Monopoles of the Dirac type and color confinement in QCD Study of the continuum limit

Tsuneo Suzuki, RCNP, Osaka University

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1. Introduction
2. Abelian monopoles of the Dirac type in QCD $J_\mu = k_\mu$
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1. Introduction

1. Color confinement in QCD is not yet solved. About half a century!!

2. 1974-75: Idea of the dual superconductor (electric $\leftrightarrow$ magnetic) as the color-confinement mechanism ('tHooft-Mandelstam): Something color magnetic must be condensed in QCD.

3. QCD is however composed of color electric quark and gluon fields alone.

4. No one could not have found such a color magnetic quantity in QCD without artificial assumptions like additional partial gauge-fixing.

5. 2014: Violation of non-Abelian Bianchi identity if exists is equal to Abelian-like magnetic currents. T. Suzuki, arXiv:1402.1294 (2014)

2. Abelian magnetic monopoles of the Dirac type in QCD

Note the Jacobi identities:

$$\epsilon_{\mu\nu\rho\sigma}[D_\nu, [D_\rho, D_\sigma]] = 0,$$

where $D_\mu \equiv \partial_\mu - igA_\mu$. Calculate explicitly:

$$\begin{aligned}
[D_\rho, D_\sigma] &= [\partial_\rho - igA_\rho, \partial_\sigma - igA_\sigma] \\
&= -ig(\partial_\rho A_\sigma - \partial_\sigma A_\rho - ig[A_\rho, A_\sigma]) + [\partial_\rho, \partial_\sigma] \\
&= -igG_{\rho\sigma} + [\partial_\rho, \partial_\sigma] \\
f_{\mu\nu} &\equiv \partial_\mu A_\nu - \partial_\nu A_\mu = (\partial_\mu A^a_\nu - \partial_\nu A^a_\mu)\sigma^a/2
\end{aligned}$$

$$\begin{aligned}
[\partial_\rho, \partial_\sigma] = 0 \quad &\rightarrow \quad D_\nu G^*_{\mu\nu} = 0 : \text{Non-Abelian Bianchi identity (NABI)} \\
&\rightarrow \quad \partial_\nu f^*_{\mu\nu} = 0 : \text{Abelian-like Bianchi identity,}
\end{aligned}$$

Jacobi identity, $[D_\nu, G_{\rho\sigma}] = D_\nu G_{\rho\sigma}$, $[\partial_\rho, \partial_\sigma] \neq 0$

$$\Downarrow$$

$$\begin{aligned} D_\nu G^*_{\mu\nu} &= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma} D_\nu G_{\rho\sigma} \\ &= -\frac{i}{2g}\epsilon_{\mu\nu\rho\sigma}[D_\nu, [\partial_\rho, \partial_\sigma]] \\ &= \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}[\partial_\rho, \partial_\sigma]A_\nu = \partial_\nu f^*_{\mu\nu} \\ J_\mu = \frac{1}{2}J^a_\mu \sigma^a = D_\nu G^*_{\mu\nu} &= \partial_\nu f^*_{\mu\nu} = \frac{1}{2}k^a_\mu \sigma^a = k_\mu \end{aligned}$$

Color magnetic monopoles= Violation of non-Abelian Bianchi identity (VNABI) :Reference T.Suzuki,arXiv:1402.1294 (2014), C. Bonati et al,, P.R.D81, 085022 (2010)

$$[\partial_\rho, \partial_\sigma]A_\nu \neq 0$$

$$\Downarrow$$

Line singularities existing in gauge fields $A_\mu(x)$ themselves!!! can be the origin of Abelian monopoles in QCD. $N^2 - 1$ monopoles of the Dirac type exist in $SU(N)$.

3. Lattice studies of the new QCD magnetic monopoles

Lattice monopole after Abelian projection

Maximize $R = \sum_{s,\mu} Re\,\mathrm{Tr}\, e^{i\theta_1(s,\mu)\lambda_1} U^\dagger(s,\mu)$

$$\Downarrow$$

$$\begin{aligned}
\theta_1(s,\mu) &= \tan^{-1}\frac{Im(U_{12}(s,\mu)+U_{21}(s,\mu))}{Re(U_{11}(s,\mu)+U_{22}(s,\mu))} \\
\theta_1(s,\mu\nu) &= \partial_\mu\theta_1(s,\nu)-\partial_\nu\theta_1(s,\mu) = \bar{\theta}_1(s,\mu\nu)+2\pi n_1(s,\mu\nu) \quad (|\bar{\theta}_1(s,\mu\nu)|<\pi) \\
k_\mu^1(s) &= -(1/2)\epsilon_{\mu\alpha\beta\gamma}\partial_\alpha\bar{\theta}_1(s+\hat{\mu},\beta\gamma)
\end{aligned}$$

$$\begin{aligned}
W_A^a &= \exp\{i\sum J_\mu^{ext,a}(s)\theta_\mu^a(s)\} = W_{mon}^a W_{ph}^a \\
W_{mon}^a &= \exp\{2\pi i\sum k_\beta^a(s)D(s-s^{'})\frac{1}{2}\epsilon_{\alpha\beta\rho\sigma}\partial_\alpha M_{\rho\sigma}^a(s^{'})\},
\end{aligned}$$

where $J_\nu^{ext,a} = \partial' M_{\mu\nu}^a(s)$.

Study of the continuum limit in SU3

(1) Perfect Abelian and monopole dominances

The Abelian dual Meissner pciture $\Longrightarrow$ $\sigma_F = \sigma_a = \sigma_m$.
(1) Studies on small lattices without additional assumptions:

Lattice size	β	σ_a/σ_F
12^4	5.6	0.87(13)
16^4	5.6	1.05(9)
12^4	5.7	0.91(8)
12^4	5.8	1.01(11)

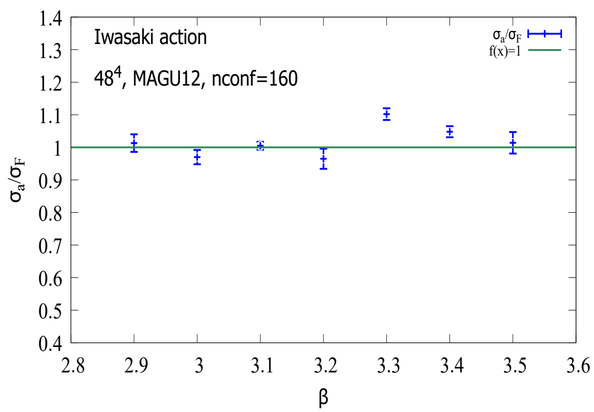
Table 1: σ_a/σ_F determined by applying the multilevel method in the Wilson action

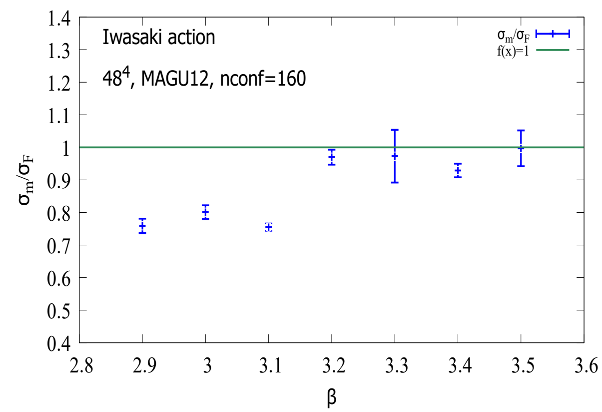
Types of the potential	σa^2
non-Abelian	0.178(1)
Abelian	0.16(3)
monopole	0.17(2)
photon	-0.0007(1)

Table 2: String tensions from Polyakov-loop correlations in the Wilson action at $\beta = 5.6$ on $24^3 \times 4$

(2) Studies on large lattice for various β in MAG gauge:

(1)160 configurations of Iwasaki gluonic action on 48^4 for $\beta = 2.9 \sim 3.5$.
(2)the maximally Abelian (MA) + $U(1) \times U(1)$ gauge (MAU12)





(2). Existence of the continuum limit

Does the continuum limit of $k^a(s,\mu)$ exist?

Study the monopole density in the continuum limit in pure SU3 QCD.

The lattice vacuum is contaminated with large amount of lattice artifact monopoles. To reduce lattice artifacts, various techniques smoothing the vacuum are introduced.

1. Iwasaki action: 48^4 at $\beta = 2.3 \sim 3.5$:

2. Introduction of smooth gauge-fixings
Maximal Abelian and $U(1)^2$ Landau gauge (MAGU12): Maximization of
$R = \sum_{s,\mu}(U_\mu^\dagger(s)\vec{H}U_\mu(s)\vec{H}) \quad \vec{H} = (\lambda_3, \lambda_8)$

3. The blockspin transformation of monopoles

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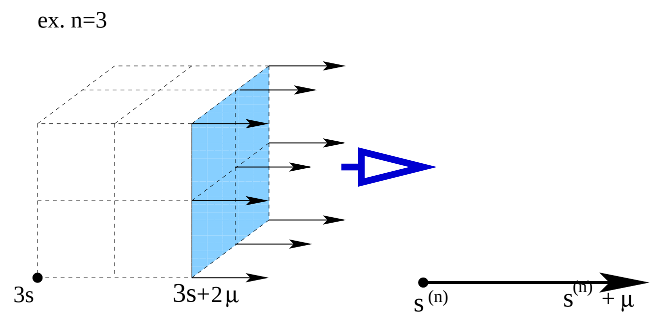

Figure 3: Blockspin definition of monopoles:
T.L. Ivanenko et al., Phys. Lett. **B252**, (1990) 631

Monopole is defined on a a^3 cube and the n-blocked monopole is defined on a cube with a lattice spacing $b = na$

$$\boldsymbol{k}_{\mu}^{(n)}(s_n) = \sum_{i,j,l=0}^{n-1} k_{\mu}(ns_n + (n-1)\hat{\mu} + i\hat{\nu} + j\hat{\rho} + l\hat{\sigma})$$

$n = 1, 2, 3, 4, 6, 8, 12$ blockings adopted on 48^4 lattice.

Evaluate a gauge-invariant density of the n-blocked monopole:

$$\boldsymbol{\rho}(a(\beta), n) = \frac{\sum_{\mu, s_n} \sqrt{\sum_a (k_\mu^{(n)a}(s_n))^2}}{4\sqrt{8} V_n b^3}$$

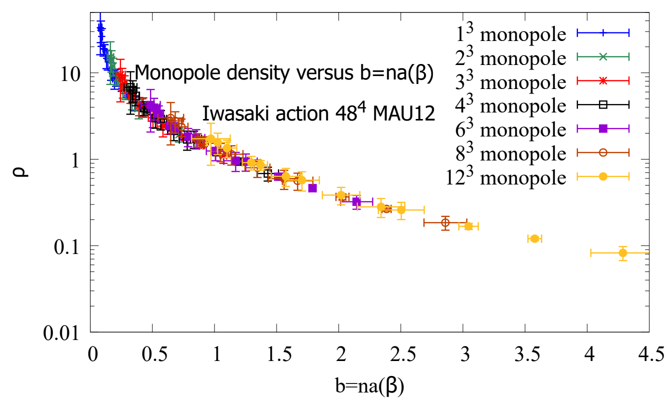

Figure 4: Monopole density versus $b = na(\beta)$

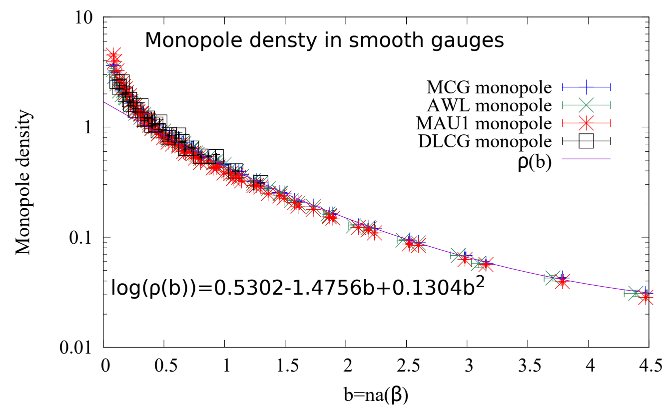

Figure 5: Monopole density behaviors versus $b = na(\beta)$ in SU2

Clear scaling behaviors are observed up to the 12-step blockspin transformations for all β adopted. The density $\rho(a(\beta), n)$ is a function of $b = na(\beta)$ alone, i.e. $\rho(b)$. $n \to \infty$ means $a(\beta) \to 0$ for fixed $b = na$.

Existence of the continuum limit!

(3) The renormalization flow studies of the monopole infrared effective action.

The effective monopole action is defined as follows:

$$
\begin{aligned}
e^{-\mathcal{S}[k]} &= \int DU(s,\mu)e^{-S(U)} \times \prod_a \delta(k^a_\mu(s) - \frac{1}{2}\epsilon_{\mu\nu\rho\sigma}\partial_\nu n^a_{\rho\sigma}(s+\hat{\mu})), \\
\mathcal{S}[k] &= \sum_i^{10} F(i)\mathcal{S}_i[k],
\end{aligned}
$$

Table 3: The quadratic interactions used for the modified Swendsen method.

coupling $\{F(i)\}$	distance	*type*
$F(1)$	(0,0,0,0)	$k_\mu(s)k_\mu(s)$
$F(2)$	(1,0,0,0)	$k_\mu(s)k_\mu(s+\hat{\mu})$
$F(3)$	(0,1,0,0)	$k_\mu(s)k_\mu(s+\hat{\nu})$
$F(4)$	(1,1,0,0)	$k_\mu(s)k_\mu(s+\hat{\mu}+\hat{\nu})$
$F(5)$	(0,1,1,0)	$k_\mu(s)k_\mu(s+\hat{\nu}+\hat{\rho})$
$F(6)$	(1,1,1,0)	$k_\mu(s)k_\mu(s+\hat{\mu}+\hat{\nu}+\hat{\rho})$
$F(7)$	(0,1,1,1)	$k_\mu(s)k_\mu(s+\hat{\nu}+\hat{\rho}+\hat{\sigma})$
$F(8)$	(2,0,0,0)	$k_\mu(s)k_\mu(s+2\hat{\mu})$
$F(9)$	(1,1,1,1)	$k_\mu(s)k_\mu(s+\hat{\mu}+\hat{\nu}+\hat{\rho}+\hat{\sigma})$
$F(10)$	(0,2,0,0)	$k_\mu(s)k_\mu(s+2\hat{\nu})$

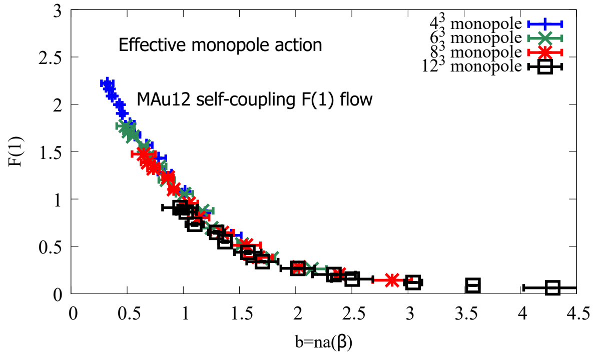

Figure 6: The self-coupling constant $F(1)$ versus $b = na(\beta)$

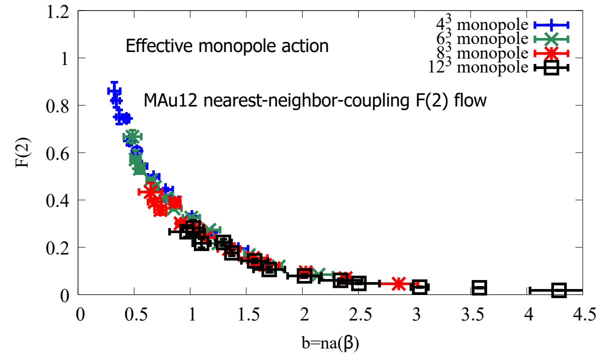

Figure 7: The nearest-neighbor coupling constant $F(2)$ versus $b = na(\beta)$.

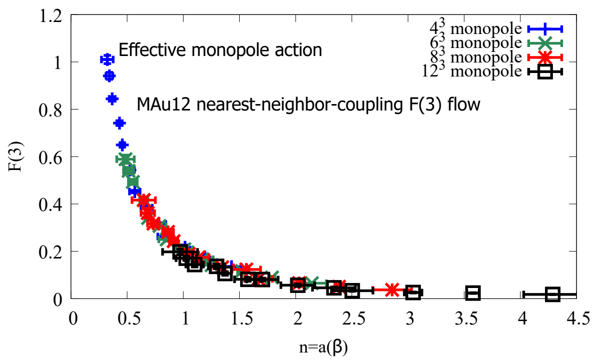

Figure 8: The The another nearest-neighbor coupling constant $F(3)$ versus $b = na(\beta)$

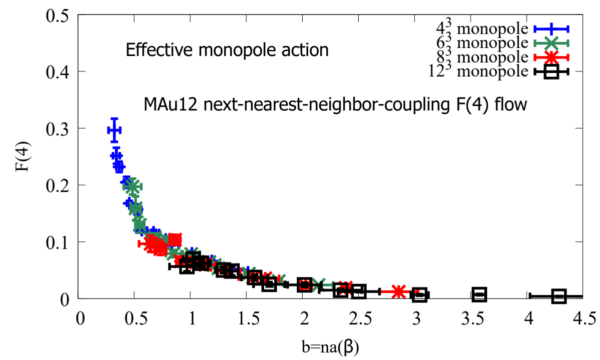

Figure 9: Next to the nearest-neighbor coupling constant $F(4)$ versus $b = na(\beta)$.

Clear scaling behaviors are observed again!

Summary and discussions

1. The perfect Abelian dominance and the perfect monopole dominance

2. The block-spin transformation studies with respect to Abelian monopoles are done. The behaviors of the monopole densities $\rho(n, a(\beta))$ of the blocked monopole currents show the beautiful scaling behavior: $\rho(n, a(\beta)) = \rho(b = na(\beta))$, i.e. ρ is a function of $b = na(\beta)$ alone. The scaling behaviors are seen here for $n = 1, 2, 3, 4, 6, 8, 12$. If on larger lattices, similar scaling behaviors are seen for $n \to \infty$, it means $a(\beta) \to 0$, the continuum limit.

3. Adopting the inverse Monte Carlo method, we determine the coupling constant flow of the effective monopole action under the blocking transformation. All coupling constants which usually a two-point function of n and $a(\beta)$ are actually found to be a function of $b = a(\beta)$ alone.

References:

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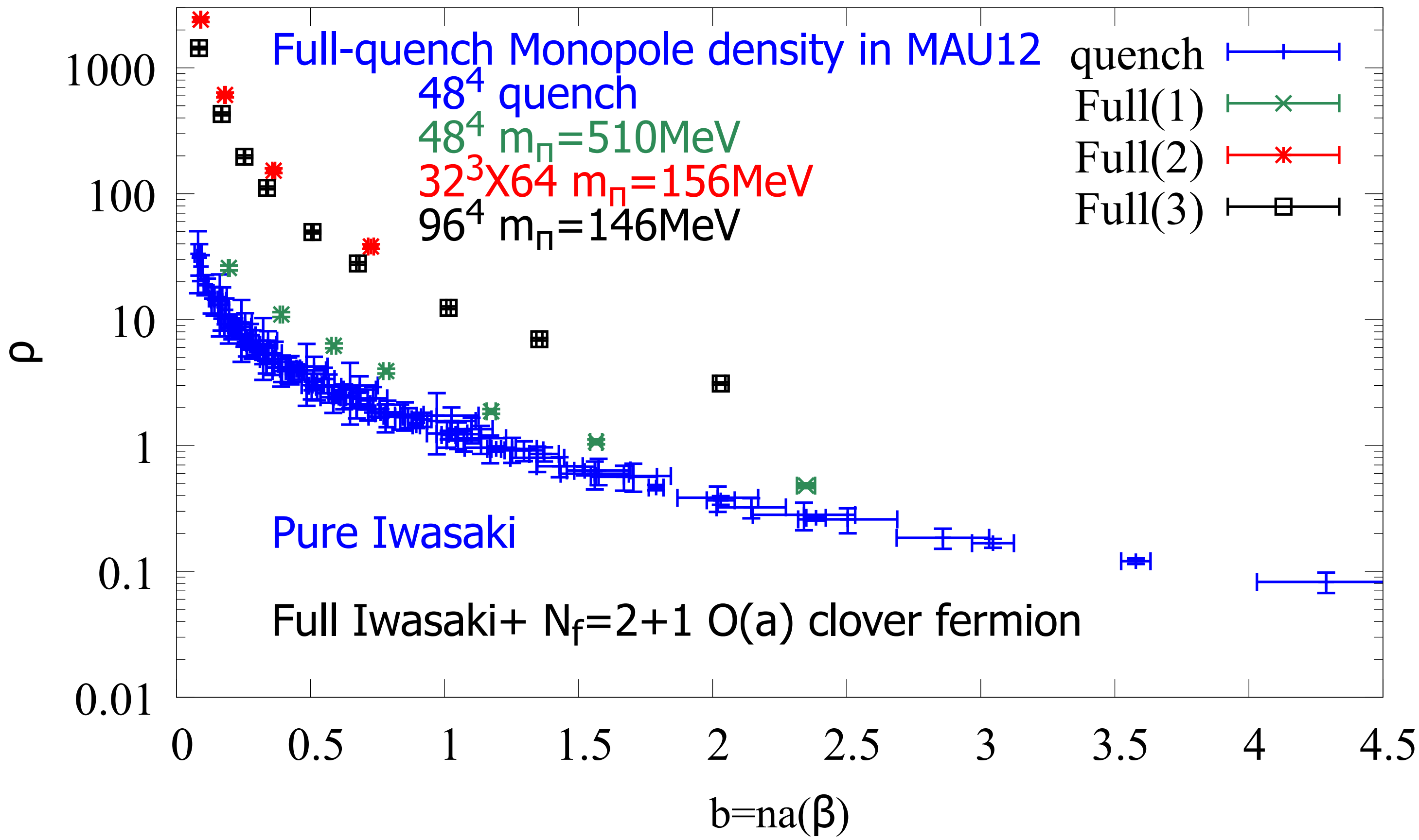

Figure 10: Comparison of monopole density in full and quench SU3