A Tale of Two studies: the $\Delta$ resonance and role of tetraquark operators for probing the $\kappa$ and $a_0(980)$ resonances

Sarah Skinner
August 2, 2023
Carnegie Mellon University
Introduction
Some of the results presented in this talk are published in


Special thanks to my collaborators:

Motivations for Resonances

In this presentation...

- $\Delta$ within $N\pi$ scattering
  - need this information for future experiments (DUNE)
  - sets us up for studying $N\pi\pi$ scattering
- $\kappa/\bar{s}u\bar{s}s$ in $K\eta-K\pi$ scattering
- $a_0(980)/\bar{u}u\bar{d}u$ in $K\bar{K}-\eta\pi$ scattering
  - see if tetraquark operators overlap with low energy states

Also check out...

- *The two-pole nature of the $\Lambda(1405)$*, Fernando Romero-López
- *The $\Lambda(1405)$ from lattice QCD*, Bárbara Cid-Mora
- *Hadron spectroscopy and few-body dynamics*, Andrew Hanlon
- *To bind or not to bind*, André Walker-Loud
Methods
Notes on Operator/Correlator Construction

Operator Notes:

• Gluons $\rightarrow$ Stout smearing
• Quarks $\rightarrow$ LapH smearing

Correlator Notes:

• stochastic factorization $\rightarrow$ tensor contraction
• efficient algorithm $\rightarrow$ produce many different correlators
Correlation matrix elements in the same channel share the same FV energy levels:

\[
\langle 0| O_i(t + t_0) O_j(t_0) |0 \rangle = \sum_{n=0}^{\infty} Z_i^{(n)} Z_j^{(n)} e^{-E_n t}
\]

Separate out by solving GEVP of $N \times N$ matrix and eigenvalues are:

\[
\lim_{t \to \infty} \lambda_n(t) \approx b_n e^{-E_n t}
\]

Example ($N_{\pi}, I = 3/2, H_g(0)$):
Fitting methods:

- single-exp: $Ae^{-Et}$
- double-exp: $Ae^{-Et}(1 + Re^{-D^2t})$
- geometric: $Ae^{-Et}/(1 - Re^{-Dt})$

Ratio:

$$R(t) = \frac{\lambda_n(t)}{C_1(t)C_2(t)}$$
Phase Shifts/Amplitude Analysis

Connect finite-volume to infinite-volume via Lücher:

$$\det[\tilde{K}^{-1}(E_{cm}) - B^P(E_{cm})] = 0$$

- truncate higher waves
- $\tilde{K}$ - related to the usual scattering $K$-matrix
- $B^P$ ('box matrix') - finite volume irreps
- only works for 2-2 scattering
Results
• CLS Lattice
• Dim \((x^3 \times t)\): 
  \[64^3 \times 128\]
• \(a = 0.064\text{fm}\)
• \(m_\pi = 200\text{ MeV}\)
• \(m_K = 480\text{ MeV}\)
• 2000 configurations
• open temporal boundary conditions
• \(N_f = 2 + 1\)
Delta Resonances

\[ N_{\pi} \to N_{\pi} \]

Correlation Matrix Information:

\[ a_{N_{\pi}}^{l=1/2} \]

- \( l = 1/2 \)
- operators:
  - \( N \)
  - \( N_{\pi} \)
- momenta: \( d^2 = 0, 1, 2, 3, 4 \)

\[ \Delta(1232), a_{N_{\pi}}^{l=3/2} \]

- \( l = 3/2 \)
- operators:
  - \( \Delta \)
  - \( N_{\pi} \)
- momenta: \( d^2 = 0, 1, 2, 3, 4 \)
$I = 1/2 \quad N_\pi$

- Grey bands: noninteracting scattering levels ($N, \pi$ correlators)
- Green dots: interacting levels ($N_\pi, N$ correlators)
- Filled green dots: levels used for constraining $a_{N_\pi}^{I=1/2}$

\[ I = 1/2 \quad N_\pi \]
$I=3/2 \ N\pi, \ \Delta(1232)$

- Grey bands: noninteracting scattering levels ($N, \pi$ correlators)
- Green dots: interacting levels ($N\pi, \Delta$ correlators)
- Filled green dots: levels used for calculating $a_{I=3/2}^{N\pi}$
Phase Shifts

\[
\delta_{\pi/2}^{1/2} - G_{1u}(0) - G_{1}(1) - G(2) - G(3)
\]

\[
\delta_{\pi/2}^{2/2} + H_g(0)
\]

\[
\delta_{3/2}^{1/2} - G_{1u}(0) - G_{2}(1) - F_{1}(3) - F_{2}(3) - G_{2}(4)
\]

\[
\delta_{3/2}^{2/2} + H_g(0)
\]
Phase Shifts

Physical point
- Fukugita et al. 1995
- Lang and Verduci 2012
- Bulava et al. 2022
- Silvi et al. 2021
- Alexandrou et al. 2023

$m_\pi a_0^{1/2}$ vs. $m_\pi$ (MeV)

$m_\Delta$ (MeV)

$g_{\Delta N\pi}$
How important was the Δ operator?
\( l = 3/2 \)

\[ G_1(1) \]

\[ E_{cm}/m_\pi \]

\[ N_\pi \]

\[ N_{\pi\pi} \]

\[ w/o \ Delta \]

\[ w/ \ Delta \]

\[ |Z^{(n)}|^2 \]

Level number \( n \)
Lattice #2 Computational Details

• Dim \((x^3 \times t)\):
  \[32^3 \times 256\]

• \(a_s = 0.11\, fm\)

• \(a_t = 0.033\, fm\)

• \(m_\pi = 230\, MeV\)

• \(m_K = 490\, MeV\)

• 412 configurations

• periodic temporal boundary conditions

• \(N_f = 2 + 1\)
Tetraquark Resonances

Two coupled-channel scattering channels investigated:

\[ K\pi, K\eta \rightarrow K\pi, K\eta \quad K\bar{K}, \pi\eta \rightarrow K\bar{K}, \pi\eta \]

- resonance: \( \kappa \)
- \( I = 1/2 \)
- operators:
  - \( K \)
  - \( K\pi \)
  - \( K\eta \ (\eta = u\bar{u} + d\bar{d}) \)
  - \( K\phi \ (\phi = s\bar{s}) \)
  - \( \bar{u}u\bar{d}d \) (diquark-antidiquark)
- momentums: \( d^2 = 0 \)

- resonance: \( a_0(980) \)
- \( I = 1 \)
- operators:
  - \( \pi \)
  - \( K\bar{K} \)
  - \( \pi\eta \ (\eta = u\bar{u} + d\bar{d}) \)
  - \( \pi\phi \ (\phi = s\bar{s}) \)
  - \( \bar{u}u\bar{d}d \) (diquark-antidiquark)
- momentums: \( d^2 = 0 \)
Meson-Meson Spectrums

$\kappa$ channel
TQ = $\bar{s}u\bar{s}s$

$a_0$ channel
TQ = $\bar{u}u\bar{d}u$

$E_{cm}/m_{\bar{K}}$

$A_{1g}(0)$

$\bar{K}\pi$

$\bar{K}\eta$

$\bar{K}\eta'$

$2\bar{K}$

$\pi\eta$

$E_{cm}/m_{\bar{K}}$

$A_{1g}^{-}(0)$

PRELIMINARY
Amplitudes

$K\pi-K\eta$ Spectrum ($\kappa$ channel)

- Without tetraquark $\rightarrow$ no resonance (fit to 5 levels)
- With tetraquark $\rightarrow$ resonance at $\sim 2.1m_K$ (fit to 5+TQ levels)

$K\bar{K}-\pi\eta$ Spectrum ($a_0$ channel)

- Without tetraquark $\rightarrow$ no resonance (fit to 3 levels)
- With tetraquark $\rightarrow$ virtual bound state (fit to 2+TQ levels)
Conclusions

• Extracted scattering information in the $\Delta$ channel → needed $\Delta$ operators to extract accurate spectrum
• Tetraquark operators are needed to study resonances in $K\pi - K\eta$ and $K\bar{K} - \pi\eta$ channels

Future Work

• More statistics
• Multiple-lattice spacing
• Cutoff effects
• Investigate more complicated operators
Thanks for listening!

IN MY PAST, I’VE MADE MISTAKES.

BUT IN MY FUTURE...

I WILL MAKE DIFFERENT MISTAKES.