
- It is most convenient to expose a solution to this equation by introducing a Sudakov form factor, Δ(t).

$$\Delta(t) = \exp\left[-\int_{t_0}^{t} \frac{dt'}{t'} \int dz \left(\frac{\alpha_s}{2\pi}\right) P_{ab}(z)\right]$$

- Hence we can rewrite as:

$$t\,\frac{\partial f(x,t)}{\partial t} = \int \frac{dz}{z} \left(\frac{\alpha_s}{2\pi}\right) P_{ab}(z) f(x/z,t) + \frac{f(x,t)}{\Delta(t)} \frac{t\,\partial \Delta(t)}{\partial t}$$

$$\Longrightarrow\ t\,\frac{\partial}{\partial t}\left(\frac{f(x,t)}{\Delta(t)}\right) = \frac{1}{\Delta(t)} \int \frac{dz}{z} \left(\frac{\alpha_s}{2\pi}\right) P_{ab}(z) f(x/z,t)$$

The Sudakov form factor

- Integrate up to find solution given boundary condition at $t=t_0$:

$$f(x,t) = \Delta(t) f(x,t_0) + \int_{t_0}^{t} \frac{dt'}{t'} \frac{\Delta(t)}{\Delta(t')} \int \frac{dz}{z} \left(\frac{\alpha_s}{2\pi}\right) P_{ab}(z) f(x/z,t)$$

no branching between t_0 and t

integrate over multiple branchings; for each value of t', no branching between t' and t

- Interpret Sudakov form factor as the probability for no parton emission
  - better: no resolvable parton emission. We must cut off the *z*-integration as $z \to 0,1$ to avoid the singularities found before. Above cutoff unresolvable.
- The Sudakov interpretation lends itself to Monte Carlo methods (universally used in parton showers):
  - pick a random number *r* in [0,1] and determinate t_2 from t_1 and $\frac{\Delta(t_2)}{\Delta(t_1)} = r$
  - can generate *z* according to integral over correct P_{ab} for splitting.

Exponentiation

- Limits on emission depend on definition of “resolvable” for the shower
  - simplest case: $\frac{t'}{t_0} < z < 1 - \frac{t'}{t_0}$
  - Now consider probability of no resolvable branchings from a quark:

$$\Delta_q(t) = \exp\left[-\int_{t_0}^{t} \frac{dt'}{t'} \int_{t_0/t'}^{1-t_0/t'} dz \left(\frac{\alpha_s}{2\pi}\right) P_{qq}(z)\right]$$

$$\sim \exp\left[-C_F \left(\frac{\alpha_s}{2\pi}\right) \int_{t_0}^{t} \frac{dt'}{t'} \int_{t_0/t'}^{1-t_0/t'} \frac{dz}{1-z}\right]$$

$$\sim \exp\left[-C_F \left(\frac{\alpha_s}{2\pi}\right) \log^2 \frac{t}{t_0}\right]$$

- Exponentiation sums all terms with greatest number of logs per power of α_s
  - hence the terminology, leading log parton shower

Evolution variables

- We have used the virtuality, t as the evolution variable here, but other choices are possible.
- Recall the form of the evolution equation: $\frac{dt}{t}$

  and the relations:

$$t = z(1-z)E_b^2\theta^2$$

$$p_T^2 = \theta_a^2 E_a^2 = z^2(1-z)^2 E_b^2 \theta^2$$

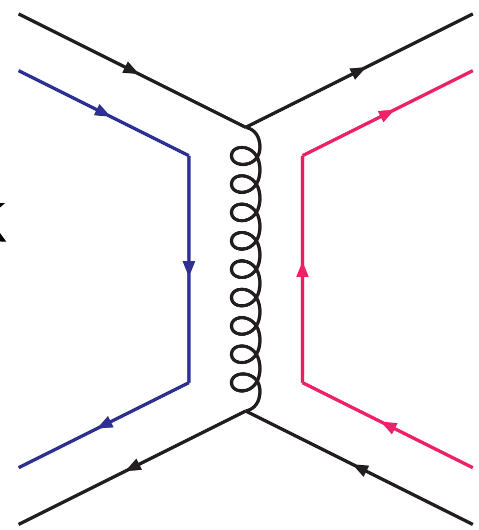
  which, for constant *z*, imply that: $\frac{dt}{t} = \frac{d\theta^2}{\theta^2} = \frac{dp_T^2}{p_T^2}$

- Different choices of variable - virtuality, angle x energy, transverse momentum - are equivalent in the collinear limit
  - but give different results away from that region
  - most modern showers p_T-ordered (except Herwig, angular-ordered)

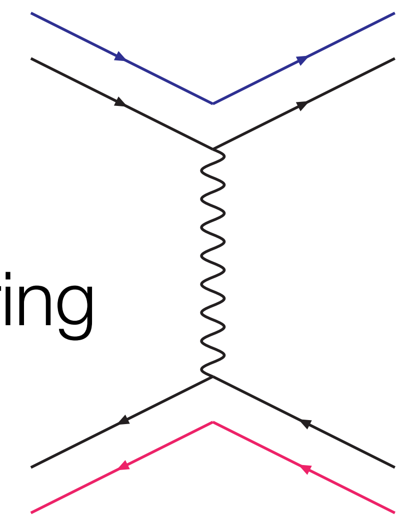
Color effects

- Parton shower is initiated by emission from a “color line” consisting of two color-connected partons
    - formally, large-color limit ($N_c \to \infty$), where gluon color = quark-antiquark color

quark-antiquark
QCD scattering



quark-antiquark
electroweak scattering



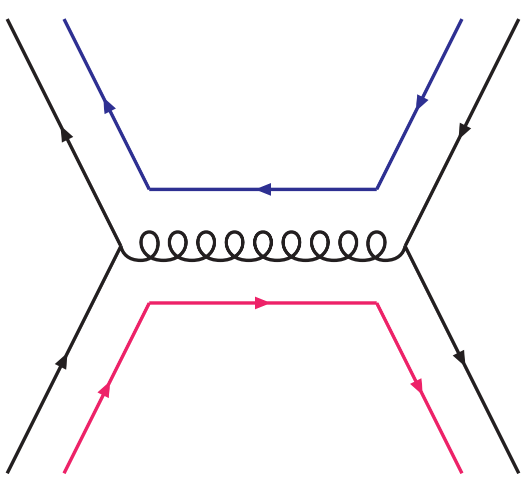
small-angle scattering: radiation throughout the event | radiation only at small angles

- Color coherence even stronger: each successive emission occurs at a smaller opening angle than the last emission → shower exact to next-to-leading log.
    - automatic in angular ordering, otherwise additional work required

Color coherence at work

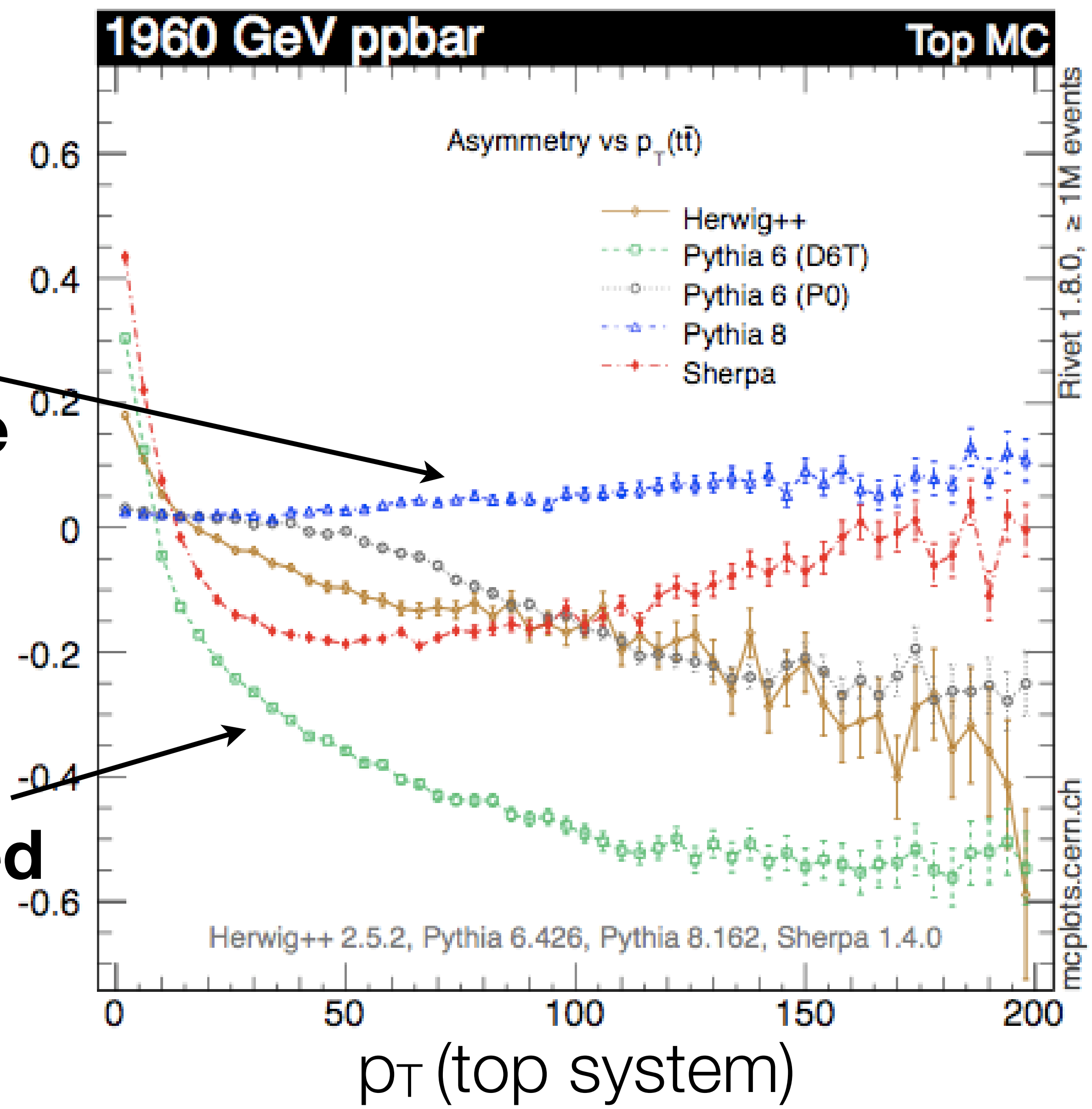
- Top forward-backward asymmetry: $$A^{t\bar{t}}_{\text{lab}} = \frac{\sigma(y_t > 0) - \sigma(y_t < 0)}{\sigma(y_t > 0) + \sigma(y_t < 0)}$$
- QCD theory: non-zero only beyond LO.
- Parton shower only LO → zero? No!

color coherence: negative asymmetry at large top system recoil



no coherence

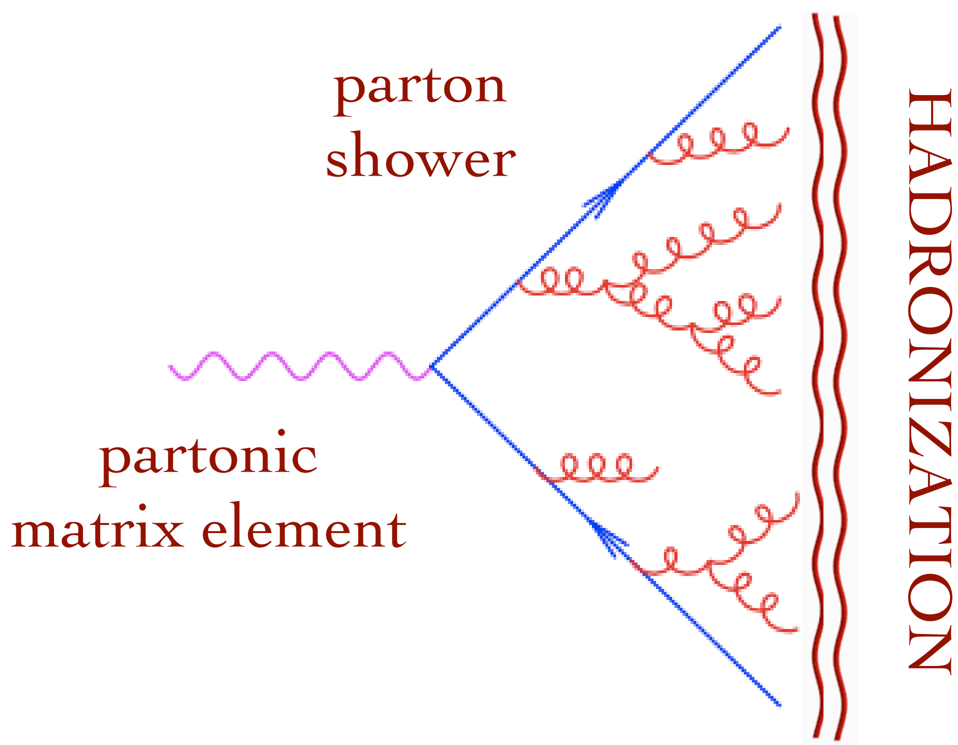
over-estimated



Ending the shower

- Eventually the evolution will bring us to a very small scale of *t* at which we no longer believe in the perturbation theory (say ~ 1 GeV). Beyond that point we no longer perform any branching.

- All partons produced in this shower are showered further, until same condition.



- Once this point is reached, no more perturbative evolution possible.

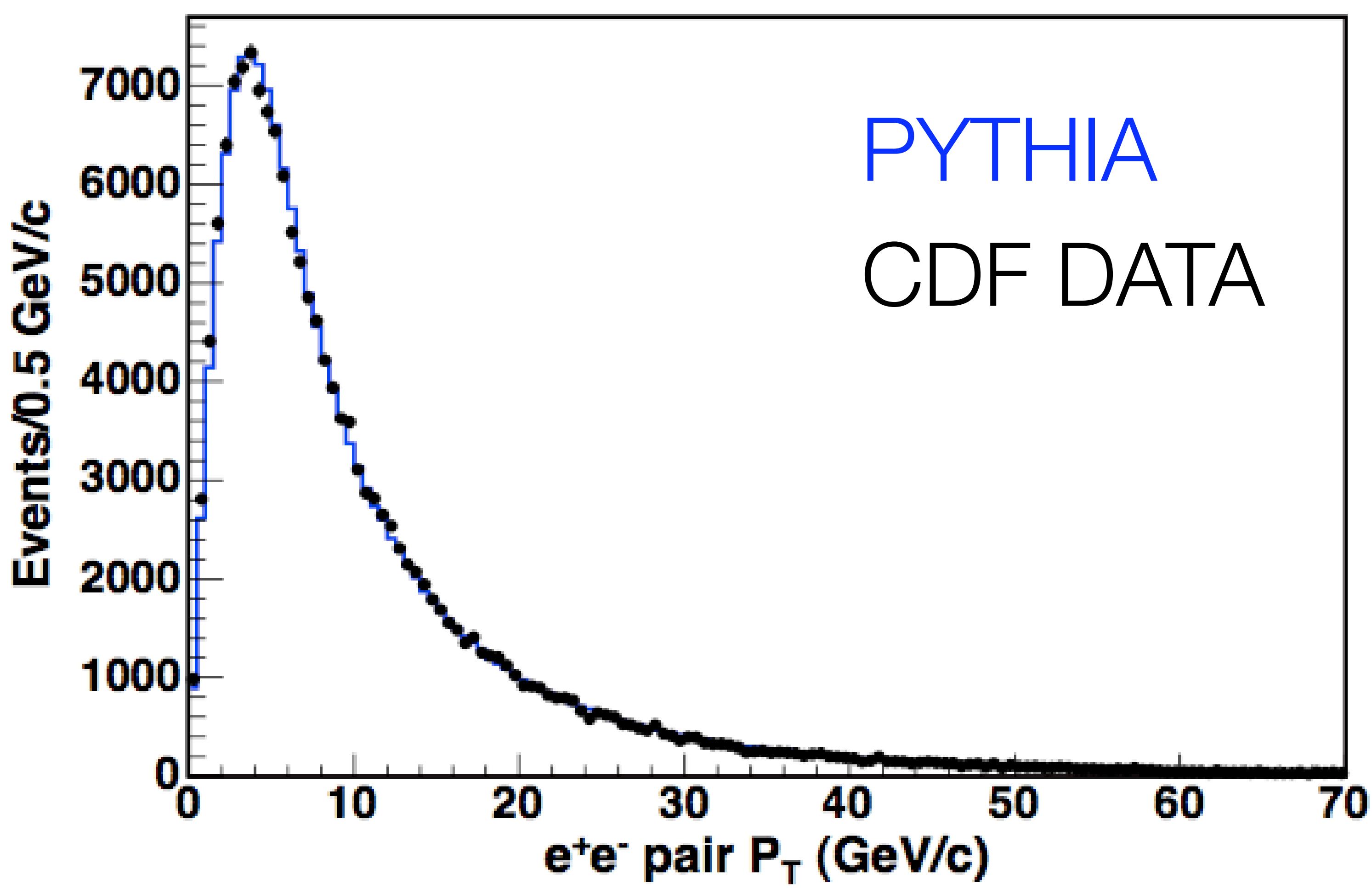
- Partons should be interpreted as hadrons according to a hadronization model.
  - examples: string model, cluster model.

- Most importantly: these are all phenomenological models.

- They require inputs that cannot be predicted from the QCD Lagrangian ab initio and must therefore be tuned by comparison with data (mostly LEP).

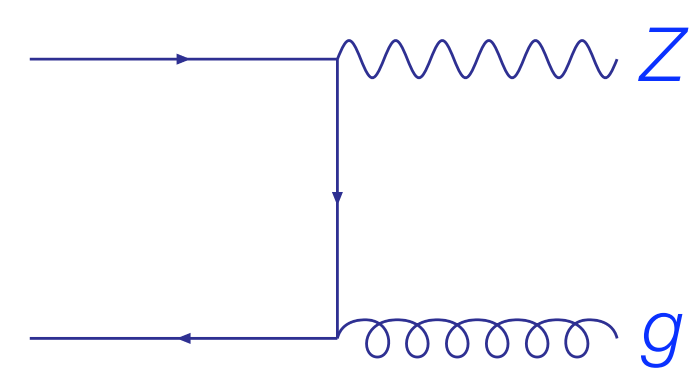
Parton shower advantages

- A parton shower allows us to (attempt to) describe features of the whole event: the output is high multiplicity final states containing hadrons.
- Very flexible framework. In principle, start with any hard scattering (e.g. any theorist's latest and greatest model) and the PS takes care of QCD radiation.



Z boson transverse momentum

- In contrast to a pure leading order prediction, a parton shower can be matched to data even at low p_T.

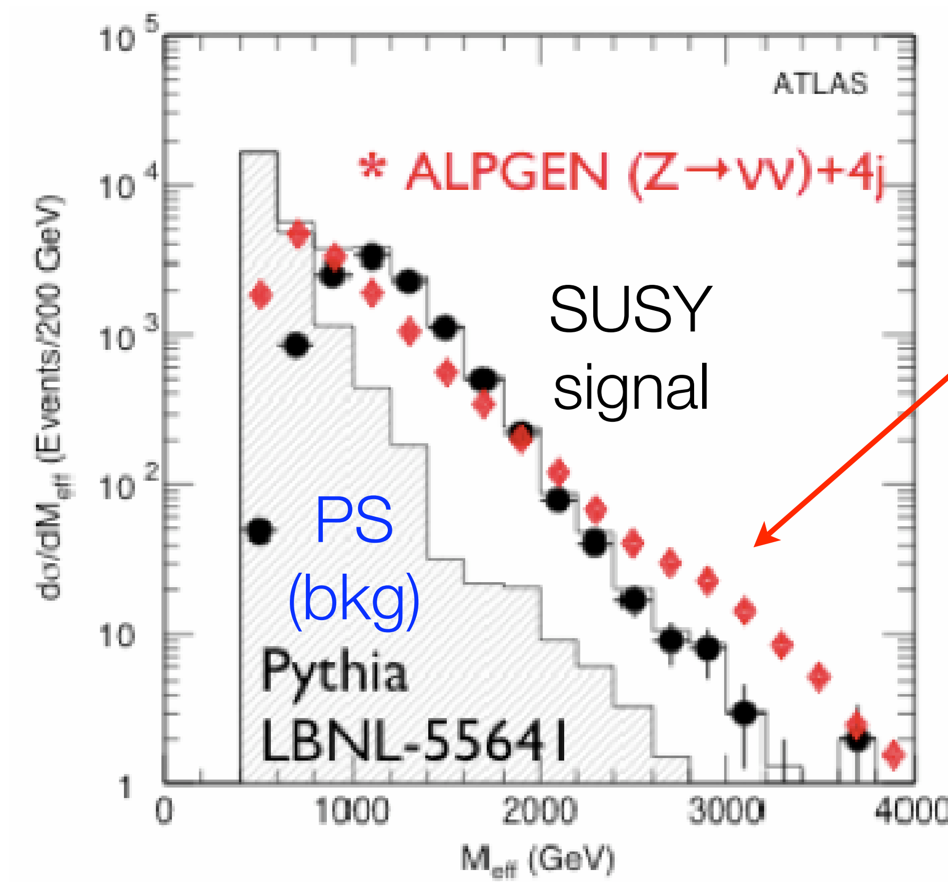


- This is true in general: broader region of applicability.

Warnings

- By construction, a parton shower is correct only for successive branchings that are collinear or soft (i.e. only leading/next-to-leading logs).

- Should therefore take care when describing final states in which there is either manifestly multiple hard radiation, or its effects might be important.
  - example: simulation of background to a SUSY search in the ATLAS TDR.



improved background calculation

$$M_{\text{eff}} = \Sigma_i |p_{T(i)}| + \not{E}_T$$

- Higher-order corrections are not included.

- Uncertainty can only be estimated by comparison with data and/or between different parton shower implementations.
  - exact details of each shower differ, possibility for significant differences.

Recap

- There are many tools capable of producing leading order cross section predictions from scratch.
- They are limited only by computer power: as a result, cannot generate more than 10 particles in the final state (program/process specific).
- The factorization of both QCD matrix elements and phase space, in the soft and collinear limits, allows us to generate arbitrarily many such branchings.
  - factorization of matrix elements: universal Altarelli-Parisi splitting functions
  - factorization of phase space: small angle approximation.
- Such a formalism leads to a DGLAP evolution equation for the probability of finding a given parton within the branching process.
- Introducing a Sudakov form factor leads to an interpretation which is easy to implement as a parton shower (e.g. Pythia, Herwig, Sherpa).
  - can describe exclusive final states (hadrons), even down to small scales;
  - in regions of hard radiation the soft/collinear approx. may not be sufficient.