Non-perturbative RG $\beta$-function of 8-flavor SU(3) gauge theory

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in collaboration with Anna Hasenfratz
Why $N_f = 8$?

- Popular BSM model
  - e.g., LatKMI*, LSD* and Appelquist et al.*
    - Consistent with both conformal hyperscaling and dilaton $\chi$PT
- No evidence of chiral symmetry breaking even at much stronger couplings*
- Must simulate in strongly-coupled regime to better understand infrared dynamics
  - Limited by bulk first-order phase transition (PT)

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Objective

- Access strongly-coupled regime
  - Pauli-Villars improvement†
- Untangle strongly-coupled behavior
  - Continuous $\beta$-function$x$

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  *[LatKMI PRD 96, 014508 (2017)]
  *[(LSD Collaboration PRD 99, 014509 (2019))
  *[Appelquist, T., Ingoldby, J., Piai, M. PRD 126, 191804 (2021)]
  *[(LSD Collaboration, arXiv:2305.03665)
  *[(Hasenfratz, A. PRD 106, 014513 (2022))]

†[(Hasenfratz, Mon. 13:30]
  †[(Shamir, Mon. 13:50]
  †[(Witzel, Mon. 14:30]
  †[(Kuti, 14:50]
Realizing our objective

Access strongly-coupled regime

- **Problem**: bulk first-order phase transitions (PT)
  - Triggered by strong UV fluctuations
  - Occur at finite $g_0^2$
- **Partial resolution**: introduce heavy Pauli-Villars bosons*
  - Counteract UV fluctuations
  - Push PT to larger $g_0^2$

Untangle strongly-coupled behavior

Continuous $\beta$-function method (CBFM)*

\[
g^{2}_{GF}(t; L, g_0^2) \sim \langle t^2 E(t) \rangle^* \\
\beta_{GF}(t; g_0^2) \equiv -t \frac{d}{dt} g^{2}_{GF}(t; g_0^2)
\]

1. $L/a \to \infty$ extrapolation of $g^{2}_{GF}(t; L, g_0^2)$ at fixed $\beta_b$ & $t/a^2$
2. $a^2/t \to 0$ extrapolation of $\beta_{GF}(t; g_0^2)$ at fixed $g^{2}_{GF}$

* $E(t)$ is the Yang-Mills energy density; we consider Wilson & clover “operators”

* [Hasenfratz, A., Shamir, Y., Svetitsky, B. PRD 104, 074509 (2021)]
The $\beta$-function

Finite-size scaling* suggests renormalization group (RG) $\beta$-function just touches zero

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*Hasenfratz, A. PRD 106, 014513 (2022)

*Artz, Harlander, Lange, Neumann, Prausa JHEP 06 (2019) 121

*Hasenfratz, A., Witzel, O., Rebbi, C. PRD 107, 114508 (2023)
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Simulation details

- nHYP-smeared staggered fermions &
  - Pauli-Villars (PV) improvement
  - Use MILC* & Quantum EXpressions* (QEX)
  - Symmetric volumes ($L/a = 24, 32, 36, 40$)
    - (Anti-““)periodic BC’s for
    - fermion(gauge)
  - $8.8 \leq \beta_b \equiv 6/g_0^2 \leq 9.9$ (8 total)
  - $am_f = 0$

- Gauge flows (GF) run with MILC & QEX
  - Run Wilson flow & modified rectangle flow*
  - Measure Wilson & clover operator

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**Pauli-Villars action**

<table>
<thead>
<tr>
<th>8 degenerate PV/fermion</th>
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<tbody>
<tr>
<td>$am_{PV} = 0.75$</td>
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**Flow action**

$$S_f = c_p S_p + c_r S_r \quad (c_p + 8c_r = 1)$$

- $c_p = 1 \rightarrow$ “Wilson flow”
- $c_p = 1/3 \rightarrow$ “C13 flow”

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*David Schaich’s modified MILC code: github.com/daschaich/KS_nHYP_FA

*QEX main branch: github.com/jcosborn/qex

*Curtis Peterson’s fork of QEX: github.com/ctpeterson/qex

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5/9 Lattice 2023
Infinite volume extrapolation

Extrapolate \( g_{GF}^2(t; L, \gamma_0^2) \)
linearly in \( (a/L)^4 \rightarrow 0 \) at fixed \( \beta_b \) and \( t/a^2 \)

C13 flow
(see supplement for Wilson flow)
Interpolation

Interpolate $\beta_{GF}(t; g_0^2)$ in $g_{GF}^2(t; g_0^2)$ at fixed $t/a^2$

We use a cubic polynomial

Extended reach
Continuum Extrapolation

- Linear extrapolation in $a^2/t$ at fixed $\tilde{g}^2_{GF}(t; \tilde{g}^2_0)$
  - $2.6 \leq t/a^2 \leq 4.2$

- WW & C13W consistent
  - WC also consistent
  - C13C plagued by nonlinear discretization effects
    - Under investigation

![Graph showing extrapolation results]

Preliminary results Lattice 2023
Conclusions & future directions

- Range of $g_{GF}^2$ extended 2x
  - Closest IRFP possibly at $g_{GF}^2 > 22$
  - No signs of $\chi$SB up to $g_{GF}^2 \sim 22$
  - Negative curvature $8 \lesssim g_{GF}^2 \lesssim 22$
- Overlap between 8PV staggered and previously published results
- Generating & analyzing step-scaling ensembles
- Simulating on strong-coupling side of phase transition
- Extending finite-size scaling ensembles

*Artz, Harlander, Lange, Neumann, Prausa JHEP 06 (2019) 121
*Hasenfratz, A., Witzel, O., Rebbi, C. PRD 107, 114508 (2023)
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GF $\beta$-function

- GF describes a real-space RG transformation in infinite volume when combined with appropriately-defined coarse graining step

- Define a renormalized running coupling $(\mu^2 \propto 1/8t)$
  - Common choice in LGT studies is to use the flowed Yang-Mills energy density, since it does not renormalize

$$g_{GF}^2(t; g_0^2) \equiv \mathcal{N}\langle t^2 E(t) \rangle$$

- Describes flow along renormalized trajectory with corresponding $\beta$-function

$$\beta_{GF}(t; g_0^2) = -t \frac{d}{dt} g_{GF}^2(t; g_0^2)$$

*\(\mathcal{N} = 128\pi^2/3(N^2 - 1)\) chosen such that the GF coupling matches $\overline{\text{MS}}$ at tree level

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[Lüscher, M., JHEP 08 (2010) 71]
[Makino, H., Morikawa, O., Suzuki, H. PTEP 05, 099201 (2021)]
Infinite volume extrapolation

Extrapolate \( g_{GF}^2(t; L, g_0^2) \) linearly in \( (a/L)^4 \to 0 \) at fixed \( \beta_b \) and \( t/a^2 \).