# QED Corrections to Meson Masses

Joshua Swaim

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University of Connecticut

# Motivation

- Lattice QCD calculations are becoming more precise
- QED and isospin breaking corrections are becoming important
- However, introducing QED naively leads to large finite volume errors
- The infinite-volume reconstruction method suppresses these errors
- We use this method to calculate meson and quark mass corrections

# Summary

- Introduce a perturbative method for treating QCD+QED
- Introduce the infinite volume reconstruction method
- Demonstrate that this method works for calculating meson masses
- Discuss a method for dealing with quark mass renormalization

# Perturbative QED

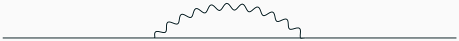
- We introduce QED perturbatively
- By expanding the path integral in the electric charge $e$, we get

$$\langle \mathcal{O}(T)\mathcal{O}(-T)\rangle_{\mathrm{QCD+QED}} = \langle \mathcal{O}(T)\mathcal{O}(-T)\rangle_{\mathrm{QCD}}$$

$$+\frac{e^2}{2}\int d^4x d^4y \langle \mathcal{O}(T) J_\mu(x) J_\nu(y) \mathcal{O}(-T)\rangle_{\mathrm{QCD}} S_{\mu\nu}(x-y) + ...,$$

where $S_{\mu\nu}(x-y)$ is the photon propagator.

- We can represent the correction diagrammatically as



# Perturbative QED

- From this expansion, we get

$$\Delta m = \frac{e^2}{2} \int d^4x \mathcal{H}_{\mu\nu}(x) S_{\mu\nu}(x),$$

- where, on the lattice,

$$\mathcal{H}_{\mu\nu}(x) = L^3 \frac{\langle \mathcal{O}(t+T) J_\mu(x) J_\nu(0) \mathcal{O}(-T) \rangle}{\langle \mathcal{O}(t+T) \mathcal{O}(-T) \rangle}.$$

- With infinite-volume hadron states, this definition corresponds to

$$\mathcal{H}_{\mu\nu}(x) = \frac{1}{2m} \langle \pi | J_\mu(x) J_\nu(0) | \pi \rangle$$

# Finite-Volume Effects

- Even at large distances, when $t >> |\vec{x}|$, $\mathcal{H}_{\mu\nu}(x)$ is order 1.
- On the other hand, the photon propagator $S_{\mu\nu}(t, \vec{x})$ is not exponentially suppressed at large $t$ because the photon is massless.
- Therefore, our finite volume errors in $\Delta m = \frac{1}{2} \int d^4x \mathcal{H}_{\mu\nu}(x) S_{\mu\nu}(x)$ will only be power-law suppressed.

# Infinite-Volume Reconstruction

- To get exponentially suppressed finite volume effects, we can reconstruct the large-distance contributions to $\Delta m$.*
- At large current separation, $\mathcal{H}_{\mu\nu}(x)$ is dominated by contributions from the lowest energy states.
- We choose some cutoff time $t_s$ that is large enough that $\mathcal{H}_{\mu\nu}(t_s, \vec{x})$ is dominated by the single-meson states, but small enough to be computed on the finite-volume lattice.
- Then we get

$$\mathcal{H}_{\mu\nu}(t, \vec{x}) \approx \int d^3\vec{x}\mathcal{H}_{\mu\nu}(t_s, \vec{x})$$

$$\times \int \frac{d^3\vec{p}}{(2\pi)^3} e^{-i\vec{p}\cdot(\vec{x}'-\vec{x})} e^{-(E_{n,\vec{p}}-m_\pi)(t-t_s)},$$

with corrections to this formula exponentially suppressed.

* Xu Feng, Luchang Jin (2019)

# Meson Mass Corrections

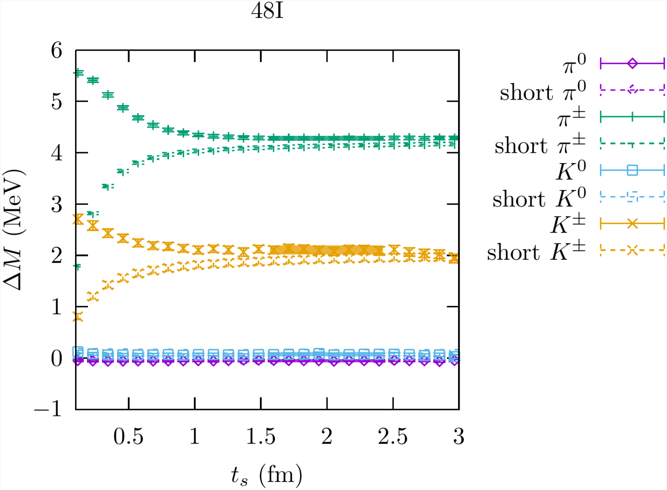



**Figure 1:** $\Delta M$ versus $t_s$ on a $48^3 \times 96$ lattice using Iwasaki gauge action and domain-wall fermions. "Short" means including only $|t| < t_s$ contributions.

# Meson Mass Corrections

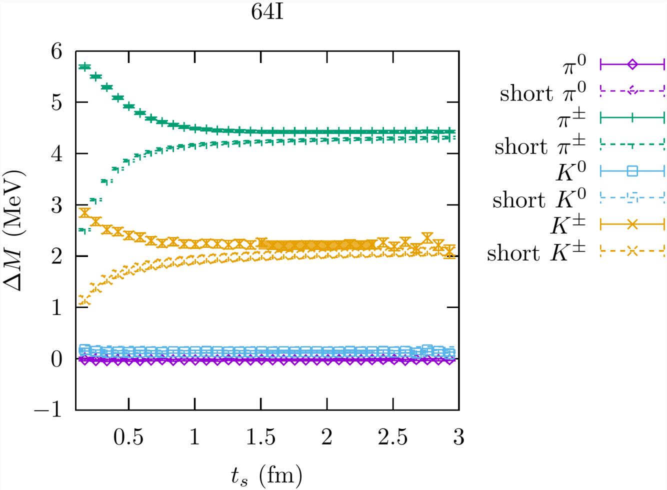



**Figure 2:** $\Delta M$ versus $t_s$ on a $64^3 \times 128$ lattice using Iwasaki gauge action and domain-wall fermions. "Short" means including only $|t| < t_s$ contributions.

# Future Work

- We need to match our simulation parameters to the physical world
- We need to perform a continuum extrapolation
- To get quark mass corrections, we need to determine QED renormalization factors

# Quark Mass Renormalization

- In QCD, the quark masses renormalize by a multiplicative constant

$$m_f^{\overline{\text{MS}}} = Z_m m_f .$$

- In QCD+QED, this renormalization is modified. We define $Z_{\text{QED}}$ by

$$m_f^{\overline{\text{MS}}} = Z_m(1 + e_f^2 Z_{\text{QED}}) m_f .$$

- To get $Z_{\text{QED}}$, we note that hadron masses are renormalization-invariant.
- Therefore, if we calculate a meson mass shift in $\overline{\text{MS}}$ versus on the lattice, we should get the same result.
- For example, let $\Delta m_\pi^{\overline{\text{MS}}}$ and $\Delta m_\pi^{\text{lat}}$ be the shifts in pion mass caused by equivalent shifts in the $\overline{\text{MS}}$ and lattice quark masses respectively.
- Then we should have

$$\Delta m_\pi^{\overline{\text{MS}}} = \Delta m_\pi^{\text{lat}} .$$

# Quark Mass Renormalization

- To the leading order, the change in hadron mass, $m_H$, due to a change in the quark mass $m_f$ and introducing an electric charge $e$ is

$$\Delta m_H = \frac{e^2}{2} \int d^4x \mathcal{H}_{\mu\nu}(x) S_{\mu\nu}(x) + \Delta m_f \mathcal{H}_f^{\text{3pt}},$$

- where $\mathcal{H}$ is the four-point function, and (in the lattice normalization)

$$\mathcal{H}_f^{\text{3pt}} = L^3 \frac{\langle \mathcal{O}_H(T) \bar{\psi}_f(0) \psi_f(0) \mathcal{O}_H(-T) \rangle}{\langle \mathcal{O}_H(T) \mathcal{O}_H(-T) \rangle}.$$

- In $\overline{\text{MS}}$, we can calculate the divergent part of the integral using the operator product expansion.
- We can compare this with the small-distance (high-momentum) contribution to this integral from the lattice.