Localization of source for interferometer

Notations

We focus on the **1D case with two antennas**:

1. the sample rate τ ($\frac{1}{250\ \mathrm{MHz}} = 4\ \mathrm{ns}$ for Tianlai)
2. Total bandwidth $\Delta\nu_B = \frac{1}{2\tau}$ ($125\ \mathrm{MHz}$ for Tianlai)
3. Channel width $\Delta\nu_c$ ($125\ \mathrm{MHz}/1024 \approx 0.122\ \mathrm{MHz}$)
4. FFT length $\tau_{\mathrm{FFT}} = \frac{1}{\Delta\nu_c}$
5. Baseline length d
6. Zenith angle of the source θ
7. Zenith angle of the synthesized beam θ_b

Time delay measurement

The time delay is

$$\tau_d = \frac{d}{c}\sin\theta$$

Differentiate it and the difference in time delay for two angular position should be **larger than the sample rate τ** to be distinguishable:

$$\Delta\tau_d = \frac{d}{c}\cos\theta\Delta\theta \geq \tau$$

Therefore the angular resolution is:

$$\cos\theta\Delta\theta \geq \frac{c\tau}{d} \approx 0.4' \frac{250\ \mathrm{MHz}}{1/\tau}\frac{d}{10\ \mathrm{km}}$$

Visibility measurement

In visibility

- Time delay $\Rightarrow$ phase
- Short time information $\Rightarrow$ spectrum (by FFT)

The **phase** ϕ of a source will change over the frequency range, the phase change between in $\Delta\nu$ is:

$$\Delta\phi = 2\pi \frac{d}{c} \Delta\nu \sin\theta$$

We require the **difference of phase change** in the bandwidth $\Delta\nu_B$ to reach π for two angular position to be distinguishable (phase change $\Delta\phi_B$ for θ and $\Delta\phi_B + \pi$ for $\theta + \Delta\theta$):

$$2\pi \frac{d}{c} \Delta\nu_B \cos\theta\Delta\theta \geq \pi$$

The angular resolution:

$$\cos\theta\Delta\theta \geq \frac{c}{2d\Delta\nu_B} = \frac{c\tau}{d}$$

where we have used the fact that $\Delta\nu_B = \frac{1}{2\tau}$.

But we have **another requirement**, the phase change in the channel width $\Delta\nu_c$ must be smaller than 2π

$$2\pi \frac{d}{c} \Delta\nu_c \sin\theta \leq 2\pi$$

and we get

$$\frac{d\sin\theta}{c} \leq \frac{1}{\Delta\nu_c} = \tau_{\mathrm{FFT}}$$

where we have use the fact that $\Delta\nu_c = 1/\tau_{\mathrm{FFT}}$. Therefore, this requirement is equivalent to that **the FFT sequence must overlap**.

And we can evaluate it

$$d \leq \frac{c}{\Delta\nu_c \sin\theta} = 4915 \text{ m} \frac{0.122 \text{ MHz}}{\Delta\nu_c} \frac{\sin 30^\circ}{\sin\theta}$$

Beam-forming measurement

We consider the beam-forming of **two antennas**:

$$B = A_1 + A_2 + 2\sqrt{A_1 A_2} \cos(2\pi \frac{d}{c} \nu(\sin\theta - \sin\theta_b))$$

If $\theta \neq \theta_b$, B will vary periodically with ν, we assume that when the number of period over the bandwidth $\Delta\nu_B$ of two angular positions is larger than $1/2$, then are distinguishable

$$2\pi \frac{d}{c} \Delta\nu_B \cos\theta \Delta\theta = \pi$$

and again we get

$$\cos\theta\Delta\theta \geq \frac{c}{2d\Delta\nu_B} = \frac{c\tau}{d}$$

It is a little bit surprising that the result is independent of $\sin\theta_b$. It is because if you only see e.g. one period in the whole bandwidth, the source must be close to the beam center.

We also require the amplitude variation in one frequency channel less than one period and again we get

$$\frac{d\sin\theta}{c} \leq \frac{1}{\Delta\nu_c} = \tau_{\text{FFT}}$$

Note:

we only use the cross-correlation part $2\sqrt{A_1 A_2} \cos(2\pi \frac{d}{c} \nu(\sin\theta - \sin\theta_b))$ to do the localization. The **auto-correlation part can only add noise** to the result **without**

adding localization power and therefore should be dropped.

The auto-correlation and increase the sensitivity of **event detection**, but is bad for **localization**. Therefore **VLBI never use auto**.

Caveat

The beam-forming approach lost the information of **phase** (and therefore we only have $\cos$ part) and the **intrinsic spectrum of the source** may influence the result (e.g. effect of **Rotation Measure** is also periodical for single polarization).

However, the spectrum of source are **the same for all beams** and we may use the data from different beams to break the degeneracy.