

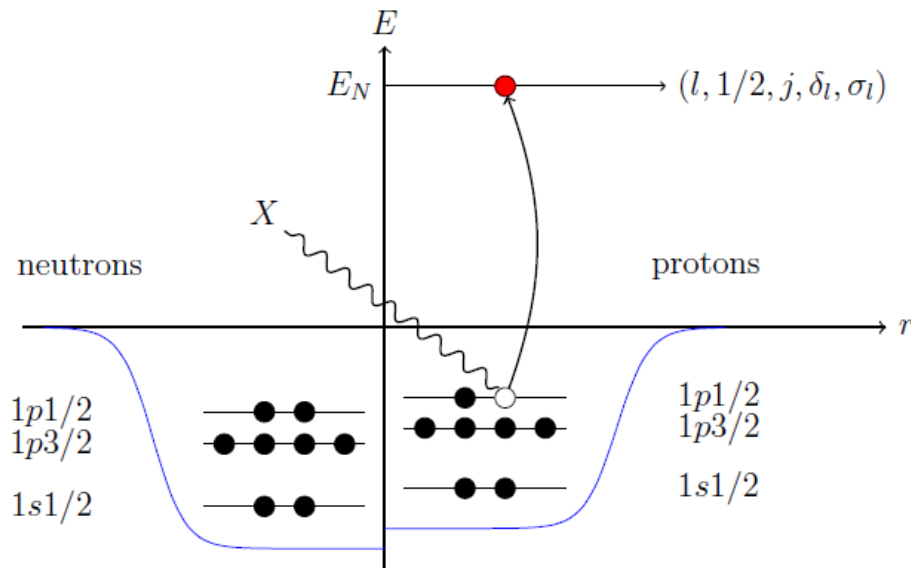
Final-State Interactions in inclusive and exclusive one-nucleon knockout

A. Nikolakopoulos (Fermilab)

In collaboration with

R. Gonzalez-Jimenez, J. M. Udias (UC Madrid), N. Jachowicz, V. Pandey (U Gent),
K. Niewczas (U Wroclaw), S. Dolan (CERN), F. Sanchez (U Geneva)

Self-consistent mean field



RMF

Mean field nucleus

- Mean field potential
- Single-particle wavefunctions
- Binding energies
- Orthogonal states (\rightarrow Pauli-blocking)

HF-SKE2

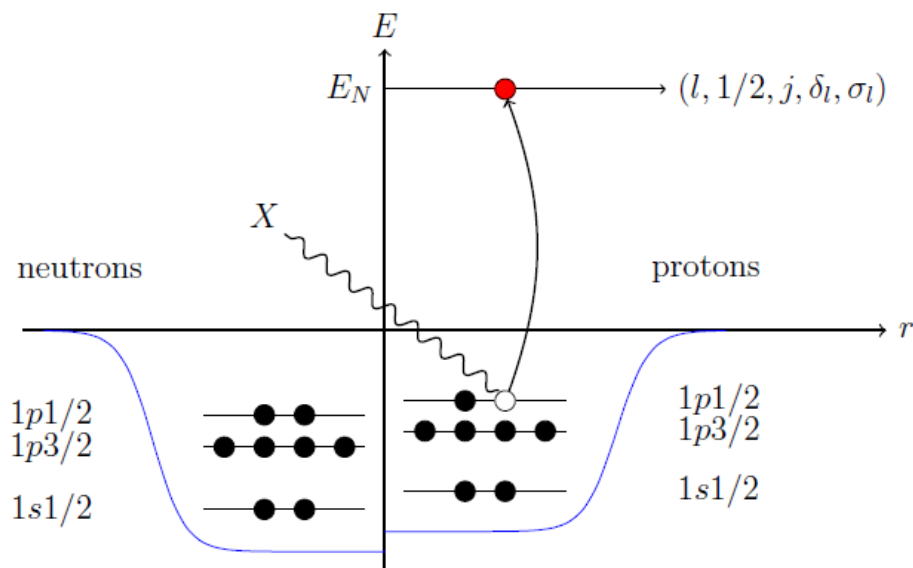
- Non-linear extended sigma-omega model

Extension of the original
 σ - ω Walecka model
(Ann. Phys.83,491 (1974)).

- Hartree-Fock with extended Skyrme force

M. Waroquier et al. / Effective Skyrme-type interaction (I)
Nuclear Physics **A404** (1983) 269–297

Self-consistent mean field



Mean field nucleus

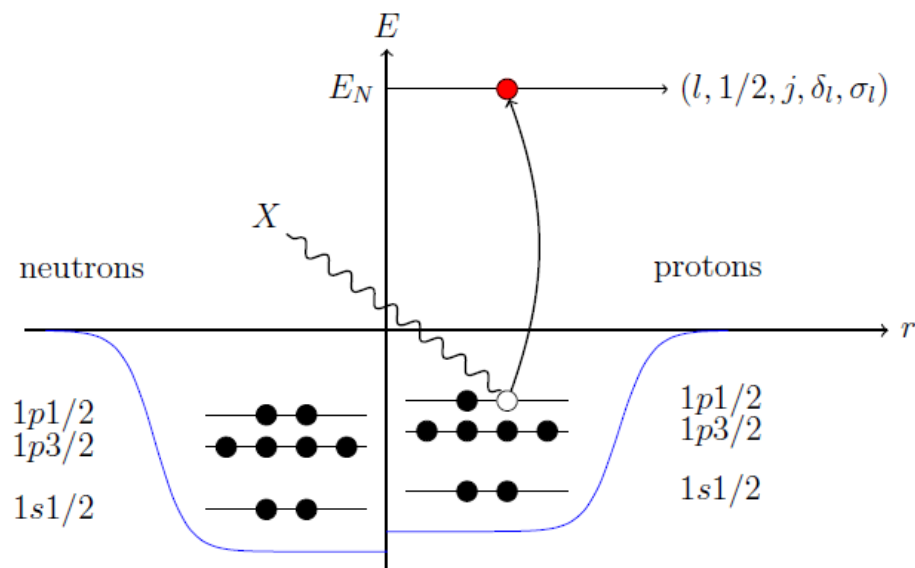
- Mean field potential
- Single-particle wavefunctions
- Binding energies
- Orthogonal states (\rightarrow Pauli-blocking)

Effective interactions constrained by properties of nuclei and nuclear matter

	t_4 (MeV \cdot fm ⁸)	K (MeV)	$(E/A)_{n.m.}$ (MeV)	k_F (fm ⁻¹)	m^*/m	a_τ (MeV)
SkE2	-15808.79	200	-16.0	1.33	0.72	29.7
SkE4	-12258.97	250	-16.0	1.31	0.75	30.0
SkIII	0.0	356	-15.87	1.29	0.76	28.2

	E/A	r_p	r_n	r_c	E/A	r_p	r_n	r_c
		¹⁶ O				⁴⁰ Ca		
SkE2	-7.92	2.63	2.60	2.68	-8.56	3.37	3.31	3.42
SkE4	-7.96	2.65	2.62	2.70	-8.59	3.40	3.35	3.46
SkIII	-8.03	2.64	2.61	2.70	-8.57	3.41	3.36	3.46
exp	-7.98			2.71 ^{a)}	-8.55		3.36 ^{e)}	3.48 ^{b)}
		⁹⁰ Zr				¹³² Sn		
SkE2	-8.67	4.17	4.24	4.21	-8.36	4.62	4.84	4.66
SkE4	-8.71	4.22	4.29	4.26	-8.36	4.68	4.89	4.71
SkIII	-8.69	4.26	4.31	4.30	-8.36	4.73	4.90	4.78
exp	-8.71			4.27 ^{c)}	-8.36			

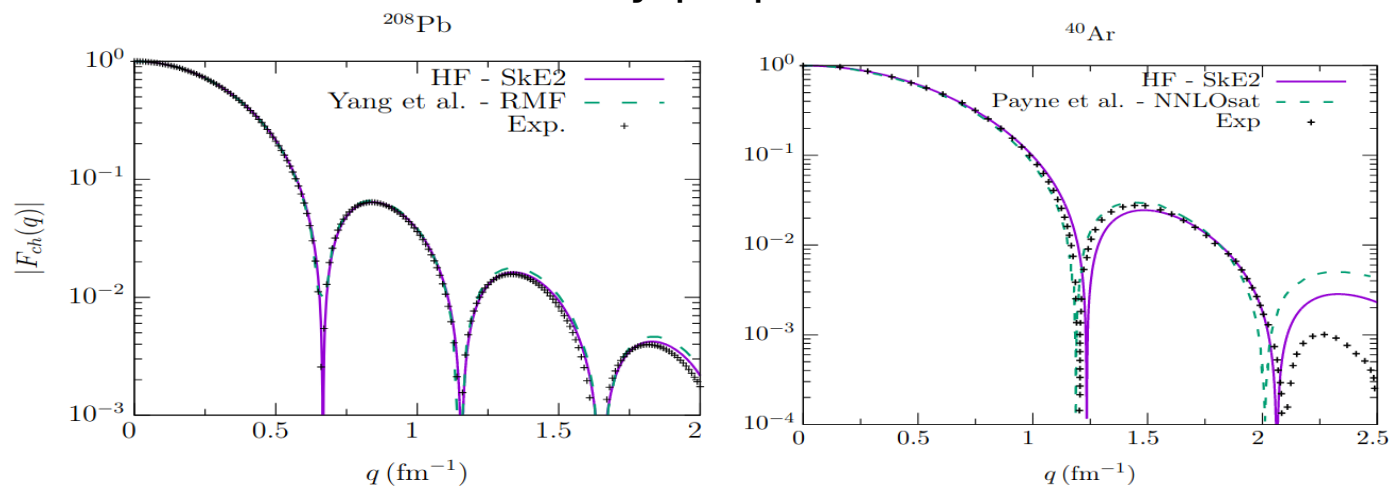
Self-consistent mean field



Mean field nucleus

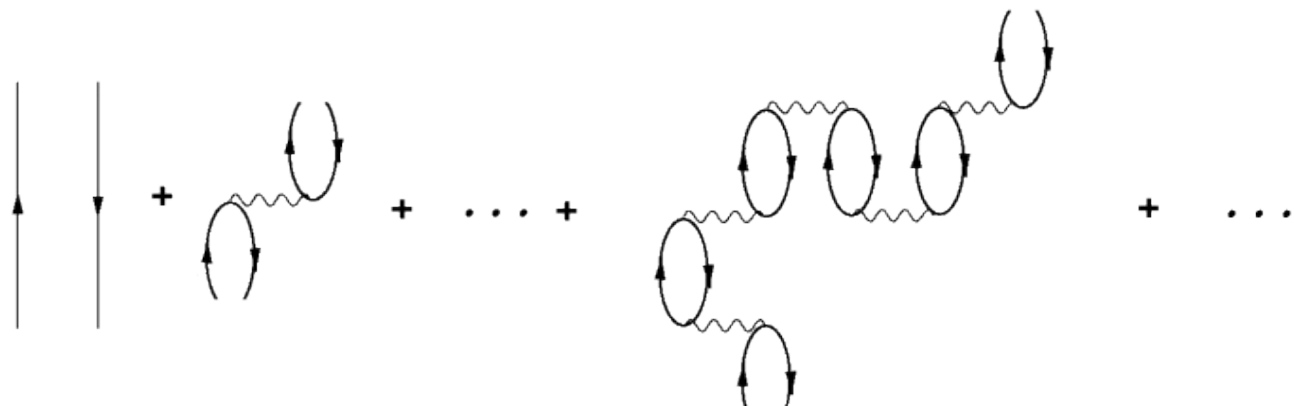
- Mean field potential
- Single-particle wavefunctions
- Binding energies
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Effective interactions constrained by properties of nuclei and nuclear matter



Charge (and weak) form factors in [N. Van Dessel et al. Arxiv:2007.03658]

HF-Skyrme mean field + Continuum RPA



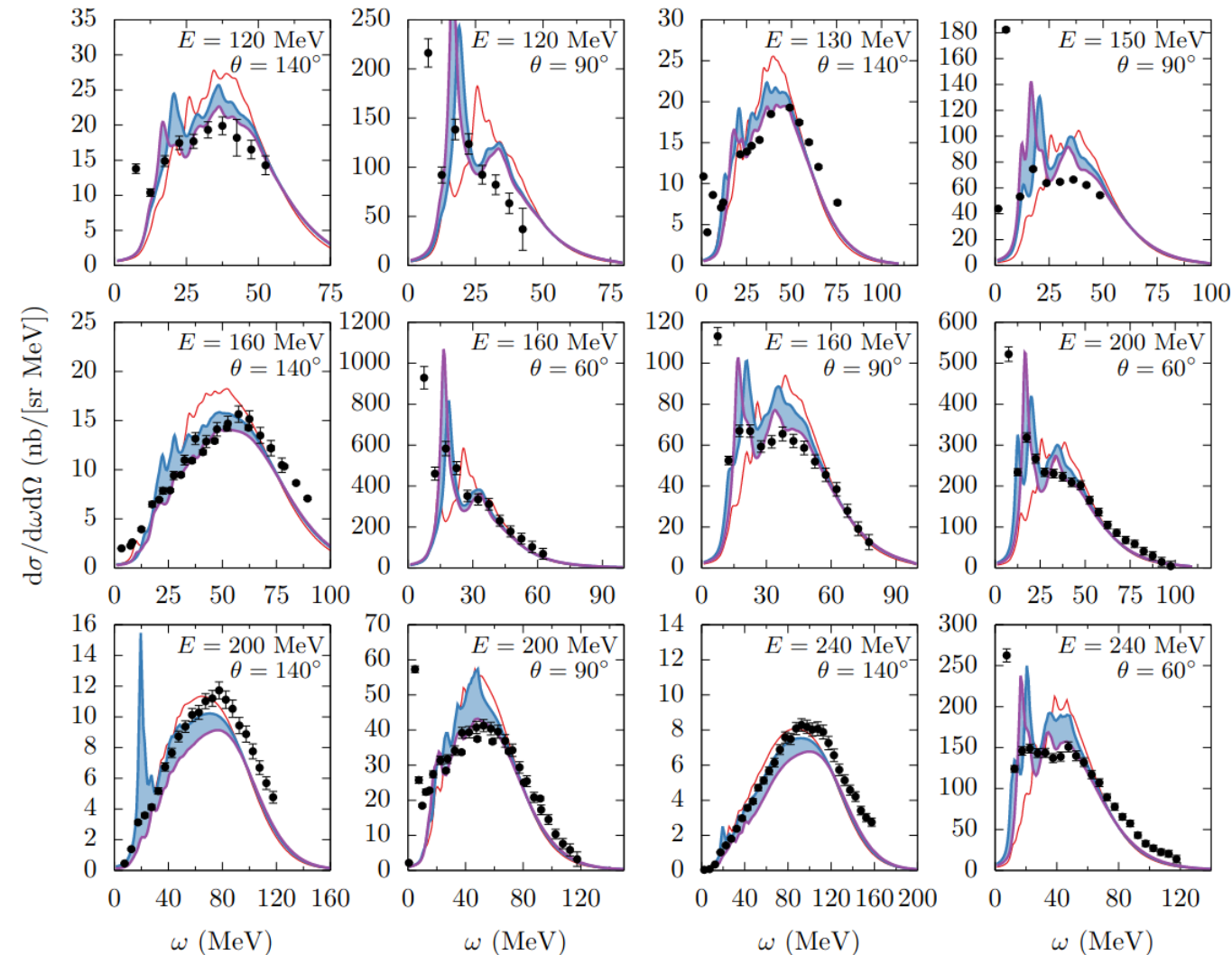
$$\begin{aligned} \Pi^{(RPA)}(x_1, x_2; \omega) &= \Pi^{(0)}(x_1, x_2; \omega) \\ &+ \frac{1}{\hbar} \int dx \int dx' \Pi^{(0)}(x_1, x; \omega) \tilde{V}(x, x') \Pi^{(RPA)}(x', x_2; \omega) \end{aligned}$$

Coordinate space formulation → No cut-off in momenta (full continuum)
Use the same Skyrme interaction for HF nucleus and in RPA

Takes into account collective d.o.f. : Giant Resonance region
: Quenching of QE peak at low Q^2

$$|\Psi_{RPA}\rangle = \sum_c \left\{ X_{(\Psi, C)} |ph^{-1}\rangle - Y_{(\Psi, C)} |hp^{-1}\rangle \right\}$$

Electron scattering ^{40}Ca



[A. Nikolakopoulos et al. PRC 103 (2021) 6, 064603]

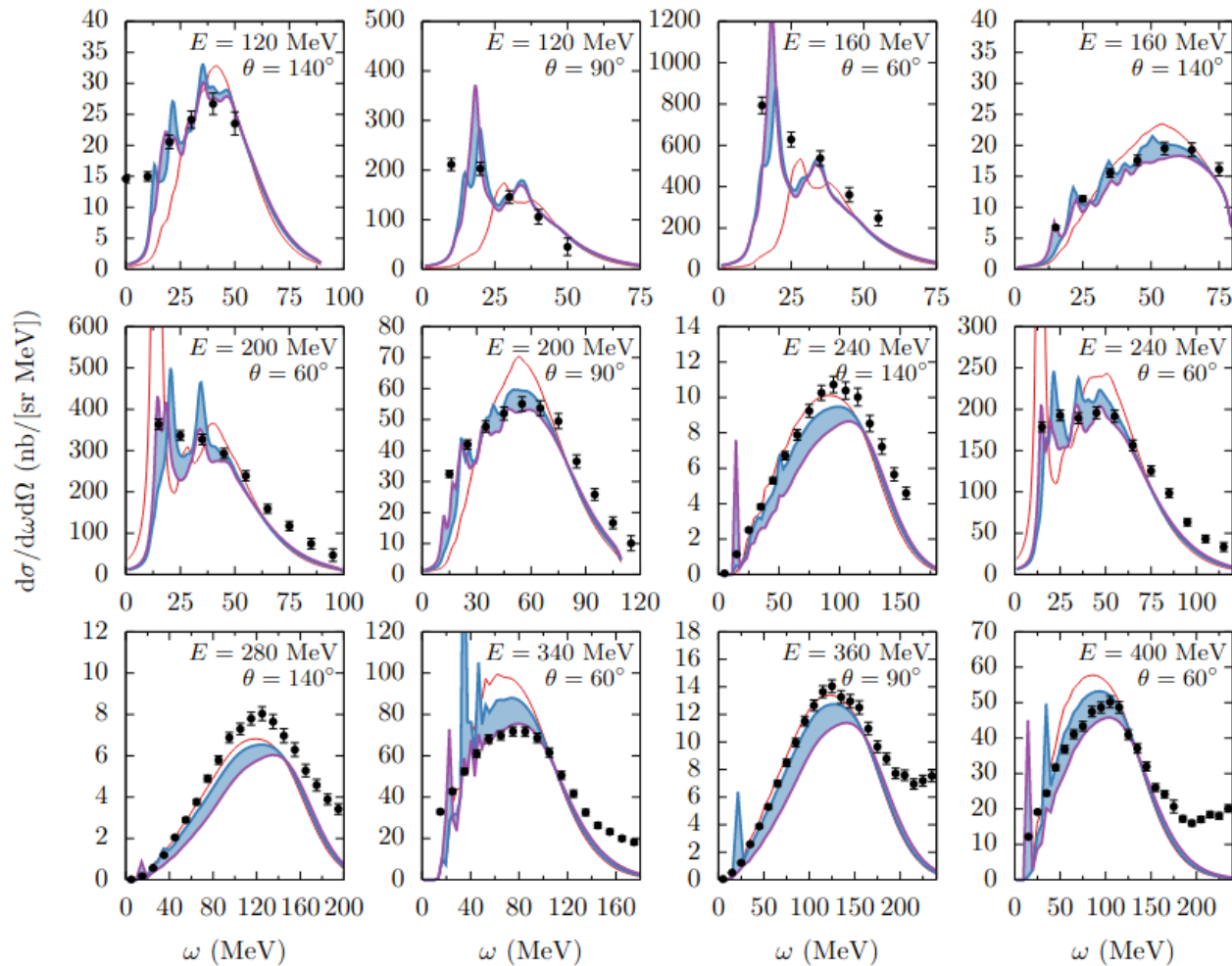
(e, e') off Calcium

- Blue band uncertainty due to residual interaction in CRPA

$$V \rightarrow \frac{V}{(1 + Q^2/\Lambda)^2}$$

- Cut off determined in [V. Pandey, et al Phys. Rev. C 92, 024606 (2015)]

Electron scattering ^{56}Fe



(e,e') off Iron

- Blue band uncertainty due to residual interaction in CRPA

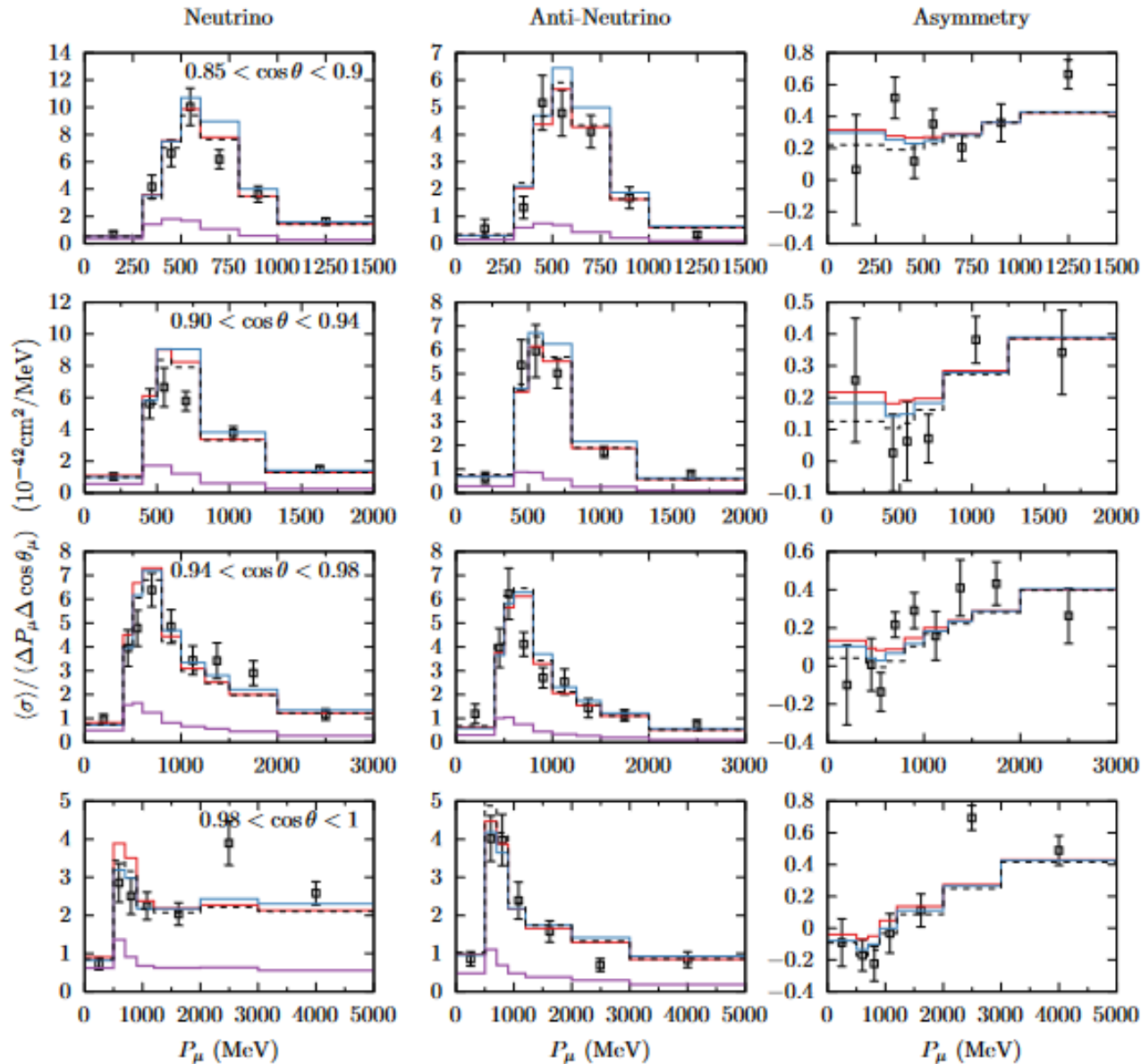
$$V \rightarrow \frac{V}{(1 + Q^2/\Lambda)^2}$$

- Cut off determined in

[V. Pandey, et al Phys. Rev. C 92, 024606 (2015)]

[A. Nikolakopoulos et al. PRC 103 (2021) 6, 064603]

T2K data



Asymmetry

T2K measurement

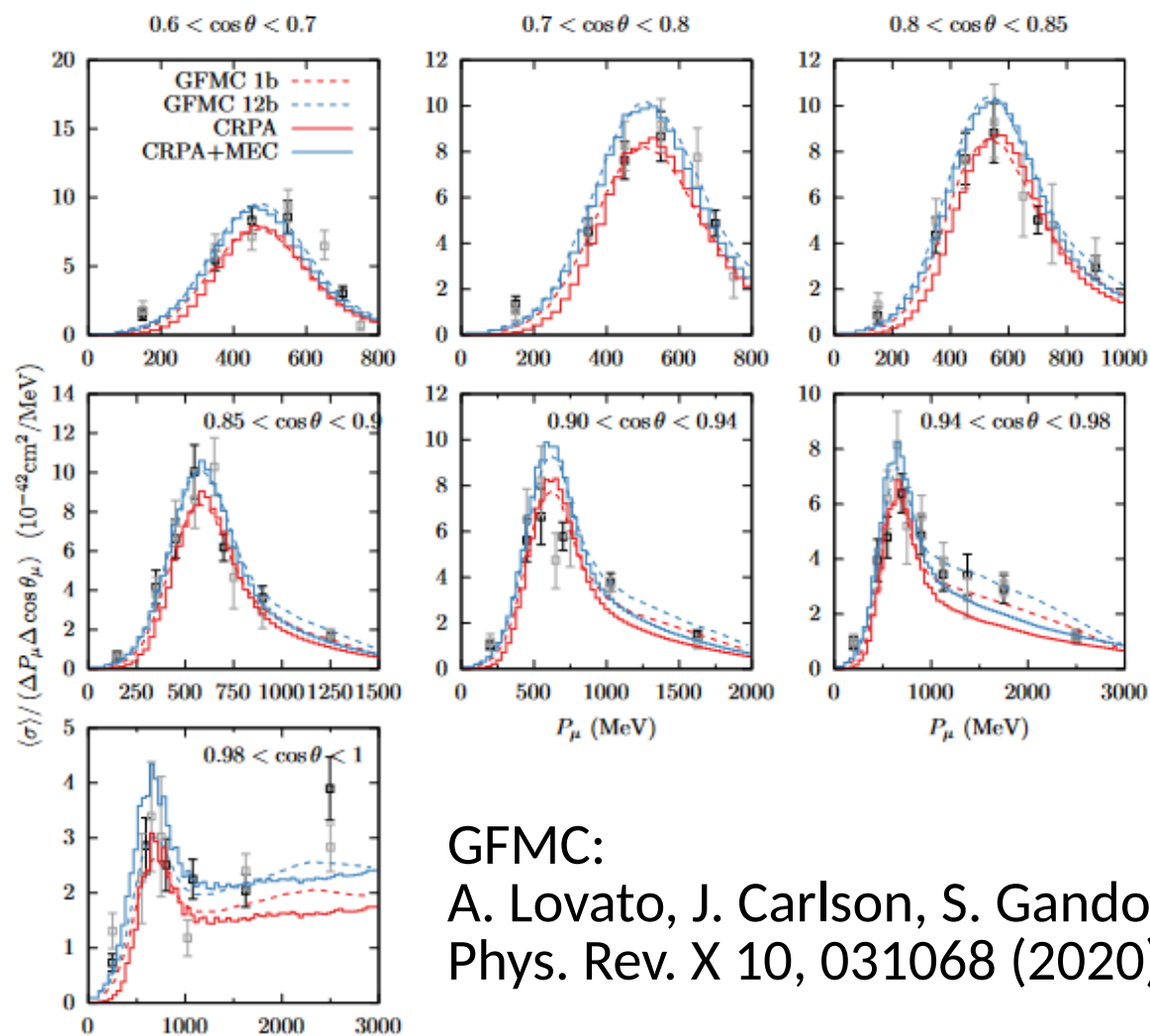
SuSAv2 MEC (RFG calculation)
[G. D. Megias et al. PRD91, 073004 (2015)]

Add Hydrogen in
anti-neutrino reactions

Dashed lines: assumption
of isospin symmetry
(neglect Coulomb effects)

Asymmetry quite
model-independent

GFMC ab-initio results



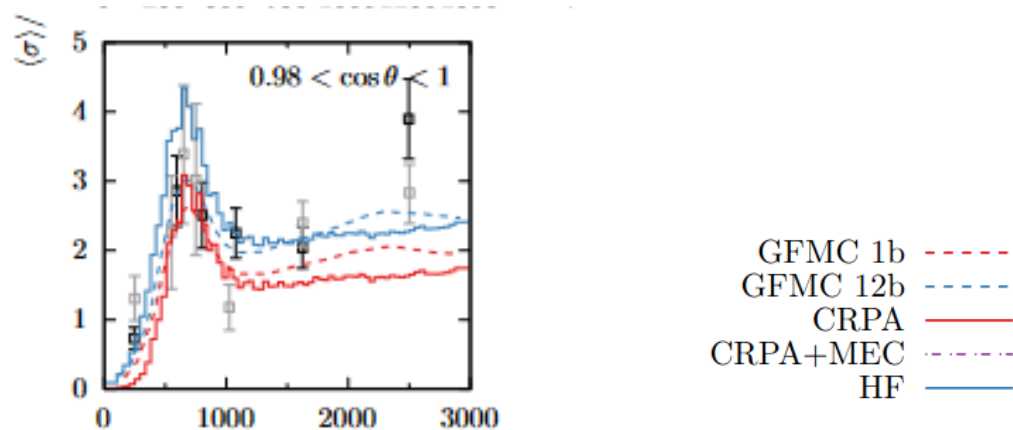
GFMC:

A. Lovato, J. Carlson, S. Gandolfi, N. Rocco, and R. Schiavilla,
 Phys. Rev. X 10, 031068 (2020)

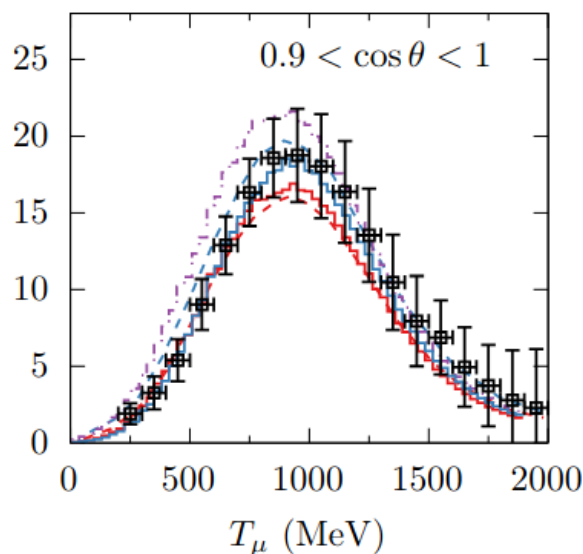
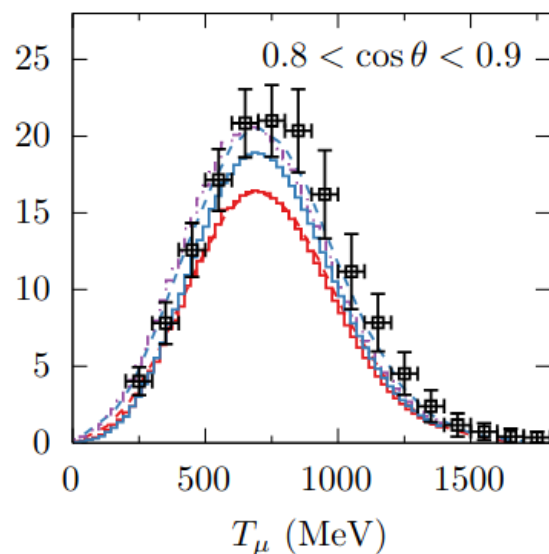
GFMC results

- Consistent treatment of one- and two-body currents!
- CRPA & GFMC 1b are similar
- CRPA + SuSAv2 MEC & GFMC 1+2b Similar in backward bins
- Discrepancies in forward low- P_μ region

GFMC ab-initio results



MiniBooNE



GFMC results

- Consistent treatment of one- and two-body currents!
- CRPA & GFMC 1b are similar
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GFMC:

[A. Lovato et al. Phys. Rev. X 10, 031068 (2020)]

Treatment of final-state nucleon

Same initial-state nucleon, different treatments of final-state wavefunction

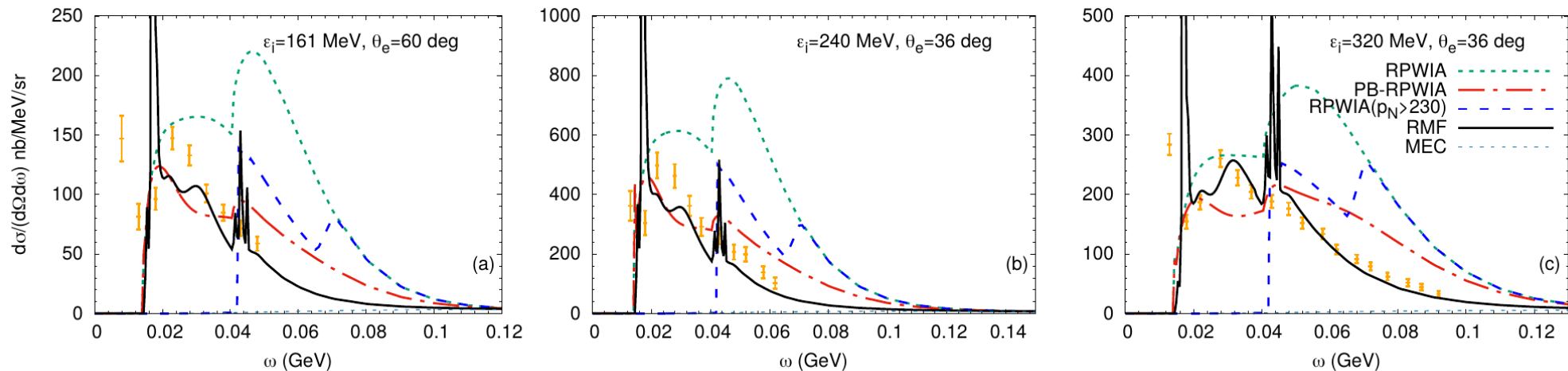
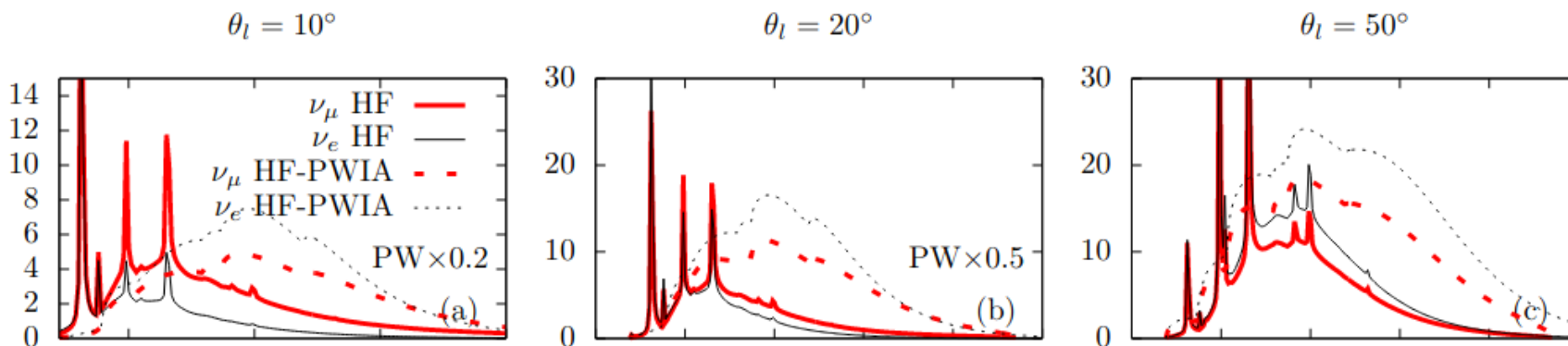


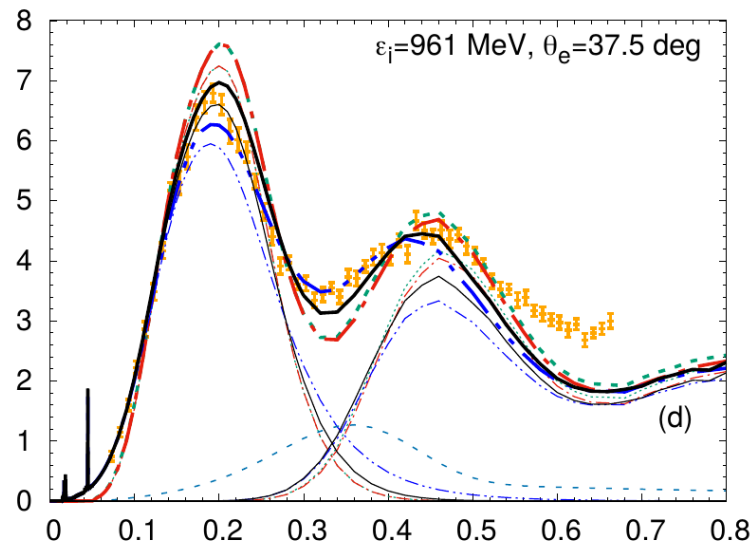
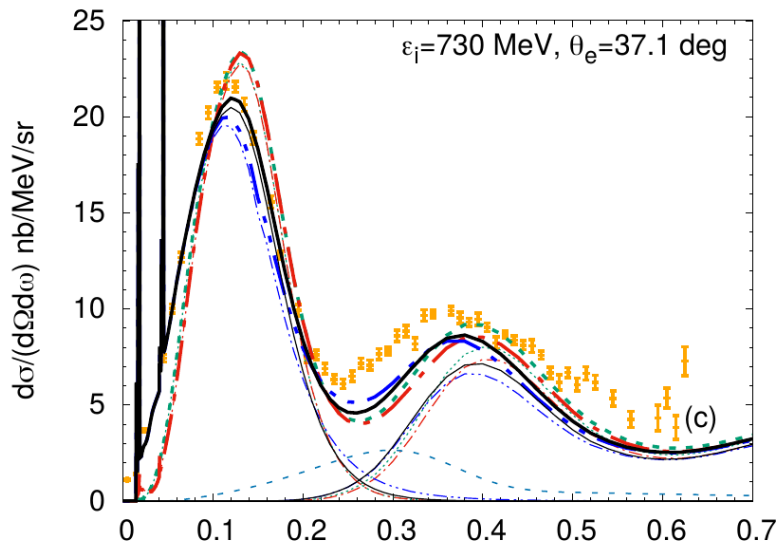
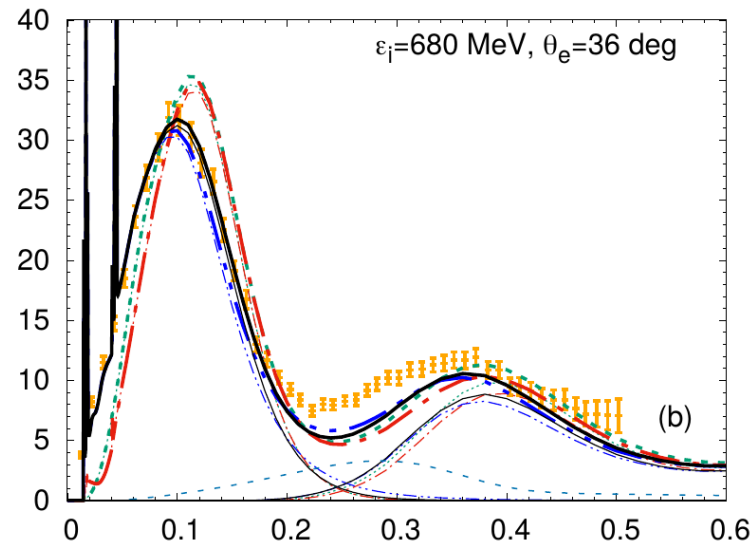
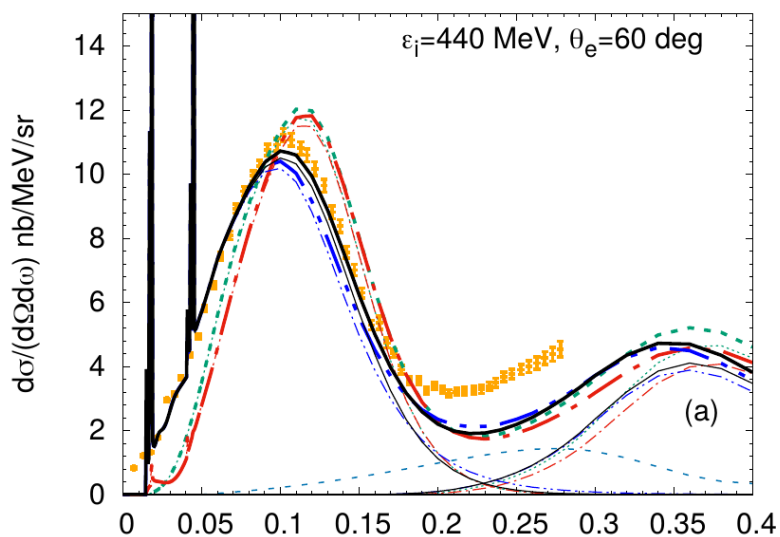
FIG. 4: QE predictions for the process $^{12}\text{C}(e, e')$ with the RPWIA, PB-RPWIA, RPWIA($p_N > 230$), and RMF



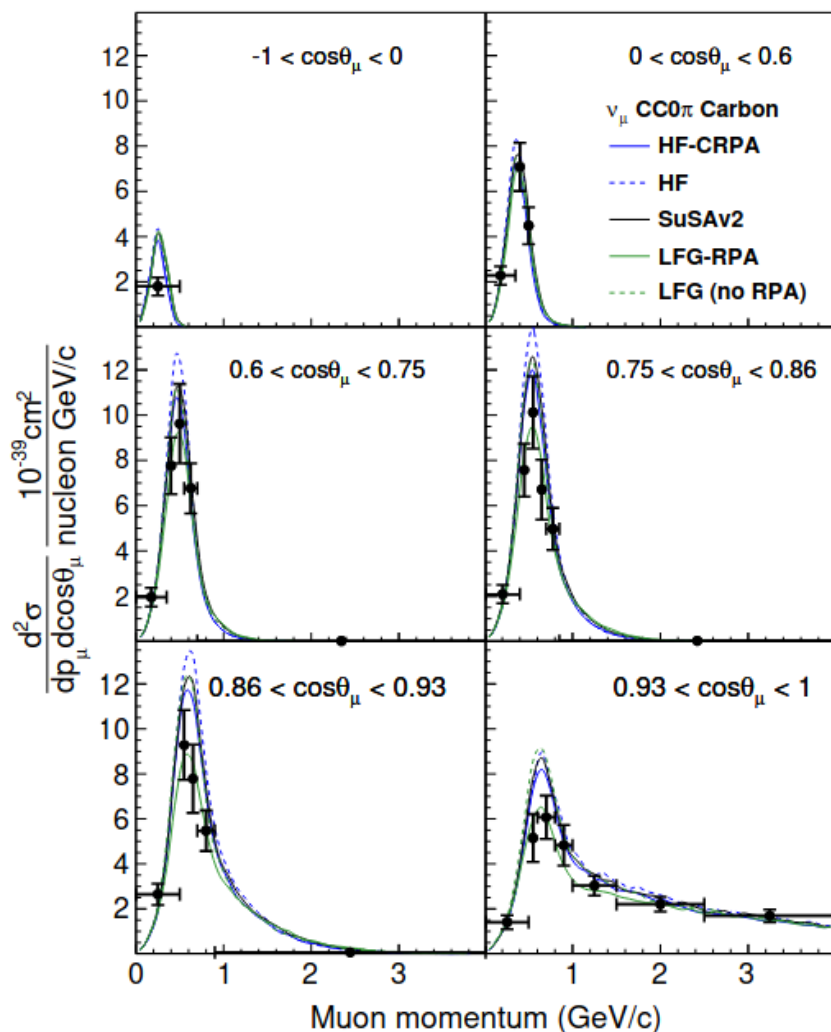
$\nu + {}^{40}\text{Ar} \text{ (CC) at } E_\nu = 200 \text{ MeV}$

Treatment of final-state nucleon

Same initial-state nucleon, different treatments of final-state wavefunction



Inclusive cross sections in generators



Implementation of the CRPA model in the GENIE event generator and analysis of nuclear effects in low-energy transfer neutrino-nucleus interactions

S. Dolan,¹ A. Nikolakopoulos,² O. Page,³ S. Gardiner,⁴ N. Jachowicz,² and V. Pandey⁵

¹CERN, European Organization for Nuclear Research, Geneva, Switzerland*

²Department of Physics and Astronomy, Ghent University, Proeftuinstraat 86, B-9000 Gent, Belgium†

³School of Physics, University of Bristol, Bristol BS8 1TL, United Kingdom

⁴Fermi National Accelerator Laboratory, Batavia, IL 60502, USA

⁵Department of Physics, University of Florida, Gainesville, FL 32611, USA‡

(Dated: November 2, 2021)

Implementation of the SuSAv2-MEC 1p1h and 2p2h models in GENIE and analysis of nuclear effects in T2K measurements

S. Dolan,^{1,2,3} G.D. Megias,^{1,2,4} and S. Bolognesi²

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³CERN, European Organization for Nuclear Research, Geneva, Switzerland

⁴University of Tokyo, Institute for Cosmic Ray Research, Research Center for Cosmic Neutrinos, Kashiwa, Japan

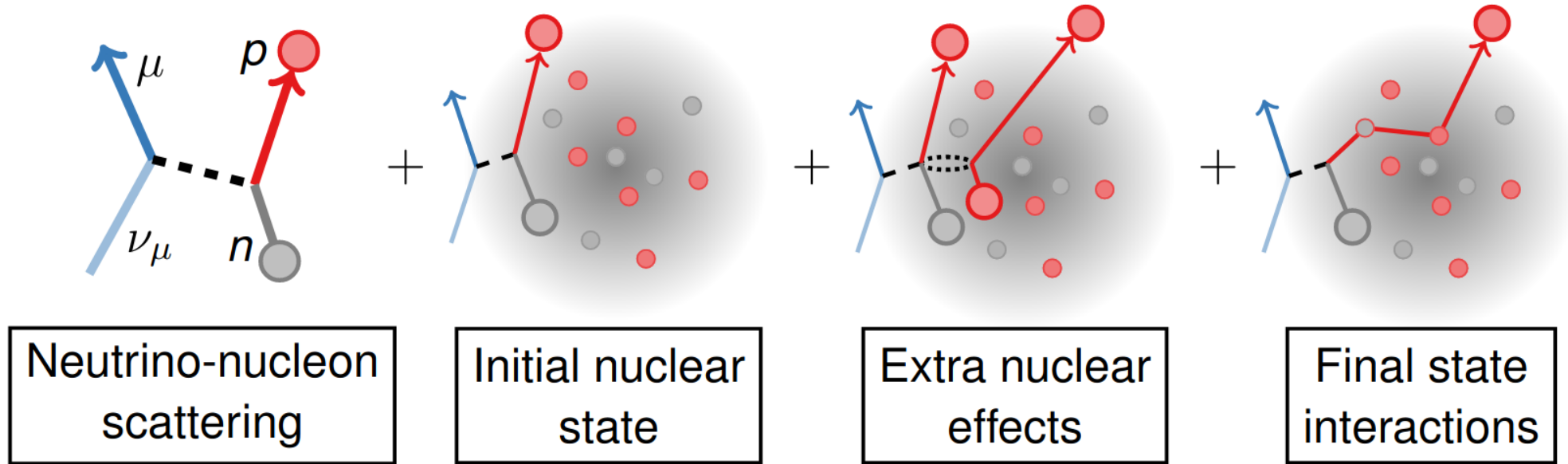
(Dated: February 21, 2020)

Reproduce exactly the inclusive results

But lose information about nucleon observables

Inclusive cross sections in generators

The idealized version of the event generator:

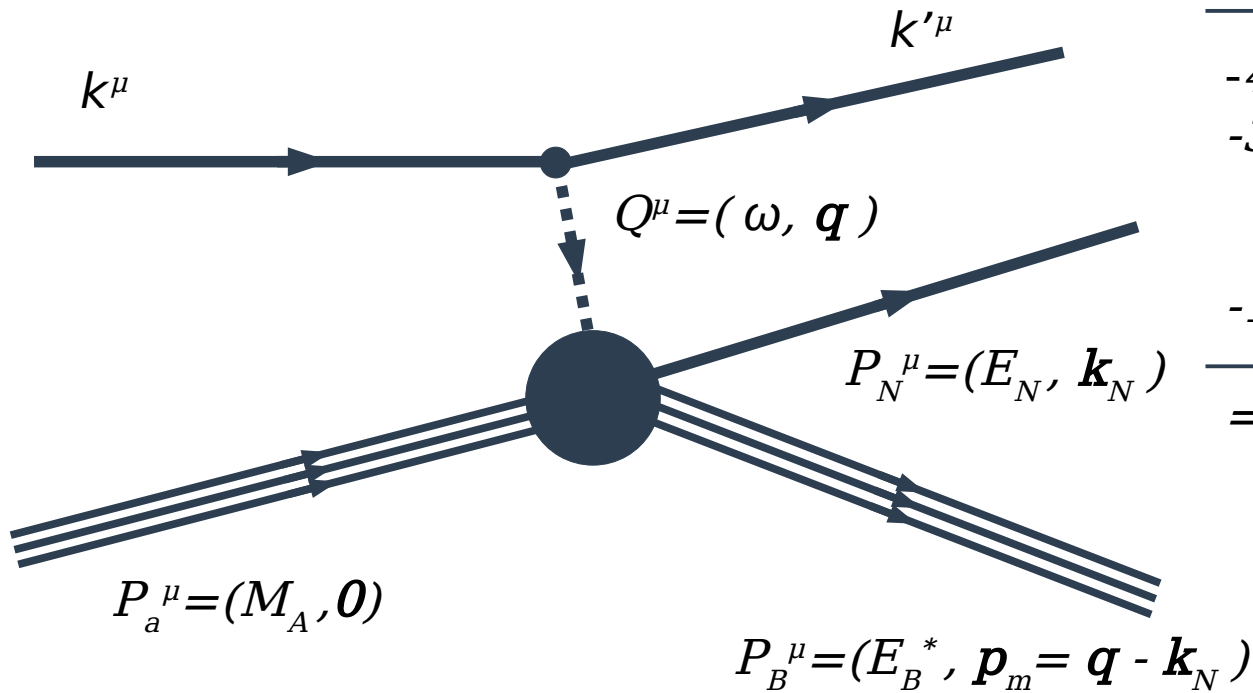


[Fig. From K. Niewczas]

When an inclusive calculation is implemented:
(part of) the initial-state, the 'extra effects' and the FSI are included

It is not possible to obtain the hadron information from inclusive CS

Kinematics of single-nucleon knockout



Given k^μ and P_a^μ : 3·4
=12 d.o.f

-4 : $k^\mu + P_a^\mu = k'^\mu + P_N^\mu + P_B^\mu$

-3 : $k'^2 = m'^2$,

$P_N^2 = M_N^2$,

$P_B^2 = M_B^{*2}$

-1 : $R^\nu{}_\mu k^\mu = k^\nu$ (φ' trivial)

= 4 d.o.f

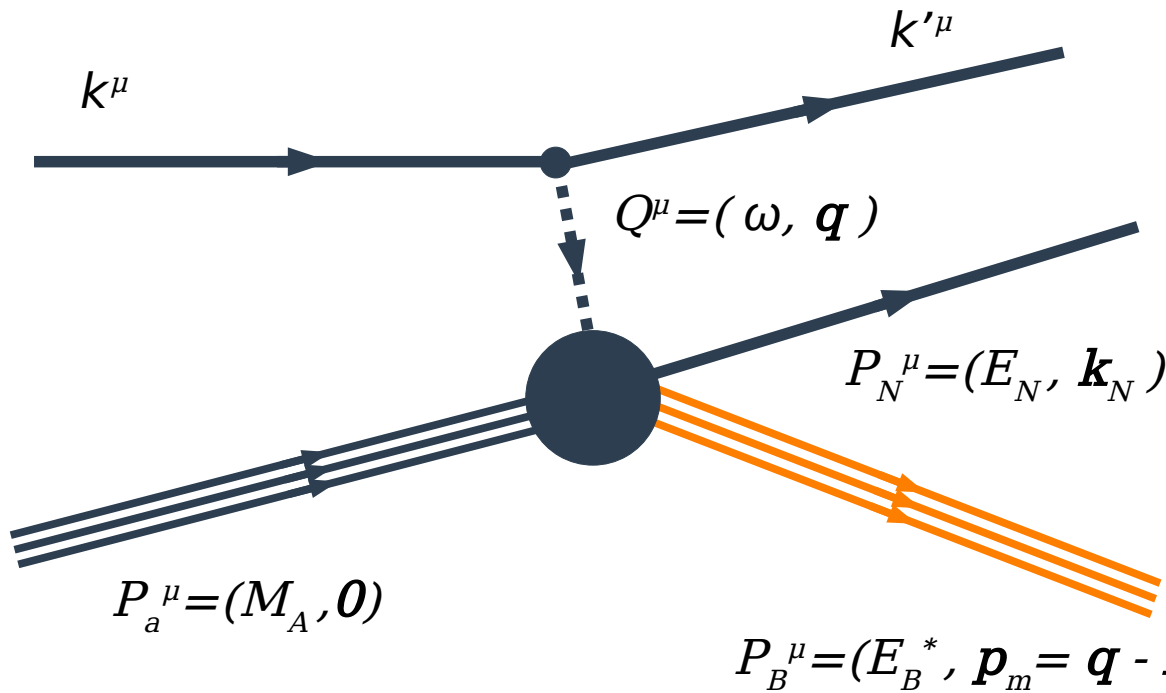
$E', \cos\theta', \cos\theta_N, \varphi_N$

All particles:

$$P_X \cdot P_X = M_X^2$$

No 'off-shell' initial nucleon

Kinematics of single-nucleon knockout



Given k^μ and P_a^μ : 3·4
=12 d.o.f

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$E', \cos\theta', \cos\theta_N, \varphi_N$

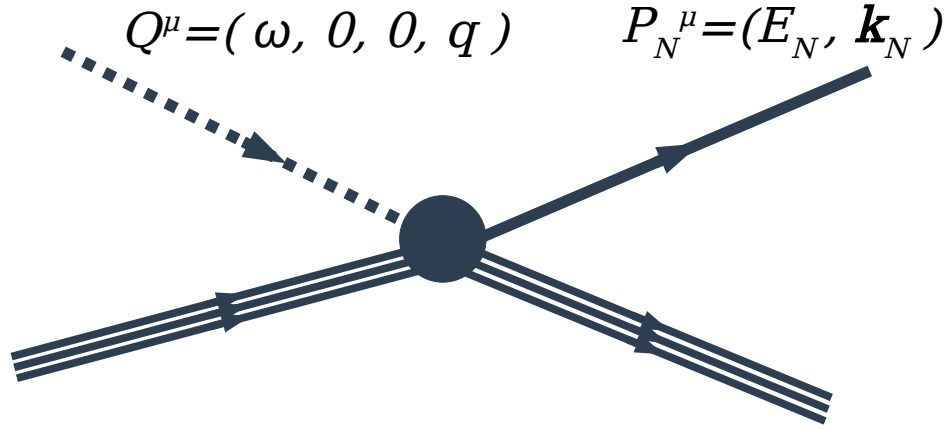
Non-trivial parameter M_B^2

All particles:

$$P_X \cdot P_X = M_X^2$$

No 'off-shell' initial nucleon

Kinematics of single-nucleon knockout



given ω q $\cos \theta_N$ φ_N

$$E_m = E_i - E_f - T_N - T_B = M_B + M_N - M_A. \quad (2)$$

The total energy of the residual system is

$$E_B = T_B + M_B = \sqrt{M_B^2 + |\vec{p}_m|^2}, \quad (3)$$

$$P_a^\mu = (M_A, \mathbf{0})$$

$$P_B^\mu = (E_B, \mathbf{p}_m = \mathbf{q} - \mathbf{k}_N) \text{ and its momentum is the missing momentum}$$

$$\vec{p}_m = \vec{k}_i - \vec{k}_f - \vec{k}_N. \quad (4)$$

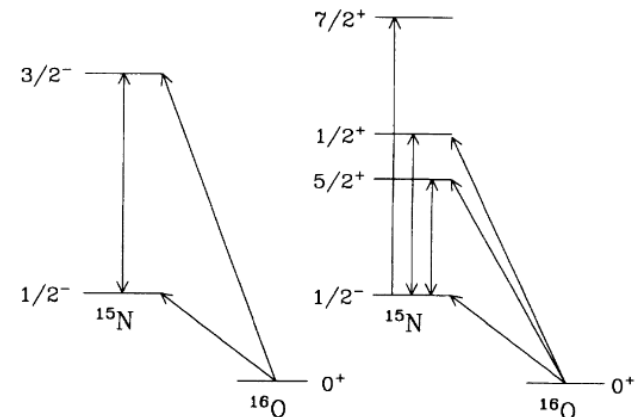
All particles:

$$P_X \cdot P_X = M_X^2$$

$$M_B = M_B^0 + E_{ex} \approx (M_A - M_N) + E_{ex}$$

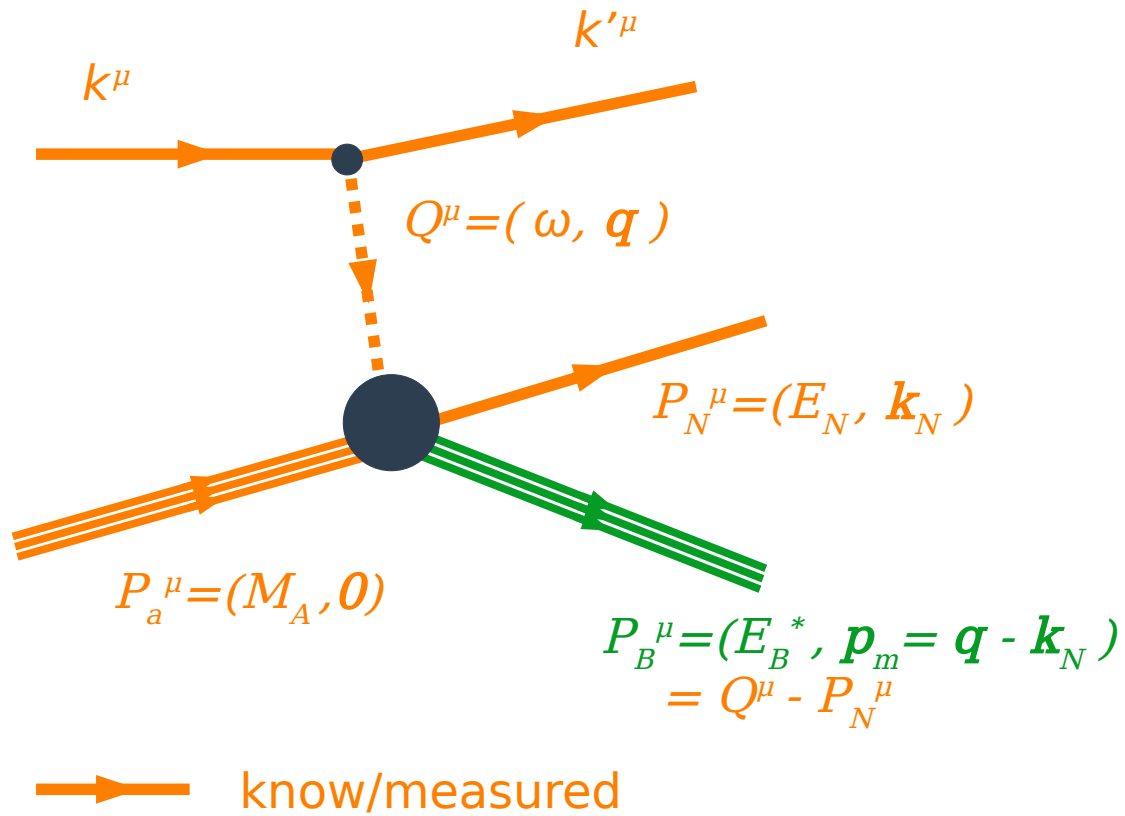
$$\rightarrow E_{ex} \approx E_m$$

+Nuclear model
→ Available states

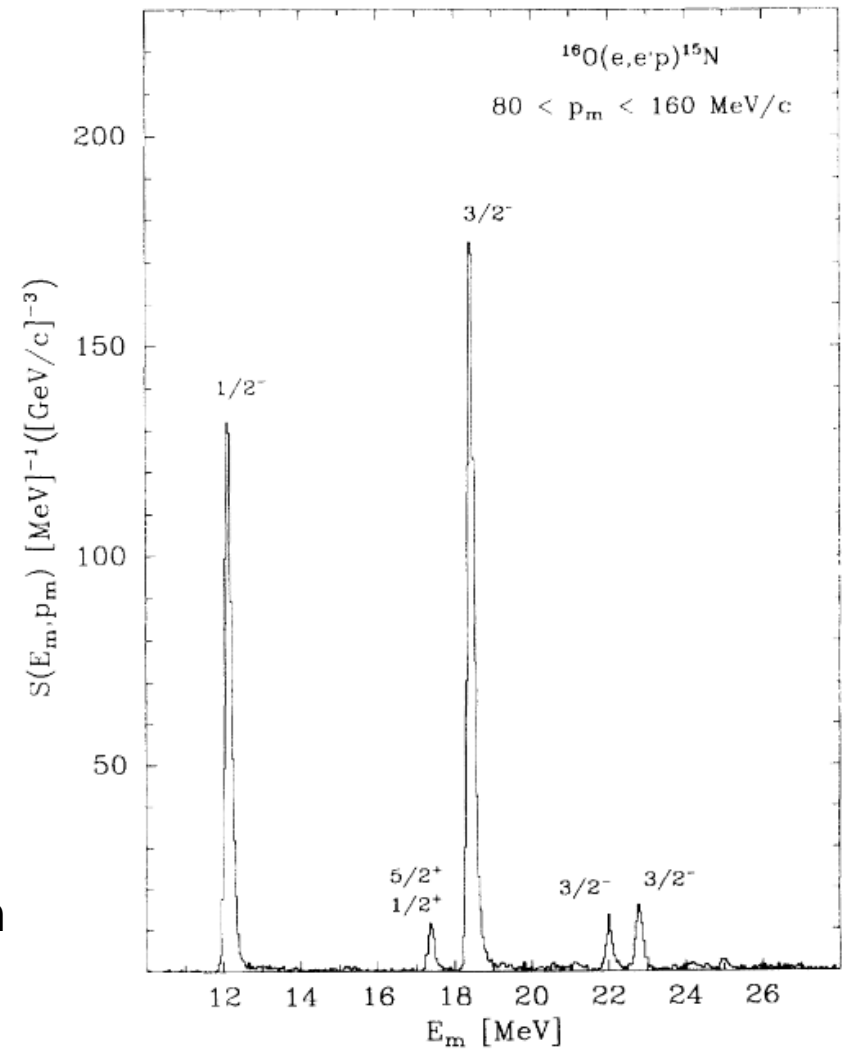


[M. Leuschner et al. PRC49, 955 (1994)]

Exclusive electron scattering

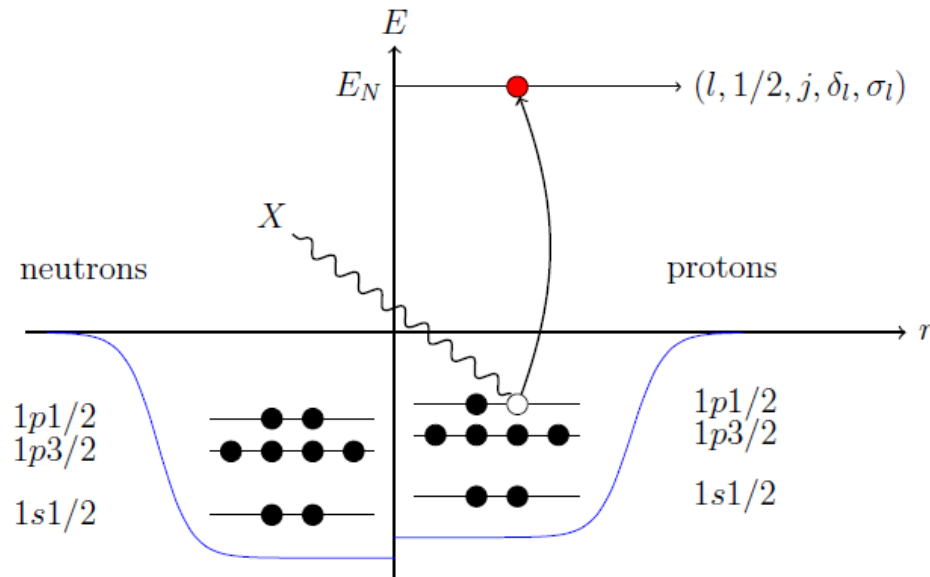


Exclusive conditions : everything is known



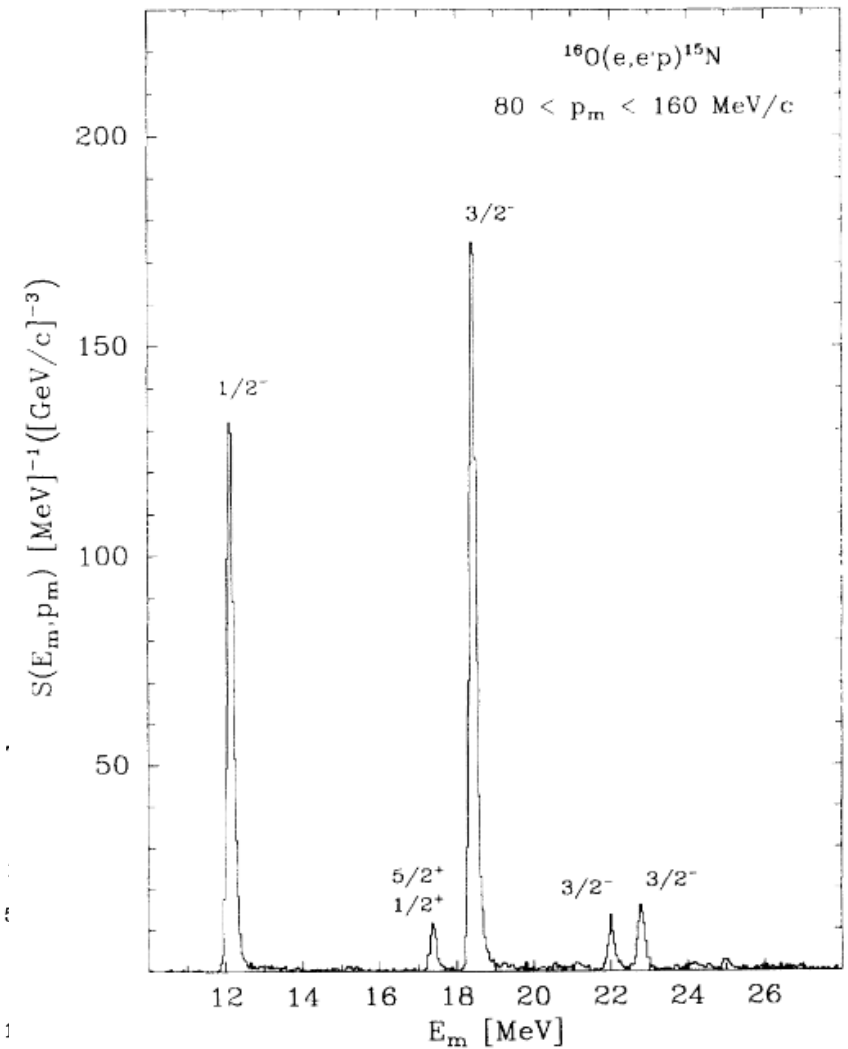
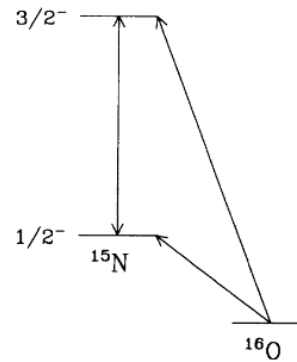
[M. Leuschner et al. PRC49, 955 (1994)]

Independent particle model



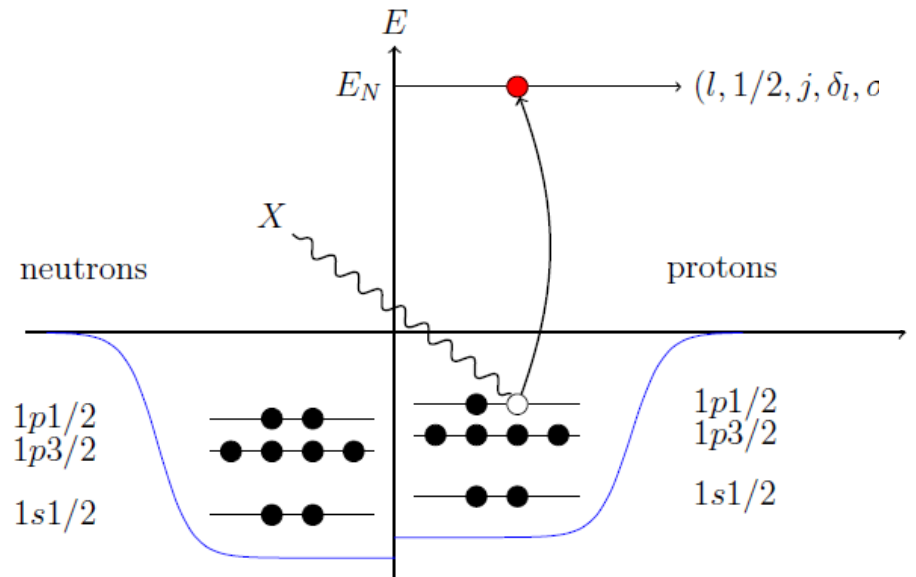
Quantum numbers:
angular momentum
Energy

No momentum eigenstates



[M. Leuschner et al. PRC49, 955 (1994)]

Independent particle model



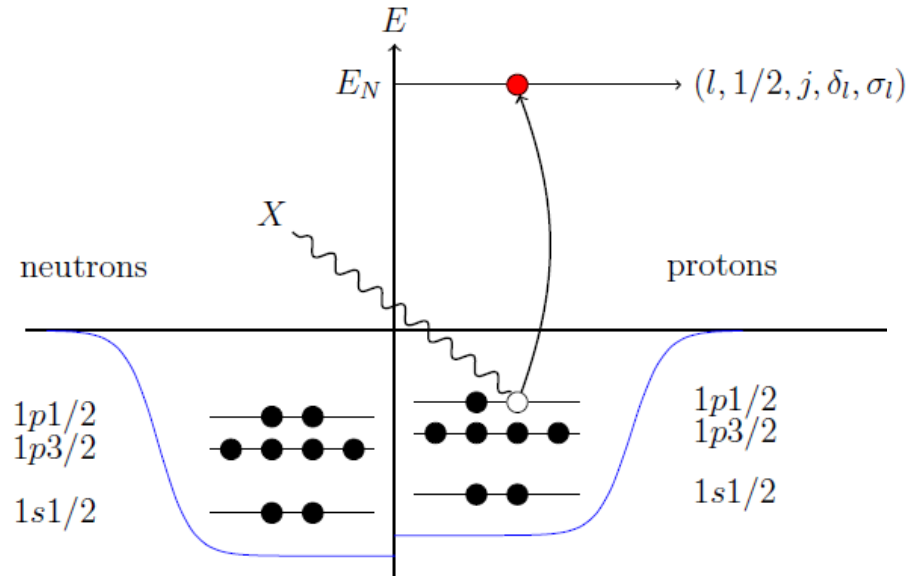
$$J_{\kappa, n}^{\mu}(m_j, s_N, Q, k_N) = \int d\vec{r} e^{i\vec{q}\cdot\vec{r}} \bar{\Psi}(\vec{r}, s_N, k_N) \mathcal{O}^{\mu}(Q) \psi_{\kappa}^{m_j}(\vec{r}).$$

$$[E - V(r)] \Psi(\vec{r}, s_N, k_N) = [\alpha \cdot \hat{\mathbf{p}}(\vec{r}) - \beta(S(\vec{r}) - M_N)] \Psi(\vec{r}, s_N, k_N)$$

$$V_{strong}(r) \rightarrow 0 \quad S_{strong}(r) \rightarrow 0 \quad \text{for } r \rightarrow \infty (\approx 5 \text{ fm})$$

Wave function behaves like a momentum state only for large radius

Relativistic plane-wave impulse approximation



$$J_{\kappa, n}^{\mu}(m_j, s_N, Q, k_N) = \int d\vec{r} e^{i\vec{q}\cdot\vec{r}} \bar{\Psi}(\vec{r}, s_N, k_N) \mathcal{O}^{\mu}(Q) \psi_{\kappa}^{m_j}(\vec{r}).$$

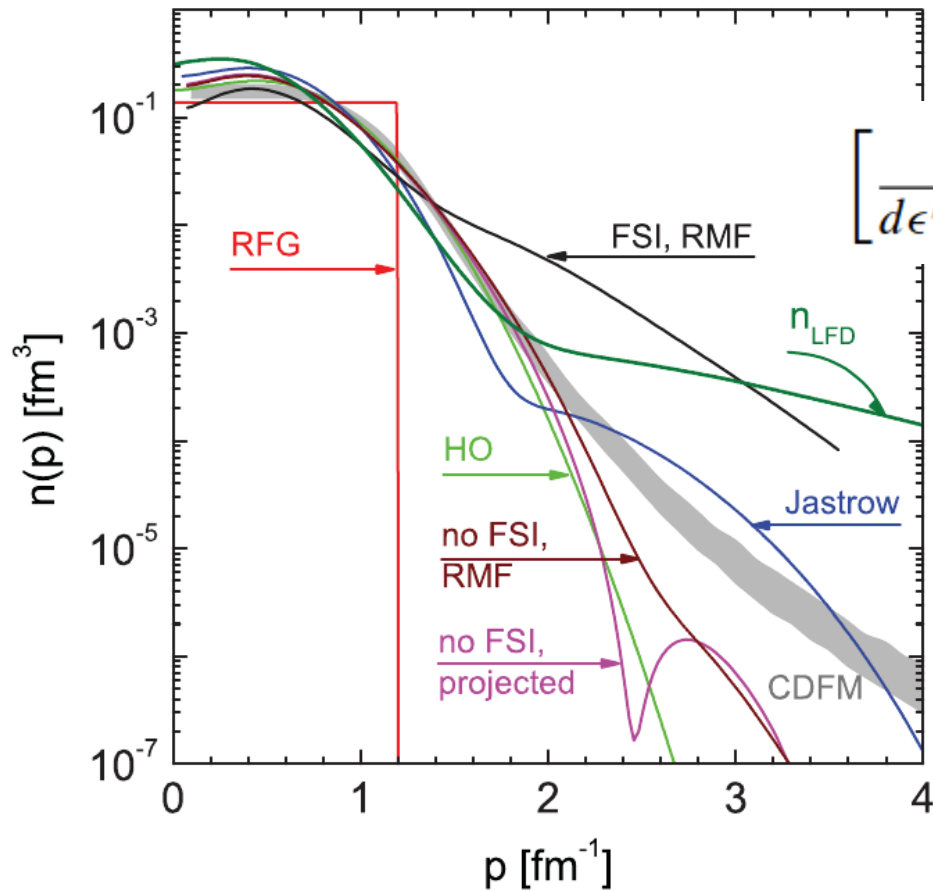
With plane-wave $\bar{\Psi}_{pw}(\vec{r}, k_N, s) = \bar{u}(k_N, s) e^{-i\vec{r}\cdot\vec{k}_N}$

$$J_{n, \kappa}^{\mu} = (2\pi)^{3/2} \bar{u}(k_N, s) \mathcal{O}^{\mu} \phi_{\kappa}^{m_j}(\vec{p}_m)$$

The squared matrix element is 'proportional' to $n(p_m) = \sum_{\kappa, m_j} |\phi_{\kappa}^{m_j}(p_m)|^2$

With distorted-wave : probe a broader region in momentum space

Factorization in PWIA and (R)DWIA



[A. N. Antonov et al. PRC83, 045504]

In the PWIA there is exact factorization

$$\left[\frac{d\sigma}{d\epsilon' d\Omega' dp_N d\Omega_N} \right]_{(e, e' N)}^{\text{PWIA}} = K \sigma^{eN}(q, \omega; p, \mathcal{E}, \phi_N) S(p, \mathcal{E}),$$

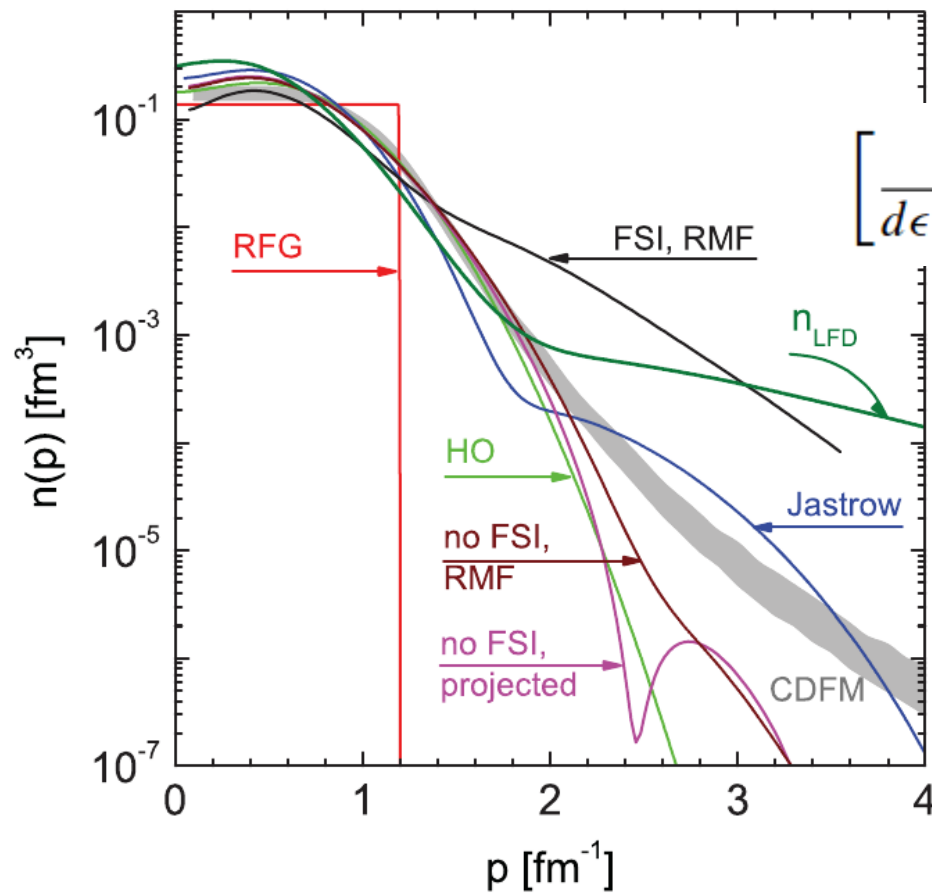
Can define 'distorted' $n(p)$

$$\rho(p) = n^{\text{dist}}(p) = \frac{\left[\frac{d\sigma}{d\Omega' d\epsilon' d\Omega_N} \right]_{\text{FSI}}}{K \sigma^{eN}}.$$

(Depends on $\omega, q, T_N, \cos\theta_N, \phi_N$ in principle)

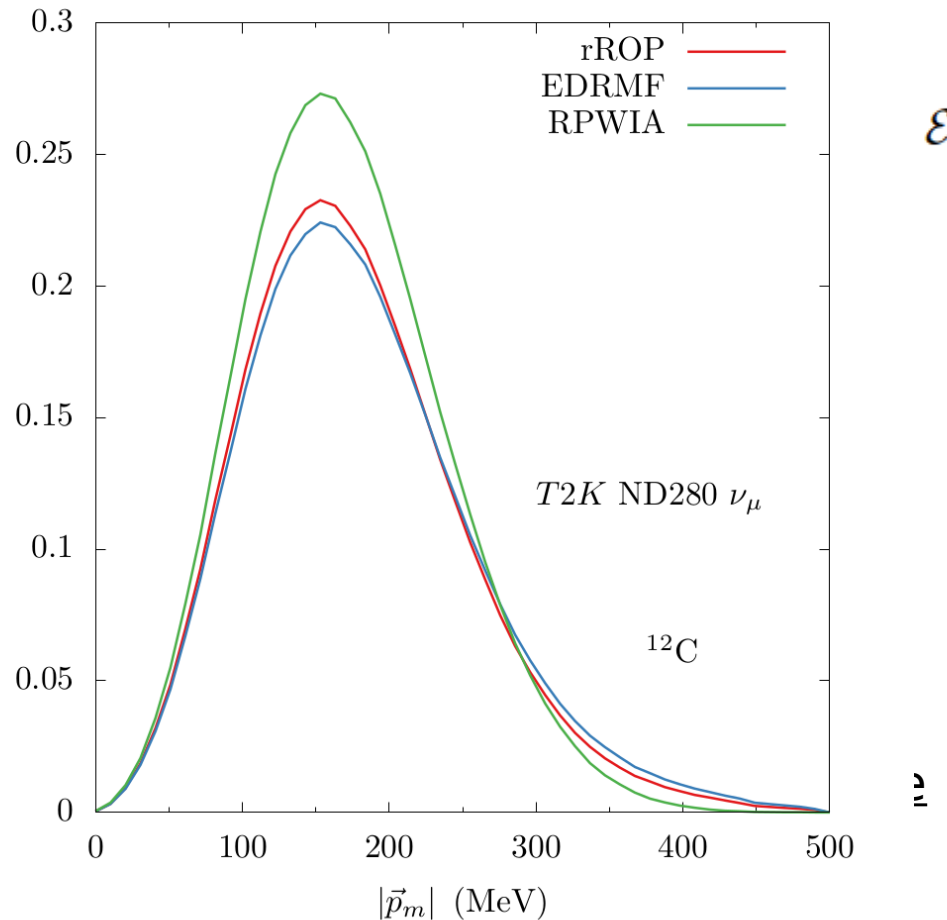
Different models with same initial state

Factorization in PWIA and (R)DWIA



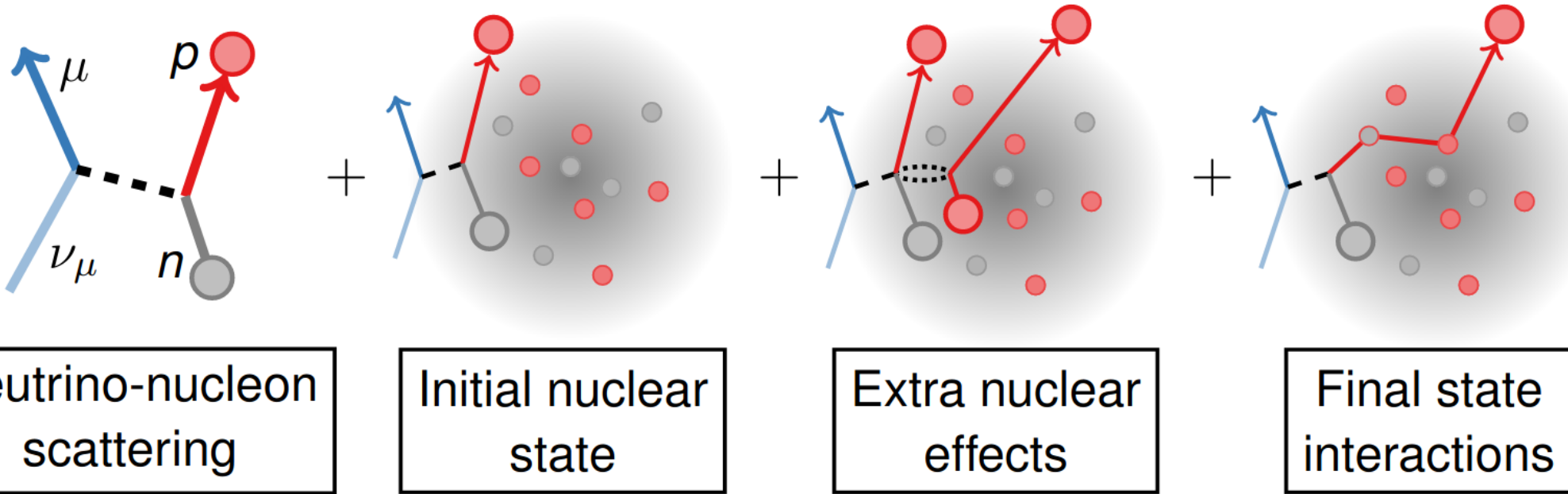
[A. N. Antonov et al. PRC83, 045504]

Missing momentum distribution for T2K γ



Different models with same initial state

From inclusive to exclusive



[Fig. from K. Niewczas]

Input to the generator here:

$$\frac{d\sigma(E_\nu)}{dE_l d\cos\theta_l} = G^2 \frac{k_l}{E_\nu} L_{\mu\nu} \int d\Omega_N \sum_{n,\kappa} H_{n,\kappa}^{\mu\nu}(\omega, q, \Omega_N, E_{n,\kappa})$$

$$G^2 \frac{k_l}{E_\nu} \left[L_{00} \tilde{H}^{00} - 2L_{03} \tilde{H}^{03} + L_{33} \tilde{H}^{33} + \frac{L_{11} + L_{22}}{2} (\tilde{H}^{11} + \tilde{H}^{22}) + 2L_{12} \text{Im} \tilde{H}^{12} \right]$$

As function of ω, q

Nucleon variables in GENIE

Input to the generator is inclusive cross section:

$$\frac{d\sigma(E_\nu)}{dE_l d\cos\theta_l} = G^2 \frac{k_l}{E_\nu} L_{\mu\nu} \int d\Omega_N \sum_{n,\kappa} H_{n,\kappa}^{\mu\nu}(\omega, q, \Omega_N, E_{n,\kappa})$$

Lost nucleon information → Need to generate it in GENIE

1. Draw initial nucleon \mathbf{p}_m from $p^2 n(p)$ (e.g. LFG)

!! 2. Compute $E_m^2 = \mathbf{p}_m^2 + M_N^2$

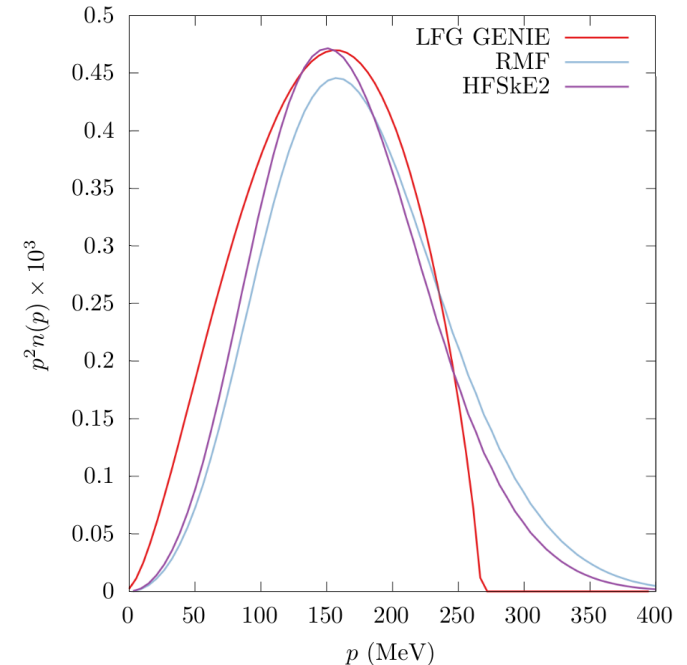
3. $E_N = E_m + \omega - E_b(q)$

4. $k_N^2 = E_N^2 - M_N^2$

!! $|\mathbf{p}_m + \mathbf{q}| \neq k_N = \sqrt{E_N^2 - M_N^2}$

→ $\mathbf{k}_N = \frac{k_N}{|\mathbf{p}_m + \mathbf{q}|} (\mathbf{p}_m + \mathbf{q})$

5. Give residual momentum to remnant

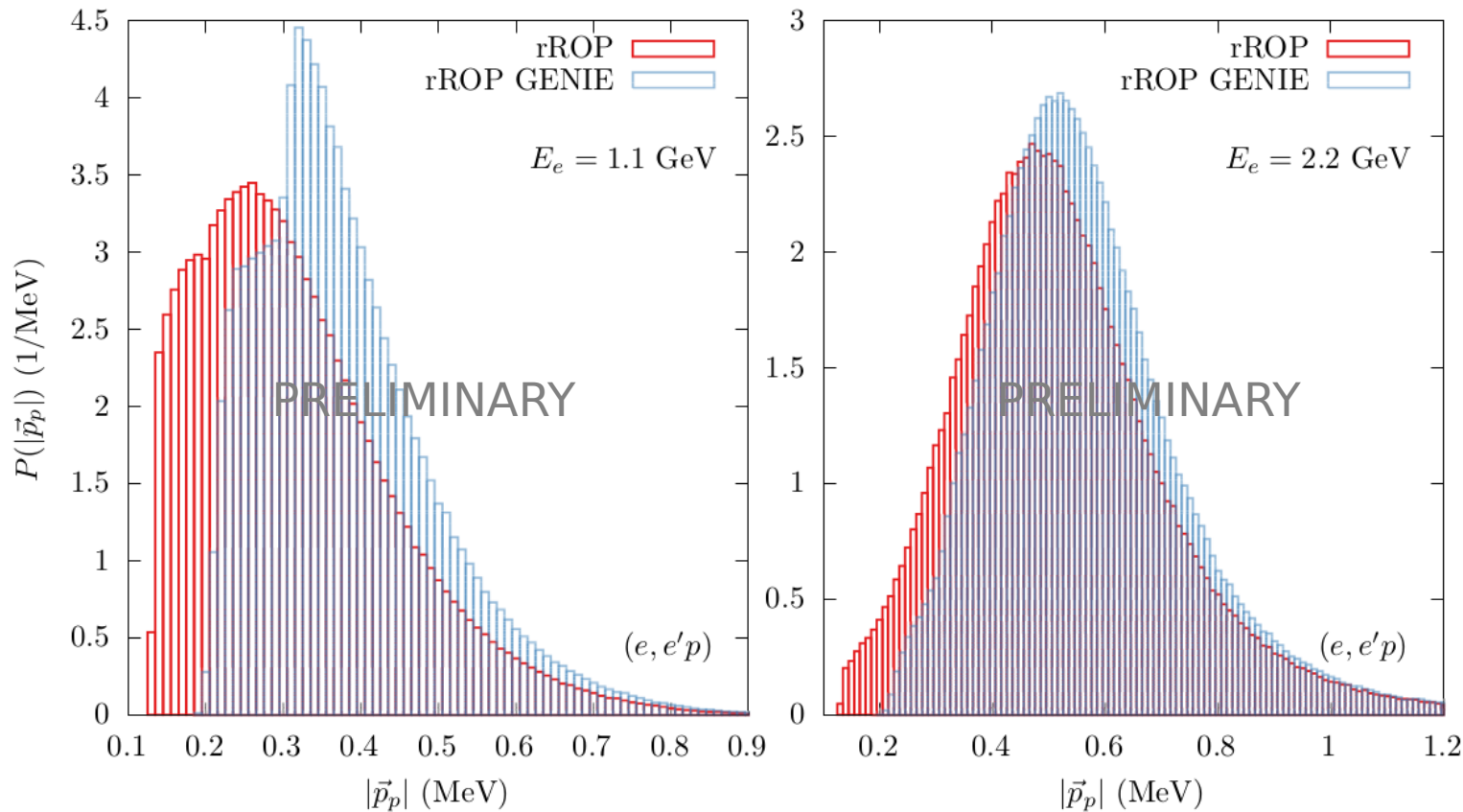


Nucleon variables in GENIE and RDWIA

Input to the generator is inclusive cross section:

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Same inclusive cross sections different nucleon observables



(e, e'p) at fixed
Incoming energy

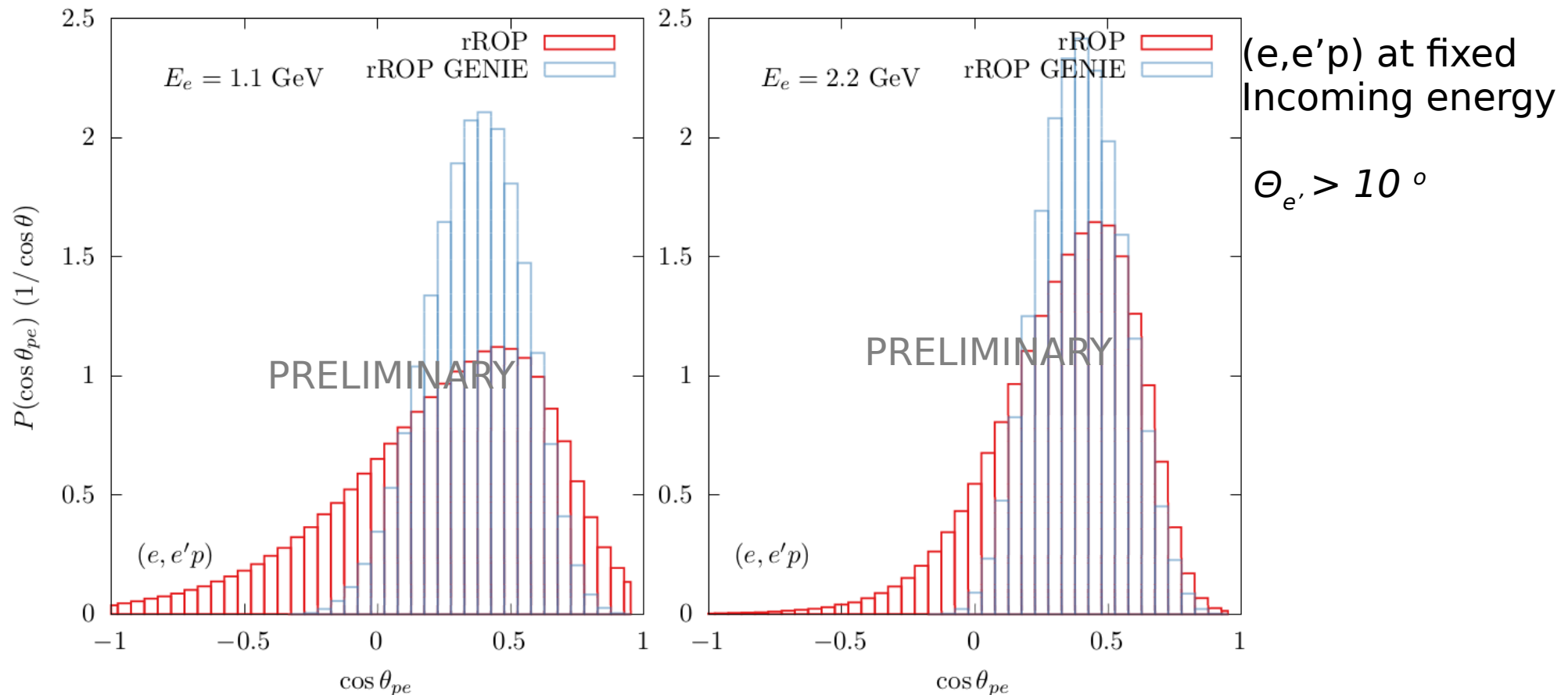
$$\Theta_{e'} > 10^\circ$$

Nucleon variables in GENIE and RDWIA

Input to the generator is inclusive cross section:

$$\frac{d\sigma(E_\nu)}{dE_l d\cos\theta_l} = G^2 \frac{k_l}{E_\nu} L_{\mu\nu} \int d\Omega_N \sum_{n,\kappa} H_{n,\kappa}^{\mu\nu}(\omega, q, \Omega_N, E_{n,\kappa})$$

Same inclusive cross sections different nucleon observables



Benchmarking intra-nuclear cascade models for neutrino scattering with relativistic optical potentials.

A. Nikolakopoulos,^{1,2,*} R. González-Jiménez,³ N. Jachowicz,¹ K. Niewczas,^{1,4} F. Sánchez,⁵ and J. M. Udías³

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³*Grupo de Física Nuclear, Departamento de Estructura de la Materia,
Física Térmica y Electrónica, Facultad de Ciencias Físicas,*

Universidad Complutense de Madrid and IPARCOS, CEI Moncloa, Madrid 28040, Spain

⁴*University of Wrocław, Institute of Theoretical Physics, Plac Maza Borna 9, 50-204 Wrocław, Poland*

⁵*University of Geneva, Section de Physique, DPNC, Geneva, Switzerland*

arXiv:2202.01689v1 [nucl-th] 3 Feb 2022

A direct comparison of RDWIA with relativistic optical potential (ROP) with (NEUT) intra-nuclear cascade (INC) model

- 1.) Differences and similarities between RDWIA and INC
- 2.) Consistent input from the RDWIA for the INC
- 3.) Event selection to compare ROP and INC
- 4.) The actual comparison

Empirical relativistic optical potential (ROP)

PHYSICAL REVIEW C

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Global Dirac phenomenology for proton-nucleus elastic scattering

E. D. Cooper, S. Hama, and B. C. Clark

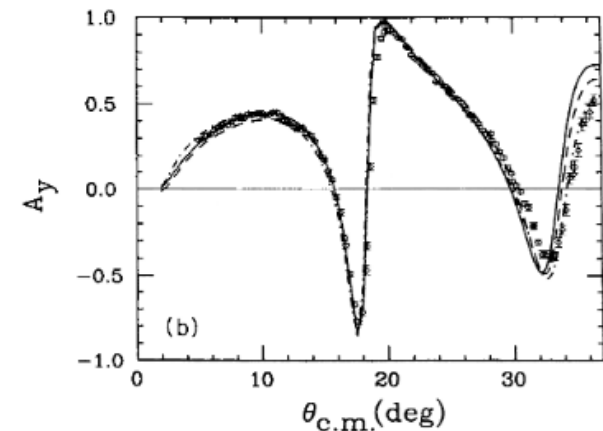
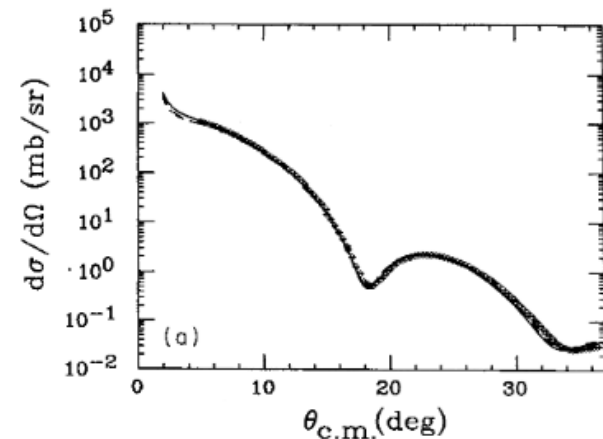
Department of Physics, The Ohio State University, Columbus, Ohio 43210

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(Received 31 August 1992)

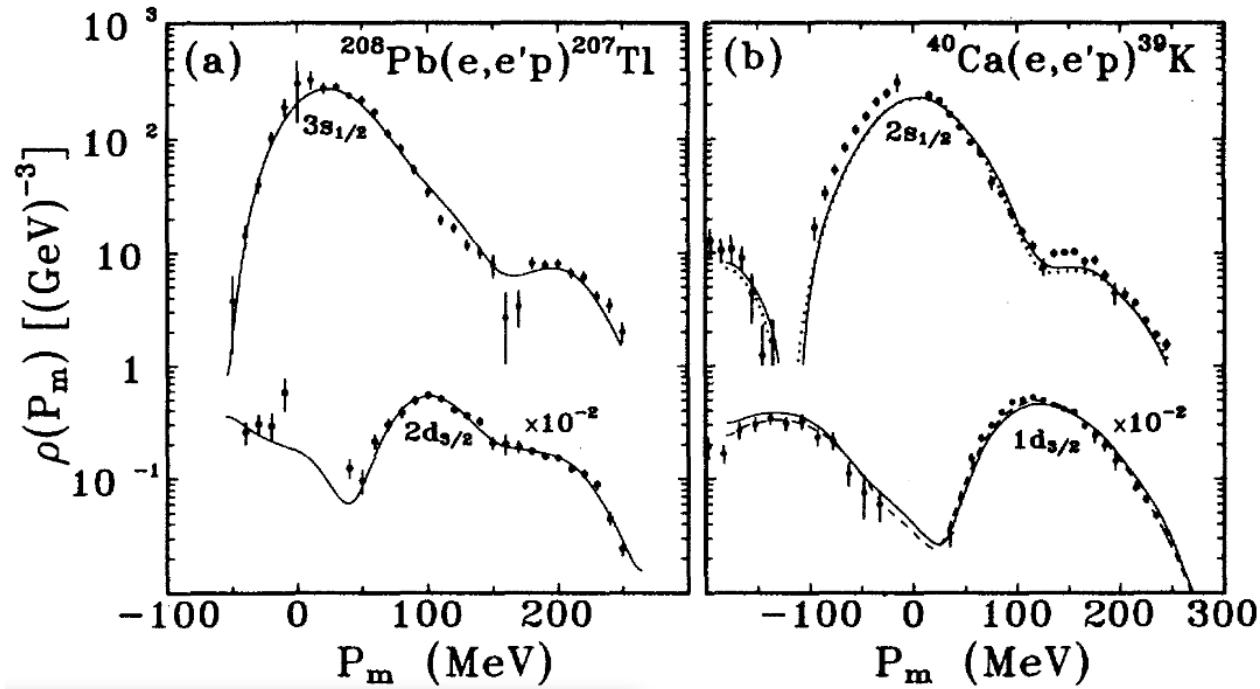
Target	T_p (MeV)	σ_R (mb)				Reference
		EDAI-fit	EDAD-fit			
			fit 1	fit 2	fit 3	
^{12}C	29.00	420.2	435.5	433.1	422.7	[6]
	30.30	415.9	429.0	425.6	414.2	[7]
	49.00	358.8	363.0	348.4	327.7	[6]
	49.48	357.4	361.8	347.0	326.1	[8]
	61.40	323.3	335.6	317.0	294.8	[9]
	65.00	313.5	329.0	309.7	287.4	[10]
	122.00	202.2	269.0	254.4	230.5	[11]
	160.00	177.8	252.3	246.4	215.2	[11]
	200.00	177.6	243.0	243.9	205.0	[11-13]
	300.00	201.1	233.0	235.4	194.9	[14]
	398.00	215.8	227.4	218.6	199.1	[15]
	494.00	227.2	223.7	203.0	211.6	[16]
	797.50	238.4	235.3	209.9	250.0	[17,18]
	1040.00	198.6	259.4	243.8	232.2	[19,20]



Nucleus-specific and nucleus-dependent Potentials fit to elastic proton-nucleus scattering

RDWIA with ROP for exclusive (e,e'p)

[Udias et al. PRC48, 2731]



[Meucci et al. PRC64, 014604]

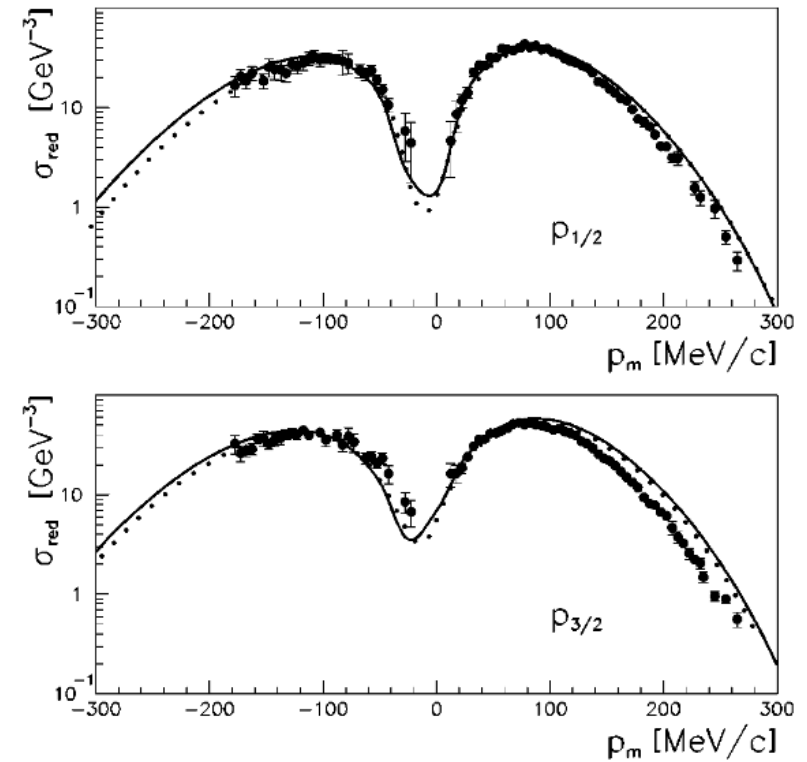
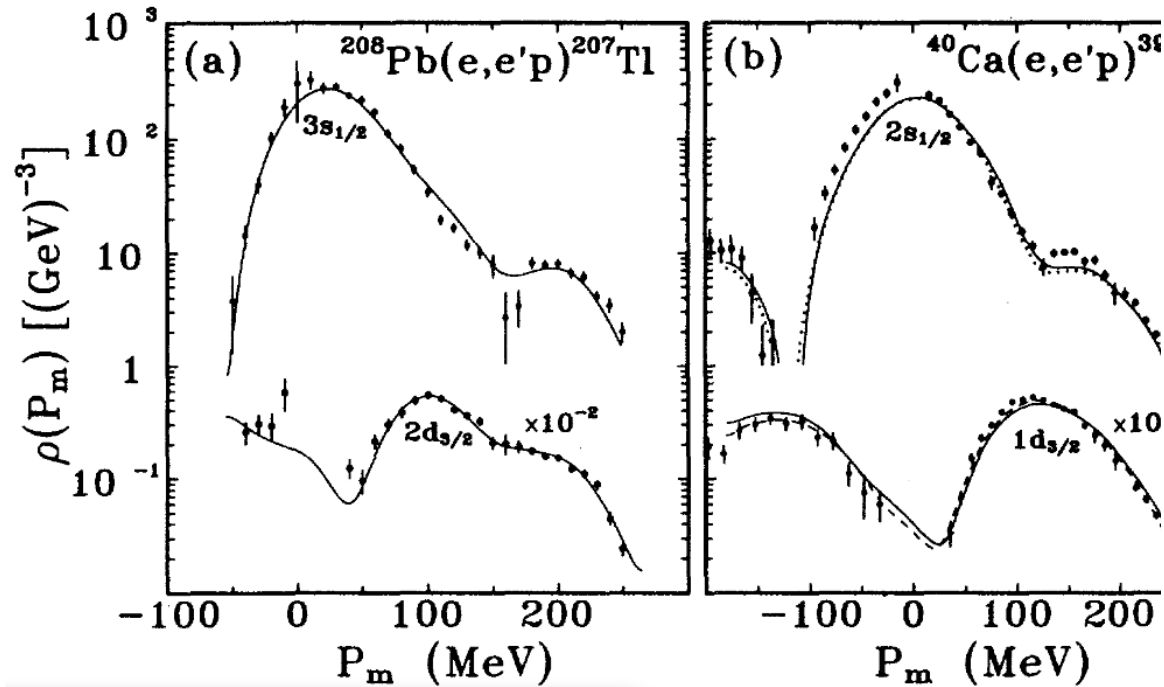


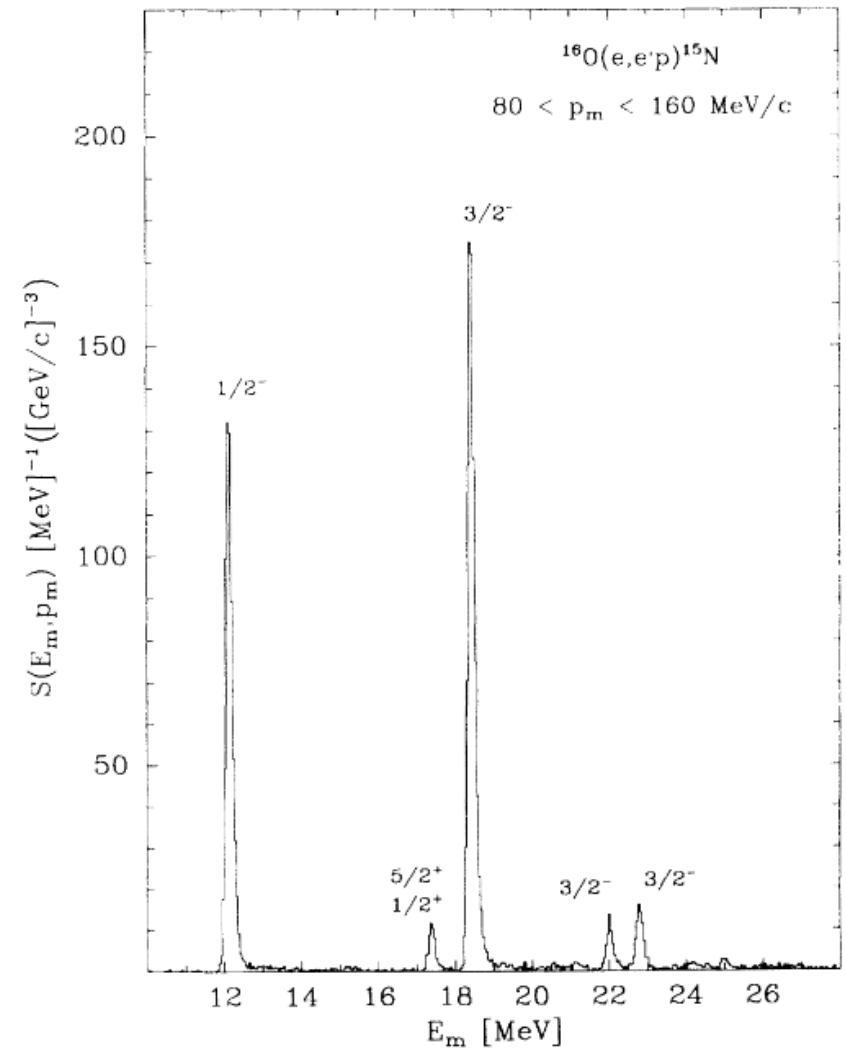
FIG. 11. The reduced cross section (σ_{red}) of the $^{16}\text{O}(e,e'p)$ reaction as a function of the recoil momentum p_m for the transitions to the $1/2^-$ ground state and to the $3/2^-$ excited state of ^{15}N , in

RDWIA with ROP for exclusive (e,e'p)

[Udias et al. PRC48, 2731]



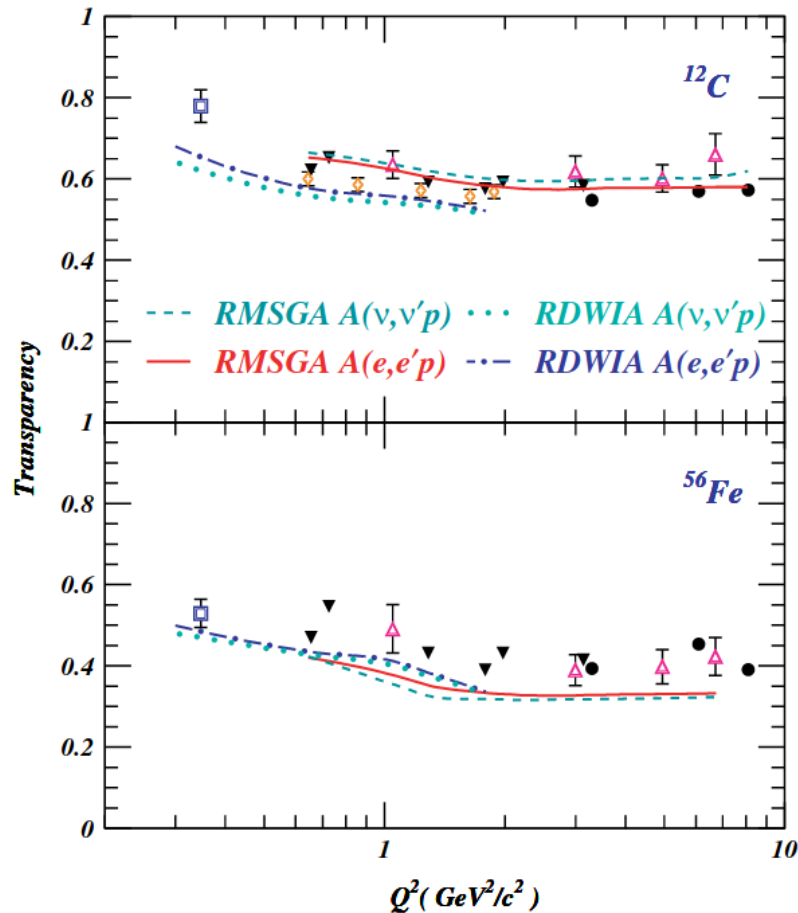
Imaginary part of the potential removes
 Strength lost in inelastic FSI
 i.e. FSI which changes E_m considerably



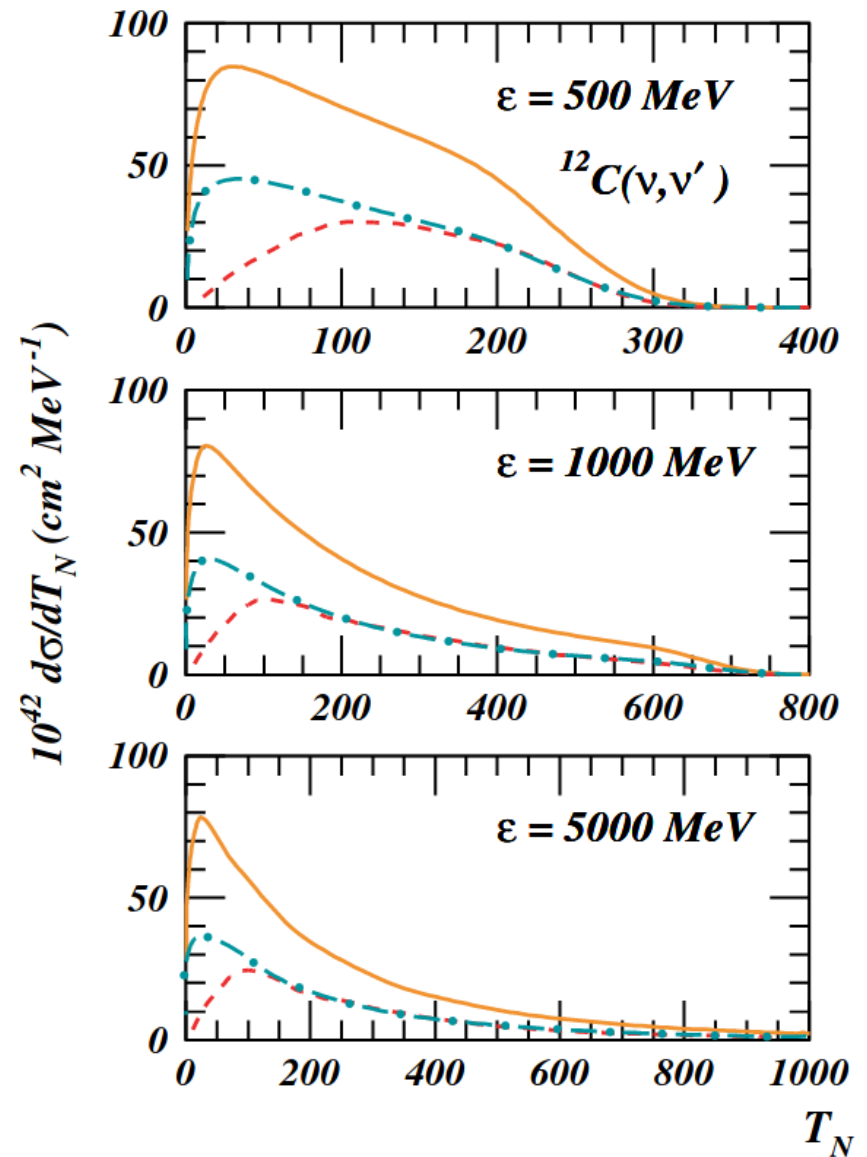
[M. Leuschner et al. PRC49, 955 (1994)]

RDWIA with ROP for exclusive (e,e'p)

[M.C. Martinez et al. PRC73 024607]



Imaginary part of the potential removes
Strength lost in inelastic FSI
i.e. FSI which changes E_m considerably



NEUT Cascade model

- 1.) Nucleon propagates in straight lines with step of 0.2 fm
- 2.) Check for interaction based on density and in-medium nucleon-nucleon CS
- 3.) Pauli-blocking: Reaction products must be above p_{Fermi}
- 4.) Track created particles on the way Eur. Phys. J. Spec. Top. (2021) 230:4469–4481

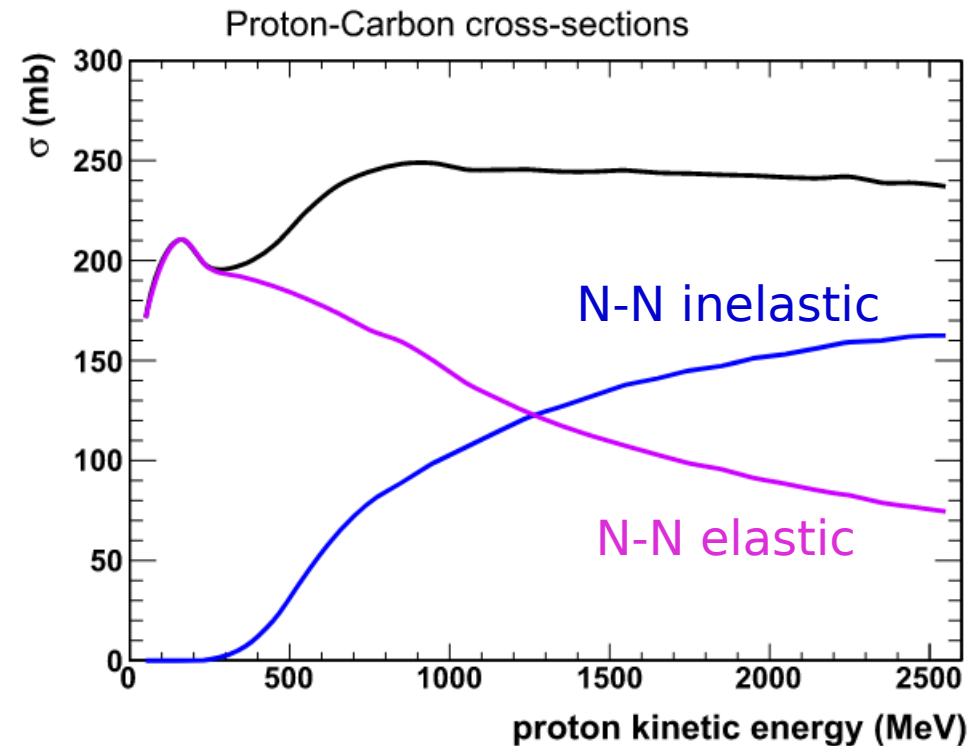
Differences with ROP:

Explicit description of reaction products

No elastic FSI

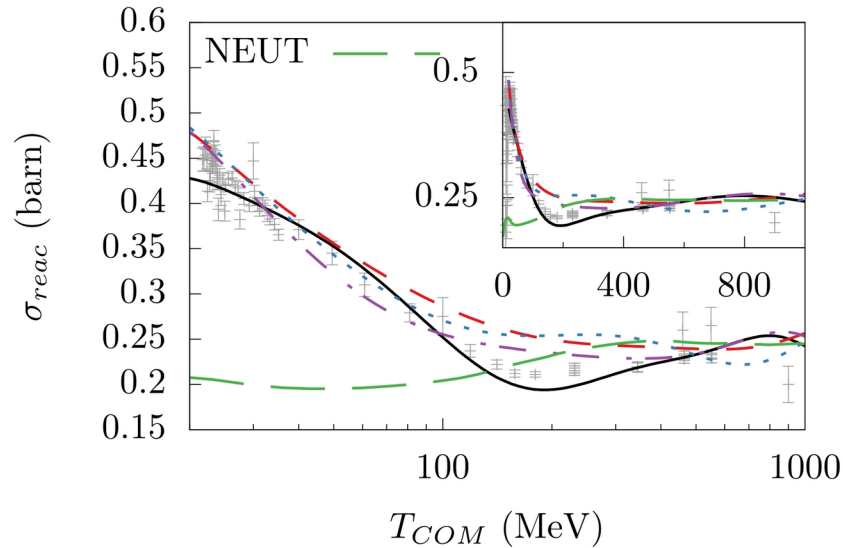
Interactions only with constituent nucleons

Cascade does not affect inclusive CS

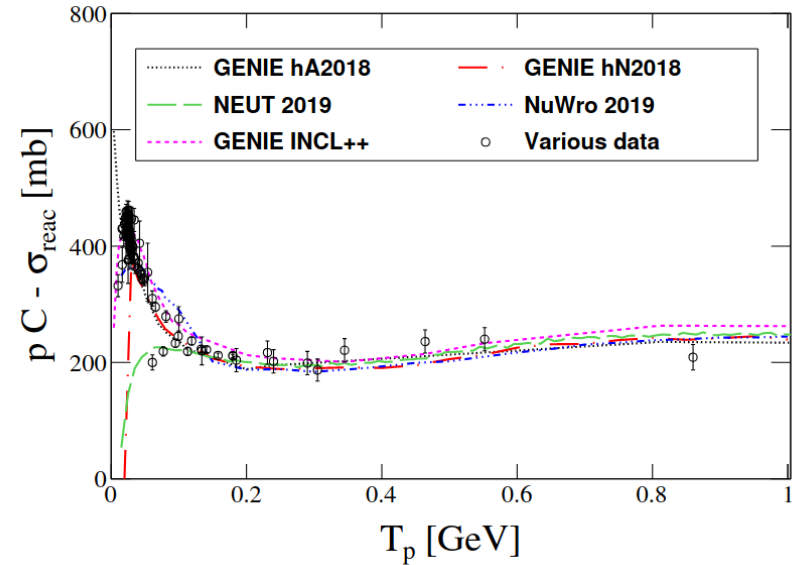


Nucleon - nucleus scattering

Used as benchmark (and/or input) to INC

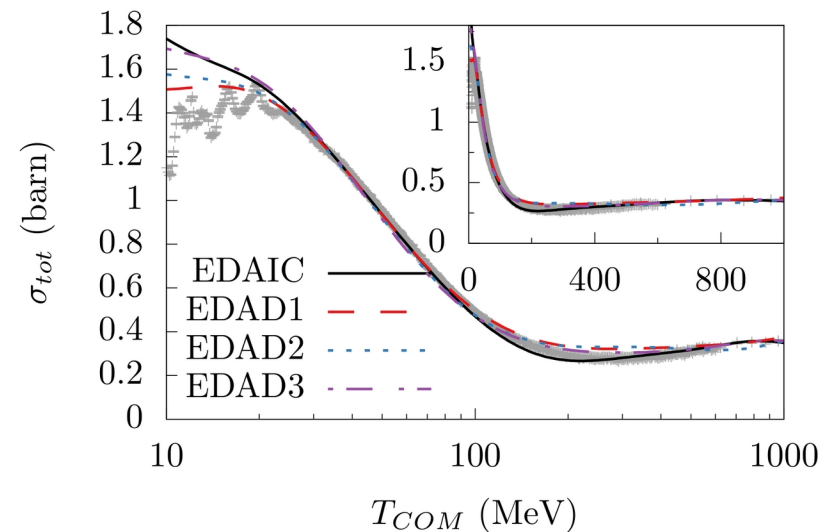


[Dytman et al. PRD104, 053006]



$$\sigma_{tot} = \sigma_{reac} + \sigma_{el}$$

In NEUT σ_{el} is not modeled



Avoiding incl-excl mismatch: rROP input

Generated unfactorized RDWIA events as input to cascade

Events are 1p 1 μ with T2K flux
Distributed according to \rightarrow

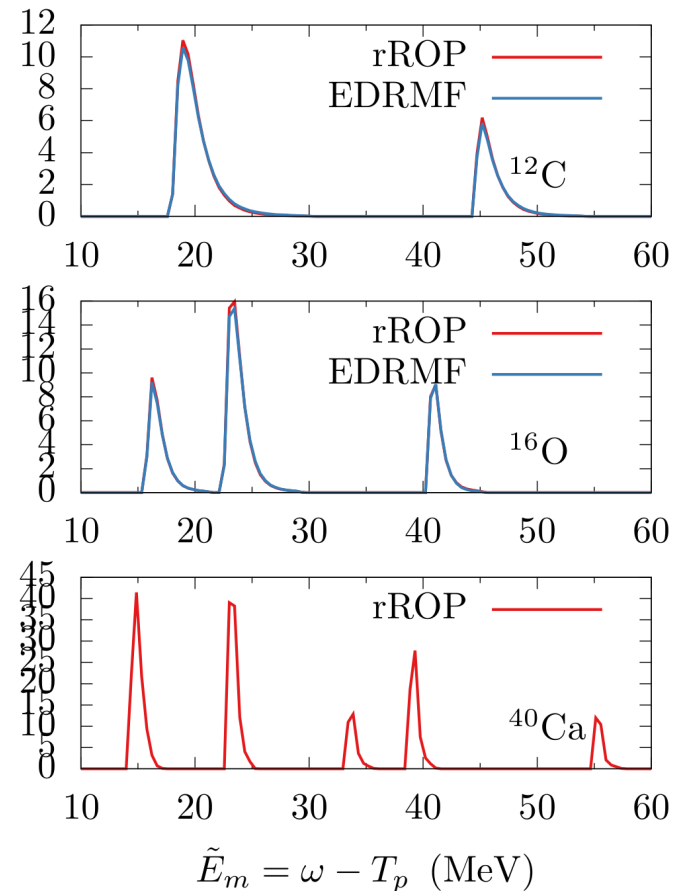
$$\left\langle \frac{d^6\sigma}{dk_l d\Omega_l dp_N d\Omega_N} \right\rangle = \int dE_m \phi(E)$$
$$\times \mathcal{F} \frac{k_l^2 p_N^2 M_B^*}{(2\pi)^5 E_B f_{rec}} \ell_{\mu\nu} H^{\mu\nu},$$

Avoiding incl-excl mismatch: rROP input

Generated unfactorized RDWIA events as input to cascade

Events are 1p 1 μ with T2K flux

Single-particle states for GS
are included \rightarrow



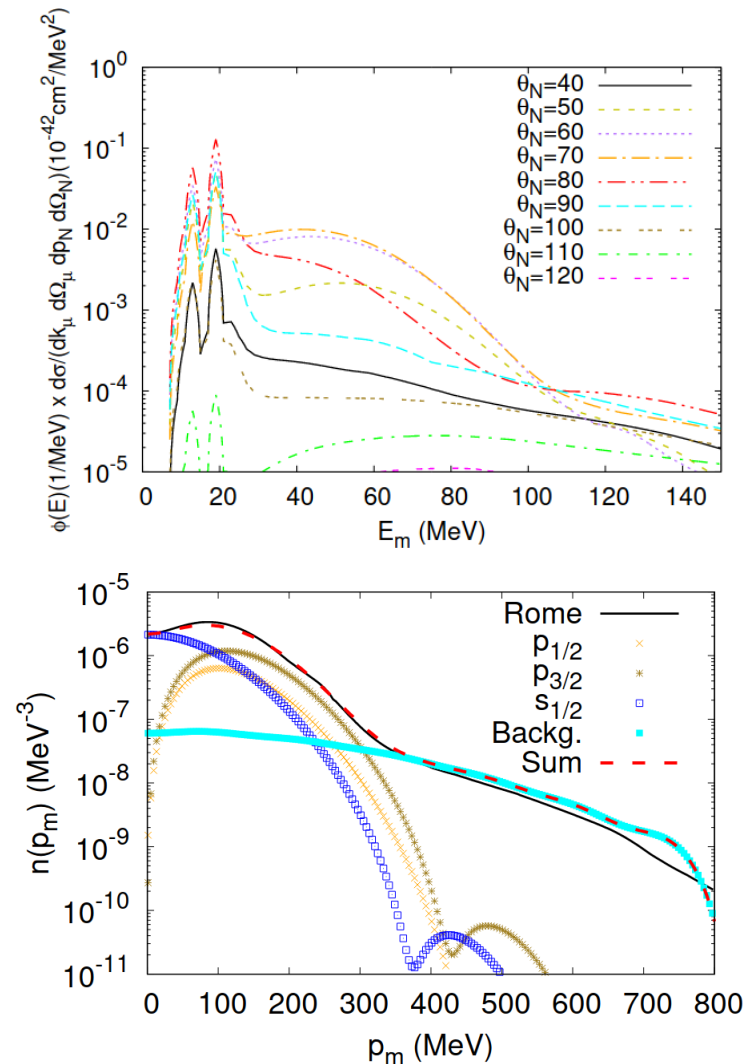
Avoiding incl-excl mismatch: rROP input

Generated unfactorized RDWIA events as input to cascade

Events are 1p 1 μ with T2K flux

Single-particle states for GS

See [RGJ et al. PRC105, 025502] \rightarrow
For inclusion of effective SF



Avoiding incl-excl mismatch: rROP input

Generated unfactorized RDWIA events as input to cascade

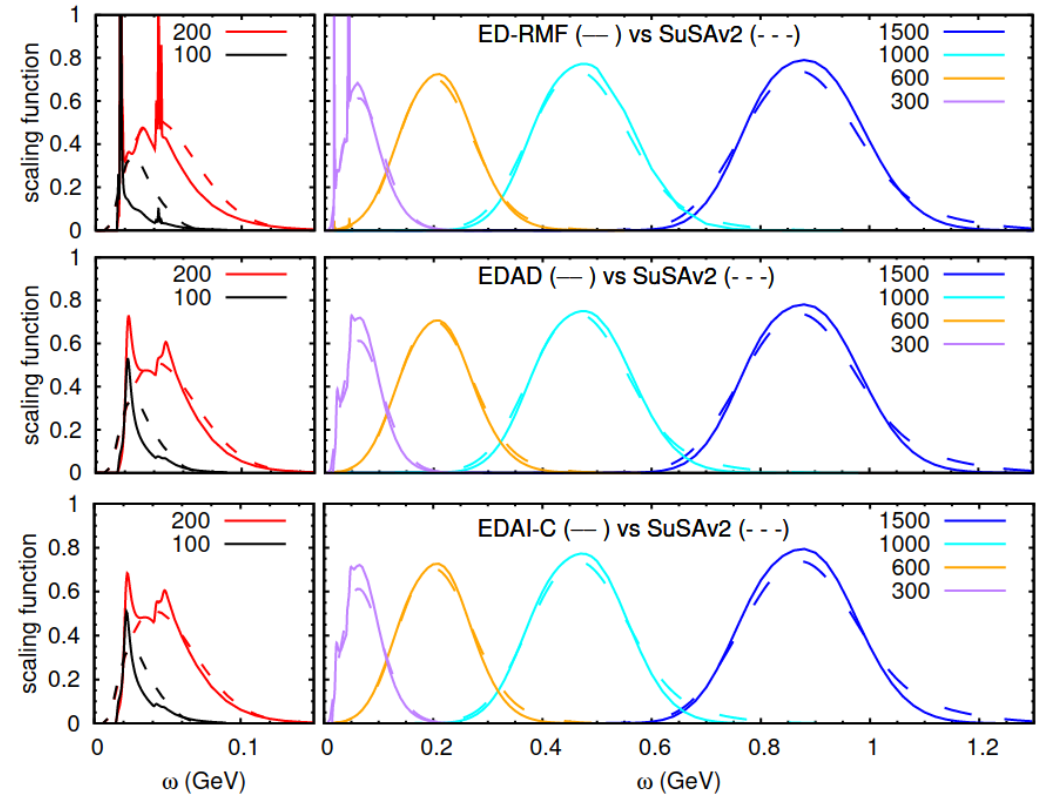
Events are 1p 1 μ with T2K flux

Single-particle states for GS

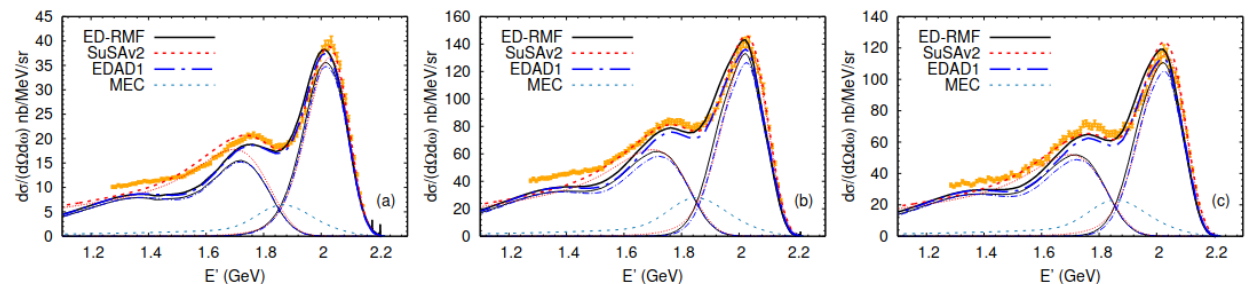
Motivation:

Realistic inclusive cross section is retained with cascade

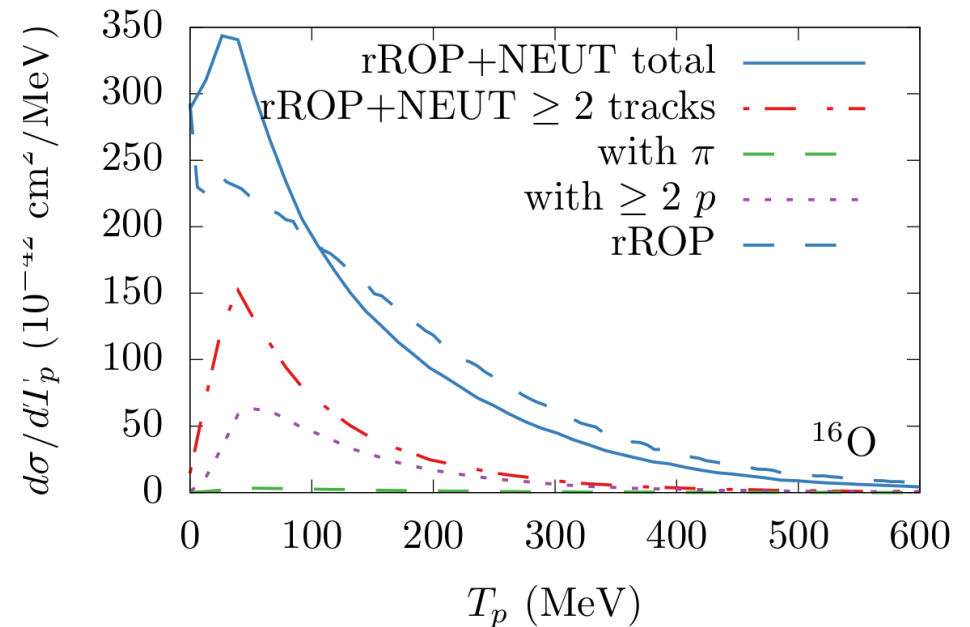
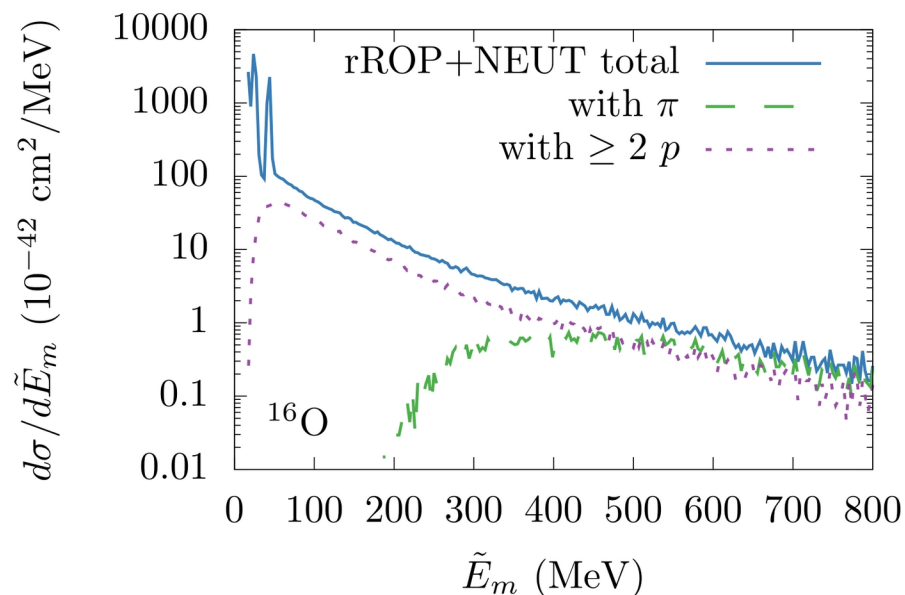
rROP modifies dispersion relation in nucleus \rightarrow not included in cascade 'elastic FSI'



[RG] et al. PRC101, 015503]



NEUT Cascade with rROP input



Simplified missing energy:

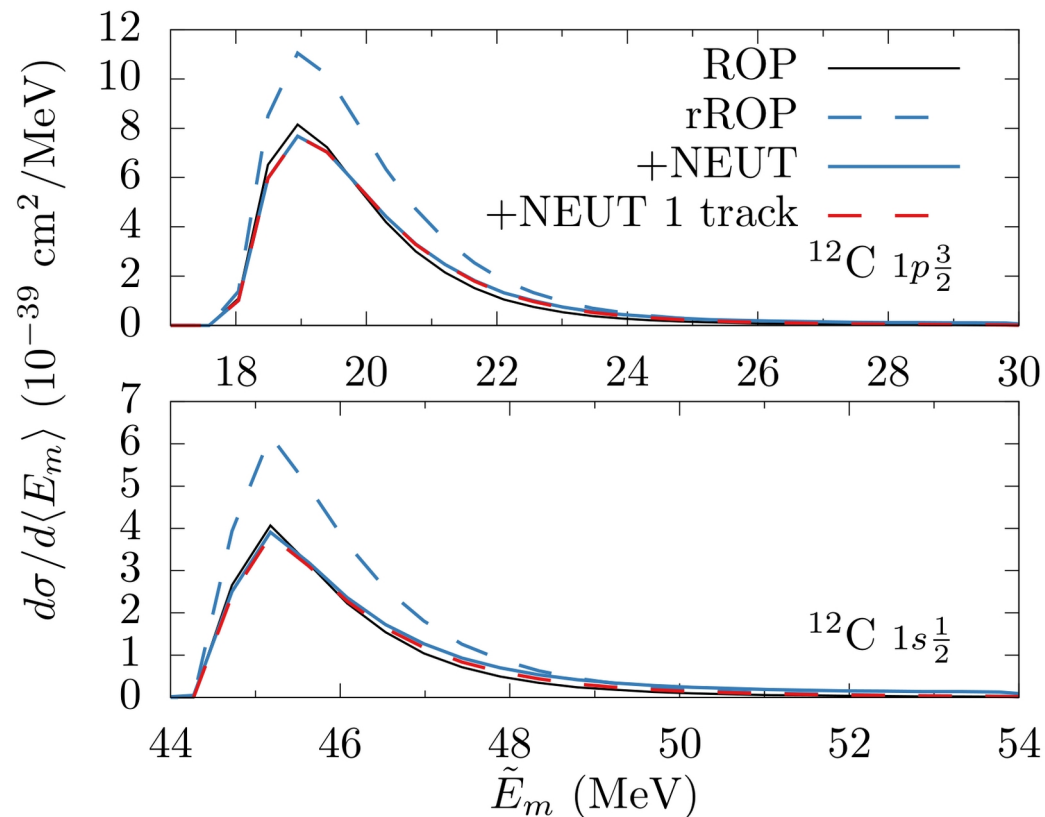
$$\tilde{E}_m = E_i - E_l - T_p$$

T_p is leading proton

RDWIA with ROP removes inelastic FSI from signal

→ Need to remove it from NEUT output

Selecting 'elastic' events



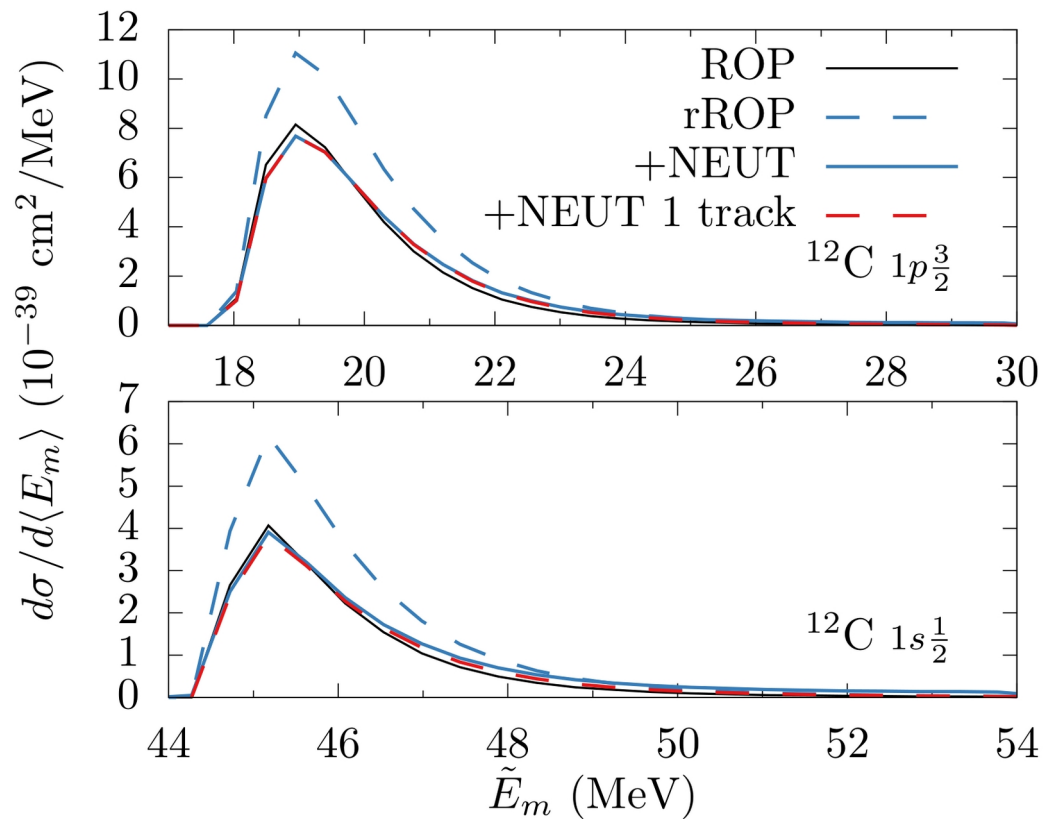
In NEUT any interaction will produce additional particle tracks

→ 1 track events 'nothing happens'

→ Is equivalent to selecting missing energy from the shell-model region

A cut on E_m makes NEUT and ROP comparable

Selecting 'elastic' events

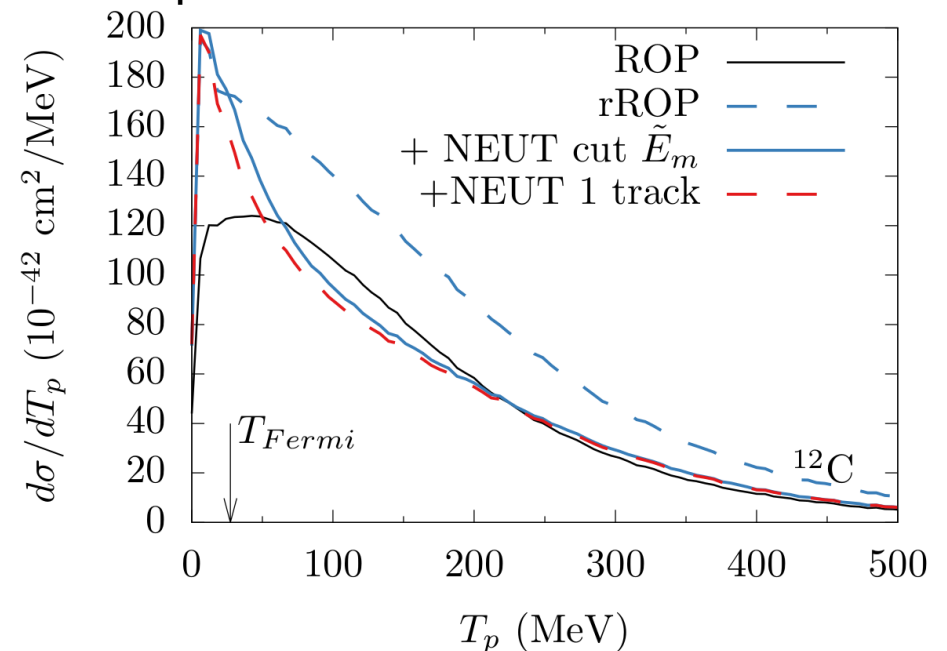


In NEUT any interaction will produce additional particle tracks

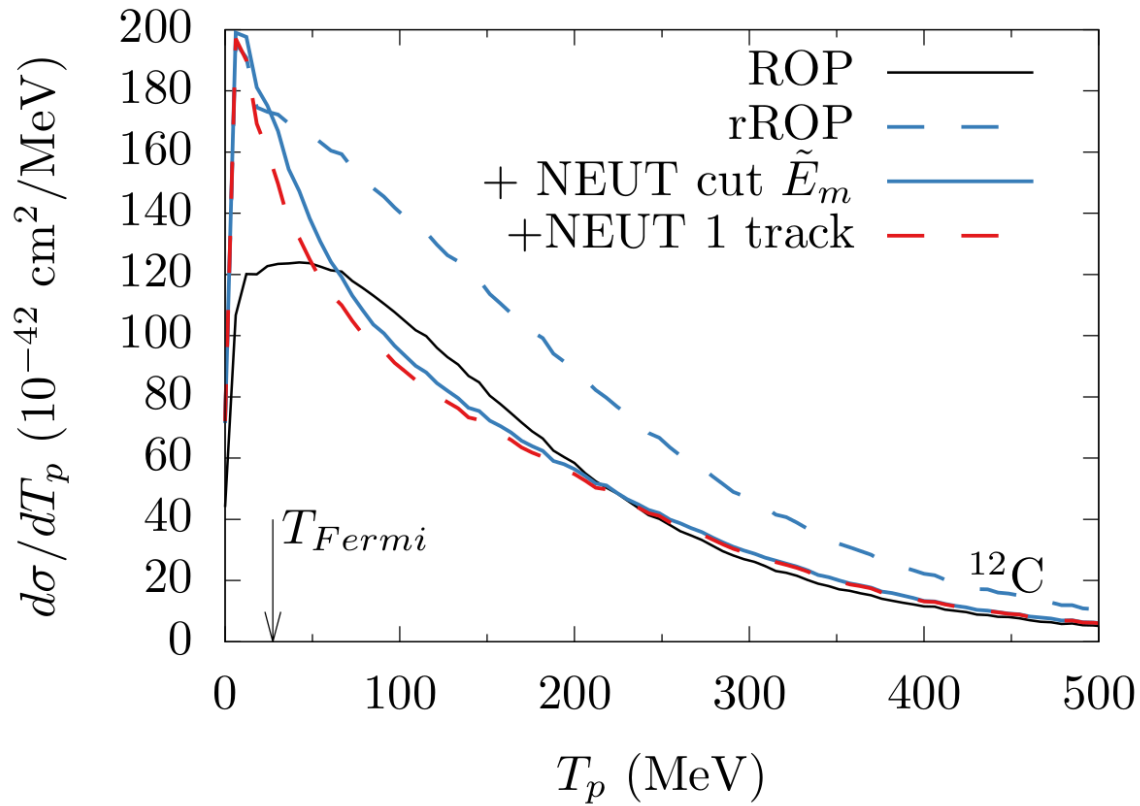
→ 1 track events 'nothing happens'

→ Is equivalent to selecting missing energy from the shell-model region

A cut on E_m makes NEUT and ROP comparable



rROP+NEUT and ROP for carbon

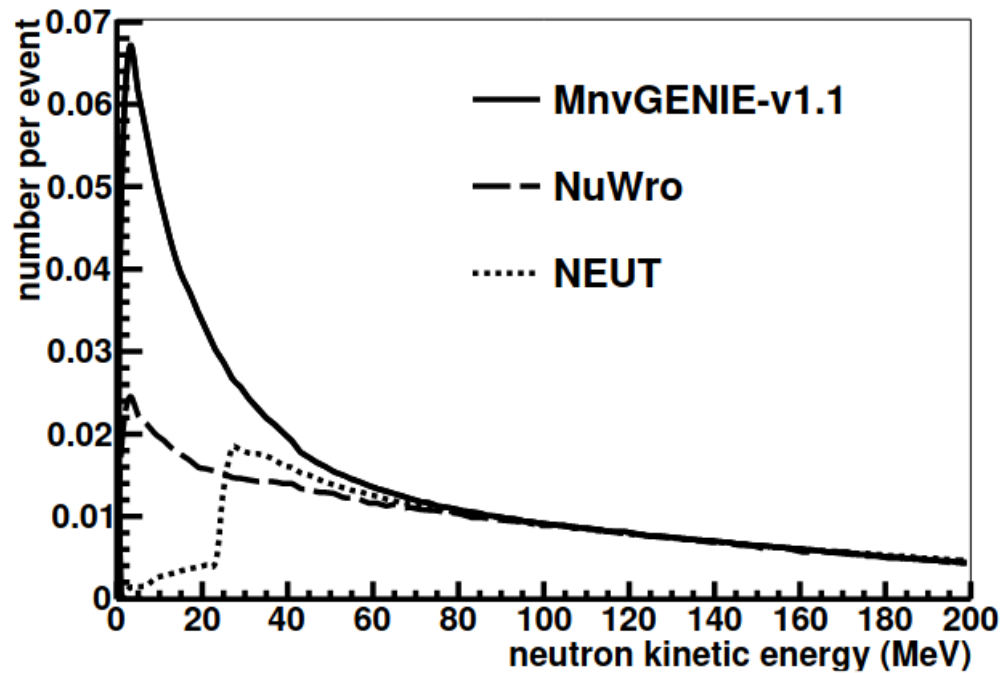


Agreement for T_p large
($T_p > 150$ MeV)

Disagreement at small T_p

Below (local) T_F : Pauli-blocking
The cascade lets all nucleons
escape without interaction

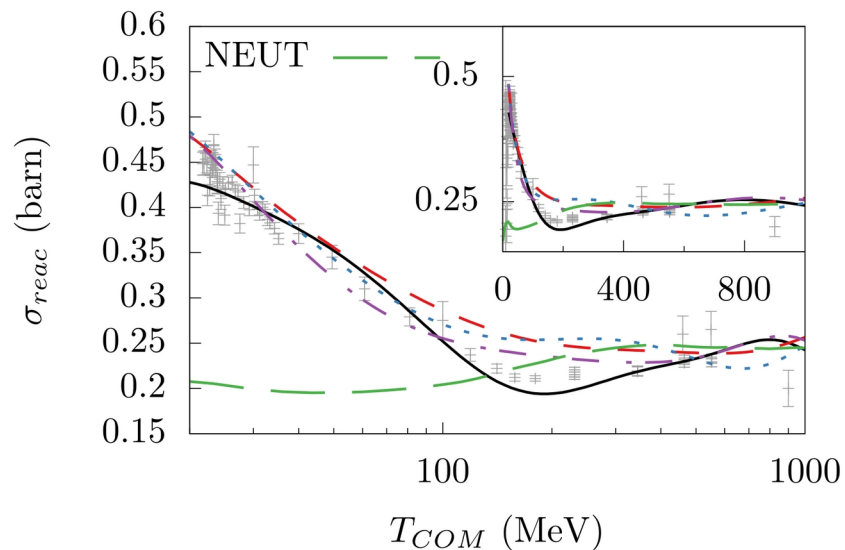
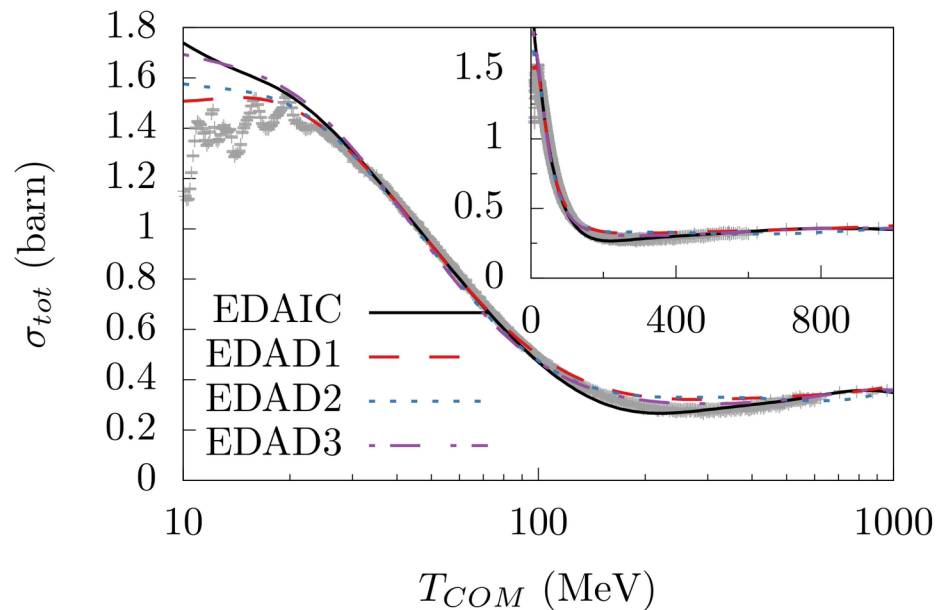
rROP+NEUT and ROP for carbon



Important:
E.g. large differences in
produced neutrons at low T_n

[MINERvA PRD100, 052002]

rROP+NEUT and ROP for carbon

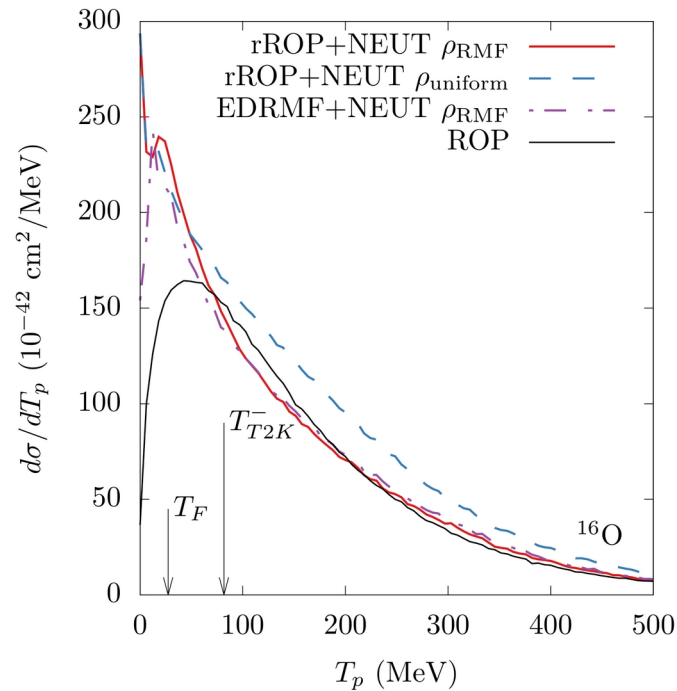
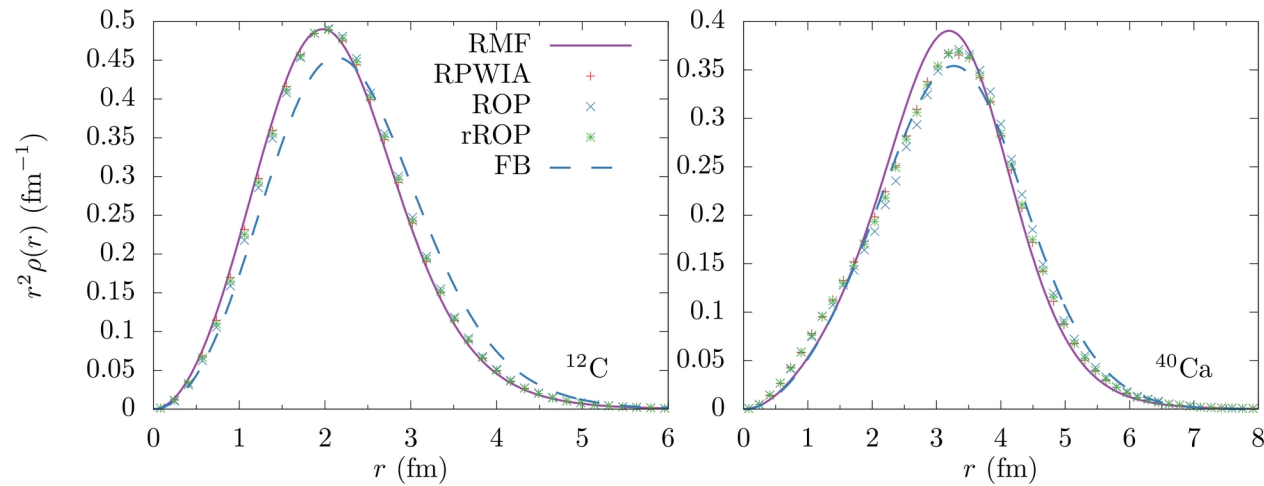


Important:
E.g. large differences in
produced neutrons at low T_n

At small energy optical model
'breaks down'

Should be more suitable than
INC

Dependence on radius

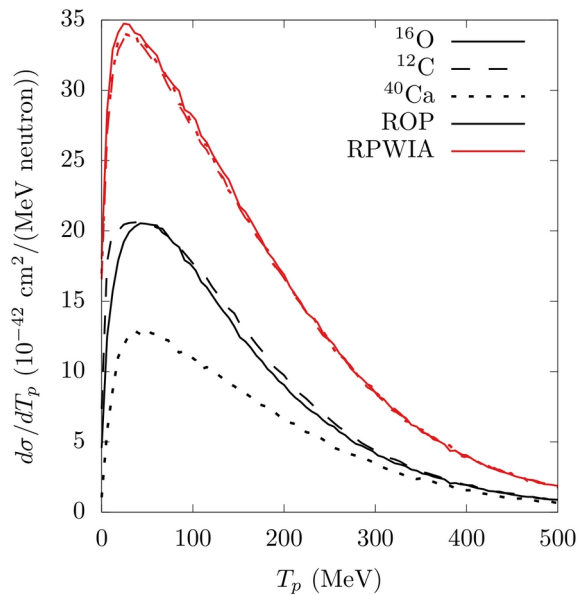


Introduce events at CS-weighted density

→ Resulting distribution similar to GS density

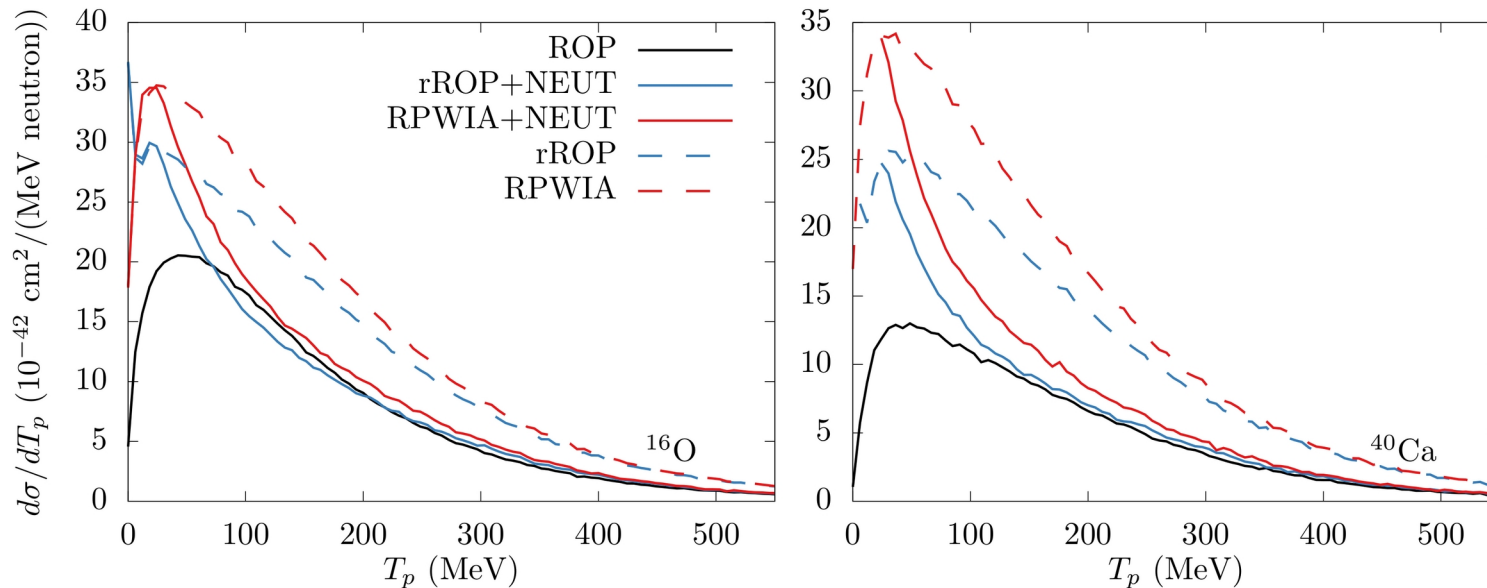
→ Agreement between ROP and INC is lost when uniform spherical density is used

A-dependence



RPWIA cross section scales per active nucleon
 ROP cross section does not (transparency decreases with A)

NEUT results similar for C, O, and Ca
 Slightly better for rROP compared to RPWIA

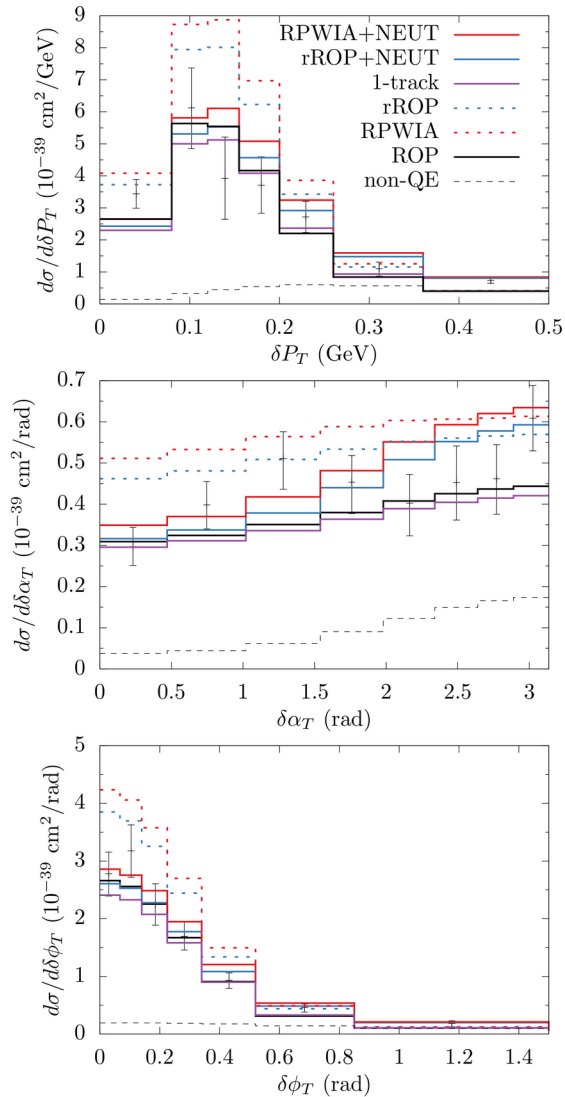


Transverse Kinematic Imbalance

Because $p_p > 450$ MeV low energy differences are not seen

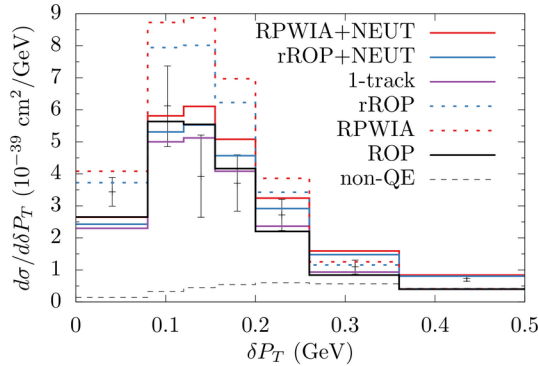
Effect of non-elastic FSI visible in P_T and α_T

Large non-QE 'background' not separable from FSI effects



Non-QE from [Bourguille et al. JHEP04(2021)004]

Transverse Kinematic Imbalance



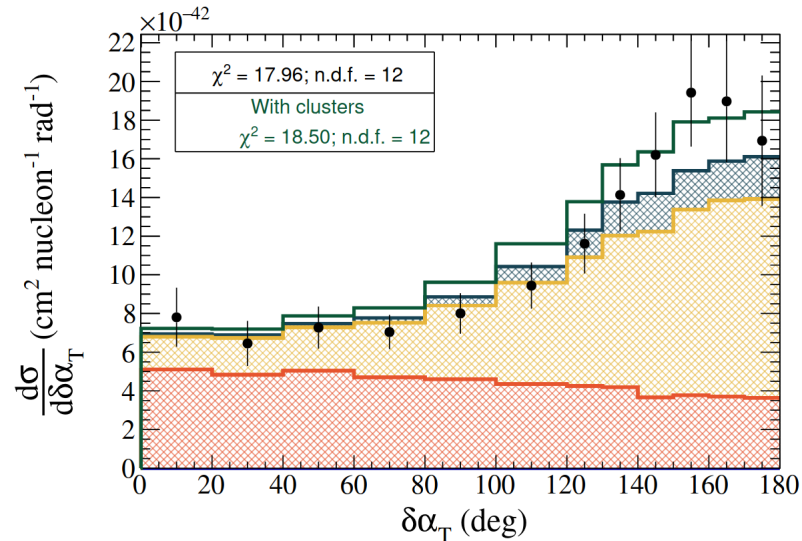
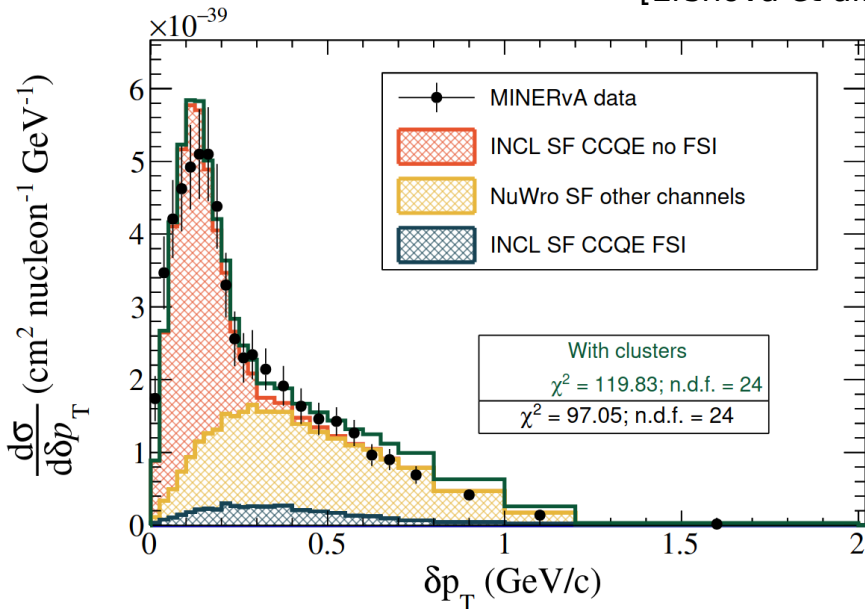
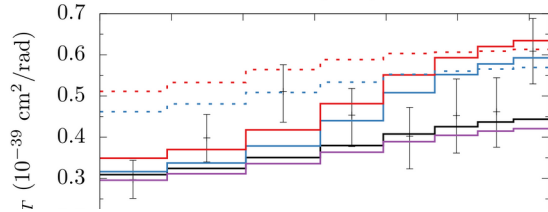
Because $p_p > 450$ MeV low energy differences are not seen

Effect of non-elastic FSI visible in P_T and α_T

Large non-QE 'background' not separable from FSI effects

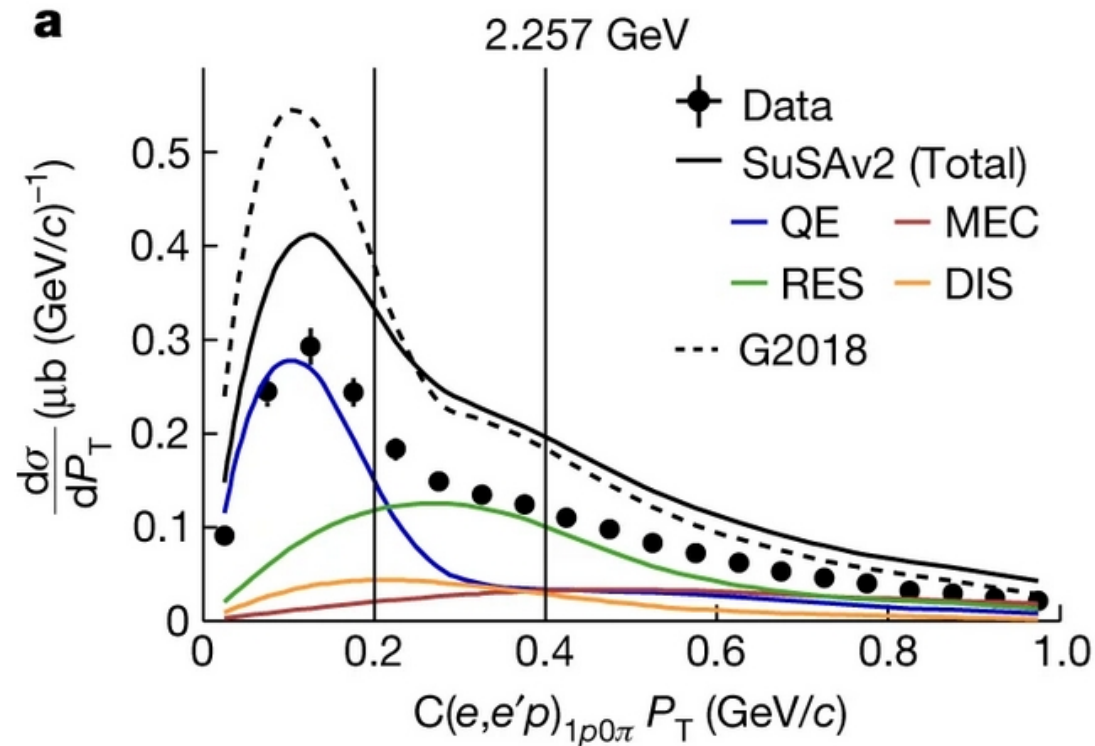
Similar degeneracy in MINERvA data:

[Ershova et al. ArXiv:2202.10402]

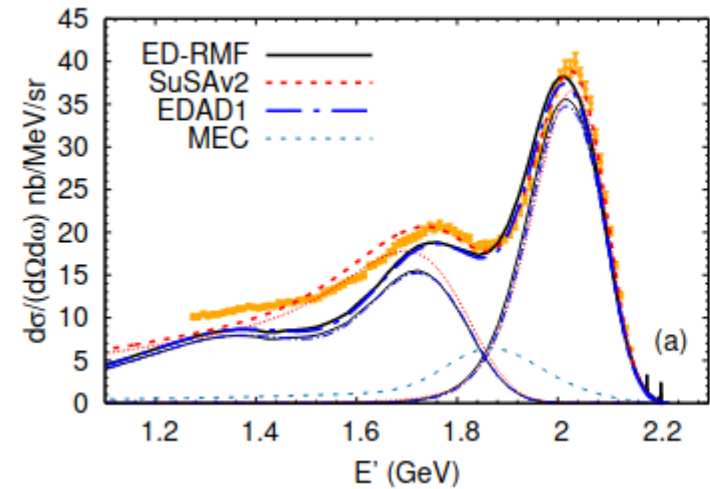


Degeneracy of interaction channels

Is reduced, but not removed by fixing incoming energy



[M. Khachatryan et al. Nature 599, 565]

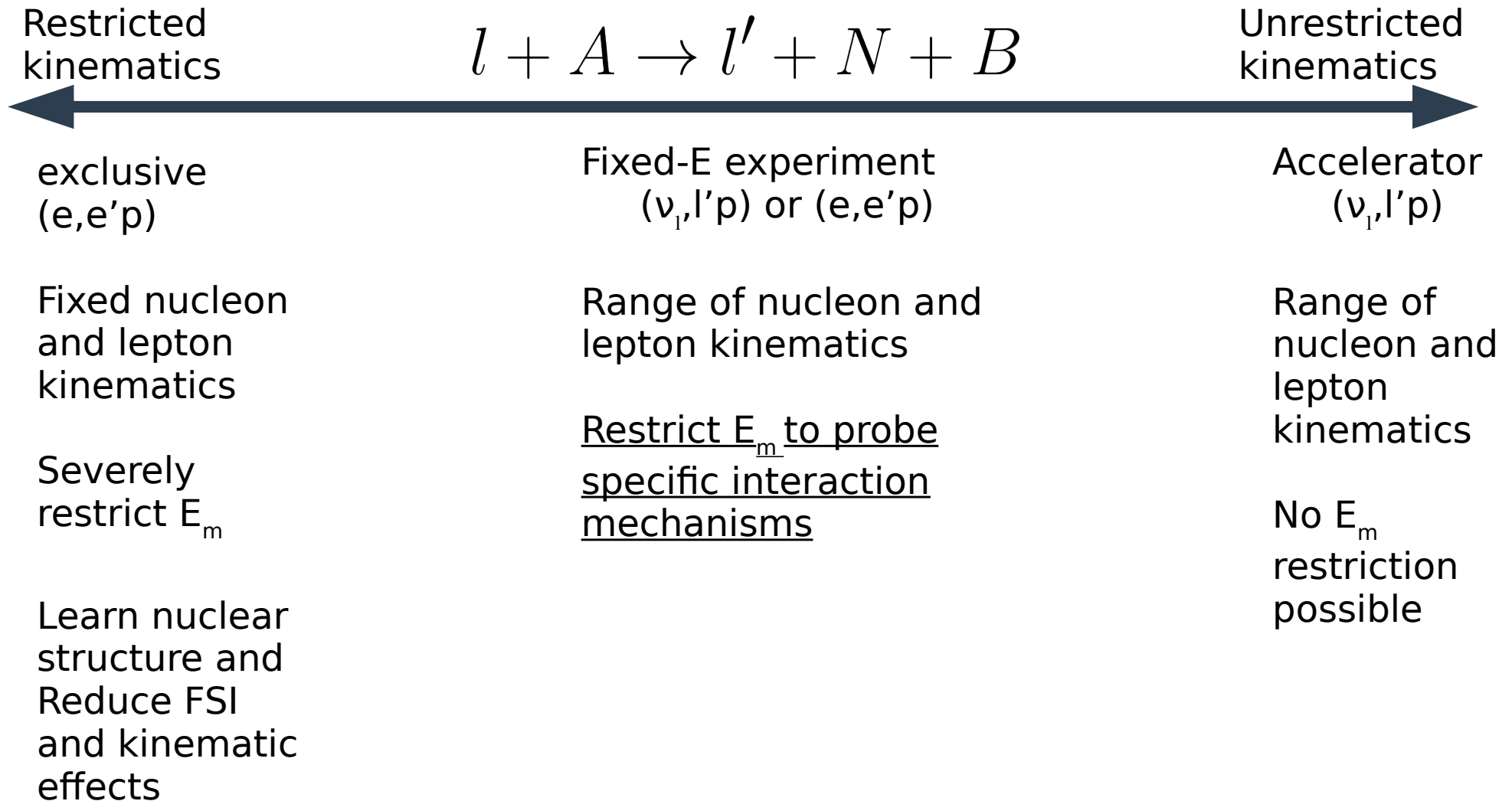


[RG] et al. PRC101, 015503]

Degeneracy of interaction channels

Is reduced, but not removed by fixing incoming energy

Is removed by restricting the energy/momentum of residual system



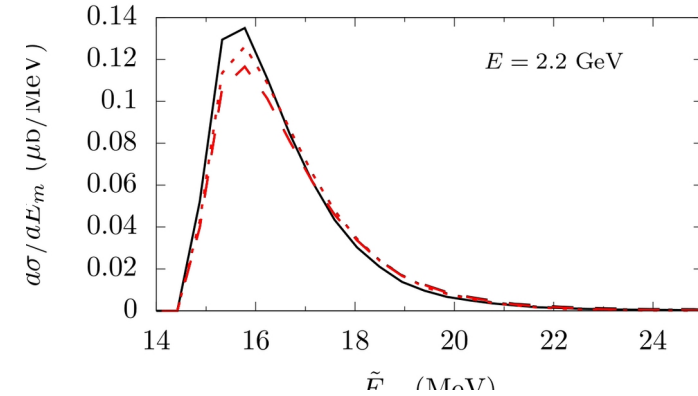
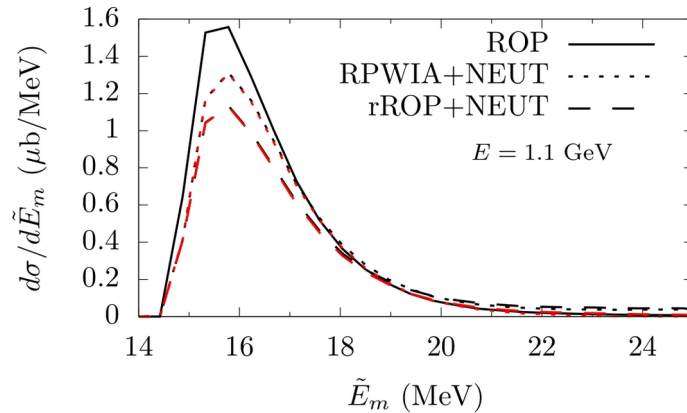
Degeneracy of interaction channels

Fixed-E experiment
(e,e'p) in CLAS

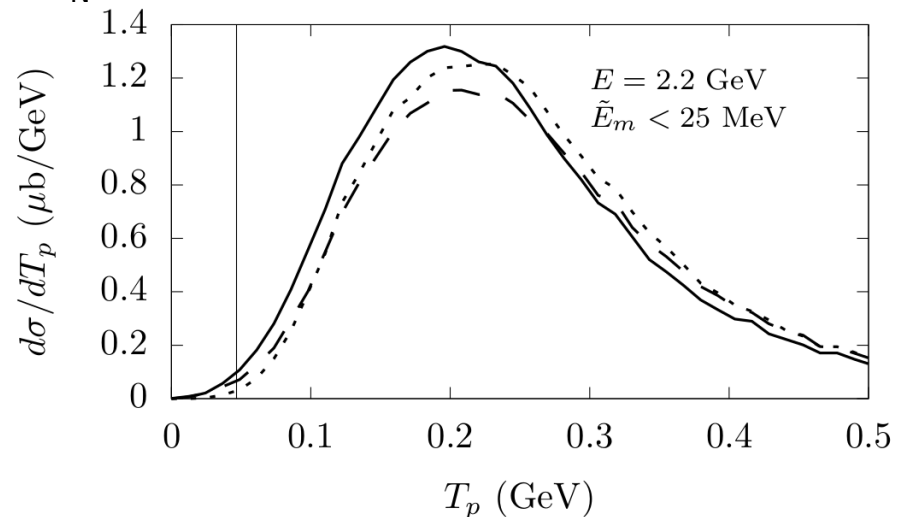
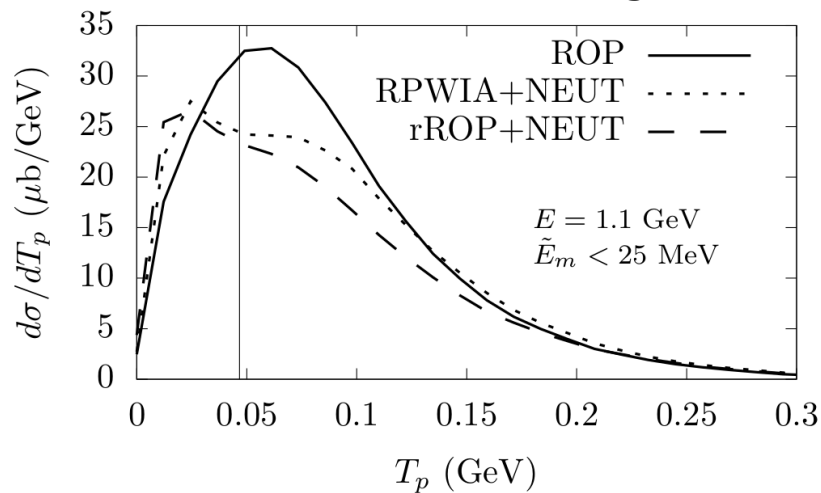
Range of nucleon and
lepton kinematics

Restrict \tilde{E}_m to probe
specific interaction
mechanisms

$\tilde{E}_m < 25 \text{ MeV} \longrightarrow$ No 2p2h, RES, inelastic FSI



But with full range of ω , q , T_N to study FSI



Conclusions

- (R)DWIA calculations in mean-field provide a robust description of inclusive electron scattering over a large mass range and reasonable agreement with neutrino data
- Serves as the basis for additional nuclear effects like in RPA
- Implementation of inclusive cross sections in event generators do not provide nucleon observables. The nucleon distributions obtained in the generator do not agree with exclusive RDWIA calculations
- Unfactorized 1 nucleon knockout calculations over the whole phase space can be used to generate events
- Direct comparison of optical potential approach and nuclear cascade is possible and sets constraints on cascade model from elastic p-A
- Agreement for high Energy and different targets validates the cascade
- Disagreement for low energy requires assessment!
- Restricting energy of the residual hadron system through fixed incoming energy can isolate interaction mechanisms and provide further insight in FSI

Hartree-Fock with effective Skyrme interaction

$$-\nabla \left[\frac{\hbar^2}{2m_q^*(\mathbf{r})} \nabla \phi_{\alpha_q}(\mathbf{r}) \right] + [U_q(\mathbf{r}) - i\mathbf{W}_q(\mathbf{r}) \cdot (\nabla \times \sigma)] \phi_{\alpha_q}(\mathbf{r}) = \varepsilon_{\alpha_q}^{\text{HF}} \phi_{\alpha_q}(\mathbf{r}).$$

Density dependent effective mass and potential:

$$\begin{aligned} \frac{\hbar^2}{2m_q^*(\mathbf{r})} &= \frac{\hbar^2}{2m_q} + \frac{1}{4}(t_1 + t_2)\rho_{\text{tot}}(\mathbf{r}) + \frac{1}{8}(t_2 - t_1)\rho_q(\mathbf{r}) + \frac{1}{24}t_4(\rho_{\text{tot}}^2(\mathbf{r}) - \rho_q^2(\mathbf{r})). \\ U_q(\mathbf{r}) &= t_0[(1 + \frac{1}{2}x_0)\rho_{\text{tot}} - (\frac{1}{2} + x_0)\rho_q] + \frac{1}{4}(t_1 + t_2)\tau_{\text{tot}} + \frac{1}{8}(t_2 - t_1)\tau_q \\ &\quad + \frac{1}{8}(t_2 - 3t_1)\nabla^2\rho_{\text{tot}} + \frac{1}{16}(3t_1 + t_2)\nabla^2\rho_q + \frac{1}{4}t_3(\rho_{\text{tot}}^2 - \rho_q^2) \\ &\quad - \frac{1}{2}W'_0(\nabla \cdot \mathbf{J}_{\text{tot}} + \nabla \cdot \mathbf{J}_q) + \delta_{qp}V^C(\mathbf{r}) + \frac{1}{24}t_4[2\rho_{\text{tot}}\tau_{\text{tot}} - 2\rho_q\tau_q \\ &\quad + \frac{5}{2}\rho_q\nabla^2\rho_q - \frac{5}{2}\rho_{\text{tot}}\nabla^2\rho_{\text{tot}} + \frac{5}{4}(\nabla\rho_q)^2 - \frac{5}{4}(\nabla\rho_{\text{tot}})^2 + \frac{1}{2}J_q^2], \end{aligned} \quad (2.11)$$

With density:

$$\rho_q(\mathbf{r}) = \sum_{\alpha_q \gamma_q} \rho_{\alpha_q \gamma_q}^{(q)} \phi_{\alpha_q}^*(\mathbf{r}) \phi_{\gamma_q}(\mathbf{r}), \quad \longrightarrow \quad \text{Iterate for self-consistent solution}$$

Relativistic mean field with nls ω interaction

$$\begin{aligned} \mathcal{L} = & \bar{\Psi} (i\gamma_{\mu}\partial^{\mu} - M) \Psi + \frac{1}{2} (\partial_{\mu}\sigma\partial^{\mu}\sigma - m_{\sigma}^2\sigma^2) - U(\sigma) \\ & - \frac{1}{4}\Omega_{\mu\nu}\Omega^{\mu\nu} + \frac{1}{2}m_{\omega}^2\omega_{\mu}\omega^{\mu} - \frac{1}{4}\mathbf{R}_{\mu\nu}\mathbf{R}^{\mu\nu} + \frac{1}{2}m_{\rho}^2\rho_{\mu}\rho^{\mu} - \frac{1}{4}F_{\mu\nu}F^{\mu\nu} \\ & - g_{\sigma}\bar{\Psi}\sigma\Psi - g_{\omega}\bar{\Psi}\gamma_{\mu}\omega^{\mu}\Psi - g_{\rho}\bar{\Psi}\gamma_{\mu}\boldsymbol{\tau}\boldsymbol{\rho}^{\mu}\Psi - g_e\frac{1+\tau_3}{2}\bar{\Psi}\gamma_{\mu}A^{\mu}\Psi. \end{aligned}$$

Extension of the original
 σ - ω Walecka model
(Ann. Phys.83,491 (1974)).

$$U(\sigma) = \frac{1}{3}g_2\sigma^3 + \frac{1}{4}g_3\sigma^4$$

$$[\hat{\alpha} \cdot \hat{\mathbf{p}} + \beta (m_N + S(r)) - (E - V(r))] \psi = 0,$$

Where:

$$S(r) = g_{\sigma}\sigma(r)$$

$$V(r) = g_{\omega}\omega^0(r) + g_{\rho}\tau_3\rho_3^0(r) + e\frac{1+\tau_3}{2}A^0(r).$$

$$(\nabla^2 - m_{\sigma}^2)\sigma(r) = g_{\sigma}\rho_s(r) + g_2\sigma^2(r) + g_3\sigma^3(r),$$

$$(\nabla^2 - m_{\omega}^2)\omega^0(r) = g_{\omega}\rho_B(r),$$

$$(\nabla^2 - m_{\rho}^2)\rho_3^0(r) = g_{\rho}\rho_I(r),$$

$$\nabla^2\sigma(r) = -e\rho_e(r),$$

Main approximations:

1) Mean-field approximation:

$$\omega_{\mu} \rightarrow \langle \omega_{\mu} \rangle \quad \sigma \rightarrow \langle \sigma \rangle \quad \rho_{\mu} \rightarrow \langle \rho_{\mu} \rangle$$

2) Static limit:

$$\partial^0\omega_0 = \partial^0\rho_0 = \partial^0\sigma = 0 \quad \omega_{\mu} = \delta_{\mu 0}\omega_0, \quad \rho_{\mu} = \delta_{\mu 0}\rho_0$$

3) Spherical symmetry for finite nuclei:

$$\omega_0 = \omega_0(r) \quad \rho_0 = \rho_0(r) \quad \sigma = \sigma(r)$$

Solving the RPA equations in coordinate space

One gets coupled self-consistent integral equation for the radial transition densities :

$$\begin{aligned} \langle \Psi_0 || X_{\eta J} || \Psi_C(J; E) \rangle_r &= - \langle h || X_{\eta J} || p(\varepsilon_{ph}) \rangle_r \\ &+ \sum_{\mu, \nu} \int dr_1 \int dr_2 U_{\mu\nu}^J(r_1, r_2) \mathcal{R} \left(R_{\eta\mu; J}^{(0)}(r, r_1; E) \right) \langle \Psi_0 || X_{\nu J} || \Psi_C(J; E) \rangle_{r_2} \end{aligned}$$

Solved numerically by discretizing on a mesh in coordinate space
 Translates into a matrix inversion for the transition densities:

$$\rho_C^{RPA} = - \frac{1}{1 - R U} \rho_C^{HF}$$