A Tale of Two studies: the Δ resonance and role of tetraquark operators for probing the κ and $a_0(980)$ resonances

2023 LATTICE

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Introduction

Acknowledgements

Some of the results presented in this talk are published in

J. Bulava et al., Elastic nucleon-pion scattering at m_π=200 MeV from lattice QCD, *Nuclear Physics B. 987 (2023) 116105.* doi:10.1016/j.nuclphysb.2023.116105.

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Motivations for Resonances

In this presentation...

- **Δ** within $N\pi$ scattering
  - need this information for future experiments (DUNE)
  - sets us up for studying $N\pi\pi$ scattering
- $\kappa/\bar{s}u\bar{s}s$ in $K\eta$-$K\pi$ scattering
- $a_0(980)/\bar{u}u\bar{d}u$ in $K\bar{K}$-$\eta\pi$ scattering
  - see if tetraquark operators overlap with low energy states

Also check out...

- *The two-pole nature of the Λ(1405)*, Fernando Romero-López
- *The Λ(1405) from lattice QCD*, Bárbara Cid-Mora
- *Hadron spectroscopy and few-body dynamics*, Andrew Hanlon
- *To bind or not to bind*, André Walker-Loud

Methods

Notes on Operator/Correlator Construction

Operator Notes:

- Gluons $\rightarrow$ Stout smearing
- Quarks $\rightarrow$ LapH smearing

Correlator Notes:

- stochastic factorization $\rightarrow$ tensor contraction
- efficient algorithm $\rightarrow$ produce many different correlators

GEVP

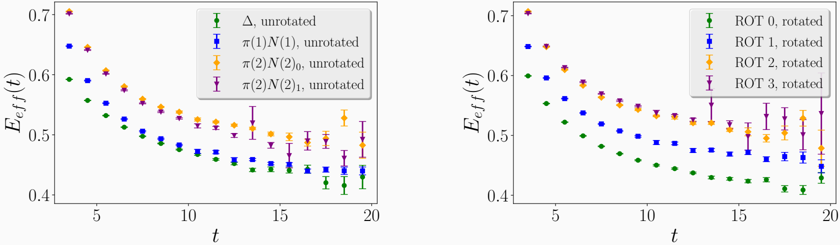
Correlation matrix elements in the same channel share the same FV energy levels

$$\langle 0|\mathcal{O}_i(t+t_0)\overline{\mathcal{O}}_j(t_0)|0\rangle = \sum_{n=0}^{\infty} Z_i^{(n)} Z_j^{(n)} e^{-E_n t}$$

Separate out by solving GEVP of $N \times N$ matrix and eigenvalues are

$$\lim_{t\to\infty} \lambda_n(t) \approx b_n e^{-E_n t}$$

Example ($N\pi$, $I = 3/2$, $H_g(0)$):



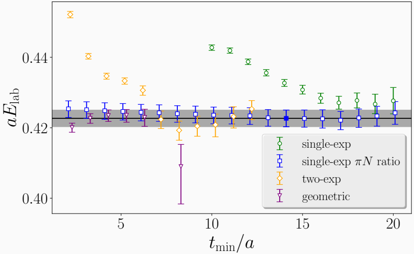
Finite-Volume Energy Spectrum

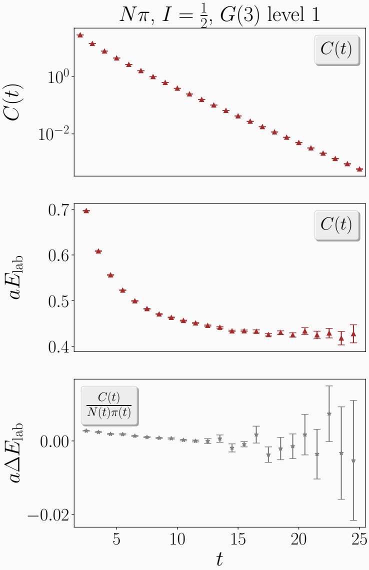
Fitting methods:

- single-exp: Ae^{-Et}
- double-exp: $Ae^{-Et}(1 + Re^{-D^2t})$
- geometric: $Ae^{-Et}/(1 - Re^{-Dt})$

Ratio:

$$R(t) = \frac{\lambda_n(t)}{C_1(t)C_2(t)}$$



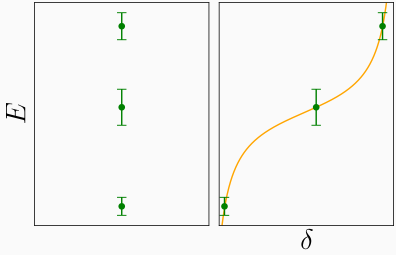


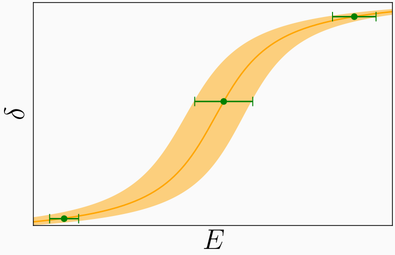
Phase Shifts/Amplitude Analysis

Connect finite-volume to infinite-volume via Lücsher:

$$\det[\widetilde{K}^{-1}(E_{\mathrm{cm}}) - B^{P}(E_{\mathrm{cm}})] = 0$$

- truncate higher waves
- $\widetilde{K}$ - related to the usual scattering K-matrix
- B^P ('box matrix') - finite volume irreps
- only works for 2-2 scattering





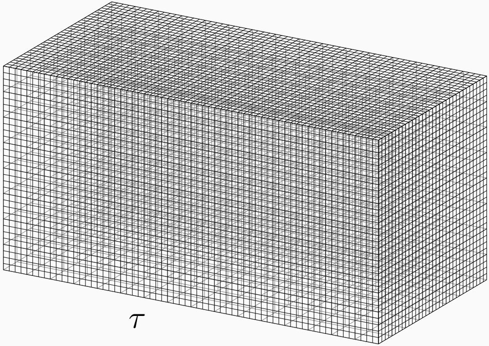
Results

D200 Computational Details

- CLS Lattice
- Dim ($x^3 \times t$):

$$64^3 \times 128$$

- $a = 0.064fm$
- m_π = 200 MeV
- m_K = 480 MeV
- 2000 configurations
- open temporal boundary conditions
- $N_f = 2 + 1$



Delta Resonances

$$N\pi \to N\pi$$

Correlation Matrix Information:

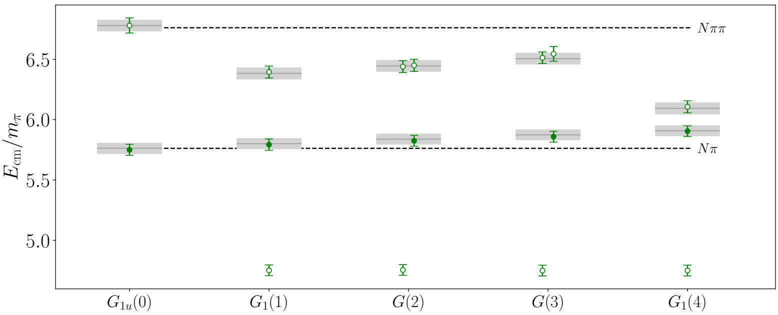
$a_{N\pi}^{I=1/2}$

- $I = 1/2$
- operators:
  - N
  - $N\pi$
- momenta: $\mathrm{d}^2 = 0, 1, 2, 3, 4$

$\boldsymbol{\Delta}(1232)$, $a_{N\pi}^{I=3/2}$

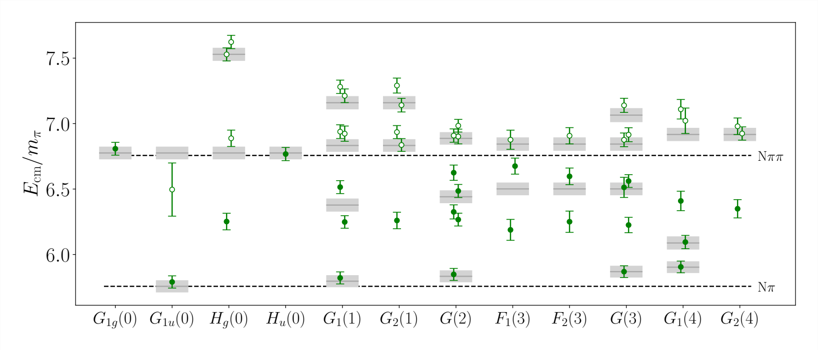
- $I = 3/2$
- operators:
  - $\boldsymbol{\Delta}$
  - $N\pi$
- momenta: $\mathrm{d}^2 = 0, 1, 2, 3, 4$

I=1/2 $N\pi$



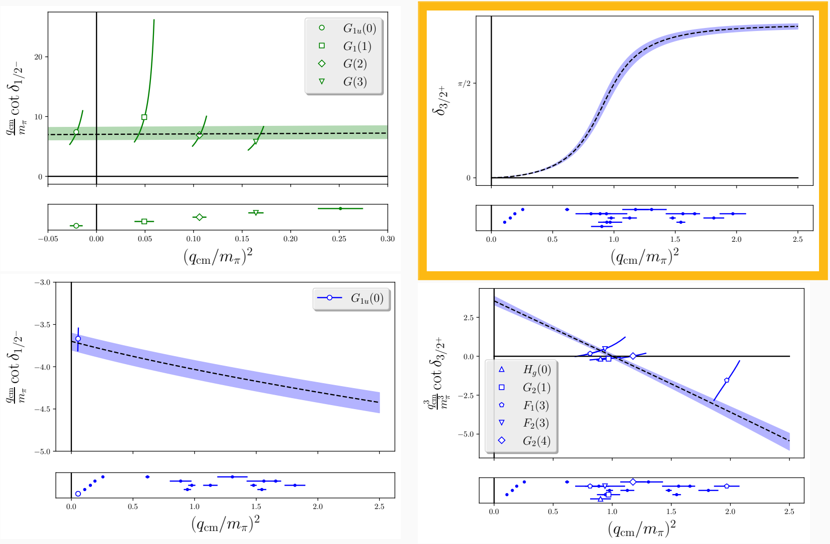
- Grey bands: noninteracting scattering levels (N, π correlators)
- Green dots: interacting levels ($N\pi$, N correlators)
- Filled green dots: levels used for constraining $a_{N\pi}^{I=1/2}$

I=3/2 $N\pi$, $\Delta(1232)$

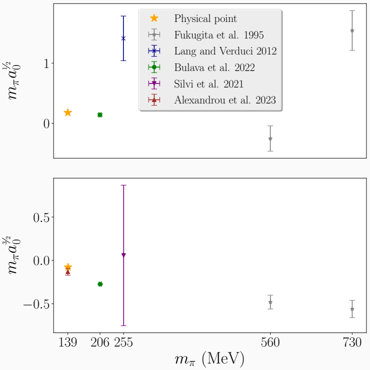


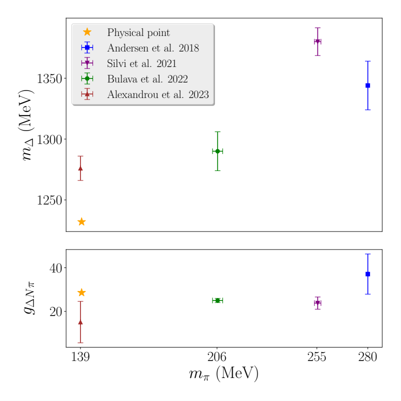
- Grey bands: noninteracting scattering levels (N, π correlators)
- Green dots: interacting levels ($N\pi$, Δ correlators)
- Filled green dots: levels used for calculating $a_{N\pi}^{I=3/2}$

Phase Shifts



Phase Shifts



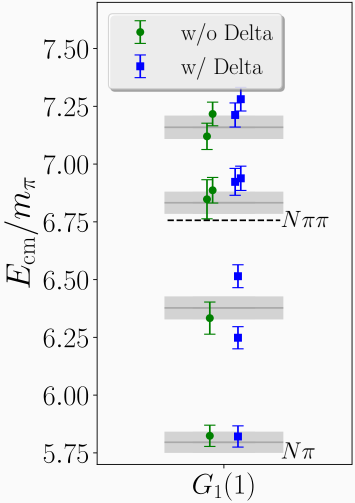


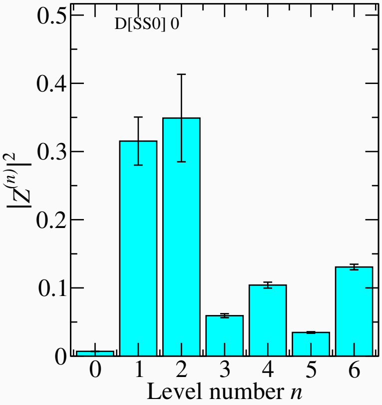
Δ Operator's Impact

How important was the **Δ** operator?

Δ Operator's Impact

$I = 3/2$



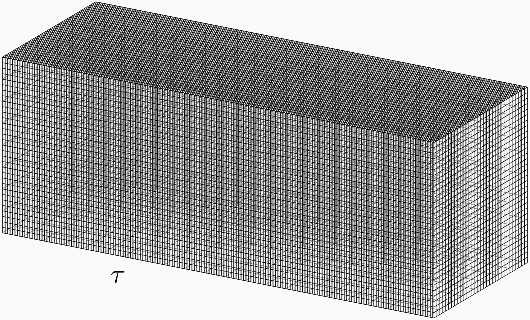


Lattice #2 Computational Details

- Dim ($x^3 \times t$):

  $32^3 \times 256$

- $a_s = 0.11fm$
- $a_t = 0.033fm$
- m_π = 230 MeV
- m_K = 490 MeV
- 412 configurations
- periodic temporal boundary conditions
- $N_f = 2 + 1$



Tetraquark Resonances

Two coupled-channel scattering channels investigated:

$$K\pi, K\eta \to K\pi, K\eta$$

- resonance: κ
- $I = 1/2$
- operators:
  - K
  - $K\pi$
  - $K\eta$ ($\eta = u\bar{u} + d\bar{d}$)
  - $K\phi$ ($\phi = s\bar{s}$)
  - $\bar{s}u\bar{s}s$ (diquark-antidiquark)
- momentums: $\mathrm{d}^2 = 0$

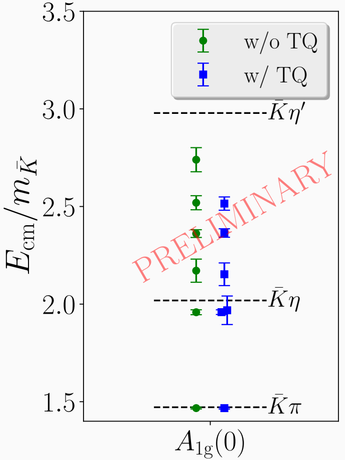
$$K\bar{K}, \pi\eta \to K\bar{K}, \pi\eta$$

- resonance: $a_0(980)$
- $I = 1$
- operators:
  - π
  - $K\bar{K}$
  - $\pi\eta$ ($\eta = u\bar{u} + d\bar{d}$)
  - $\pi\phi$ ($\phi = s\bar{s}$)
  - $\bar{u}u\bar{d}u$ (diquark-antidiquark)
- momentums: $\mathrm{d}^2 = 0$

Meson-Meson Spectrums

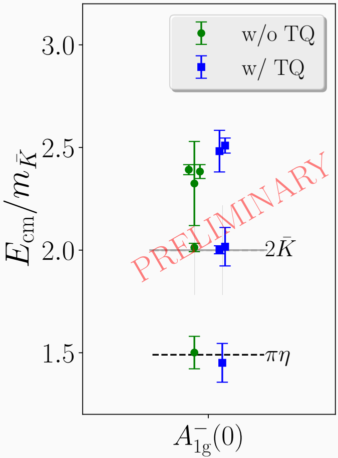
κ channel

TQ = $\bar{s}u\bar{s}s$



a_0 channel

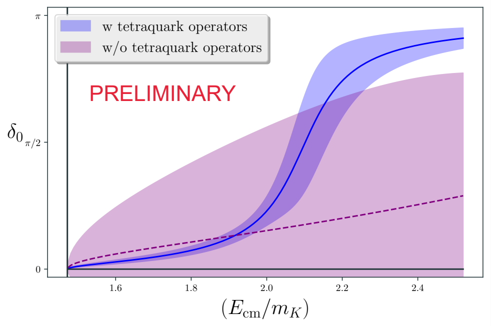
TQ = $\bar{u}u\bar{d}u$



Amplitudes

$K\pi$–$K\eta$ Spectrum (κ channel)

- Without tetraquark $\rightarrow$ no resonance (fit to 5 levels)
- With tetraquark $\rightarrow$ resonance at $\sim 2.1 m_K$ (fit to 5+TQ levels)



$K\bar{K}$–$\pi\eta$ Spectrum (a_0 channel)

- Without tetraquark $\rightarrow$ no resonance (fit to 3 levels)
- With tetraquark $\rightarrow$ virtual bound state (fit to 2+TQ levels)

Conclusions and Future Work

Conclusions

- Extracted scattering information in the $\boldsymbol{\Delta}$ channel $\rightarrow$ needed $\boldsymbol{\Delta}$ operators to extract accurate spectrum
- tetraquark operators are needed to study resonances in $K\pi$–$K\eta$ and $K\bar{K}$–$\pi\eta$ channels

Future Work

- more statistics
- multiple-lattice spacing
- cutoff effects
- investigate more complicated operators

Thanks for listening!