



SRF cavities and Superconducting qubits for Gravitational Waves and Dark Photons detection

Roberto Menta

Supervisors : Anna Grassellino, Roni Harnik and Asher Berlin

04/10/2022

OUTLINE

● Gravitational Waves

- Theory
- Gertsenshtein effect
- Heterodyne experiment
- Noise and Sensitivity

● Dark Photons

- Theory
- Dixit et al.'s experiment
- Experiments of photons parity measurements at SQMS?

Preface

- This talk is mainly based on :

Sebastian Ellis, *Revisiting Gravitational Wave Detection in a SRF Cavity* (DESY-Talk, March 11, 2021)

Asher Berlin et al., *Detecting High-Frequency Gravitational Waves with Microwave Cavities* (2021)

Asher Berlin et al., *Axion Dark Matter Detection by Superconducting Resonant Frequency Conversion* (2019)

Asher Berlin et al., *Searches for New Particles, Dark Matter, and Gravitational Waves with SRF cavities* (2022)


Akash Dixit et al., *Searching for Dark Matter with a Superconducting Qubit* (2020)

... and ideas developed on the road in this two months

Detecting High-Frequency Gravitational Waves with Microwave Cavities

Asher Berlin,^{1,2,3} Diego Blas,^{4,5} Raffaele Tito D'Agrolo,⁶ Sebastian A. R. Ellis,^{7,8}
Roni Harnik,^{7,9} Yonatan Kahn,^{9,9,3} and Jan Schütte-Engel^{9,9,3}

¹Center for Cosmology and Particle Physics, Department of Physics,
New York University, New York, NY 10003, USA
²Theoretical Physics Division, Fermi National Accelerator Laboratory, Batavia, IL 60510, USA
³Superconducting Quantum Materials and Systems Center (SQMS),
Fermi National Accelerator Laboratory, Batavia, IL 60510, USA
⁴Grup de Física Teòrica, Departament de Física,
Universitat Autònoma de Barcelona, 08193 Bellaterra, Spain
⁵Institut de Física d'Altes Energies (IFAE), The Barcelona Institute of Science and Technology,
Campus UAB, 08193 Bellaterra (Barcelona), Spain
⁶Université Paris-Saclay, CEA, Institut de Physique Théorique, 91191, Gif-sur-Yvette, France
⁷Département de Physique Théorique, Université de Genève,
21 quai Ernest Ansermet, 1211 Genève 4, Switzerland
⁸Department of Physics, University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA
⁹Illinois Center for Advanced Studies of the Universe,
University of Illinois at Urbana-Champaign, Urbana, IL 61801, USA
(Date: December 23, 2021)

FERMILAB-PUB-22-150-SQMS-T 

Searches for New Particles, Dark Matter, and Gravitational Waves with SRF Cavities

Asher Berlin,^{2,1} Sergey Belomestnykh,^{1,3,4} Diego Blas,^{5,6} Daniil Frolov,¹ Anthony J. Brady,^{7,1}
Caterina Braggio,^{8,9,1} Marcela Carena,^{2,10,1} Raphael Cervantes,¹ Mattia Checchin,¹ Cispin Contreras-Martinez,^{1,3}
Raffaele Tito D'Agrolo,¹¹ Sebastian A. R. Ellis,¹² Grigory Ermeev,^{1,3} Christina Gao,^{15,2,1} Bianca Giaccone,¹
Anna Grassellino,^{1,3} Roni Harnik,^{1,2,7} Matthew Hollister,^{1,5} Ryan Janish,^{2,1} Yonatan Kahn,^{15,1} Sergey Kazakov,¹
Doga Murat Kurkcuoglu,^{14,1} Zhen Liu,^{15,1} Andrei Lunin,¹ Alexander Netepenko,^{1,3} Oleksandr Melnychuk,¹
Roman Pilipenko,^{1,3} Yuriy Pischalnikov,^{1,3} Sam Posen,^{1,3,11} Alex Romanenko,^{1,3} Jan Schütte-Engel,^{13,1}
Changqing Wang,¹ Vyacheslav Yakovlev,^{1,3} Kevin Zhou,¹⁶ Silvia Zorzetti,¹ and Quntao Zhuang^{7,1,17}

Axion Dark Matter Detection by Superconducting Resonant Frequency Conversion

Asher Berlin
Center for Cosmology and Particle Physics, Department of Physics,
New York University, New York, NY 10003, USA.

Raffaele Tito D'Agrolo
Institut de Physique Théorique, Université Paris Saclay, CEA, F-91191 Gif-sur-Yvette, France

Sebastian A. R. Ellis, Christopher Nantais, Jeffrey Nelson,
Philip Schuster, Sani Tanaka, Natalia Toro, and Kevin Zhou
SLAC National Accelerator Laboratory, 2575 Sand Hill Road, Menlo Park, CA 94025, USA

We propose an approach to search for axion dark matter with a specially designed superconducting radio frequency cavity, targeting axions with masses $m_a \lesssim 10^{-8}$ eV. Our approach exploits axion-induced transitions between nearly degenerate resonant modes of frequency \sim GHz. A scan over axion mass is achieved by varying the frequency splitting between the two modes. Compared to traditional approaches, this allows for parametrically enhanced signal power for axions lighter than a GHz. The projected sensitivity covers unexplored parameter space for QCD axion dark matter for 10^{-8} eV $\lesssim m_a \lesssim 10^{-9}$ eV and axion-like particle dark matter as light as $m_a \sim 10^{-14}$ eV.



Revisiting Gravitational Wave Detection in an SRF Cavity

Sebastian A. R. Ellis
IPHT, CEA Saclay

Based on:
210x.xxxxxx
A. Berlin, R. T. D'Agrolo, SARE

PHYSICAL REVIEW LETTERS **126**, 141302 (2021)

Featured in Physics

Searching for Dark Matter with a Superconducting Qubit

Akash V. Dixit^{1,2,3,4}, Srivatsan Chakram,^{1,2,4} Kevin He^{1,2}, Ankur Agrawal^{1,2,3}, Ravi K. Naik⁵,
David I. Schuster,^{1,2,6} and Aaron Chou⁷

¹James Franck Institute, University of Chicago, Chicago, Illinois 60637, USA

²Department of Physics, University of Chicago, Chicago, Illinois 60637, USA

³Kavli Institute for Cosmological Physics, University of Chicago, Chicago, Illinois 60637, USA

⁴Department of Physics and Astronomy, Rutgers University, Piscataway, New Jersey 08854, USA

⁵Department of Physics, University of California Berkeley, Berkeley, California 94720, USA

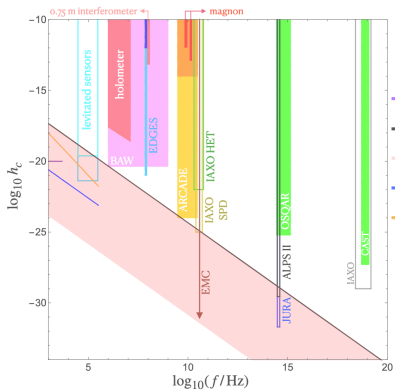
⁶Pritzker School of Molecular Engineering, University of Chicago, Chicago, Illinois 60637, USA

⁷Fermi National Accelerator Laboratory, Batavia, Illinois 60510, USA

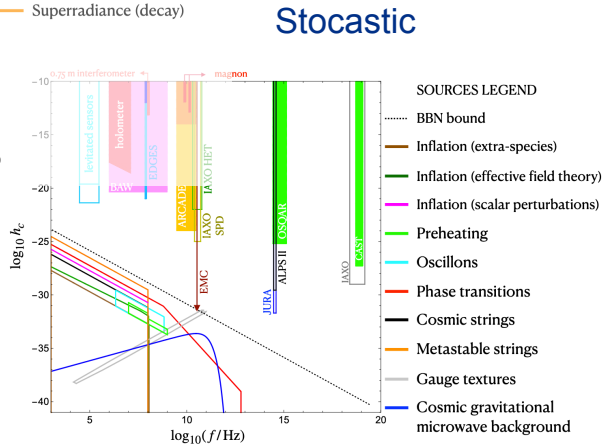
(Received 19 October 2020; revised 5 February 2021; accepted 22 February 2021; published 8 April 2021)

Gravitational Waves, a long story...

- Theorized by A. Einstein in the Theory of General Relativity, 1915.
- First observation by LIGO and Virgo in the regime of Hz-kHz, 2016.



Coherent



Stochastic

- The Universe is expected to be populated by GW over decades in frequency.
- ➔ Development of RF cavities for GW detection to explore a larger regime of frequencies. *Pegorato et al. (1978)*.
- In the last few years, Superconducting RF cavities for GW and Dark Matter detection. *Berlin et al.*

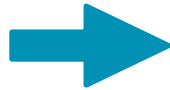
Gravitational Waves - Theory

The *linearized theory* of GR is invariant under the Poincaré group

$$g_{\mu\nu} = \eta_{\mu\nu} + h_{\mu\nu} \quad |h_{\mu\nu}| \ll 1$$

Einstein equations :

$$R_{\mu\nu} - \frac{1}{2}g_{\mu\nu}R = \frac{8\pi G}{c^4}T_{\mu\nu}$$



GW equation of motion :

$$\begin{cases} \square \bar{h}_{\mu\nu} = -\frac{16\pi G}{c^4}T_{\mu\nu} \\ \partial^\nu \bar{h}_{\mu\nu} = 0 \quad (\text{Lorentz gauge}) \end{cases}$$

A GW can most easily described in the TT-gauge :

$$\partial_\mu h^{\mu\nu} = 0, \quad h_\mu^\mu = 0, \quad h_{00} = h_{0i} = 0$$

$$h_{\mu\nu}^{TT} = H_{\mu\nu} e^{i\omega(t-z)}, \quad H_{\mu\nu} = \begin{pmatrix} 0 & 0 & 0 & 0 \\ 0 & h_+ & h_\times & 0 \\ 0 & h_\times & h_+ & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$$R_{0i0j} = -\frac{1}{2}\ddot{h}_{ij}^{TT}$$



Monochromatic
GW in z-direction

Gravitational Waves - Theory

Electromagnetism in presence of gravity : $\partial_\mu \rightarrow \nabla_\mu$

$$\begin{cases} \nabla_\mu F^{\mu\nu} = -\frac{4\pi}{c} J^\nu \\ \nabla_{[\mu} F_{\nu\rho]} = 0 \end{cases} \longrightarrow \begin{aligned} \partial_\mu F^{\mu\nu} &\simeq J^\nu \left(1 + \frac{h_\alpha^\alpha}{2} \right) - h^{\nu\alpha} J_\alpha + \frac{\partial_\mu (h_\alpha^\alpha F^{\mu\nu})}{2} + \\ &+ \partial_\mu (h^{\mu\alpha} F_\alpha^\nu + h^{\nu\alpha} F_\alpha^\mu) \end{aligned}$$

A variation of metric (GWs) acts as EM sources

- GR-EM interaction is encapsulated in the Einstein-Hilbert action :

$$\mathcal{S} = -\frac{1}{4} \int d^4x \sqrt{-g} g^{\mu\alpha} g^{\nu\beta} F_{\mu\nu} F_{\alpha\beta} \propto \int h F^2 \longrightarrow \text{Inverse-Gertsenshtein effect}$$

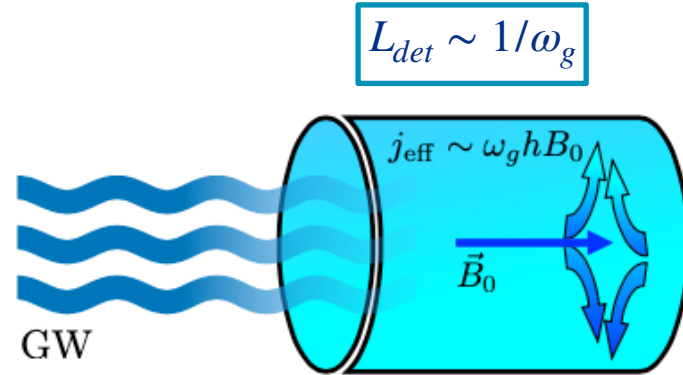

Gertsenshtein effect, a classical interpretation

- Formalism of *effective* current

$$\mathcal{S}[\mathcal{O}(h)] = -\frac{1}{2} \int d^4x j_{\text{eff}}^\mu A_\mu$$

$$j_{\text{eff}}^\mu \equiv \partial_\nu \left(\frac{1}{2} h F^{\mu\nu} + h^\nu_\alpha F^{\alpha\mu} - h^\mu_\alpha F^{\alpha\nu} \right)$$

(not invariant under gauge transf.)



GW - cavity interaction

- Direct interaction : inverse-Gertsenshtein effect
- Indirect or mechanical interaction : GW perturbs the cavity wall, $\omega_0 \sim \mathcal{O}(1)/L_{\text{det}}$

- GW is on-resonant with an eigenmode of the cavity and couples to a static B-field
- Good method to detect high-frequency GW, $f \sim \mathcal{O}(\text{GHz})$
- Method already established in ADMX

Gertsenshtein effect - electromagnetic interaction

- Coupling of GW to electromagnetic field can be described in weak field limit by

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} - \frac{1}{2}j_{\text{eff}}^{\mu}A_{\mu}$$

- The GW induced current was given by $j_{\text{eff}}^{\mu} \equiv \partial_{\nu} \left(\frac{1}{2}hF^{\mu\nu} + h^{\nu}_{\alpha}F^{\alpha\mu} - h^{\mu}_{\alpha}F^{\alpha\nu} \right)$
- We define a normalized current $\vec{j}_{+,x}$ as $\vec{j}_{\text{eff}}(\vec{x}) := B_0\omega_g^2 V_{\text{cav}}^{1/3} \left(h_+ \vec{j}_+(\vec{x}) + h_{\times} \vec{j}_{\times}(\vec{x}) \right)$
- A GW on resonant with a cavity mode \vec{E}_n induces a signal

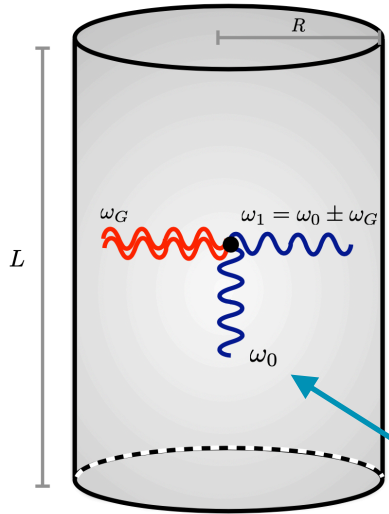
$$P_{\text{sig}} = \frac{1}{2} Q \omega_g^3 V_{\text{cav}}^{5/3} (\eta_n h_0 B_0)^2$$

$$\eta_n \equiv \frac{\left| \int_V d^3\mathbf{x} \mathbf{E}_n^* \cdot \hat{\mathbf{j}}_{+,x} \right|}{V^{1/2} \left(\int_V d^3\mathbf{x} |\mathbf{E}_n|^2 \right)^{1/2}}$$

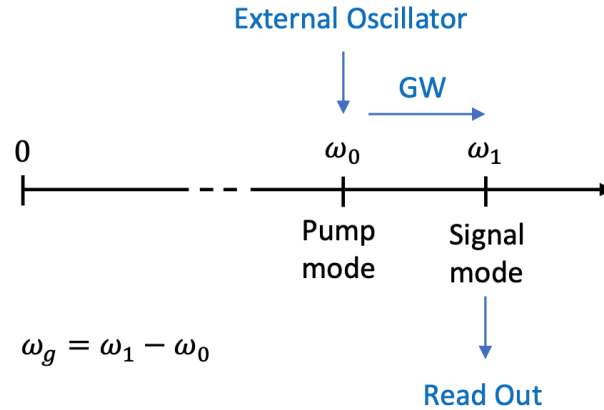
$h_0 = h_+ \text{ or } h_{\times}$

Heterodyne experiments

- The GW is on-resonant with the frequency difference of two cavity modes and couples to both E- and B- field.



Cylinder for illustrative purposes only!



Resonant B-field
in the cavity

- $\omega_0 \sim \mathcal{O}(\text{GHz})$
- $Q_{int} \simeq 10^9 \div 10^{12}$
- Tunability $\delta\omega \sim \text{MHz}$

The effect of \vec{B}_{sig} are different depending on the \vec{B}_0 :

$$\Delta B_{sig} \sim hB_0\sqrt{Q} \min\left[\frac{\omega_g^2}{\omega_0^2}, 1\right] \text{ (oscill)}$$

$$\Delta B_{sig} \sim hB_0\sqrt{Q} \min\left[\frac{\omega_g^2}{\omega_0^2}, \frac{\omega_g}{\omega_0}, 1\right] \text{ (static)}$$

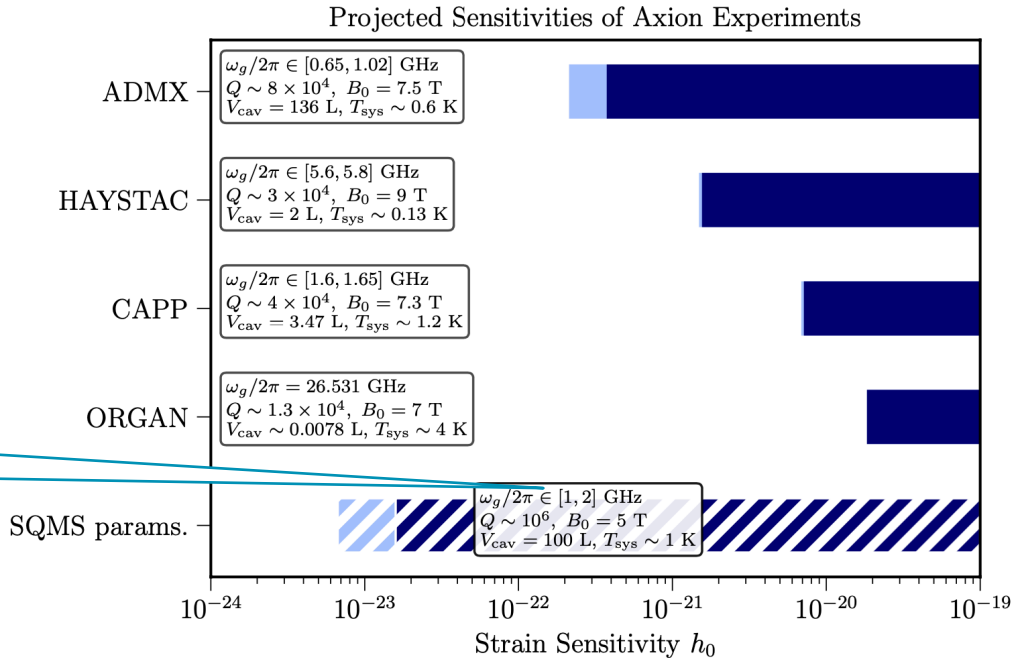
High Frequency Sensitivity

- Estimation of the the projected sensitivity of axion cavity haloscope experiments to high-frequency coherent gravitational waves.

$$\omega_g/2\pi \in [1, 2] \text{ GHz}$$

$$Q \sim 10^6 \quad B_0 = 5 \text{ T}$$

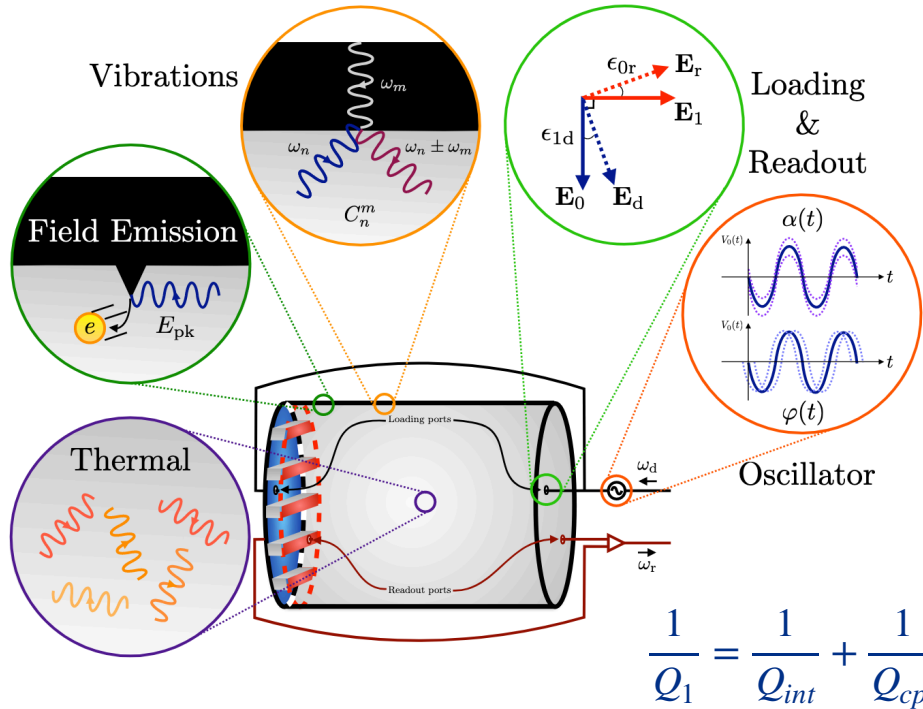
$$V_{cav} = 100 \text{ L} \quad T_{sys} \sim 1 \text{ K}$$



A.Berlin et al.

Noise sources

- Advantage of heterodyne setup: noise sources are well investigated



- Every noise sources drives additional power into the signal mode described by PSD.
 - Thermal Noise (cavity walls):
- $$S_{th}(\omega) = \frac{Q_1}{Q_{int}} \frac{4\pi T k_B (\omega \omega_1 / Q_1)^2}{(\omega^2 - \omega_1^2)^2 + (\omega \omega_1 / Q_1)^2}$$
- Amplifier Noise:

$$S_{ql}(\omega) = 4\pi \hbar \omega_1$$

Noise sources

- Phase Noise (Oscillator):

$$S_{\text{phase}}(\omega) \simeq \frac{1}{2} \epsilon_{1d}^2 \underbrace{S_{\phi}(\omega - \omega_0)}_{\text{Input Oscillator}} \underbrace{\frac{(\omega\omega_1/Q_1)^2}{(\omega^2 - \omega_1^2)^2 + (\omega\omega_1/Q_1)^2}}_{\text{Cavity response (B-W)}} \underbrace{\frac{\omega_0 Q_1}{\omega_0 Q_0} P_{\text{in}}}_{\text{Overall Normalization}}$$

- Mechanical Noise :

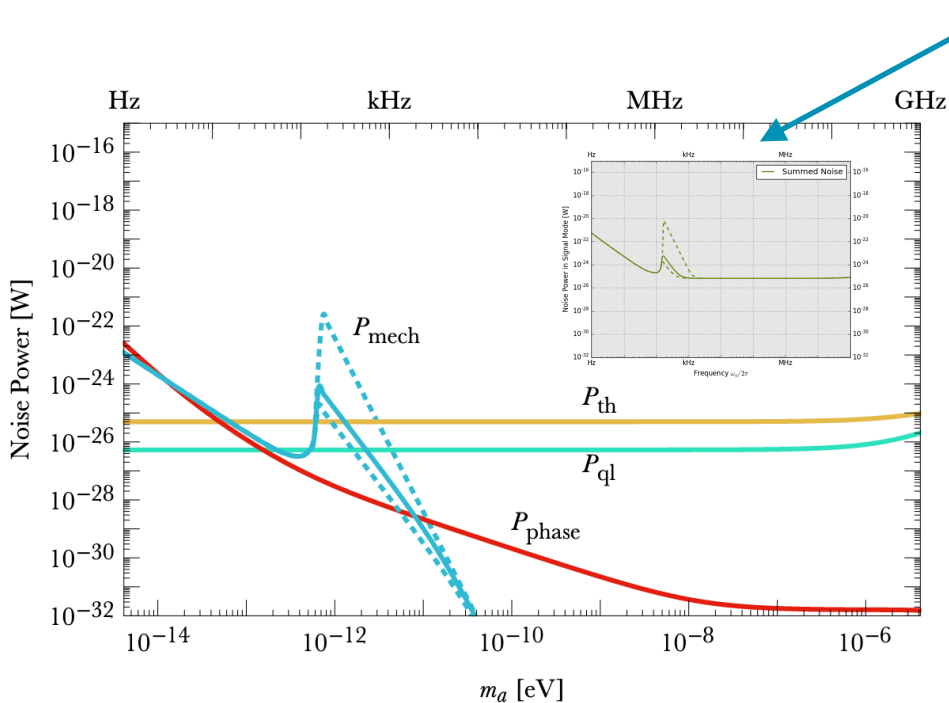
$$S_{\text{mech}}(\omega) = \sum_{n=0,1} S_{\text{mech}}^{(n)}(\omega) \simeq \frac{\epsilon_{1d}^2}{4} \frac{\omega_0}{Q_0} P_{\text{in}} \sum_{n=0,1} \frac{\overbrace{(S_{q_m}(\omega - \omega_0) / V^{2/3})}^{\text{Wall Displacement}} (\omega_n / Q_n) \omega_n^4 \omega^2}{\underbrace{\left[(\omega^2 - \omega_n^2)^2 + (\omega\omega_n / Q_n)^2 \right] \left[(\omega_0^2 - \omega_n^2)^2 + (\omega_0\omega_n / Q_n)^2 \right]}_{\text{CavityResponse}}}$$

- All PSDs can be summed up

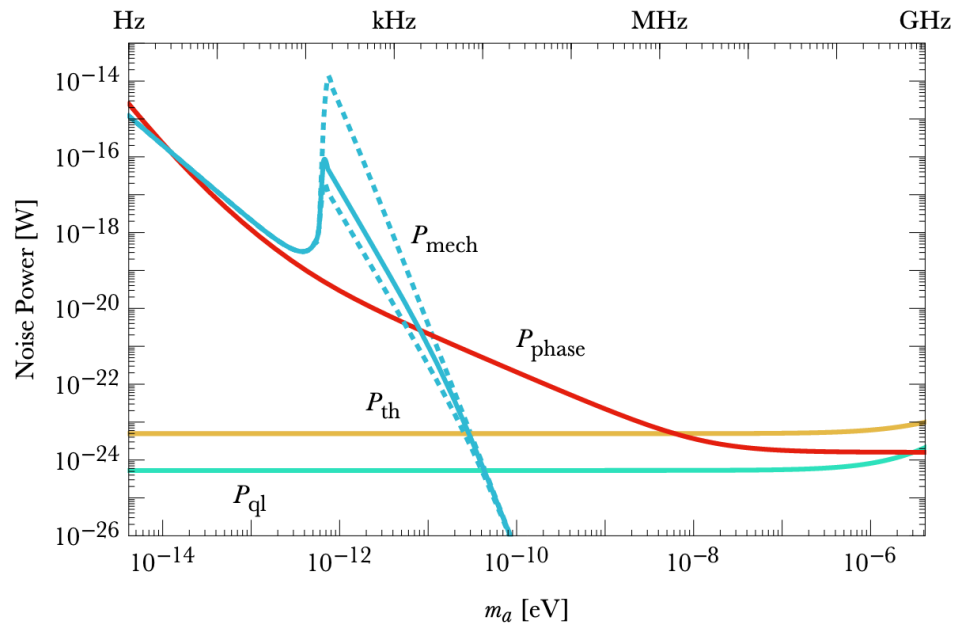
$$S_{\text{noise}}(\omega) = S_{ql}(\omega) + \frac{Q_1}{Q_{cpl}} \left(S_{th}(\omega) + S_{\text{phase}}(\omega) + S_{\text{mech}}^{(1)}(\omega) \right) + \frac{Q_0}{Q_{cpl}} S_{\text{mech}}^{(0)}(\omega)$$

Plot of different noises

Summed PSDs of noises



(a) $\epsilon_{1d} = 10^{-7}$, $Q_{int} = 10^{12}$



(b) $\epsilon_{1d} = 10^{-5}$, $Q_{int} = 10^{10}$

Quantum Sensors for GW: Cavity-qubit system and Squbit ††

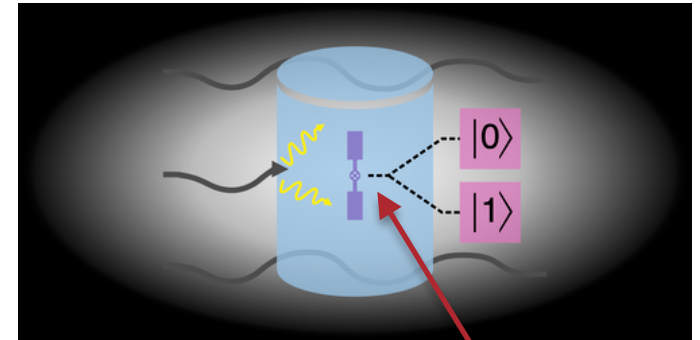
- **Cavity-qubit system** : the suggestion is that $\text{SNR} \rightarrow \text{SNR} \times \sqrt{\frac{T_{cav}}{T_R}}$

$$|\psi\rangle = |g\rangle|1\rangle + |e\rangle|0\rangle$$

Problem : not enough sensitivity in phase's measurements

- **Squbit - GW** : draft idea is to find a mapping between the spinor field of the QED in the chiral representation and the two level states of the qubit.

$$\psi = \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} \quad \psi_R \mapsto |0\rangle, \psi_L \mapsto |1\rangle$$



readout on qubit

†† ultra-preliminary

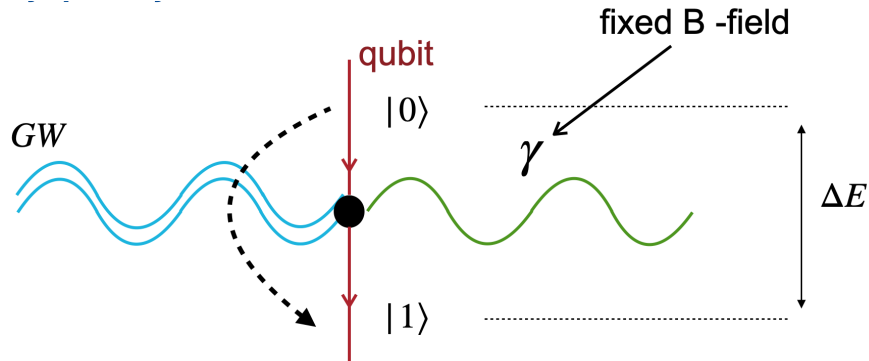
Quantum Sensors for GW: Cavity-qubit system and Squbit ††

- Squbit - GW

$$\begin{aligned} \mathcal{L}_{dip,int}^{QED} &= \Lambda \bar{\psi} \sigma^{\mu\nu} \gamma^5 \psi F_{\mu\nu} = \Lambda (\psi_R^\dagger, \psi_L^\dagger) \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \sigma^{\mu\nu} \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \begin{pmatrix} \psi_R \\ \psi_L \end{pmatrix} F_{\mu\nu} \\ &= \Lambda (\psi_L^\dagger, \psi_R^\dagger) \sigma^{\mu\nu} \begin{pmatrix} \psi_R \\ -\psi_L \end{pmatrix} \eta_{\mu\alpha} h_{\nu\beta} F^{\alpha\beta} + \dots \propto \Lambda (\psi_L^\dagger \psi_R - \psi_R^\dagger \psi_L) \eta_{\mu\alpha} h_{\nu\beta} F^{\alpha\beta} \end{aligned}$$

$$\sigma^\mu = (1, \sigma^i)$$

$$\sigma^{\mu\nu} = \frac{i}{2} (\bar{\sigma}^\mu \sigma^\nu - \bar{\sigma}^\nu \sigma^\mu)$$



†† ultra-preliminary

Dark Photons

- It is a *massive* photon m_{df} , a gauge boson of $U(1)$ symmetry.
- Group symmetry of extended SM : $SU(3) \times SU(2) \times U(1) \times U(1)$
- Any heavy particle that is charged, both photons will generate mixing.



⊃ dark photons?

- The only renormalizable interaction between the dark and visible photons that we can construct is the *kinetic mixing* term.

Nature has already ordered extra copies of fermions. Why not gauge bosons?

	1 st	2 nd	3 rd	
Quarks	u up	c charm	t top	γ' dark photon
	d down	s strange	b beauty	γ photon
	e electron	μ muon	τ tau	W^\pm W boson
Leptons	ν_e neutrino electron	ν_μ neutrino muon	ν_τ neutrino tau	Z^0 Z boson
				g gluon
				H Higgs Boson

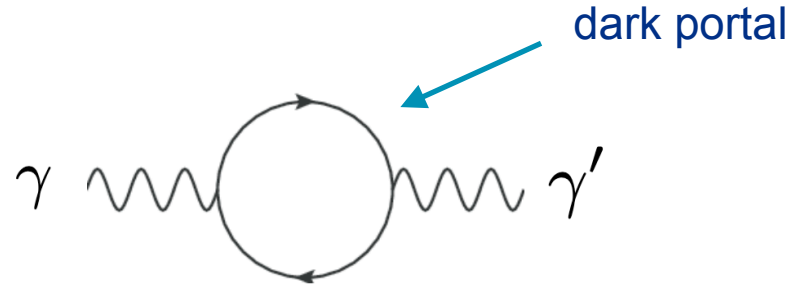
$$\rho_{DM} \simeq 0.4 \text{ GeV/cm}^3$$

Dark Photons

$$\mathcal{L} = \mathcal{L}_{SM} - \frac{1}{4} F'_{\mu\nu} F'^{\mu\nu} + \frac{1}{2} m_{A'}^2 A'_\mu A'^\mu + \epsilon F^{\mu\nu} F'_{\mu\nu} \supset \epsilon(\vec{E} \cdot \vec{E}' + \vec{B} \cdot \vec{B}')$$

$$\mathcal{L}_{int} \propto -e\epsilon J_\mu^{EM} A'^\mu$$

(kinetic mixing)

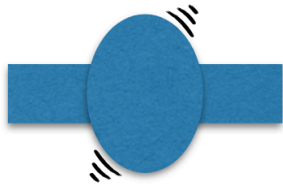


An oscillating EM field is a source of dark photons, and vice versa.

There are two different kind of experiment for dark photon search

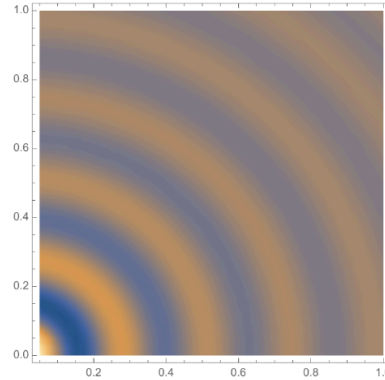
- Light-shining-through-a wall: **Dark SRF**
- Parity measurement of photons number in a SRF cavity : **Dixit Experiment**

Dark SRF : the first simple setup

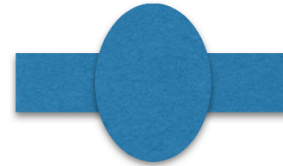


Emitter Cavity

Frequency of 1.3 GHz,
excited to ~ 35 MV/m.
Thats $\sim 10^{25}$ Photons!



a dark photon
field is radiated
at 1.3 GHz.



Receiver Cavity

Tuned to 1.3 GHz.
Responds to dark field.
Contains only thermal
noise ($T=1.4$ K).

Dixit et al.'s experiment

- **Setup:** cavity-qubit system for parity measurements of photons number.
- **Qubit-based photon counter :** QND techniques.

$$H/\hbar \simeq \omega_c a^\dagger a + \frac{1}{2}(\omega_q + 2\chi a^\dagger a)\sigma_z$$

$$\hat{n} = a^\dagger a$$

$$\bar{n} \ll 1 \Rightarrow n = 0, 1$$

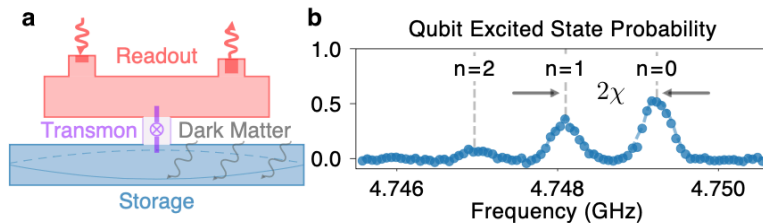
Frequency qubit shift

(1) We go to the *rotating* ω_q - frame

$$|g\rangle \text{ or } |e\rangle \xrightarrow{\pi/2 - \text{pulse}} \frac{|g\rangle + |e\rangle}{\sqrt{2}}$$

precedes with a frequency $|2\chi|$ if there is one photon.

Ramsey measurements



$$\hbar\omega_c = m_{df}c^2$$

Expt. Parameter	Θ	σ_Θ
Quantum efficiency	$\eta = 0.409$	$\sigma_\eta = 0.055$
Storage cavity frequency	$\omega_s = 6.011 \text{ GHz}$	$\sigma_{\omega_s} = 205 \text{ Hz}$
Storage quality factor	$Q_s = 2.06 \times 10^7$	$\sigma_{Q_s} = 8.69 \times 10^5$
Storage cavity volume	$V = 11.8 \text{ cm}^3$	$\sigma_V = 0.2 \text{ cm}^3$
Storage form factor	$G = 0.22$	$\sigma_G = 0.003$

Dixit et al.'s experiment

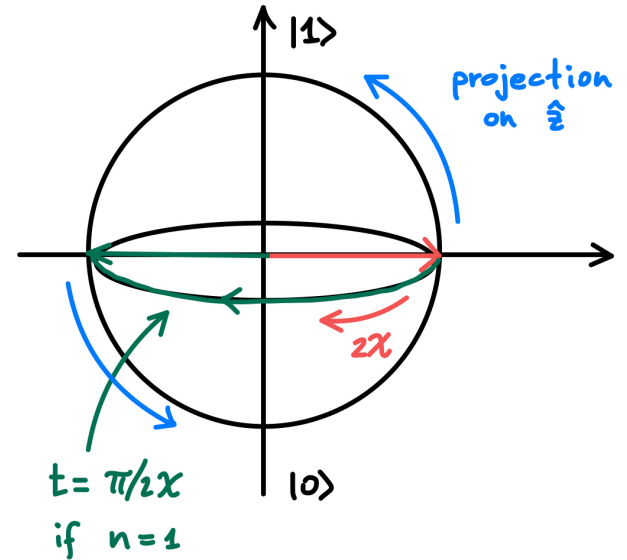
- (2) if there is the photon, after a time $t = \pi/2\chi$ the state was flipped in the plane.
- (3) we make a projection on \hat{z} axis, with a $-\pi/2$ pulse.

If there are zero photons in the cavity, the qubit remains in its initial state. If there is one photon in the cavity, the qubit state is flipped ($|g\rangle \leftrightarrow |e\rangle$).

$$V = 11.8 \text{ cm}^3$$

$$\rho_{DM} \simeq 0.4 \text{ GeV/cm}^3$$

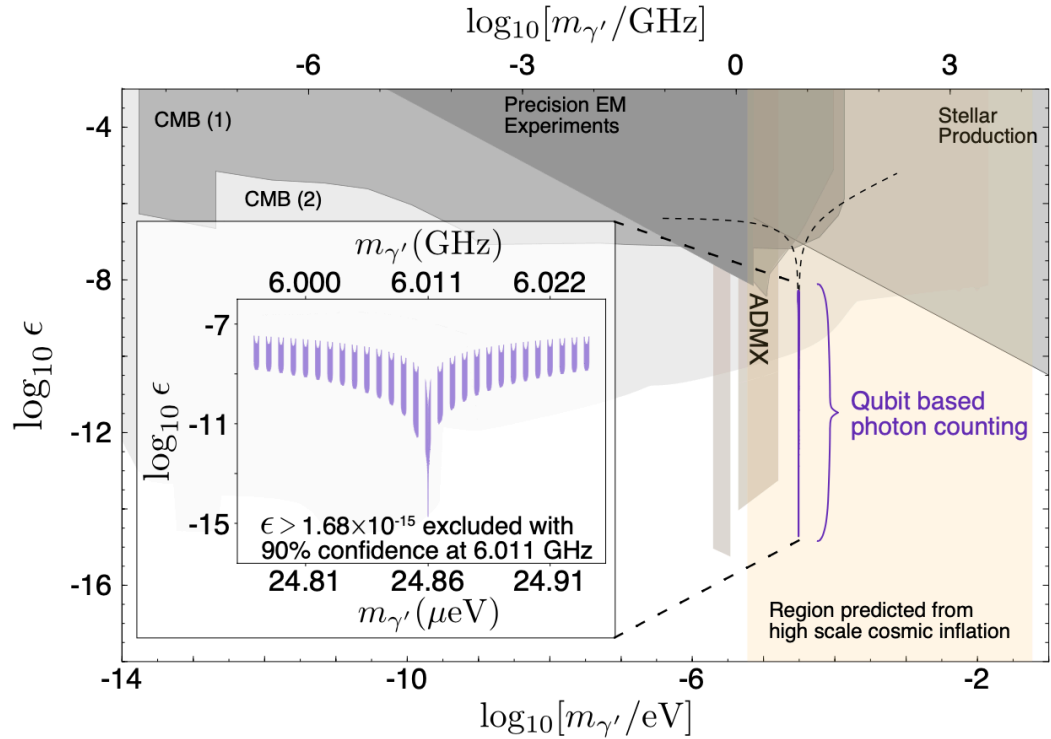
$$n_{df} \simeq \rho_{DM}/m_{df} \Rightarrow N_{df} \simeq n_{df} \cdot V \simeq \left(\frac{0.4 \text{ GeV}}{\text{cm}^3}\right) \times \left(\frac{1}{\mu\text{eV}}\right) \times 11.8 \text{ cm}^3 \simeq 10^{15}$$



Dixit et al.'s experiment

The experiment has excluded dark photon candidates with mass centred around $m_{df} \sim 24.86 \mu\text{eV}$ (6.011 GHz) and $\epsilon \geq 1.68 \times 10^{-15}$ by using a superconducting qubit to repeatedly measure the same photon.

Integration time : $T_1^s = 8.33 \text{ s}$



What about using Fock states of photons?

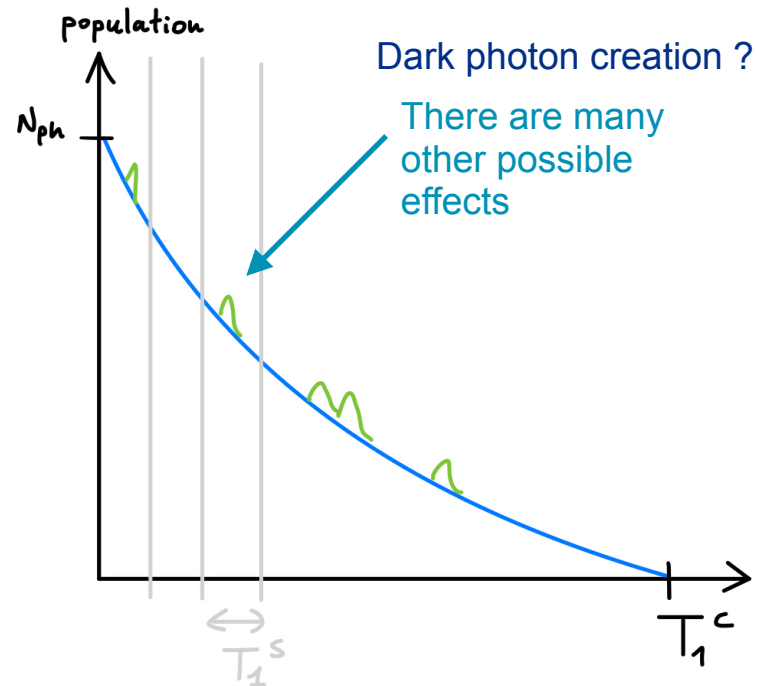
- We prepare the cavity with a N_{ph} photons (Fock state).
- After an amount of time T_1^c the photons population is expected to go to zero.
- We can do $\simeq T_1^c/T_1^s$ parity measurements.

Hold on...is there an advantage using this protocol?

$$\begin{aligned} \frac{df_\gamma}{dt} &= \Gamma_{\gamma' \rightarrow \gamma}(1 + f_\gamma) - \Gamma_{\gamma \rightarrow \gamma'}(1 + f_{\gamma'}) \\ &\propto f_{\gamma'}(1 + f_\gamma) - f_\gamma(1 + f_{\gamma'}) \\ &= f_{\gamma'} - f_\gamma \simeq f_{\gamma'} \end{aligned}$$

$$N_{df} \gg N_{ph} \Rightarrow$$

Answer: NO



Conclusion and next steps...

● Gravitational Waves

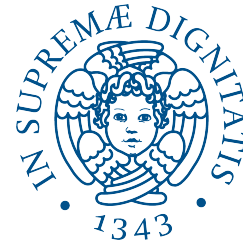
- More studies to understand is the cavity-qubit system can have an advantage in the sensitivity
- Theoretical work: understand the qubit-GW interaction. It means to unify linearized GR with quantum formalism of qubit.

● Dark Photons

- No advantage in using Multiphotons Fock states.
- Reproduce the Dixit experiment : use SQMS cavities of different sizes to explore a larger regime of m_{df} . The biggest cavity has a volume of 209.367 cm^3 , i.e. a resonant frequency of $\simeq 3 \text{ GHz} \Rightarrow N_{df} \simeq 10^{17}$.

Thank you for listening !

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Northwestern
University