



## Neutrino Event Generation

Joshua Isaacson

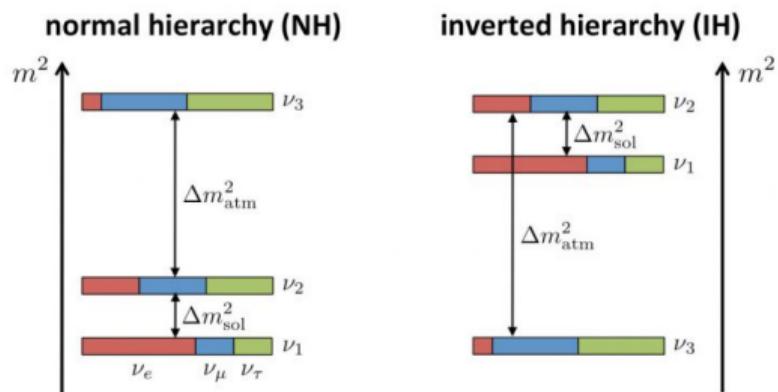
CTEQ Fall Meeting 2022

15 November 2022

# Neutrino Physics

## Open Questions:

- ① Source of neutrino masses
- ② CP violation in the neutrino sector
- ③ Are there other types of neutrinos?  
(Sterile neutrinos, etc.)
- ④ Are neutrinos Dirac or Majorana particles?
- ⑤ Mass ordering (Normal or inverted)?



Credit: JUNO Collaboration / JGU-Mainz

# Neutrino Oscillations

## PMNS Matrix

$$U_{PMNS} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & c_{23} & s_{23} \\ 0 & -s_{23} & c_{23} \end{pmatrix} \begin{pmatrix} c_{13} & 0 & s_{13}e^{-i\delta_{CP}} \\ 0 & 1 & 0 \\ -s_{13}e^{i\delta_{CP}} & 0 & c_{13} \end{pmatrix} \begin{pmatrix} c_{12} & s_{12} & 0 \\ -s_{12} & c_{12} & 0 \\ 0 & 0 & 1 \end{pmatrix}$$

$$H_{ij}^0 = (p + m_i^2/2p)\delta_{ij} \quad H = U_{PMNS}^\dagger H^0 U_{PMNS} \quad P(\nu_\alpha \rightarrow \nu_\beta; L) = |\langle \nu_\beta | e^{-iHL} | \nu_\alpha \rangle|^2$$

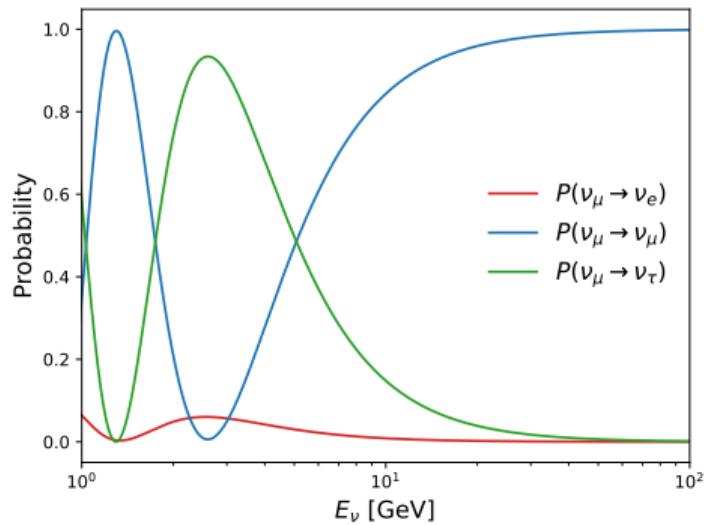
- Three mixing angles and one CP violating phase
- Since  $m_\nu \approx 0$ , we can take  $E = p$
- Above does not include corrections for propagating through matter

- $H^0$  is the Hamiltonian in the mass basis,  $H$  is the Hamiltonian in the flavor basis
- Probability depends on distance the neutrinos propagate ( $L$ )
- Only sensitive to mass squared differences ( $\Delta m_{ij}^2 = m_i^2 - m_j^2$ )

# Neutrino Oscillations

## 2-Flavor Approximation Oscillation Probability

$$P(\nu_\alpha \rightarrow \nu_\beta; L) = \sin^2(2\theta) \sin^2 \left( \frac{\Delta m^2 L}{4E} \right) \quad P(\nu_\alpha \rightarrow \nu_\alpha; L) = 1 - P(\nu_\alpha \rightarrow \nu_\beta; L)$$



# Measuring Oscillation Parameters

## Experimental Events

$$\frac{N_{FD}}{N_{ND}}(E_{\text{reco}}) \propto \frac{\int dE_\nu \frac{d\phi_\alpha^{\text{FD}}}{dE_\nu} P(\nu_\alpha \rightarrow \nu_\beta; E_\nu) \sigma_\beta(E_\nu) \mathcal{M}_\alpha^{\text{FD}}(E_\nu, E_{\text{reco}})}{\int dE_\nu \frac{d\phi_\alpha^{\text{ND}}}{dE_\nu} \sigma_\alpha(E_\nu) \mathcal{M}_\alpha^{\text{ND}}(E_\nu, E_{\text{reco}})}$$

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- Number of events in the near/far detector

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- Probability to oscillate from  $\alpha$  to  $\beta$

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- Neutrino-nucleus interaction cross section

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- Number of events in the near/far detector
- Probability to oscillate from  $\alpha$  to  $\beta$
- Neutrino-nucleus interaction cross section
- Migration matrix. Depends on topology of detected event  
(i.e. number of protons, etc.)

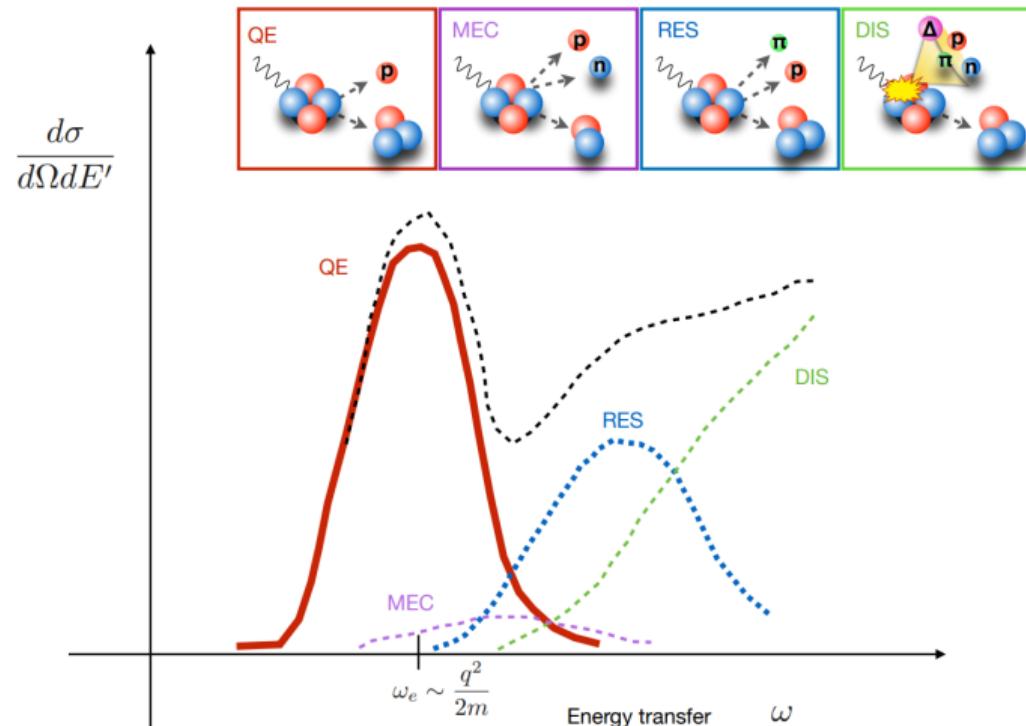
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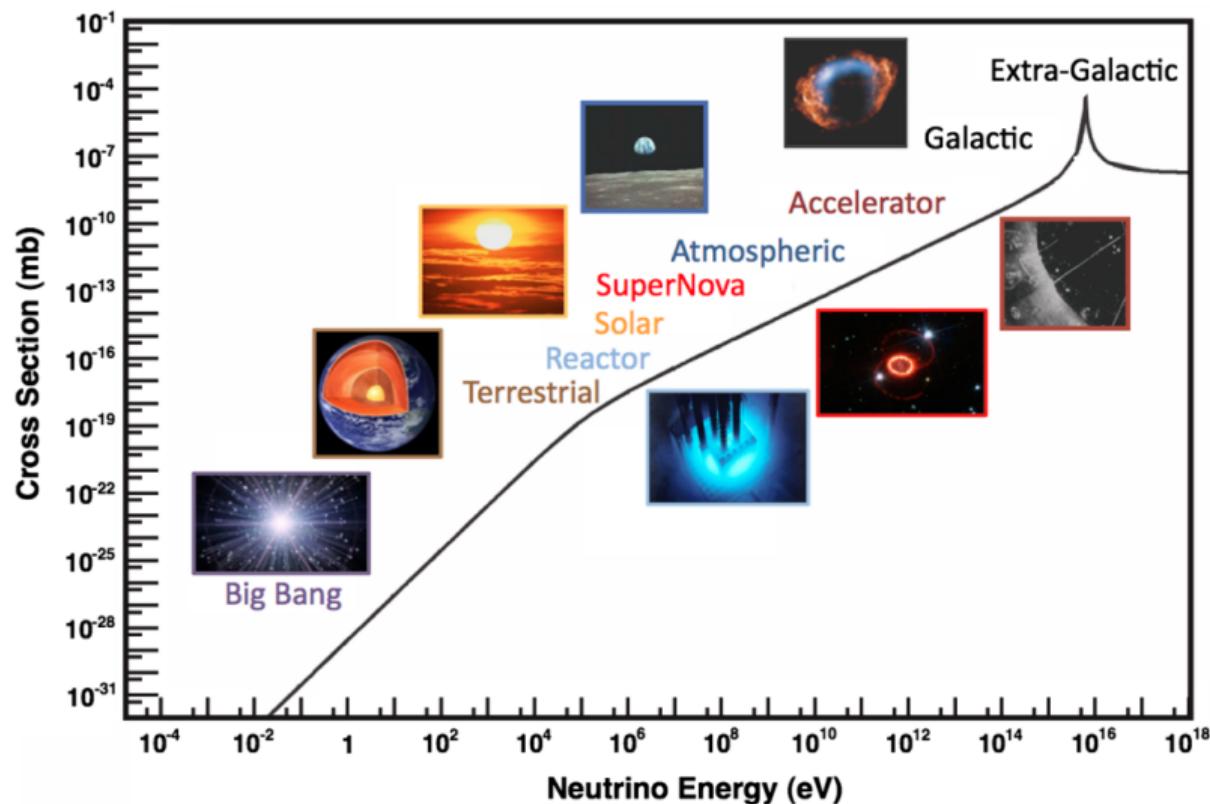
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- Probability to oscillate from  $\alpha$  to  $\beta$
- Neutrino-nucleus interaction cross section
- Migration matrix. Depends on topology of detected event  
(i.e. number of protons, etc.)
- Ratio helps control systematics, but cross sections do not cancel out from ratio → requires theory predictions

# Lepton-Nucleus reaction processes

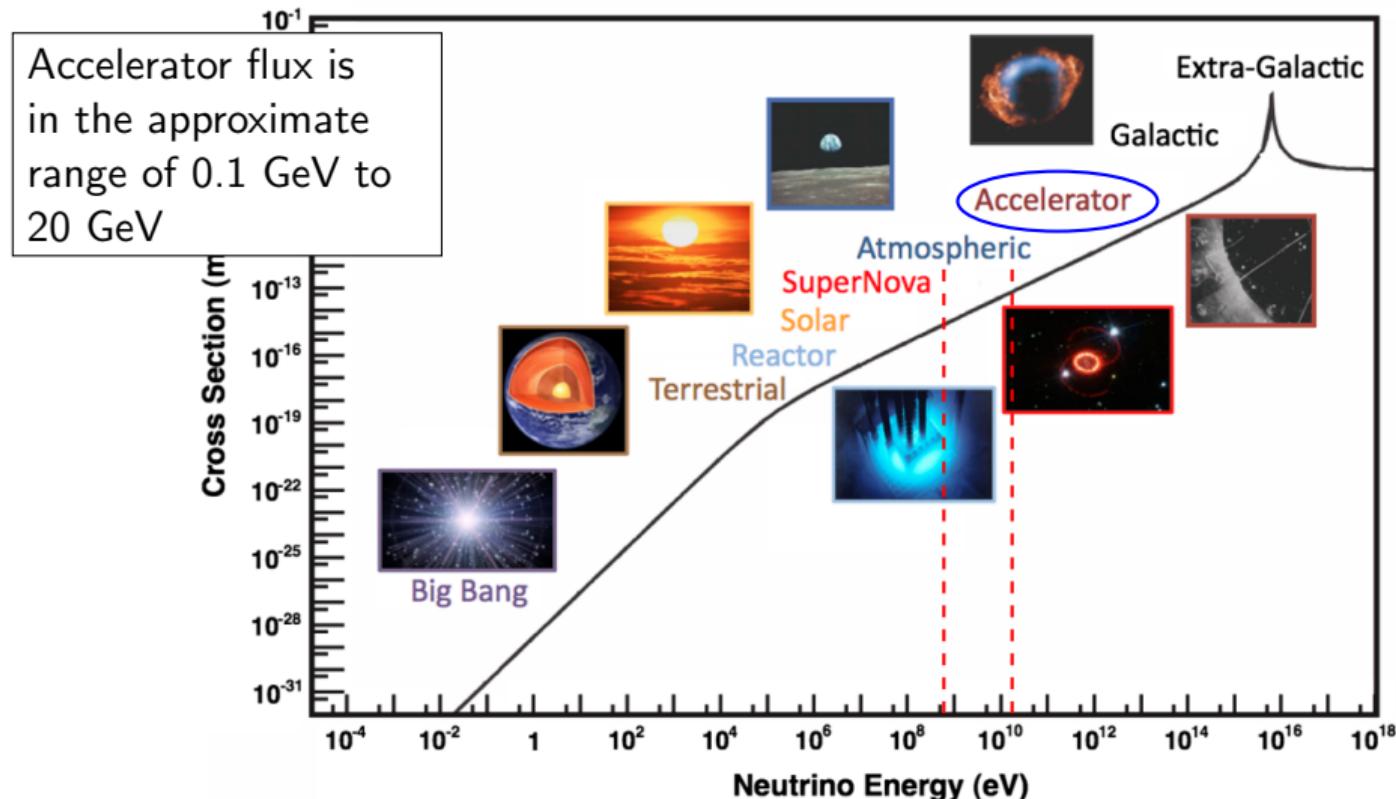


Credit: Noemi Rocco

# Neutrino Experiments Overview

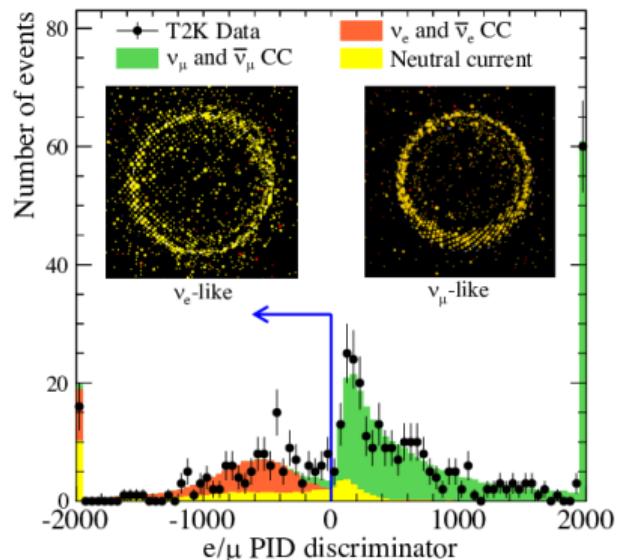


# Neutrino Experiments Overview



# Cherenkov Detectors

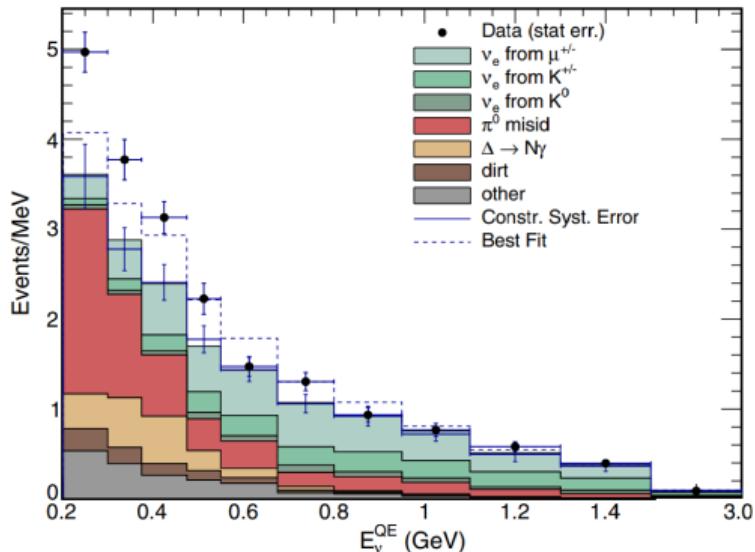
- Detect particles via Cherenkov radiation
- Muons form distinct ring, electrons form fuzzy ring
- Can't detect protons or neutrons
- Experiments: MiniBooNE, T2K, HyperK, IceCube, ORCA



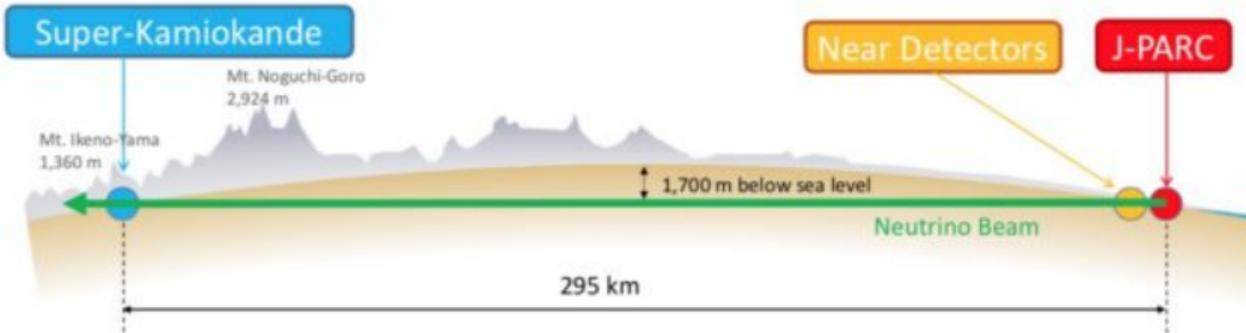
[T2K Collaboration. 1910.03887]

# MiniBooNE

- Experiment located at FNAL
- Short baseline experiment
- Water Cherenkov detector
- Has an excess of events at low reconstructed neutrino energy
- Many BSM explanations for this process (sterile neutrinos, dark neutrinos, etc.)



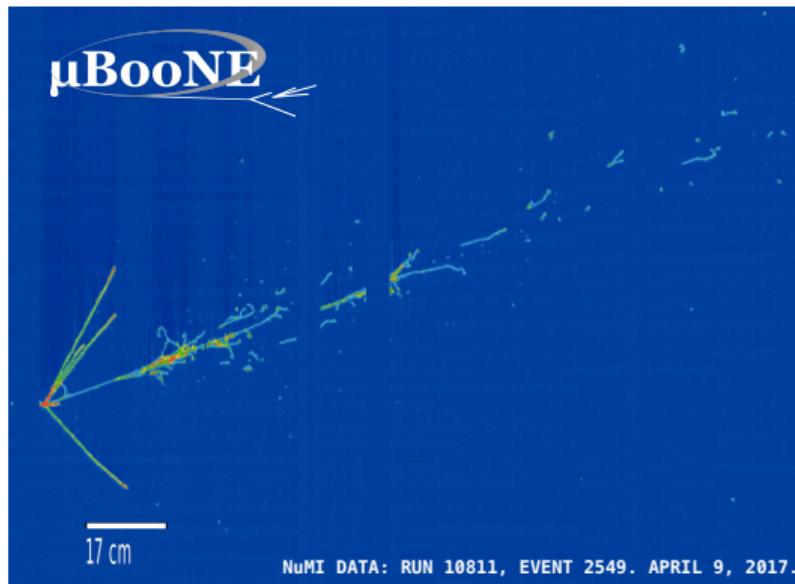
# HyperK / T2K



- Experiment located in Japan (Tokai to Kamiokande)
- Long baseline experiment
- Water Cherenkov far detector
- Goal to measure hierarchy and CP phase
- HyperK aiming for **1-2%** systematic uncertainties
- General purpose near detector can be used for BSM physics



# Calorimetric Detectors



- Collect energy deposits as charged particles propagate
- Can select events based on topology
- Neutrons are difficult to detect
- Experiments: NOvA, the SBN program, DUNE, T2K near detector

Credit: MicroBooNE collaboration

# MINERvA

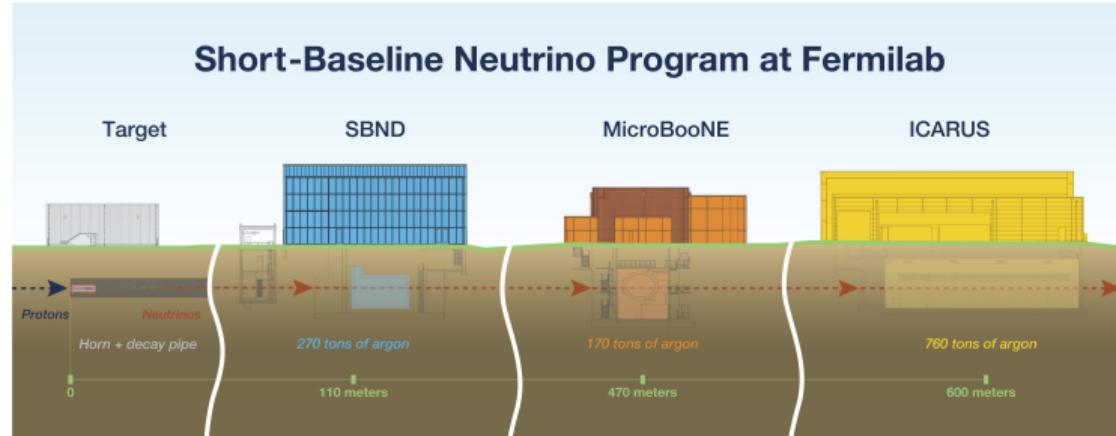


- Goal: To provide high-precision measurements of neutrino interactions on various nuclei in the 1-10 GeV range.

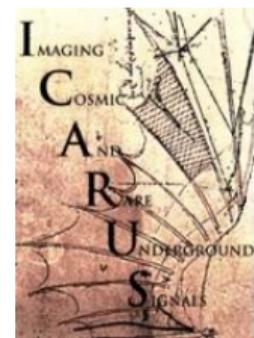


Credit: Fermilab

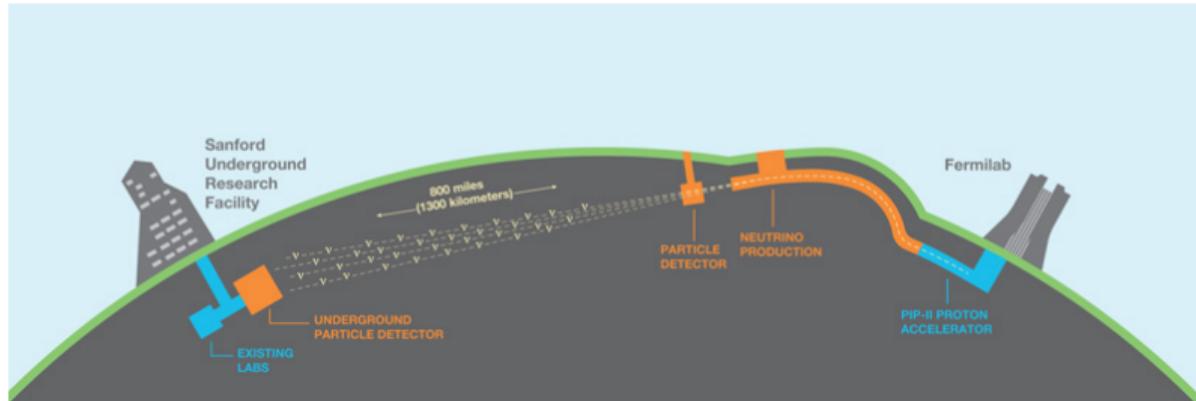
# The SBN Program



- Consists of 3 experiments
- SBND under construction
- Goals:
  - Search for BSM physics
  - Study neutrino-nucleus interactions at GeV scale
  - Advance Liquid Argon detector technology



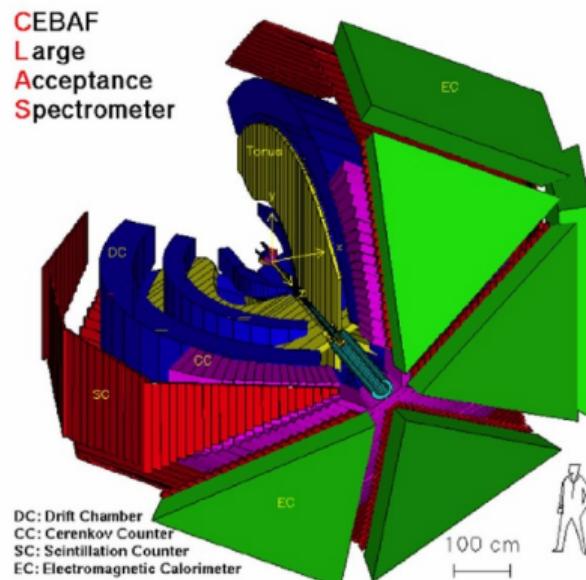
# DUNE



- Next generation US neutrino experiment
- Goals are to measure CP phase and mass hierarchy
- Claim an uncertainty on interaction rates  $> 1\%$  will substantially degrade sensitivity
- General purpose near detector useful for BSM searches

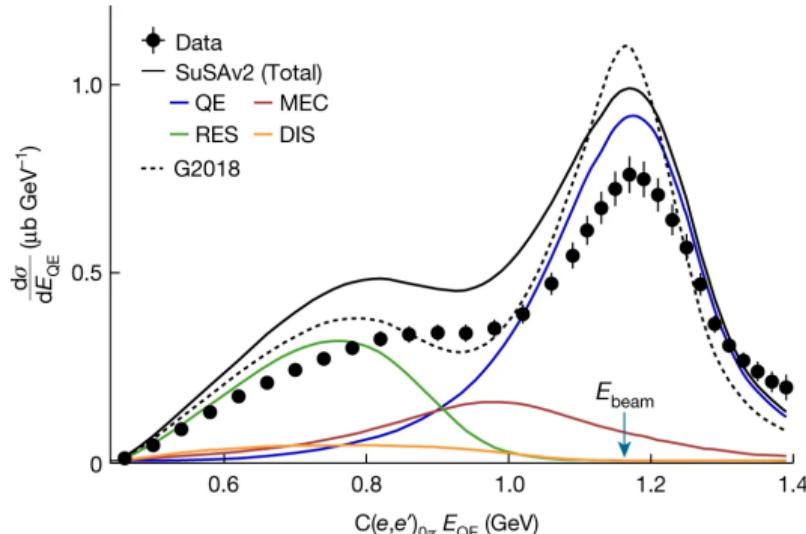
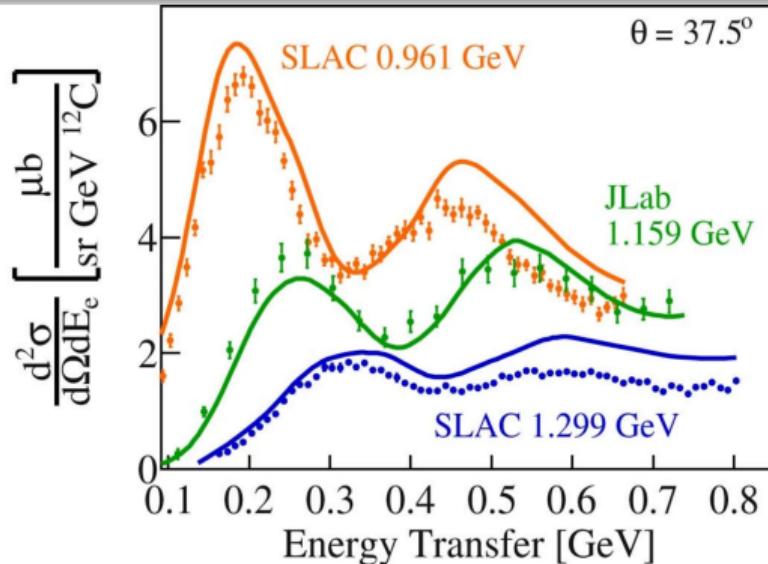
**DUNE**  
DEEP UNDERGROUND  
NEUTRINO EXPERIMENT

# CLAS / e4v Experiment



- Electron-nucleus scattering experiment
- Analysis treated like neutrino beam
- Use to improve theory predictions

# Generator Comparison (CLAS / e4v)



- Differential distributions independent of nuclear physics work ok

- Reconstruction of the neutrino energy needs significant work

[Nature 599, 565–570 (2021)]

# Theory Uncertainty (NOvA)

Source of Uncertainty	$\nu_e$ signal (%)	Total beam background (%)
Cross-section and FSI	7.7	8.6
Normalization	3.5	3.4
Calibration	3.2	4.3
Detector response	0.67	2.8
Neutrino flux	0.63	0.43
$\nu_e$ extrapolation	0.36	1.2
Total systematic uncertainty	9.2	11
Statistical uncertainty	15	22
Total uncertainty	18	25

[M. A. Acero, et al. NOvA collaboration, Phys. Rev. D 98, 032012]

- Cross section uncertainty one of dominant uncertainties
- NOvA systematics and statistical uncertainty equal
- DUNE and HyperK will have significantly more events

# Why is a generator needed?

## Oscillation Measurements

- Only measure events and not fluxes directly
- Fit oscillation parameters by taking ratio of number of events in  $E_{reco}$  bins
- Cross sections do not exactly cancel in ratio, thus they are crucial
- Requires fully differential predictions (Migration matrices):
  - Requires fully-exclusive predictions (*i.e* keep track of all particles in event simulation)
- DUNE and HyperK require precision on the cross sections of about 1%

## Other Measurements

- The SBN program, and both DUNE and HyperK near detectors are general purpose
- Leverage them for BSM searches
- Requires both SM and BSM fully differential predictions

# Separating Primary Interaction and Cascade

## General Lepton-Nucleus Scattering Cross Section

$$d\sigma = \left( \frac{1}{|v_A - v_\ell|} \frac{1}{4E_A^{\text{in}} E_\ell^{\text{in}}} \right) |\mathcal{M}|^2 \prod_f \frac{d^3 p_f}{(2\pi)^3} (2\pi)^4 \delta^4 \left( k_A + k_\ell - \sum_f p_f \right)$$

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## Matrix Element Schematically

$$|\mathcal{M}(\{k\} \rightarrow \{p\})|^2 = \left| \sum_{p'} \mathcal{V}(\{k\} \rightarrow \{p'\}) \times \mathcal{P}(\{p'\} \rightarrow \{p\}) \right|^2$$

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- Primary interaction

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- Primary interaction
- Evolution out of nucleus

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## Matrix Element Schematically Approximation

$$|\mathcal{M}(\{k\} \rightarrow \{p\})|^2 = \sum_{p'} |\mathcal{V}(\{k\} \rightarrow \{p'\})|^2 \times |\mathcal{P}(\{p'\} \rightarrow \{p\})|^2$$

- Primary interaction
- Evolution out of nucleus
- Approximate as incoherent product of primary interaction and cascade

[JL, et. al. 2205.06378]

# Aside: Relationship to LHC event generation

Neutrino Event Generation		LHC Event Generation
Primary neutrino interaction	$\longleftrightarrow$	Hard interaction
Intranuclear cascade	$\longleftrightarrow$	Parton shower

**Intranuclear Cascade:**

$$\exp \left\{ -i \sum_{j=2}^A \int_0^t d\tau \Gamma_{k_i}(|\mathbf{r}_1 + \mathbf{v}\tau - \mathbf{r}_j|) \right\}$$

**Parton Shower:**

$$\exp \left\{ -i \sum_{i=1}^k \int d^4x_i j_a^\mu(x_i) A_\mu^a(x_i) \right\}$$

- Neutrino event generators can benefit from history of the LHC event generators push for precision

# Achilles: A CHicago Land Lepton Event Simulator

## Project Goals:

- Theory driven
  - Develop modular neutrino event generator
  - Provide means for easy extension by end users
  - Provide automated BSM calculations for neutrino experiments
  - Evaluate theory uncertainties

```

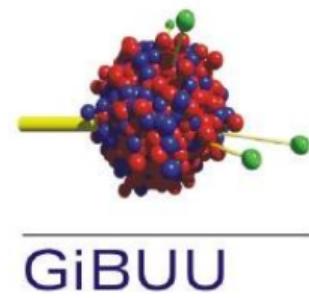
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```

Version: 1.0.0  
Authors: Joshua Isaacson, William Jay, Alessandro Lovato,  
Pedro A. Machado, Noemí Rocco

# Other Generators

- GENIE: <http://www.genie-mc.org/>
- NuWro: <https://nuwro.github.io/user-guide/>
- NEUT: Closed source T2K code
- GiBUU: <https://gibuu.hepforge.org/trac/wiki>

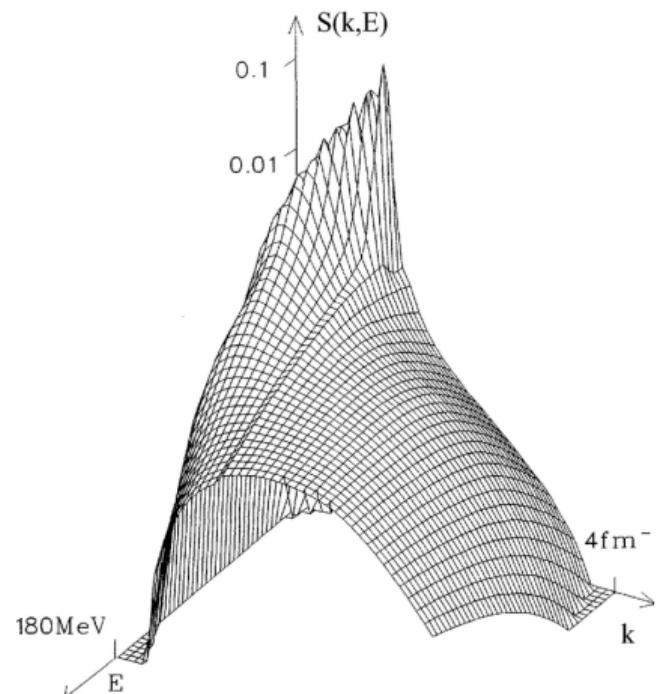


# Factorization

- For Quasielastic scattering, factorize primary interaction as:  $|\Psi_f\rangle = |p\rangle \otimes |\Psi_f^{A-1}\rangle$
- Initial state given via spectral function (probability distribution of removing a bound nucleon):

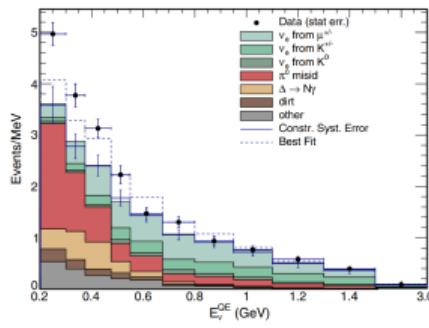
$$S_h(\mathbf{k}_h, E') = \sum_{f_{A-1}} |\langle \Psi_0 | k \rangle \otimes |\Psi_f^{A-1}\rangle|^2 \delta(E' + E_0^A - E_f^{A-1})$$

- Spectral function based on correlated basis function theory [[Phys. A 579, 493 \(1994\)](#)]
- All but DIS implemented in this formalism [[Phys. Rev. C 100 \(2019\) 4,045503](#)] just need to interface with Achilles.
- Achilles provides general purpose interface to allow for other nuclear models

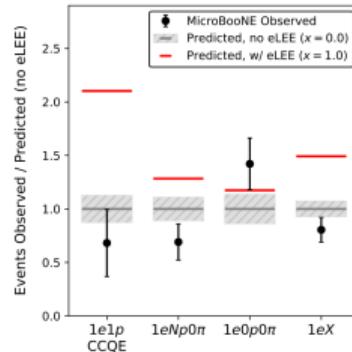


[[Rev. Mod. Phys. 80, 189 \(2008\)](#)]

# BSM Motivation: MiniBooNE and MicroBooNE

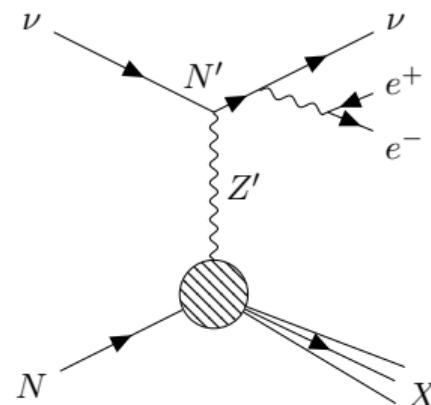


[PRL 121, 221801]



[arXiv:2110.14054]

- High intensity beams enable probes of weakly coupled BSM
- Probe different mass region than LHC
- MiniBooNE sees excess of events (MicroBooNE does not for single electrons)
- Event generators can not simulate these processes



# Using Currents

Hadronic tensor ( $W^{\mu\nu}$ ) given by most general Lorentz structure

$$W^{\mu\nu} = \left( -g^{\mu\nu} + \frac{q^\mu q^\nu}{q^2} \right) W_1 + \frac{\hat{p}_a^\mu \hat{p}_a^\nu}{p_a \cdot q} W_2 - i\epsilon^{\mu\nu\alpha\beta} \frac{q_\alpha p_{a\beta}}{2p_a \cdot q} W_3$$

Extending to BSM becomes complex to track all interferences:

$$\frac{d\sigma}{d\Phi_n} = \sum_{i,j} L_{\mu\nu}^{(ij)} W^{(ij)\mu\nu} = L_{\mu\nu}^{(\gamma\gamma)} W^{(\gamma\gamma)\mu\nu} + L_{\mu\nu}^{(\gamma Z)} W^{(\gamma Z)\mu\nu} + L_{\mu\nu}^{(Z\gamma)} W^{(Z\gamma)\mu\nu} + L_{\mu\nu}^{(ZZ)} W^{(ZZ)\mu\nu} + \dots$$

Approaches such as the spectral function formalism can directly calculate currents:

$$W^\mu = \langle \psi_f^A | \mathcal{J}^\mu | \psi_0^A \rangle \rightarrow \sum_{p_a} \left[ \langle \psi_f^{A-1} | \otimes \langle p_a | \right] | \psi_0^A \rangle \langle p_a + q | \sum_i \mathcal{J}_i^\mu | p_a \rangle,$$

**This enables the automatic handling of interferences**

$$\frac{d\sigma}{d\Phi_n} = \left| \sum_i L_\mu^{(i)} W^{(i)\mu} \right|^2$$

[JI, et. al. Phys. Rev. D 105 (2022) 9, 096006]

# Final State Interactions

## Modify Primary Interaction:

- Captures rate change from FSI
- Loses all information about hadronic final state
- Primarily done using folding functions
- **Note:** Both approaches attempt to capture effects from nuclear potential. Therefore, can only use one or the other to avoid double counting effects.
- **Note:** Intranuclear cascade is only method to provide fully exclusive final states required by experiments

## Intranuclear Cascade:

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[JL, et. al. 2205.06378]

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# Algorithm Overview

## Algorithm Overview:

- Propagate struck nucleons
- Determine interactions based on impact parameter and cross-section
- Pauli blocking used to restrict final state phase space

## Interaction Probabilities:

$$P_{\text{cyl}}(b) = \Theta(\sigma/\pi - b^2)$$

$$P_{\text{Gau}}(b) \equiv \exp\left(-\frac{\pi b^2}{\sigma}\right)$$

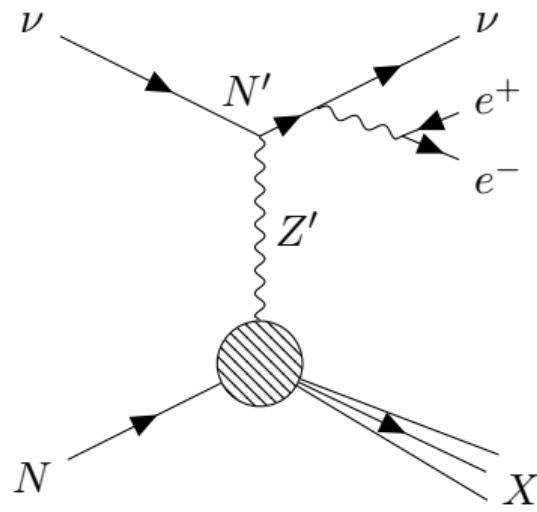
✗ hit?

✗

propagating

[JI, et. al. Phys. Rev. C 103(2021) 1, 015502] , [JI, et. al. 2205.06378]

# Dark Neutrino

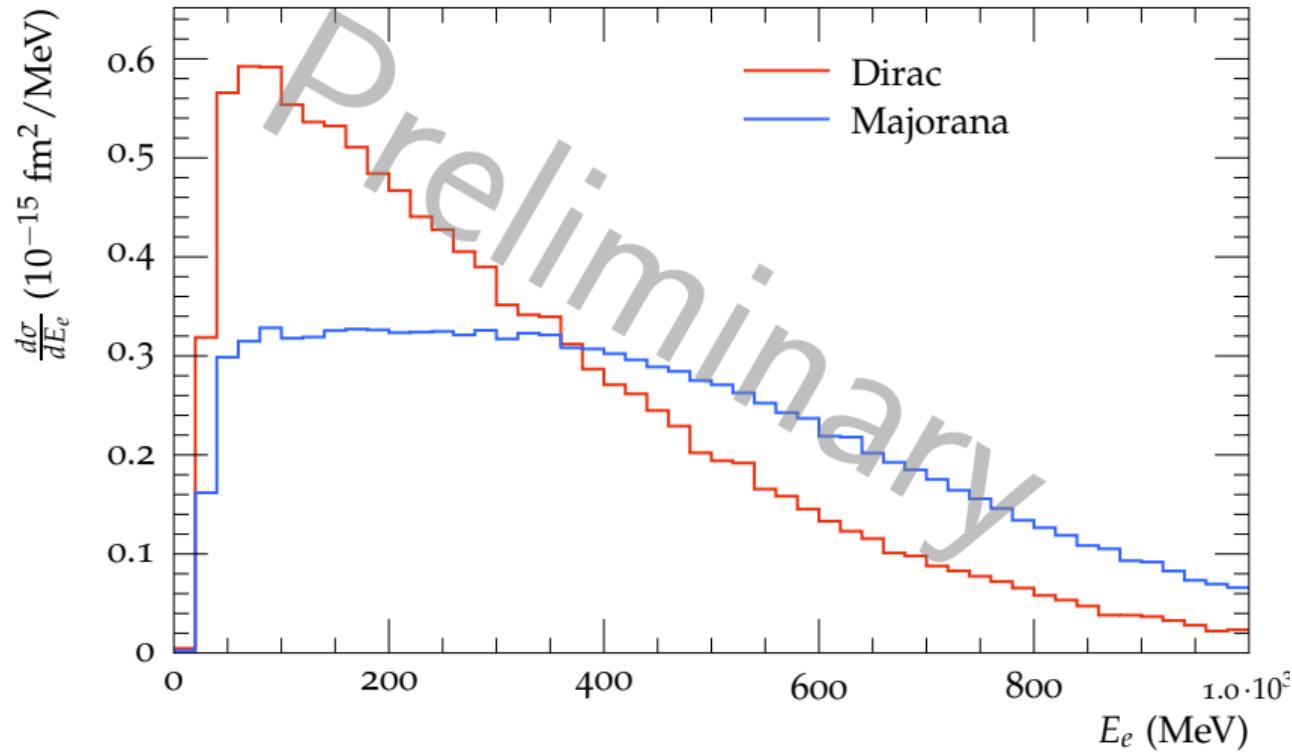


## Parameters:

- $m_{N'} = 420 \text{ MeV}$
- $m_{Z'} = 30 \text{ MeV}$
- $\alpha_D = 0.25$
- $\alpha\epsilon^2 = 2 \times 10^{-10}$
- $|U_{42}^\mu| = 9 \times 10^{-7}$
- Handles both Dirac and Majorana fermions
- Results are flux-averaged over the MiniBooNE / MicroBooNE neutrino flux

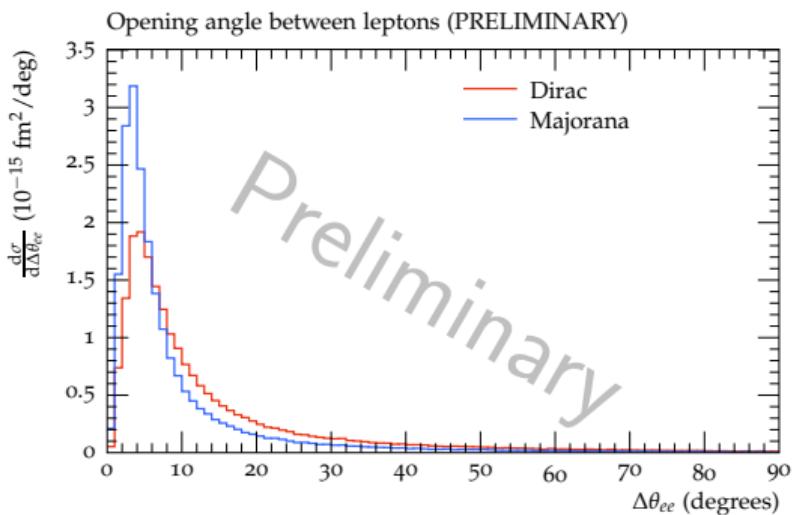
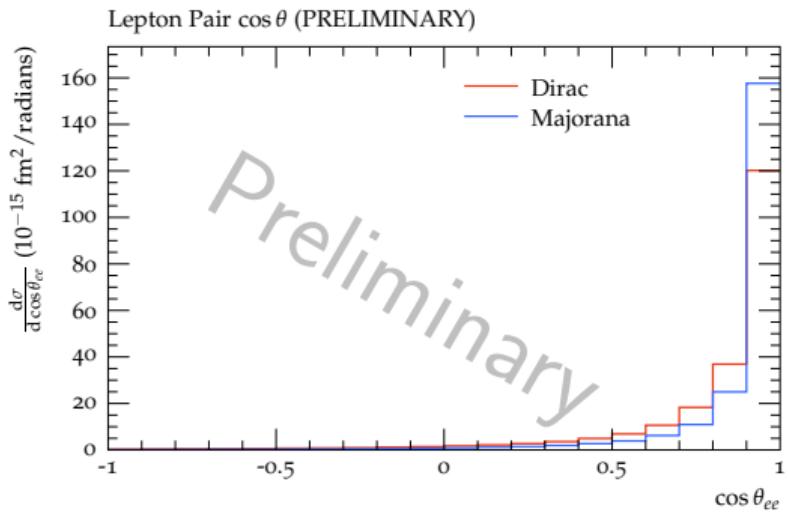
# Dark Neutrino

Energy of leading lepton (PRELIMINARY)



# Dark Neutrino

- No cuts applied yet
- Typical opening angle around 5-6 degrees
- MiniBooNE needs separation of about 10 degrees to distinguish 1 or 2 electrons



- Need to include background to compare to MiniBooNE data
- Simulate possible MicroBooNE limits

# MicroBooNE Simulation

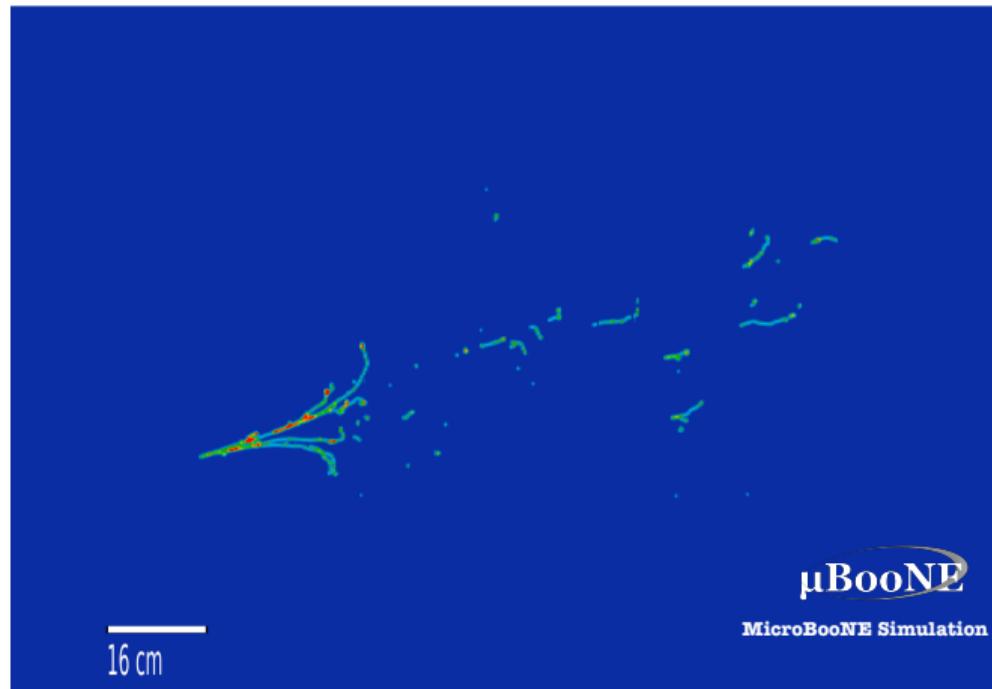


Image generated by the MicroBooNE collaboration using Achilles

# CLAS/e4v Comparison

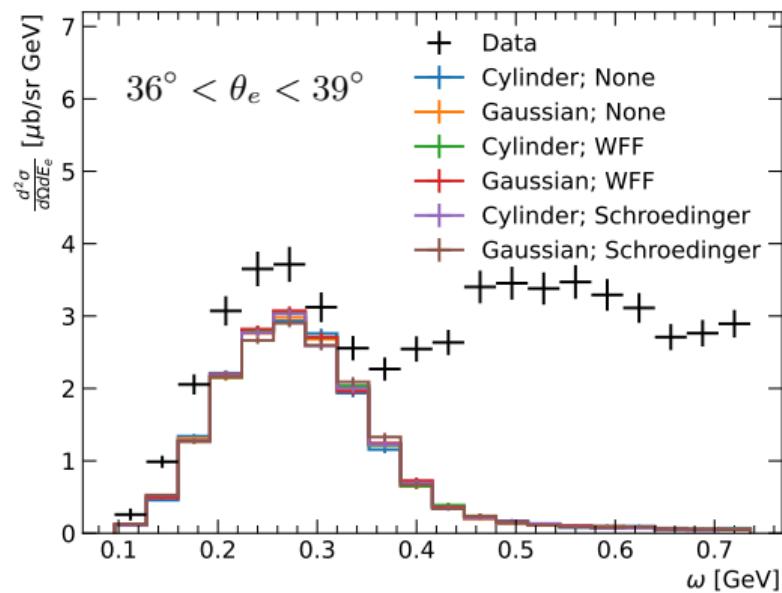
CLAS/e4v cuts:

- Select 1p0 $\pi$  events
- Protons:  $p_p > 300$  MeV,  $12^\circ < \theta_p$ .
- electrons:  $E_e > 0.4, 0.55, 1.1$  GeV,  
 $\theta_e^i > \theta_0^i + \frac{\theta_1^i}{p_e[\text{GeV}]}$ ,  $\theta_0^i = 17^\circ, 16^\circ, 13.5^\circ$ ,  
 $\theta_1^i = 7^\circ, 10.5^\circ, 15^\circ$  for  
 $E_{\text{beam}} = 1.159, 2.257, 4.453$  GeV  
respectively.

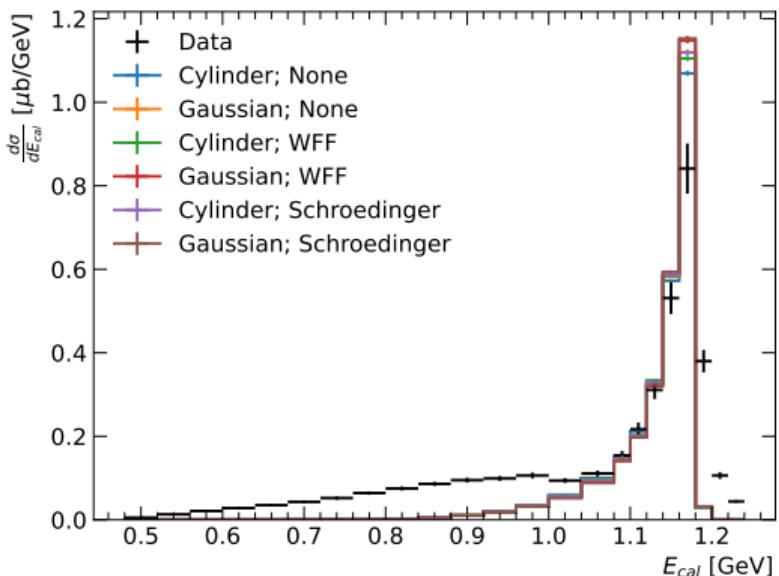
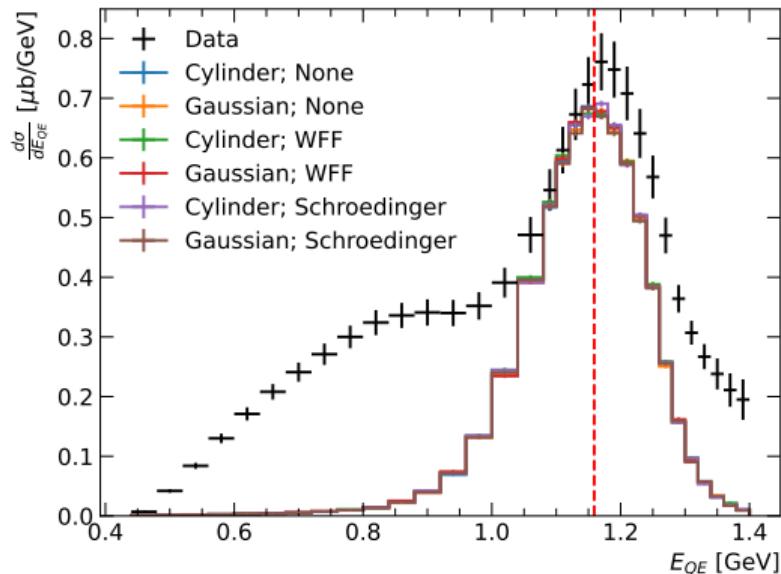
Simulation details:

- ACHILLES only has Quasielastic channel so far
- Events are reweighted by  $Q^4/\text{GeV}^4$  (as done in the analysis)

Inclusive Results:



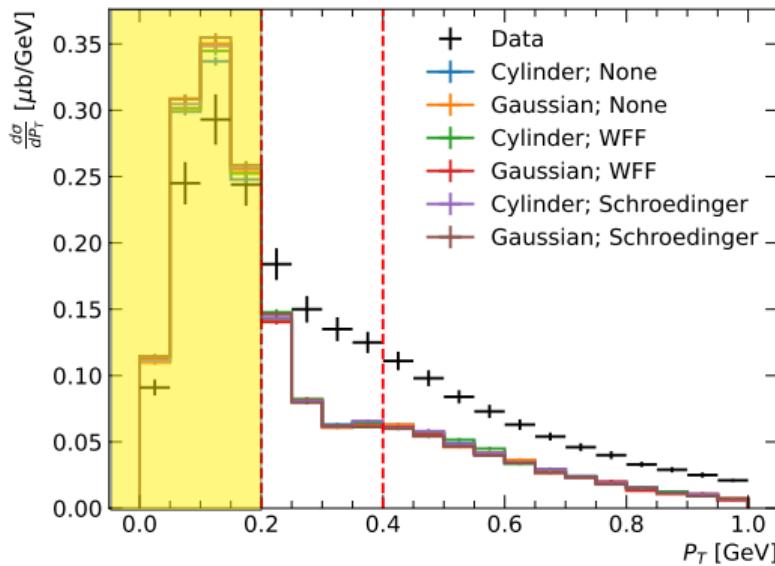
# CLAS/e4v Comparison



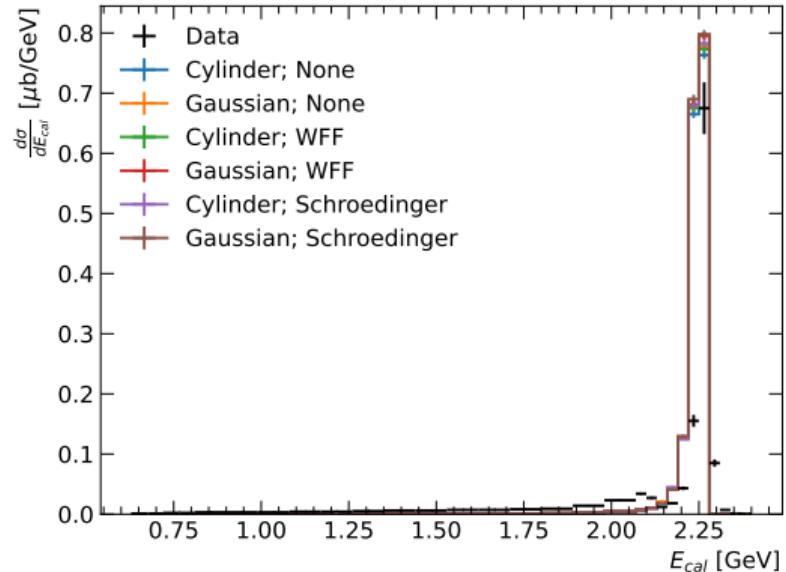
- $E_{QE} = \frac{2m_N\epsilon + 2m_NE_\ell - m_\ell^2}{2(m_N - E_\ell + p_\ell \cos \theta_\ell)}$
- $\epsilon = 21$  MeV
- Mimics Cherenkov detectors

- $E_{cal} = \sum_i (E_i + \epsilon_i)$   
Sum over all ionizing particles
- Mimics LArTPC detectors

# CLAS/e4v Comparison

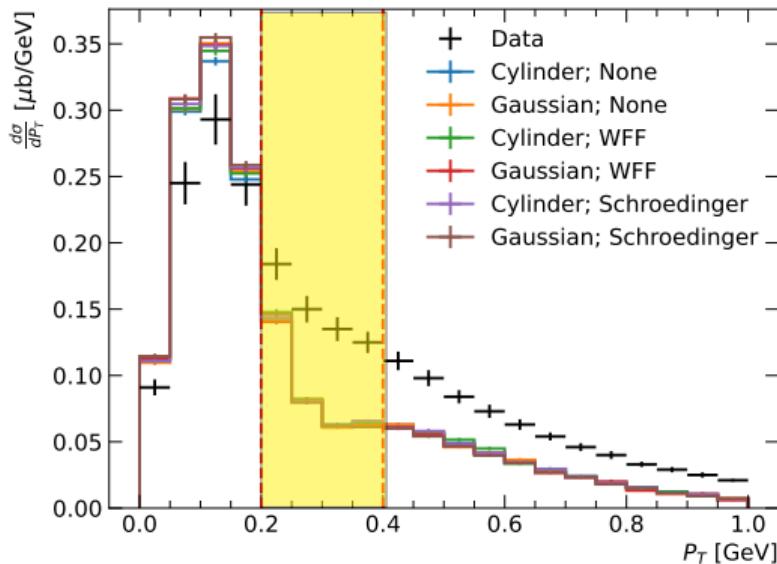


$$\mathbf{p}^T = \mathbf{p}_e^T + \mathbf{p}_p^T$$

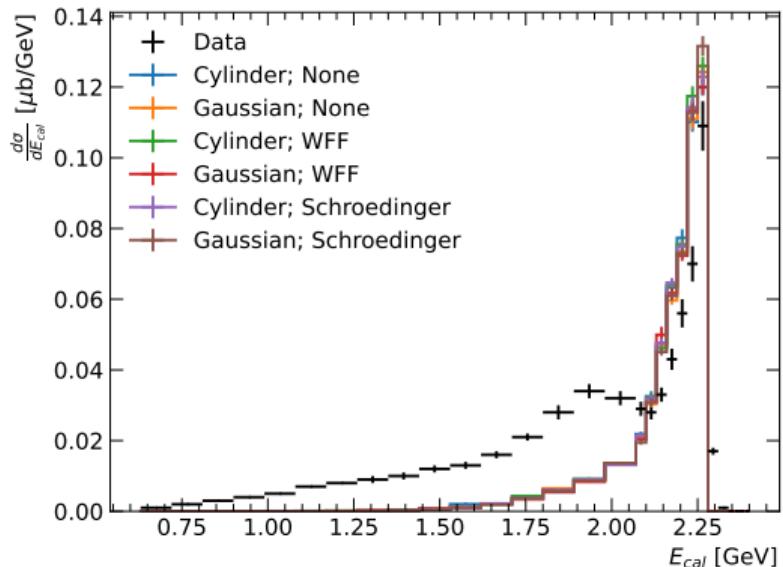


[JI, et. al. 2205.06378]

# CLAS/e4v Comparison

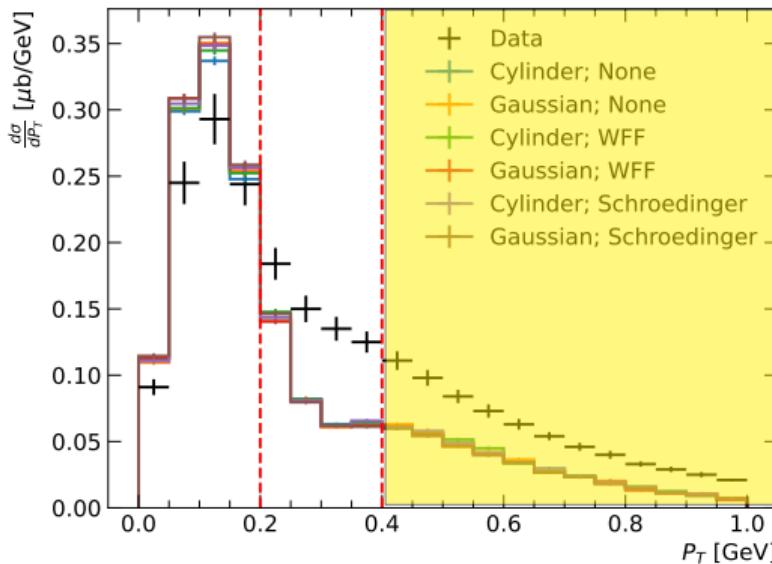


$$\mathbf{p}^T = \mathbf{p}_e^T + \mathbf{p}_p^T$$

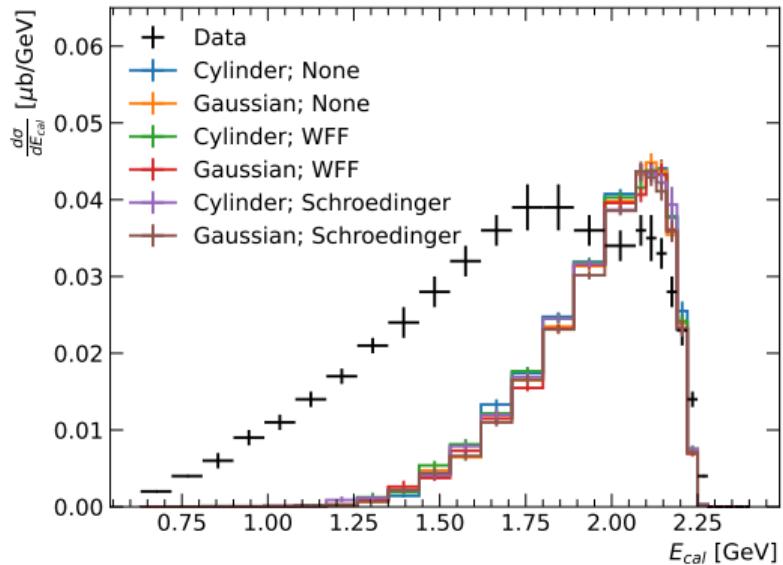


[JI, et. al. 2205.06378]

# CLAS/e4v Comparison



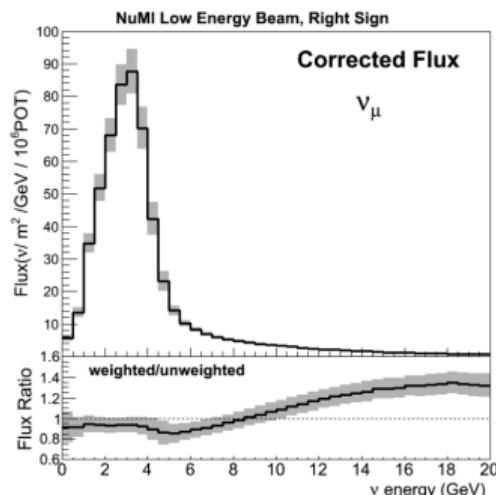
$$\mathbf{p}^T = \mathbf{p}_e^T + \mathbf{p}_p^T$$



[JI, et. al. 2205.06378]

# MINERvA Comparison (arxiv:1811.02774)

- Neutrino Flux



[JPS Conf. Proc. 12, 010006 (2016)]

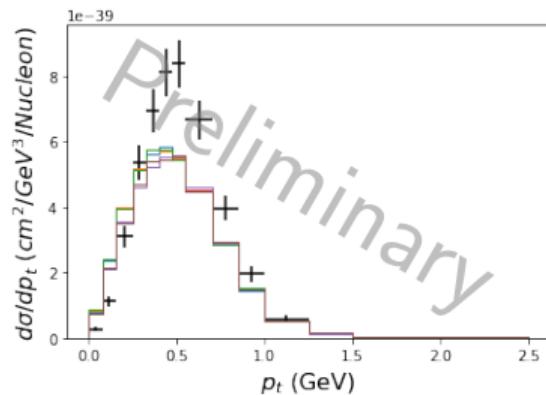
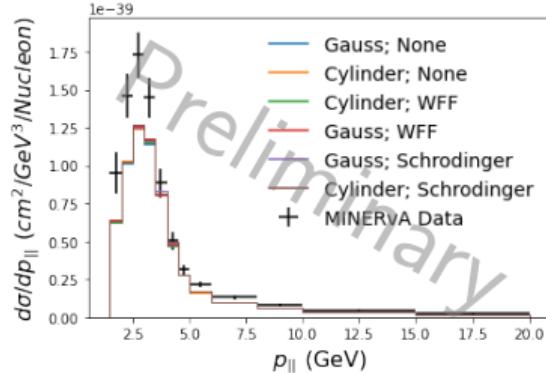
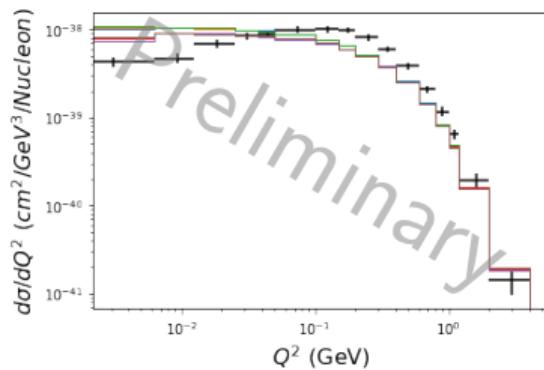
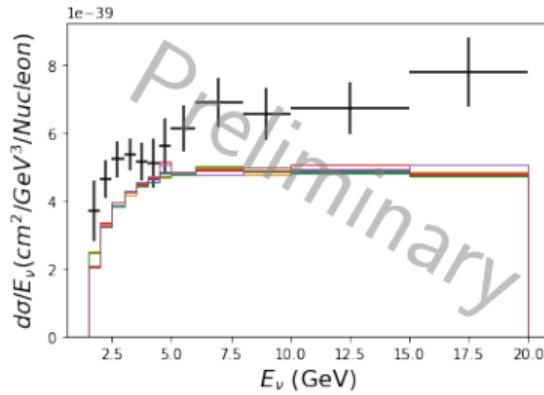
- Signal Definition:

- One muon within  $20^\circ$  of neutrino beam
- Any number of protons and neutrons
- No mesons
- No heavy or excited baryons
- Any number of photons with energy  $\leq 10$  MeV

- Observables:

- $E_{\nu,QE} = \frac{M_n^2 - (M_p - E_b)^2 - M_\mu^2 + 2(M_p - E_b)E_\mu}{2(M_p - E_b - E_\mu + P_\mu \cos(\theta_\mu))}$
- $Q_{QE}^2 = 2E_\nu(E_\mu - P_\mu \cos(\theta_\mu)) - M_\mu^2$
- Transverse momentum of muon ( $p_T$ )
- Longitudinal momentum of muon ( $p_z$ )

# MINERvA Comparison (arxiv:1811.02774)



# Conclusions

## Current Status:

- Current and next generation experiments require a cross section uncertainty of 1-2%
- Achilles aims to be a modular theory driven generator to address these needs
- BSM important for the current and next generation neutrino experiments
- Robust BSM program requires automating theory calculations
- Comparison of Achilles with CLAS/e4v and MINERvA experiments
- Preliminary results for CC tau interactions with decays

## Future Steps:

- Implement QED showers to handle radiative corrections
- Interface with LArSoft
- Finalize Achilles calculations for MEC and Resonance
- Investigate spectral function approach in DIS region
- Use LHC techniques to improve cascade modeling
- Implement nuclear de-excitation models

Achilles code can be found at: <https://github.com/AchillesGen/Achilles>

# Collaborators

## Undergraduates:

- Diego Lopez Gutierrez: Involved in automating BSM (SIST mentor 2020,2021, Senior thesis co-advisor)
- Sherry Wang: Involved in tau polarization (SULI mentor 2022)
- Russell Farnsworth: Involved in tau polarization (SULI co-mentor 2022)
- Antu Santanu: Developing general purpose geometry driver (SIST mentor 2022)

## Achilles authors:

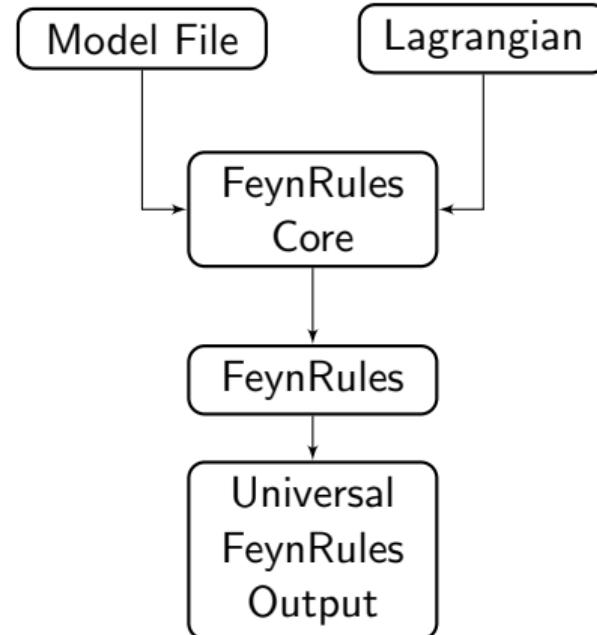
- William Jay (Lattice, MIT)
- Alessandro Lovato (Nuclear, ANL)
- Pedro Machado (Neutrino Pheno, FNAL)
- Luke Pickering (Neutrino exp, Royal Holloway)
- Noemi Rocco (Nuclear, FNAL)

## External Contributors:

- Sherpa Interface:  
Stefan Hoeche (FNAL)
- Marley Interface:  
Steven Gardiner (FNAL)

# FeynRules

- *Mathematica* Program
- Takes model file and Lagrangian as input
- Calculates the Feynman rules
- Outputs in Universal FeynRules Output (UFO) format



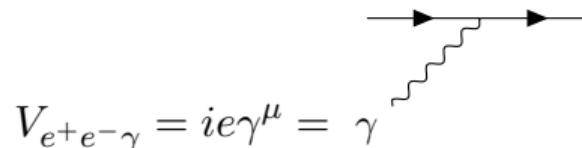
[arXiv:0806.4194, arXiv:1310.1921]

# Universal FeynRules Output (UFO)

Example QED ( $e^+e^-\gamma$  Vertex):

- Python output files
- Contains model-independent files and model-dependent files
- Contains all information to calculate any tree level matrix element
- Has parameter file to adjust model parameters to scan allowed regions

$$\mathcal{L} = -\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \bar{\psi}(iD^\mu\gamma_\mu - m)\psi$$



$$V_{e^+e^-\gamma} = ie\gamma^\mu = \gamma$$

[arXiv:1108.2040]

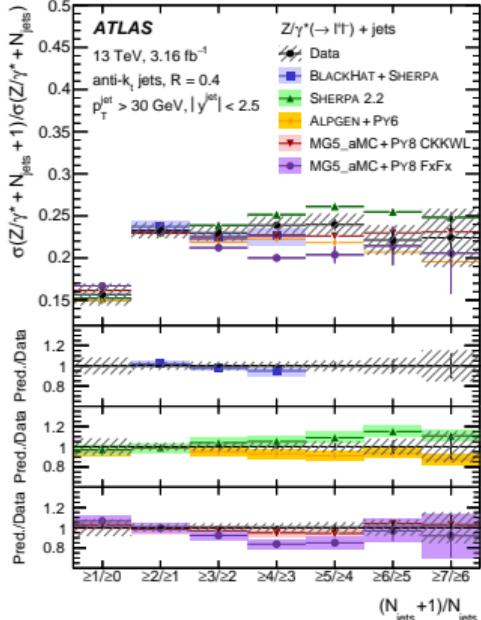
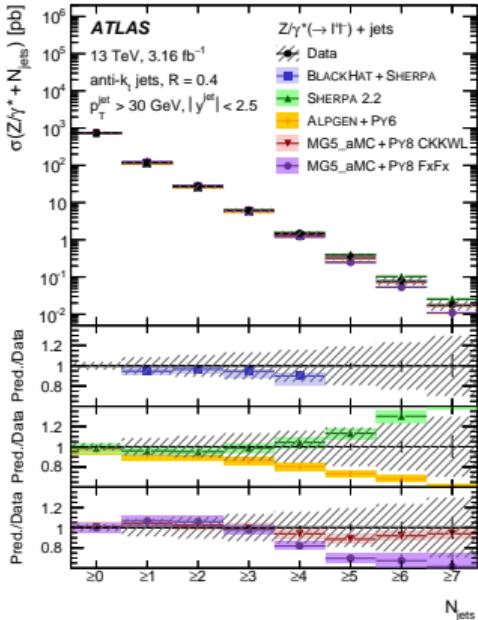
# Universal FeynRules Output (UFO)

## Example for photon-electron vertex

```
e__minus__ = Particle(pdg_code=11, name='e-', antiname='e+',  
                      spin=2, color=1, mass=Param.ZERO,  
                      width=Param.ZERO, texname='e-',  
                      antitexname='e+', charge=-1,  
                      GhostNumber=0, LeptonNumber=1,  
                      Y=0)  
  
V_77 = Vertex(name='V_77',  
               particles=[ P.e__plus__, P.e__minus__, P.a ],  
               color=[ '1' ], lorentz=[ L.FFV1 ],  
               couplings={(0,0):C.GC_3})  
  
FFV1 = Lorentz(name='FFV1', spins=[ 2, 2, 3 ],  
                structure='Gamma(3,2,1)')  
  
GC_3 = Coupling(name='GC_3', value='-(ee*complex(0,1))',  
                 order={'QED':1})
```

# Tree Level Matrix Element Generators

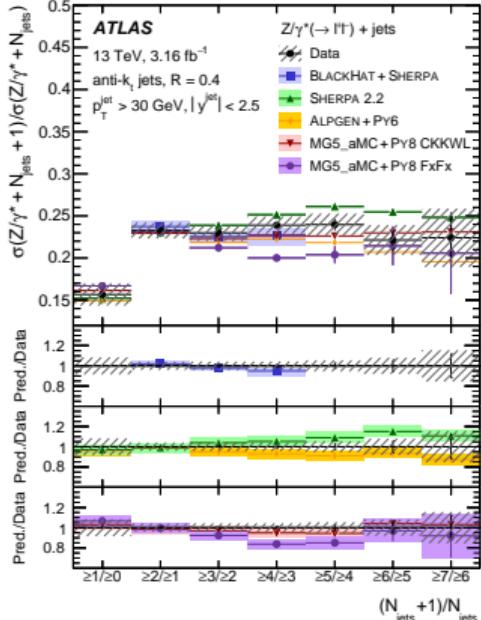
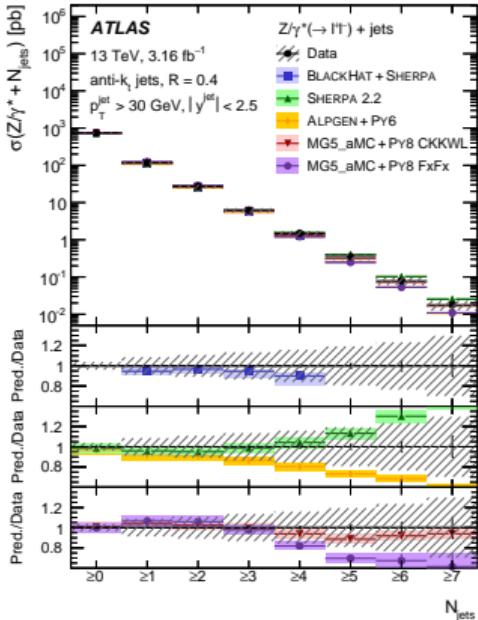
- ALPGEN [[arXiv:hep-ph/0206293](#)]
- AMEGIC [[arXiv:hep-ph/0109036](#)]
- COMIX [[arXiv:0808.3674](#)]
- CALCHEP [[arXiv:1207.6082](#)]
- HERWIG [[arXiv:0803.0883](#)]
- MADGRAPH  
[[arXiv:1405.0301](#)]
- WHIZARD [[arXiv:0708.4233](#)]
- etc.



[[arXiv:1702.05725](#)]

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- etc.



[[arXiv:1702.05725](#)]

## $2 \rightarrow 2$ Phase Space Example

Consider  $l + {}^{12}C \rightarrow l' + N + X$  in the quasielastic regime.

$$d\sigma \propto d\Phi_2(a, b; 1, 2) \quad d^4 p_a \quad d^3 p_b$$

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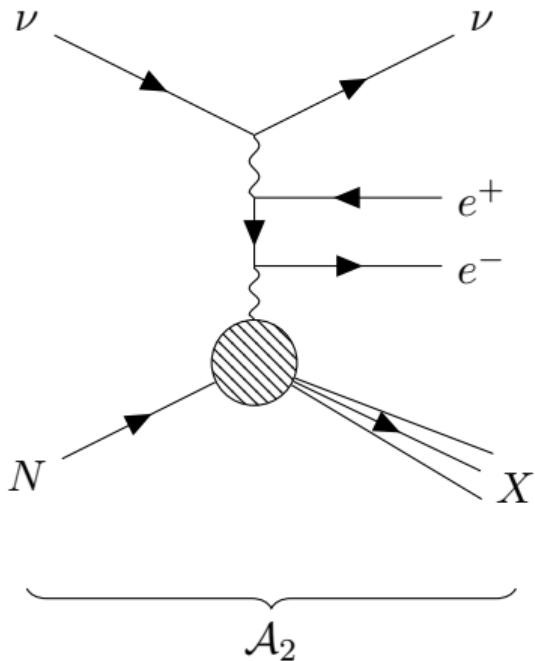
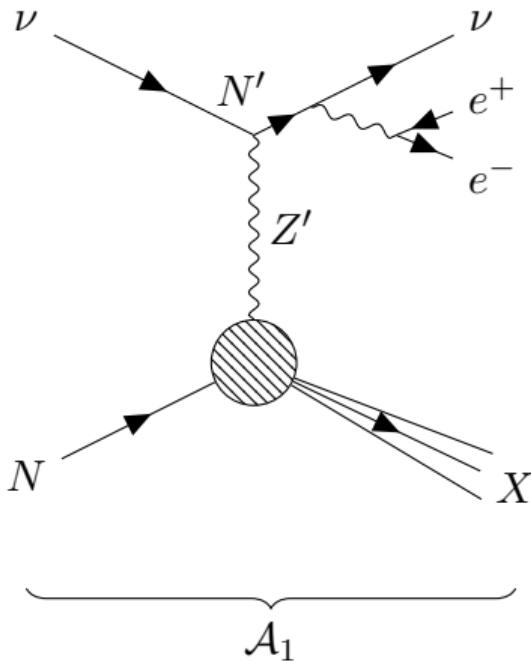
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Quasielastic Delta Function:  $\delta(E_b - E_1 - E_r + m - E_2)$

Phase Space Delta Function:  $\delta(E_a + E_b - E_1 - E_2)$

Define initial nucleon energy as  $E_a = m - E_r$ . Allows use of phase space tools developed at LHC.

# Multi-channel Integration



- Both diagrams contribute to cross section
- They have different pole structures
- Need method to sample these structures efficiently  
(i.e.  $|\mathcal{A}_1 + \mathcal{A}_2|^2$ )

# Multi-channel Integration and VEGAS

## Multi-channel Integration

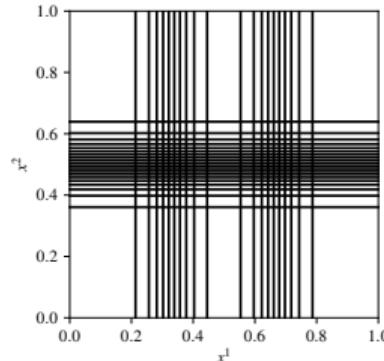
- Generate PS efficiently for  $|\mathcal{A}_1|^2$  or  $|\mathcal{A}_2|^2$
- Do not know how to efficiently sample  $2Re(\mathcal{A}_1\mathcal{A}_2^\dagger)$
- Define channels:  $C_1$  and  $C_2$
- Generate events according to distributions  $g_i$  for channel  $i$

$$\int d\vec{x} f(\vec{x}) = \sum_i \alpha_i \int d\vec{x} g_i(\vec{x}) \frac{f(\vec{x})}{g_i(\vec{x})}$$

- Optimize  $\alpha_i$  to minimize variance

## VEGAS

- Adaptive importance sampling
- Use this to get interference terms more accurately



VEGAS grid for  $\int_0^1 d^4x \left( e^{-100(\vec{x}-\vec{r}_1)^2} + e^{-100(\vec{x}-\vec{r}_2)^2} \right)$

[J.Comput.Phys. 27 (1978) 291, 2009.05112]

# Handling Form Factors

Nuclear one-body current operators:

$$\mathcal{J}^\mu = (\mathcal{J}_V^\mu + \mathcal{J}_A^\mu)$$

$$\mathcal{J}_V^\mu = \gamma^\mu \mathcal{F}_1^a + i\sigma^{\mu\nu} q_\nu \frac{\mathcal{F}_2^a}{2M}$$

$$\mathcal{J}_A^\mu = -\gamma^\mu \gamma_5 \mathcal{F}_A^a - q^\mu \gamma_5 \frac{\mathcal{F}_P^a}{M}$$

Coherent Form Factors (spin-0 nucleus):

$$\mathcal{J}^\mu = (p_{\text{in}} + p_{\text{out}})^\mu \mathcal{F}_{\text{coh}}$$

Standard Model Form Factors:

$$\mathcal{F}_i^{\gamma(p,n)} = F_i^{p,n}, \quad \mathcal{F}_A^\gamma = 0$$

$$\mathcal{F}_i^{W(p,n)} = F_i^p - F_i^n, \quad \mathcal{F}_A^W = F_A$$

$$\mathcal{F}_i^{Z(p)} = \left( \frac{1}{2} - 2 \sin^2 \theta_W \right) F_i^p - \frac{1}{2} F_i^n,$$

$$\mathcal{F}_i^{Z(n)} = \left( \frac{1}{2} - 2 \sin^2 \theta_W \right) F_i^n - \frac{1}{2} F_i^p$$

$$\mathcal{F}_A^{Z(p)} = \frac{1}{2} F_A, \quad \mathcal{F}_A^{Z(n)} = -\frac{1}{2} F_A$$

[JI, et. al. Phys. Rev. D 105 (2022) 9, 096006]

# Recursive Matrix Element Generation

$$\mathcal{J}_\alpha(\pi) = P_\alpha(\pi) \sum_{V_\alpha^{\alpha_1, \alpha_2}} \sum_{\mathcal{P}_2(\pi)} \mathcal{S}(\pi_1, \pi_2) V_\alpha^{\alpha_1, \alpha_2}(\pi_1, \pi_2) \mathcal{J}_{\alpha_1}(\pi_1) \mathcal{J}_{\alpha_2}(\pi_2)$$

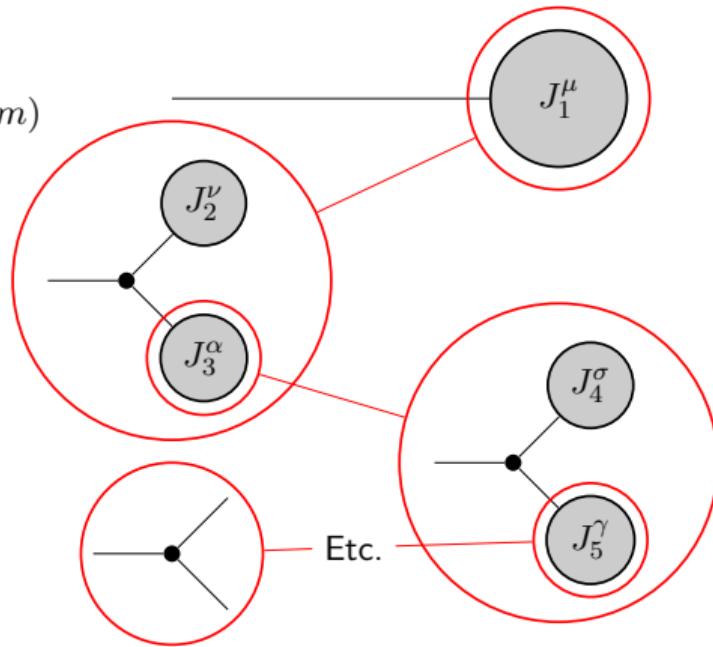
$$L_\mu^{(i)}(1, \dots, m) = \mathcal{J}_\mu^{(i)}(1, \dots, m)$$

$$L_{\mu\nu}^{(i,j)}(1, \dots, m) = \mathcal{J}_\mu^{(i)}(1, \dots, m) \mathcal{J}_\nu^{(j)\dagger}(1, \dots, m)$$

## Berends-Giele Recursion

- Reuse parts of calculation
- Most efficient for high multiplicity
- Reduces computation from  $\mathcal{O}(n!)$  to  $\mathcal{O}(3^n)$

[Nucl. Phys. B306(1988), 759]



# Phase Space Generation

$$d\Phi_n(a, b; 1, \dots, n) = \delta^{(4)} \left( p_a + p_b - \sum_{i=1}^n p_i \right) \left[ \prod_{i=1}^n \frac{d^4 p_i}{(2\pi)^3} \delta(p_i^2 - m_i^2) \Theta(p_{i_0}) \right]$$

**The above phase space definition does not contain the handling of initial states.**

Algorithms for  $n$ -body phase space generation

- RAMBO [[Comput. Phys. Commun. 40\(1986\) 359](#)]
- Multi-channel techniques [[hep-ph/9405257](#)]
- Recursive Phase Space [[arXiv:0808.3674](#)]

# Recursive Phase Space Decomposition

Phase space can be decomposed as:

$$d\Phi_n(a, b; 1, \dots, n) = d\Phi_{n-m+1}(a, b; m+1, \dots, n) \frac{ds_\pi}{2\pi} d\Phi_m(\pi; 1, \dots, m)$$

Iterate until only 1 → 2 phase spaces remain.

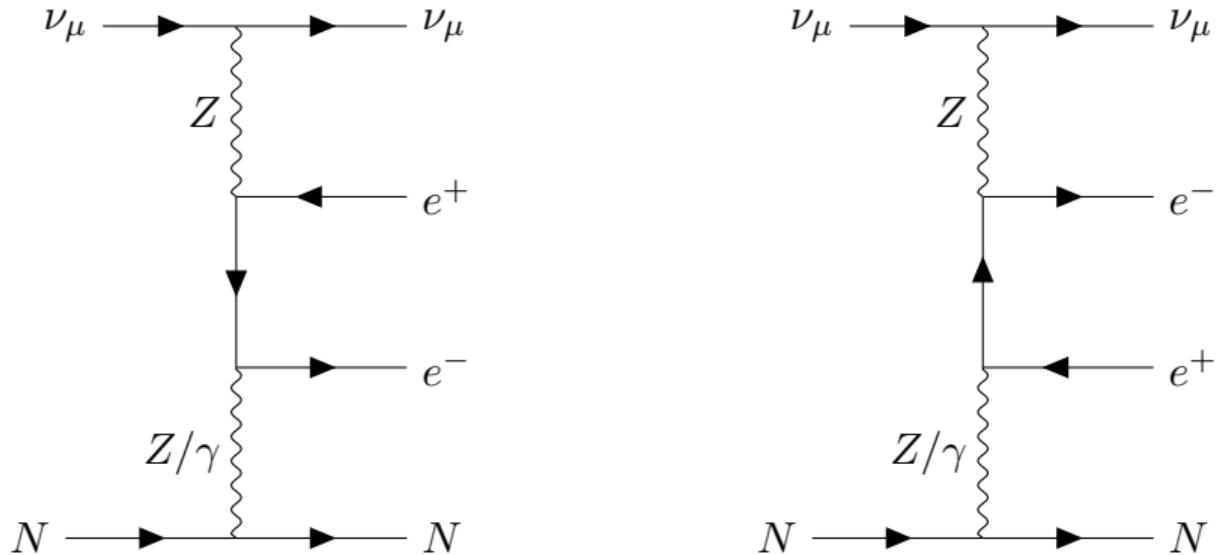
Basic building blocks:

$$S_\pi^{\rho, \pi \setminus \rho} = \frac{\lambda(s_\pi, s_\rho, s_{\pi \setminus \rho})}{16\pi^2 2 s_\pi} d\cos\theta_\rho d\phi_\rho$$

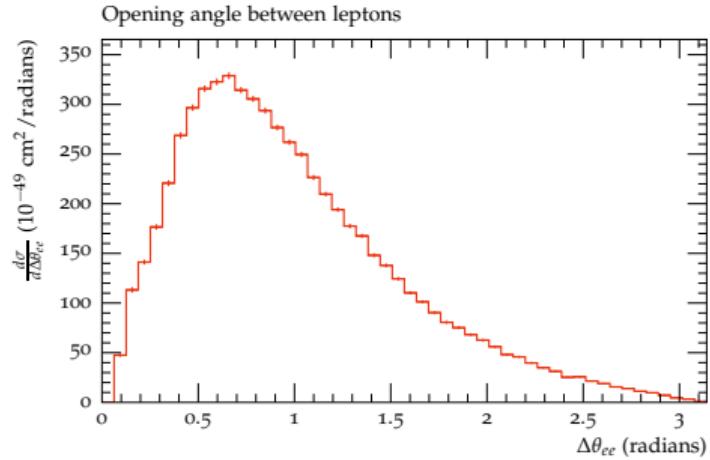
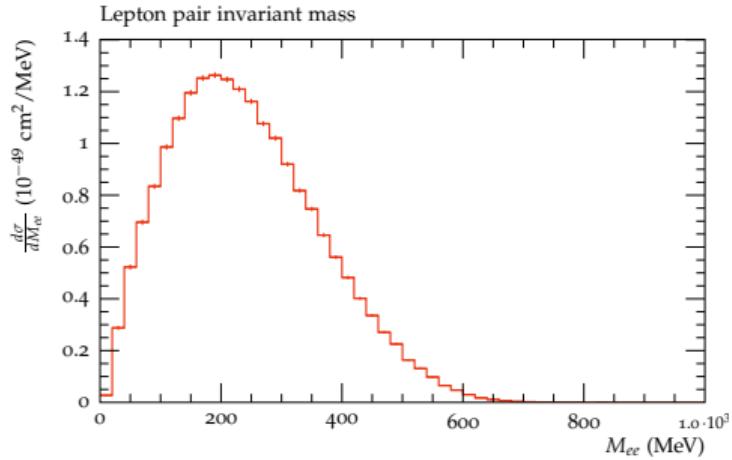
$$T_{\alpha, b}^{\pi, \overline{\alpha b \pi}} = \frac{\lambda(s_{\alpha b}, s_\pi, s_{\overline{\alpha b \pi}})}{16\pi^2 2 s_{\alpha b}} d\cos\theta_\pi d\phi_\pi$$

Momentum conservation:  $(2\pi)^4 d^4 p_{\overline{\alpha b}} \delta^{(4)}(p_\alpha + p_b - p_{\overline{\alpha b}})$

# Neutrino Tridents

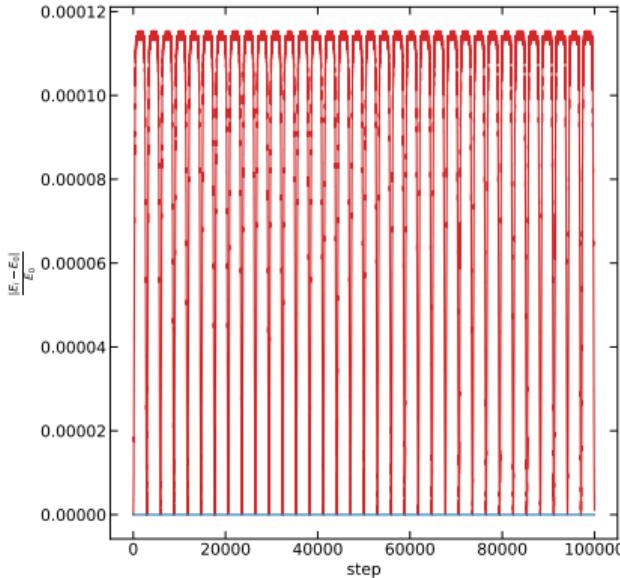


# Neutrino Tridents



# Propagation with Potential

**Initial Momentum:** 250 MeV



- Blue: Non-relativistic potential  
 $(E = \sqrt{p^2 + m^2} + V)$

[Phys. Rev. C. 38, 2967]

- Propagation using symplectic integrator for non-separable Hamiltonians [1609.02212]
- Energy is conserved to a high degree of precision
- Extremely stable

- Red: Relativistic potential  
 $(E = \sqrt{p^2 + (m + S)^2} + V)$

[Phys. Rev. C. 80, 034605]

# Symplectic Integration for non-separable Hamiltonians

- Create copy of Hamiltonian:

$$\bar{H}(q, p, x, y) \equiv H_A(q, y) + H_B(x, p) + \omega H_C(q, p, x, y)$$

$$H_C(q, p, x, y) = |q - x|^2/2 + |p - y|^2/2$$

- Time step for each Hamiltonian:

$$\phi_{H_A}^\delta : \begin{bmatrix} q \\ p \\ x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} q \\ p - \delta \partial_q H(q, y) \\ x + \delta \partial_y H(q, y) \\ y \end{bmatrix}, \quad \phi_{H_B}^\delta : \begin{bmatrix} q \\ p \\ x \\ y \end{bmatrix} \rightarrow \begin{bmatrix} q + \delta \partial_p H(x, p) \\ p \\ x \\ y - \delta \partial_x H(x, p) \end{bmatrix},$$

$$\phi_{H_C}^\delta : \begin{bmatrix} q \\ p \\ x \\ y \end{bmatrix} \rightarrow \frac{1}{2} \begin{bmatrix} \binom{q+x}{p+y} + R(\delta) \binom{q-x}{p-y} \\ \binom{q+x}{p+y} - R(\delta) \binom{q-x}{p-y} \end{bmatrix},$$

- Full second order time step:

$$\phi_2^\delta = \phi_{H_A}^{\delta/2} \circ \phi_{H_B}^{\delta/2} \circ \phi_{\omega H_C}^\delta \circ \phi_{H_B}^{\delta/2} \circ \phi_{H_A}^{\delta/2}.$$

# Results

## Processes Considered:

- Electron-Carbon Scattering
- Neutrino-Carbon Scattering
- Dirac/Majorana Dark neutrino  
[[1807.09877](#)]

## Experimental Setup:

- Target Nucleus: Carbon (Argon for Dark Neutrino)
- Electron: 961 MeV and 1299 MeV
- Neutrino: 1000 MeV
- Validating beam fluxes

**NOTE:** All processes are fully differential

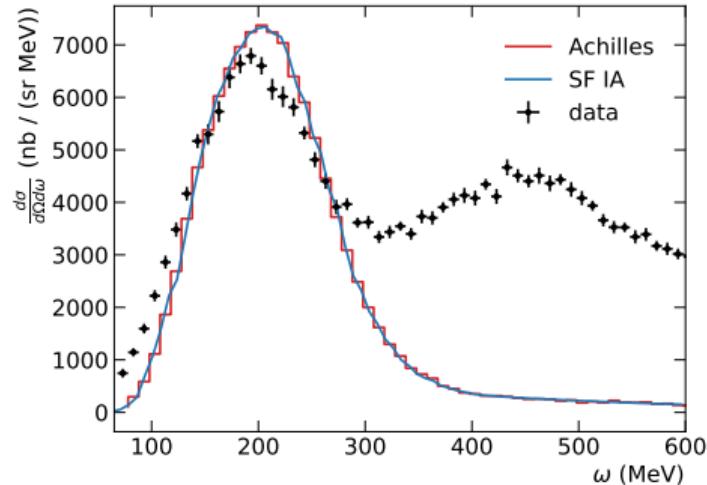
## Parameters:

- Only quasielastic scattering (coherent for Dark Neutrino) is included and no FSI
- EM Form Factors:  
Kelly [[PRC 70, 068202 \(2004\)](#)]
- Coherent Form Factor: Lovato [[1305.6959](#)]
- Axial Form Factor:
  - Dipole
  - $M_A = 1.0 \text{ GeV}$
  - $g_A = 1.2694$
- $\alpha = 1/137$
- $G_F = 1.16637 \times 10^{-5}$
- $M_Z = 91.1876 \text{ GeV}$

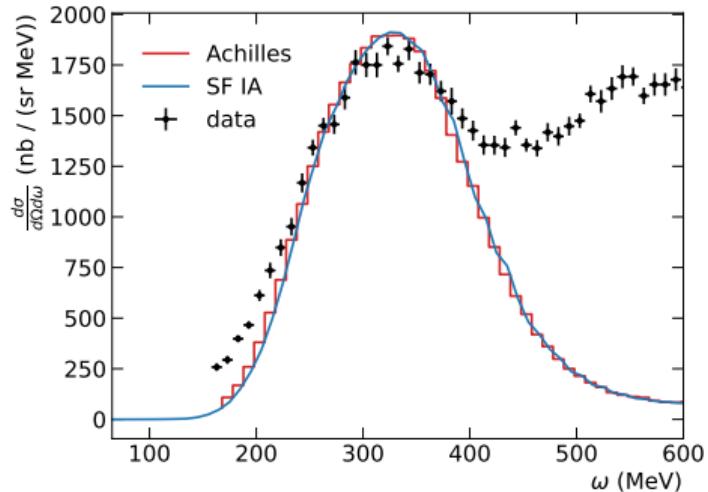
[J. et. al. Phys. Rev. D 105 (2022) 9, 096006]

# Electron Scattering

$E_e = 961 \text{ MeV}, \theta = 37^\circ$



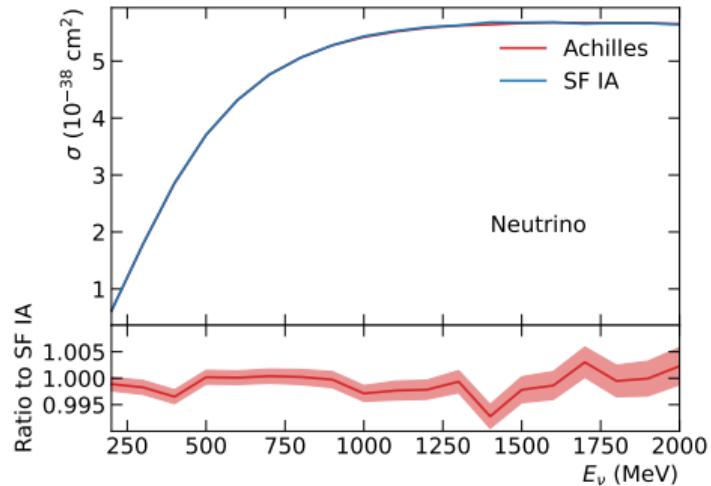
$E_e = 1300 \text{ MeV}, \theta = 37^\circ$



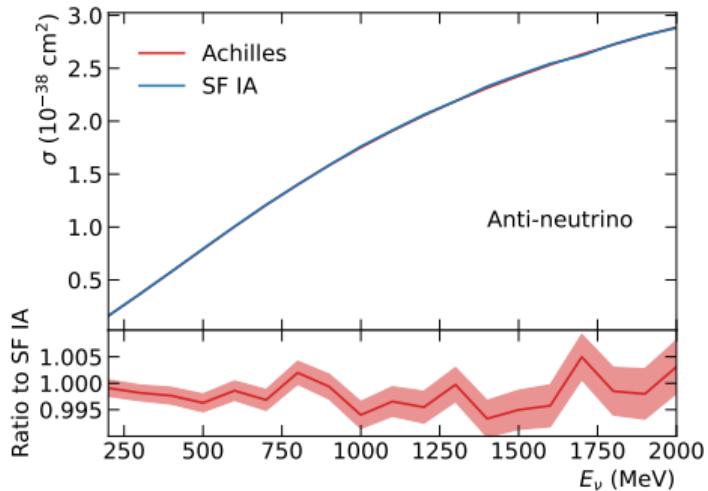
**Observable:** Double differential cross section with respect to outgoing electron angle and energy transfer from electron to nuclear system ( $\omega = E_{\text{in}}^e - E_{\text{out}}^e$ )

[JI, et. al. Phys. Rev. D 105 (2022) 9, 096006]

# Neutrino Total Cross Section



Neutrino



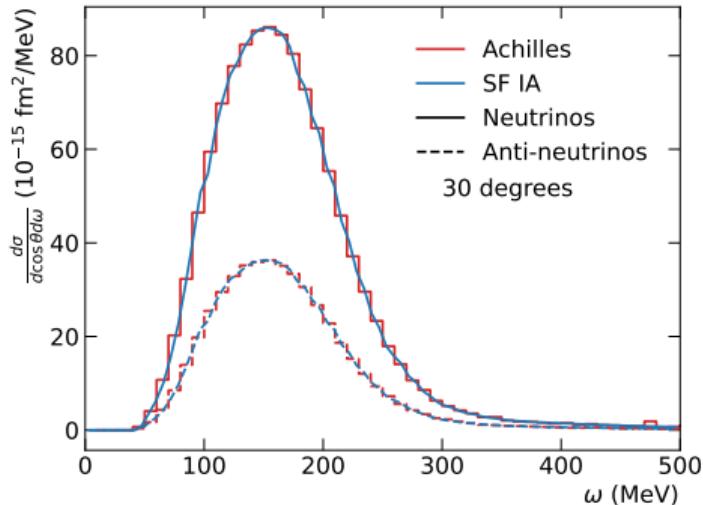
Anti-neutrino

**Observable:** Total neutrino-Carbon cross section versus neutrino energy

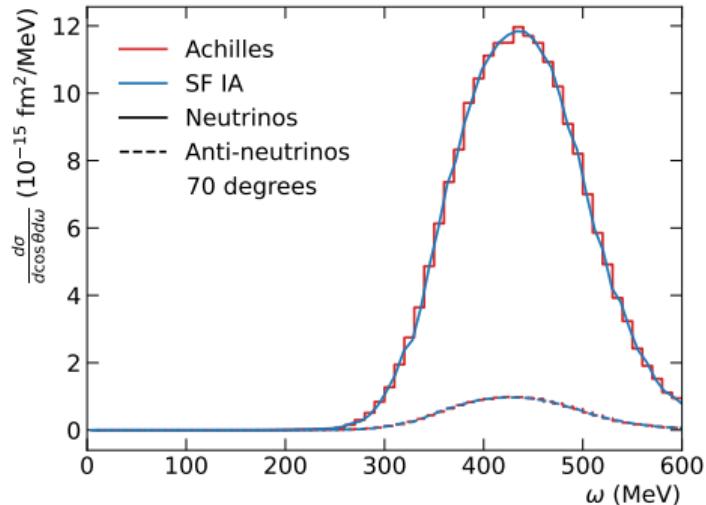
[J.I. et. al. Phys. Rev. D 105 (2022) 9, 096006]

# Neutrino Differential Cross Section

$E_\nu = 1000 \text{ MeV}$ ,  $\theta = 30^\circ$



$E_\nu = 1000 \text{ MeV}$ ,  $\theta = 70^\circ$



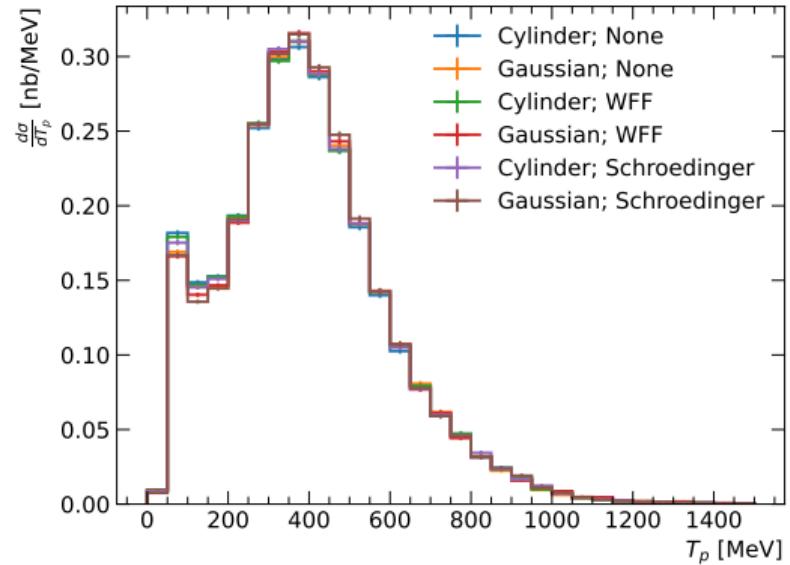
**Observable:** Double differential cross section with respect to outgoing electron angle and energy transfer from neutrino to nuclear system ( $\omega = E_\nu - E_e$ )

[JI, et. al. Phys. Rev. D 105 (2022) 9, 096006]

# New Observables

## New Observables:

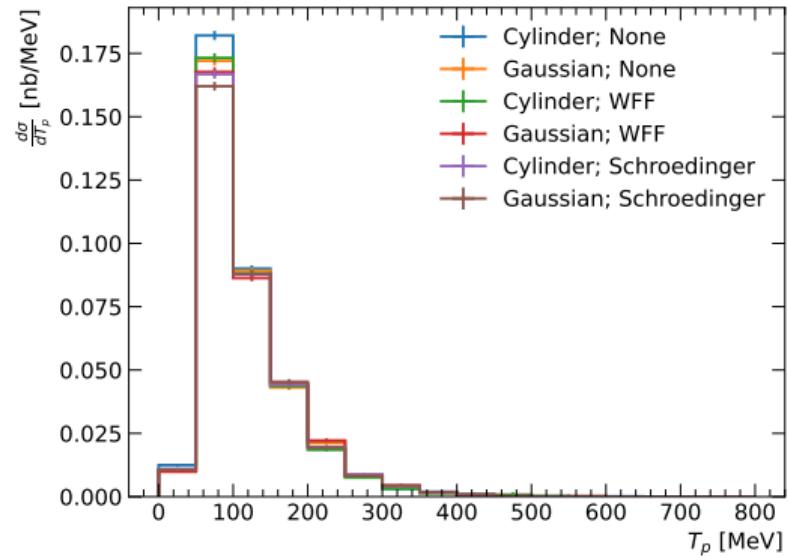
- Momentum of 1st proton



# New Observables

## New Observables:

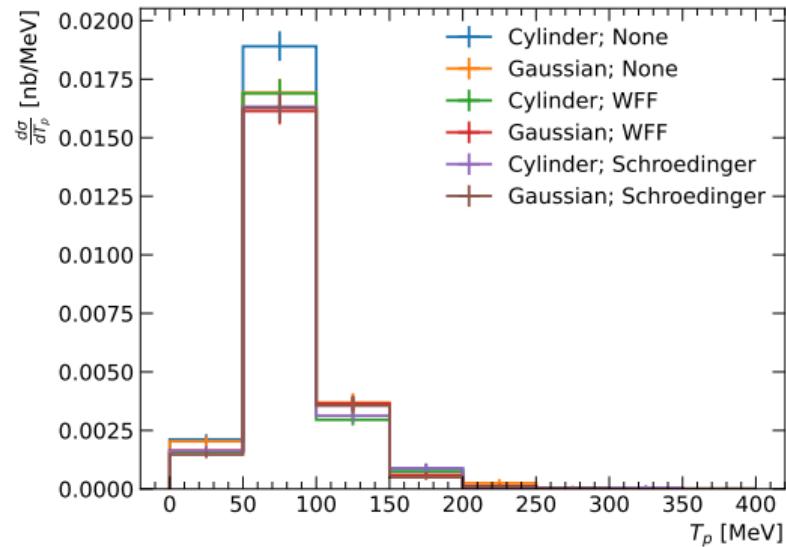
- Momentum of 1st proton
- Momentum of 2nd proton



# New Observables

## New Observables:

- Momentum of 1st proton
- Momentum of 2nd proton
- Momentum of 3rd proton



# New Observables

## New Observables:

- Momentum of 1st proton
- Momentum of 2nd proton
- Momentum of 3rd proton
- Reconstructed beam direction:

$$\cos \theta_{\text{rec}} \equiv \frac{\hat{\mathbf{k}}_e \cdot \mathbf{p}_{\text{out}}}{|\mathbf{p}_{\text{out}}|}$$

