Muon Colliders and $(g-2)_\mu$

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+Rodolfo Capdevilla, David Curtin, Yonatan Kahn

2101.10334, 2006.16277, 2112.08377

Muon Collider Physics and Detector Workshop, FNAL 12/14/22

Overview

Current Status of g-2

Singlet Models

Electroweak Models

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Singlet Models

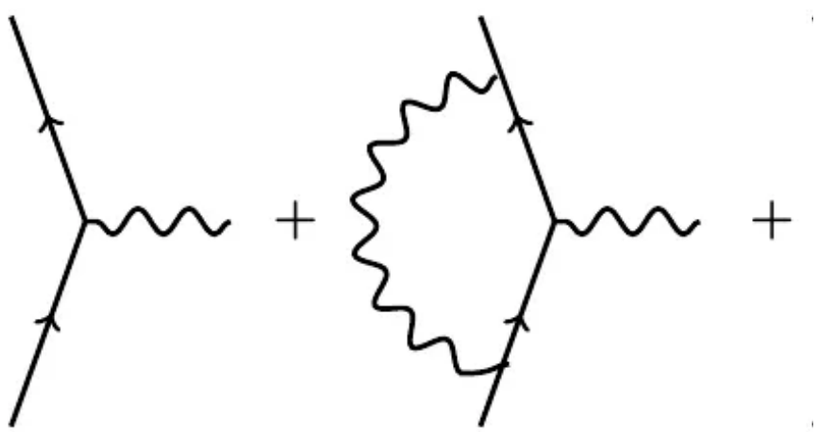
Electroweak Models

Muon Anomalous Magnetic Moment

Lepton dipole moment $\vec{\mu}_\ell = \pm g_\ell \frac{e}{2m_\ell}\vec{S}$ $\qquad a \equiv \frac{g-2}{2}$

Tree level QED prediction: $a = 0$

Quantum loop corrections: $a \neq 0$



$+ \cdots \rightarrow g = 2 + \frac{\alpha}{2\pi} + \cdots$

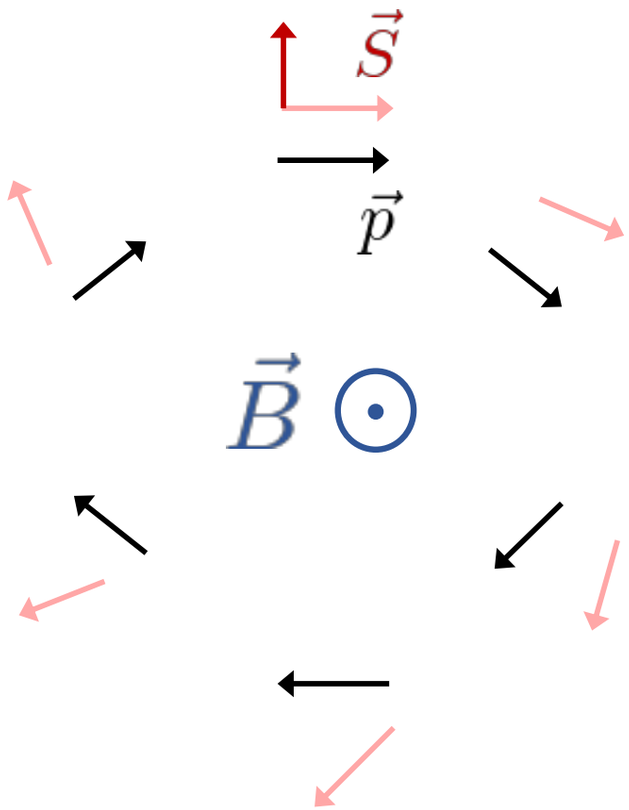
Sensitive to all known ***and unknown*** particles coupled to leptons

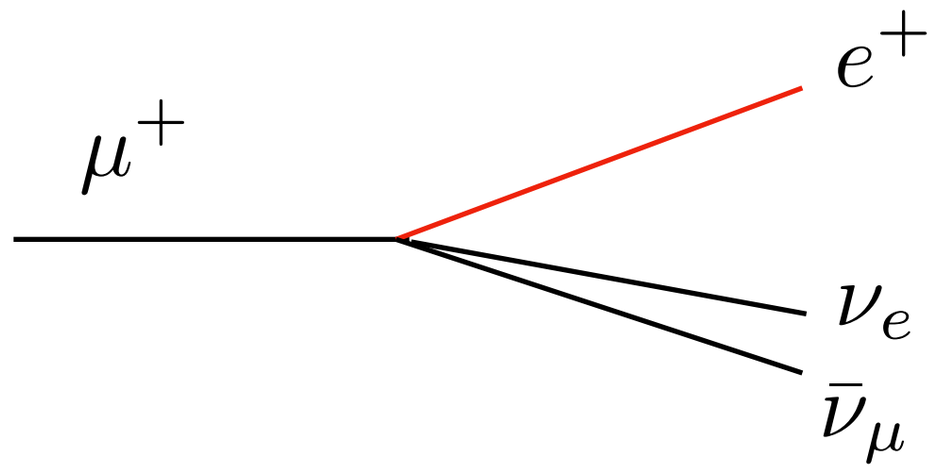
For electrons agrees SM to ~ 12 decimals, best prediction in history

Spin precession in a uniform B field

$$\omega_a = a_\mu \frac{eB}{2m_\mu}$$

Lab Frame





$$\frac{d\Gamma}{dE} = A\left[1 - B(\vec{s}\cdot\vec{p})\right] \qquad \vec{s}\cdot\vec{p} \propto \cos\omega_a t$$

Upon $\mu^+ \to e^+\nu_e\bar{\nu}_\mu$ decay e^+ emitted preferentially along $\vec{S}$
Asymmetry in e^+ energy distribution measures ω_a

Graphic: Ryan Janish

Spin precession in a uniform B field

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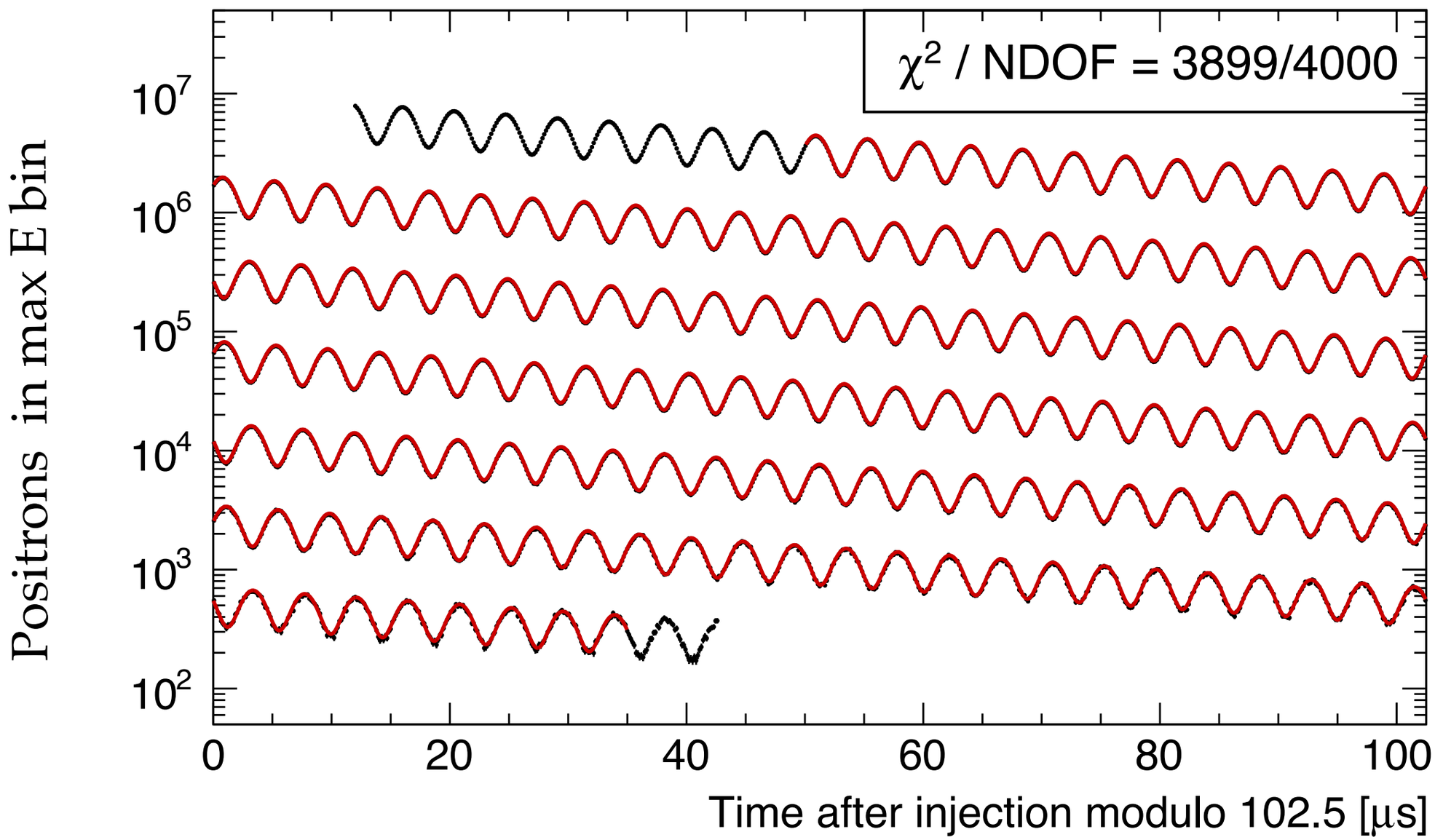
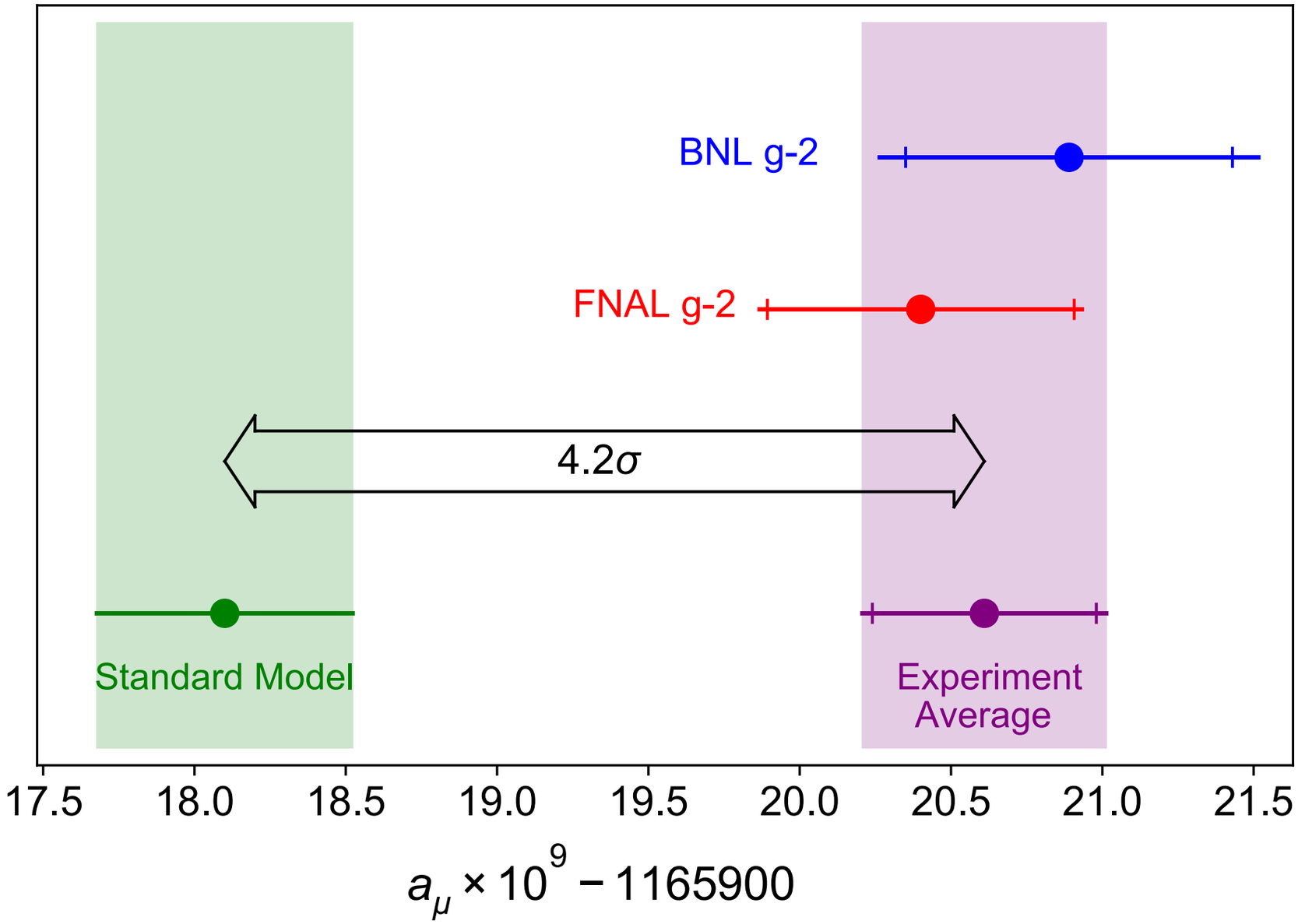


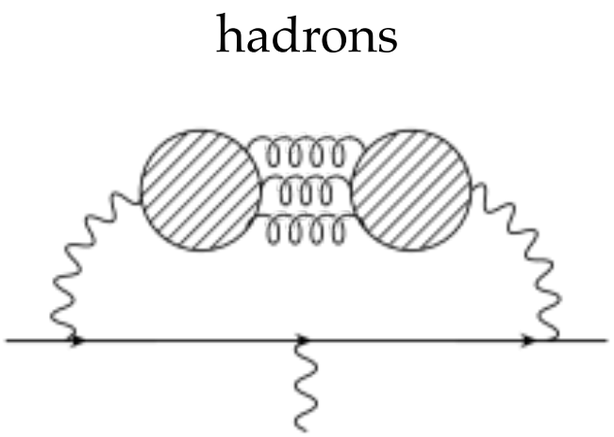
$$N_e(t) \propto e^{-t/\tau_\mu}\left(1 + B' \cos \omega_a t\right)$$

g-2 collaboration 2104.03247

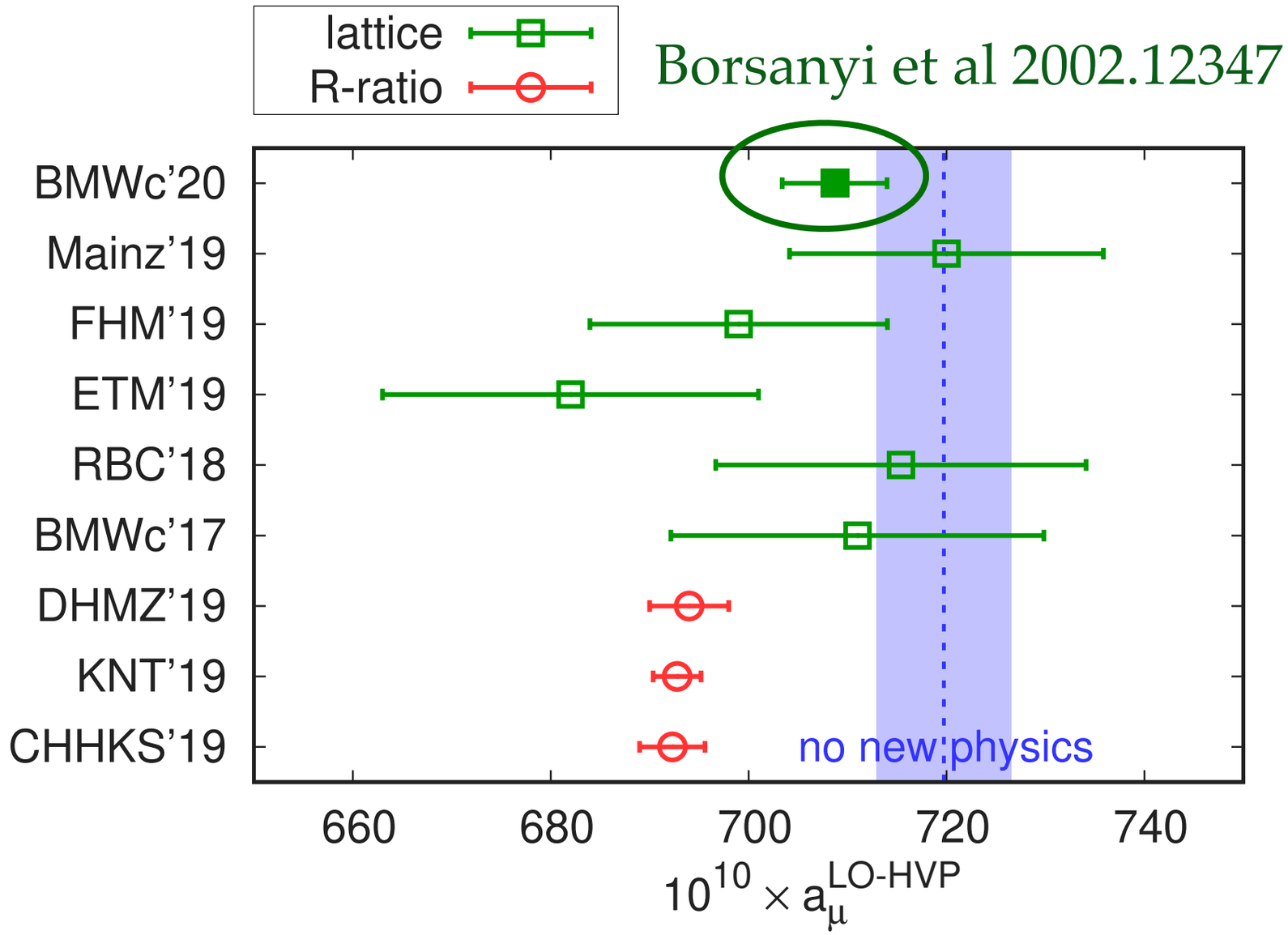
Theory vs. Experiment



Theory uncertainty driven by
non-perturbative QCD corrections



Comparing SM Theory Calculations

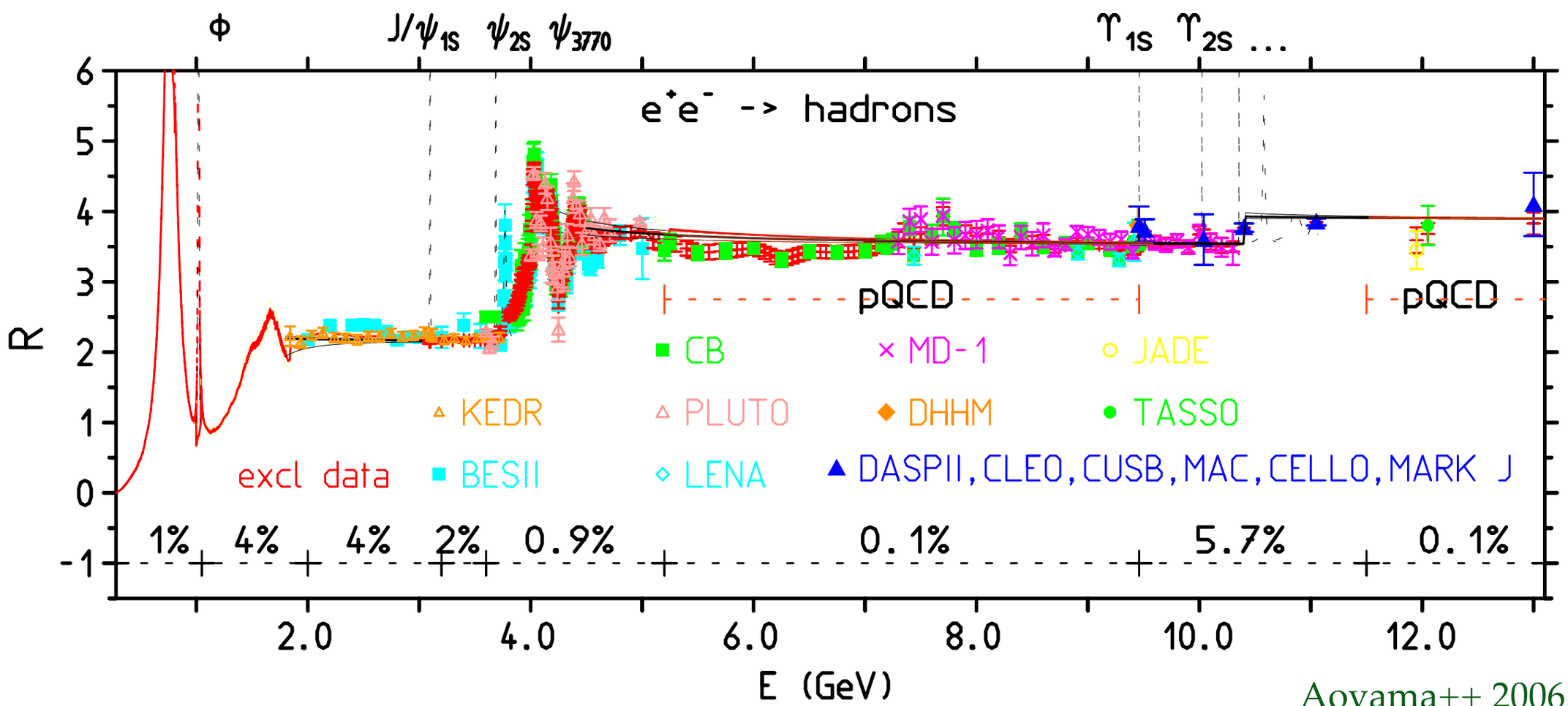


Recent lattice BMWc result in tension with data driven R-ratio method

… but it's closer to experiment

R-Ratio Calculations

Hadronic contributions can be extracted from $e^+e^- \to$ hadrons data

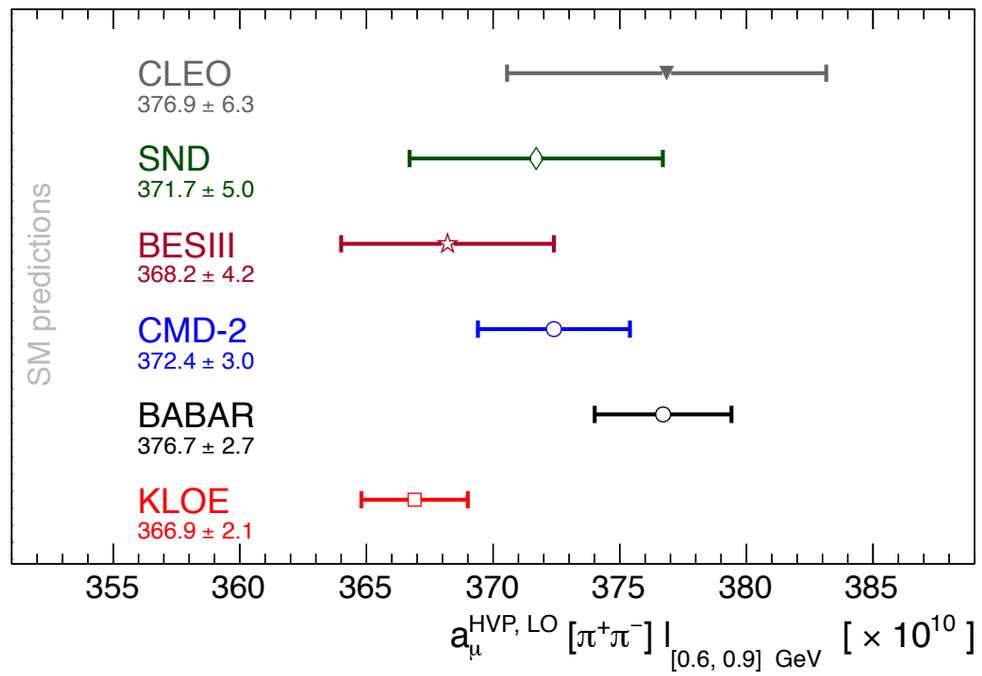


Aoyama++ 2006.04822

$$a_\mu^{\text{HVP, LO}} = \frac{\alpha^2}{3\pi^2}\int_{M_\pi^2}^{\infty}\frac{K(s)}{s}R(s)\,ds$$

$$R(s) \propto \sigma(e^+e^- \to \text{hardons})$$

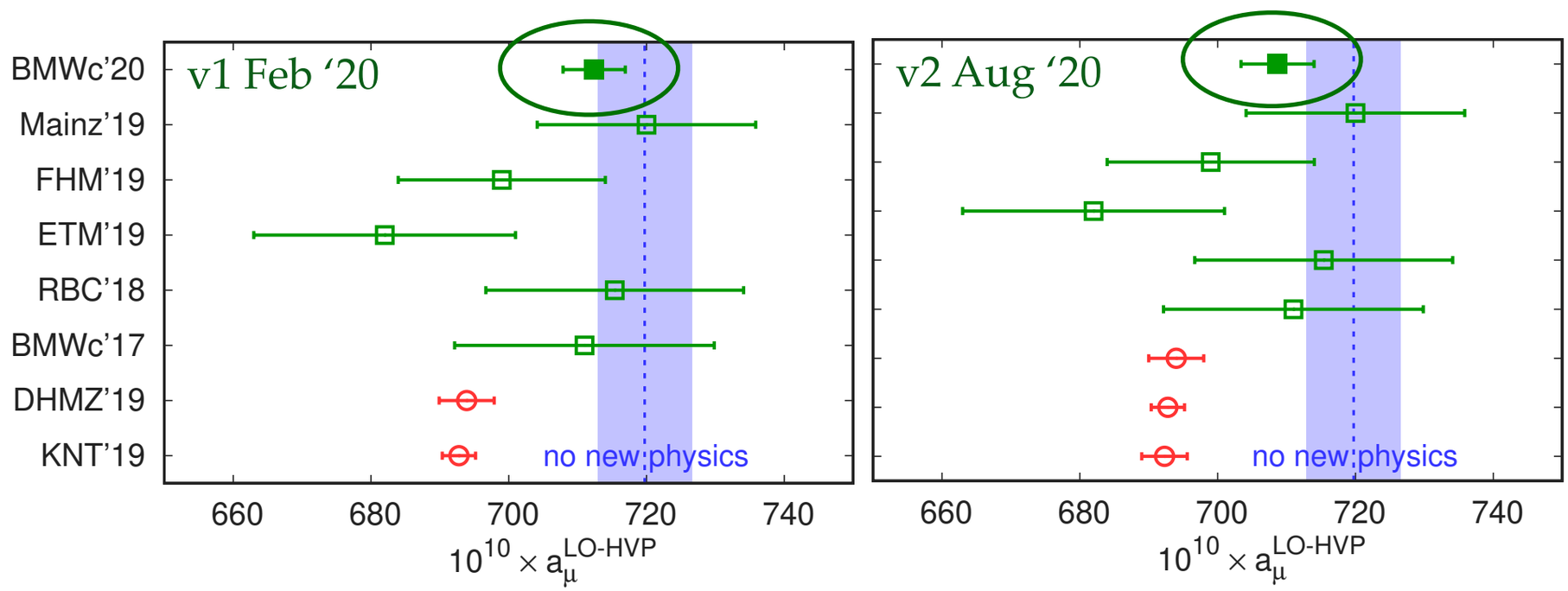
R-Ratio Possible issue of internal consistency across full data set?



Aoyama++ 2006.04822

Borsanyi++ 2002.1234

Lattice



Possible issue of extrapolating to continuum limit?

BMW also makes electroweak fit worse and in tension with $e^+e^- \to \pi\pi$

What should we believe?

1) Issue with with R-ratio calculations?

Possible, but nothing obvious (maybe tension in data?)

2) Issue with lattice calculations?

Need confirmation from other groups

Possible hint from tension in electroweak fit?

Crivellin, Hoferichter, Manzari, Montull 2003.04886

3) R-ratio correct, but unknown experimental systematic?

After new data, this is extremely unlikely

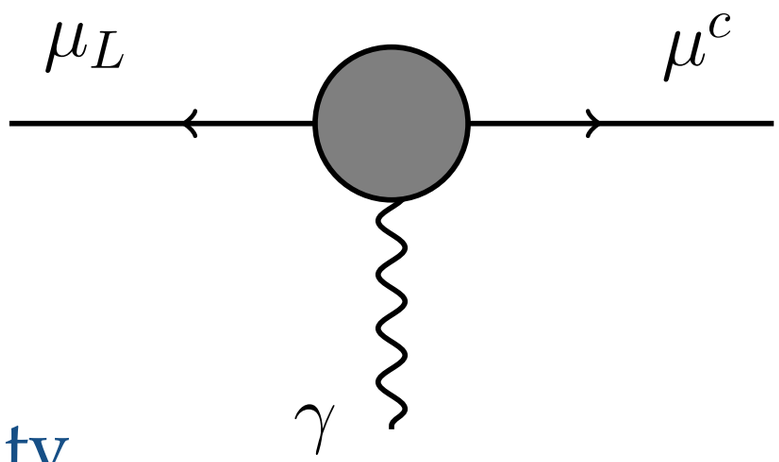
This is the main new thing we have learned

4) New particles interacting with the muon?

Effective Operator Analysis

Generic interaction to g-2 can be written as

$$\mathcal{L}_{\text{eff}} = \frac{1}{\Lambda}\left(\mu_L \sigma^{\nu\rho} \mu^c\right) F_{\nu\rho}$$



In 2 component notation: arrows track chirality

Arrows point in **opposite** direction **(chiral flip)**

but like with Yukawas, this is not gauge invariant

$$\mathcal{L}_{\text{mass}} = y_\mu H^\dagger L \mu^c \;\rightarrow\; y_\mu v \mu_L \mu^c \equiv m_\mu \, \mu_L \mu^c$$

… so also need an **EWSB insertion**

$$\psi_{\text{dirac}} = \begin{pmatrix} \mu_L \\ \mu^{c\dagger} \end{pmatrix}$$

Effective Operator Analysis

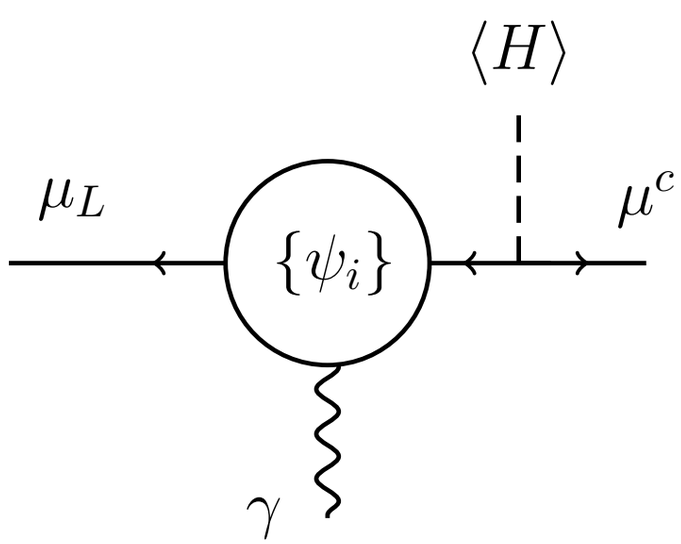
$$\mathcal{L}_{\text{eff}} = C_{\text{eff}} \frac{v}{M^2} (\mu_L \sigma^{\nu\rho} \mu^c) F_{\nu\rho} + \text{h.c.}$$

Do EWSB + chiral flip come from muon mass insertion?

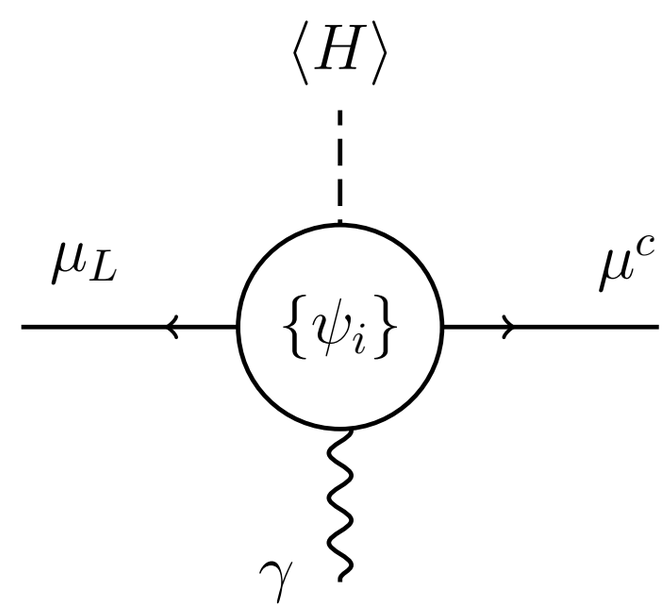
YES

NO

Singlet Models



Electroweak Models



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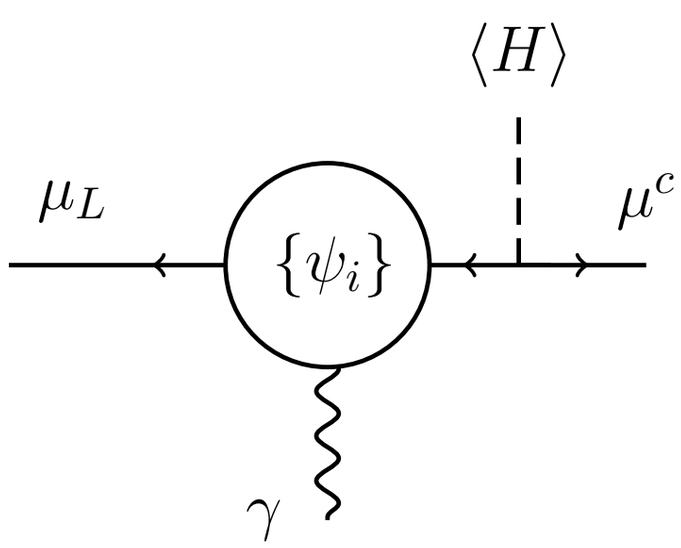
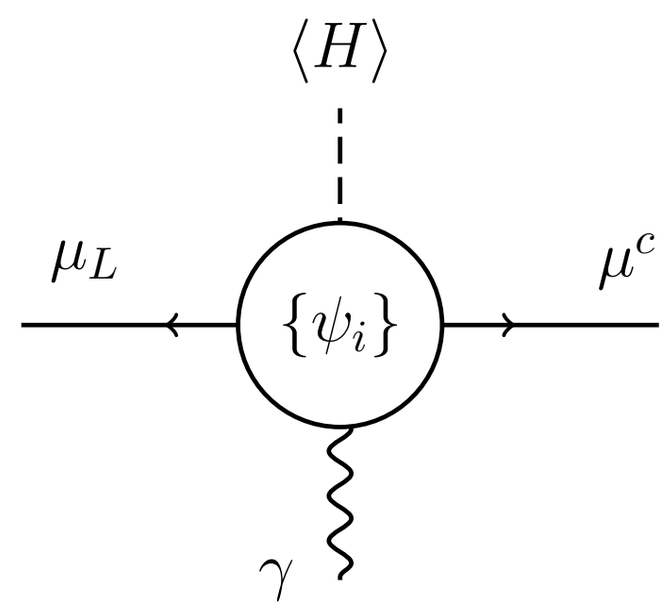


Electroweak Models



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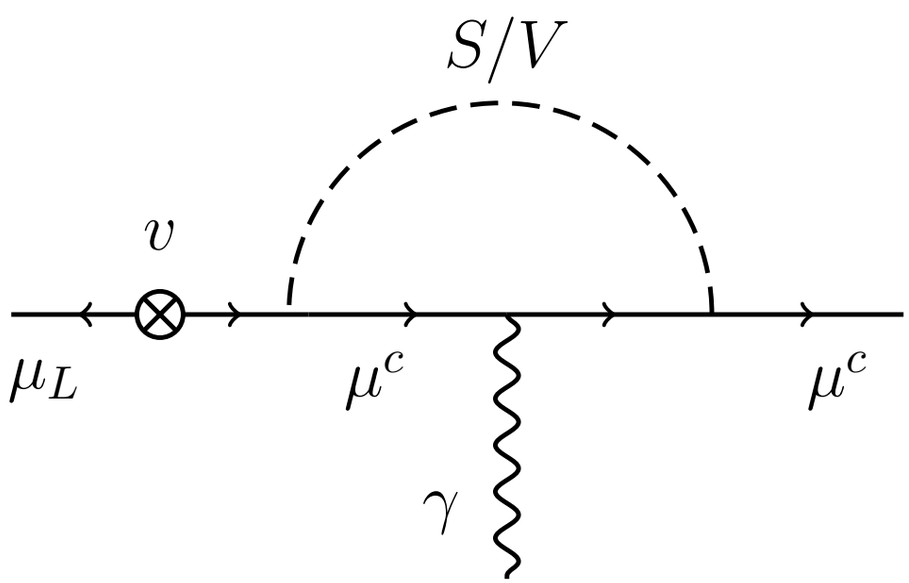
Chiral flip and EWSB on muon line



Simple BSM Landscape

Must be scalar (S) or vector (V)

Must be SM gauge singlet

Must be MeV-TeV (BBN/unitarity)

$$\Delta a_\mu^V = \frac{g_V^2}{4\pi^2}\int_0^1 dz \frac{m_\mu^2 z(1-z)^2}{m_\mu^2(1-z)^2 + m_V^2\, z} \simeq 1.3\times 10^{-10}\left(\frac{g_V}{10^{-4}}\right)^2 \quad (m_V \ll m_\mu)$$

Pospelov 0811.1030

Pseudo scalar or axial vector make g-2 worse

Options For Singlets

1) Mix S/V with neutral SM bosons

2) Couple S/V to heavy states that mix with the muon

3) V is the gauge boson of a new U(1) SM extension

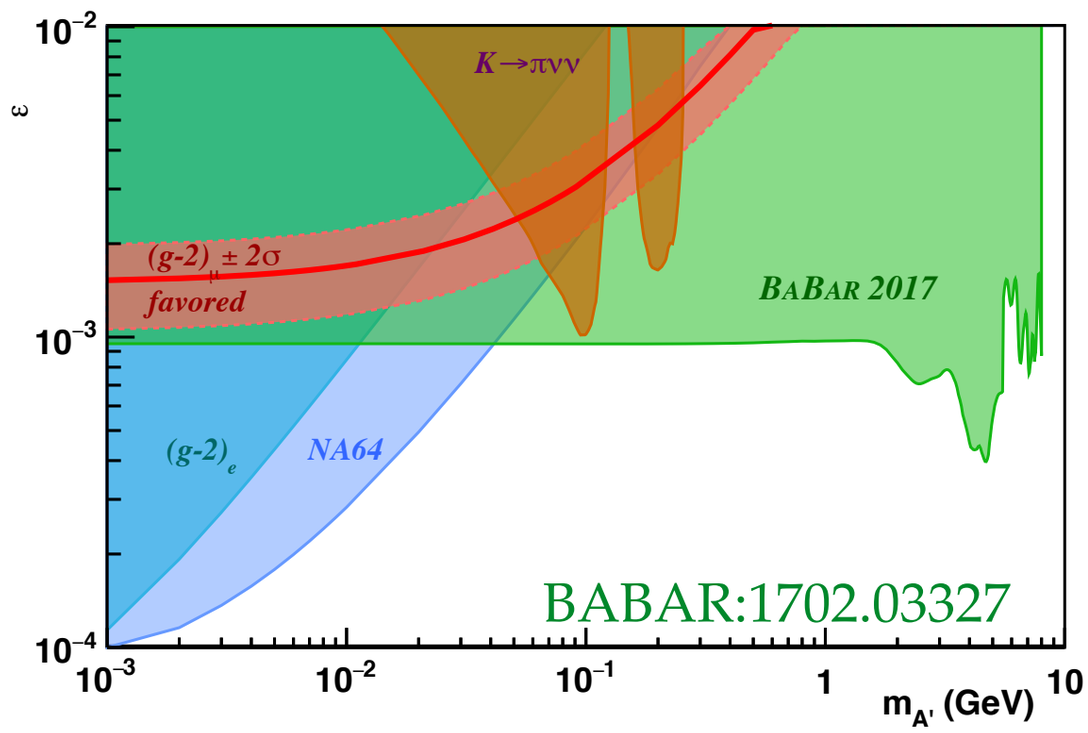
Options For Singlets

1) Mix S/V with neutral SM bosons

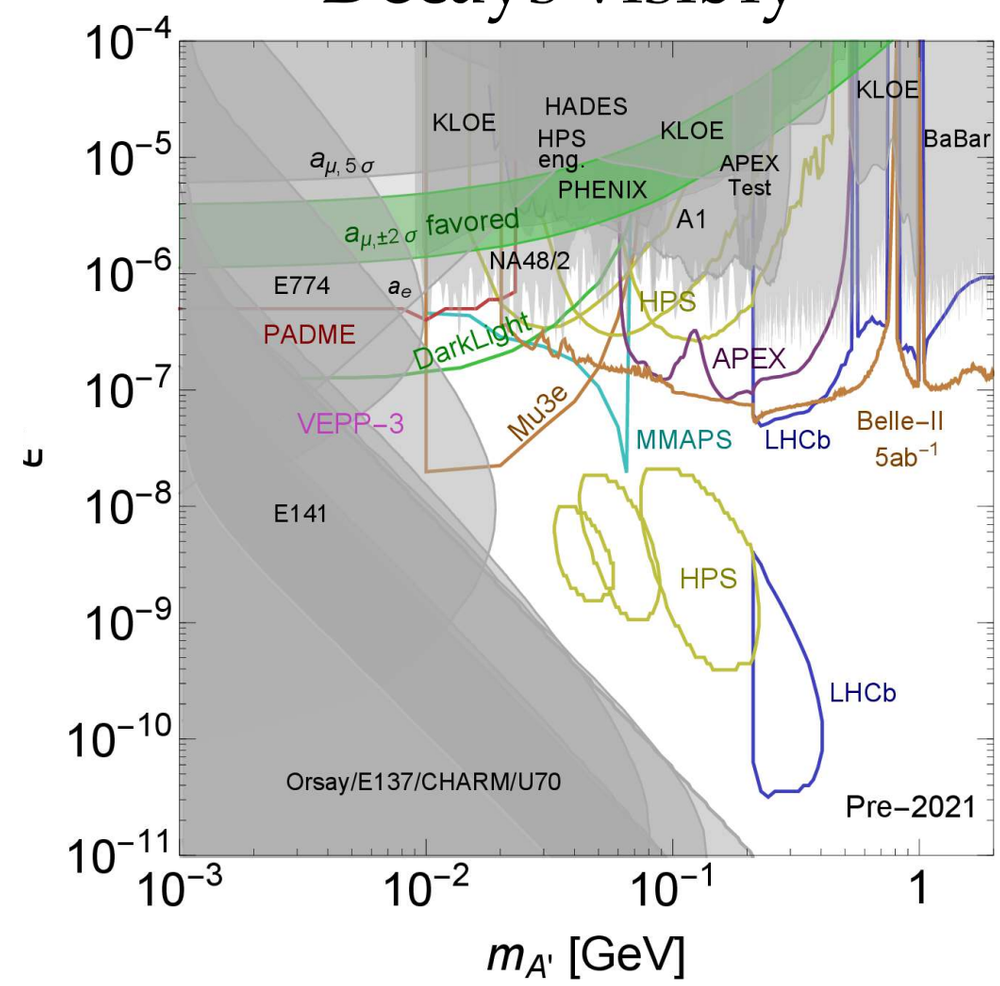
Kinetically mixed dark photon A' ruled out

$$\mathcal{L}_{\rm int} = \epsilon e A'_\mu J^\mu_{\rm EM}$$

Decays invisibly



Decays visibly



See Mohlabeng for semi-visible decays 1809.07768

1707.04591

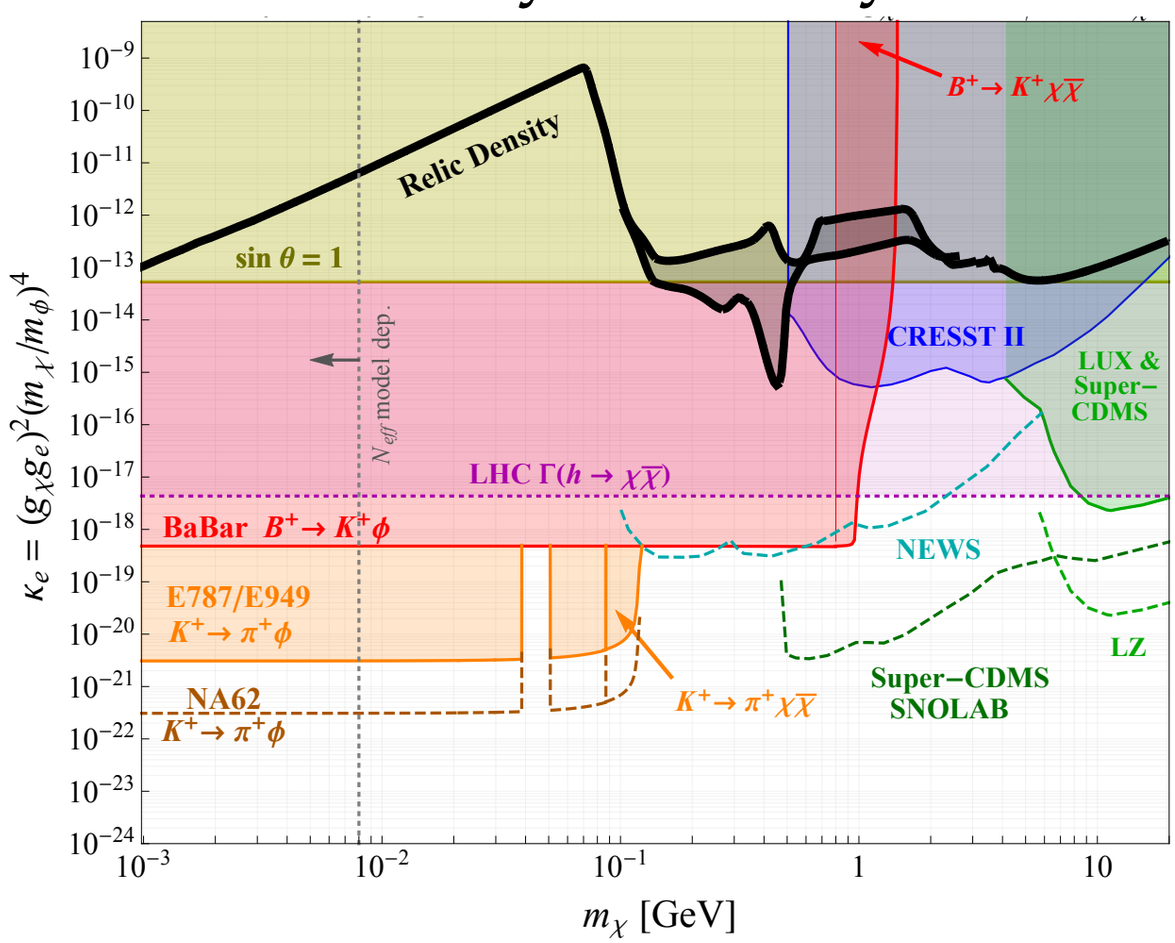
Options For Singlets

1) Mix S/V with neutral SM bosons

Higgs mixed scalar ϕ ruled out

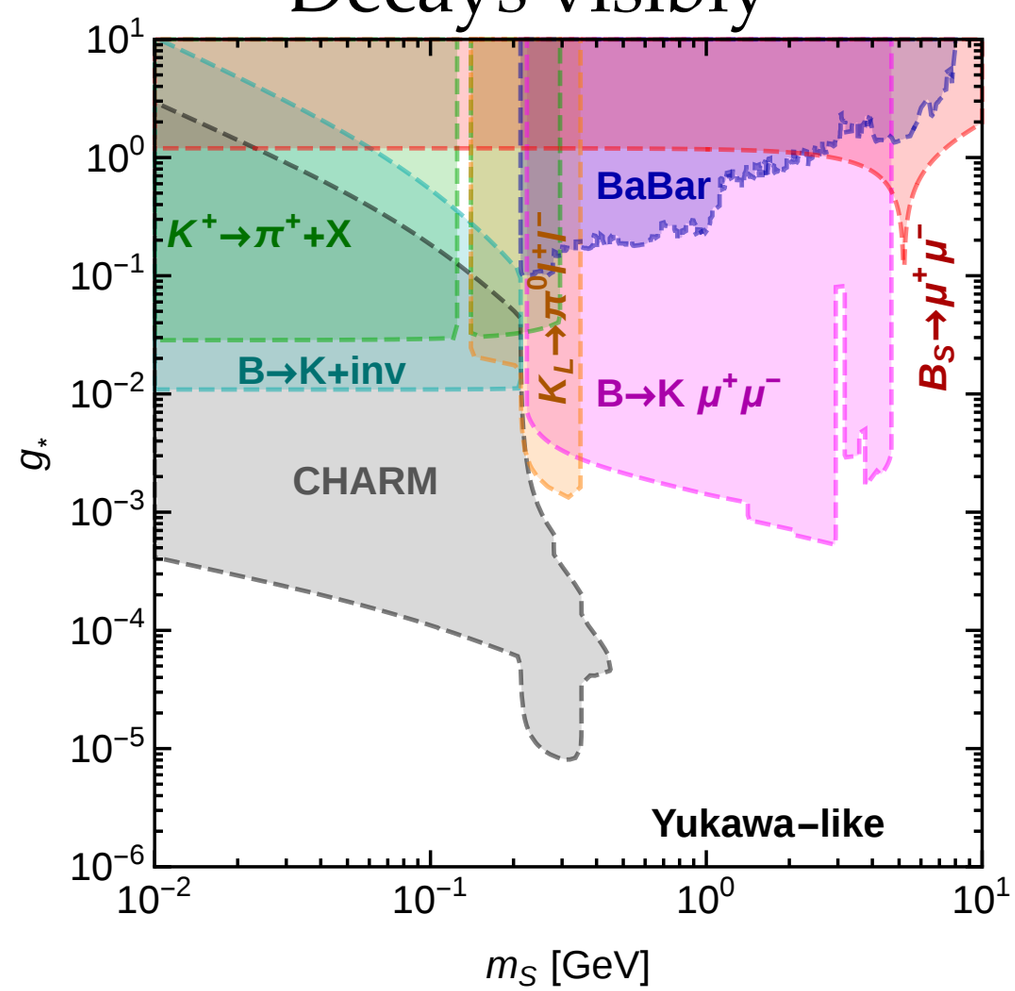
$$\mathcal{L}_{\rm int} = \sin\theta\,\phi\,\frac{m_f}{v}\,\bar{f}f$$

Decays invisibly



GK 1512.04119

Decays visibly



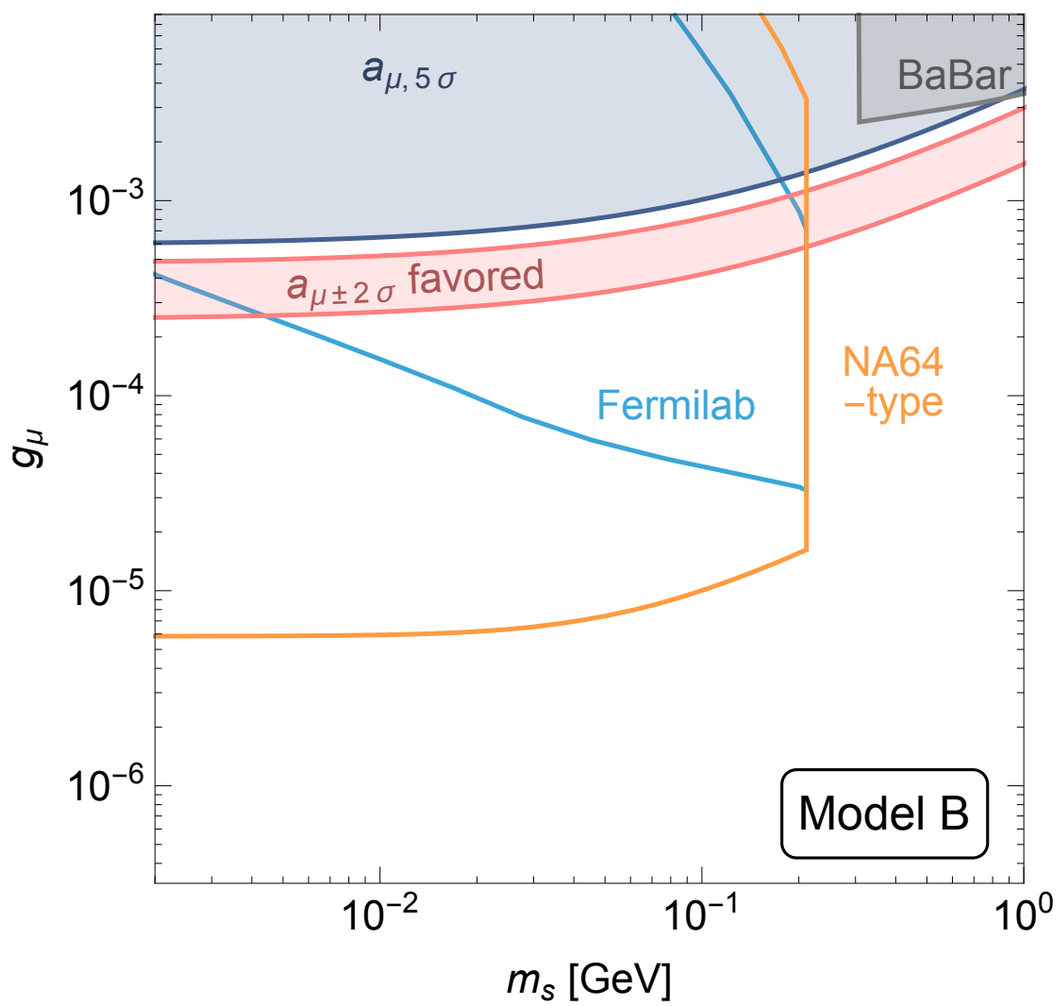
SHiP collab 1504.04855

Options For Singlets

2) Couple S to heavy states that mix with the muon

$$\mathcal{L} \supset -\frac{1}{2}m_S^2 - \left(y_\mu H^\dagger L \mu^c + \frac{c_s}{M} S H^\dagger L \mu^c\right) \to \mathcal{L}_{\text{eff}} = g_\mu S \mu \mu^c$$

$S \to \gamma\gamma$ via muon loop



Muon only coupling

Decays through loop

… or decays invisibly

Chen, Pospelov, Zhong, 1701.07437
Batell, Freitas, Ismail, McKeen 1712.10022

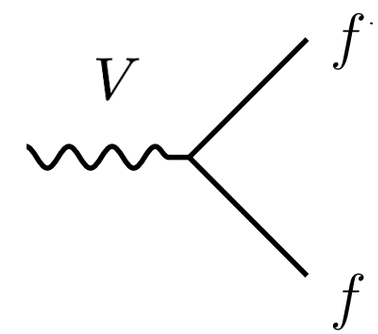
Egana-Uvanovic, Homiller, Meade 1911.10203

Options For Singlets

3) V is the gauge boson of a new U(1) SM extension

SM particles now carry a new gauge quantum number

$$\mathcal{L} \supset g V_\mu J^\mu_{\rm SM} \quad , \quad J^\mu_{\rm SM} \equiv \sum_f Q_f \bar{f} \gamma^\mu f$$



Only anomaly free possibilities:

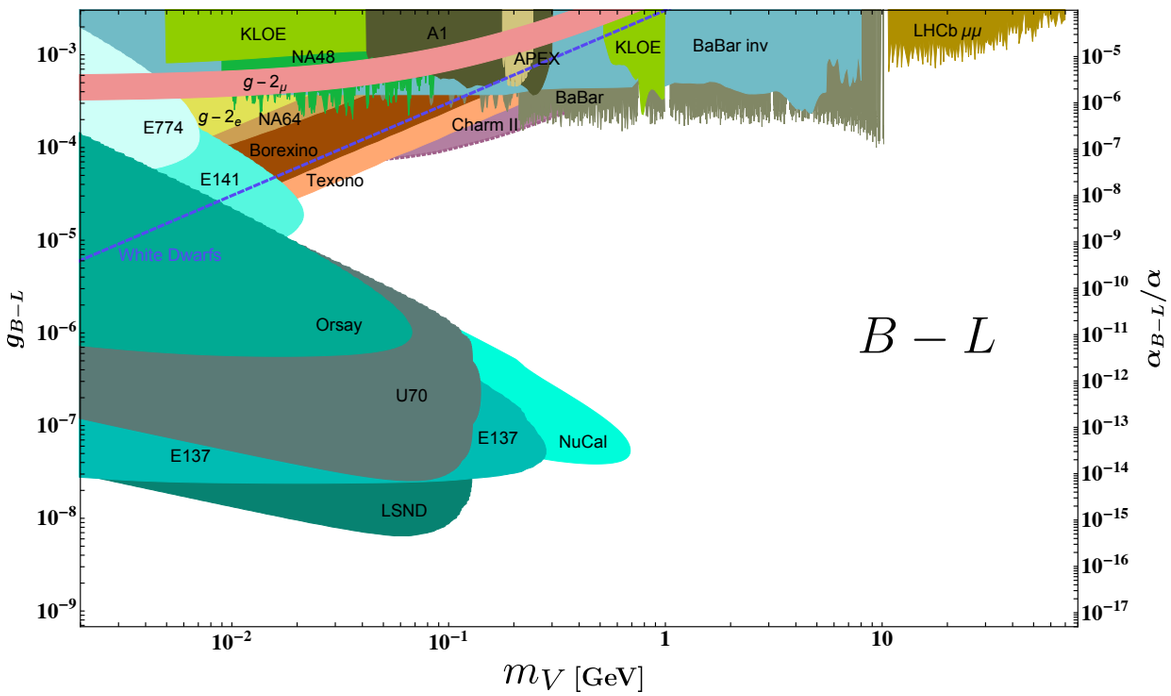
$$U(1)_{B-L} \ , \ U(1)_{L_i - L_j} \ , \ U(1)_{B-3L_i}$$

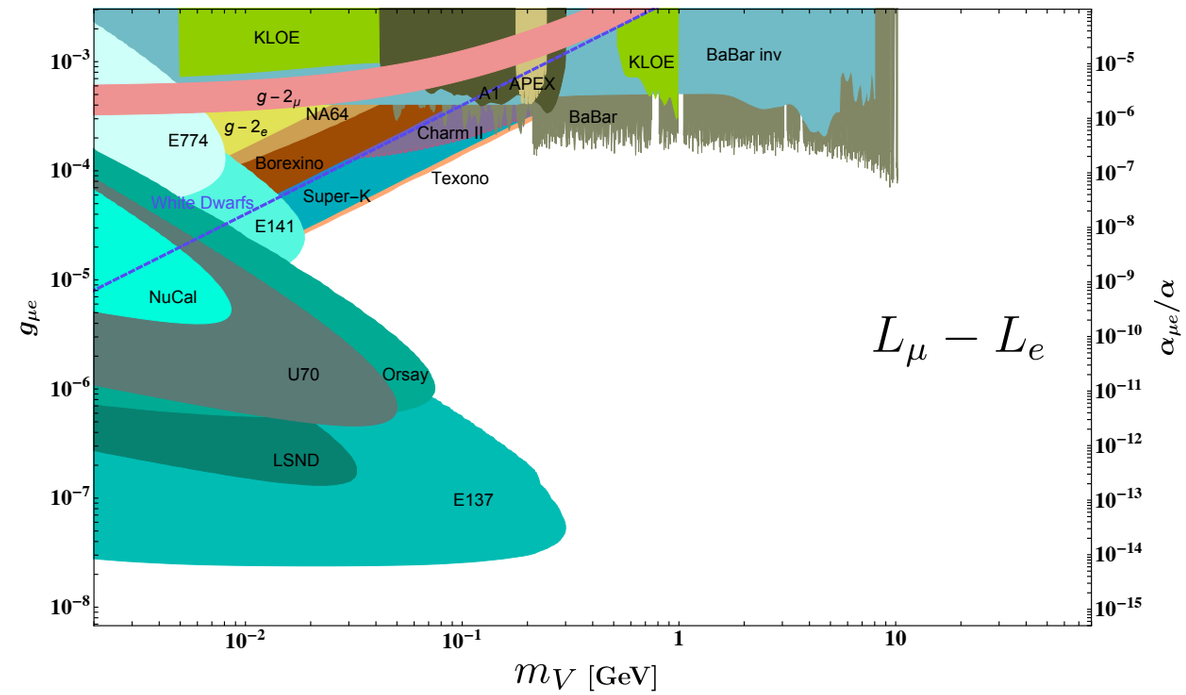
All similar, but some differences in bounds

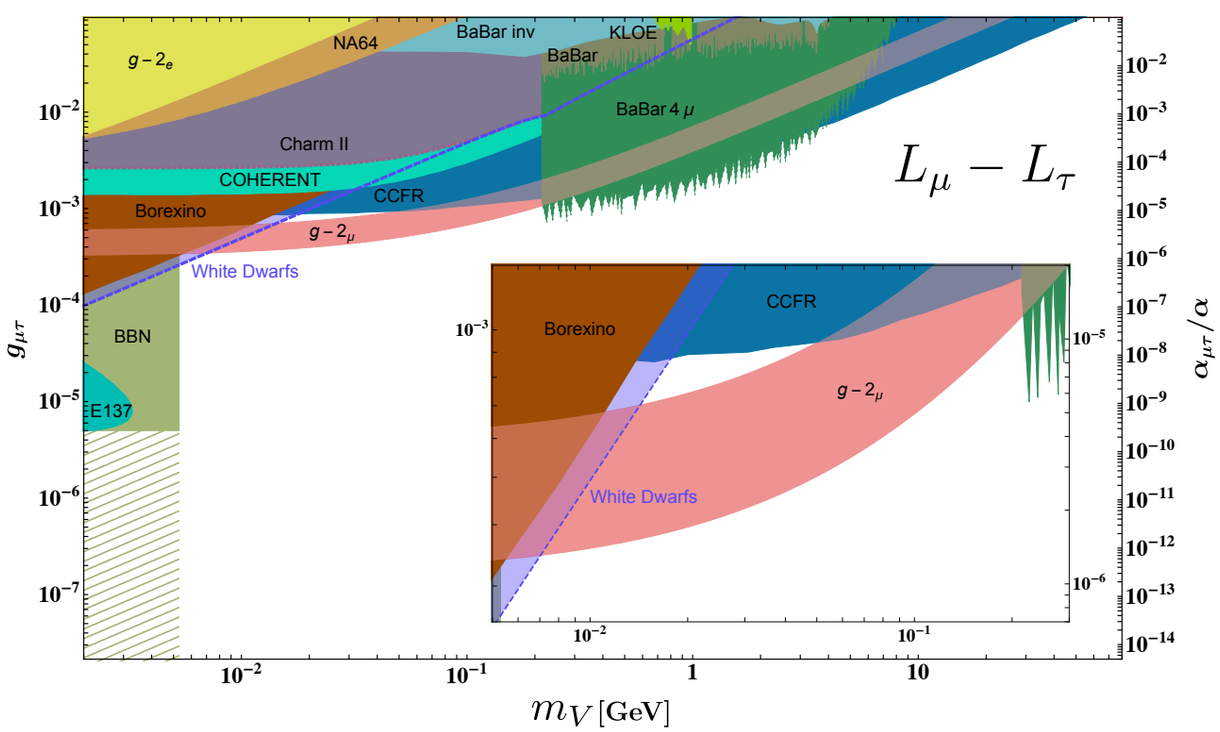
Two parameter family of models: $\{g, m_V\}$

Options For Singlets

3) V is the gauge boson of a new U(1) SM extension



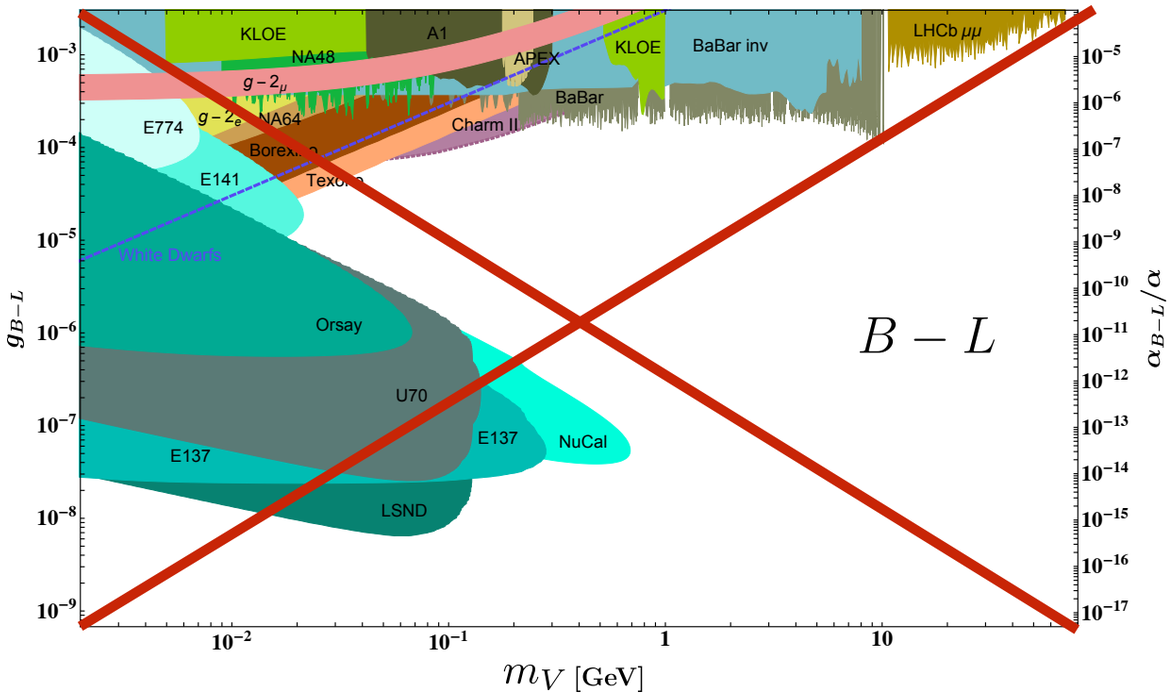


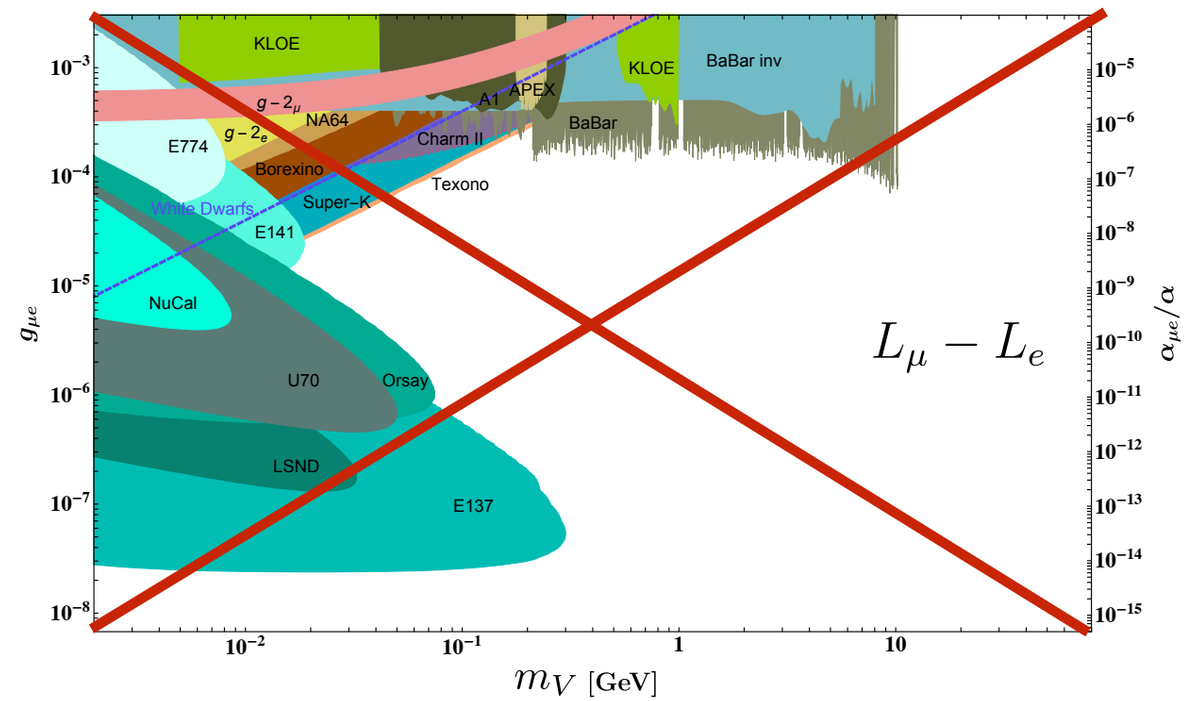


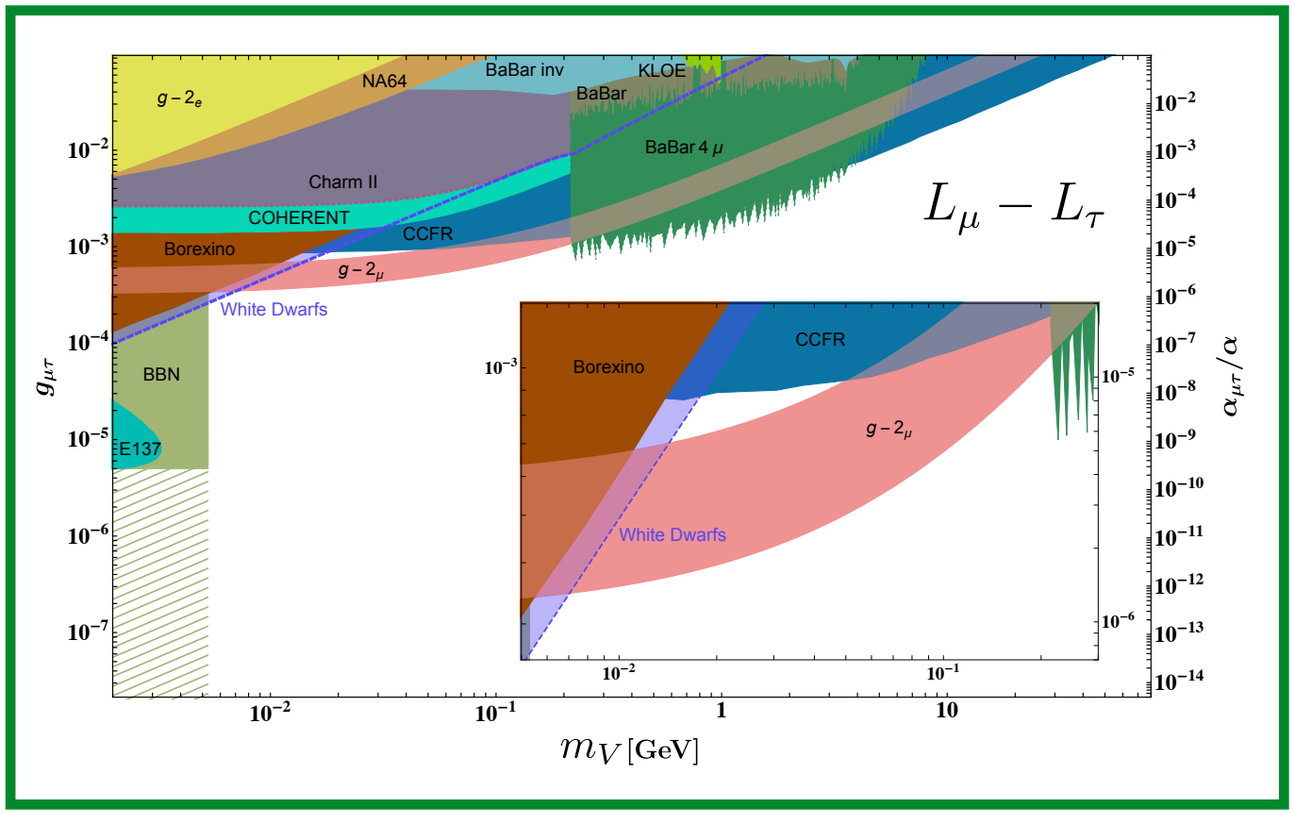
Bauer, Foldenauer, Jaeckel, 1803.05466

Options For Singlets

3) V is the gauge boson of a new U(1)







Only one possibility left

Viable between $\mathrm{MeV} - 2m_\mu$

$<$ MeV spoils BBN

$> 2m_\mu$ decays to muons

Bauer, Foldenauer, Jaeckel, 1803.05466

Summary of Singlet Models

Experimental bounds require muon-philic forces

Scalar model

$$\mathcal{L} \supset \frac{1}{2}(\partial_\mu S)^2 - \frac{1}{2}m_S^2 S^2 - \sum_{\ell=e,\mu,\tau} g_\ell S\bar{\ell}\ell,$$

Generically need $g_{e,q} \ll g_\mu$

Need extra SM charged fields in UV

Model dependent decays — **invisible = safest**

Vector model

$U(1)_{\mu-\tau}$

$$\mathcal{L} \supset \frac{m_V^2}{2} V_\alpha V^\alpha + g_V V_\alpha J^\alpha_{\mu-\tau}$$

$$J^\alpha_{\mu-\tau} \equiv \bar{\mu}\gamma^\alpha\mu + \bar{\nu}_\mu\gamma^\alpha P_L\nu_\mu - (\mu \to \tau)$$

For viable mass range < 200 MeV, V always* decays invisibly

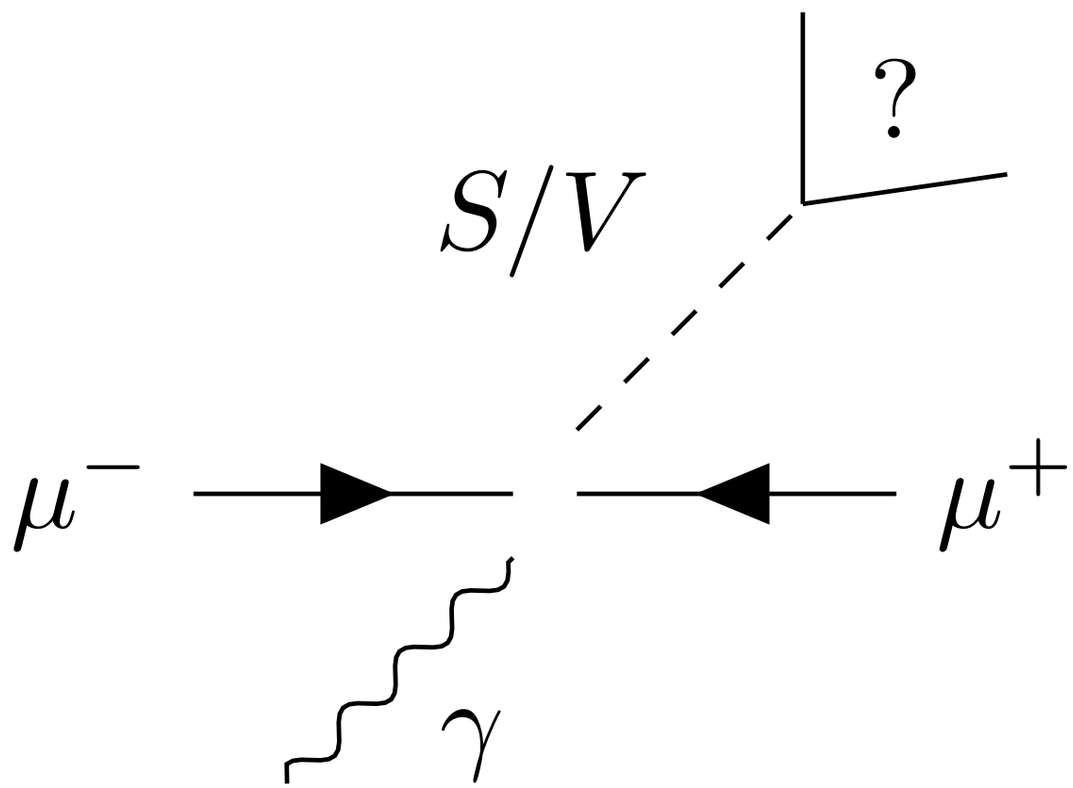
Overview

Current Status of g-2

Discovering Singlet Models
@ Future Muon Colliders

Electroweak Models

Inclusive MuC Search Strategy

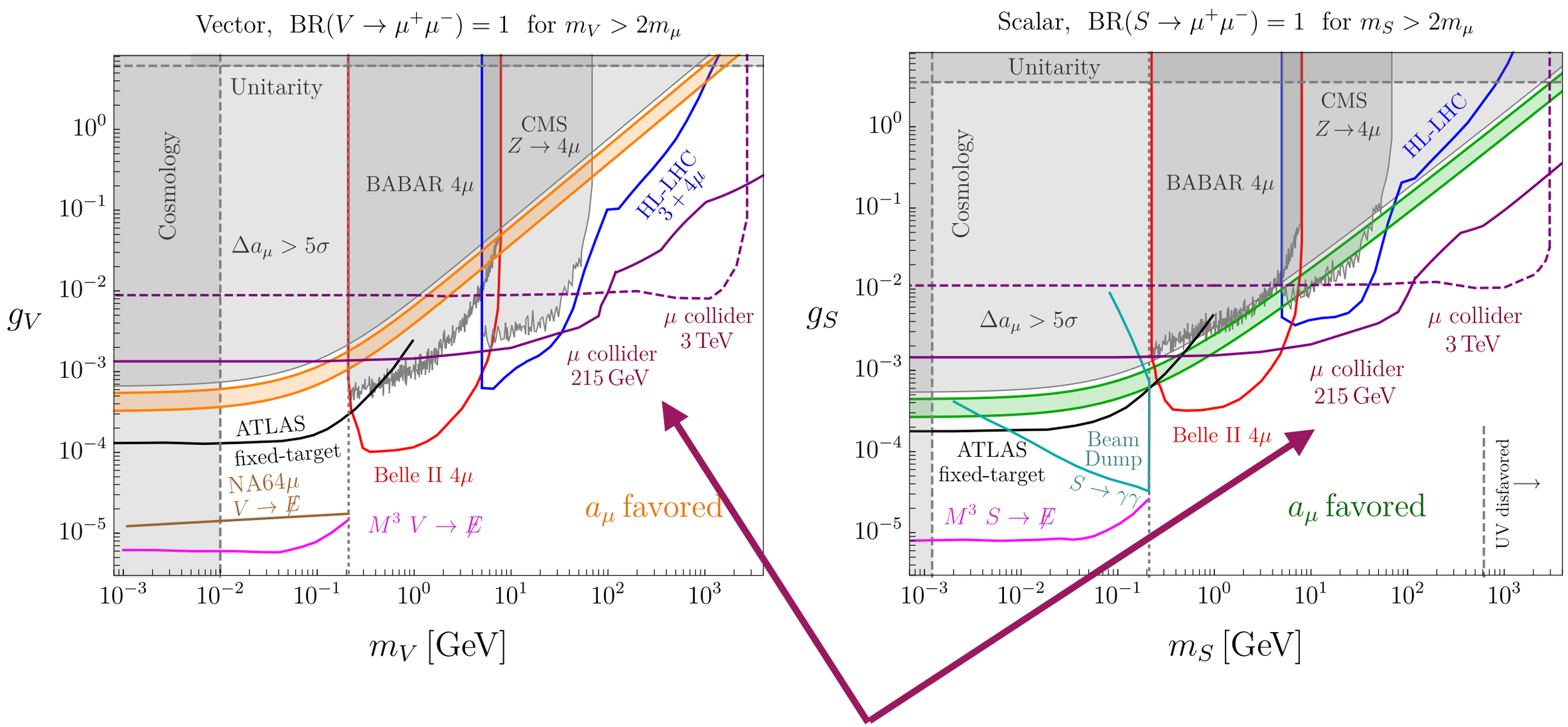


Back-to-back production of photon & singlet $\mu^+\mu^- \to \gamma S/V$

Trigger on photon, demand hemisphere isolation for photon/Z

Independent of how S/V decays — only assumes 2-body production

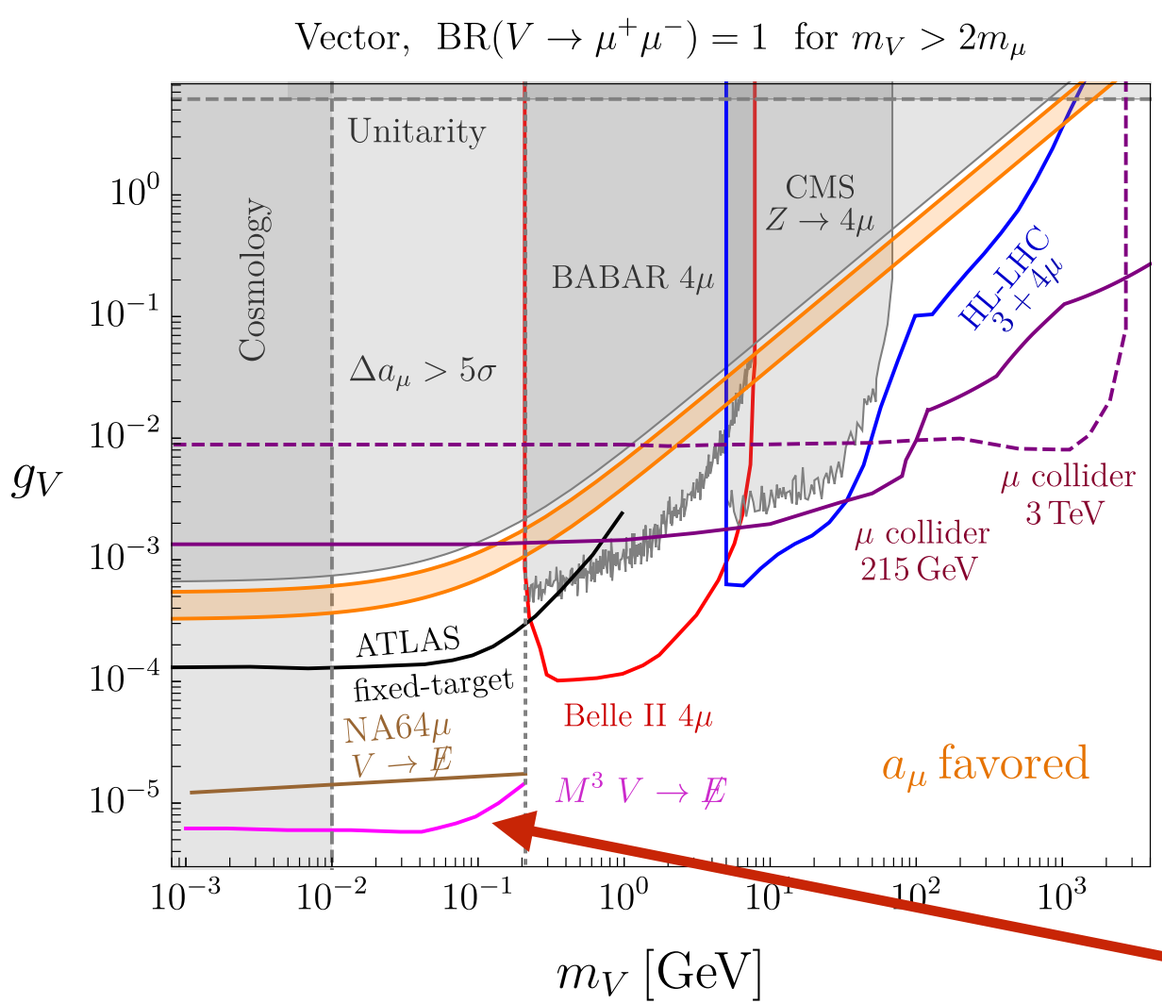
Maximum Dimuon Branching

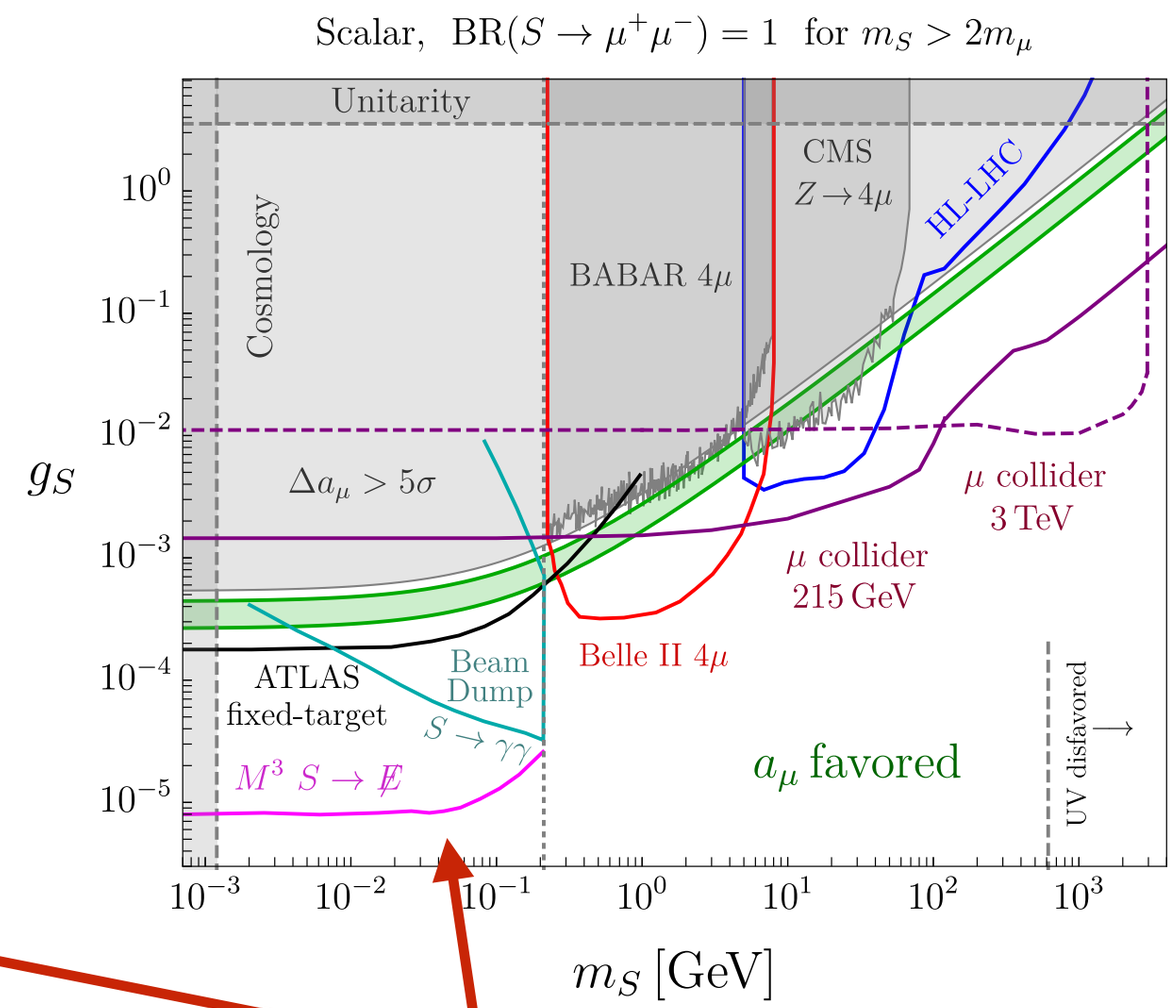


Representative muon collider concepts

215 GeV + 0.4/ab 3 TeV + 1/ab

Maximum Dimuon Branching



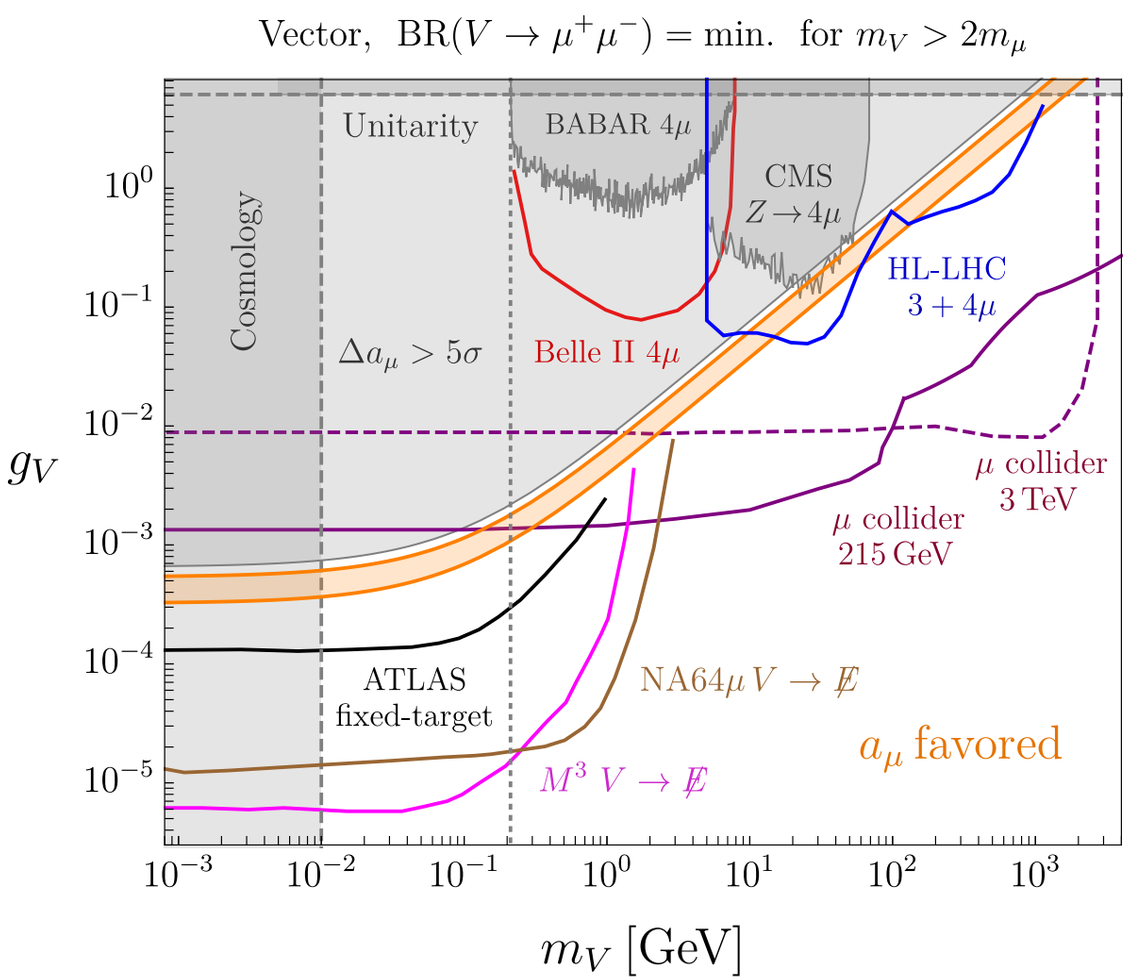


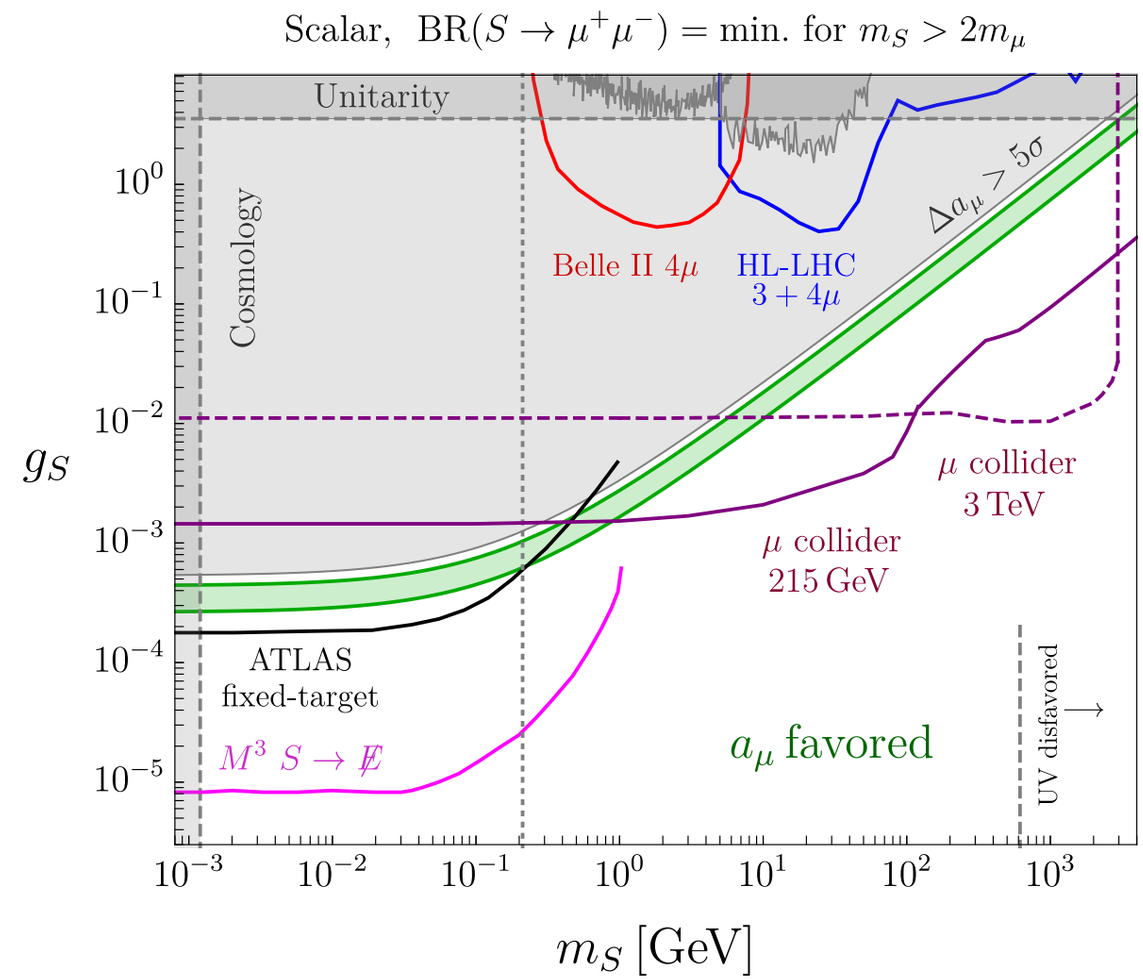
Complementary with muon beam ***fixed target*** experiments

Important part of collider R&D program and commissioning

See Cari Cesarotti's talk

Minimal Dimuon Branching





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Effective Operator Analysis

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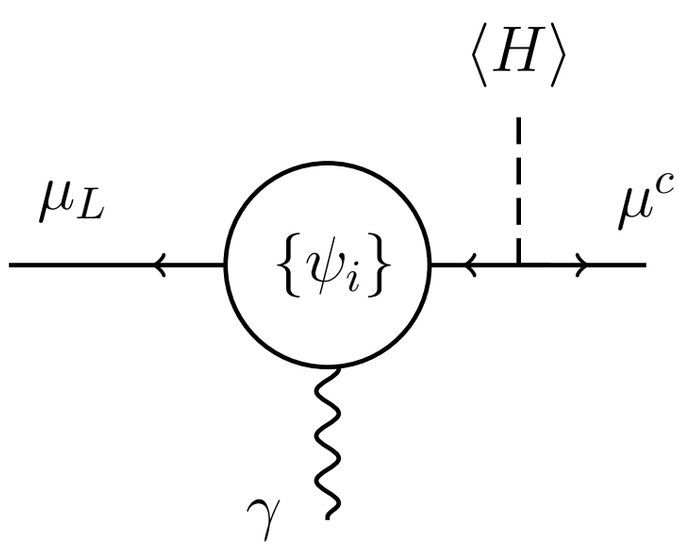
Do EWSB + chiral flip come from muon mass insertion?

YES

NO

Singlet Models



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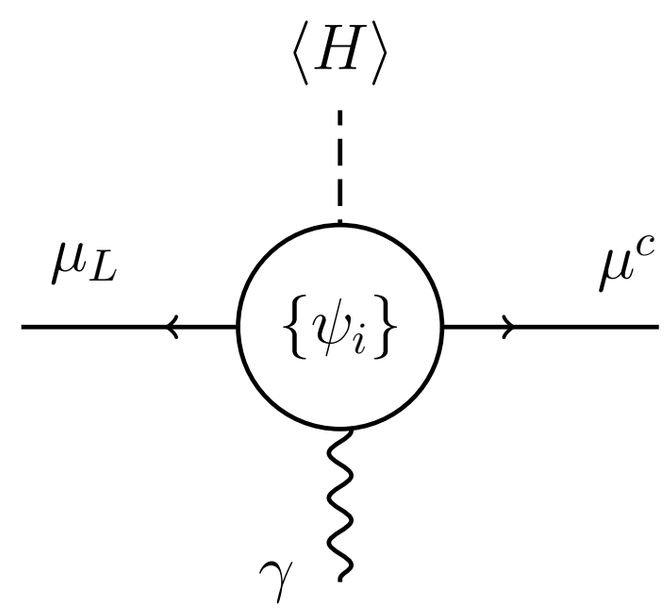
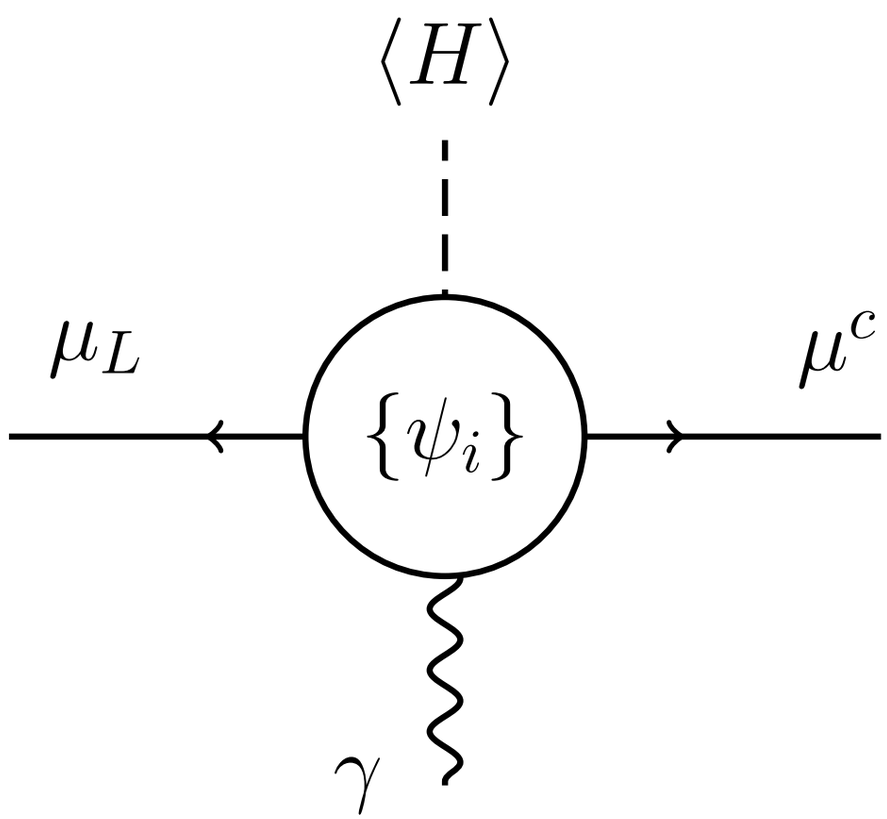


Electroweak Models

Chiral flip and EWSB on BSM lines



Hard to fully catalog BSM

Most new EW stuff will work
M > 100 GeV from LEP limits

Many free params (eg MSSM)
Many models testable @ LHC

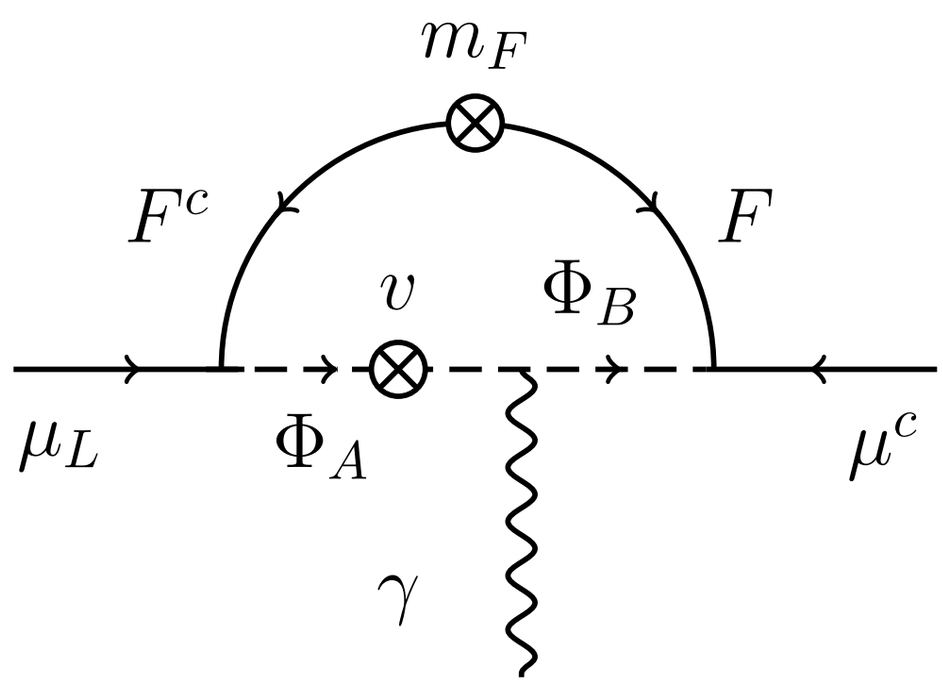
"Nightmare" Heavy BSM

Muon-philic~100 TeV & O(1) couplings
Need *muon* collider to fully test these

Capdevilla, Curtin, Kahn, GK 2006.16277,2101.10334

Electroweak Models

Representative ultra-heavy model



$$\mathcal{L}_{\rm SSF} \supset -y_1 F^c L_{(\mu)} \Phi_A^* - y_2 F \mu^c \Phi_B - \kappa H \Phi_A^* \Phi_B$$
$$-m_A^2 |\Phi_A|^2 - m_B^2 |\Phi_B|^2 - m_F F F^c + \text{h.c.}\ .$$

What is **maximum** mass of the ***lightest*** charged particle?

Algorithm for finding heaviest BSM charged particle

$$M^{\max}_{\text{BSM,charged}} \equiv \max_{\substack{\text{BSM theory space} \\ \Delta a_\mu = \Delta a_\mu^{\text{obs}}}} \left\{ \min_{i \in \text{BSM spectrum}} \left(m^{(i)}_{\text{charged}} \right) \right\}$$

1. Fix muon couplings to resolve anomaly

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1. Fix muon couplings to resolve anomaly

2. Calculate mass spectrum of theory

Algorithm for finding heaviest BSM charged particle

$$M^{\max}_{\text{BSM,charged}} \equiv \max_{\substack{\text{BSM theory space} \\ \Delta a_\mu = \Delta a_\mu^{\text{obs}}}} \left\{ \min_{i \in \text{BSM spectrum}} \left(m^{(i)}_{\text{charged}} \right) \right\}$$

1. Fix muon couplings to resolve anomaly

2. Calculate mass spectrum of theory

3. Identify **lightest** charged particle for each spectrum

Algorithm for finding heaviest BSM charged particle

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4. Vary model parameters, repeat 1-3 over all spectra in theory space

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1. Fix muon couplings to resolve anomaly

2. Calculate mass spectrum of theory

3. Identify **lightest** charged particle for each spectrum

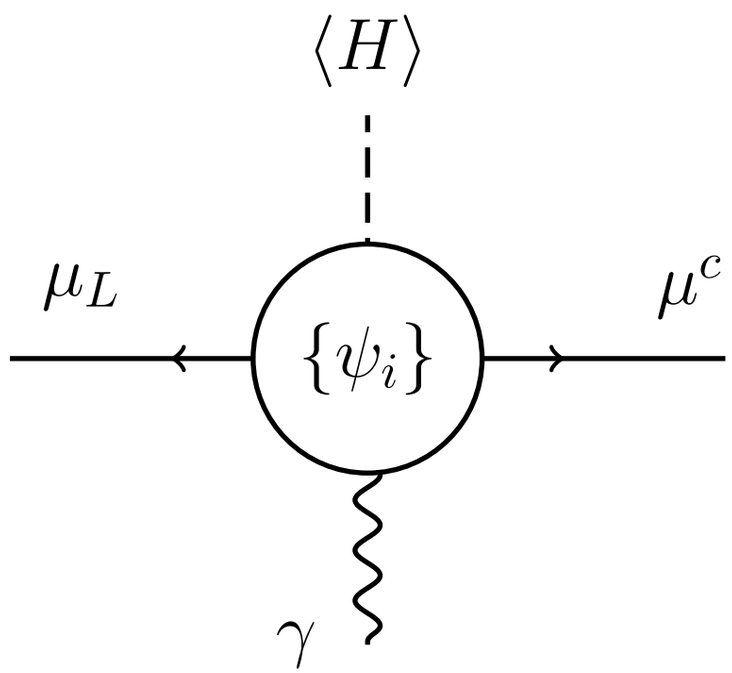
4. Vary model parameters, repeat 1-3 over all spectra in theory space

5. Keep the **heaviest** among all these (*e.g.* hardest to discover)

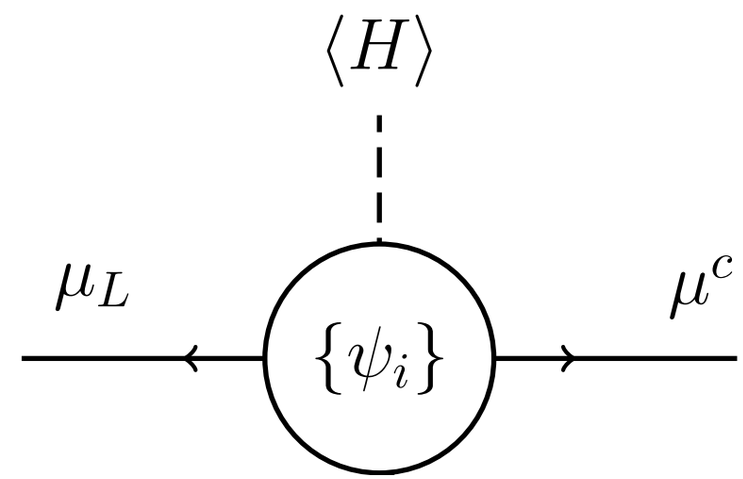
Model	R	R_A	R_B	Highest possible mass (TeV) of lightest charged BSM state: Unitarity only, $N_{\rm BSM}$: 1	10
SSF	1_{-1}	$2_{1/2}$	1_0	65.2	241
	1_{-2}	$2_{3/2}$	1_1	85.9	321
	1_0	$2_{-1/2}$	1_{-1}	46.2	176
	1_1	$2_{-3/2}$	1_{-2}	81.8	302
	$2_{-1/2}$	3_0	$2_{-1/2}$	21.4	107
	$2_{-3/2}$	3_1	$2_{1/2}$	83.7	308
	$2_{1/2}$	3_{-1}	$2_{-3/2}$	95.5	356
	$2_{-1/2}$	1_0	$2_{-1/2}$	65.2	241
	$2_{-3/2}$	1_1	$2_{1/2}$	85.9	321
	$2_{1/2}$	1_{-1}	$2_{-3/2}$	44.8	155
	3_{-1}	$2_{1/2}$	3_0	95.4	359
	3_0	$2_{-1/2}$	3_{-1}	39.4	144
FFS	1_{-1}	$2_{1/2}$	1_0	37.3	118
	1_{-2}	$2_{3/2}$	1_1	67.3	213
	1_0	$2_{-1/2}$	1_{-1}	59.1	187
	1_1	$2_{-3/2}$	1_{-2}	73.2	231
	$2_{-1/2}$	3_0	$2_{-1/2}$	40	126
	$2_{-3/2}$	3_1	$2_{1/2}$	56.3	178
	$2_{1/2}$	3_{-1}	$2_{-3/2}$	82.3	260
	$2_{-1/2}$	1_0	$2_{-1/2}$	37.3	118
	$2_{-3/2}$	1_1	$2_{1/2}$	67.3	213
	$2_{1/2}$	1_{-1}	$2_{-3/2}$	46.2	146
	3_{-1}	$2_{1/2}$	3_0	71	225
	3_0	$2_{-1/2}$	3_{-1}	23.4	75
$M^{\rm max}_{\rm BSM,charged}$ (max in each column)				**95.5**	**359**

This seems pretty bad, but …

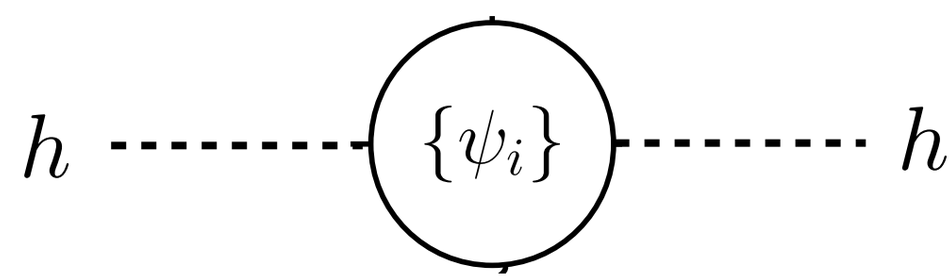
Makes the hierarchy problem real



Required for anomaly



Muon mass

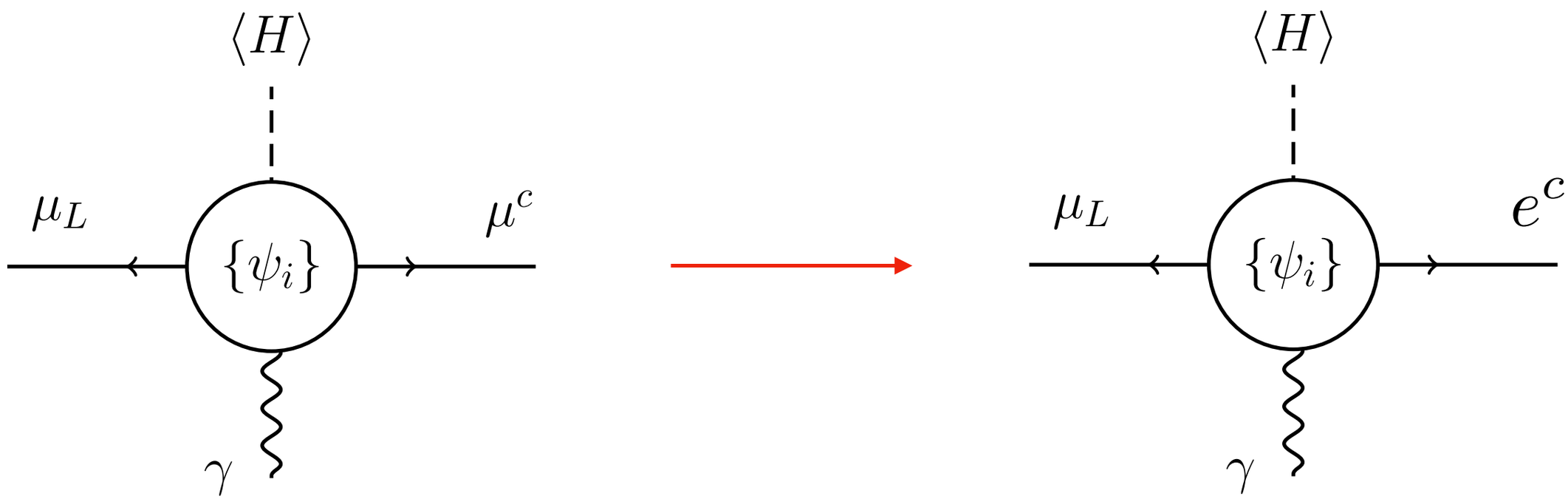


Higgs mass

Finite calculable ~ 100 TeV corrections to Higgs/muon masses

NOT like LHC and SUSY!

Important connection to flavor physics



Required for anomaly

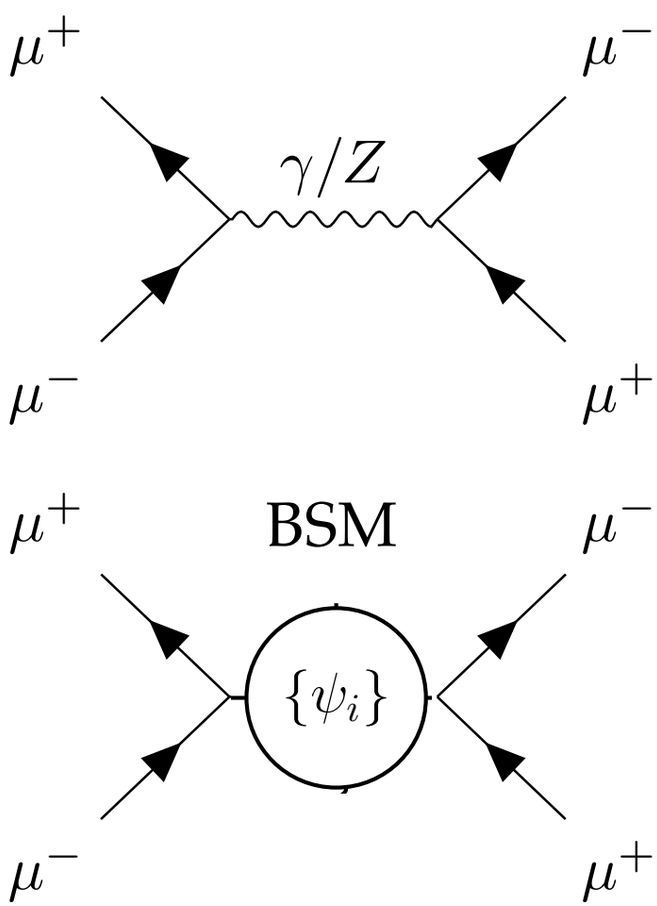
$$\mathrm{Br}(\mu \to e\gamma) < 4.2 \times 10^{-13}$$
$$\mathrm{Br}(\tau \to \mu\gamma) < 4.4 \times 10^{-8}$$
$$\mathrm{Br}(\tau \to e\gamma) < 3.3 \times 10^{-8}$$

Dangerous FCNC without MFV or tuning

Model	R	R_A	R_B	Highest possible mass (TeV) of lightest charged BSM state: Unitarity only, $N_{\rm BSM}$: 1	Unitarity only, $N_{\rm BSM}$: 10	Unitarity + MFV, $N_{\rm BSM}$: 1	Unitarity + MFV, $N_{\rm BSM}$: 10	Unitarity + Naturalness, $N_{\rm BSM}$: 1	Unitarity + Naturalness, $N_{\rm BSM}$: 10	Unitarity + Naturalness + MFV, $N_{\rm BSM}$: 1	Unitarity + Naturalness + MFV, $N_{\rm BSM}$: 10
SSF	1_{-1}	$2_{1/2}$	1_0	65.2	241	12.9	47.1	11.5	11.5	6.54	10.1
	1_{-2}	$2_{3/2}$	1_1	85.9	321	18.1	64.8	19.2	19.2	8.41	12.3
	1_0	$2_{-1/2}$	1_{-1}	46.2	176	9.41	34.1	15.6	17.5	5.93	8.56
	1_1	$2_{-3/2}$	1_{-2}	81.8	302	17.1	63.7	19.3	19.3	8.38	12.1
	$2_{-1/2}$	3_0	$2_{-1/2}$	21.4	107	4.2	15.5	7.47	8.99	3.23	5.0
	$2_{-3/2}$	3_1	$2_{1/2}$	83.7	308	16.6	60.7	13.4	13.4	7.06	10.6
	$2_{1/2}$	3_{-1}	$2_{-3/2}$	95.5	356	18.3	67.8	15.6	15.6	7.75	11.3
	$2_{-1/2}$	1_0	$2_{-1/2}$	65.2	241	12.9	47.1	11.5	11.5	6.54	10.1
	$2_{-3/2}$	1_1	$2_{1/2}$	85.9	321	18.1	64.8	19.2	19.2	8.41	12.3
	$2_{1/2}$	1_{-1}	$2_{-3/2}$	44.8	155	8.8	32.3	10.9	10.9	5.64	8.56
	3_{-1}	$2_{1/2}$	3_0	95.4	359	19.4	73	20.1	30	7.75	11.5
	3_0	$2_{-1/2}$	3_{-1}	39.4	144	7.82	28.6	10.8	15.1	4.14	6.08
FFS	1_{-1}	$2_{1/2}$	1_0	37.3	118	8.87	28	12.3	18.7	4.6	7.04
	1_{-2}	$2_{3/2}$	1_1	67.3	213	15.8	50	13.5	18.8	4.86	6.93
	1_0	$2_{-1/2}$	1_{-1}	59.1	187	13.2	41.8	12.4	17.2	4.02	6.28
	1_1	$2_{-3/2}$	1_{-2}	73.2	231	17.4	55	13.9	19.7	5.04	7.25
	$2_{-1/2}$	3_0	$2_{-1/2}$	40	126	9.38	29.7	8.0	11.5	2.88	4.34
	$2_{-3/2}$	3_1	$2_{1/2}$	56.3	178	13.6	42.9	11.8	16.2	4.26	6.1
	$2_{1/2}$	3_{-1}	$2_{-3/2}$	82.3	260	19.2	60.6	13.6	19	4.93	7.0
	$2_{-1/2}$	1_0	$2_{-1/2}$	37.3	118	8.87	28	12.3	18.7	4.6	7.04
	$2_{-3/2}$	1_1	$2_{1/2}$	67.3	213	15.8	50	13.5	18.8	4.86	6.93
	$2_{1/2}$	1_{-1}	$2_{-3/2}$	46.2	146	11.2	35.4	9.83	13.8	3.49	5.18
	3_{-1}	$2_{1/2}$	3_0	71	225	17	53.6	13.1	18.1	4.04	6.97
	3_0	$2_{-1/2}$	3_{-1}	23.4	75	5.29	16.9	7.3	7.69	2.73	4.03
$M^{\rm max}_{\rm BSM,charged}$ (max in each column)				**95.5**	**359**	**19.4**	**73**	**20.1**	**30**	**8.41**	**12.3**

~ 10 TeV within scope of muon collider studies [Delahaye et al 1901.06150]

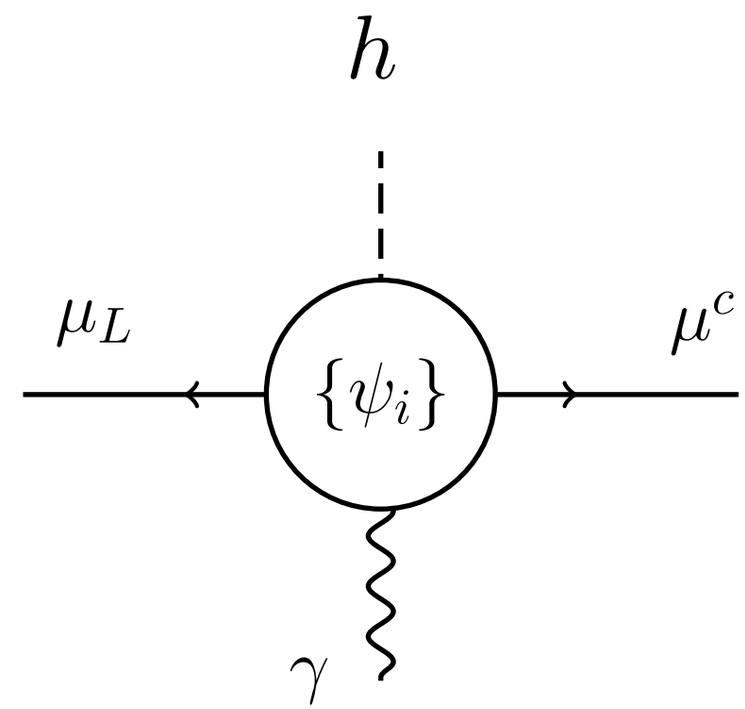
Guaranteed BSM at Muon Colliders



Muonic Bhabha scattering

$$\mu^+\mu^- \to \mu^+\mu^-$$

Capdevilla, Curtin, Kahn, GK 2006.16277



Higgs + photon production

$$\mu^+\mu^- \to h\gamma$$

Buttazzo Paradisi 2012.02769

Concluding Remarks

Exciting time for g-2 , new results soon (FNAL + JPARC)

If anomaly is due to SM singlets

Must be muon-philic scalar or vector

If anomaly is due to EW BSM

Unitarity < 100 TeV, but ~ few-10 TeV from naturalness/flavor

Agnostic search strategies for BSM

Bhabha scattering, higgs/photon, hemisphere analysis

Complementarity with light experiments

BSM searches at intermediate steps towards MuC

Backup Slides

Constraints: Big Bang Nucleosynthesis

V is in chemical equilibrium with SM in early universe

$$n_V \propto \begin{cases} T^3 & (T \gg m_V) \\ e^{-m/T} & (T \ll m_V) \end{cases}$$

When T < *m*, the V decays transfer entropy to SM particles
Must happen before neutrinos decouple from photons

$$m \gtrsim T_{\nu,\mathrm{dec}} \approx 2\,\mathrm{MeV}$$

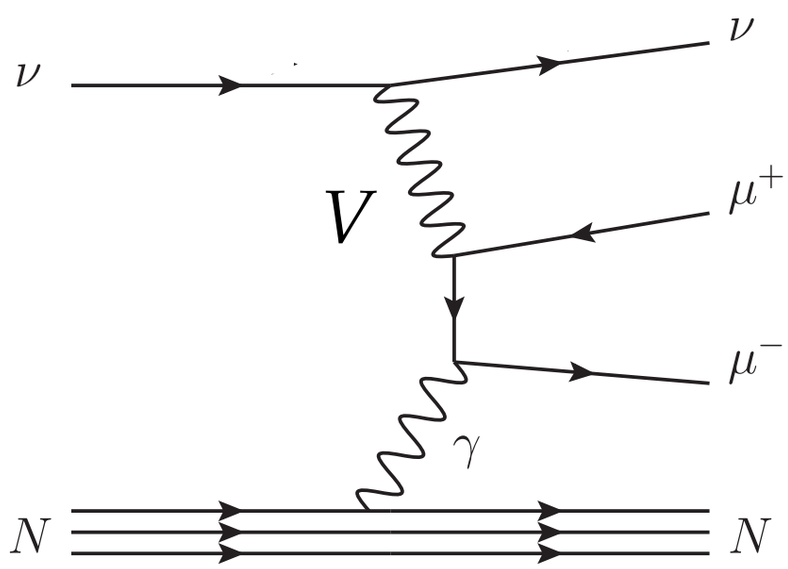
Otherwise V decays heat neutrinos not CMB $\rightarrow \Delta N_{\mathrm{eff}} \gtrsim 0.5$

Spoils BBN element yields

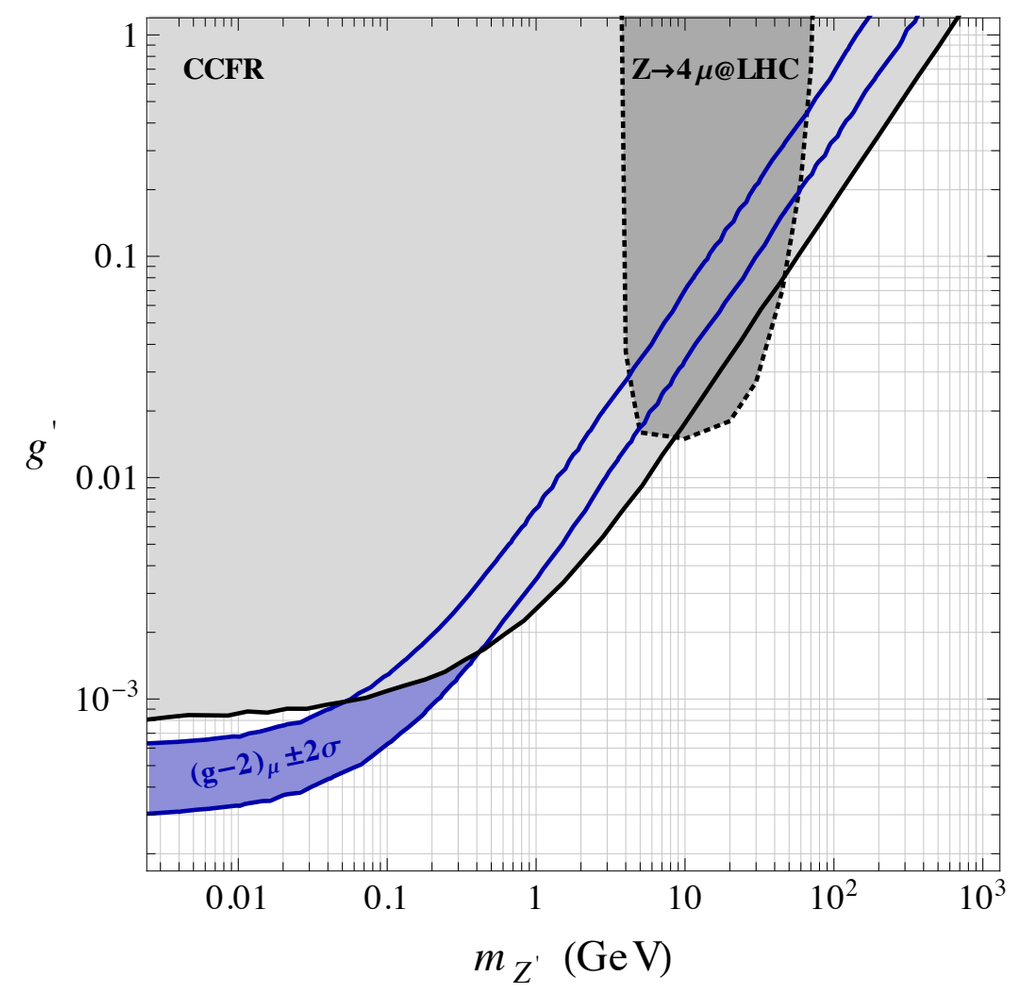
*mild contribution for *m*~ few MeV may reduce Hubble tension

Escudero, Hooper, GK, Pierre, 1902.02010

Constraints: Neutrino Tridents, CCFR + CHARM II



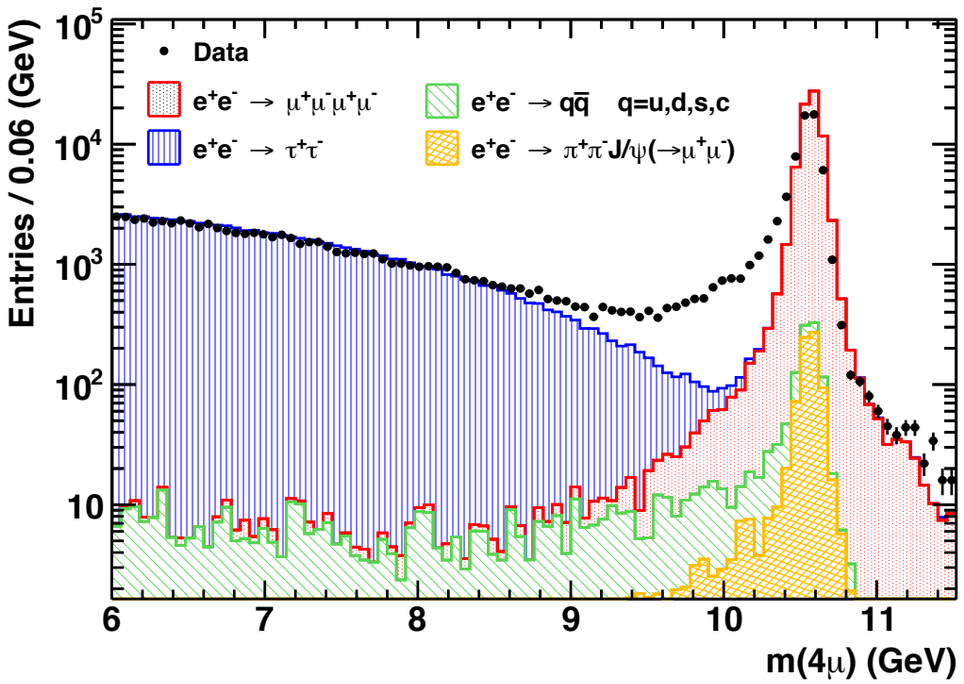
$$\frac{\sigma^{\rm CCFR}}{\sigma^{\rm SM}} = 0.82 \pm 0.28.$$

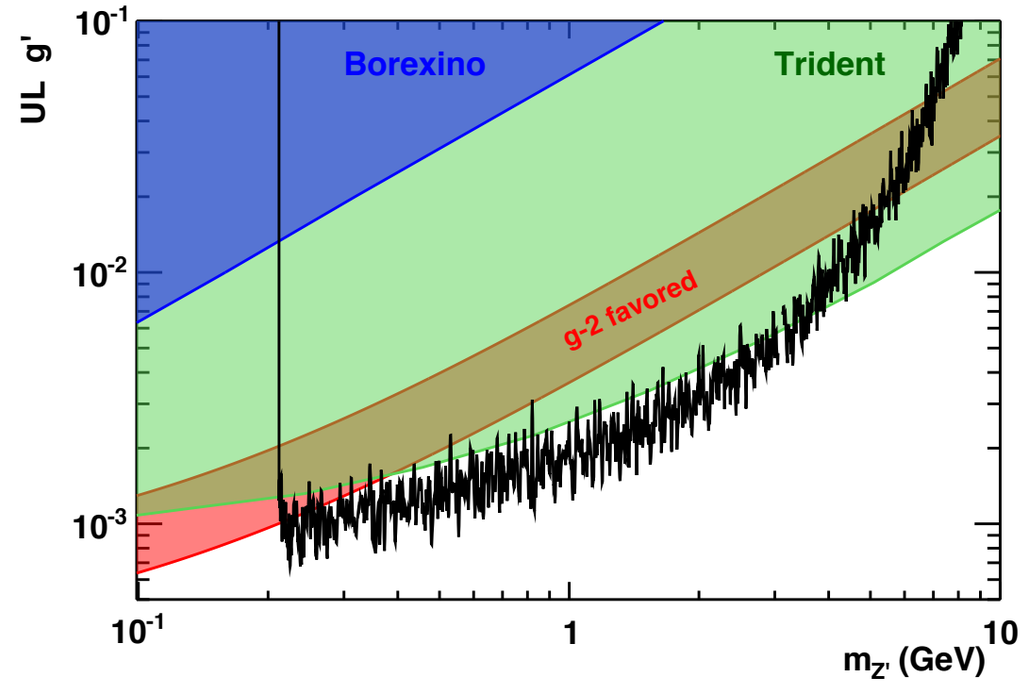


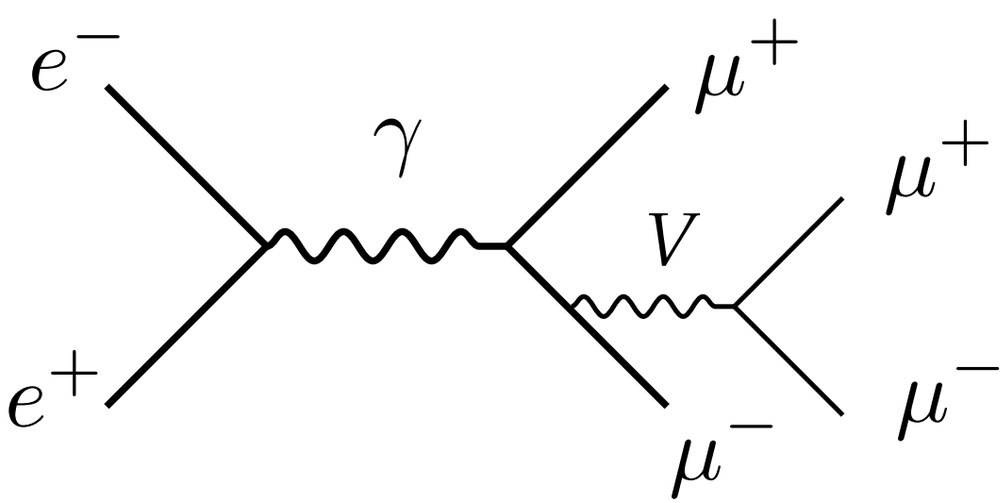
S. Mishra et al. (CCFR Collaboration), Phys.Rev.Lett. 66, 3117 (1991)

Altmanshoffer, Pospelov, Gori, Yavin 1406.2332

Constraints: BABAR Experiment





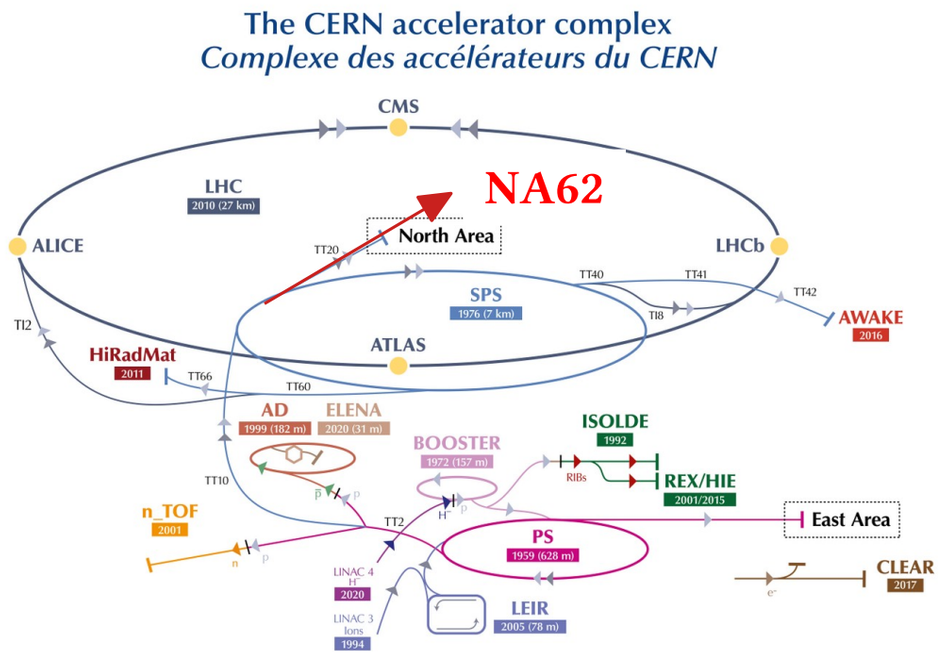


Search for 4 muon excess

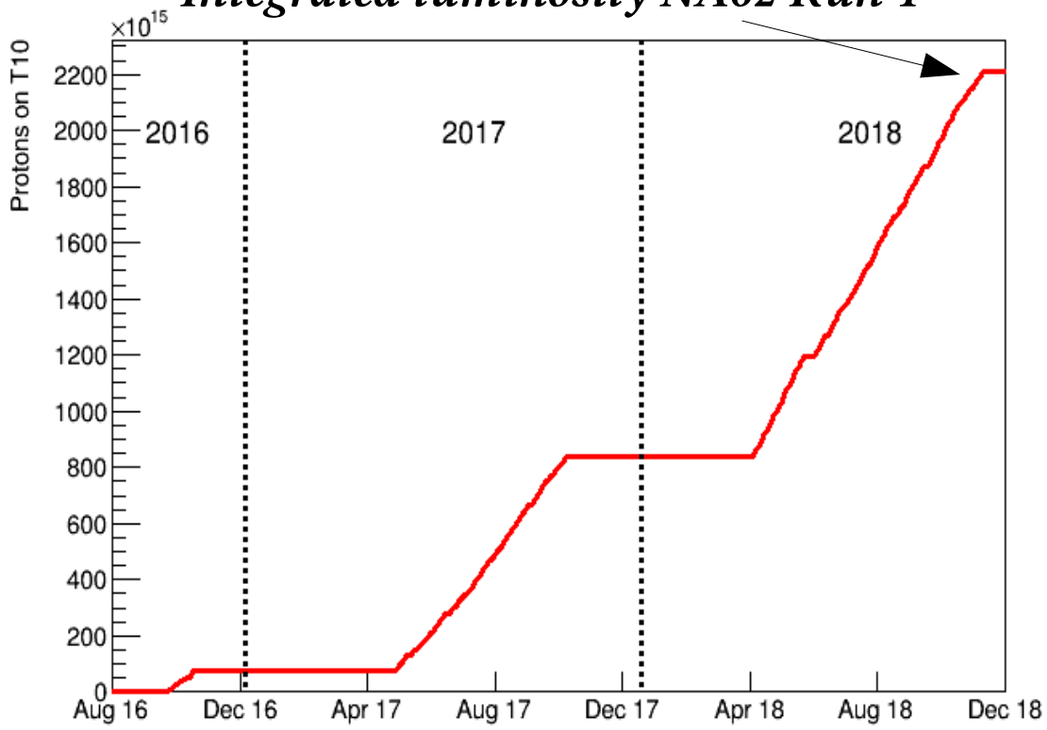
Excludes g-2 for $m > 200$ MeV

BABAR Collaboration 1606.03501

State-of-the-art $K^+ \to \pi^+ \nu\bar{\nu}$ experiments



Integrated luminosity NA62 Run 1



- **Past experiments (E787/E949 @ BNL)**
  - ★ Kaon decay-at-rest technique

    $BR(K^+ \to \pi^+ \nu\bar{\nu}) = (1.73^{+1.15}_{-1.05}) \times 10^{-10}$

    Phys. Rev. D 79, 092004 (2009)

    Phys. Rev. D 77, 052003 (2008)

- **Present state-of-the-art $K^+ \to \pi^+ \nu\nu$ experiments**
  - ★ Kaon decay-in-flight technique
  - ★ NA62 experiment (this talk)

Run 1 statistics

1.9×10^{12} proton per spill on target

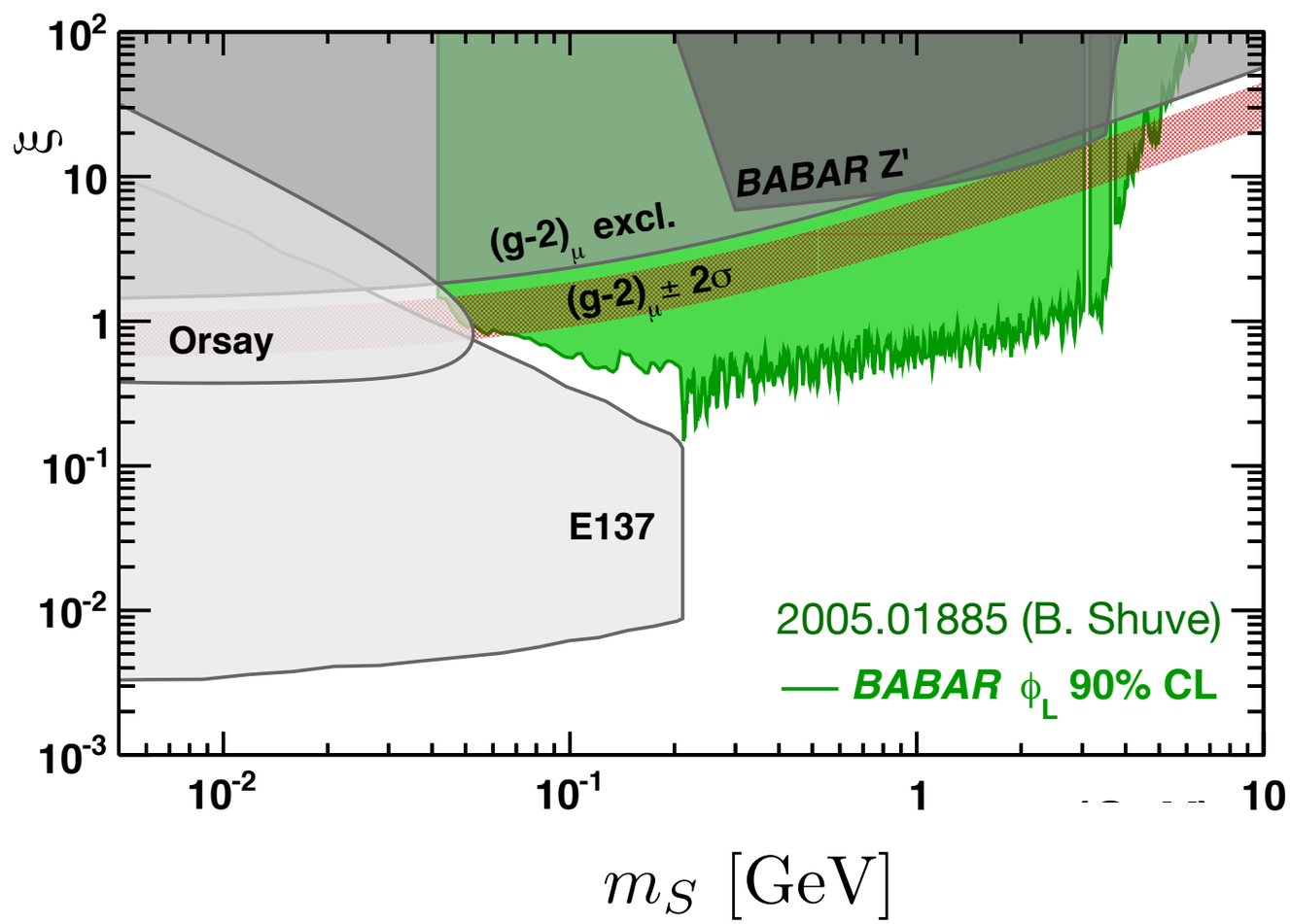
~ 2.2×10^{18} POT collected in Run 1

Options For Singlets

Fun variation: mix S with neutral BSM bosons

Leptophilic scalar
(eg mix with 2HDM)

$$\mathcal{L} = -\xi \sum_{\ell=e,\mu,\tau} \frac{m_\ell}{v} \bar{\ell}\ell S$$



LHC *probably* excludes > 10 GeV region… or will soon 1808.03684