Computation of the Kugo-Ojima function from lattice simulations

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Outline

1. Introduction and Motivation
2. Results
3. Conclusions and outlook
Faddeev-Popov quantization procedure

- effective Lagrangian

\[ \mathcal{L}_{\text{eff}} = \mathcal{L}_{\text{QCD}} + \mathcal{L}_{\text{GF}} + \mathcal{L}_{\text{FP}} \]

- not gauge invariant anymore!
- however, invariant under BRST transformations
- BRST charge

\[ Q_B = \int d^3 x J^0(x) \]

- \( J^\mu \) is the BRST Noether current
Kugo-Ojima confinement mechanism

- assumes a unbroken BRST charge $Q_B$
  - allows to define the subspace of the physical states $|\text{phys}\rangle$

\[ \mathcal{V}_{\text{phys}} = \{ |\text{phys}\rangle : Q_B |\text{phys}\rangle = 0 \} \]

- total space $\mathcal{V}$ has indefinite metric and contains physical states (like baryons and mesons) as well as non-physical states (e.g. free gluons and ghosts)

- $\mathcal{V}_{\text{phys}}$ only contains color singlet states, if the charge $Q^a$ of global gauge symmetry is unbroken and BRST-exact

\[ \langle \Phi | Q^a | \Phi' \rangle = 0 \]

for any physical states $|\Phi\rangle$ in $\mathcal{V}_{\text{phys}}$. 
in such scenario, the Kugo-Ojima confinement parameter $u^{ab}$ should satisfy

$$u^{ab} = -\delta^{ab}.$$ 

infrared limit of the function $u^{ab}(p^2)$:

$$u^{ab} = \lim_{p^2 \to 0} u^{ab}(p^2)$$ 

$u^{ab}(p^2)$ defined from

$$\int d^4x e^{ip(x-y)} \langle D^{ae}_{\mu} c^e(x) g_0 f^{bcd} A^{d}_{\nu}(y) \bar{c}^c(y) \rangle = \left( \delta^{\mu\nu} - \frac{p_\mu p_\nu}{p^2} \right) u^{ab}(p^2).$$

Lattice calculation of the Kugo-Ojima function

\[ U^{ab}_{\mu\nu}(k) = \left\langle \sum_x \sum_{c,d,e} e^{-ik \cdot (x-y)} D^{ae}_\mu \left( M^{-1} \right)^{ec}_{xy} f^{bcd} A^d_\nu(y) \right\rangle_U \]

\[ u(k) = \frac{1}{(N_d - 1)(N_c^2 - 1)} \sum_{\mu,a} U^{aa}_{\mu\mu}(k) \]

- for practical reasons, we use a point source in the inversion

\[ U^{ab}_{\mu\nu}(k) = \left\langle \sum_x \sum_{c,d,e} e^{-ik \cdot (x-y_0)} D^{ae}_\mu \left( M^{-1} \right)^{ec}_{xy} f^{bcd} A^d_\nu(y_0) \right\rangle_U \]
Lattice Kugo-Ojima cooking recipe

1. prepare the source

\[
f_{abc} A^c_\mu(x) = -\frac{1}{2} \text{Tr} \left\{ \left( U^\dagger_{x,-\mu} + U_{x,\mu} \right) - \left( U^\dagger_{x,-\mu} + U_{x,\mu} \right)^\dagger \right\} [t^a, t^b] \]

2. Solve the system, taking care of zero modes

\[
MY = M\phi_{b,\nu} \quad ; \quad M\psi_{b,\nu} = Y
\]

3. apply the covariant derivative

\[
(D_\mu[U])^{ab}_{xy} = 2 \text{Re} \text{Tr} \left[ t^b t^a U_{x,\mu} \right] \delta_{x+\hat{\mu},y} - 2 \text{Re} \text{Tr} \left[ t^a t^b U_{x,\mu} \right] \delta_{x,y}
\]

4. apply FFT, including correction due to point source
Lattice setup

- Wilson gauge action, $\beta = 6.0$ ($a \sim 0.1\text{fm}$)
- $32^4, 48^4, 64^4,$ and $80^4$ ($3\text{fm}^4 < V < 8\text{fm}^4$)
- 100 configurations, 1 point source
  - 50 configurations for the largest volume
  - several point sources for the smallest volume
- Chroma and PFFT libraries
- simulations performed on Navigator supercomputer Coimbra
- single configuration, point source: 32 (double) inversions
Introduction and Motivation

Results

Conclusions and outlook

Results

Low momenta

High momenta
Checking consistency

Longitudinal component

Imaginary part
Statistics issues, Lattice artifacts

Adding more point sources

Without momentum cuts

![Graph 1: Adding more point sources](image1)

![Graph 2: Without momentum cuts](image2)
Lattice artifacts

Without momentum cuts ("dressing function") $32^4$

Without momentum cuts ("dressing function") $64^4$
lattice scalar quantity $F$
function of $H(4)$ invariants

$$p^{[n]} = \sum_{\mu} p^\mu_n, \ n = 2, 4, 6, 8$$

small lattice corrections:

$$F_{Lat} = F(p^{[2]}, p^{[4]}, p^{[6]}, p^{[8]})$$
$$\sim F(p^{[2]}, 0, 0, 0) + \ldots$$
Comparison with SDE and perturbative results

- Renormalized at $\mu = 4.3$ GeV ($u(\mu) = 0$)
- good agreement with 1-loop for high $q$
- non-perturbative effects below 3 GeV
Conclusions and outlook

- Lattice computation of the Kugo-Ojima function
- several lattice volumes up to \((8\text{fm})^4\)
- checked tensor structure, lattice artifacts
- good agreement with outcome from SDE
- does not seem to be compatible with Kugo-Ojima scenario \((u(0) = -1)\)

Outlook:
- increase statistics
- larger lattice volumes
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