SCATTERING IN A DARK SECTOR DESCRIBED BY SP(4) GAUGE THEORY

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40TH INTERNATIONAL SYMPOSIUM ON LATTICE FIELD THEORY, FERMILAB

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MOTIVATION

- Why dark matter?
- Why SIMP?
- Why Sp(4)?

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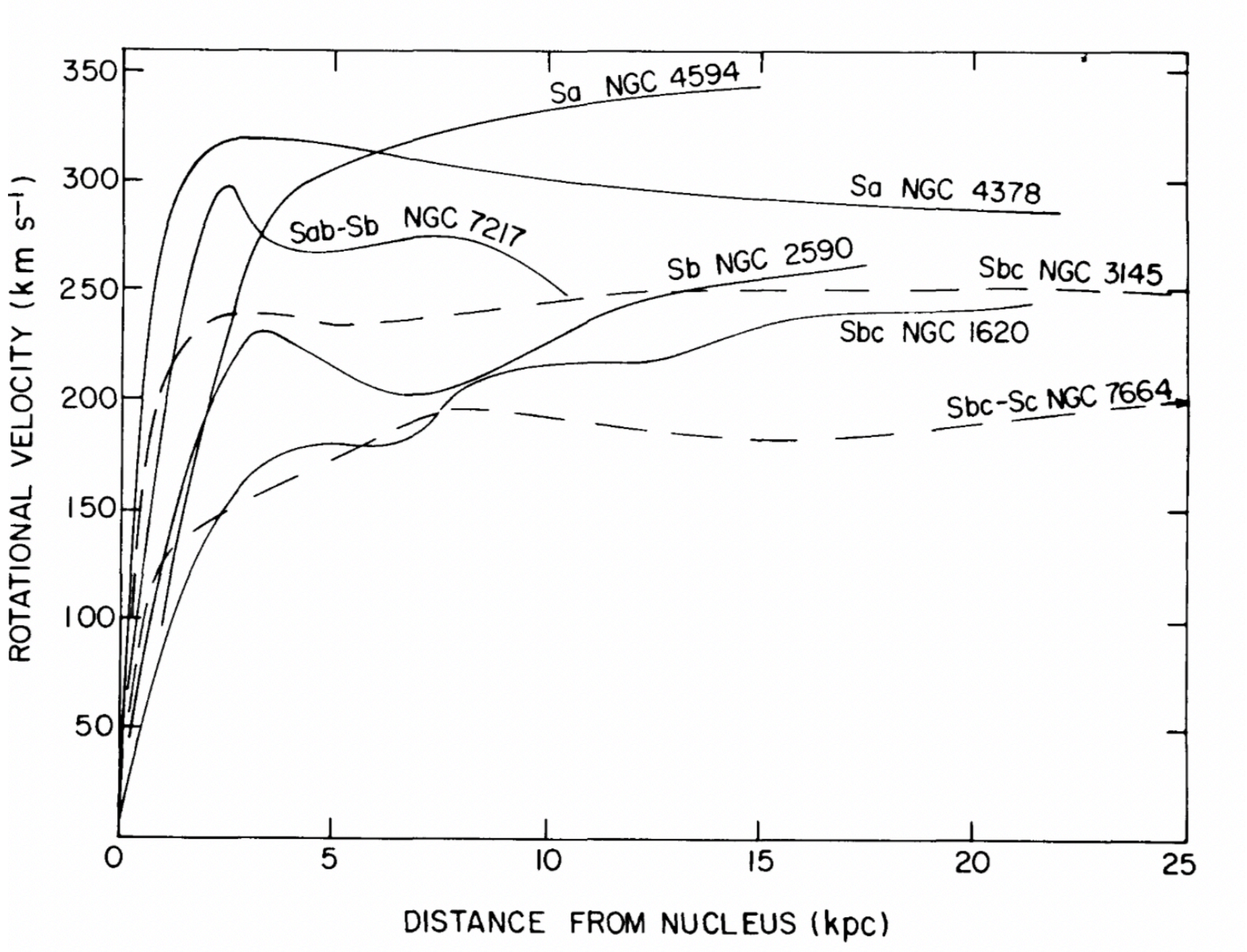
- Why dark matter?
- Why SIMP?
- Why Sp(4)?
    - Note: „Sp“ stands for symplectic group

DARK MATTER

- Collection of astrophysical phenomena
- Rotational Curves
- Modified Gravity?

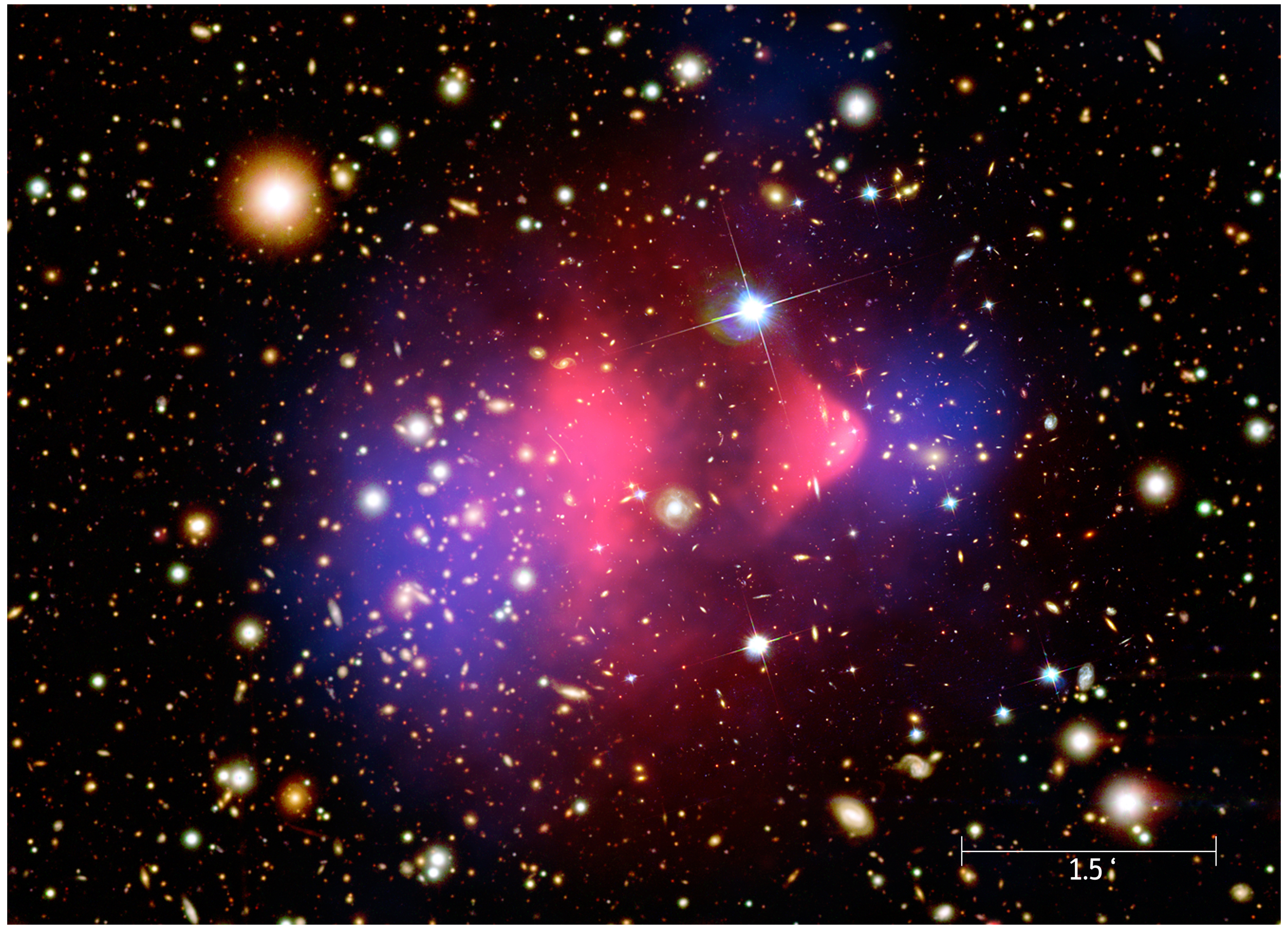


Rotational curves

Bertone et al.: Phys.Rept.405 (2005)
Rubin et al.: Ap.J.L. 225 (1978)
Chandra X-ray Observatory
Tulin, Yu: arXiv:1705.02358 (2017)
Kaplinghat et al: Phys. Rev. Lett. 116 (2016)

DARK MATTER

- Collection of astrophysical phenomena
- Rotational Curves
- Particle picture preferred
  - No standard model candidate



„Bullet" cluster

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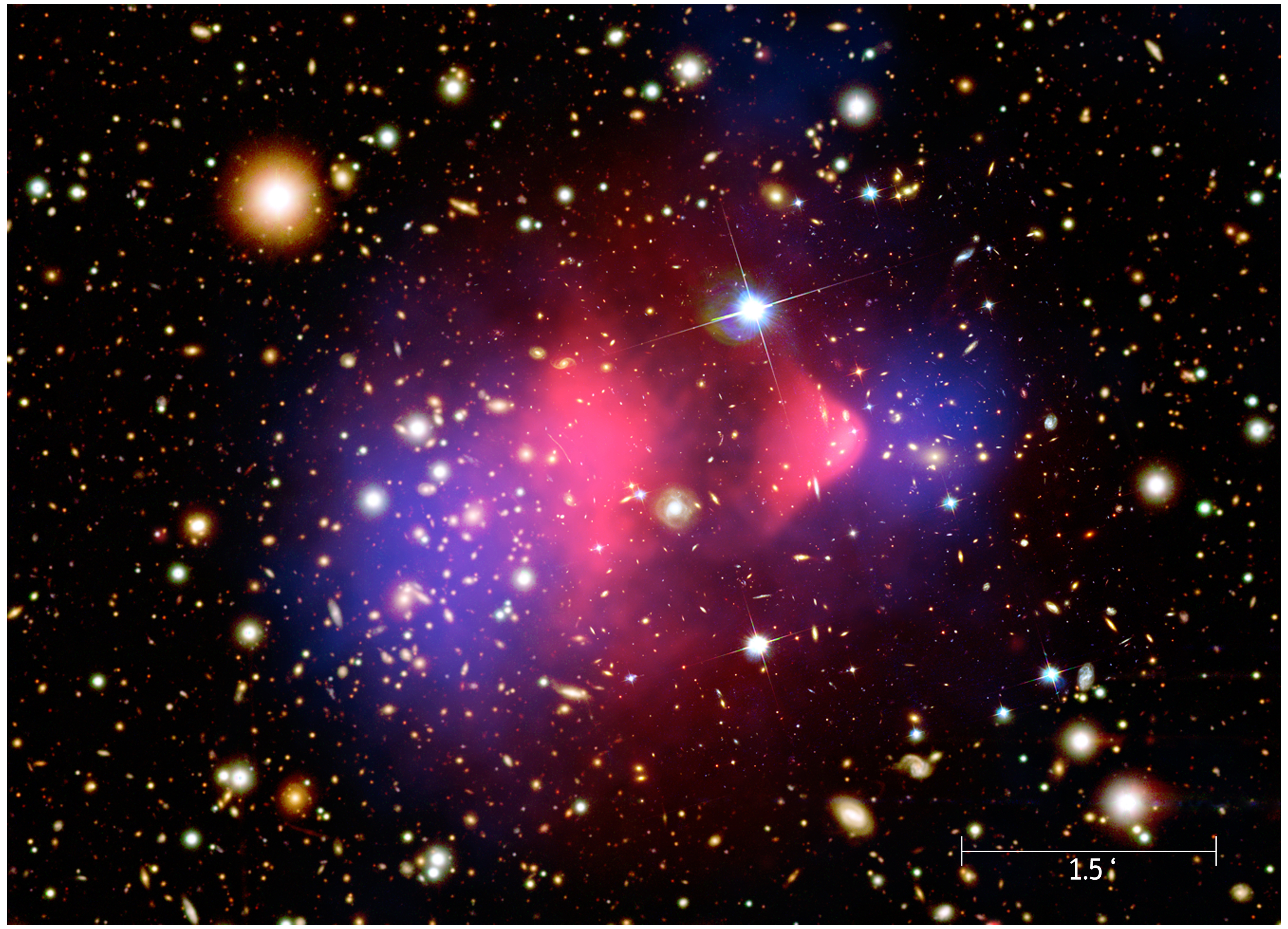

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  - No candidate from standard model
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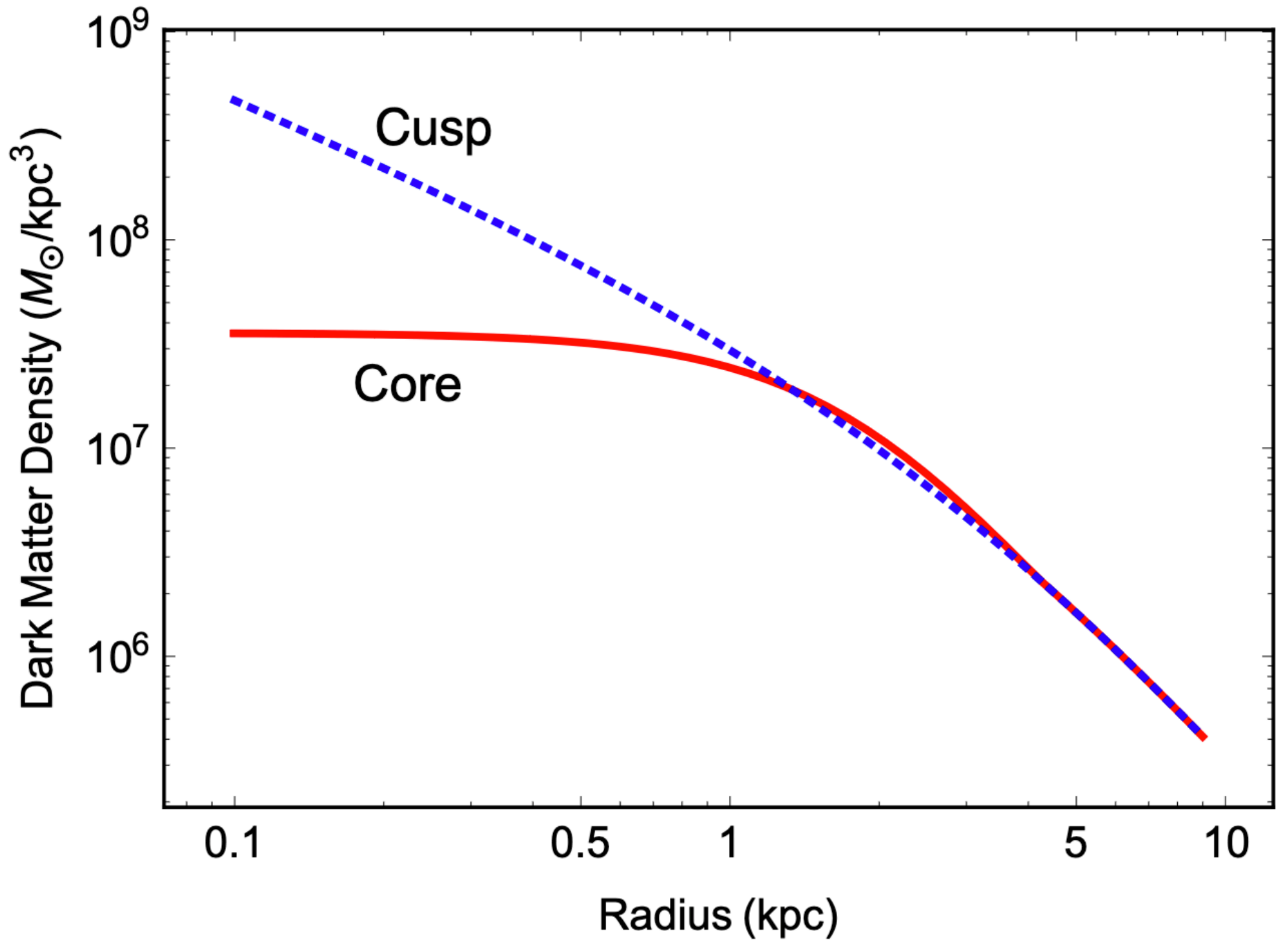

Cusp vs. core

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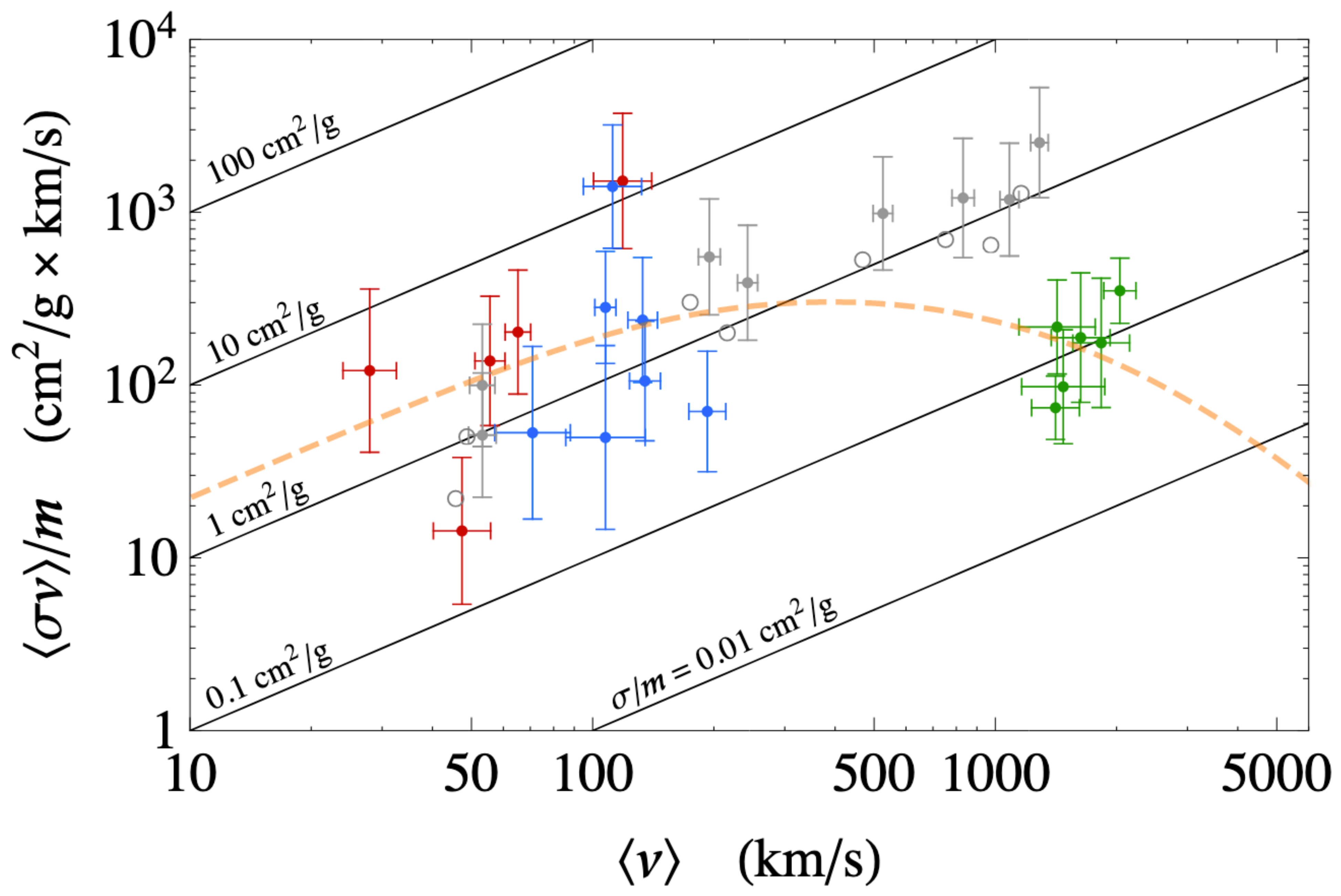
DARK MATTER

- Collection of astrophysical phenomena
- Rotational Curves
- Particle picture preferred
  - No candidate from standard model
- Large scale simulations hint towards dark matter self-interaction
- Velocity dependence?



„DM Halos as particle colliders"

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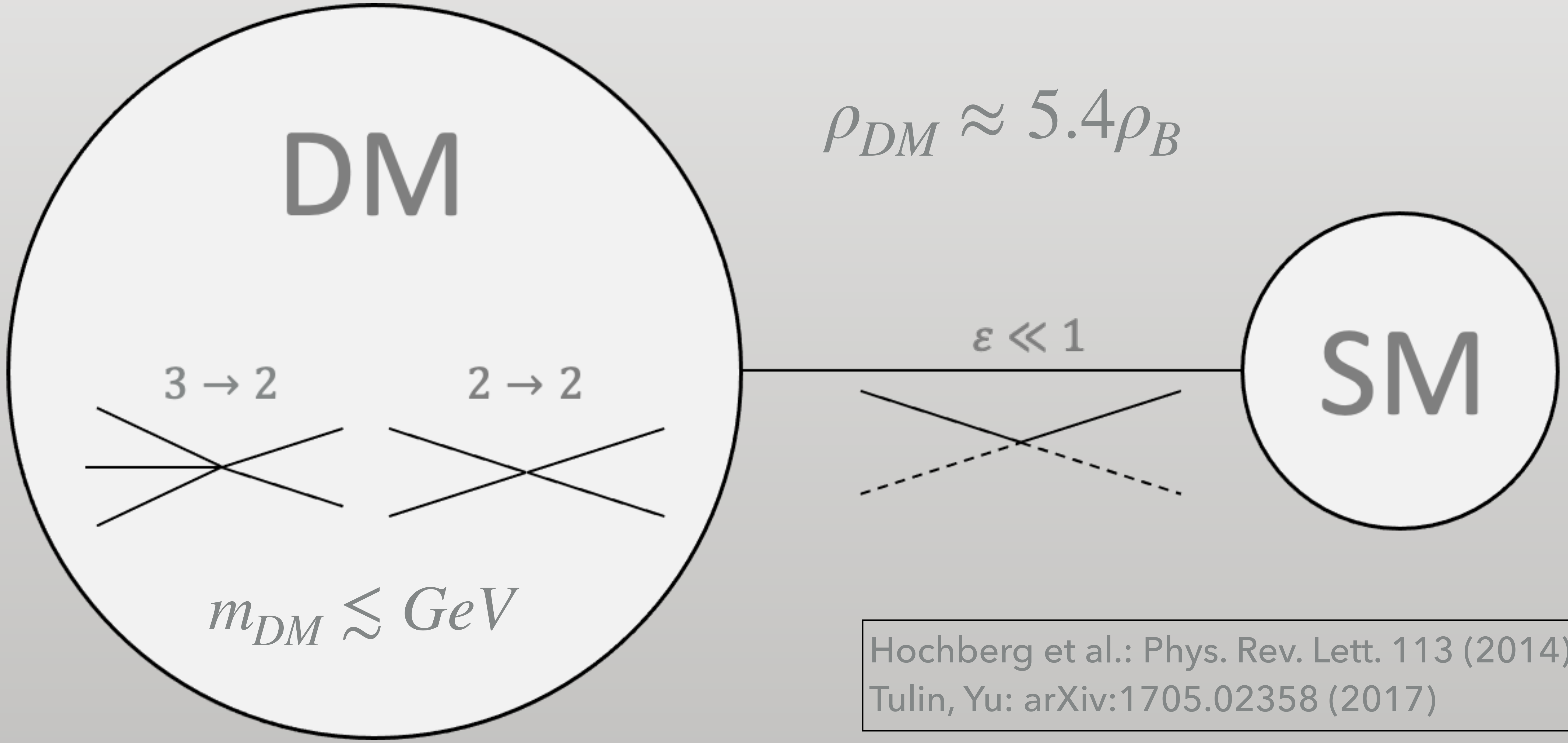
OUR MODEL – SIMP (STRONGLY INTERACTING MASSIVE PARTICLES)

- Paradigm for DM as a thermal relic from early universe via freeze-out



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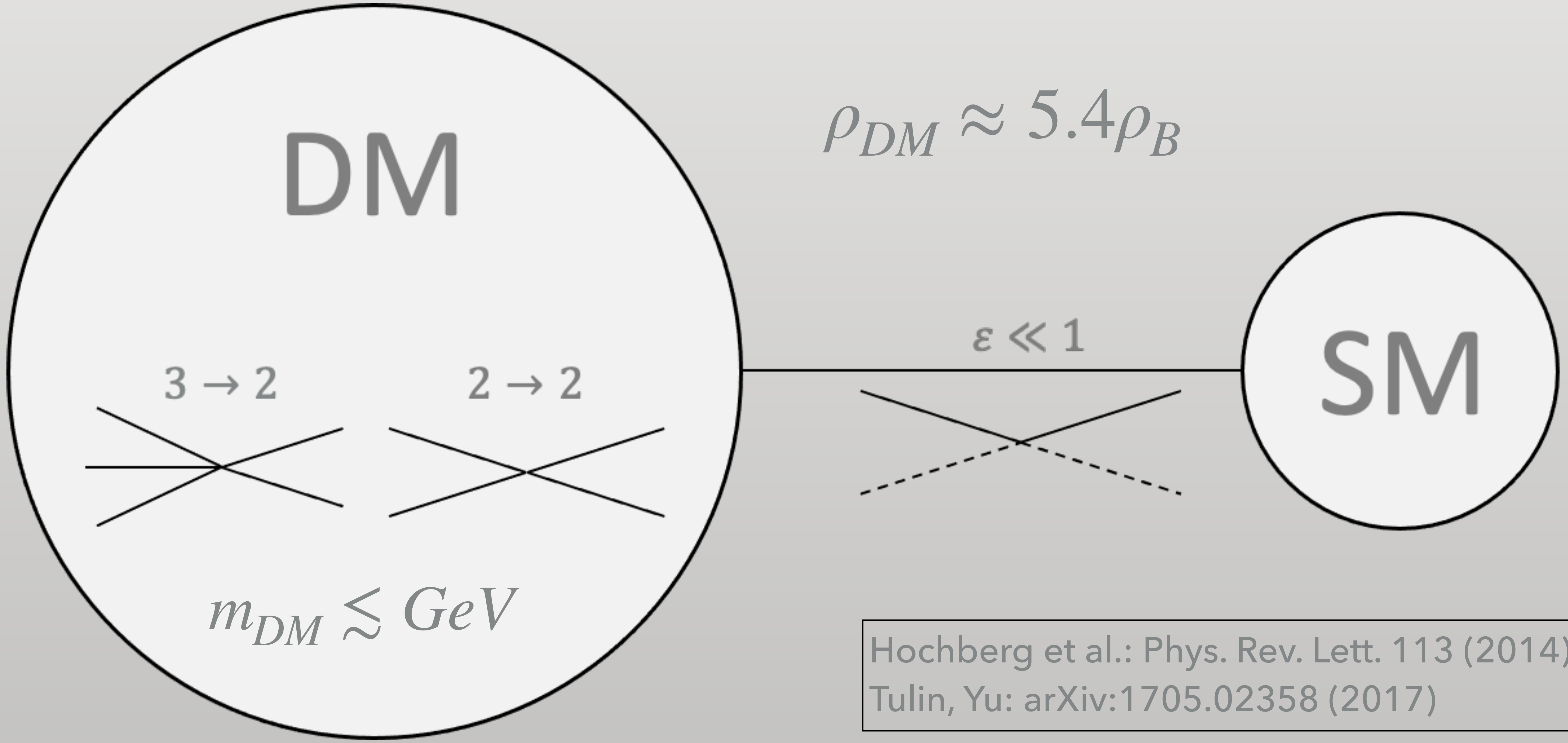
  - Thermal equilibrium in early universe
- Number changing process in dark sector ($3 \to 2$)



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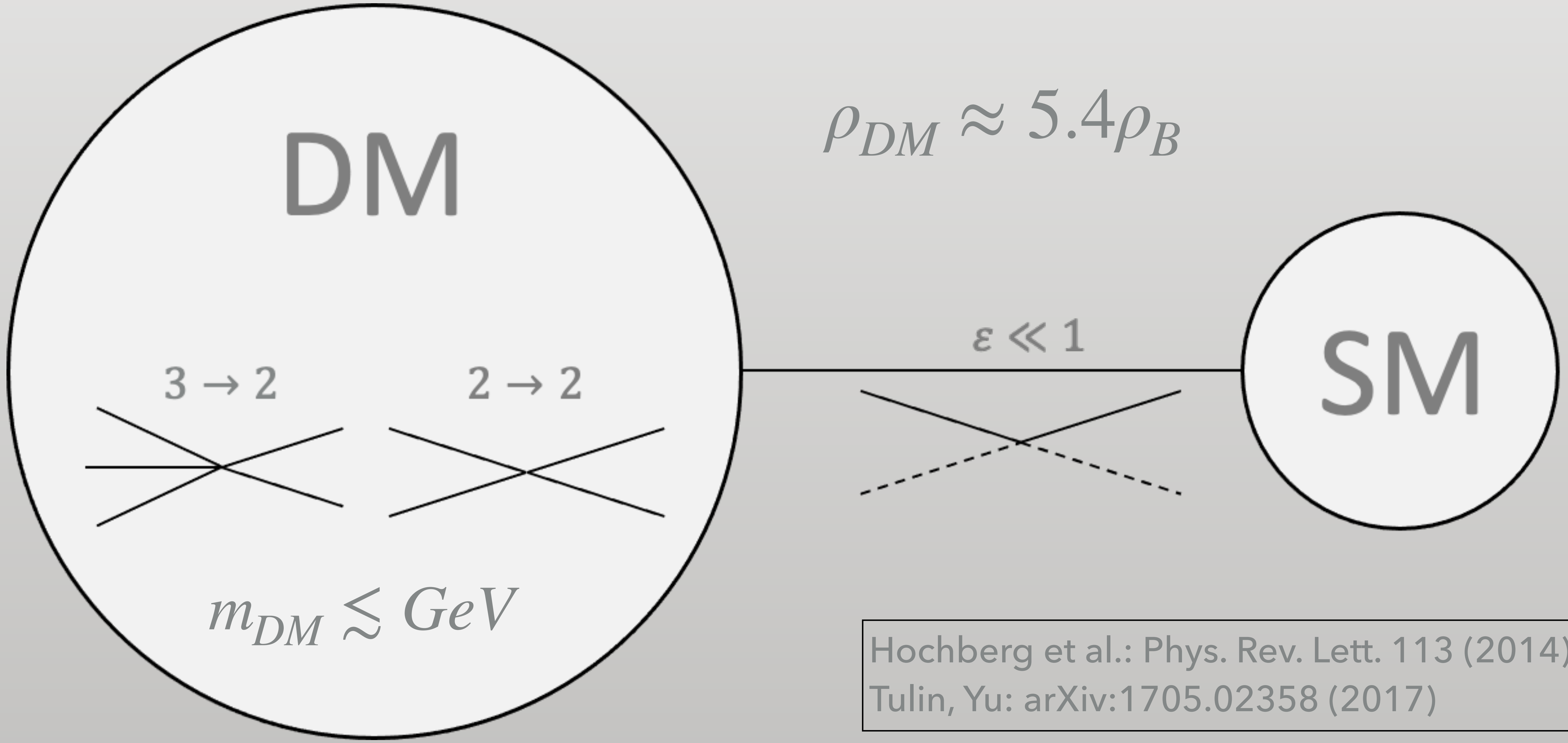
- Paradigm for DM as a thermal relic from early universe via freeze-out
- Mediator with the standard model
  - Thermal equilibrium in early universe
- Number changing process in dark sector ($3 \to 2$)
- Self interaction adresses structure formation
- Mediator enables direct detection



$\rho_{DM} \approx 5.4 \rho_B$

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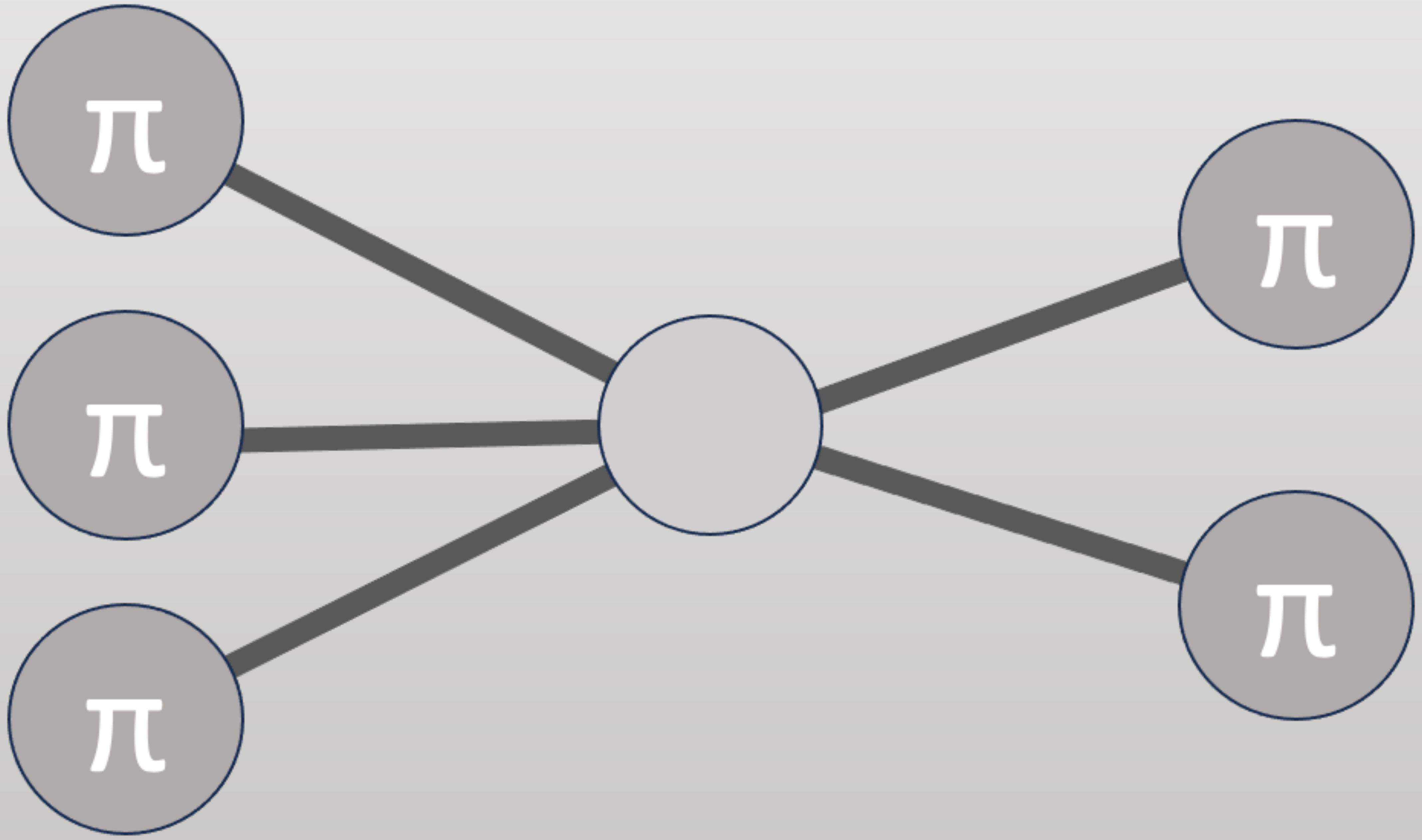
WHY SP(4)

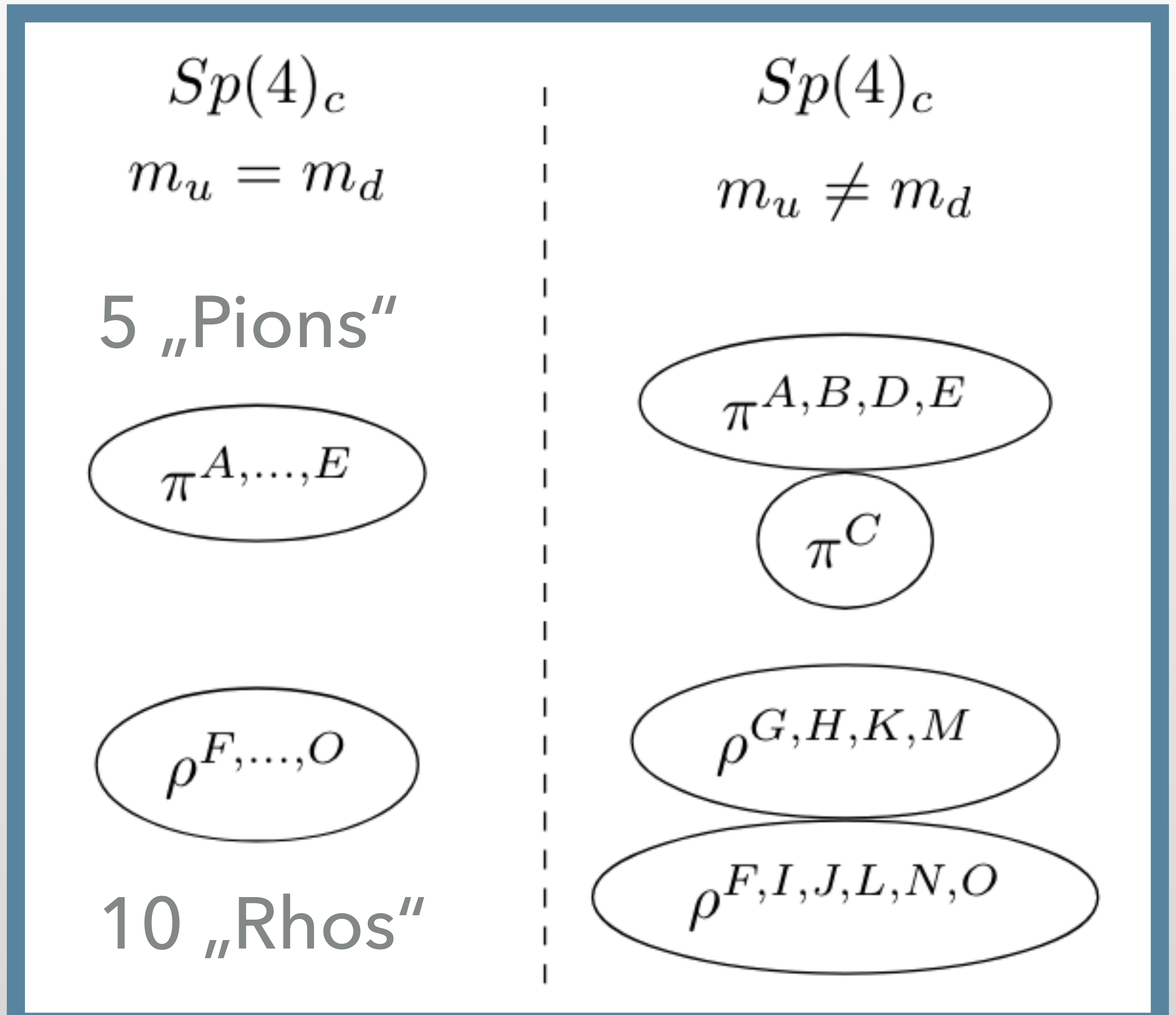
- A minimal realisation of SIMP:
  - Sp(4) gauge with N_f=2 flavours („dark quarks")
- DM candidate: pNGB from chiral symmetry breaking
  - 5 „dark Pions"
  - $3 \rightarrow 2$ process possible



Kulkarni et al.: SciPost Phys. 14 (2023)

SP(4)

- Fermions in fundamental representation (pseudo-real)
- 5 „dark Pions"
  - 3 Pions like in SU(3)
  - 2 „diquarks"

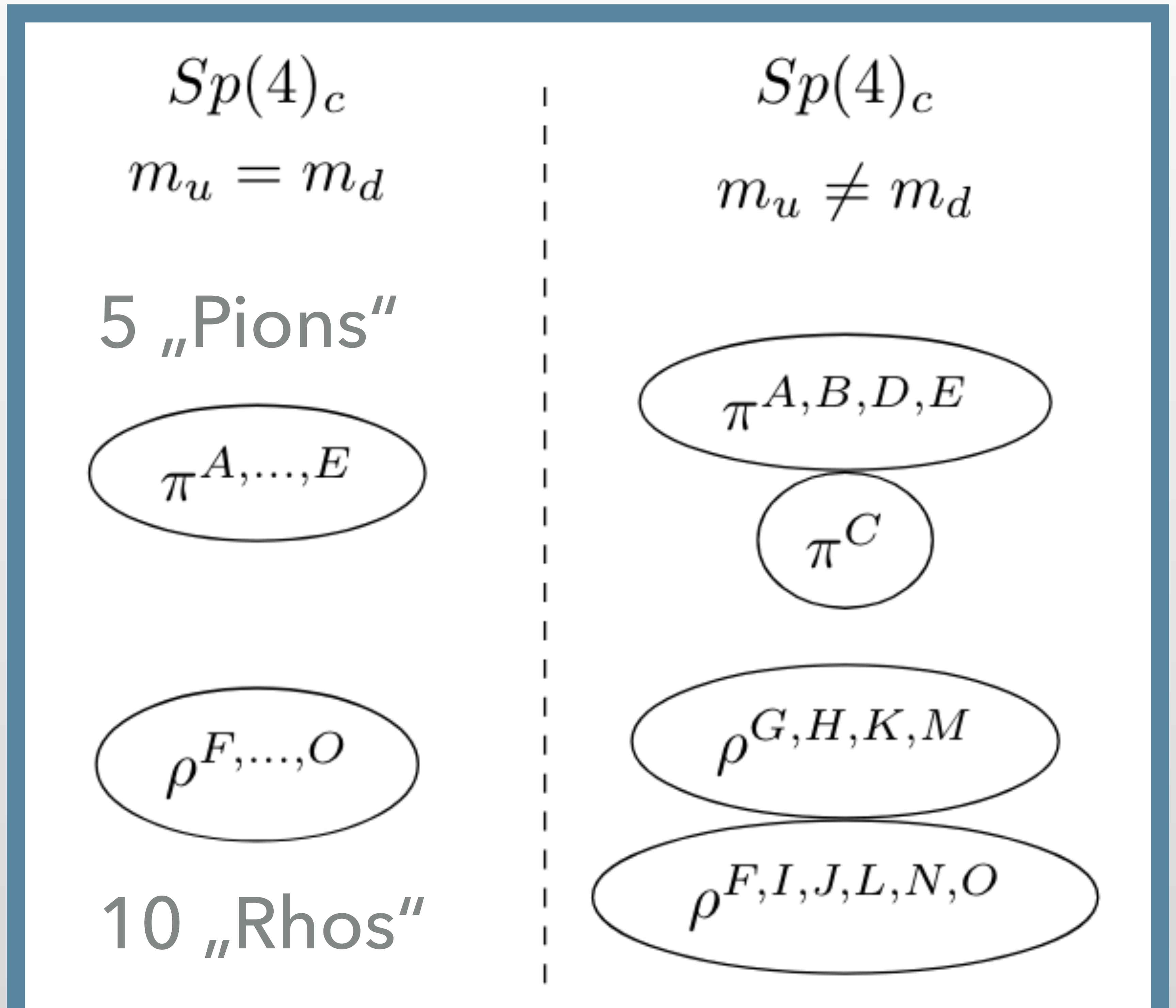


Pseudo-scalar and vector mesons in Sp(4)

Kulkarni et al.: SciPost Phys. 14 (2023)

SP(4)

- Fermions in fundamental representation (pseudo-real)
- 5 „dark Pions"
  - 3 Pions like in SU(3)
  - 2 „diquarks"
- No fermionic bound states (even number of colours)
- Rich hadron sector like QCD



Pseudo-scalar and vector mesons in Sp(4)

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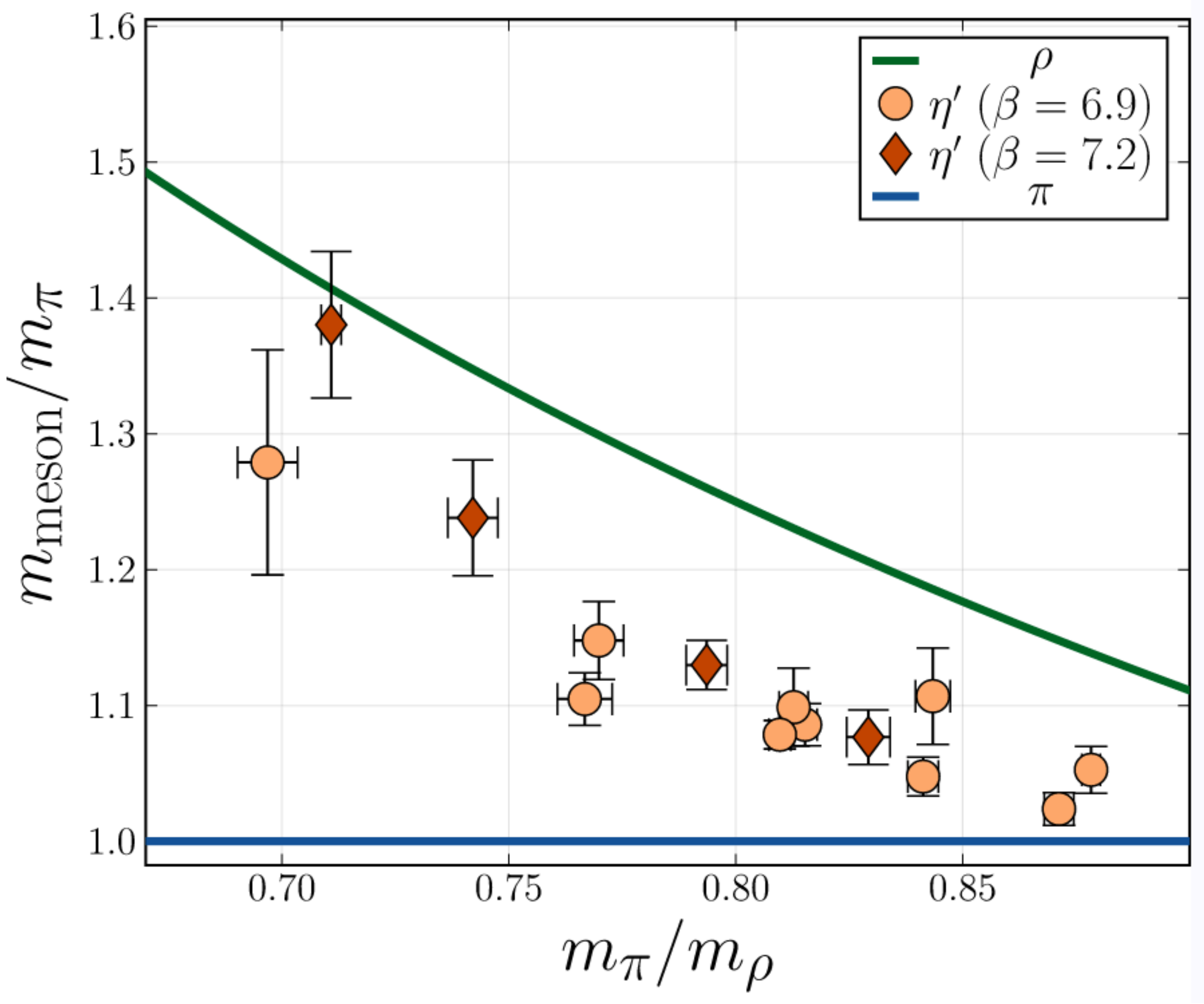
LATTICE SETUP

- Calculation with HiRep:
  - Wilson-Fermion-Gauge-Action with Wilson-Fermions
- Parameters:
  - Inverse gauge coupling β
  - „Dark quark" masses m_u, m_d
- Energy levels from Correlator fitting
- Errors with bootstrap

Debbio et al.: Phys.Rev.D81 (2010)
Blum et al: arXiv:2301.09286 (2023)
Drach et al.: arXiv:2107.09974 (2022)

MASSES

- Mass spectrum of mesons
  - For details: [2304.07191]

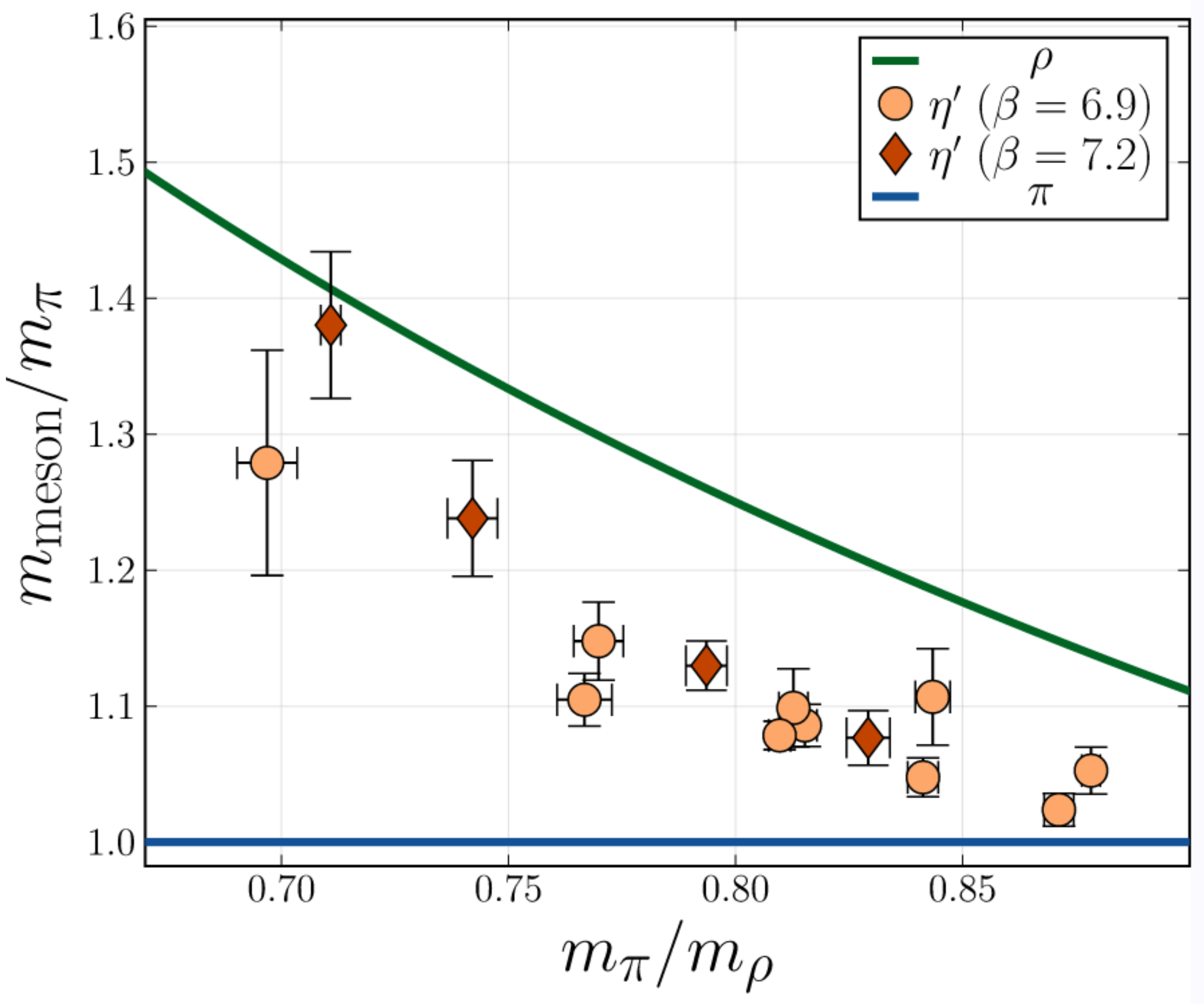


η′ mass in Sp(4)

Bennett et al.: arXiv:2304.07191 (2023)
F. Zierler: PHD thesis

MASSES

- Mass spectrum of mesons
  - For details: [2304.07191]
- Spectrum differs from QCD
- Example:
  - Light η′
  - Effects scattering: $\eta' \leftrightarrow \pi\pi$



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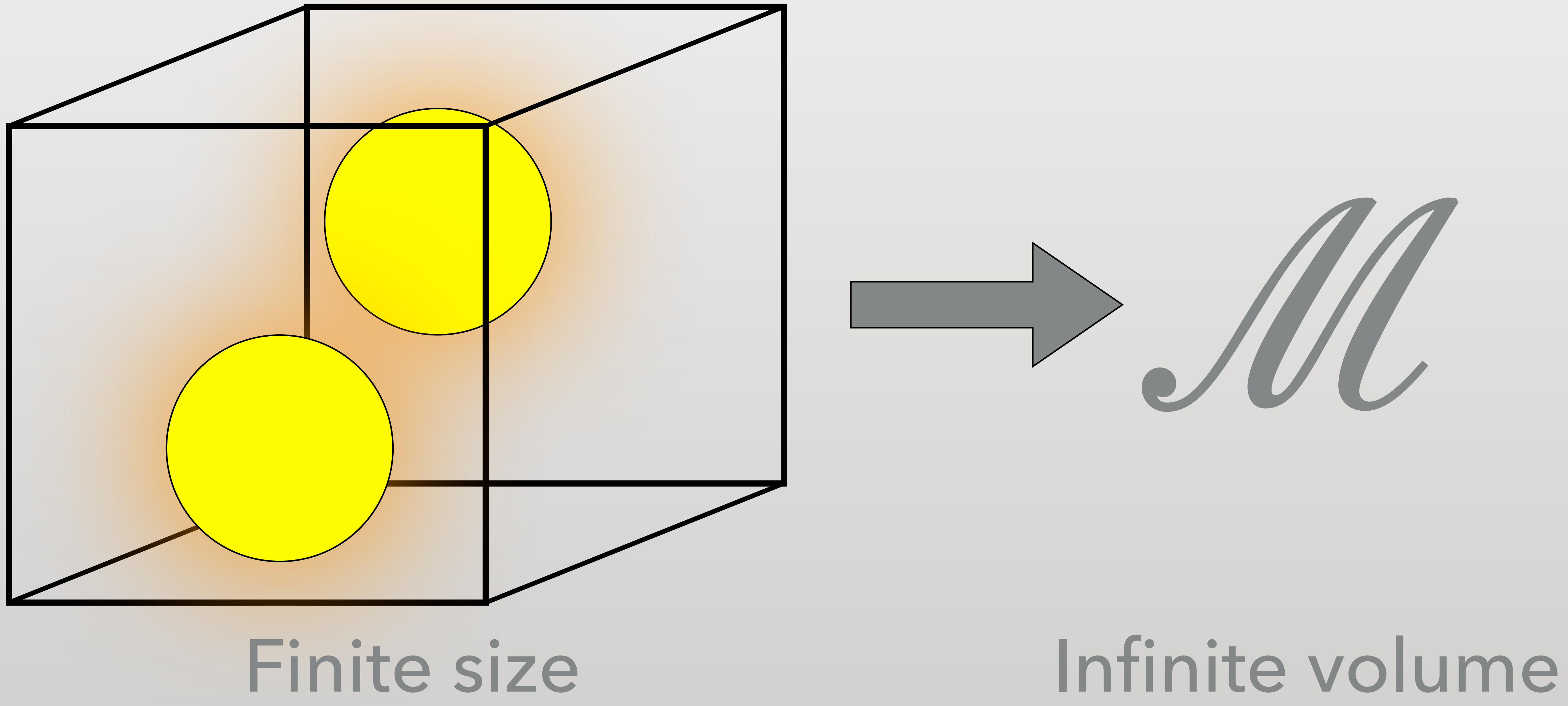
SCATTERING ON THE LATTICE

- Relate finite size effects to infinite volume scattering properties



Lüscher et al.: Commun. Math. Phys. 104/105 (1986)
Jenny et al.: Phys. Rev. D 105 (2022)
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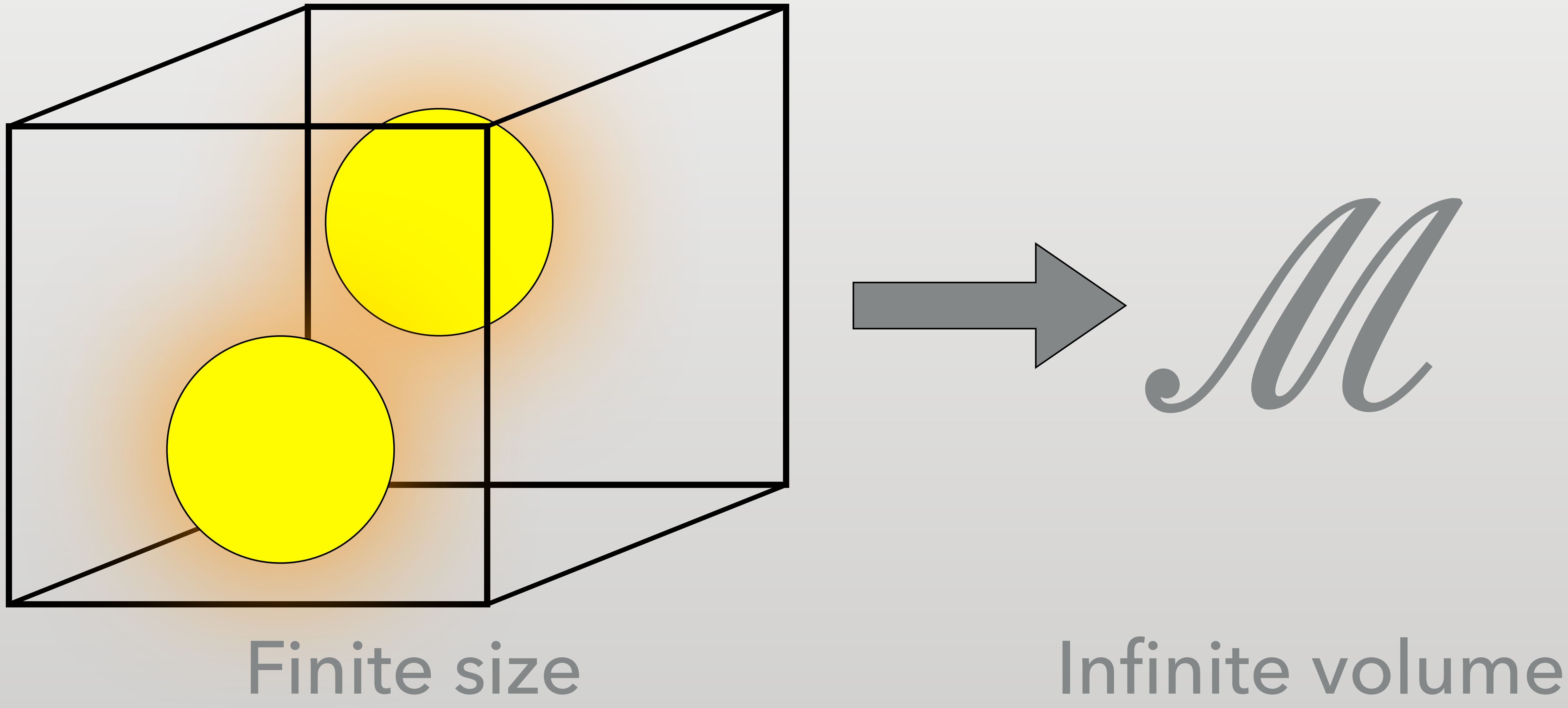
Prelovsek et al.: arXiv:1110.4520 (2011)
Briceño eta al.: Rev. Mod. Phys. 90 (2018)
Hansen et al.: arXiv:2101.10246 (2021)

SCATTERING ON THE LATTICE

- Relate finite size effects to infinite volume scattering properties
- Phase shift from $\pi\pi$-energy levels
  - $C_{\pi\pi} = \left\langle \mathcal{O}^{\dagger}_{\pi\pi} \mathcal{O}_{\pi\pi} \right\rangle$
  - $\mathcal{O}_{\pi\pi} = \mathcal{O}_{\pi^+} \mathcal{O}_{\pi^+} = \left(\bar{d}\gamma_5 u\right)\left(\bar{d}\gamma_5 u\right)$
- $m_u = m_d$ & maximal isospin (I=2)



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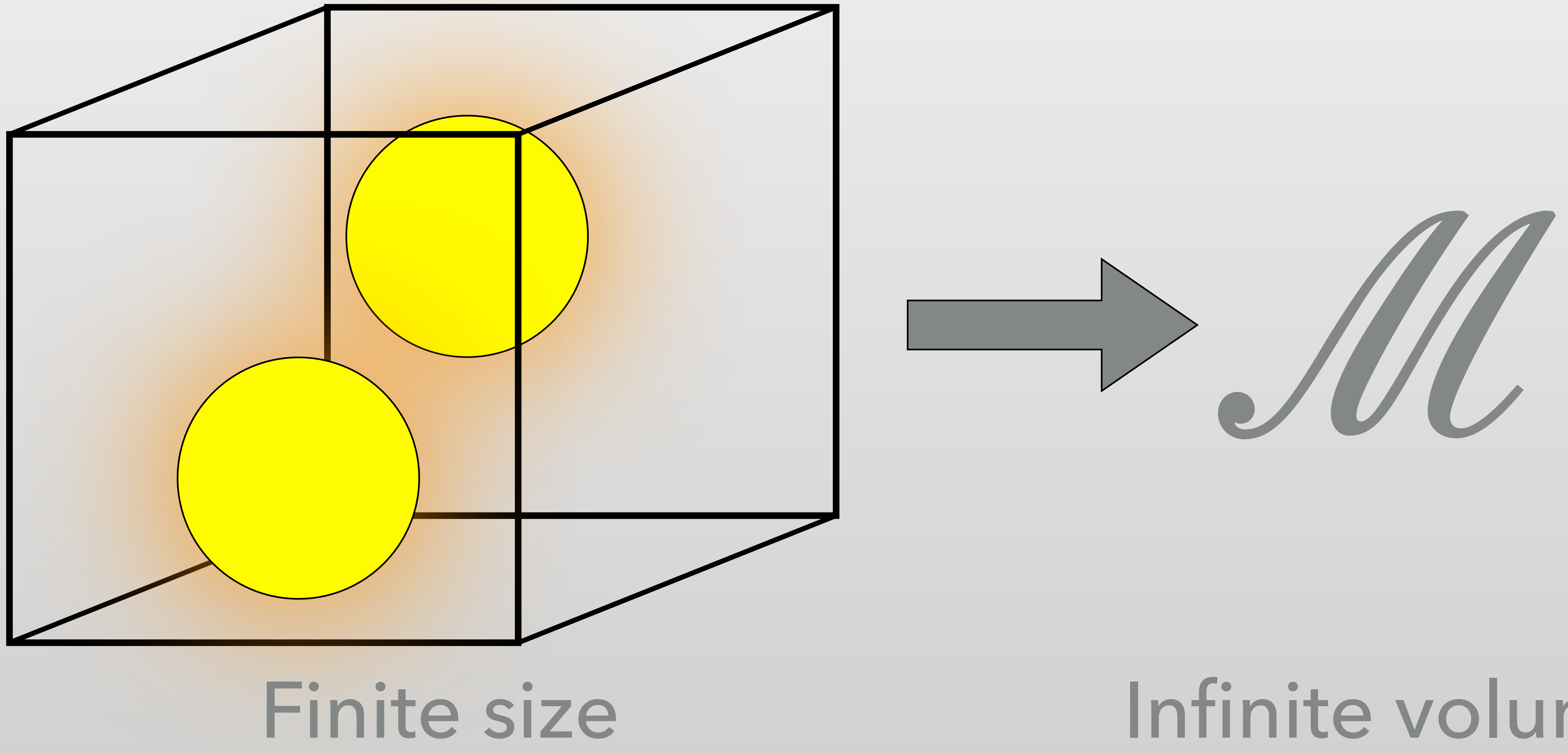
- Lüscher's finite size method:
  - $\tan(\delta_0(q)) = \dfrac{\pi^{\frac{3}{2}} q}{\mathcal{Z}^0_{00}(1,q^2)}$



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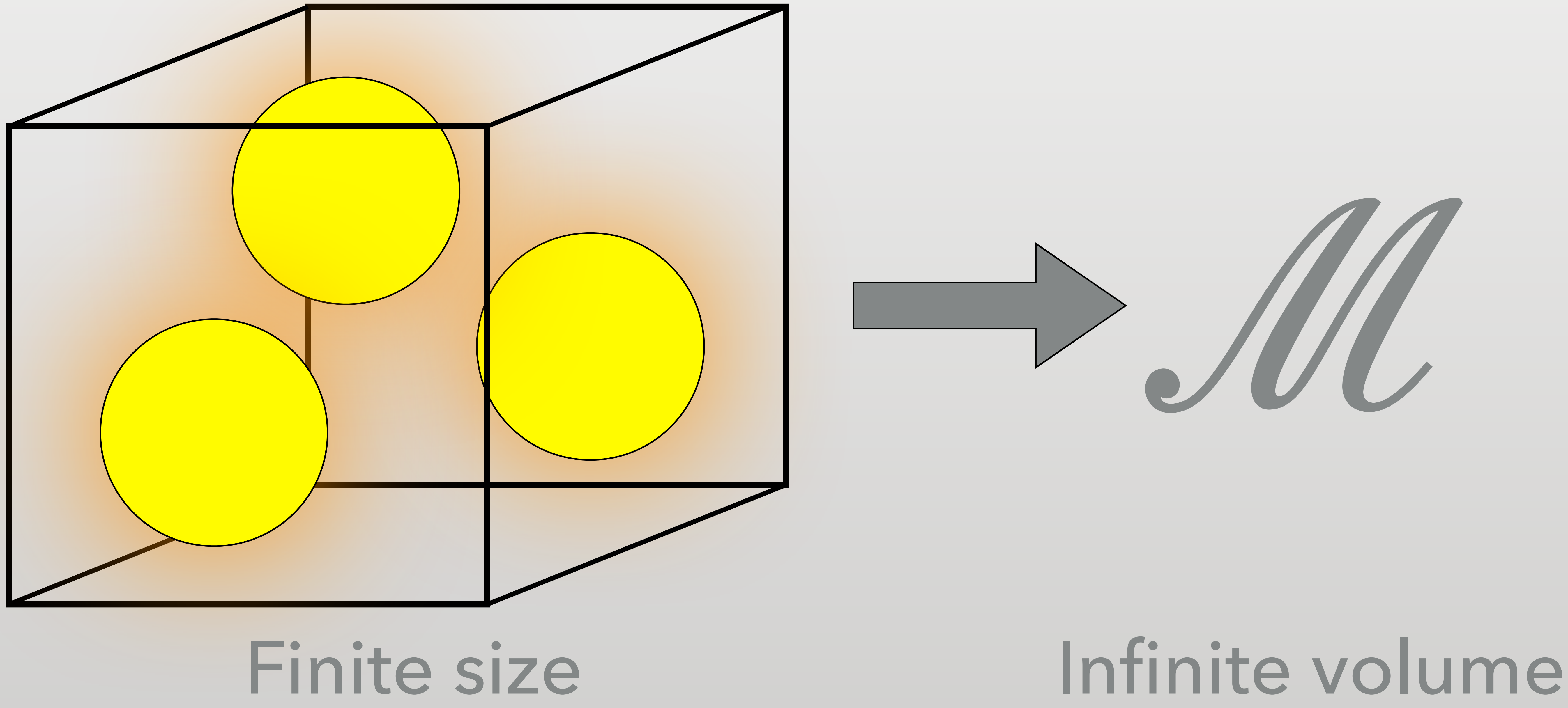
3 PARTICLE SCATTERING

- For $3 \to 2$ process
  - Full $2 \to 2$ information needed
- 3 particle quantization condition
  - $det[F_3^{-1} + \mathcal{K}_3] = 0$
  - Hansen, Romero-López, Sharpe arXiv:2101.10246 [hep-lat]



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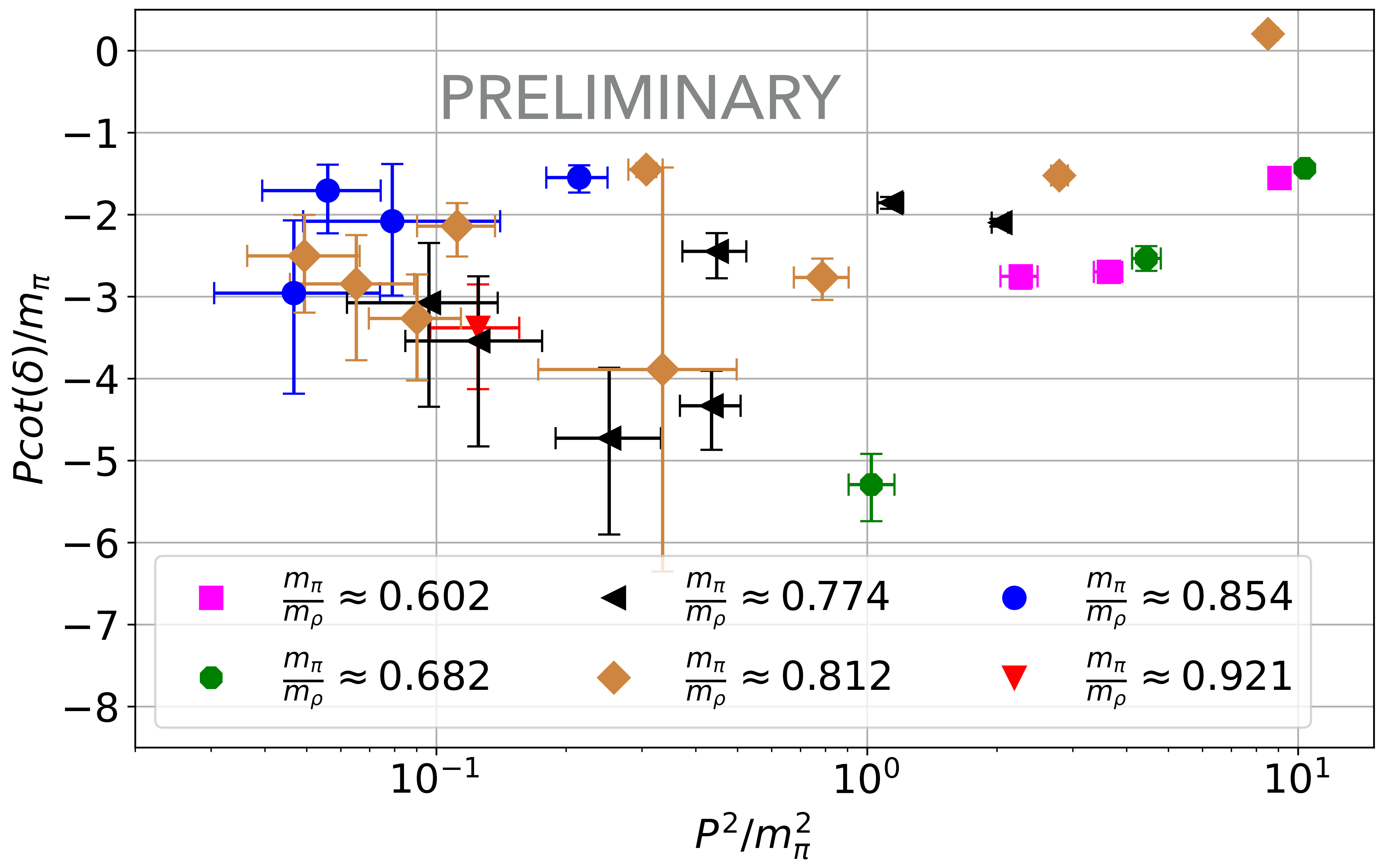
RESULTS – PHASE SHIFT δ_0

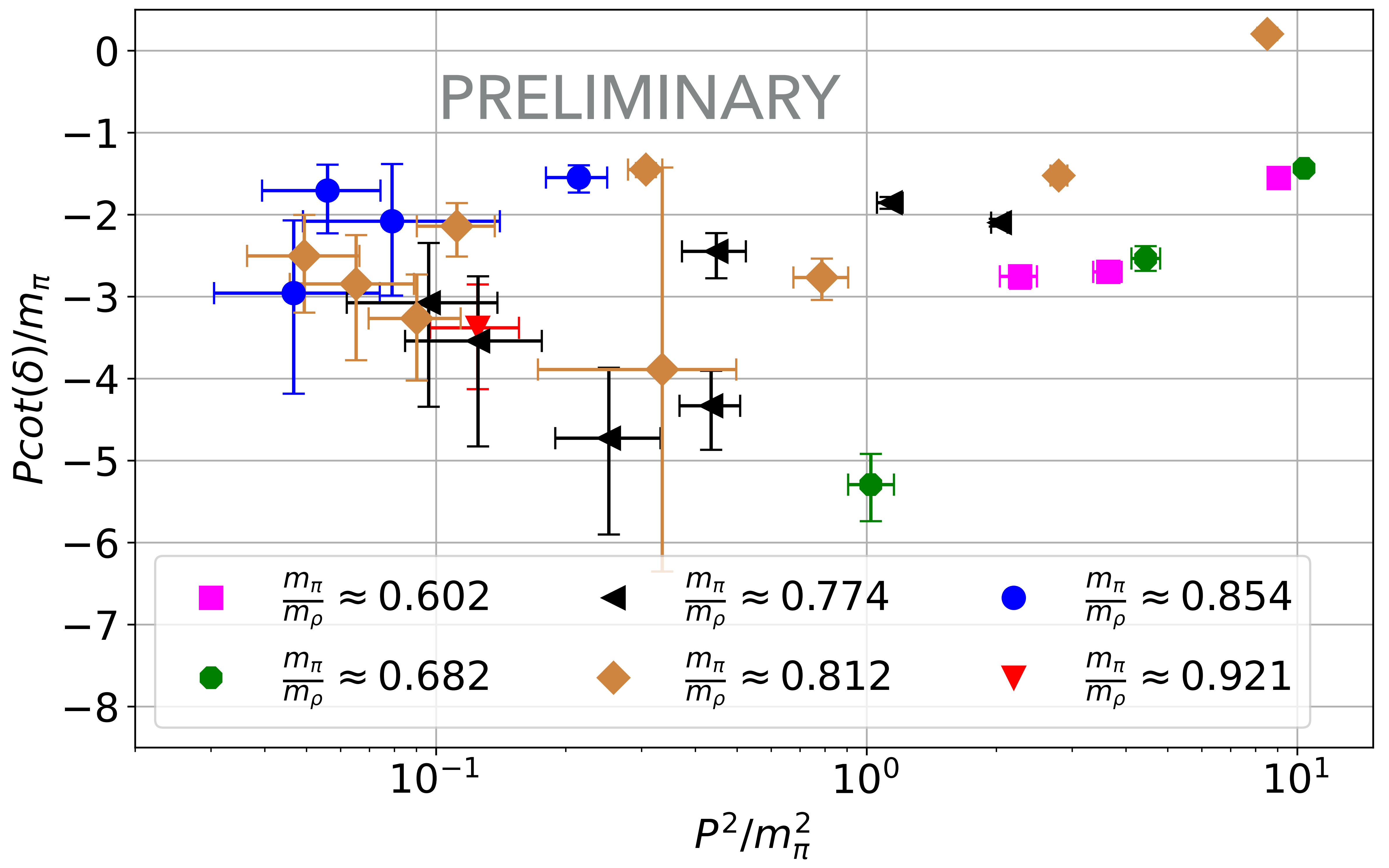
- $|\vec{p}|\cot(\delta_0(q)) = \frac{1}{a_0} + \mathcal{O}(|\vec{p}|^2)$
- Fitting difficult



$p\ cot(\delta_0)$

RESULTS – PHASE SHIFT δ_0

- $|\vec{p}|\cot(\delta_0(q)) = \frac{1}{a_0} + \mathcal{O}(|\vec{p}|^2)$
- Threshold ($|\vec{p}| \to 0$):
  - Information about a potential bound state



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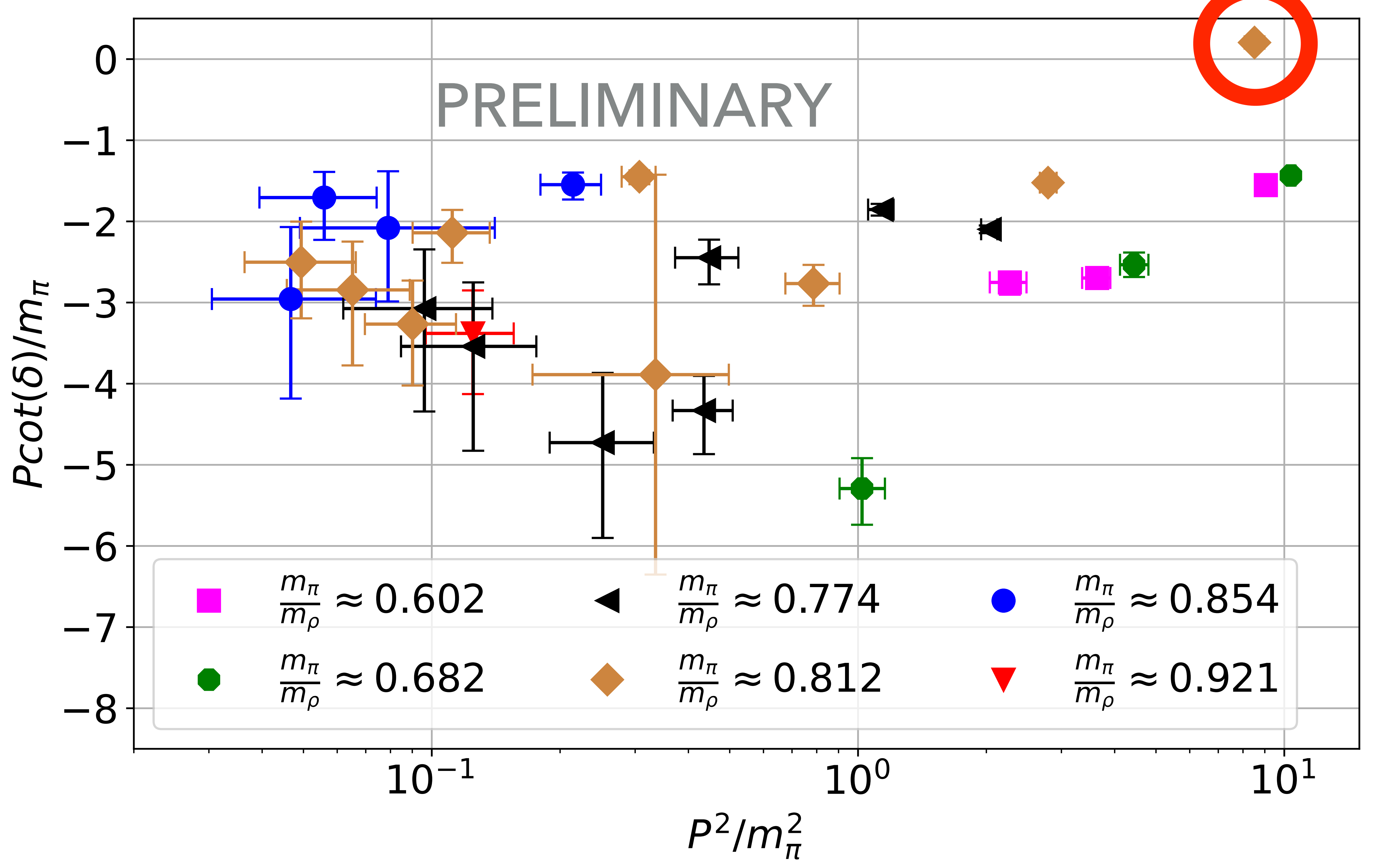
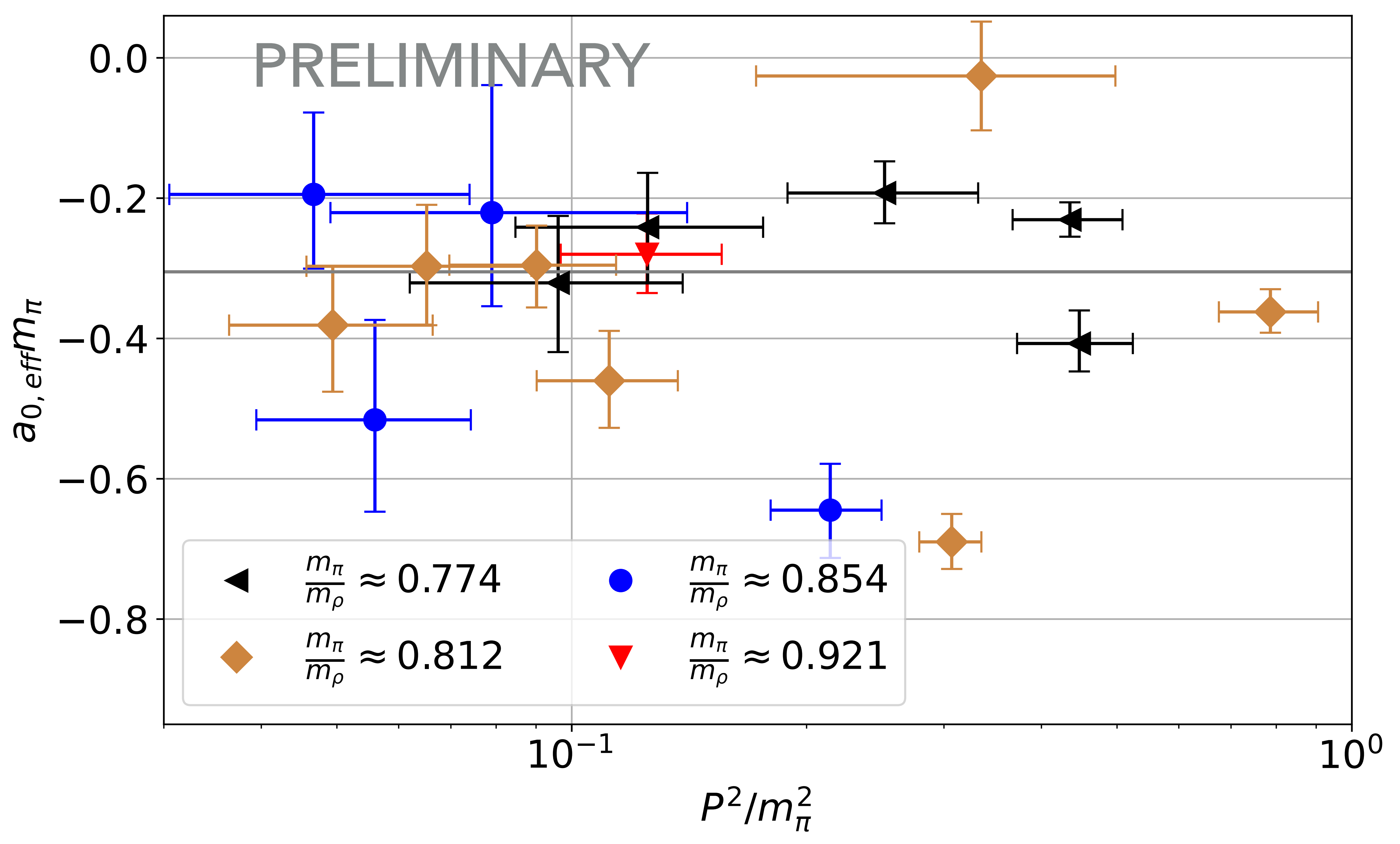
- Threshold ($|\vec{p}| \to 0$):
  - Information about a potential bound state
- Zero crossing:
  - Information about a potential resonance



$p\ cot(\delta_0)$

RESULTS – EFFECTIVE a_0

- $a_{0,eff} = \lim_{|\vec{p}|\to 0} \frac{tan(\delta_0)}{|\vec{p}|}$
- $a_0 m_\pi = -0.30^{-0.17}_{+0.06}$
  - $a_0 < 0 \rightarrow$ “scattering state“



Effective scattering length

Eckert et al.: A&A 666, A41 (2022)

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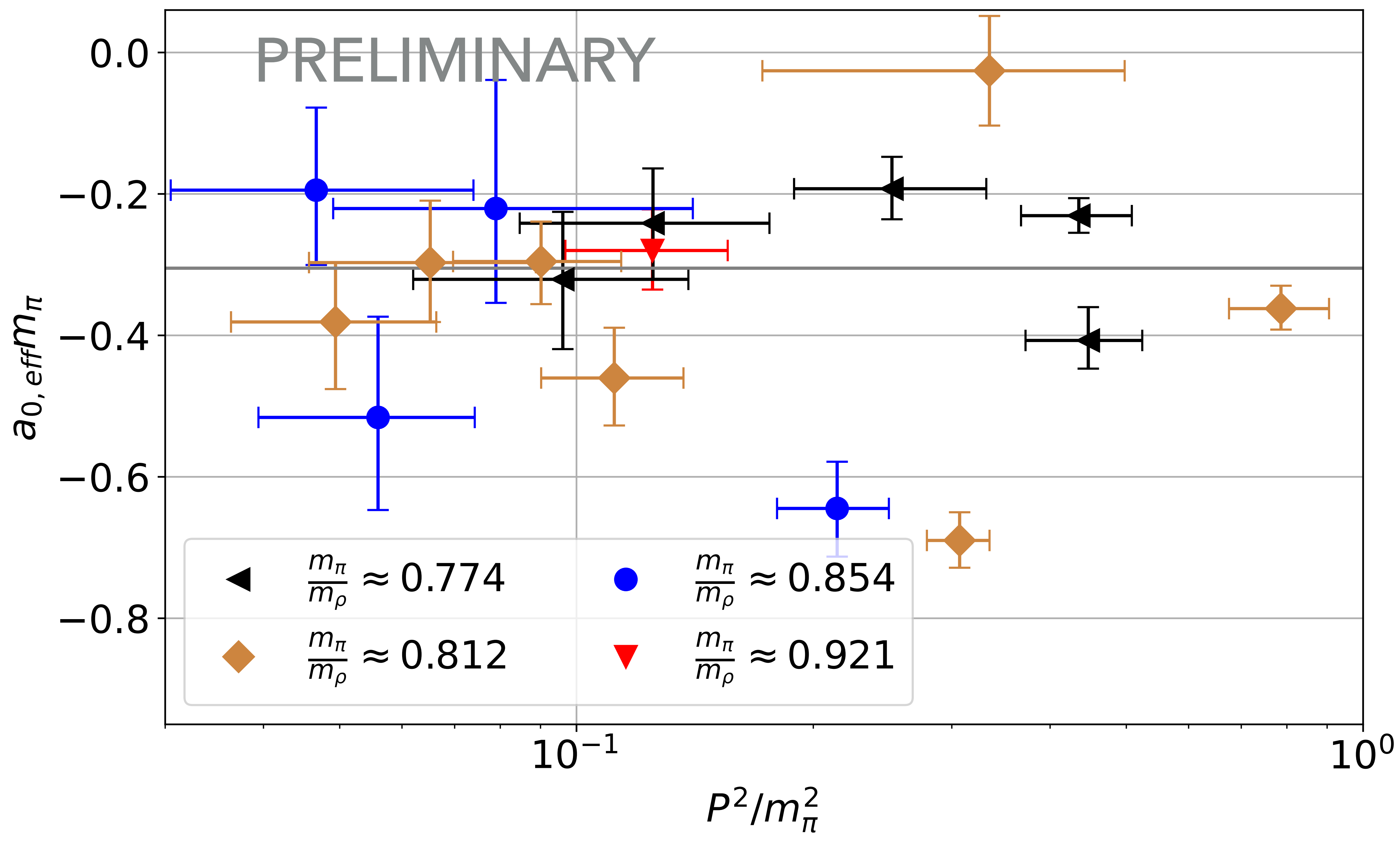
- $a_0 m_\pi = -0.30^{-0.17}_{+0.06}$
  - $a_0 < 0 \rightarrow$ "scattering state"
- Cross-section: $\sigma \approx \pi a_0^2$
- Constraint from density profiles of galaxy clusters: $\frac{\sigma}{m} < 0.19cm^2/g$
- $m_{DM} > 75MeV$



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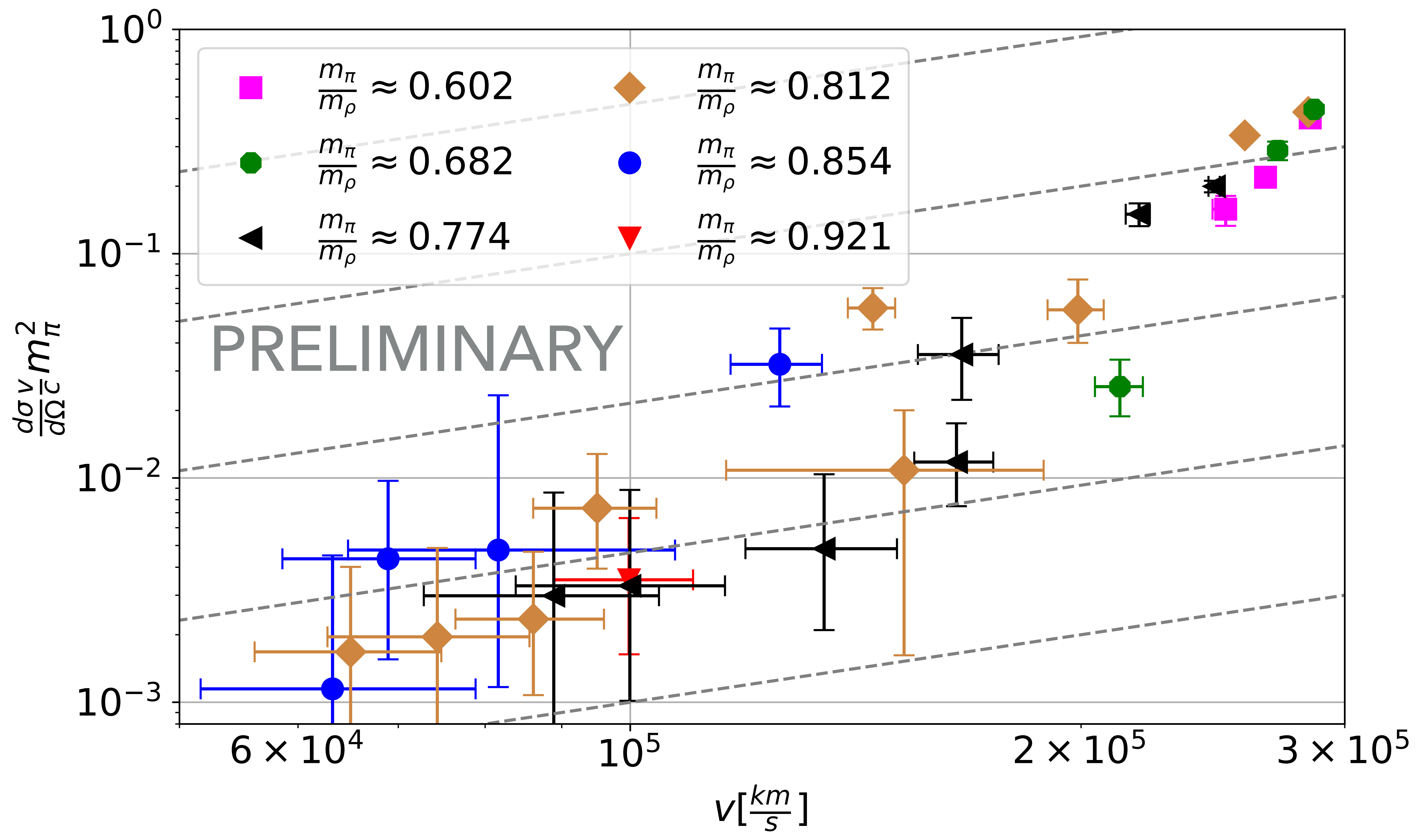
RESULTS – CROSS SECTION

- $\frac{d\sigma}{d\Omega} \propto \sigma$ (s-wave)
- $\frac{d\sigma}{d\Omega}\frac{v}{c}m_\pi^2$ vs velocity
- Do we see a hint for velocity dependence?
  - Yes, but not the one from the motivation



Cross-section vs velocity

SUMMARY & OUTLOOK

- Mass spectrum of Sp(4)
- Full Lüscher Analysis of „dark Pion" $2 \to 2$ scattering
- Constraint on dark matter particle mass

- Solve GEVP with relative momenta
- All isospin channels
- $3 \to 2$ scattering

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- All isospin channels
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THANK YOU!

BACKUP – SCATTERING (LATTICE MOMENTA)

- Scattering phase-shift from „lattice momenta"
  - Finite volume effects correspond to scattering properties
  - $q = |\vec{p}|\frac{L}{2\pi}, \quad \cosh\left(\frac{E_{\pi\pi}}{2}\right) = \cosh(m_\pi) + 2\sin^2\left(\frac{|\vec{p}|}{2}\right)$
  - Valid for $2m_\pi < E_{\pi\pi} < 4m_\pi$
- $\tan(\delta_0(q)) = \frac{\pi^{\frac{3}{2}} q}{\mathcal{Z}_{00}^{0}(1,q^2)}$

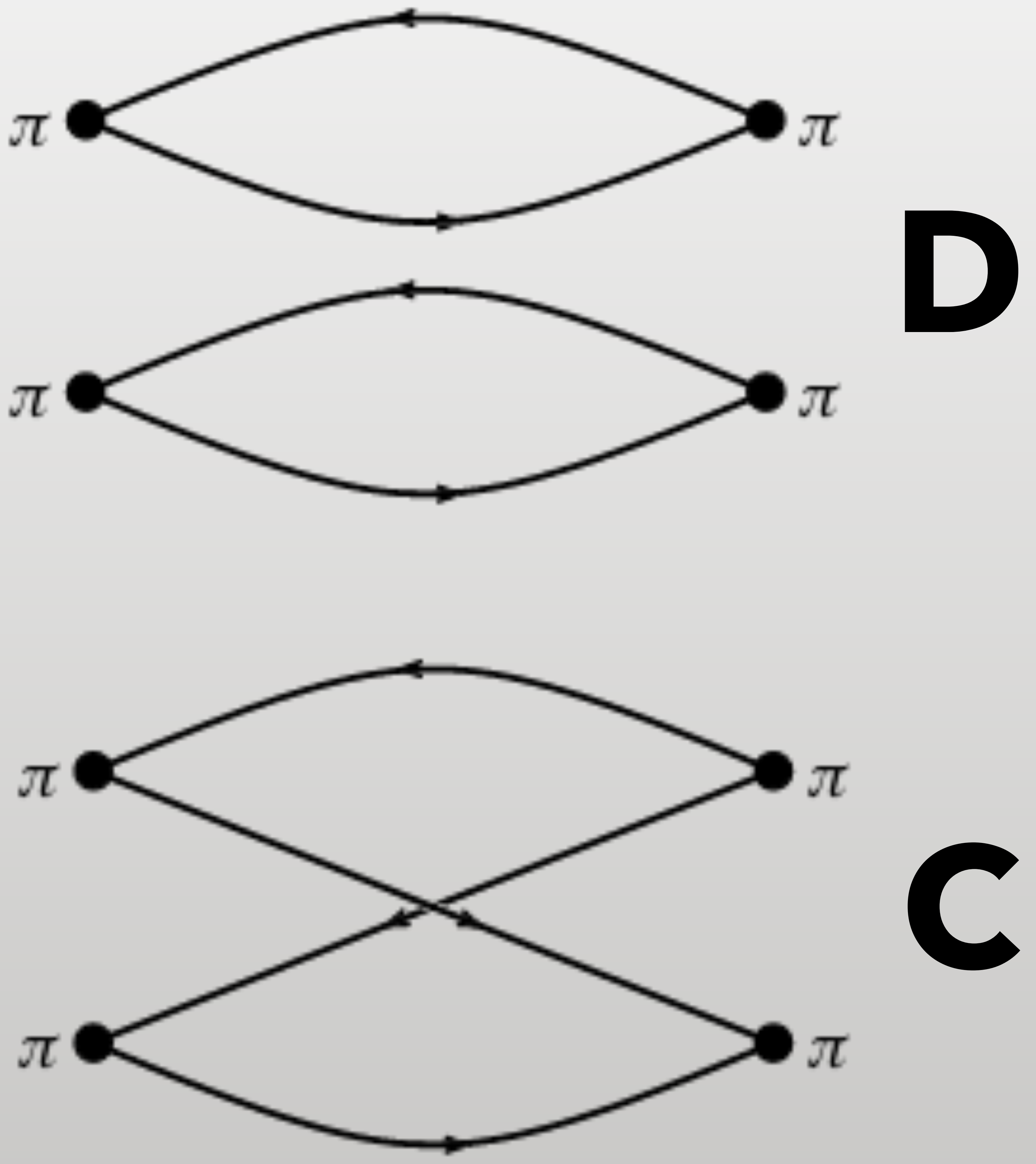
BACKUP – SIMP

BACKUP - WICK-CONTRACTIONS - 2-π-SCATTERING

- In $Sp(4)_F$
  - 5-plet of pseudo scalar (5 dark „Pions")
  - $\pi^+, \pi^-, \pi^0, \Pi_{ud}, \Pi_{\bar{u}\bar{d}}$
- 3 Isospin channels:
  - $5 \otimes 5 = 1 \oplus 10 \oplus 14$ (Isospin I=0,1,2)

BACKUP - WICK-CONTRACTIONS - I=2

- For I=2: One Operator sufficient
  - π^+ for example:
- $\mathcal{O}_{\pi\pi} = \mathcal{O}_{\pi^+}\mathcal{O}_{\pi^+} = \left(\bar{d}\gamma_5 u\right)\left(\bar{d}\gamma_5 u\right)$
- $C_{\pi\pi} = < \mathcal{O}_{\pi\pi}\mathcal{O}^{\dagger}_{\pi\pi} > = 2D - 2C$
- Same as in QCD



Blum et al. - [2301.09286]

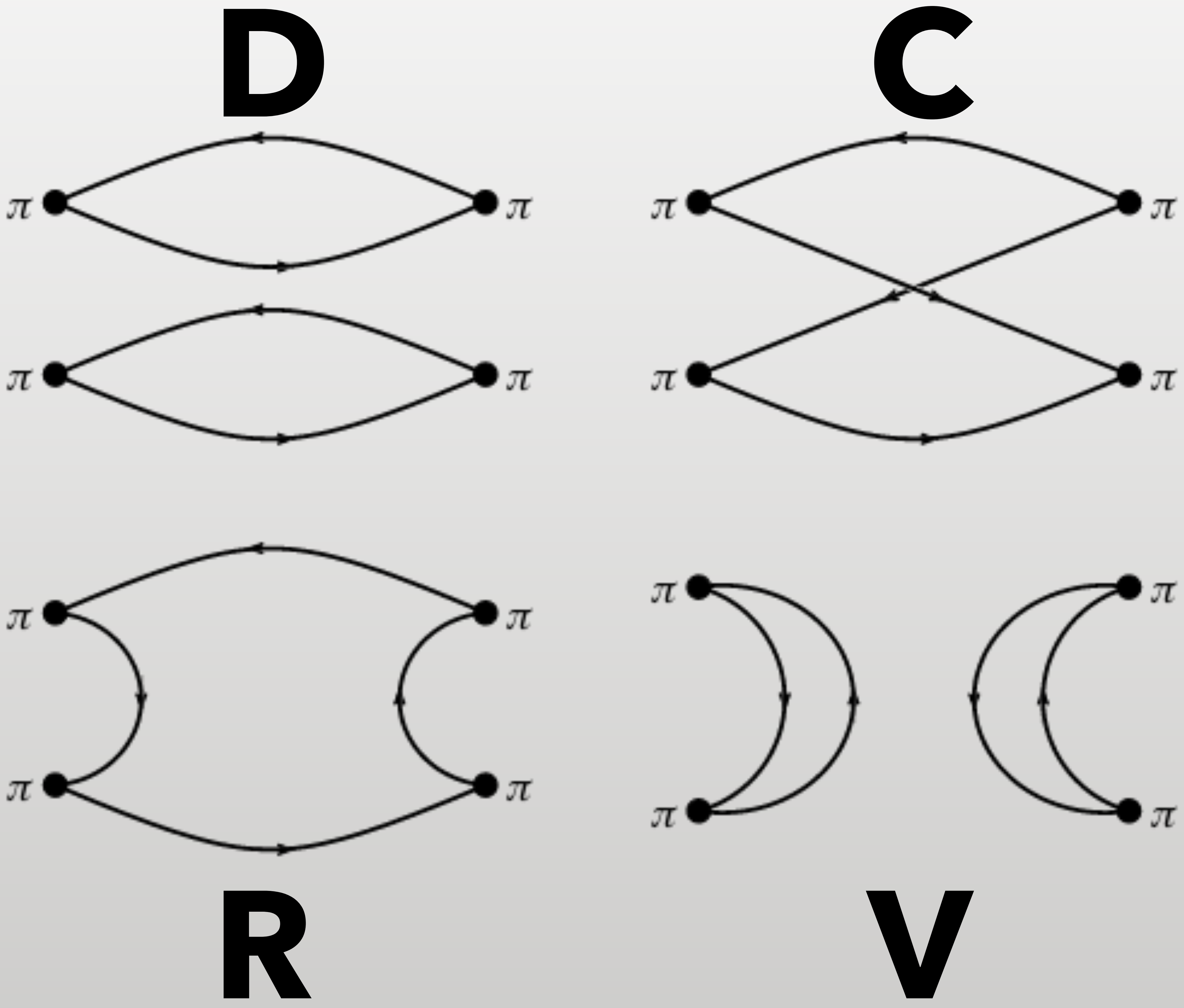
BACKUP - WICK-CONTRACTIONS - I=0

- For I=0: Contraction of all 5 „Pions
  - $\mathcal{O}(\pi\pi, I=0) = \frac{1}{\sqrt{5}}(\pi^+\pi^- + \pi^-\pi^+ - \pi^0\pi^0 + \Pi_{ud}\Pi_{\bar{u}\bar{d}} + \Pi_{\bar{u}\bar{d}}\Pi_{ud})$
- 25 Terms with 4 ψ's and 4 $\bar{\psi}$'s
  - 6 similar groups
  - 344 terms
- $C_{\pi\pi}^{I=0} = 2D + 3C - 10R + 5V$

Blum et al. - [2301.09286]

BACKUP - WICK-CONTRACTIONS - I=0

- We confirm the result of [2107.09974]
  - $C_{\pi\pi}^{I=0} = 2D + 3C - 10R + 5V$
- QCD case:
  - $C_{\pi\pi,QCD}^{I=0} = 2D + C - 6R + 3V$
- „Noisy" vacuum term V



Blum et al. - [2301.09286]

BACKUP – MOMENTUM ON THE LATTICE

- Momentum via Fourier transform:
  - $C(t) = \sum_{\vec{x}\vec{y}} e^{i(\vec{x}\vec{p}_x - \vec{y}\vec{p}_y)} \left\langle \mathcal{O}(\vec{x})\mathcal{O}^\dagger(\vec{y}) \right\rangle$
- In 2->2 Scattering 4 π involved
  - -> 4 momenta ($P_{in} = P_{out}$)
  - 3 independent momenta

BACKUP – MOMENTUM ON THE LATTICE

- Total momentum $\sum \vec{p} \neq 0$
  - Probes different energy levels
  - Introduces noice
- Relative momenta between π $\sum \vec{p} = 0$
  - Probes the same energy
  - Might yield higher overlap with states

BACKUP – VARIATIONAL ANALYSIS

- Extraction of energy states via Generalized Eigenvalue Problem (GEVP)
- Calculate cross-correlator matrix from an operator basis:
  - $C_{ij}(t) = \left\langle \mathcal{O}_i(t)\mathcal{O}_j^\dagger(0) \right\rangle$
  - $\mathcal{O}_i = \mathcal{O}_{\pi\pi}(t, \vec{p} = \vec{p}_i)$
  - $\vec{p}_0 = (0,0,0), \vec{p}_1 = (1,0,0), \vec{p}_1 = (1,1,0) \dots$
- $\lambda_k(t) \propto e^{-E_k t}(1 + \mathcal{O}(e^{-\Delta E t}))$, for Eigenvalues of C_{ij}

Blum et al. - [2301.09286]

BACKUP – DARK EFT (CHI PT)

- Low energy description of dark sector
  - „Dark Pions“ are the fundamental degrees of freedom (χPT)
- Low energy constants from lattice
  - Masses, decay constants, etc.
- 3->2 interaction via Wess-Zumino-Witten Term

BACKUP - LATTICE

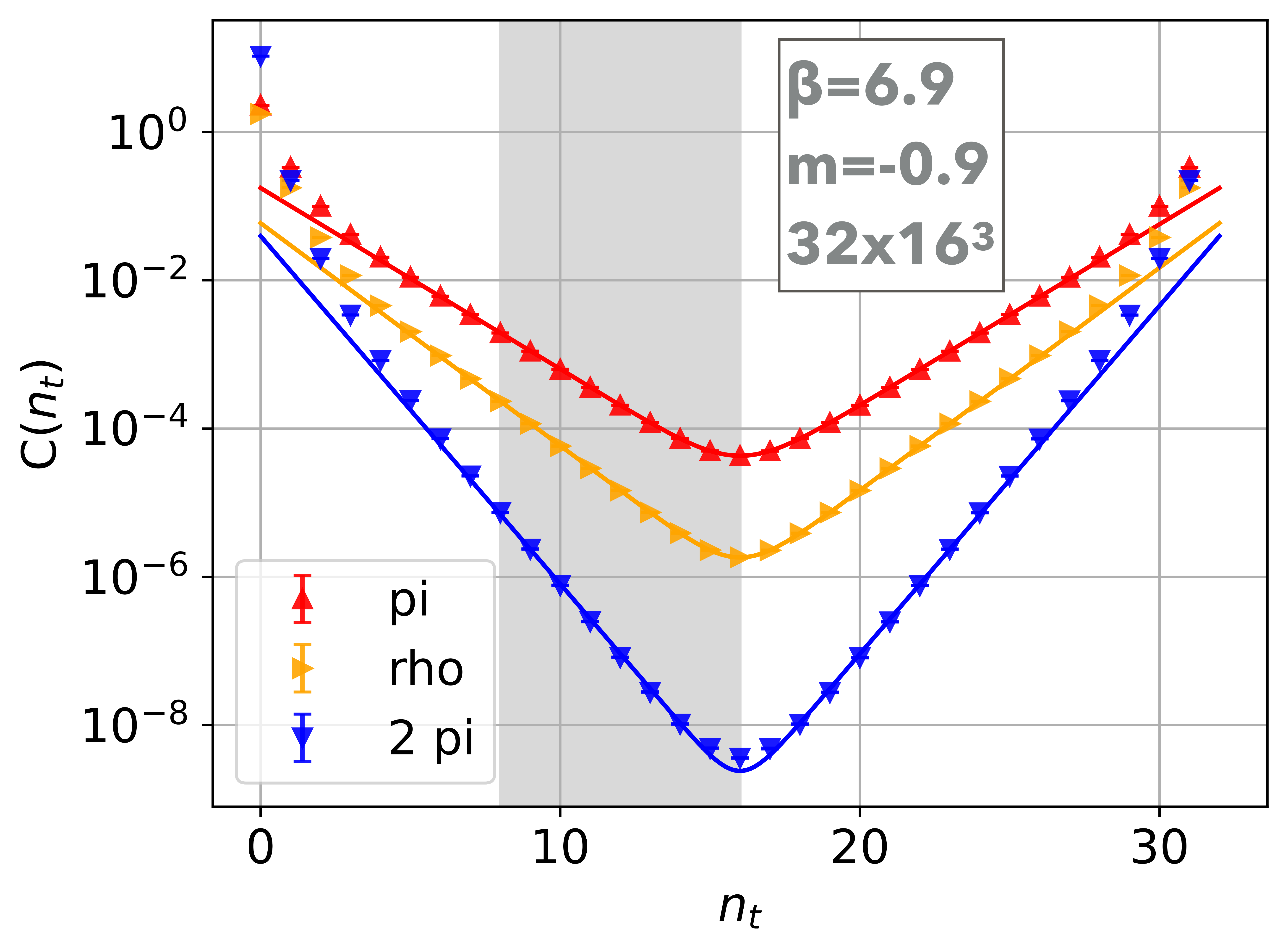
- Two sets of parameters:
  - $\beta = 6.9$, $m_u = m_d = -0.9$
    - $\frac{m_\pi}{m_\rho} \approx 0.81$
  - $\beta = 7.2$, $m_u = m_d = -0.78$
    - $\frac{m_\pi}{m_\rho} \approx 0.78$

LxT\β	6.9	7.2
20×10^3	1273	195
24×12^3	2904	150
24×14^3	942	425
32×16^3	546	265
48×12^3	251	X
64×12^3	94	X

- Number of uncorrelated configurations per choice of parameters

BACKUP – CORRELATOR

- Correlation function
  - Only for the largest lattice (32×16^3)
- Fit works fine for π and ρ but not for ππ
- Correlator of ππ has lowest values



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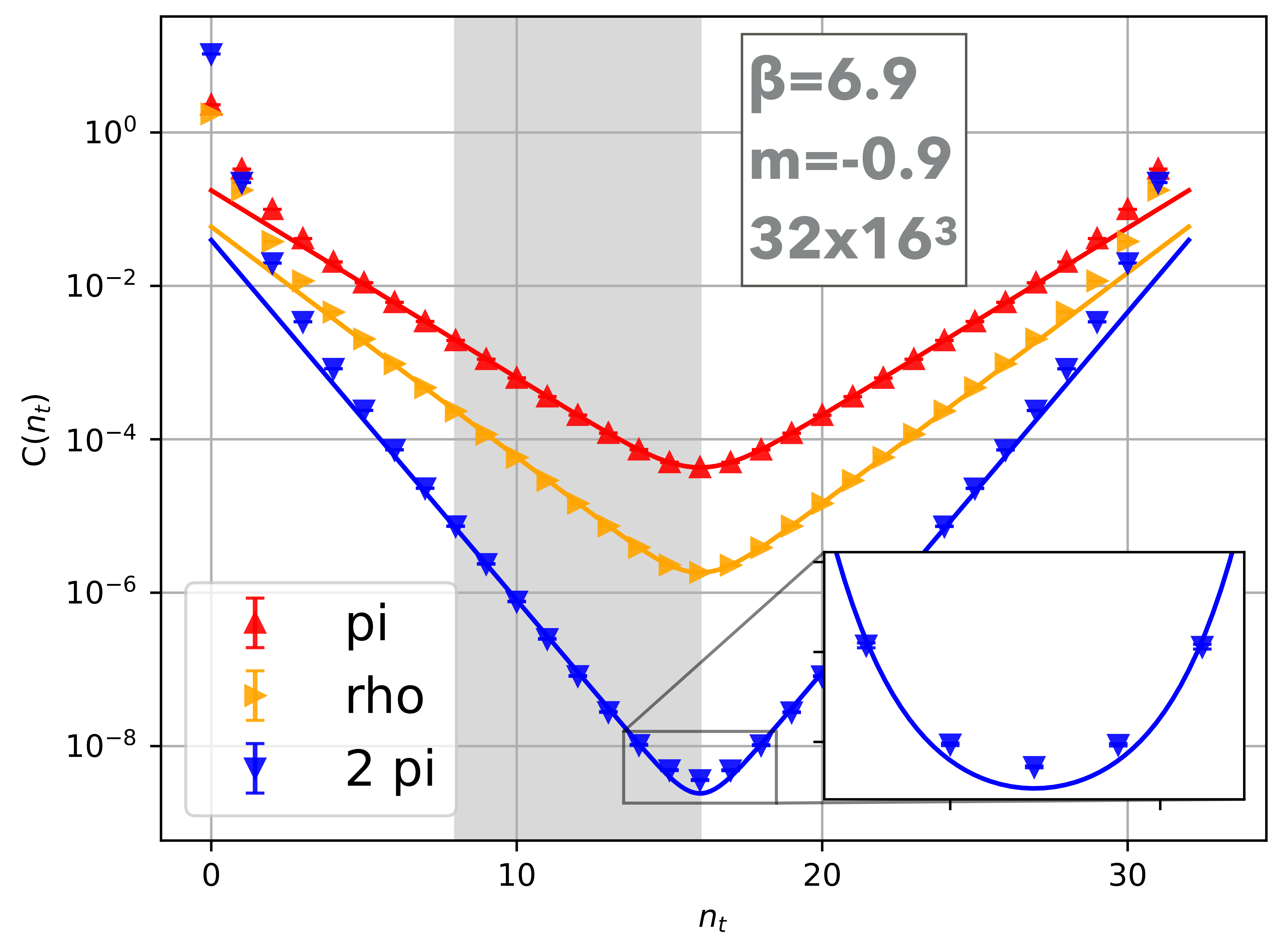
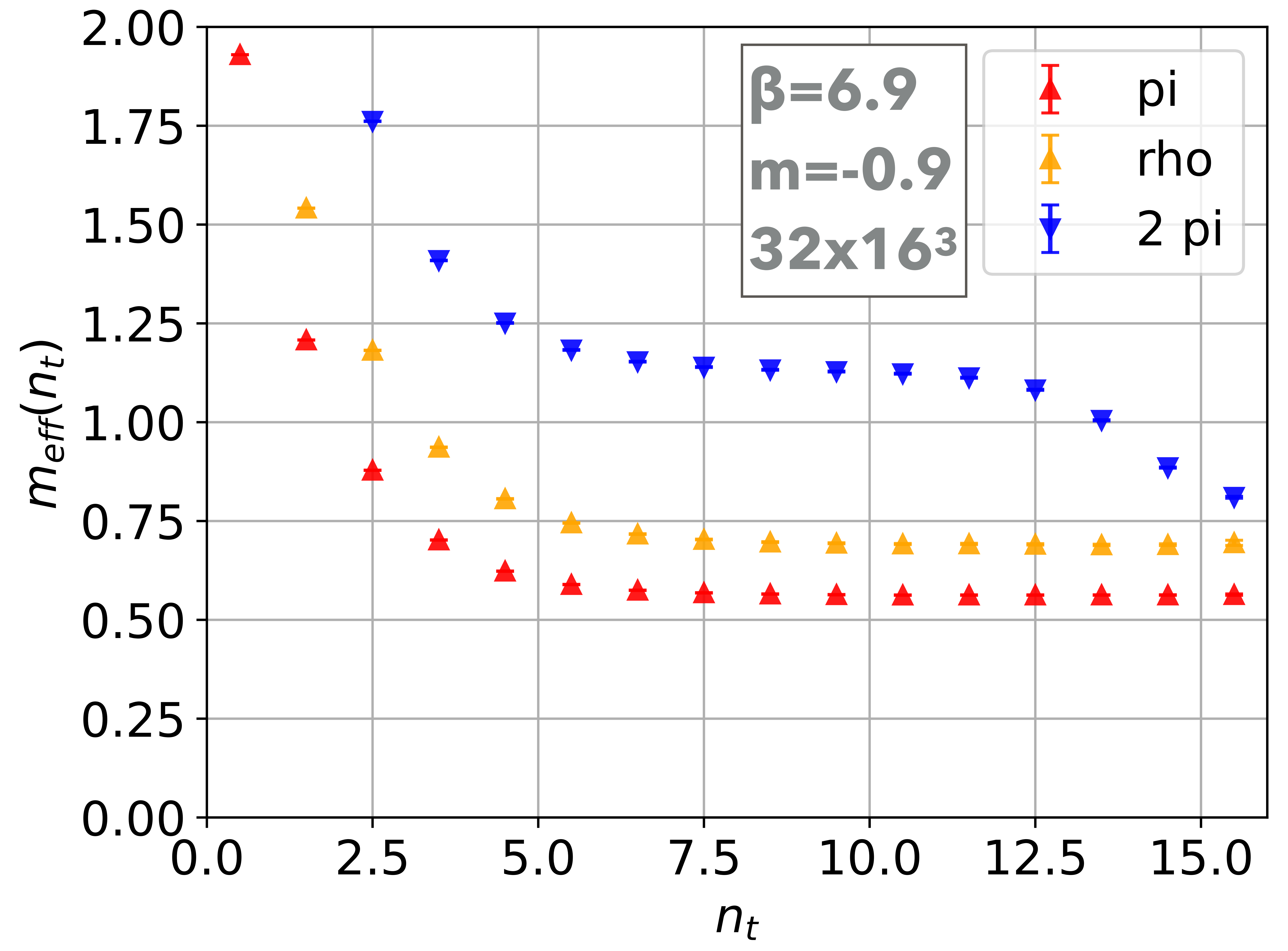


BACKUP – EFFECTIVE MASS

- Effective mass should show plateau when ground state dominates
- Second drop off closer to the center of the lattice
  - Low values of the correlator
  - Needs further investigation



BACKUP – DERIVATIVE METHOD

- Redefinition of C with its derivative
- Constant cancels

$$\tilde{C}(n_t+1) = C(n_t) - C(n_t+2)$$
$$\tilde{C}(n_t+1) \propto sinh((\frac{N_T}{2} - n_t)E_0)$$

- Downside: Loss of two time-steps

BACKUP – EFFECTIVE MASS

- Now: Nice plateaus for every correlator