Resummation in MCFM

CTEQ Fall Meeting, November 15, 2022

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(work with Keith Ellis, Tobias Neumann, Satyajit Seth)

MCFM

Ellis, Neumann, Williams, JC + more: mcfm.fnal.gov

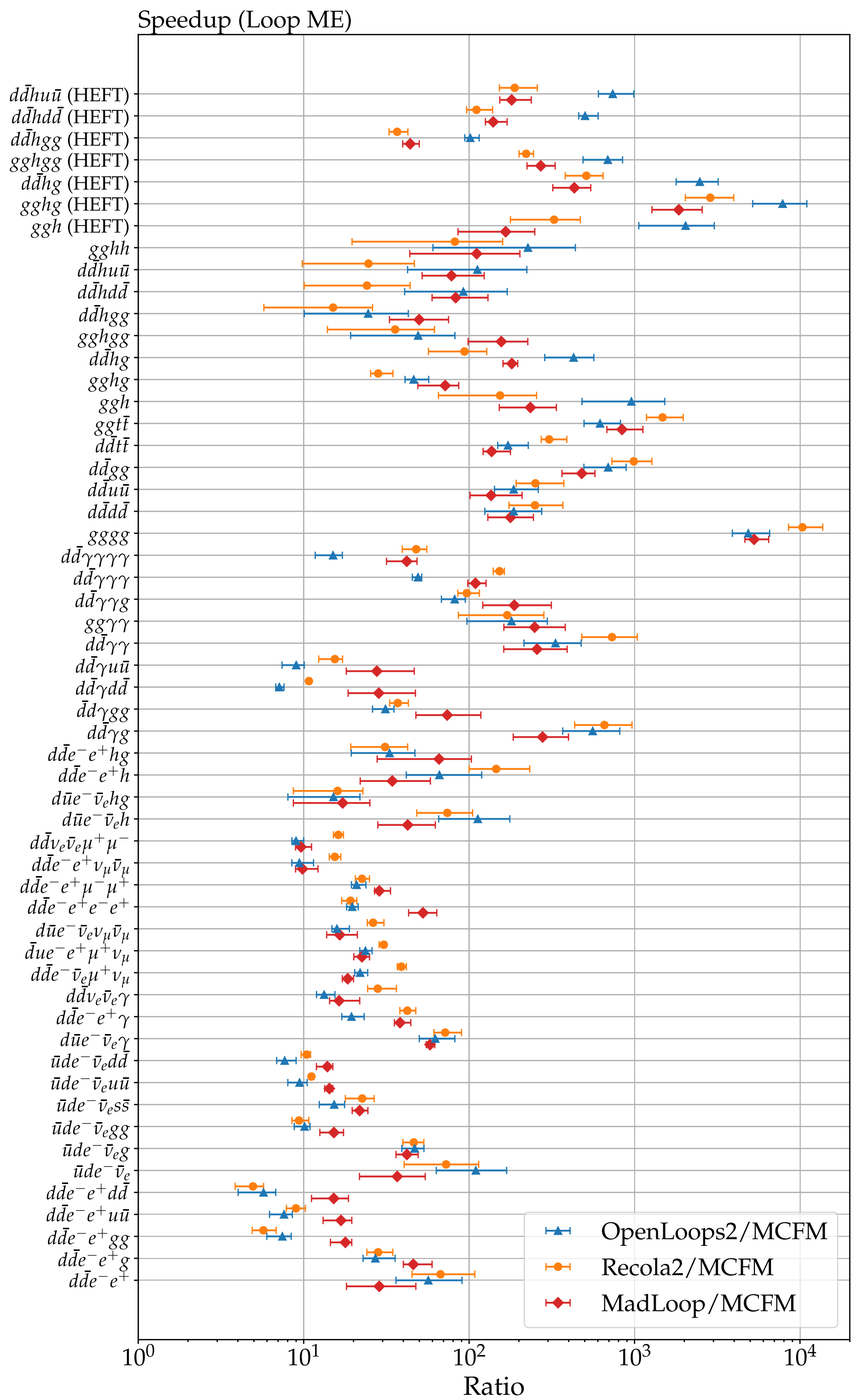
- MCFM contains about 350 processes at hadron-colliders evaluated at NLO.
- Since matrix elements are calculated using analytic formulae one can expect better performance, in terms of stability and computer speed, than fully numerical codes.
- In addition MCFM contains a number of process evaluated at NNLO using both the jettiness and the q_T slicing schemes.
  - NNLO results for pp → X require process pp → X + 1 parton at NLO, and two loop matrix elements for pp → X, so mostly limited to color-singlet processes.
- Recent(ish) additions to virtual matrix elements:
  - H+4 partons with full mass effects at one-loop Budge, De Laurentis, Ellis, Seth, JC, 2107.04472
  - Vector boson pair production at one loop: simplified analytic results for the process $q\bar{q}\ell\bar{\ell}\ell'\bar{\ell}'g$ De Laurentis, Ellis, JC, 2203.17170

MCFM 1-loop library

- Analytic 1-loop matrix elements from MCFM are also available in the form of a standalone library.

  Hoeche, Preuss, JC, 2107.04472

  - easily accessed in a similar way as, e.g. OpenLoops, through a C++ interface.
  - potential for significant speed gains vs. a numerical one-loop provider, either as component of higher-order calculation or parton shower.



NNLO in MCFM

- Attempt to document all the hadron collider processes calculated at NNLO (as of Feb. 2022).
- About 50% are available in MCFM.
- Note that in some cases N^{3}LO is now the start of the art

VBF: Dreyer, Karlberg, 1811.07906
Higgs: Chen et al, 2102.07607
DY: Chen et al, 2203.01565
VH, W: Baglio et al, 2209.06138

Process	MCFM	Process	MCFM
$H + 0$ jet [9, 10, 16–20]	✓ [21]	$W^{\pm} + 0$ jet [22–24]	✓ [21]
$Z/\gamma^* + 0$ jet [9, 23–25]	✓ [21]	ZH [26]	✓ [27]
$W^{\pm}\gamma$ [24, 28, 29]	✓ [30]	$Z\gamma$ [24, 31]	✓ [31]
$\gamma\gamma$ [24, 32–34]	✓ [35]	single top [36]	✓ [37]
$W^{\pm}H$ [38, 39]	✓ [27]	WZ [40, 41]	✓
ZZ [1, 24, 42–46]	✓	W^+W^- [24, 47–50]	✓
$W^{\pm} + 1$ jet [51, 52]	[8]	$Z + 1$ jet [53, 54]	[11]
$\gamma + 1$ jet [55]	[12]	$H + 1$ jet [56–61]	[13]
$b\bar{b} \to H$+jet	[14]		
$t\bar{t}$ [62–67]		$Z + b$ [68]	
$W^{\pm}H$+jet [69]		ZH+jet [70]	
Higgs WBF [71, 72]		$H \to b\bar{b}$ [73–75]	
top decay [37, 76, 77]		dijets [78–80]	
$\gamma\gamma$+jet [81]		$W^{\pm}c$ [82]	
$b\bar{b}$ [83]		$\gamma\gamma\gamma$ [84, 85]	
HH [86]		HHH [87]	

Comparative study of jettiness and q_T slicing

Ellis, Seth, JC, 2202.07738

- Leading log behavior of a color singlet cross section integrated up to a small value of q_T

$$\Sigma_T = \sigma_0 \exp\left[-\frac{\alpha_s C_F}{2\pi}\ln^2\left(\left(q_T^{\rm cut}\right)^2/Q^2\right)\right] = \sigma_0 \exp\left[-\frac{2\alpha_s C_F}{\pi}\ln^2\left(q_T^{\rm cut}/Q\right)\right]$$

- Corresponding LL formula for zero-jettiness

$$\Sigma_\tau = \sigma_0 \exp\left[-\frac{\alpha_s C_F}{\pi}\ln^2\frac{\tau^{\rm cut}}{Q}\right]$$

- Hence a similar size of the above-cut integral, and thus the computing time required to reach a given error estimate, is obtained when

$$\frac{\tau^{\rm cut}}{Q} \simeq \left(\frac{q_T^{\rm cut}}{Q}\right)^{\sqrt{2}}$$

Comparison of NNLO slicing methods

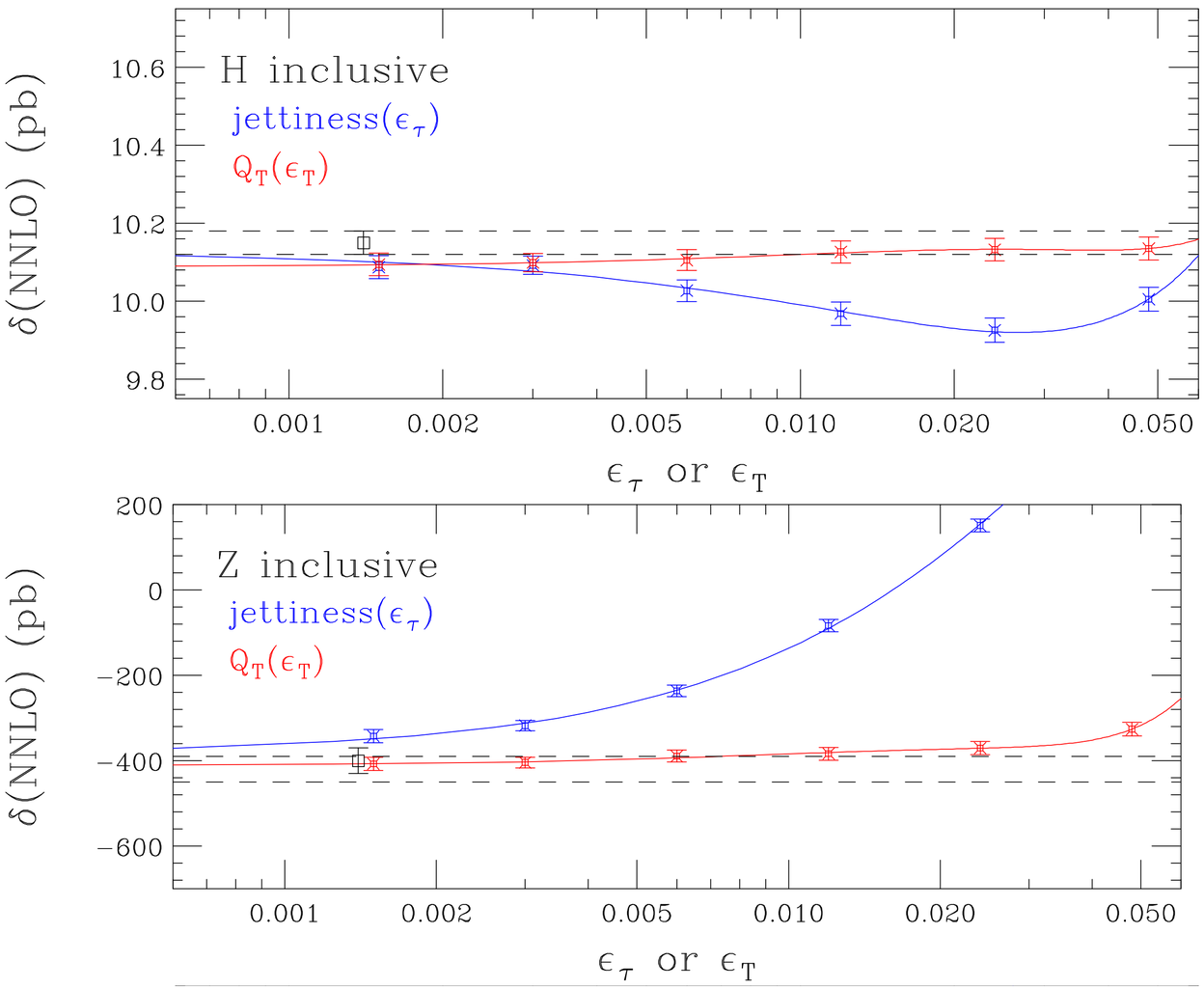
q_T $\quad \epsilon_T = q_T^{\rm cut}/Q$

jettiness $\quad \epsilon_\tau = (\tau^{\rm cut}/Q)^{\frac{1}{\sqrt{2}}}$

- q_T slicing method appears to have smaller power corrections in most cases for equal computational burden.
- However jettiness has the proven ability to deal with final states containing a jet.

  W+jet: Boughezal et al, 1504.02131
  Z+jet: Boughezal et al, 1512.01291
  H+jet: Boughezal et al, 1505.03893,
  JC et al, 1906.01020

- c.f. attempt to develop formalism for new slicing variables ("k_T-ness"), so far only to NLO. **Buonocore et al, 2201.11519**



q_T resummation

- Use the SCET-based "collinear anomaly" q_T resummation formalism:

  Becher, Neubert, +Hager, Wilhelm, 1109.6027, 1212.2621, 1904.08325

$$\mathrm{d}\sigma_{ij}(p_1,p_2,\{\underline{q}\}) = \int_0^1 \mathrm{d}z_1 \int_0^1 \mathrm{d}z_2\, \mathrm{d}\sigma^0_{ij}(z_1p_1,z_2p_2,\{\underline{q}\})\, \mathcal{H}_{ij}(z_1p_1,z_2p_2,\{\underline{q}\},\mu)$$
$$\times\, \frac{1}{4\pi}\int \mathrm{d}^2x_\perp\, e^{-iq_\perp x_\perp} \left(\frac{x_T^2Q^2}{b_0^2}\right)^{-F_{ij}(x_\perp,\mu)} \times B_i(z_1,x_\perp,\mu)\cdot B_j(z_2,x_\perp,\mu)\,,$$

- All universal ingredients (beam functions, B_i , B_j and collinear anomaly exponent F_{ij}) known up to 3 loops.
  - piggyback existing machinery of NNLO calculations in MCFM to reach N^{3}LL+NNLO accuracy for important processes.
  - implemented as "CuTe-MCFM", results for DY, Higgs, VH, $\gamma\gamma$, $Z\gamma$.

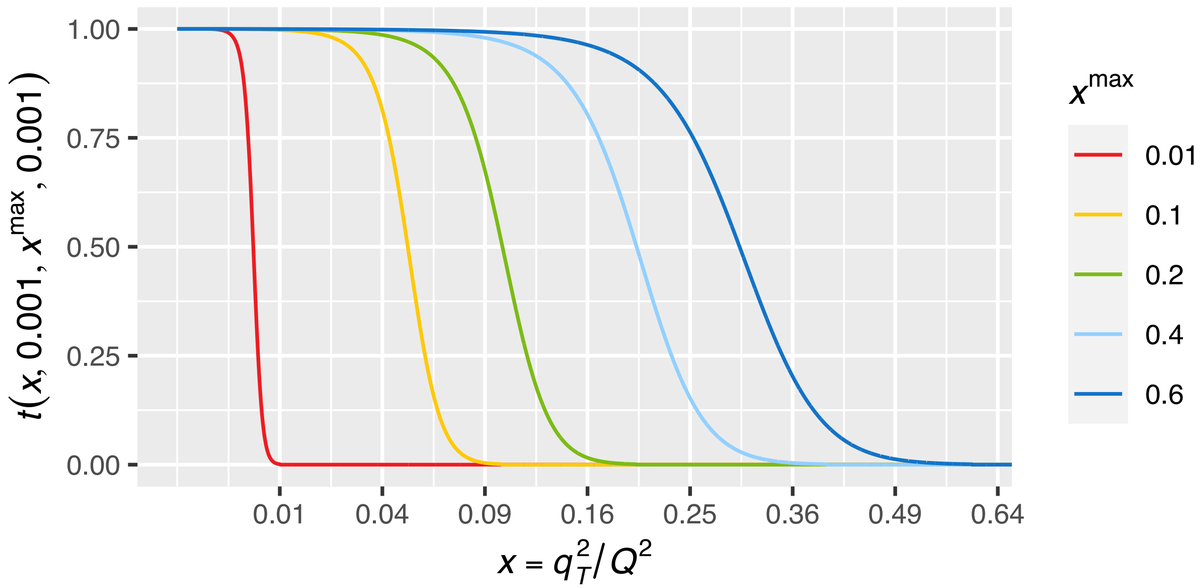
    Becher, Neumann 2009.11437

Matching to fixed order

- Fixed order result recovered up to higher order terms, which can induce unphysical behavior at large q_T.
- Match by expanding resummed result and replacing with fixed-order one — but computationally demanding at small q_T (introduce cutoff q_0).
- Implement a transition function to smoothly pass between resummed and fixed-order domains, choosing its parameters on a case-by-case basis.

Becher, Neumann 2009.11437

$$\left.\frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}q_T}\right|_{\text{naively matched to NNLO}} = \frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}q_T} + \underbrace{\frac{\mathrm{d}\sigma^{\mathrm{NNLO}}}{\mathrm{d}q_T} - \left.\frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}q_T}\right|_{\text{exp. to NNLO}}}_{\text{matching correction } \Delta\sigma}$$

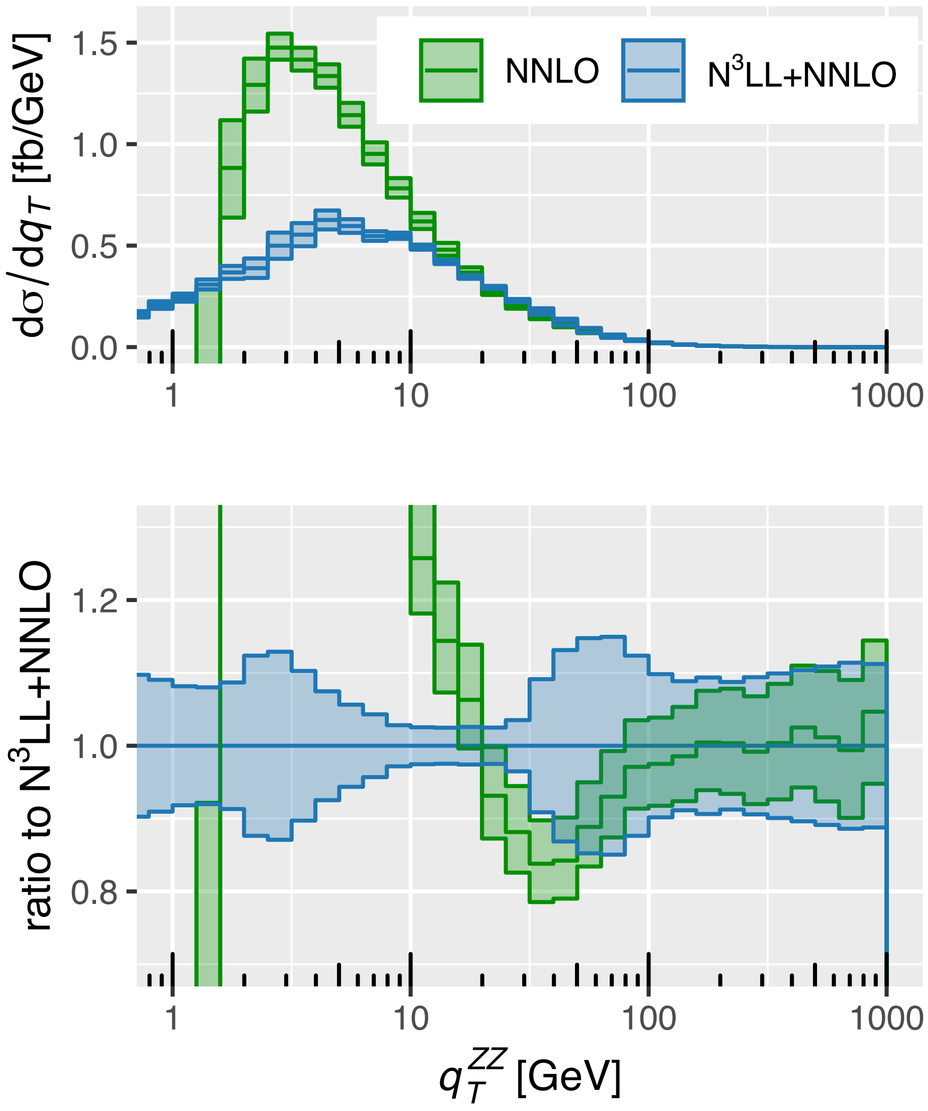


$$\left.\frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}q_T}\right|_{\text{matched to NNLO}} = t(x)\left(\frac{\mathrm{d}\sigma^{\mathrm{N^3LL}}}{\mathrm{d}q_T} + \Delta\sigma|_{q_T>q_0}\right) + (1-t(x))\frac{\mathrm{d}\sigma^{\mathrm{NNLO}}}{\mathrm{d}q_T}$$

Vector boson pair production at small q_T

Ellis, Neumann, Seth, JC, 2210.10724

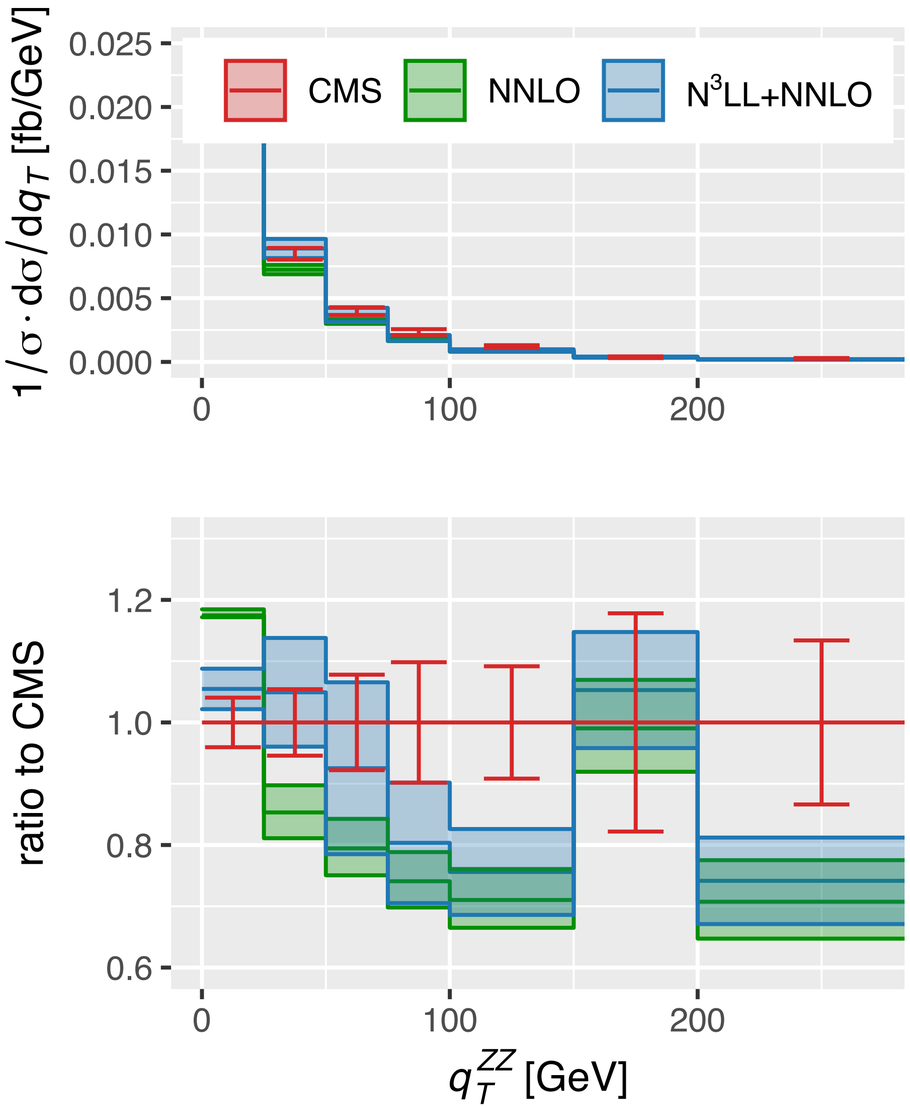
- Resummation effects are potentially more important for vector boson pair production at the same q_T since Q is larger.
- Transition between about 50 and 100 GeV, $(q_T/Q)^2 \sim [0.05,\ 0.2]$, leading to total uncertainty up to 15% in that region.
- Resummation at N³LL+NNLO becomes important below those scales, small uncertainties until ~ 5 GeV.



Transverse momentum distribution of the ZZ pair at NNLO and NNNLL+NNLO using CMS cuts at $\sqrt{s} = 13.6$ TeV

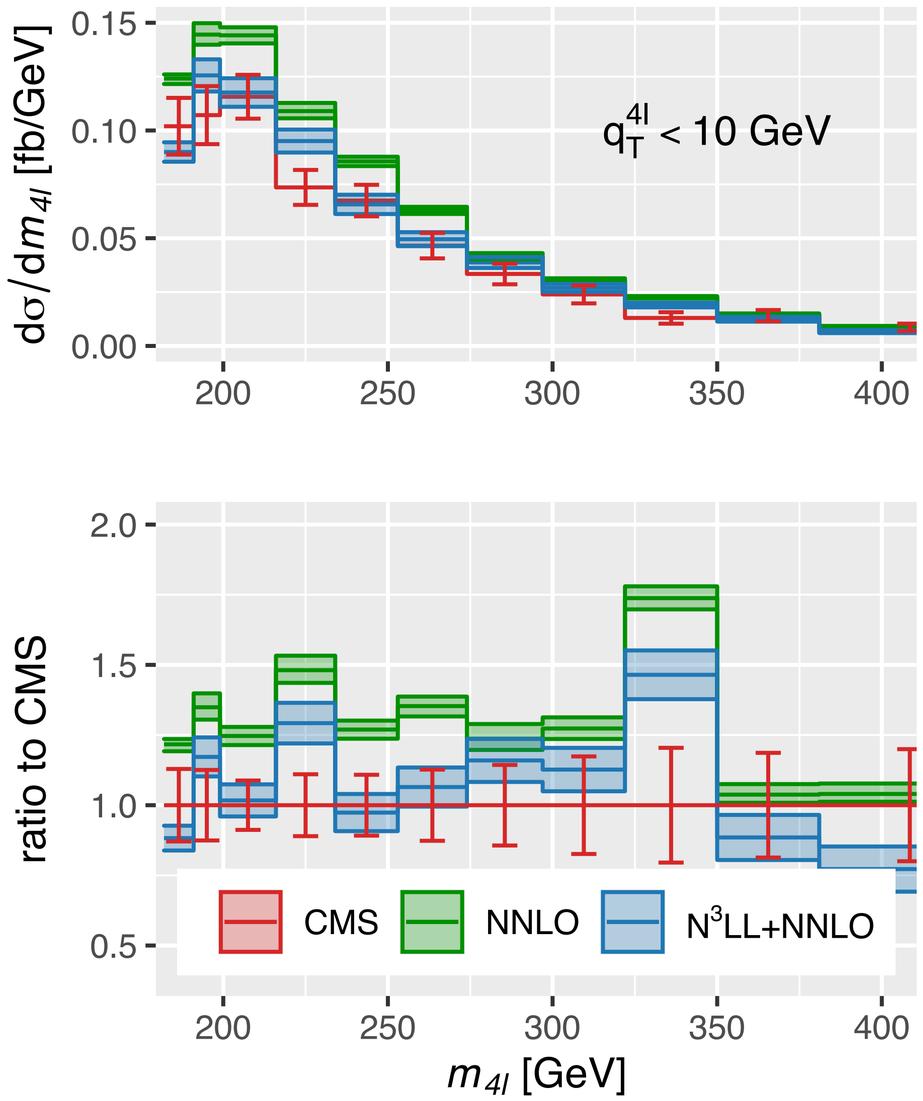
Comparison with CMS data at 13 TeV

- We simplify the CMS analysis, by applying the same cuts to both electrons and muons and neglect (tiny) identical particle effects.
- Resummation improves description below $q_T \sim 75$ GeV.
- More data will allow finer binning, so the resummation effects will be ever more necessary.



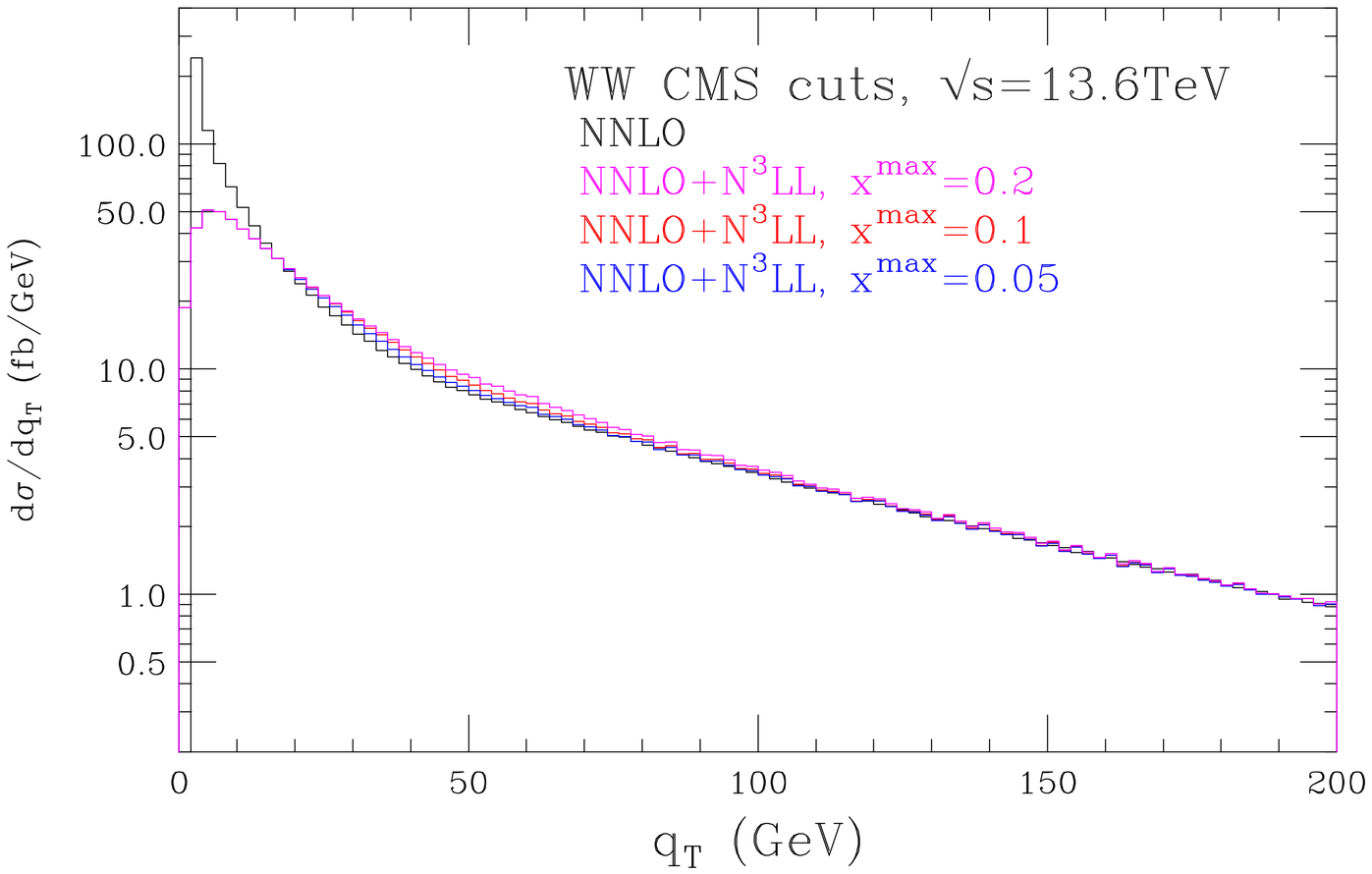
ATLAS data ZZ

- The ATLAS collaboration (2103.01918) performed measurements of the m_{4l} distribution in five slices of q_T^{4l}
- Expectation is that resummation should improve agreement with the data, as m_{4l} increases, as observed.



Truth WW cross section

- WZ and WW q_T distributions show similar pattern but of course not directly measurable.
- Much more important for WW is the p_T^{veto} cross section to reduce background from $t\bar{t}$



Jet veto cross section

see, for example, Becher et al, 1307.0025 , Stewart et al, 1307.1808

- Well-developed formalism, primarily focussed on (important) Higgs case;
  - jets defined using sequential recombination jet algorithms.
- Jet vetos generate large logarithms, as codified in factorization formula; however logarithms are smaller, typical value of $p_T^{\rm veto} \sim 25\ \text{GeV}$.
- Beam and soft functions for leading jet p_T recently calculated at two-loop order using an exponential regulator by Abreu et al.
- Jet veto cross sections are simpler than the q_T resummed calculation (no b-space, directly in p_T).

$$d_{ij} = \min(p_{Ti}^n, p_{Tj}^n)\frac{\sqrt{\Delta y_{ij}^2 + \Delta\phi_{ij}^2}}{R}, \qquad d_{iB} = p_{Ti}^n$$

Rapidity regulator ν

$$\frac{d^2\sigma(p_T^{veto})}{dM^2dy} = \sigma_0 \left|C_V(-M^2,\mu)\right|^2$$
$$\left[\mathscr{B}_c(\xi_1, M, p_T^{veto}, R^2, \mu, \nu)\,\mathscr{B}_{\bar{c}}(\xi_2, M, p_T^{veto}, R^2, \mu, \nu) \times \mathscr{S}(p_T^{veto}, R^2, \mu, \nu)\right]$$

Beam functions
Abreu et al, 2207.07037

Soft function
Abreu et al, 2204.03987

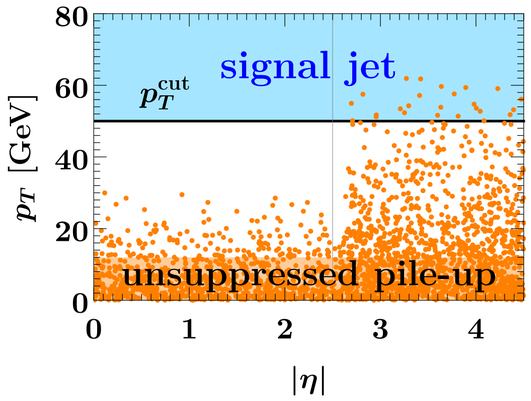
$$\sigma_0 = \frac{4\pi\alpha^2}{3N_c M^2 s} \qquad \xi_{1,2} = (M/\sqrt{s})\, e^{\pm y}$$

Jet veto in a limited rapidity range

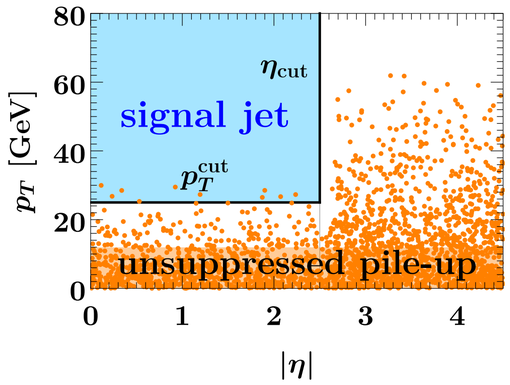
- Formula on last slide is valid for jet cross sections which are vetoed for all values of the jet rapidity.
- Experimental analyses actually perform jet rapidity cuts, i.e. $\eta < \eta_{\rm cut}$.
- Can identify three theoretical regions:

  Michel, Pietrulewicz, Tackmann, 1810.12911

  - $\eta_{\rm cut} \gg \ln(Q/p_T^{\rm veto})$ (standard jet veto resummation)
  - $\eta_{\rm cut} \sim \ln(Q/p_T^{\rm veto})$ ($\eta_{\rm cut}$-dependent beam functions)
  - $\eta_{\rm cut} \ll \ln(Q/p_T^{\rm veto})$ (collinear non-global logs)



Current theory calculation



Typical Experimental cuts

Strategy: determine where resummation is potentially important, before considering limited rapidity range resummation

Refactorize à la Becher-Neubert

$$\left[\mathscr{B}_c(\xi_1, Q, p_T^{veto}, R^2, \mu, \nu)\, \mathscr{B}_{\bar{c}}(\xi_2, Q, p_T^{veto}, R^2, \mu, \nu\ )\mathcal{S}(p_T^{veto}, R, \mu, \nu)\right]_{q^2=Q^2}$$

$$= \left(\frac{Q}{p_T^{veto}}\right)^{-2F_{qq}(p_T^{veto},R,\mu)} e^{2h^F(p_T^{veto},\mu)}\, \bar{B}_q(\xi_1, p_T^{veto}, R)\, \bar{B}_{\bar{q}}(\xi_2, p_T^{veto}, R)$$

"Collinear anomaly"

- Collinear anomaly expansion: $F_{qq}(p_T^{\rm veto}, \mu) = a_S F_{qq}^{(0)} + a_S^2 F_{qq}^{(1)} + a_S^3 F_{qq}^{(2)} + \dots\ , \quad a_S = \dfrac{\alpha_S}{4\pi}$

$$F_{qq}^{(0)} = \Gamma_0^F L_\perp + d_1^{\rm veto}(R, F)$$

$$F_{qq}^{(1)} = \frac{1}{2}\Gamma_0^F \beta_0 L_\perp^2 + \Gamma_1^F L_\perp + d_2^{\rm veto}(R, F)$$

$$F_{qq}^{(2)} = \frac{1}{3}\Gamma_0^F \beta_0^2 L_\perp^3 + \frac{1}{2}(\Gamma_0^F \beta_1 + 2\Gamma_1^F \beta_0) L_\perp^2 + (\Gamma_2^F + 2\beta_0 d_2^{\rm veto}(R, F)) L_\perp + d_3^{\rm veto}(R, F)$$

$$L_\perp = 2 \ln \frac{\mu}{p_T^{\rm veto}}$$

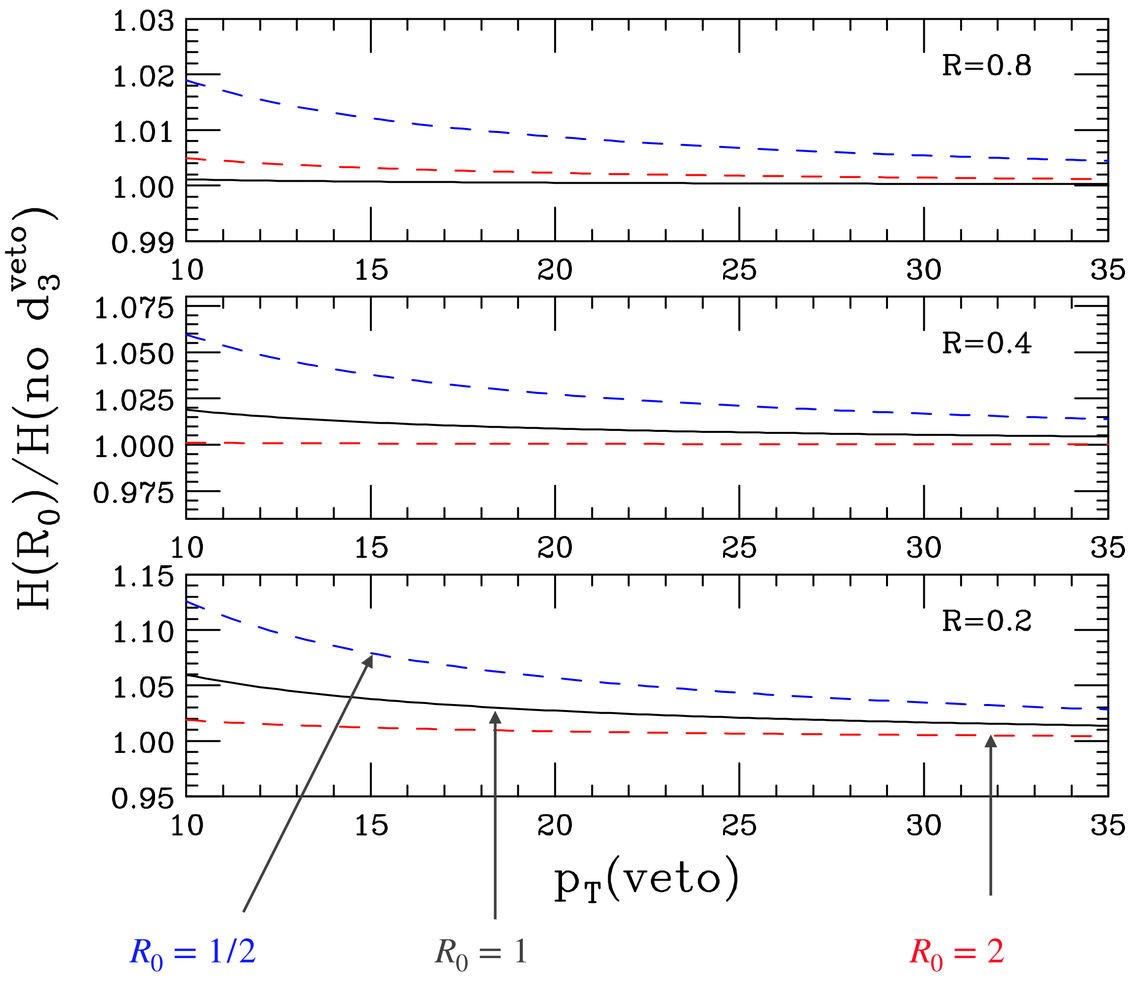
- Full N³LL requires (R-dependent) coefficient $d_3^{\rm veto}$, which is currently unknown.
  - Extracted in small-R limit — good to O(25%) in $d_2^{\rm veto}$ (for typical R) $\longrightarrow$ only claim N³LL$_p$.

Banfi et al, 1511.02886

Dependence on approximate $d_3^{\rm veto}$

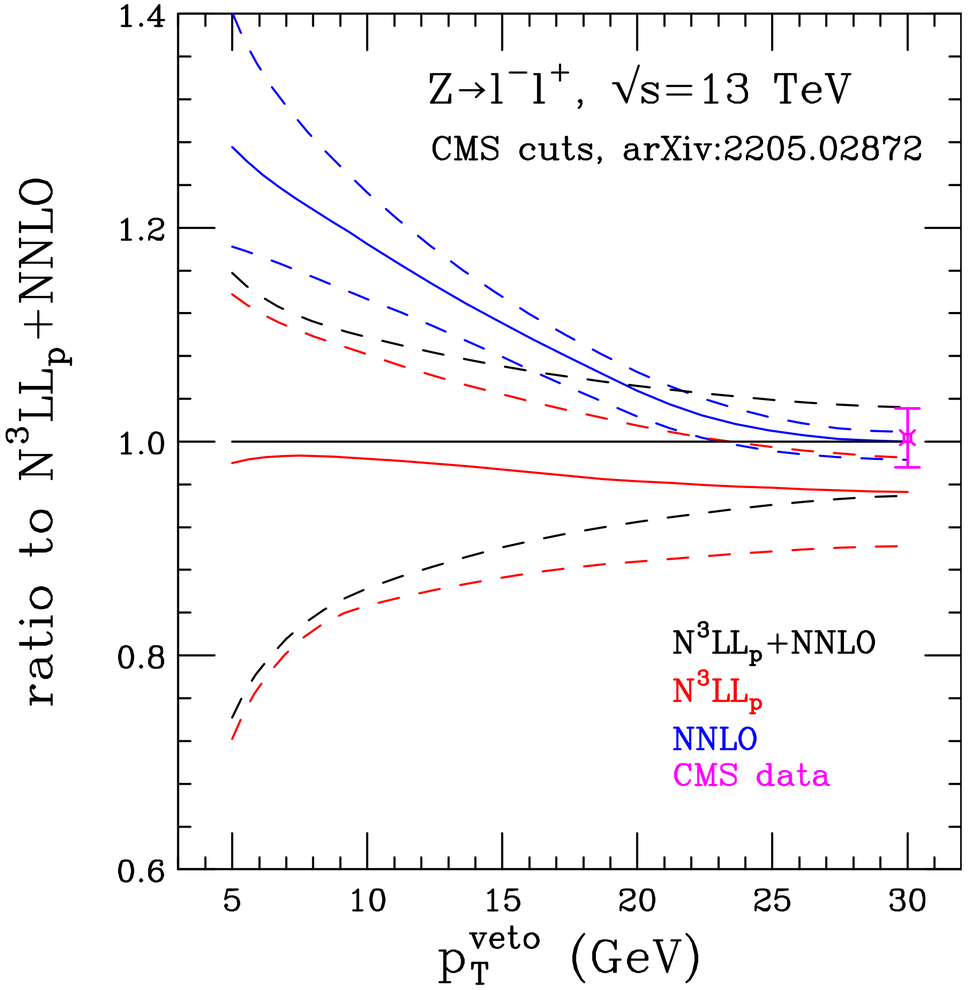
Ellis, Neumann, Seth, JC, to appear

- $d_3^{\rm veto} \sim -8.4 \times 64 C_B \ln^2(R/R_0)$
- R_0 varied as an uncertainty: for R=0.4, varying between 0.5 and 2 scales $d_3^{\rm veto}$ in the range [0.06,3].
- Contributes as $\left(\frac{m_H}{p_T^{\rm veto}}\right)^{-2\left(\frac{\alpha_s(\mu)}{4\pi}\right)^3 d_3^{\rm veto}}$ so in this approximation $d_3^{\rm veto} < 0$ and it increases the cross section.
- Estimate $\leq 2.5\,\%$ uncertainty at $p_T^{\rm veto}$= 25 GeV and $R = 0.4$.



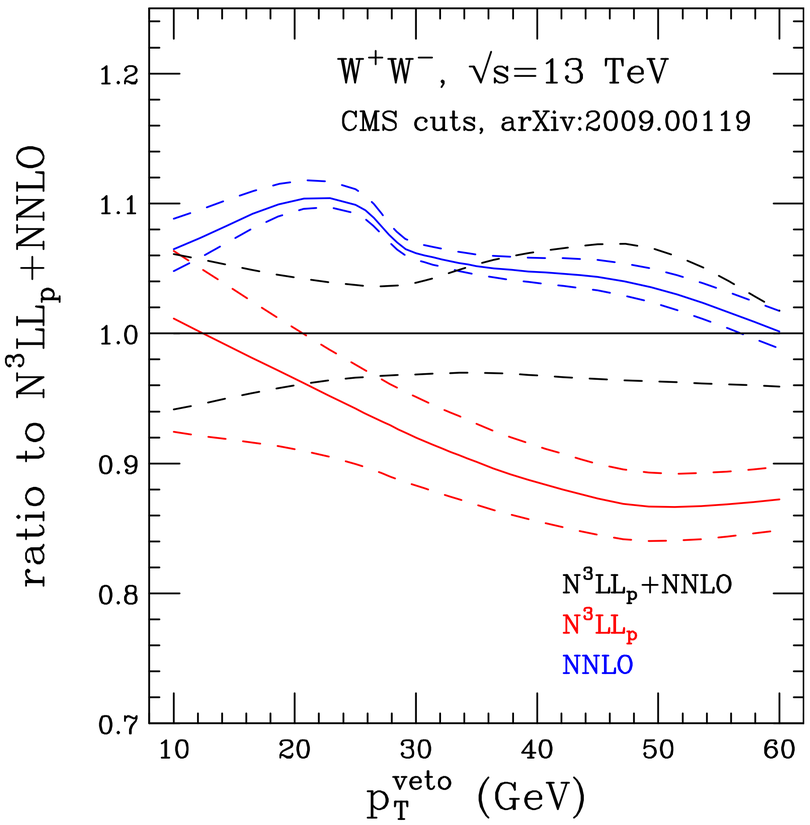
Jet veto in Z production vs. CMS

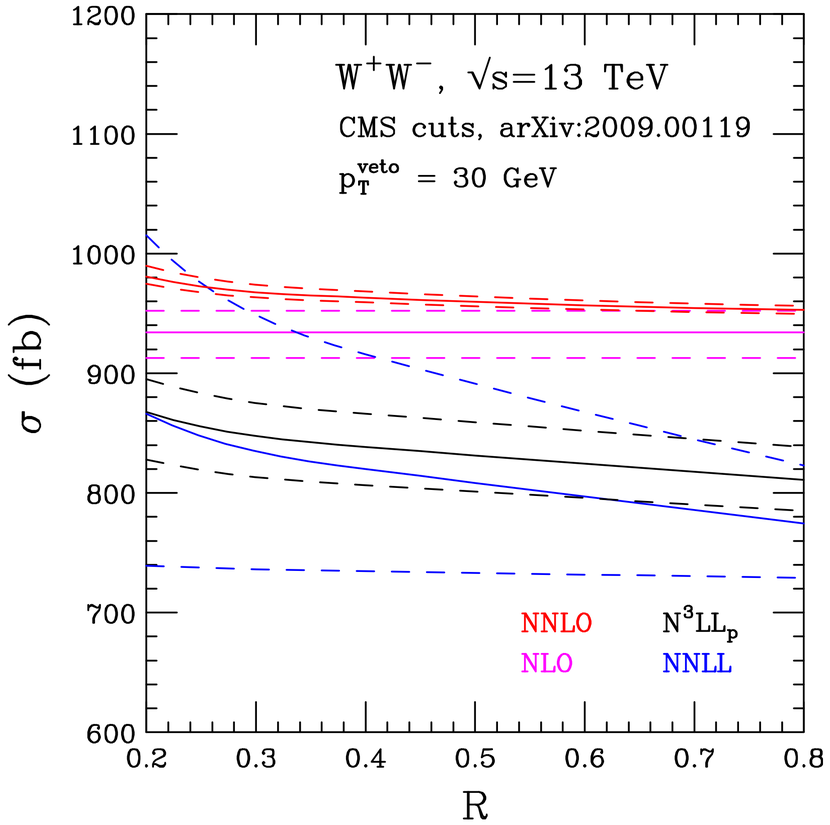
- At $p_T^{\rm veto} \sim 25 - 30$ all calculations agree within errors.
- However error estimates differ between NNLO and N^3LL_p +NNLO.
- For $p_T^{\rm veto} = 30$ GeV, $(\ln(Q/p_T^{\rm veto} = 1.1) \ll (\eta_{\rm cut} = 2.4)$
- As expected, at (probably irrelevant) small $p_T^{\rm veto}$ resummed calculations show significant deviations from fixed order.
- Jet veto resummation probably not so necessary here.



Jet veto in W^+W^- production

- Evidence that neither NNLO nor N^3LL is sufficient, especially around $p_T^{\rm veto} = 20 - 40$ GeV
- R dependence is modest.
- $|\eta_{\rm cut}| < 4.5$, so we can argue that $(\ln(Q/p_T^{\rm veto}) = (1.3 - 2.2) \ll 4.5$

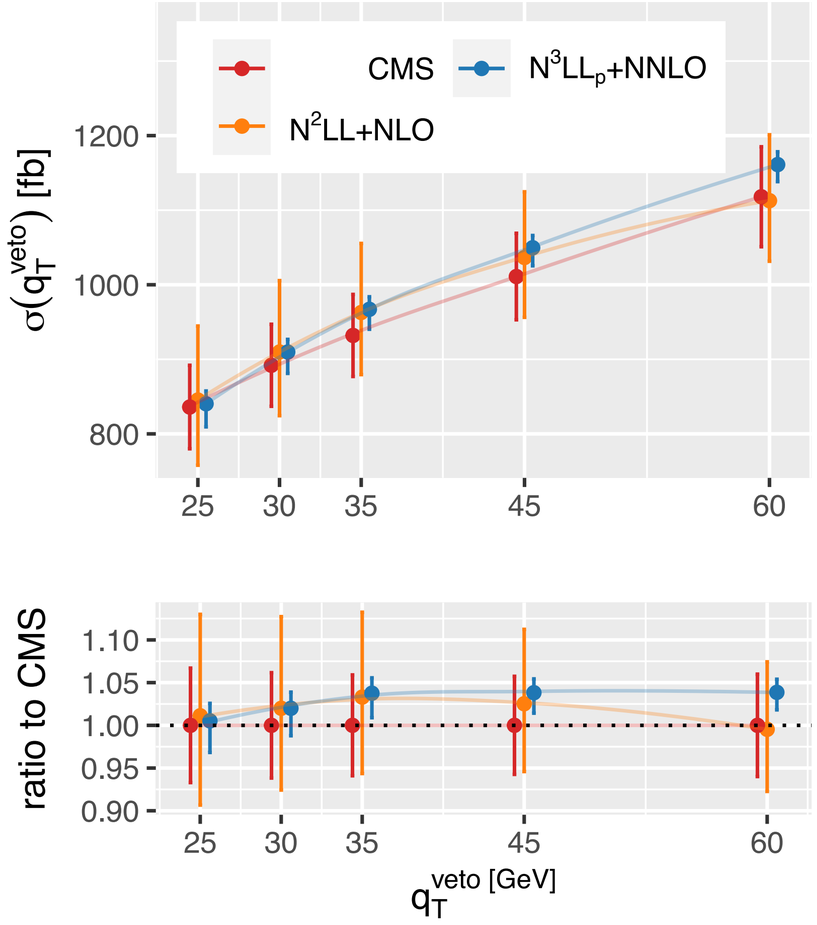




Jet veto in W^+W^- production vs. CMS

Ellis, Neumann, Seth, JC, 2210.10724

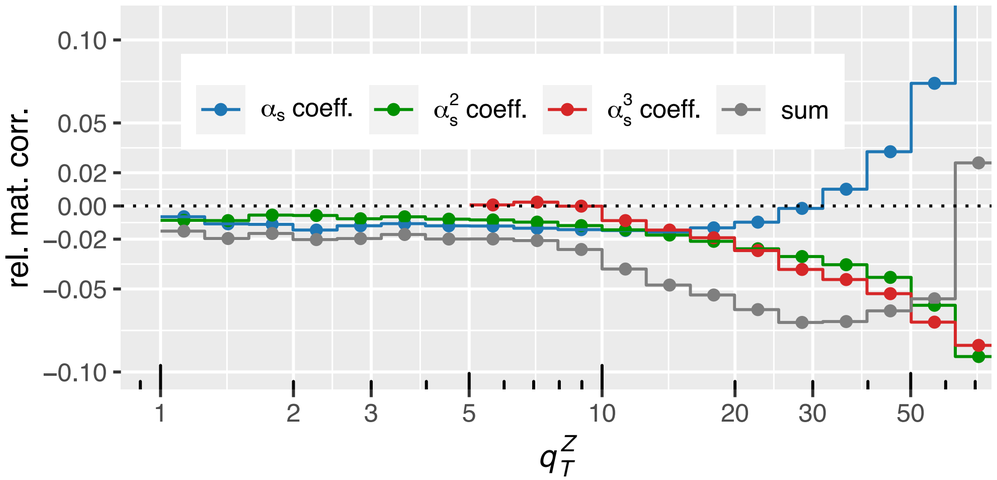
- Compare with CMS data taken from 2009.00119.
- Errors improve going from N^2LL+NNLO to N^3LL_p+NNLO
- Theoretical errors smaller than experimental ⟶ interesting to see more data (only 36/fb).



q_T resummation at $N^4LL_p+N^3LO$

Neumann, JC, 2207.07056

- Use recent calculations to push logarithmic accuracy to next order.
  - 3-loop beam functions 1912.05778, 2006.05329, 2012.03256, Luo et al. and Ebert et al.
  - 4-loop rapidity anomalous dimension Duhr et al., 2205.02242; Moult et al., 2205.02249
- "p": 5-loop cusp estimated (negligible) and missing unknown N^3LO PDFs.
- Combine with MCFM Z+jet calculation at NNLO to also reach N^3LO accuracy for Drell-Yan process.
- Performing pure fixed-order calculation tough at very low q_T but in practice only need to be convinced that matching corrections approach zero and are sufficiently small.



Comparison with CMS

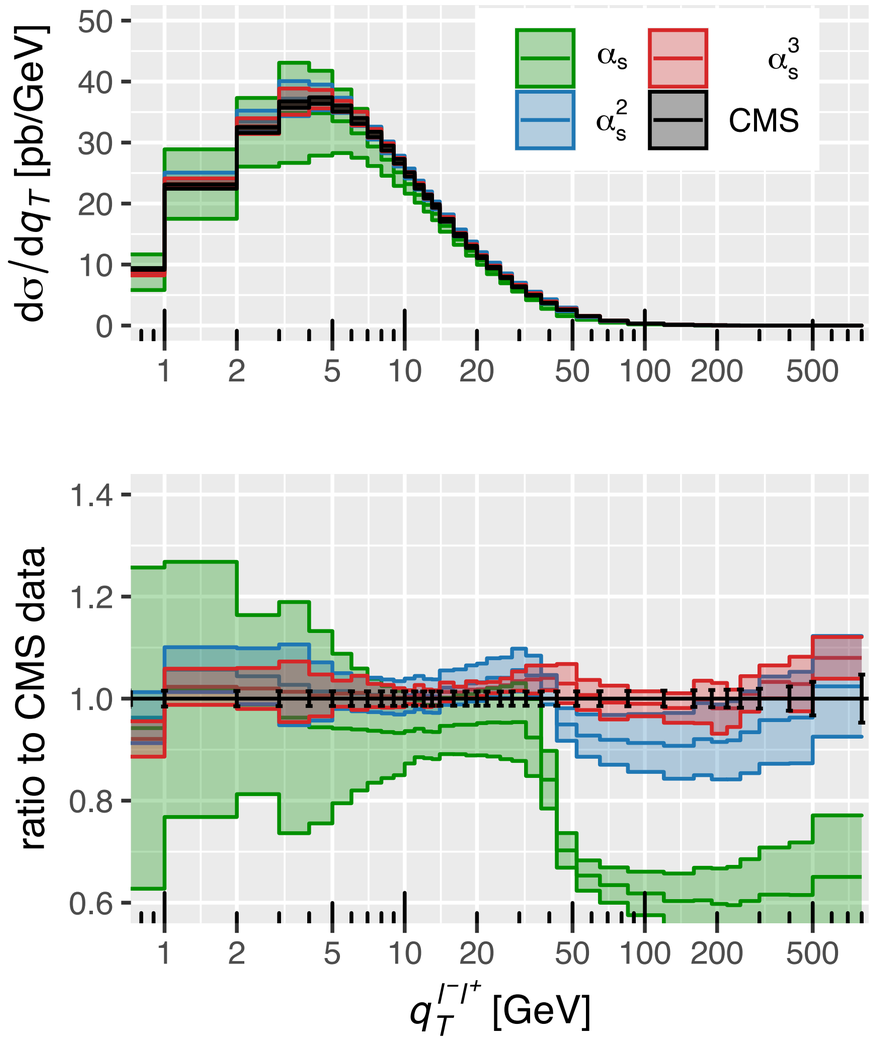
- Excellent agreement with CMS data at the highest order, noticeable improvement at both low and high q_T.
- Integrate over spectrum for a cross-section comparison.

Order k	fixed-order α_s^k	res. improved α_s^k
0	694^{+85}_{-92}	—
1	732^{+19}_{-30}	$637 \pm 8_{\text{mat.}} \pm 70_{\text{sc.}}$
2	720^{+4}_{-3}	$707 \pm 3_{\text{mat.}} \pm 29_{\text{sc.}}$
3	$700^{+4}_{-6} \pm 1_{\text{slicing}}$	$702 \pm 1_{\text{mat.}} \pm 1_{\text{m.c.}} \pm 17_{\text{sc.}}$

$699 \pm 5\,(\text{syst.}) \pm 17\,(\text{lumi.})$ (e, μ combined) [3]

- Total uncertainty larger by factor 2 than RadISH+NNLOJET

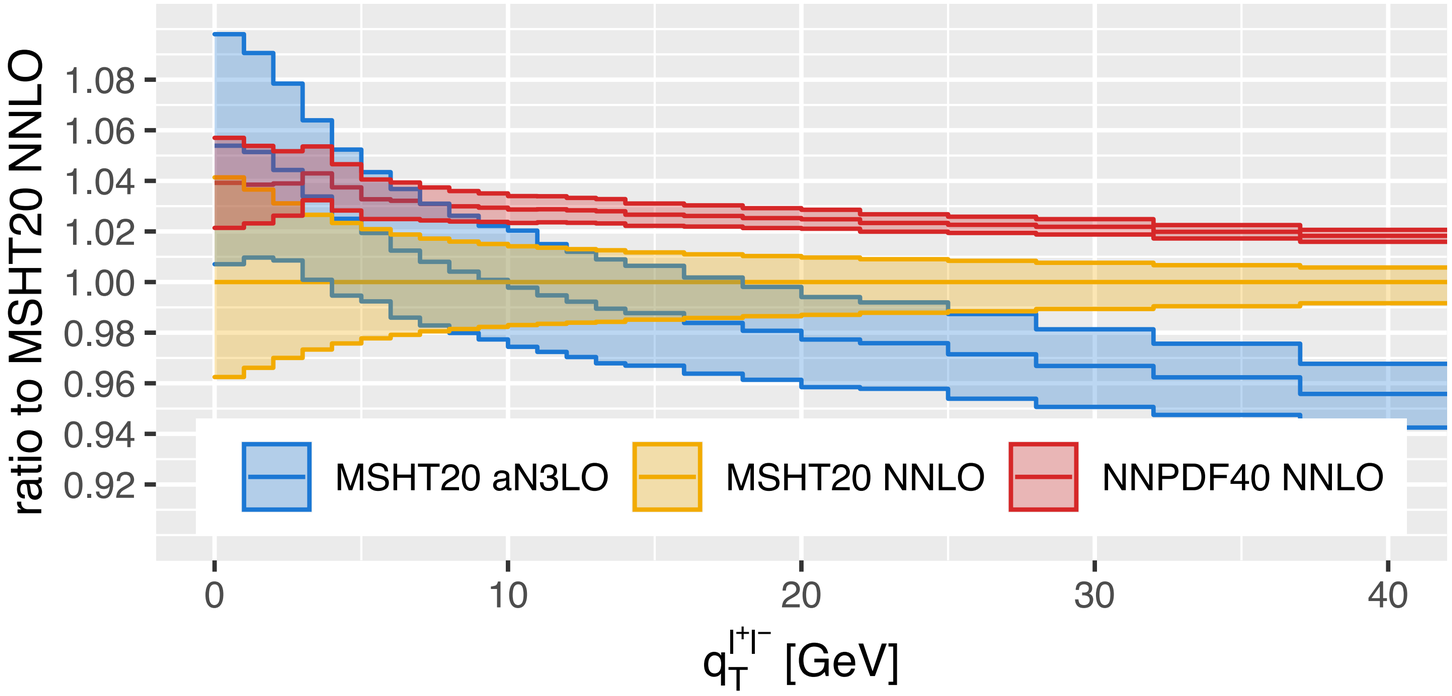
Chen et al., 2203.01565



Impact of PDFs

- For illustration, resummed contribution only: for approximate N^{3}LO of MSHT, NNLO of the same and (our default) NNPDF4.0 NNLO.

  MSHT20aN^{3}LO: McGowan et al., 2207.04739

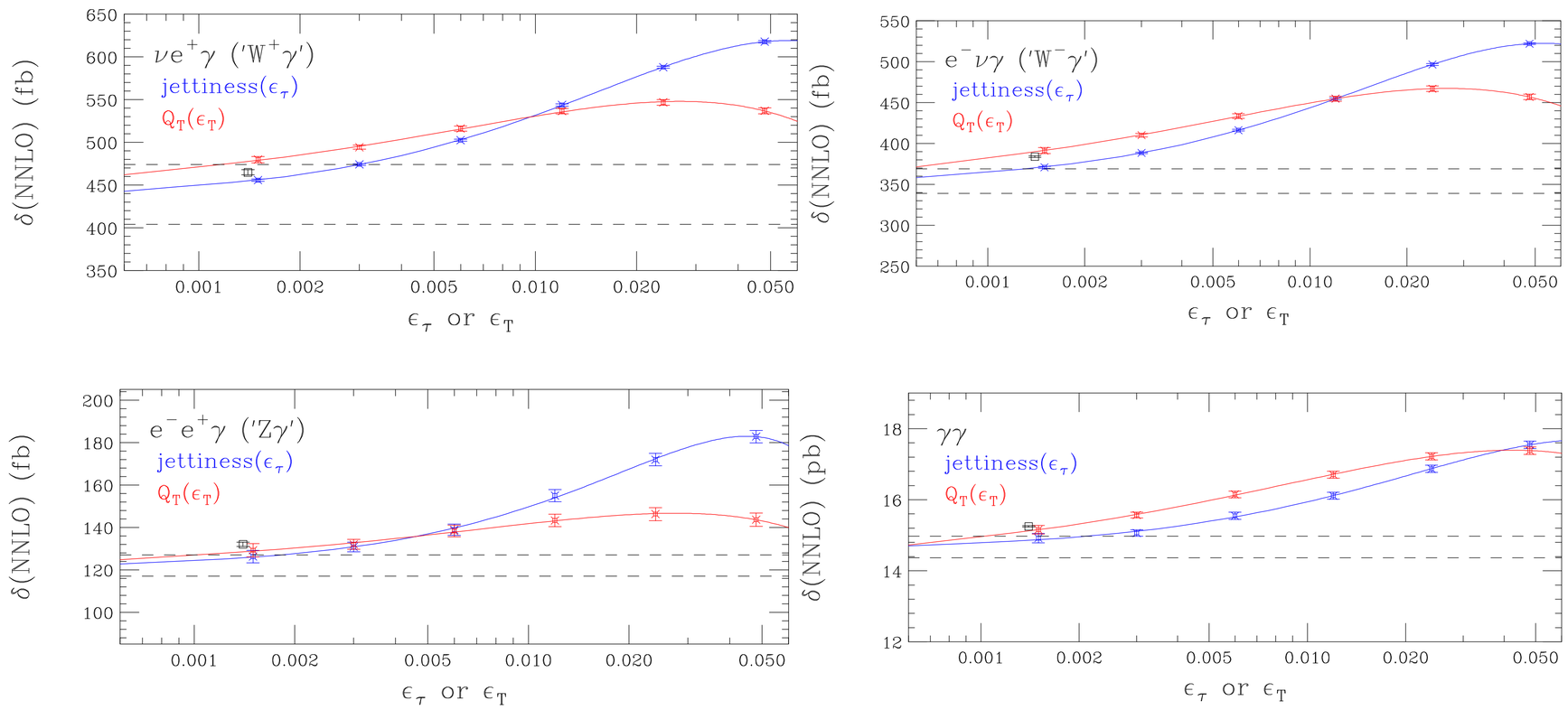


Conclusion

- Calculations at NNLO show mainly smaller power corrections for q_T slicing than for zero-jettiness slicing, with computing times roughly equal.
- The small q_T resummation in CuTe-MCFM, accurate to N^3LL +NNLO, has been extended to all color singlet final states with pairs of massive vector bosons — public release soon.
- We have compared our predictions with the available data but the fine-grained experimental study of vector boson pair processes where the resummation effects will be crucial is, in the main, still to come.
- Extension to N^4LL_p + N^3LO for Z production CPU-intensive but complete (public soon).
- We have also resummed cross sections at N^3LL_p +NNLO for all color singlet final state processes with a $p_T^{\rm veto}$ at all rapidities. Necessary for Higgs production and for vector boson pair production.

Backup material

q_T vs. jettiness: photon processes



- Much more similar for photon cases, jettiness perhaps slightly favored.
- Photon isolation induces significant power corrections in both approaches

Ebert, Tackmann 1911.08486; Becher, Neumann 2009.11437; Becher et al, 2208.01544

Uncertainty estimate

- Estimate the perturbative truncation uncertainty by varying the renormalization/hard scale and the factorization/resummation scale by the multipliers

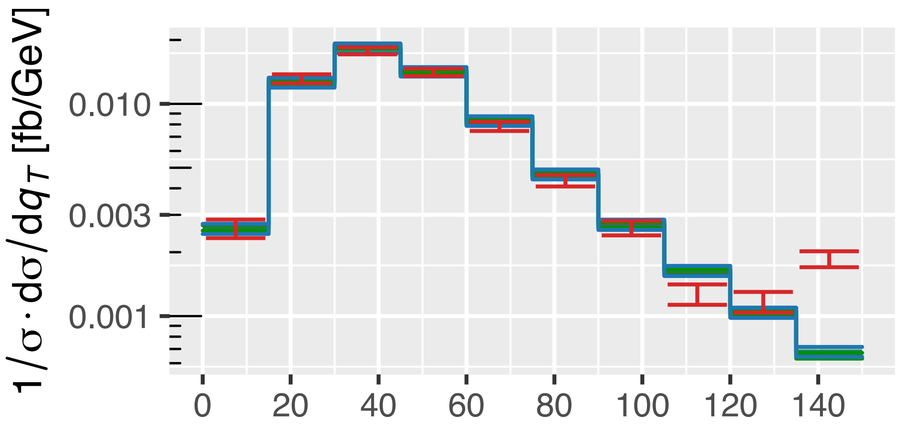
$$(k_F; k_R) \in \left\{(2,2), (0.5,0.5), (2,1), (1,1), (0.5,1), (1,2), (1,0.5)\right\}.$$

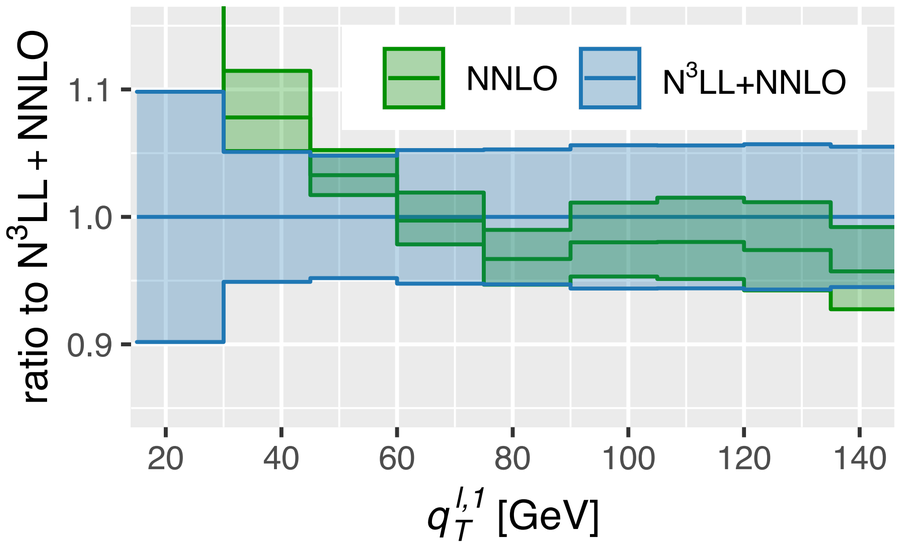
- For fixed order $\mu_F = k_F \hat{Q}$, $\mu_R = k_R \hat{Q}$.
- Hard scale is $k_R \hat{Q}$. To set the resummation scale, first calculate characteristic scale $q^* = Q^2 \exp(-\pi/C_i/\alpha_s(q^*))$ and then set $\mu = \max\{k_F \times qT + q^* \exp(-q_T/q^*), 2\text{ GeV}\}$ so that for small qT, μ approaches q* and it remains in the perturbative region.
- Additional important resummation uncertainties:
  - reintroduce rapidity scale dependence (fixed-order remnant of analytic regulator)
  - vary parameters in transition function.

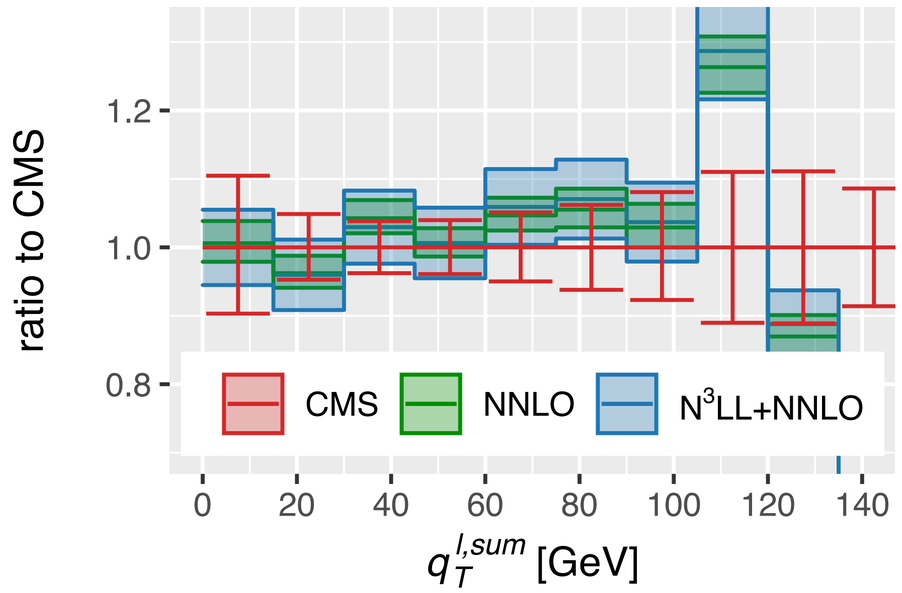
Jaiswal, Okui, 1506.07529

CMS results on lepton q_T in ZZ

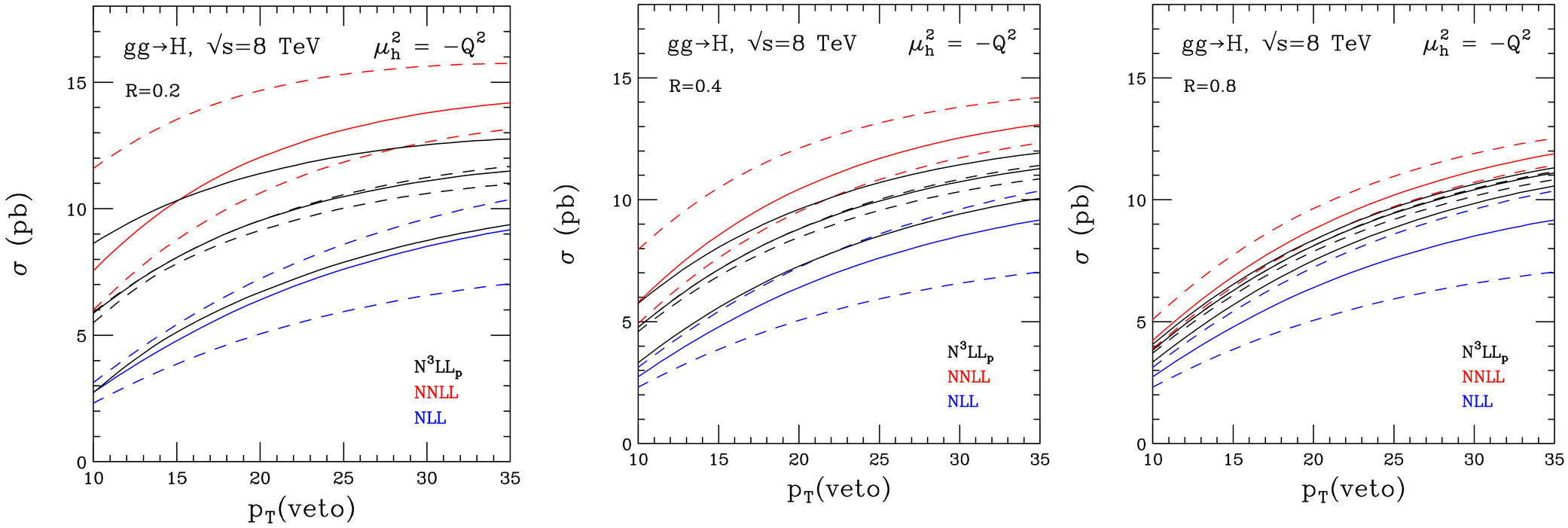
- CMS also present results on the lepton q_T^l (summed over all leptons). Here the effect of resummation is minimal since
- However the q_T of the leading lepton ($q_T^{l,1}$) shows an effect.







Jet-veto in Higgs production



- In the main the perturbative series is well-behaved at moderate R and successive orders lie within the band of the preceding order
- Resummation important in this case.